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Average Inflation Targeting in Calvo Model with Imperfect Knowledge and Learning

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Abstract

Properties of average inflation targeting under imperfect knowledge and learning have been studied only for the Rotemberg NK model, where price stickiness arises from adjustment costs in price setting. This note fills the gap by studying average inflation targeting in the NK model with Calvo price stickiness. It is shown that in this setting the two models have the same basic properties, unless the central bank aggressively stabilizes output instead of average inflation.

1 Introduction

The US Federal Reserve Board reformed its monetary policy strategy in August 2020 by changing the inflation targeting framework to average inflation targeting (AIT) to have better response to developments leading to the zero lower bound (ZLB) for policy interest rates. The reformed monetary policy adopted an opaque AIT strategy. (Powell (2020)) clarified at the 2020 Jackson Hole Symposium that the Federal Reserve would not be tying their hands "to a particular mathematical formula that defines the average" and that their "approach could be viewed as a flexible form of average inflation targeting". The AIT regime is currently maintained.

The research literature about AIT mostly relies on the rational expectations (RE) hypothesis despite the opacity of the Fed framework.[1] Honkapohja and McClung (2024) introduce the realistic assumption of imperfect knowledge about the interest rate policy of an average inflation targeting central bank. If agents do not know the RE equilibrium, they must engage in learning about the dynamics of the variables that must be forecasted as part of agents' intertemporal decision-making. A fundamental issue is whether the learning dynamics converge or fail to converge to the inflation target RE equilibrium.[2]

Honkapohja and McClung (2024) develop their analysis using the assumption that there are adjustment costs in price setting due to Rotemberg (1982). The Rotemberg model is one standard New Keynesian (NK) model with price stickiness. Opaque AIT can result in non-robust performance in the sense that learning behavior does necessarily not converge to an intertemporal equilibrium even under standard assumptions about the interest rate rule.

[1]See Honkapohja and McClung (2024) for discussion and references of the literature.

[2]For general discussions of adaptive learning, see e.g. Evans and Honkapohja (2001), Evans and Honkapohja (2009) and Evans and McGough (2020).

The Calvo model, developed in Calvo (1983), is probably the most widely used NK framework for modelling price stickiness.[3] This raises the question of whether, under AIT policy, the properties of Calvo and Rotemberg models under learning are similar. It is shown that the dynamics of learning with AIT monetary policy are indeed very similar. However, in cases where the central bank aggressively stabilizes output instead of average inflation, stability concerns are mitigated under the Calvo pricing friction but not under Rotemberg.

2 The Calvo model with Imperfect Knowledge

We employ a standard New Keynesian model with Calvo price stickiness and imperfect knowledge, see Preston (2005) and Preston (2008). The two behavioral equations, linearized at the target steady state, are:[4]
(i) the aggregate demand curve

$$\hat{y}_t = -\sigma \hat{\imath}_t + \hat{E}_t \sum_{i=0}^{\infty} \beta^i [(1-\beta)\hat{y}_{t+1+i} - \sigma(\beta \hat{\imath}_{t+1+i} - \hat{\pi}_{t+1+i})] + r_t^n,$$

where r_t^n is a demand shock.
(ii) the Phillips curve

$$\hat{\pi}_t = \kappa \hat{y}_t + \hat{E}_t \sum_{i=0}^{\infty} (\alpha\beta)^i [\kappa\alpha\beta \hat{y}_{t+1+i} + (1-\alpha)\beta \hat{\pi}_{t+i+1}].$$

Here hat denotes the proportional deviation of the variable from its value at the target steady state. Thus, for example $\hat{y}_t = (y_t - y^*)/y^*$ is aggregate demand, $\hat{\pi}_t$ is inflation and $\hat{\imath}_t$ is nominal interest rate for the target. β is the subjective discount rate, σ is the intertemporal elasticity of substitution in utility function, κ is a parameter indicating degree of price stickiness and α is the fraction of firms which cannot change the price in a period. Note that $\kappa \to \infty$ if $\alpha \to 0$.

In the model there is also
(iii) the linearized AIT interest rate rule[5]

$$\hat{\imath}_t = \bar{\psi}_p \sum_{k=0}^{L-1} \hat{\pi}_{t-k} + \bar{\psi}_y \hat{y}_t.$$

The denominators π^* and y^* are incorporated into ψ_p and ψ_y, so $\bar{\psi}_p = \tilde{\psi}_p/\pi^*$ and $\bar{\psi}_y = \psi_y/y^*$. ($\pi^*$ and y^* denote the inflation and output levels at the target steady state.)

[3] For example, see the treatises by Gali (2008) and Woodford (2003).

[4] See Preston (2005), equations (18) and (19).

[5] Note that $\hat{\imath}_t = (i_t - i^*)/i^* = \hat{R}_t(1+i^*)/i^*$ as $R = 1 + i$. So $\tilde{\psi}_p = \psi_p(i^*/(1+i^*))$, where ψ_p is the policy rule parameter in the undeviated model.

Let $x_t = (\hat{y}_t, \hat{\pi}_t, \hat{\imath}_t)^T$. The economic system can be written as

$$\begin{pmatrix} 1 & 0 & \sigma \\ -\kappa & 1 & 0 \\ -\bar{\psi}_y & -\bar{\psi}_p & 1 \end{pmatrix} x_t = \sum_{k=1}^{L-1} \left[\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \bar{\psi}_p & 0 \end{pmatrix} x_{t-k} \right] + \begin{pmatrix} r_t^n \\ 0 \\ 0 \end{pmatrix} + \tag{1}$$
$$+ \sum_{i=0}^{\infty} \beta^i \begin{pmatrix} (1-\beta) & \sigma & -\sigma\beta \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \hat{E}_t x_{t+1+i}$$
$$+ \sum_{i=0}^{\infty} (\alpha\beta)^i \begin{pmatrix} 0 & 0 & 0 \\ \kappa\alpha\beta & (1-\alpha)\beta & 0 \\ 0 & 0 & 0 \end{pmatrix} \hat{E}_t x_{t+1+i}.$$

The model can be written in the form

$$x_t = M_1 \sum_{i=0}^{\infty} \beta^i \hat{E}_t x_{t+1+i} + M_2 \sum_{i=0}^{\infty} (\alpha\beta)^i \hat{E}_t x_{t+1+i} + \tag{2}$$
$$\sum_{k=1}^{L-1} [N_k x_{t-k}] + \begin{pmatrix} r_{t+i}^n & 0 & 0 \end{pmatrix}^T,$$

where T denotes matrix transpose. M_1, M_2 and N_k $(k = 1, ..., L-1)$ are 3×3 matrices

$$M_1 = S \begin{pmatrix} (1-\beta) & \sigma & -\sigma\beta \\ \kappa(1-\beta) & \kappa\sigma & -\kappa\sigma\beta \\ (\bar{\psi}_y + \kappa\bar{\psi}_p)(1-\beta) & \sigma(\bar{\psi}_y + \kappa\bar{\psi}_p) & -\sigma\beta(\bar{\psi}_y + \kappa\bar{\psi}_p) \end{pmatrix},$$

$$M_2 = S \begin{pmatrix} -\alpha\kappa\sigma\beta\bar{\psi}_p & -\sigma\beta\bar{\psi}_p(1-\alpha) & 0 \\ \alpha\kappa\beta(\sigma\bar{\psi}_y + 1) & \beta(\sigma\bar{\psi}_y + 1)(1-\alpha) & 0 \\ \alpha\kappa\beta\bar{\psi}_p & \beta\bar{\psi}_p(1-\alpha) & 0 \end{pmatrix}$$

and

$$N_k = S \begin{pmatrix} 0 & -\sigma\bar{\psi}_p & 0 \\ 0 & -\kappa\sigma\bar{\psi}_p & 0 \\ 0 & \bar{\psi}_p & 0 \end{pmatrix}, \text{ where } S = (\sigma\bar{\psi}_y + \kappa\sigma\bar{\psi}_p + 1)^{-1}.$$

Assume for simplicity that r_{t+i}^n is an exogenous and stationary *iid* process: $r_t^n = \varepsilon_{s,t}$.

It is useful to stack the system (2) into a first order form. We write

$$X_t = \hat{M}_1 \sum_{i=0}^{\infty} \beta^i \hat{E}_t X_{t+1+i} + \hat{M}_2 \sum_{i=0}^{\infty} (\alpha\beta)^i \hat{E}_t X_{t+1+i} +$$
$$\hat{N} X_{t-1} + \begin{pmatrix} r_{t+i}^n & 0 & \ldots & 0 \end{pmatrix}^T. \tag{3}$$

Here $\hat{M}_i$ are zero matrices of 3×3 blocks, except that the $(1,1)$ block given by with M_i for $i = 1, 2$, and $\hat{N}$ is blockwise $(L-1) \times (L-1)$ and has blocks $N_i = N, i = 1, ..., L-1$ in its first row and the remaining blocks form $(L-1) \times (L-1)$ identity matrix.

The expectations $\hat{E}_t x_{t+1+i}$ are determined by an adaptive learning process. Agents have a forecasting model, called perceived law of motion (PLM). If there is transparency about the functional form of the AIT rule, the PLM is

$$x_t = A + \sum_{k=1}^{L-1} C_k x_{t-k} + \varepsilon_{s,t}, \tag{4}$$

where ε_t is a disturbance term. Write the PLM in first order form

$$\begin{pmatrix} x_t \\ x_{t-1} \\ \vdots \\ x_{t-(L-1)} \end{pmatrix} = \begin{pmatrix} A \\ 0 \\ \vdots \\ 0 \end{pmatrix} + \begin{pmatrix} C_1 & 0 & \cdots & C_{L-1} \\ I & 0 & \cdots & 0 \\ \vdots & \ddots & \cdots & \vdots \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \cdots & I \end{pmatrix} \begin{pmatrix} x_{t-1} \\ x_{t-2} \\ \vdots \\ x_{t-L} \end{pmatrix} + \begin{pmatrix} \varepsilon_{s,t} \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{5}$$

and, with new notation, the system (5) is written as

$$X_t = \Delta + \Theta X_{t-1} + \Xi_t,$$

where $\Delta = (A^T, 0..., 0)^T$ and $\Xi_t = (\varepsilon_{s,t}^T, 0, ..., 0)^T$. We get

$$\begin{aligned} \hat{E}_t(X_{t+1+i}) &= (\sum_{k=0}^{i+1} \Theta^k)\Delta + \Theta^{i+2} X_{t-1} \\ &= (I-\Theta)^{-1}(I-\Theta^{i+2})\Delta + \Theta^{i+2} X_{t-1}. \end{aligned}$$

Here Δ is $3L$ dimensional vector and Θ is $3L \times 3L$ matrix. The summation terms in (3) are

$$\begin{aligned} F_\beta &= \sum_{i=0}^{\infty} \beta^i \hat{E}_t X_{t+1+i} = \sum_{i=0}^{\infty} \beta^i \hat{E}_t[(I-\Theta)^{-1}(I-\Theta^{i+2})\Delta + \Theta^{i+2} X_{t-1}] \\ &= (I-\Theta)^{-1} \left(I(1-\beta)^{-1} - (I-\beta\Theta)^{-1}\Theta^2 \right) \Delta + (1-\beta\Theta)^{-1}\Theta^2 X_{t-1} \end{aligned}$$

and

$$\begin{aligned} F_\alpha &= \sum_{i=0}^{\infty} (\alpha\beta)^i \hat{E}_t X_{t+1+i} = (I-\Theta)^{-1} \left(I(1-\alpha\beta)^{-1} - (I-\alpha\beta\Theta)^{-1}\Theta^2 \right) \Delta \\ &\quad + (1-\alpha\beta\Theta)^{-1}\Theta^2 X_{t-1}. \end{aligned}$$

This system will be used further in Proposition 2 below.

3 Rational Expectations Equilibrium

A policymaker would not select a policy which does not, in turn, select a unique equilibrium. Consequently, we begin by assessing conditions for the uniqueness of rational expectations equilibrium (REE). In the case of flexible prices, a version of the Taylor Principle is necessary and sufficient for existence of a unique equilibrium.[6]

[6]The Rotemberg and Calvo formulations imply identical log-linearized equilibrium conditions under rational expectations and the conditions for determinacy discussed in this section apply to either model. We note that the condition $L\bar{\psi}_p > 1$ is only generically necessary and sufficient condition, as a finite number of non-generic values of $\bar{\psi}_p$ are omitted in the proof of Proposition 1. For comparison to earlier work, this analysis studies a log-linear approximation of the model around the inflation target steady state, whereas Honkapohja and McClung (2024) study a system that is linearized (first-order Taylor series approximation) around the target steady state, and therefore the relevant determinacy condition differs slightly between the two frameworks.

Proposition 1 *Consider AIT policy and suppose fully flexible prices. A unique bounded REE exists if and only if* $L\bar{\psi}_p > 1$.

The proof of the proposition is in the appendix. For the case of sticky prices, numerical analysis suggests the following condition for determinacy:

Assumption 1. $L\bar{\psi}_p > 1 - (1-\beta)(\kappa)^{-1}\bar{\psi}_y$.

In what follows, we maintain Assumption 1.

4 E-stability of REE

Under Assumption 1, a unique REE exists and the law-of-motion for endogenous variables can be written in the minimum state variable (MSV) form:

$$x_t = \sum_{k=1}^{L-1} \Omega_k x_{t-1} + \Gamma r_t^n.$$

Agents can, in principle, learn the REE law-of-motion, and therefore learn to forecast rationally, by estimating the transparency PLM (4) recursively (i.e., if $A \to 0$ and $C_k \to \Omega_k$ asymptotically). We remark that the MSV solution is E-stable for standard calibrations of the model, just as the MSV solution of the corresponding Rotemberg model studied in Honkapohja and McClung (2024) is E-stable (provided that L is not too large).

However, as pointed out in the introduction, the Federal Reserve has not communicated the window length, L, and consequently there are questions about whether agents would know L and include the correct number of lags in their PLM when learning to forecast. Hence, just as in Honkapohja and McClung (2024), we are chiefly interested in the case of "opacity" in which agents exclude lags of inflation from their forecasting models due to ignorance of various aspects of the policy framework.

5 The Case of Full Opacity

We consider the case of full opacity, in which private agents continue to do steady state learning as they do not introduce lagged inflation variables into their forecasting model.[7] Full opacity is the extreme case of insufficient lags in the PLM, because the central bank has not provided any information about the formula for computing AIT. The PLM is

$$\begin{aligned} \begin{pmatrix} x_t \\ x_{t-1} \\ \vdots \\ x_{t-(L-1)} \end{pmatrix} &= \begin{pmatrix} A \\ 0 \\ \vdots \\ 0 \end{pmatrix} + \begin{pmatrix} \varepsilon_t \\ 0 \\ \vdots \\ 0 \end{pmatrix} \text{ or} \\ X_t &= \Delta + \Xi_t. \end{aligned}$$

Then

$$\hat{E}_t X_{t+1+i} = \hat{E}_t X_t \text{ for all } i = 0, ..., \infty$$

[7] Under usual inflation targeting policy, the steady state learning setup above has the correct functional form in the standard NK model.

and (2) becomes

$$\begin{aligned} X_t &= \hat{M}_1 \sum_{i=0}^{\infty} \beta^i \Delta + \hat{M}_2 \sum_{i=0}^{\infty} (\alpha\beta)^i \Delta + \hat{N} X_{t-1} + \hat{r}_t^n \\ &= \hat{M}(\hat{E}_t X_t) + \hat{N} X_{t-1} + \hat{r}_t^n, \end{aligned}$$

where

$$\hat{M} = (1-\beta)^{-1} \hat{M}_1 + (1-\alpha\beta)^{-1} \hat{M}_2.$$

Also recall

$$N_k = \hat{N} = \psi_p S \begin{pmatrix} 0 & -\sigma & 0 \\ 0 & -\kappa\sigma & 0 \\ 0 & 1 & 0 \end{pmatrix}.$$

Let $\hat{E}_t X_t = X_t^e$ and introduce steady state learning (with constant gain)

$$X_t^e = (1-\omega) X_{t-1}^e + \omega X_{t-1}. \tag{6}$$

State dynamics becomes

$$\begin{aligned} X_t &= \hat{M} X_t^e + \hat{N} X_{t-1} + \hat{r}_t^n \qquad (7) \\ & (1-\omega)\hat{M} X_{t-1}^e + (\omega \hat{M} + \hat{N}) X_{t-1} + \sum_{k=2}^{L-1} \left[\hat{N} X_{t-k} \right]. \end{aligned}$$

and note $\hat{N}_k$ are identical, so $\hat{N}_k = \hat{N}$ and the first and third columns of $\hat{N}$ are zero.

Proposition 2 *Consider AIT policy and full opacity. The target REE is locally stable under learning with full opacity for $\kappa > 0$ and $\omega > 0$ sufficiently small.*

The proof is outlined in the Appendix. We remark that the Calvo model under AIT is formally identical to the general form of the Rotemberg model, but the coefficient matrices $\hat{M}$ and $\hat{N}$ are different from the corresponding matrices of the Rotemberg model. When $\alpha \to 0$ (i.e., full price flexibility) the Calvo model is identical to the Rotemberg model with zero price adjustment costs and the outcome is unstable.[8] This implies:

Proposition 3 *In the flexible price limit of the Calvo model the target REE is unstable under learning with full opacity.*

As in Honkapohja and McClung (2024), the fact that stability obtains with sticky prices, but not flexible prices, begs an important question: will an opaque AIT framework anchor expectations for reasonable calibrations of the learning model? In empirical models adaptive learning is usually studied using constant gain algorithms. Then the magnitude of the gain parameter becomes an important issue and a reasonable range of values for the gain is something like $[0.002, 0.04]$. The calibration of model parameters is as follows: $\beta = 0.99$, $\sigma = 1$, $\kappa = \alpha^{-1}(1-\alpha)(1-\alpha\beta)$. We consider different values of α to illustrate the importance

[8] Proposition 2(ii) of Honkapohja and McClung (2024) shows that in the Rotemberg model the flexible price limit is not stable under full opacity. The result holds in the flexible price limit of the log-linearized Calvo framework as well.

of price rigidity for stability, as in Honkapohja and McClung (2024). Honkapohja and McClung (2024) show that the target REE in the Rotemberg model is not robustly stable as convergence takes place only with gain values below 0.01, i.e. the speed of learning is very small.

In the next table ω_0 is maximal value of the gain parameter such that learning diverges for all $\omega > \omega_0$. The results for the Calvo model are:[9]

Table 1: Least upper bounds ω_0 when $\bar{\psi}_p = 1.5$, $\bar{\psi}_y = 0.125$

α	0.66	0.75	0.9
ω_0 (IT)	0.13618	0.15681	0.17895
ω_0 (PLT)	0.03467	0.20753	0.04287
ω_0 (AIT with $L = 6$)	0.01158	0.01938	0.10152
ω_0 (AIT with $L = 20$)	0.00166	0.00265	0.01044
ω_0 (AIT with $L = 32$)	0.00073	0.00121	0.00439

Thus, in the Calvo model with AIT policy rule convergence of learning is robustly stable when there is significant price stickiness (α is large) and L is not very high. However, just as in the case of Rotemberg price stickiness, there are stability concerns when prices are less rigid or L is high.[10]

While the stability outcomes under opacity are similar under Rotemberg and Calvo models for standard calibrations of the policy rule, it turns out that the central bank can achieve more robust stability under Calvo pricing by targeting output aggressively. Table 2 shows the robust stability results when $\bar{\psi}_p = 1.5/L$ and $\bar{\psi}_y = 1$. In this case, the REE is more robustly stable for different values of L and α in the Calvo model. In the Rotemberg case, however, stability concerns remain for higher values of L (details available on request).

Table 2: Least upper bounds for $\bar{\psi}_\pi = 1.5/L$, $\bar{\psi}_y = 1$

α	0.66	0.75	0.9
ω_0 (AIT with $L = 6$)	0.04797	0.04341	0.04039
ω_0 (AIT with $L = 20$)	0.04797	0.04341	0.04039
ω_0 (AIT with $L = 32$)	0.04797	0.04341	0.04039

6 Concluding Remarks

Various further properties of AIT policy in the Rotemberg model are studied in Honkapohja and McClung (2024). For example, the convergence results remain unchanged if agents introduce some, but insufficient number of lagged inflation variables into their PLM. However, if agents introduce more than $L - 1$ inflation lags to the PLM, then they can learn the value of L and the economy can converge to the MSV REE. We conjecture that these results would be similar for the Calvo model with AIT.

Honkapohja and McClung (2024) also consider variations of the AIT rule, including cases of exponentially declining weights in the AIT rule and of asymmetric AIT rules for achieving E-stability and improving performance of the economy under ZLB. The latter issues have not been considered in the linearized Calvo model. Study of the Calvo model is a very relevant open issue for situations where nonlinearities due to the ZLB prevail.

[9] IT refers to inflation targeting, i.e. $L = 1$ and PLT to price level targeting.

[10] See Table 1 in Honkapohja and McClung (2024) for analogous results under Rotemberg model.

A Appendix

Proof of Proposition 1: The model under flexible prices, AIT and RE is given by:

$$E_t \pi_{t+1} = \bar{\psi}_p \sum_{k=0}^{L-1} \hat{\pi}_{t-k}$$

which is obtained by combining the Euler equation and interest rate rule. Define $z_t := (\hat{\pi}_t, \ldots, \hat{\pi}_{t-L+1})^T$. The system can be expressed in the first-order form: $E_t z_{t+1} = Q_{RE} z_t$. The model admits a unique bounded REE if and only if one eigenvalue of Q_{RE} is outside the unit circle and all the remaining eigenvalues are inside the unit circle.[11] The roots of Q_{RE} solve the characteristic polynomial:

$$R(\lambda) = \lambda^L - \bar{\psi}_p \sum_{k=0}^{L-1} \lambda^k$$

Define $J(k) := 1 + k(\bar{\psi}_p)^2/((k-1)\bar{\psi}_p - 1)$ for $k = 1, \ldots, L$. Following the Jury stability criterion (e.g., see Honkapohja and McClung (2024)), one root of Q_{RE} is outside the unit circle and the rest are inside the unit circle if and only if $J(l) < 0$ for some $l \in \{1, \ldots, L\}$ and $J(m) > 0$ for all $m \in \{1, \ldots, L\}$ such that $m \neq l$. Clearly, this condition holds if and only if $L\bar{\psi}_p > 1$.■[12]

Proof of Proposition 2: We write (6) and (7) as the first order system.

$$Z_t = QZ_{t-1}, \text{ where } Z_t = \begin{pmatrix} (X_t^e)^T & X_t^T & X_{t-1}^T & X_{t-2}^T & \cdots & X_{t-(L-2)}^T \end{pmatrix}^T \text{ and} \tag{8}$$

$$Q = \begin{pmatrix} (1-\omega)I_3 & \omega I_3 & 0 & \cdots & 0 & 0 \\ (1-\omega)M & \omega M + N & N & \cdots & N & N \\ 0 & I_3 & 0 & \cdots & 0 & 0 \\ 0 & 0 & I_3 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & I_3 & 0 \end{pmatrix}.$$

One can now follow the technique in the proof of Proposition 2(i) in Honkapohja and McClung (2024), but with the coefficient matrices M and N for Calvo model. For stability the roots of $P(\lambda) = Det[Q - \lambda I_{3L}]$ must be inside the unit circle. It can be shown that $P(\lambda) = \lambda^{2L-2}(1 + \omega - \lambda)\tilde{P}(\lambda)$. Taking the limit $\omega \to 0$, we have

$$\tilde{P}(\lambda) = (1-\lambda)^2 \left(\lambda^{L-1} + h \sum_{k=0}^{L-2} \lambda^k \right), \text{ where } h = \kappa\sigma\bar{\psi}_p/(\sigma\bar{\psi}_y + \kappa\sigma\bar{\psi}_p + 1).$$

It is seen that $h \in (0,1)$. We apply the stability criterion of Jury (1961): the roots of $\tilde{P}(\lambda)$ are inside the unit circle if and only if

$$kh^2/(1 + (k-1)h) < 1, \text{ for } k = 1, ..., L.$$

The criterion is true for all L. Thus the roots of $P(\lambda)$ are inside the unit circle if the derivative of the non-zero eigenvalues $\partial\lambda/\partial\omega < 0$ at $\omega = 0$ and $\lambda = 1$. The latter can be established using the technique in the proof of Proposition 1(i) in Honkapohja and McClung (2024). ■

[11] Following standard practice, we disregard non-generic cases in which one or more roots of Q_{RE} lie on the unit circle.

[12] We tacitly exclude the following values of $\bar{\psi}_p$, which form a discrete set: $\{1, 1/2, \ldots, 1/(L-1), 1/L\}$. Hence, Proposition 1 holds only generically for $L\bar{\psi}_p > 1$.

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