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TESTING THE FUNCTIONAL FORM OF FINNISH AGGREGATE IMPORTS*

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Various test procedures are used to examine the functional form of Finnish aggregate imports. None of the alternatives - the log-linear, logit and exponential forms - is found to be a satisfactory specification.

1 INTRODUCTION

Economic theory seldom offers any guidance on the choice of the appropriate functional form when specifying and estimating econometric models. This is also the case in international trade theory, where the most commonly used functional forms have assumed the linear or log-linear form. The choice between this restricted class of functional forms in the context of aggregate import demand has been assessed in several studies using the Box-Cox (1964) procedure [see Khan and Ross (1977), Boylan, Cuddy and O'Muircheartaigh (1980) and Gandolfo and Petit (1983)]. Lately, the Lagrange multiplier approach has been developed to test the linear and log-linear forms against the more general Box-Cox specification [Godfrey and Wickens (1981) and Ghosh, Gilbert and Hughes Hallet (1983)].

However, in addition to the linear, log-linear and the general Box-Cox functional forms, other plausible and interesting functional forms could be considered. This is important because the choice of the functional form has implications for forecasts and policy appraisal. The aim of this paper is to examine the functional form issue in the case of Finnish aggregate import demand using some alternative forms as discussed in section 2. In section 3, discrimination between the models is performed using nested and non-nested test procedures.

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2 THEORETICAL CONSIDERATIONS

In the conventional specification of an aggregate import demand model, the quantity of imports demanded (M^d) is related to the relative price of imports and domestic production (P) and to real domestic income (Q). Assuming nonprice rationing [Gregory (1971)] or business cycle effects approximated by the capacity utilization rate (C) [see Aurikko (1984)], the linear form of the equation is

$$M_t^d = a_0 + a_1 P_t + a_2 Q_t + a_3 C_t + a_4 D_t + e_t, \qquad (1)$$

where e_t is a random error term. In (1) it assumed that $a_1 < 0$, a_2, $a_3 > 0$, implying, in particular, that a rise in domestic capacity utilization increases the propensity to import. The dummy variable, D, captures the effects of the change in the method of compiling foreign trade statistics in 1969. In the case of a small country, import prices can be taken as exogenous and import supply as infinitely elastic, so that ordinary least squares estimation is appropriate.

Equation (1) is an equilibrium relation implying instantaneous adjustment, which, in the presence of uncertainty and various frictions, might be too restrictive. It is thus assumed that imports are adjusted according to a conventional partial adjustment mechanism

$$M_t - M_{t-1} = \alpha(M_t^d - M_{t-1}), \quad 0 \leqslant \alpha \leqslant 1, \qquad (2)$$

where α is the adjustment coefficient. Inserting (1) into (2) a "dynamic" linear import equation is obtained:

$$M_t = \alpha a_0 + \alpha a_1 P_t + \alpha a_2 Q_t + \alpha a_3 C_t + \alpha a_4 D_t + (1-\alpha) M_{t-1} + \alpha e_t. \qquad (3)$$

The log-linear equation corresponding to equation (3) is

$$\log M_t = \alpha' b_0 + \alpha' b_1 \log P_t + \alpha' b_2 \log Q_t + \alpha' b_3 \log C_t \qquad (4)$$
$$+ \alpha' b_4 D_t + (1-\alpha') \log M_{t-1} + \alpha' e_t ,$$

where α' is the adjustment coefficient and parameters $b_0, \ldots, b_4$ correspond to parameters $a_0, \ldots, a_4$ in (1).

The logit model can be considered as an alternative specification for aggregate imports. It is written as

$$\log[M_t^d/(Q_t - M_t^d)] = c_0 + c_1 \log P_t + c_2 \log C_t + c_3 D_t + u_t, \tag{5}$$

where u_t is an error term and $c_1 < 0$, $c_2 > 0$. The logit form is especially suitable in long-run forecasts and policy appraisal since the constraint $0 \leqslant M_t/Q_t \leqslant 1$ is always met. Assuming the partial adjustment mechanism, the dynamic logit specification is

$$\begin{aligned} \log[M_t/(Q_t - M_t)] = {} & \beta c_0 + \beta c_1 \log P_t + \beta c_2 \log C_t + \beta c_3 D_t \\ & + (1-\beta)\log[M_{t-1}/(Q_{t-1} - M_{t-1})] + \beta u_t , \end{aligned} \tag{6}$$

where β is the adjustment coefficient.

The last specification to be examined is the exponential form expressed as

$$M_t^d = [\exp(d_0 P_t^{d_1} C_t^{d_2} \exp(c_3 D_t))]\, Q_t \exp(v_t). \tag{7}$$

The corresponding dynamic formulation with γ denoting the adjustment coefficient is

$$\log M_t = \gamma[(d_0 P_t^{d_1} C_t^{d_2} \exp(d_3 D_t)) + \log Q_t] + (1-\gamma)\log M_{t-1} + \gamma v_t , \tag{8}$$

where v_t is an error term. With $M/Q < 1$, this implies that $d_0 < 0$, $d_1 > 0$, $d_2 < 0$.

In order to assess the properties of models (3), (4), (6) and (8), it is useful to examine their (relative) price and activity elasticities[1] as shown in Table 1.

TABLE 1. Price and activity elasticities of models (3), (4), (6) and (8).

	Price elasticity		Activity elasticity	
Model	short-run[a]	long-run[b]	short-run[a]	long-run[b]
(3)	$(\alpha a_1 P)/M$	$(a_1 P)/M$	$(\alpha a_2 Q)/M$	$(a_2 Q)/M$
(4)	$\alpha' b_1$	b_1	$\alpha' b_2$	b_2
(6)	$[\beta c_1 (Q-M)]/Q$	$[c_1 (Q-M)]/Q$	1	1
(8)	$\gamma d_0 d_1 C^{d_2} P^{d_1}$	$d_0 d_1 C^{d_2} P^{d_1}$	γ	1

a Calculated setting M_{t-1} constant
b Calculated setting $M_t = M_{t-1}$

In the linear model (3), the price and activity elasticities depend on movements in the variables P, Q and M. The price elasticity decreases over time while the activity elasticity tends towards unity. In the log-linear model (6), the elasticities are constants, which might also be theoretically too restrictive. In the logit formulation, both short- and long-run activity elasticities are equal to one and the price elasticities vary inversely with the proportion of imports in production. Finally, in the exponential model (8), the activity elasticities are constants while the price elasticities decrease in absolute value as capacity utilization rises. Moreover, the price elasticity is larger the greater the change in the relative prices. As the functional forms for import demand have different but plausible properties and there is no precise theoretical guidance for choosing among them, the issue is essentially an empirical one.

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[1] The elasticities are calculated assuming C unchanged.

3 EMPIRICAL ANALYSIS

The data utilized in estimating and testing the functional form of Finnish aggregate imports are quarterly and seasonally adjusted and cover the period 1964.1 - 1983.4. They are obtained from the data base of the BOF3 quarterly model constructed by the Research Department of the Bank of Finland.[2]

First, the linear and log-linear functional forms are considered. According to the estimation results of (3) (not reported here) and (4) (see Table 2), both models perform rather well and are stable with no first order or fourth order autocorrelation.[3] Thus, we now consider the choice of the appropriate functional form by using the Box-Cox method. In this procedure a power function is estimated in which all variables, except the constant and the dummy variable, are transformed as $X(\lambda) = (X^{\lambda} - 1)/\lambda$, reducing to a linear form for $\lambda = 1$ and to a log-linear form for $\lambda = 0$. The test statistics for parameter λ in the power function for the hypotheses $\lambda = 1$ and $\lambda = 0$ are 13.87 and 1.42, respectively. Since the test statistic is distributed as χ^2_1 with a 5 per cent critical value of 3.84, the hypothesis $\lambda = 0$, implying the log-linear form, cannot be rejected.

Only linear and log-linear models are compared above. A more general approach is to test the adequacy of the linear and log-linear forms

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[2] See Bank of Finland (1983). Variable M is the volume of total imports of goods, Q is the volume of gross domestic product in producers' prices, both in 1975 prices. Variable P = PM/PD, where PM is the unit value index of M and PD stands for output prices in manufacturing industry, with 1975 = 100 for both. The capacity utilization rate is calculated as C = Q/QP, where QP is the volume of potential gross domestic product estimated from a production function. The dummy variable D stands for the change in the method of compiling foreign trade statistics in 1969.

[3] The use of price ratios in (3) and (4) assumes homogeneity in prices. The test statistic for the homogeneity restriction (the sum of the coefficients of import prices and domestic prices is equal to zero), distributed as F(1,73), is 1.57 for (3) and .01 for (4). Since the 5 per cent critical value of F(1,73) is 3.98, the homogeneity restriction cannot be rejected. In the linear model (3), the use of the price ratio rather than the price difference is based on the approximation $PM/PD \simeq 1 + PM - PD$.

against the general Box-Cox specification. The test is performed by using the procedure due to Andrews (1971) and developed by Godfrey and Wickens (1981). The t-test statistics for the hypotheses $\lambda = 1$ and $\lambda = 0$ are 1.33 and .78, respectively, implying that neither of the hypotheses can be rejected.[4] Since the procedure provides no guidance when choosing between the linear and log-linear forms, this suggests the need to test other functional forms as well, and in the present context, namely models (6) and (8), together with specification (4), for which some empirical support was found.

TALBE 2. Maximum likelihood estimates for models (4), (6) and (8).

Model	Coefficients							
	b_0	b_1	b_2	b_3	b_4	α'	$\hat{\delta}^2$	h	L
(4)	-3.54 (.99)	-.52 (.17)	1.23 (.10)	.86 (.31)	-.11 (.04)	.62 (.07)	.0024	-1.30	127.3
	c_0	c_1	c_2	c_3		β			
(6)	-.92 (.08)	-.38 (.11)	.94 (.24)	-.23 (.03)		.52 (.08)	.0046	-1.13	101.4
	d_0	d_1	d_2	d_3		γ			
(8)	-1.24 (.02)	.21 (.10)	-.60 (.26)	.13 (.02)		.58 (.07)	.0025	-1.68	125.7

Asymptotic standard error estimates of the coefficients are in parentheses, $\hat{\delta}^2$ is the estimate of the residual variance, L is the logarithm of the value of the maximized likelihood function and h is the Durbin h-test statistics for first order autocorrelation.

According to the maximum likelihood estimation results in Table 2, all models are rather satisfactory, although residual variance is relatively high in the logit model (6). There seems to be no first order autocorrelation according to the h-statistics. However, in the case of

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[4] Because of the lagged dependent variable, the test is only asymptotically valid.

the nonlinear models, the distribution of the h-statistic is unknown. Utilizing the computed likelihood values in Table 2, the discrimination between the models is examined first by the Akaike Information Criterion (AIC) [Akaike (1978)].[5] The values of the AIC for models (4), (6) and (8) are -242.6, -192.8 and -241.4, respectively, indicating that the log-linear specification might marginally be selected.

Since the AIC rule is not especially powerful in discriminating between the models, next some non-nested test procedures are utilized. First, the encompassing principle advocated by Mizon and Richard (1983) is used to test models (4) and (6). However, because of multicollinearity of the variables Q-M and Q in the nested version of the models, the approximation $\log[M/(Q-M)] \simeq \log(M/Q)$ was adopted. F-test statistics for the non-overlapping variables, implying the restrictions leading to (4) or (6), were 1.12 and 6.43, respectively, distributed as F(1,73) and F(2,73), which suggests that the log-linear form cannot be rejected.

The exponential model (8) is highly nonlinear and cannot be nested with the other models. Thus, the non-nested procedure suggested by Pesaran and Deaton (1978) for testing nonlinear equations is used. Denoting the competing models (4), (6) and (8) as H_1, H_2 and H_3, respectively, the test is based on a modified likelihood ratio, taking H_1, H_2 and H_3 in turn as the maintained hypothesis. The N-statistics calculated for all pairs of hypotheses distributed asymptotically as N(0,1) as well as the $\hat{\delta}^2$ values in the diagonal are given in Table 3.

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[5] The AIC rule selects the model for which AIC = - 2L + 2n is the minimum, where n is the number of parameters estimated. See Judge et al. (1980) on the relative pover of this test procedure.

TABLE 3. N-statistics and $\hat{\delta}^2$ for hypotheses H_1, H_2 and H_3.

Alternative hypothesis:		H_1	H_2	H_3
Maintained hypothesis:	H_1	.0024	3.21	-1.31
	H_2	-3.12	0.0046	-4.17
	H_3	-4.15	3.86	.0025

Comparing first hypotheses H_1 and H_3, the N-statistics in Table 3 indicate that the log-linear form cannot be rejected. However, in spite of the relatively poor fit of H_2, it provides evidence to reject both H_1 and H_3. Concerning hypotheses H_2 and H_3 , it is also seen that each rejects the other. Thus, the log-linear form does not seem to be an altogether satisfactory model.[6] In H_2 and H_3 there is one less parameter to be estimated than in H_1, which might favour H_1. Estimating the exponential form without constraining the activity elasticity equal to one gives $\hat{\delta}^2$ = .0024 and L = 127.4. The N-statistics for this case with first H_1 as the maintained hypothesis and reversing the procedure are -5.15 and -3.84, respectively, suggesting that both formulations are rejected.

In summary, it can be noted that the log-linear specification for Finnish aggregate imports is not without serious contenders. The evidence provided by the logit and exponential alternatives cast some doubt on the empirical validity of the popular log-linear form. From the point of view of policy appraisals the specifications give quite different results concerning, for example, the effects of exchange rate policies, since the long-run price elasticities for H_1, H_2 and H_3 calculated using sample averages are -.52, -.28 and -.26, respectively.

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[6] The results should be interpreted with caution, being valid only asymptotically. However, no small sample corrections were made on the N-statistics.

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