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**Veera Nippala**, Researcher, Labour Institute for Economic Research LABORE
**Taina Sinivuori**, Economist, Bank of Finland

## Abstract

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†Email: veera.nippala@labore.fi

# 1 Introduction

The aim of this paper is to formulate a forecasting method for private investment in Finland with focus on medium term forecasting (1 to 2 years ahead). Our methodology builds on Kilponen and Verona (2022) who analyze in-sample fit and out-of-sample forecasting performance of investment equation using different proxies for Tobin's $Q$, controlling for cash flow, and using their frequency-decomposed components with U.S. data.

We follow the empirical literature and formulate the $Q$ using four different approaches. First, we approximate average $Q$ by stock price index relative to the price deflator of investment following Takala (1995). Secondly, we approximate $Q$ by the ratio of market value of equity and debt to fixed assets following Jovanovic and Rousseau (2014). Thirdly, we approximate "total $Q$" by Peters and Taylor (2017) by dividing market value of equity and stock minus inventories by capital stock. Lastly, we calculate a measure of $Q$ by dividing market value of equity by capital stock. Cash flow is also added as an explanatory variable in the investment equation to study its effect on the performance of the investment model, since several studies recognized that reasons behind the weak performance of $Q$ might be that $Q$ is not enough to explain private investment alone (Lee and Rabanal, 2010; Abel and Eberly, 2011; Grullon et al., 2018).

We also decompose the series to different frequency components using wavelet multiresolution analysis as done by Gallegati and Ramsey (2013), Faria and Verona (2018) and Kilponen and Verona (2022). These studies found that this method improves the investment equation or forecasting model by including in the equations only specific frequencies of a variable and allowing exclusion of "noisy" frequencies. We decompose the time series into five orthogonal components to capture the fluctuations of different frequencies. The wavelet details capture fluctuations with a period of 0.5-1 years, 1-2 years, 2-4 years and 4-8 years. The smooth component captures fluctuations with a period longer than 8 years.

We find weak evidence in favor of our proxies of $Q$ performing well in forecasting private investments in Finland. All of our models are able to produce smaller RMSFE's than the benchmark AR-model, but not in a statistically significant way. The difficulty to find statistically significant results likely stems from the relatively small sample size. We also find that both frequency decomposition and inclusion of cash flow into the forecasting equation improve the out-of-sample forecasting performance. The best overall out-of-sample results are produced by the "total $Q$" measure by Peters and Taylor (2017).

To our knowledge, there are no previous studies testing the out-of-sample performance of investment forecasting methods in Finland. Therefore, this paper contributes to the existing literature by providing an empirical study for private investment with Finnish data where the weaknesses in the empirical performance of $Q$ have been controlled.

This paper is organized as follows. Section 2 presents results from relevant literature and section 3 introduces the data. Section 4 briefly describes the methodology and section 5 presents empirical results. Section 6 concludes.

## 2 Related literature

The Tobin's $Q$-theory of investment is one of the most used approaches in empirical investment analysis. The theory argues that investment depends positively on $Q$ which is defined as the ratio between the market value of a firm and the replacement cost of the firm's tangible capital. Hence $Q$ should explain all fluctuation in private investment, since in an efficient market the share price includes all expectations of future corporate returns. (Tobin, 1969). The $Q$-theory of investment has, however, performed poorly in empirical context as the impact of $Q$ on private investment has often been found weak (Chirinko, 1993; Bond and Van Reenen, 2007; Davis, 2011). Several studies have recognized that the reasons behind the weak performance of $Q$ might be that $Q$ is not enough to explain private investment alone, $Q$ is mismeasured, or that OLS regression produces downward biased estimates for $Q$.

Adding other variables to the investment equation might be justified if $Q$ is an incomplete measure of investment expectations and decisions. Private investment is likely to be sensitive not only to access to external equity financing (captured by $Q$), but also to intra-corporate assets, such as cash flow and external debt financing, such as bank loans and corporate bonds. Thus, other financial variables are also likely to influence the firm's investment decision (Verona, 2020).

A large literature has focused on examining the effect of cash flow on private investment. However, the results are contradictory. In some studies, adding cash flow variable to the investment equation has not increased the explanatory power of the model (Gomes, 2001; Cooper and Ejarque, 2003), while other studies have found that cash flow improves the explanatory power of the model significantly, or explains even larger share of the variability in private investment than $Q$ (Lee and Rabanal, 2010; Abel and Eberly, 2011; Grullon et al., 2018). According to Gallegati and Ramsey (2013), Kilponen and Verona (2022), and Verona (2020) cash flow is related to investment in the short term, while $Q$ is related to investment in the long term. According to these studies information related to investments in $Q$ and cash flow complement each other, as $Q$ and cash flow capture the effects on investments at different frequencies.

The empirical performance of the $Q$-theory has also been improved through different formulations of $Q$. Philippon (2009) approximates $Q$ using the relative prices of corporate and sovereign bonds rather than share prices (bond $Q$). In the method of Celil and Chi (2016), the market value of a firm is approximated by the probability of its default (Merton's $Q$). In contrast to traditional $Q$-theory, Peters and Taylor (2017) proposed a methodology that also takes into account the intangible capital of firms in the formation of $Q$. Peters and Taylor (2017) $Q$ is formulated using the ratio of the firm's market value to its total capital stock (sum of tangible and intangible capital).

Bond $Q$ has been empirically found to perform significantly better compared to traditional stock market $Q$ (Shen, 2010; Gallegati and Ramsey, 2013; Lin et al., 2018). According to Gallegati and Ramsey (2013), the differences between traditional $Q$ and bond $Q$ can be explained by pricing errors that are greater in the stock market than in the bond market. According to Philippon (2009), bond $Q$ predicts investment consistently in the long and in the short term while the traditional stock market $Q$ explains investments only over the long term. Philippon (2009) also found that cash flow is no longer statistically significant variable explaining investments if Bond $Q$ is included in the regression. In Merton's $Q$ method variables used to explain investments in Philippon (2009) together with cash flow lose their explanatory power (Celil and Chi, 2016).

Downward biased estimates of $Q$ produced by OLS regression have also been offered to explain the empirical weakness of traditional $Q$-theory. Erickson and Whited (2000) showed that the GMM estimator gives $Q$ a much higher regression coefficient compared to the OLS estimator. In addition, the GMM method increases the explanation rate of the model and eliminates the statistical significance of cash flow as an explanatory variable. Erickson et al. (2014), on the other hand, estimated the investment equation using a cumulant estimator. In contrast to the large literature stating the poor performance of $Q$, using classical regression Andrei et al. (2019) found that there is strong relation between aggregate investment and Tobin's $Q$ and that the improvement in the empirical performance of $Q$ is attributed to an increase in the empirical variation in Tobin's $Q$ relative to residual factors affecting investment.

The stability of regression coefficients is an important assumption underlying the classical linear regression model (Verona, 2017). However, it is unrealistic to assume that the regression coefficients remain stable when the estimated sample includes a long period of time. In this case, ignoring structural changes can lead to incorrect and unreliable results. Gallegati and Ramsey (2013) also find that the sensitivity of investment to $Q$ changes significantly when the estimation period is changed. Similarly, McLean and Zhao (2014) show that investment dependence on $Q$ varies according to business cycles and is strongest in the upward trend. Verona (2020) uses the continuous wavelet transform method and the results suggest that the investment model is improved by allowing the model properties to change in time.

This paper also relates to the stream of literature concerning the use of wavelet methods in economic analysis and forecasting. Pioneered by Ramsey and Lampart (1998), wavelet tools have shown promise in analysing both in-sample relationship and out-of-sample forecasting accuracy of economic variables. Gallegati and Ramsey (2013) analyses the in-sample relation of investment and measures of $Q$. In forecasting context, Rua (2011) combines wavelet tools with factor augmented modeling to forecast GDP growth. Faria and Verona (2021) and Faria and Verona (2018) propose each a wavelet approach to forecast the equity risk premium and stock market returns, respectively. Very closely related to this paper, Kilponen and Verona (2022) forecast aggregate investment in the U.S. with a combination of $Q$-theory and wavelet approach.

Much of the Finnish literature on modeling investments has focused on the investment decisions of firms. An example of such studies is Pyyhtiä (1992). Less attention has been paid to modeling or forecasting investments at aggregate level. Vilmunen (2002) provided some evidence of both firm level and aggregate investment in Finland using the Euler equation. Pietiläinen (2009) estimated an investment equation based on Tobin's $Q$-theory using Bayesian estimation. Sauramo (2008) examines the relationship between foreign direct investment and domestic investments with an investment equation using savings rate and ratio of foreign direct investments to GDP as explanatory variables. Takala (1995) specifies models for manufacturing sector's investment ratio and housing investments using flexible accelerator approach which combines $Q$-theory and other variables. Finnish Ministry of Finance bases its investment forecasts mostly on a joint macro model, but additionally uses a few smaller models: one based on Tobin's $Q$, one based on neoclassical investment theory, and one based on a simple accelerator model (Ministry of Finance, 2020).

## 3 Data

Most of the data is collected from Statistics Finland's national accounts data and financial accounts[1] data. Share price index is taken from Reuters. Exact data definitions, data sources and summary statistics are available in Appendix A. The data ranges from 1999Q1 to 2020Q4[2].

According to $Q$-theory all fluctuations in private investment are captured by $Q$, the ratio between market value of additional unit of capital and replacement cost of capital. This *marginal* $Q$ is not observable, so the so-called *average* $Q$ is used instead in empirical analysis. Average $Q$ is most often defined as the ratio between market value of the firm to the replacement value of its capital. In previous studies this measure has been approximated in many different ways, with either firm level or aggregate level data. We include four aggregate level proxies of $Q$.

First, following Takala (1995) we approximate average $Q$ by stock price index relative to the price deflator of investment ($Q_1$):

$$Q_1 = \frac{all\ shares\ price\ index}{price\ deflator\ of\ investment} \tag{1}$$

Stock market data has been shown to have significant predictive power on investment by Takala (1995), Barro (1990), and Rapach and Wohar (2007) among others. Stock market data is, however, volatile compared to the other variables considered. This first proxy is similar to that used by Finnish Ministry of Finance (2020), who use price deflator of capital stock instead of price deflator of investment. Both proxies, however, behave very similarly and produce similar results.

Second, we follow Jovanovic and Rousseau (2014) and approximate $Q$ by the ratio of market value of equity and debt to fixed assets ($Q_2$):

$$Q_2 = \frac{market\ value\ of\ equity\ and\ debt}{fixed\ assets} \tag{2}$$

This measure is a simplified version of the frequently used average $Q$ by Hall (2001) and Andrei et al. (2019). Fixed assets approximate the physical capital stock of firms.

Third, we approximate "total $Q$" by Peters and Taylor (2017) by dividing market value of equity and debt minus inventories by capital stock ($Q_3$):

[1] Rahoitustilinpito in Finnish, corresponding approximately to Flow of Funds accounts in the U.S.

[2] The sample is limited by data availability.

$$Q_3 = \frac{market\ value\ of\ equity\ and\ debt - inventories}{capital\ stock} \tag{3}$$

While the denominator of $Q_2$ only accounts for physical capital stock of the firms, the denominator of $Q_3$ accounts for both physical and intangible capital stocks of firms.

Fourth, we calculate a measure of $Q$ by dividing market value of equity by capital stock ($Q_4$). The measure of capital stock is the same as in the denominator of $Q_3$ so this proxy of $Q$ also takes into account both physical and intangible capital stocks of firms.

$$Q_4 = \frac{market\ value\ of\ equity}{capital\ stock} \tag{4}$$

Private investment is defined as non-financial corporations' gross fixed capital formation. We are interested in the level of investment, so as opposed to most of the literature we do not scale the investment by capital stock. Cash flow is defined as ratio of operating surplus of private firms to real GDP, following Philippon (2009).

The time series of different proxies of $Q$, investment and cash flow are displayed in the top rows of figures C1-C6 in the Appendix C. The decomposed time series of different frequencies are reported in the remaining rows of the figures C1-C6. The different proxies of $Q$, cash flow and investment are plotted in the same graph in Figure 1.

## 4 Methodological contribution

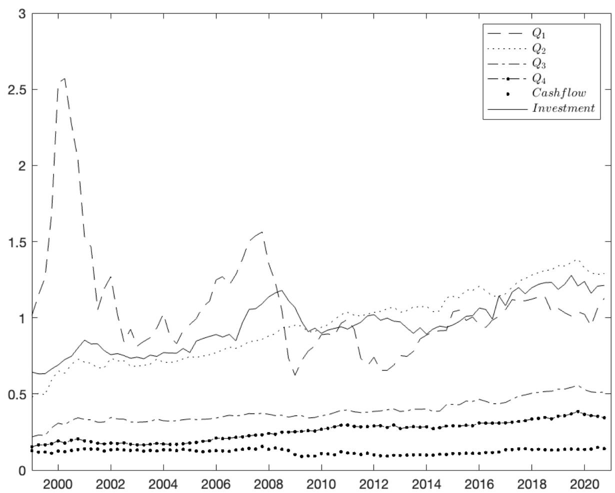
The $Q$-theory has a strong microeconomic foundation. According to the first order condition of profit-maximizing firms facing convex adjustment costs, all fluctuations in private investment are captured by $Q$, the ratio between market value of additional unit of capital and replacement cost of capital.

In addition to $Q$, we include a measure of cash flow and an autoregressive component to the forecast regression. As discussed in section 2, cash flow can be useful in capturing the short term dynamics in investments.

The $h$ period ahead out-of-sample forecasts are produced using direct forecasting and ordinary least squares (OLS) estimation. For each forecasting horizon $h$, we run the regression

$$I_{t+h} = \alpha + \phi I_t + \gamma \boldsymbol{X}_t + \epsilon_{t+h}, \tag{5}$$

Figure 1: Investment, cash flow and different proxies of $Q$ in Finland 1999Q1-2020Q4




*Notes:* The investment series is indexed to 2015=1 for graphing purposes. *Source:* Statistics Finland, Reuters

where $I$ is private investment and $X$ the regressors. Equation (5) is used to obtain the forecast $I_{T+h}$. In the first approach, called "time series" or "TS" analysis in what follows, the regressors $\boldsymbol{X}$ are simply allowed to include, depending on the case, either a proxy of $Q$, cash flow ($CF$), or both.

One purpose of this paper is to test whether complementing $Q$-theory by frequency decomposition improves the forecasting accuracy. According to Kilponen and Verona (2022) $Q$ and cash flow may have effects on investment at different frequencies, notably $Q$ at low frequencies (long term) and cash flow at high frequencies (short term). Using only specific frequencies of the time series can allow utilizing the relevant information contained in the time series without suffering from noise from the other frequencies.

Therefore, in the second approach, called "frequency domain" or "FD" in what follows, the frequency relationship between investment, $Q$ and cash flow is taken explicitly into account by wavelet multiresolution analysis. Following Kilponen and Verona (2016) we use maximum overlap discrete wavelet transform multiresolution analysis (MODWT MRA) and more specifically Daubechies least asymmetric wavelet filter of lenght 8[3] to decompose

[3] The analysis was also run with Daubechies filter lengths two and four, Symlet wavelet filter of lengths

the $Q$, investment and cash flow time series into orthogonal components as follows

$$x(t) = \sum_{j=1}^{J} D_j(t) + S_J(t), \tag{6}$$

where the $D_j(t)$ are the detail components and $S_J(t)$ is the smooth component. The smooth component represent the coarse scale level smooth behaviour of the data (the long-term behaviour or trend) and detail components represent the deviations of different frequency bands from the smooth behaviour. (Gallegati and Ramsey, 2013)

We consider four detail components, i.e. $J = 4$. Wavelet details $D_1(t)$ and $D_2(t)$ capture fluctuations with a period of 0.5-1 years and 1-2 years, respectively. These high frequency details correspond to fluctuations that are shorter lived than business cycle fluctuations. Details $D_3(t)$ and $D_4(t)$ capture fluctuations with a period of 2-4 years and 4-8 years. These details correspond broadly to the business cycle fluctuations. The smooth component $S_4(t)$ captures fluctuations with a period longer than 8 years. In the frequency domain approach, $\boldsymbol{X}$ in equation (5) is allowed to include all the wavelet details $D_1(t) - D_4(t)$ and $S_4(t)$ of $Q$, cash flow, or both.

The forecasts are produced using a sequence of expanding windows, where the first forecasts are produced using data from 1999Q1 to 2013Q4 and the out-of-sample period runs from 2014Q1 to 2020Q4. Forecasts are produced for each combination of wavelet details and the results are reported for the best combination of details for each forecasting horizon as judged by RMSFE. We do not restrict the number of explanatory variables in the forecasting equation so the equations are allowed to have as many as five explanatory variables for univariate models and ten[4] explanatory variables for bivariate models, if those combinations produce the smallest RMSFE.

# 5 Empirical results

The main interest in this paper is to test the out-of sample performance of $Q$-theory and $Q$-theory complemented by frequency decomposition. The out-of-sample performance is discussed in section 5.2. First, an in-sample analysis is conducted in order to provide information on the proxies of $Q$, such as which measures of $Q$ are potentially the best

four and eight and Haar discrete wavelet filter. The results are robust to filter changes.

[4]One or two variable times five frequency components.

Table 1: Pairwise correlations with investment

| | $Q_1$ | $Q_2$ | $Q_3$ | $Q_4$ | $Cash\ flow$ |
|---|---|---|---|---|---|
| Correlation with investment | -0.210 | 0.881 | 0.875 | 0.868 | 0.188 |

predictors of investments, and whether cash flow improves the in-sample performance of $Q$-theory based variables.

### 5.1 In-sample performance

Table 1 displays the pairwise correlations between the different proxies of $Q$ and investment. $Q_2$, $Q_3$ and $Q_4$ have remarkably high positive correlation with the investment series. $Q$-theory suggests that there's a positive relationship with investment and $Q$. Surprisingly, the correlation between $Q_1$ and investment is negative.

Table 2 presents the results of investment regressions using combinations of different measures of $Q$ and cash flow as explanatory variables. We observe that, with the exception of $Q_1$, the coefficients have expected signs and the coefficients in univariate models using $Q_2$, $Q_3$ and $Q_4$ are statistically significant. The coefficient of the univariate model using $Q_1$ or cash flow do not significantly differ from zero. We obtain similar results in multivariate regressions: the coefficients of $Q_2$, $Q_3$ and $Q_4$ are statistically significant. The coefficient of $Q_1$ is only statistically significant at 10% level. The coefficients of cash flow are only statistically significant in half of the cases in multivariate regressions, meaning that cash flow does not always bring new information besides the information provided by $Q$.

The in-sample fit of the univariate models is good. $Q_2$, $Q_3$ and $Q_4$ alone explain over 70% of the variation in investments. When cash flow is included, the models are able to explain even more of the variation. $Q_1$ alone explains only 4% of the variation in investments but including cash flow improves the performance a bit. The in-sample results show that $Q_2$ is the best in-sample proxy, as its coefficient is significant and it alone explains 78.5% of the variation in investments. In-sample analysis also reveals that $Q_1$ behaves unexpectedly: the correlation between investment and $Q_1$ is negative and the regression coefficient of $Q_1$ is negative whereas theory suggests it should be positive.

Table 2: Investment regressions 1999Q1-2020Q4

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| $Q_1$ | -0.001 | | | | | -0.001 | | | |
| | (-1.437) | | | | | (-1.963) | | | |
| $Q_2$ | | 0.004 | | | | | 0.004 | | |
| | | (11.786) | | | | | (14.545) | | |
| $Q_3$ | | | 0.012 | | | | | 0.012 | |
| | | | (12.531) | | | | | (10.093) | |
| $Q_4$ | | | | 0.014 | | | | | 0.014 |
| | | | | (10.457) | | | | | (12.630) |
| Cash flow | | | | | 0.012 | 0.021 | 0.016 | 0.004 | 0.019 |
| | | | | | (0.880) | (1.281) | (3.528) | (0.710) | (5.026) |
| N | 88 | 88 | 88 | 88 | 88 | 88 | 88 | 88 | 88 |
| Adj. $R^2$ | 0.044 | 0.785 | 0.774 | 0.762 | 0.036 | 0.138 | 0.862 | 0.788 | 0.861 |

*Notes:* T- statistics in parentheses. Newey-West standard errors, controls for autocorrelation in errors up to 12 quarters. The constant coefficients are omitted. The response variable is divided by one million for comparison purposes.

## 5.2 Out-of-sample performance

The main goal of this study is to find a forecasting method for private investment in Finland, so an out-of-sample forecasting exercise is carried out. We compare the performance of the forecasting models against an autoregressive model, the usual benchmark model. The first forecasts are produced using data from 1999Q1 to 2013Q4 and the out-of-sample period extends from 2014Q1 to 2020Q4. The analysis was also run restricting the sample to 1999Q1-2019Q4 in order to examine the possible distortions caused by the COVID-19 pandemic, but the results were the same as for the whole sample.

Table 3 reports the root mean square forecast error (RMSFE) ratios of the forecasts relative to the AR-model. The statistical significance of the difference is tested using the modified Diebold-Mariano test by Harvey et al. (1997). The test statistics are reported in Table D1 in the Appendix D. The results are reported for forecast horizons $h = 1$, $h = 4$ (one year ahead) and $h = 8$ (two years ahead). The first rows of each panel report the time series results without frequency decomposition. The second rows report the results in frequency domain, given by the optimal combination of details in the prediction regression. The optimal combination of details is reported in the third row of each panel. We allowed for including all of the variables or decomposed series of variables: if only $Q$ or cash flow are included, the models in frequency domain models are allowed to have up to five explanatory variables. Similarly, if both $Q$ and cash flow are included, ten explanatory variables are allowed in frequency domain, allowing inclusion of all of the decomposed series of both variables.

As seen in Table 3, most of the models have smaller RMSFE's than the corresponding AR-model. However, the difference is never statistically significant even at 10% level. The difficulty of obtaining statistically significant results can stem from the relatively small sample sizes in the Harvey et al. (1997) test. The error series used in calculating the Harvey et al. (1997) test statistic have lengths of 28, 25 and 21 observations for forecasting horizons 1, 4 and 8 respectively.

The model including only $Q_3$ or "total $Q$" by Peters and Taylor (2017) produces the overall most accurate forecasts. The forecasts produced using this measure of $Q$ produce most often the smallest RMSFE ratios in both univariate and bivariate cases. Models using $Q_2$ and $Q_4$ also perform well and always have RMSFE ratio below one. Cash flow and $Q_1$ perform moderately, most often having RMSFE close to one.

Including cash flow in the forecasting equation can improve the performance of the model, but usually only if it is combined with frequency decomposition and only the relevant frequencies can be chosen to the equation. With frequency decomposition the models that contain both $Q$ and cash flow perform better than their univariate counterparts and nearly all of the reported best combinations of variables include one or multiple components of cash flow series. However, the time series (TS) models with $Q$ and cash flow generally perform worse than their counterparts that include only $Q$.

We observe weak evidence in favor of frequency decomposition improving the performance of the model. The models with frequency decomposition mostly have smaller RMSFE's than their time series counterparts, but the differences are not statistically significant even at 10%. The results of the Harvey et al. (1997) test are reported Table D2 in Appendix D. The reason for the lack of statistical significance is likely, once again, the relatively small sample size.

All detail components seem to be useful in forecasting investment, but the components chosen depend on the measure of $Q$. The low frequency ($> 8$ years) components of $Q_2$ and $Q_3$ are often chosen to models but those of $Q_1$ and cash flow are rarely chosen. Contrarily, the highest frequency (6 months - 1 year) components of $Q_1$ and cash flow are often chosen but those of $Q_2$, $Q_3$ and $Q_4$ are rarely chosen. However, all components are chosen to models at least in some cases and conclusions on which components are the most or least useful cannot be drawn. Similarly, Kilponen and Verona (2022) argue that both high and low frequencies are needed in order to capture the relevant movements in investment.

Table 3: Out-of-sample results for 2014Q1-2020Q4

| | $Q_1$ | $Q_2$ | $Q_3$ | $Q_4$ | $CF$ | $[Q_1, CF]$ | $[Q_2, CF]$ | $[Q_3, CF]$ | $[Q_4, CF]$ |
|---|---|---|---|---|---|---|---|---|---|
| | h=1 | | | | | | | | |
| TS | 0.999 | 0.977 | 0.964 | 0.975 | 1.003 | 1.003 | 0.956 | 0.986 | 0.983 |
| FD | 0.997 | 0.979 | 0.974 | 0.971 | 0.984 | 0.980 | 0.927 | 0.924 | 0.916 |
| Variables | $Q_1^{D_1}$ | $Q_2^{S_4}$ | $Q_3^{S_4}$ | $Q_4^{D_1}, Q_4^{D_4}, Q_4^{S_4}$ | $CF^{D_1}, CF^{D_4}$ | $Q_1^{D_1}, Q_1^{S_4}, CF^{D_1}, CF^{D_4}$ | $Q_2^{S_4}, CF^{D_1}, CF^{D_3}, CF^{D_4}$ | $Q_3^{D_4}, CF^{D_1}, CF^{D_3}$ | $Q_4^{D_1}, Q_4^{S_4}, CF^{D_3}, CF^{D_4}$ |
| | h=4 | | | | | | | | |
| TS | 0.947 | 0.811 | 0.722 | 0.835 | 0.992 | 0.971 | 0.695 | 0.749 | 0.760 |
| FD | 0.965 | 0.799 | 0.694 | 0.821 | 0.973 | 0.965 | 0.623 | 0.553 | 0.807 |
| Variables | $Q_1^{D_1}, Q_1^{D_2}$ | $Q_2^{D_2}, Q_2^{S_4}$ | $Q_3^{D_2}, Q_3^{D_4}, Q_3^{S_4}$ | $Q_4^{D_2}, Q_4^{D_3}, Q_4^{D_4}, Q_4^{S_4}$ | $CF^{D_1}$ | $Q_1^{D_1}, Q_1^{D_2}$ | $Q_2^{D_4}, Q_2^{S_4}, CF^{D_4}, CF^{S_4}$ | $Q_3^{D_4}, Q_3^{S_4}, CF^{D_3}$ | $Q_4^{D_2}, Q_4^{D_3}, Q_4^{D_4}, Q_4^{S_4}, CF^{S_4}$ |
| | h=8 | | | | | | | | |
| TS | 1.016 | 0.748 | 0.703 | 0.864 | 1.040 | 1.052 | 0.807 | 0.759 | 0.955 |
| FD | 0.992 | 0.694 | 0.457 | 0.887 | 0.993 | 0.992 | 0.694 | 0.452 | 0.887 |
| Variables | $Q_1^{D_1}, Q_1^{D_2}$ | $Q_2^{D_2}, Q_2^{S_4}$ | $Q_3^{D_2}, Q_3^{S_4}$ | $Q_4^{D_1}, Q_4^{S_4}$ | $CF^{D_1}$ | $Q_1^{D_1}, Q_1^{D_2}, CF^{D_1}$ | $Q_2^{D_2}, Q_2^{S_4}, CF^{D_1}, CF^{D_2}$ | $Q_3^{D_2}, Q_3^{S_4}, CF^{D_1}, CF^{D_2}$ | $Q_4^{D_1}, Q_4^{S_4}, CF^{D_1}, CF^{D_4}$ |

*Notes:* RMSFE ratio relative to benchmark AR-model. Entries below (above) one indicate that the model outperforms (underperforms) the benchmark model.

## 6 Conclusion

In this paper we analyze the private investment forecasting performance of $Q$-theory utilizing frequency decomposition of variables. We approximate average $Q$ by four different measures and find that there's weak evidence for these proxies being useful in forecasting private investments in Finland. Our models are most often able to outperform the benchmark AR-model, but not in a statistically significant manner. We also find that both frequency decomposition and inclusion of cash flow into the equation can improve the out-of-sample forecasting accuracy. The best overall out-of-sample results are produced by the "total $Q$" measure by Peters and Taylor (2017).

A natural progression of this work is to extend the out-of-sample forecasting performance comparison to other methods typically used in investment forecasting, such as neoclassical investment model and accelerator model. An analysis on the relationship between private investment and business confidence indicators published by Confederation of Finnish Industries[5] would also be very interesting and useful in developing tools for investment forecasters. Further work could also undertake estimating the function to approximate the bond $Q$ by Philippon (2009) with Finnish data. Bond $Q$ has shown great potential in forecasting private investment in the U.S.

[5] Elinkeinoelämän keskusliitto in Finnish

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# Appendix A Data definitions, sources and summary statistics

*Investment*
Non-financial corporations' gross fixed capital formation from Statistics Finland's quarterly sector accounts, millions euros. As opposed to most of the literature, our variable of interest is the investment series, not the investment ratio $I/K$ so we do not scale the investment by capital stock.

*All shares price index*
NASDAQ OMX Helsinki all shares price index from Reuters. Daily data, aggregated to quarterly frequency by averaging.

*Price deflator of investment*
The ratio between investment at current prices and investment at base year 2015 prices from Statistics Finland's quarterly national accounts.

*Market value of equity and debt*
Total equity, total loans and total debt of non-financial corporations from Statistics Finland's quarterly financial accounts, millions euros.

*Inventories*
Inventories of non-financial corporations from Statistics Finland's annual financial accounts, millions euros. The annual series is disaggregated to quarterly frequency by linear interpolation. Linear interpolation is described in Appendix B.

*Capital stock*
Gross stock of non-financial assets of non-financial corporations from Statistics Finland annual national accounts, millions euros. Statistics Finland's measure of non-financial assets includes intellectual property products and therefore accounts for both tangible and intangible capital stock. The annual series is disaggregated to quarterly frequency by linear interpolation.

*Fixed assets*
Non-financial assets of non-financial corporations less intellectual property products from Statistic Finland annual national accounts, millions euros. The annual series is disaggregated to quarterly frequency by linear interpolation.

*Market value of equity*
Total equity of non-financial corporations from Statistics Finland's quarterly financial ac-

counts, millions euros.

*Cash flow*
Ratio of operating surplus of non-financial corporations to real GDP from Statistics Finland's quarterly sector accounts and quarterly national accounts, respectively, millions euros. The definition follows Philippon (2009).

Table 4: Summary statistics

| | Mean | Standard deviation | Lag 1 autocorrelation |
|---|---|---|---|
| Investment | 5650.000 | 1005.190 | -0.079 |
| $Q_1$ | 1.079 | 0.358 | 0.334 |
| $Q_2$ | 0.950 | 0.236 | 0.102 |
| $Q_3$ | 0.386 | 0.075 | 0.098 |
| $Q_4$ | 0.253 | 0.064 | -0.161 |
| Cash Flow | 0.122 | 0.016 | -0.146 |

## Appendix B Linear interpolation

The capital and inventories series are interpolated using a simple linear interpolation. The quarterly values of inventories, capital stock and fixed assets are calculated based on the corresponding annual series $X_t$ as follows:

$$Q4_{t-1} = X_{t-2} + 4 * \frac{X_{t-1} - X_{t-2}}{4} = X_{t-1} \tag{7}$$

$$Q1_t = X_{t-1} + \frac{X_t - X_{t-1}}{4} \tag{8}$$

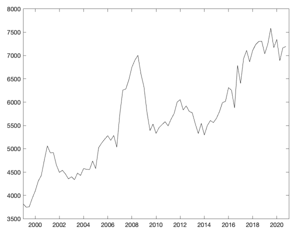
$$Q2_t = X_{t-1} + 2 * \frac{X_t - X_{t-1}}{4} \tag{9}$$

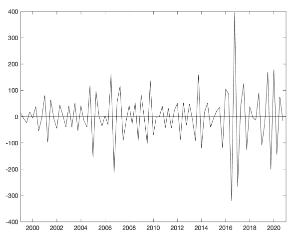
$$Q3_t = X_{t-1} + 3 * \frac{X_t - X_{t-1}}{4} \tag{10}$$

$$Q4_t = X_{t-1} + 4 * \frac{X_t - X_{t-1}}{4} = X_t \tag{11}$$

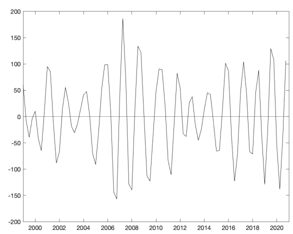
# Appendix C Decomposed data series

Figure C1: Investment and its decomposed components in Finland 1999Q1-2020Q4

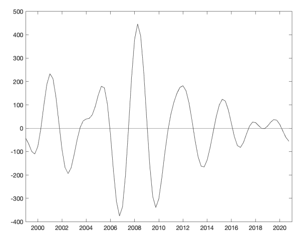




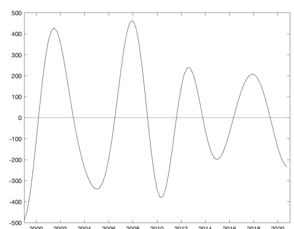
(A) $I^{D_1}$ (6m - 1y)



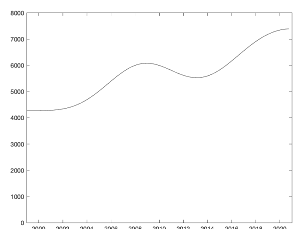
(B) $I^{D_2}$ (1y - 2y)



(C) $I^{D_3}$ (2y - 4y)



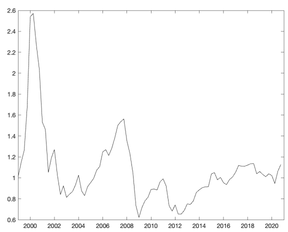
(D) $I^{D_4}$ (4y - 8y)

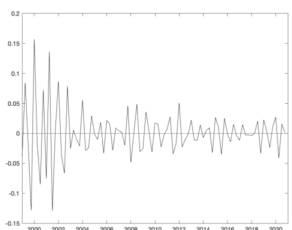


(E) $I^{S_4}$ (> 8y)

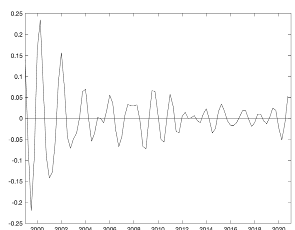
*Source:* Statistics Finland

Figure C2: Time series of $Q_1$ and its decomposed components in Finland 1999Q1-2020Q4

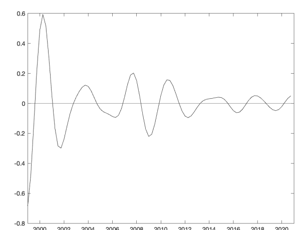




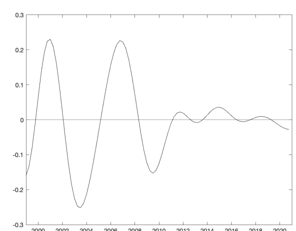
(A) $Q_1^{D_1}$ (6m - 1y)



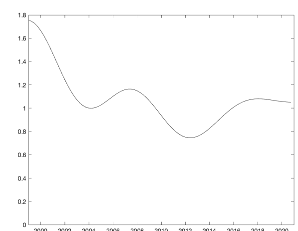
(B) $Q_1^{D_2}$ (1y - 2y)



(C) $Q_1^{D_3}$ (2y - 4y)



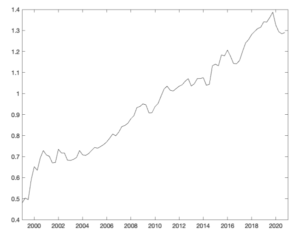
(D) $Q_1^{D_4}$ (4y - 8y)

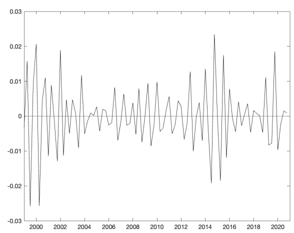


(E) $Q_1^{S_4}$ (> 8y)

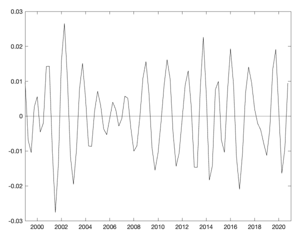
*Source:* Statistics Finland, Reuters

Figure C3: Time series $Q_2$ and its decomposed components in Finland 1999Q1-2020Q4

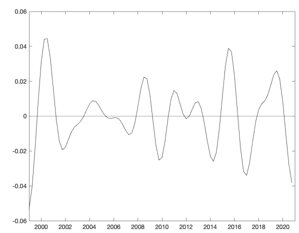




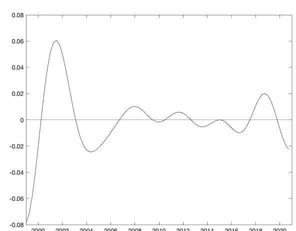
(A) $Q_2^{D_1}$ (6m - 1y)



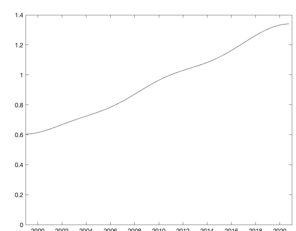
(B) $Q_2^{D_2}$ (1y - 2y)



(C) $Q_2^{D_3}$ (2y - 4y)



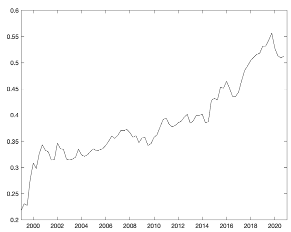
(D) $Q_2^{D_4}$ (4y - 8y)

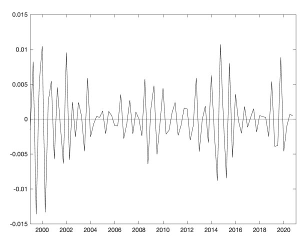


(E) $Q_2^{S_4}$ (> 8y)

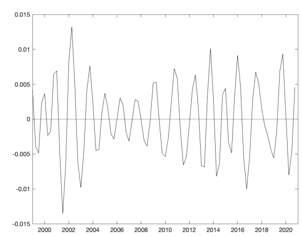
*Source:* Statistics Finland

Figure C4: Time series of $Q_3$ and its decomposed components in Finland 1999Q1-2020Q4

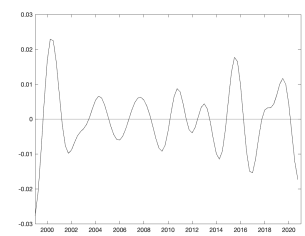




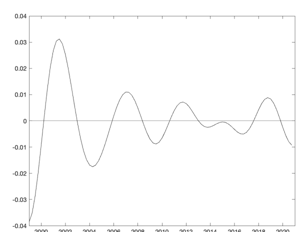
(A) $Q_3^{D_1}$ (6m - 1y)



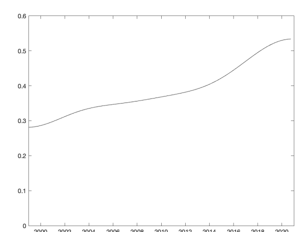
(B) $Q_3^{D_2}$ (1y - 2y)



(C) $Q_3^{D_3}$ (2y - 4y)



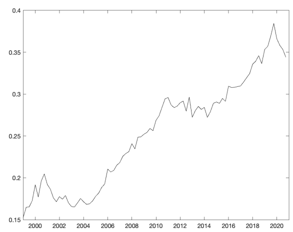
(D) $Q_3^{D_4}$ (4y - 8y)

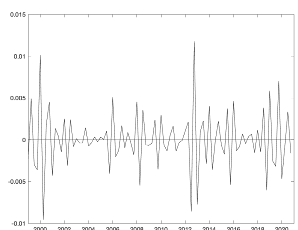


(E) $Q_3^{S_4}$ (> 8y)

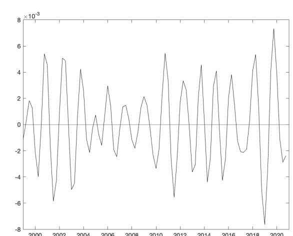
*Source:* Statistics Finland

Figure C5: Time series of $Q_4$ and its decomposed components in Finland 1999Q1-2020Q4

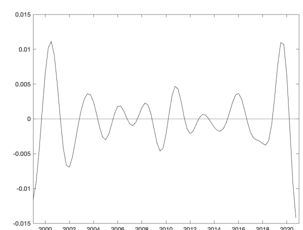




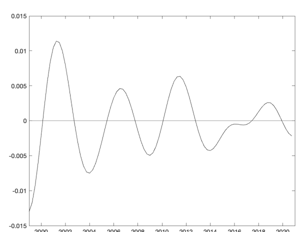
(A) $Q_4^{D_1}$ (6m - 1y)



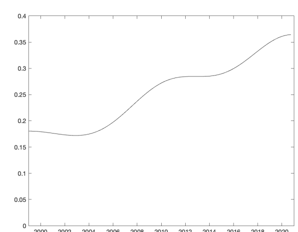
(B) $Q_4^{D_2}$ (1y - 2y)



(C) $Q_4^{D_3}$ (2y - 4y)



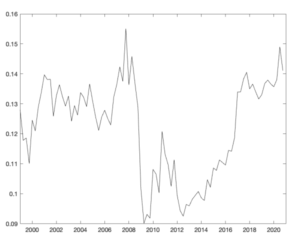
(D) $Q_4^{D_4}$ (4y - 8y)

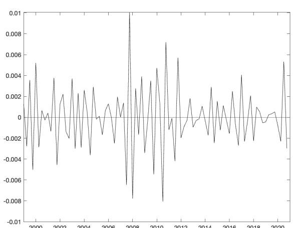


(E) $Q_4^{S_4}$ (> 8y)

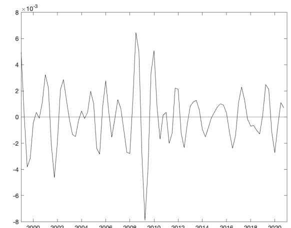
*Source:* Statistics Finland

Figure C6: Time series of cash flow and its decomposed components in Finland 1999Q1-2020Q4

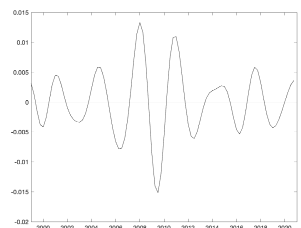




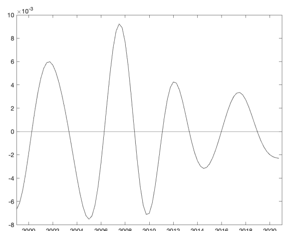
(A) $CF^{D_1}$ (6m - 1y)



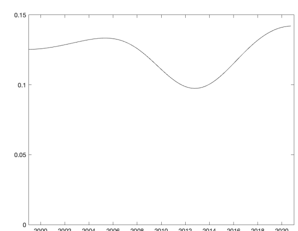
(B) $CF^{D_2}$ (1y - 2y)



(C) $CF^{D_3}$ (2y - 4y)



(D) $CF^{D_4}$ (4y - 8y)



(E) $CF^{S_4}$ (> 8y)

*Source:* Statistics Finland

# Appendix D Harvey et al. (1997) test results

Table D1: (Harvey et al., 1997) test results against corresponding AR-model

| | $Q_1$ | $Q_2$ | $Q_3$ | $Q_4$ | $CF$ | $[Q_1, CF]$ | $[Q_2, CF]$ | $[Q_3, CF]$ | $[Q_4, CF]$ |
|---|---|---|---|---|---|---|---|---|---|
| | Panel A: $h = 1$ | | | | | | | | |
| TS | 0.091 | 1.442 | 0.699 | 1.249 | -0.112 | -0.128 | 0.515 | 0.170 | 0.143 |
| FD | 0.740 | 1.347 | 0.726 | 0.846 | 0.965 | 1.161 | 1.398 | 1.204 | 0.643 |
| | Panel B: $h = 4$ | | | | | | | | |
| TS | 0.950 | 1.445 | 1.083 | 1.412 | 0.631 | 1.116 | 1.131 | 0.870 | 0.765 |
| FD | 1.325 | 1.406 | 1.249 | 1.389 | 1.252 | 1.325 | 1.129 | 1.380 | 1.404 |
| | Panel C: $h = 8$ | | | | | | | | |
| TS | -0.336 | 0.802 | 0.782 | 0.810 | -0.255 | -0.329 | 0.843 | 0.826 | 0.685 |
| FD | 0.819 | 0.779 | 0.764 | 0.770 | 0.757 | 0.802 | 0.780 | 0.764 | 0.771 |

*Notes:* T- test statistics. The t-statistics follow Student's t-distribution with 28, 25 and 21 degrees of freedom for forecasting horizons 1, 4 and 8 respectively.

Table D2: Harvey et al. (1997) test results against corresponding time series model

| | $Q_1$ | $Q_2$ | $Q_3$ | $Q_4$ | $CF$ | $[Q_1, CF]$ | $[Q_2, CF]$ | $[Q_3, CF]$ | $[Q_4, CF]$ |
|---|---|---|---|---|---|---|---|---|---|
| | h=1 | | | | | | | | |
| FD | 0.147 | -0.360 | -0.449 | 0.218 | 0.653 | 0.933 | 0.417 | 1.267 | 1.277 |
| | h=4 | | | | | | | | |
| FD | -0.553 | 0.456 | 0.520 | 0.441 | 0.994 | 0.185 | 0.412 | 1.170 | -0.184 |
| | h=8 | | | | | | | | |
| FD | 0.434 | 0.636 | 0.712 | -0.415 | 0.286 | 0.362 | 0.635 | 0.696 | 0.713 |

*Notes:* T- test statistics. The t-statistics follow Student's t-distribution with 28, 25 and 21 degrees of freedom for forecasting horizons 1, 4 and 8 respectively.

# BoF Economics Review

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