Online appendix

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Michael Funke, Petar Mihaylovski and Haibin Zhu

Monetary policy transmission in China: A DSGE model with parallel shadow banking and interest rate control

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Suomen Pankki
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Michael Funke, Department of Economics, Hamburg University and CESifo, Munich, GERMANY.
Email: michael.funke@uni-hamburg.de.

Petar Mihaylovski, Department of Economics, Hamburg University, .GERMANY
Email: PetarValentinov.Mihaylovski@wiso.uni-hamburg.de.

Haibin Zhu, JP Morgan Chase Bank, HONG KONG.
Email: haibin.zhu@jpmorgan.com.

Online appendix

This online appendix contains some additional details about the derivations that are behind key equations in the paper. In particular, we provide the complete set of equilibrium conditions.

The business cycle DSGE model aiming to capture the development of the shadow banking sector in China is solved via perturbation methods as we approximate the model around its steady state up to the first order. For this, we employ the DYNARE package using external MATLAB functions for the steady state values. Last but not least, we use Guerrieri and Iacoviello's (2014) linear first-order piecewise perturbation algorithm in order to analyze the effects of interest rate controls. For ease of notation, we adopt the following notation introduced by Verona et al. (2013):

$$q_t = \frac{Q_{\tilde{K},t}}{P_t}$$

$$\lambda_{n,t} = \lambda_t P_t$$

$$w^{e,re} = \frac{W^{e,re}}{P_t}$$

$$w^{e,se} = \frac{W^{e,se}}{P_t}$$

$$w^{sb} = \frac{W^{sb}}{P_t}$$

$$n_{t+1}^{se} = \frac{N_{t+1}^{se}}{P_t}$$

$$n_{t+1}^{re} = \frac{N_{t+1}^{re}}{P_t}$$

$$n_{t+1}^{sb} = \frac{N_{t+1}^{sb}}{P_t}$$

Intermediate good producers

The arbitrage condition for the choice of capital services implies

(1) $$\frac{r_t^{k,re}}{r_t^{k,se}} = \left(\frac{u_t^{re}\ \widetilde{K}_{t-1}^{re}}{u_t^{se}\ \widetilde{K}_{t-1}^{se}}\right)^{\rho-1}$$

Deriving the marginal cost of intermediate good producer yields

$$A = \left(- \frac{r_t^{k,re}\left(\eta\left(u_t^{re}\ \widetilde{K}_{t-1}^{re}\right)^{\rho} + (1-\eta)\left(u_t^{se}\ \widetilde{K}_{t-1}^{se}\right)^{\rho}\right)^{1-\frac{1}{\rho}}}{\alpha\left(u_t^{re}\ \widetilde{K}_{t-1}^{re}\right)^{\rho-1}\left(\frac{h_t}{\left(\eta\left(u_t^{re}\ \widetilde{K}_{t-1}^{re}\right)^{\rho} + (1-\eta)\left(u_t^{se}\ \widetilde{K}_{t-1}^{se}\right)^{\rho}\right)^{\frac{1}{\rho}}}\right)^{1-\alpha}} \right)$$

(2) $$\mathrm{B} = \rho\left(\frac{\widetilde{w}_t}{1-\alpha}\right)^{1-\frac{\alpha}{\rho+\alpha-\rho\,\alpha}}\left(\left(u_t^{re}\ \widetilde{K}_{t-1}^{re}\right)^{\rho-1}\frac{\alpha}{r_t^{k,re}}\right)^{\frac{-\alpha}{\rho+\alpha-\rho\,\alpha}}$$

$$\mathrm{C} = \left\{\left[\left(\eta\left(u_t^{re}\ \widetilde{K}_{t-1}^{re}\right)^{\rho} + (1-\eta)\left(u_t^{se}\ \widetilde{K}_{t-1}^{se}\right)^{\rho}\right)^{\frac{1}{\rho}}\right]^{\alpha} h_t^{1-\alpha}\right\}^{\frac{(\rho-1)\,\alpha}{\rho+\alpha-\rho\,\alpha}}\frac{1}{\rho+\alpha-\rho\,\alpha}$$

$$A\ B\ C = 0\ .$$

Solving for an expression for the total amount of capital services gives us

(3) $$K_t = \left[\eta\left(u_t^{re}\ \widetilde{K}_{t-1}^{re}\right)^{\rho} + (1-\eta)\left(u_t^{se}\ \widetilde{K}_{t-1}^{se}\right)^{\rho}\right]^{\frac{1}{\rho}}$$

As is common in the literature, we employ a Cobb-Douglas production function which is defined as

(4) $$Y_t = \exp\left(\mathrm{a}_t\right) h_t^{1-\alpha}\mathrm{K}_{t-1}^{\alpha}\ .$$

Capital producers

The first-order condition with respect to investment yields

(5) $$\lambda_{n,t}\ q_t\left(1 - \frac{S''}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \left(\frac{I_t}{I_{t-1}} - 1\right)\frac{S''\ I_t}{I_{t-1}}\right) - \lambda_{n,t}$$
$$+\beta\ \lambda_{n,t+1}\ q_{t+1}\ S\left(\frac{I_{t+1}}{I_t}\right)^2\left(\frac{I_{t+1}}{I_t} - 1\right) = 0$$

and the law of motion for the evolution of capital stock is

(6) $$\eta\,\widetilde{K}_t^{re} + (1-\eta)\,\widetilde{K}_t^{se} - (1-\delta)\left(\eta\,\widetilde{K}_t^{re} + (1-\eta)\,\widetilde{K}_t^{se}\right) - I_t\left(1 - \frac{S''}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) = 0.$$

Shadow banks and high-risk firms

The law of motion for optimism in the shadow banking sector is given by

(7) $$\chi_t^{sb} = \rho_\chi^{sb}\chi_{t-1}^{sb} + \left(1-\rho_\chi^{sb}\right)\alpha^{sb}\left(n_t^{re} - \bar{n}^{re}\right).$$

The first-order condition with respect to capital utilization of high-risk firm is

(8) $$r_t^{k,re} = a'(u_t^{re}).$$

The rate of return to capital of high-risk firm can be written as

(9) $$1 + R_t^{k,re} = \frac{\pi_t}{q_{t-1}}\left\{\left[u_t^{re} r_t^{k,re} - a(u_t^{re})\right] + (1-\delta)q_t\right\}.$$

The debt contract between the high-risk firm and the shadow bank requires

(10) $$E_t\left\{\left[1-\Gamma_t(\bar{\omega}_{t+1}^a)\right]\frac{1+R_t^{k,re}}{1+r_t^E} + \frac{\Gamma'(\bar{\omega}_{t+1}^a)}{\Gamma'(\bar{\omega}_{t+1}^a) - \mu G'(\bar{\omega}_{t+1}^a)}\left[\Gamma_t(\bar{\omega}_t^a) - \mu G_t(\bar{\omega}_t^a)\frac{1+R_t^{k,re}}{1+r_t^E} - 1\right]\right\} = 0$$

The associated profit condition of the shadow bank is

(11) $$\left[\Gamma_t(\bar{\omega}_t^a) - \mu G_t(\bar{\omega}_t^a)\right]\frac{q_{t-1}\widetilde{K}_t^{re}}{n_t^{re}}\frac{1+R_t^{k,re}}{1+r_t^E} = \frac{q_{t-1}\widetilde{K}_t^{re}}{n_t^{re}} - 1$$

As a result, the law of motion of high-risk firm's net worth is

(12) $$n_{t+1}^{re} = \gamma^{re}\frac{q_{t-1}}{\pi_t}\widetilde{K}_t^{re}\left[R_t^{k,re} - r_t^E - \mu\int_0^{\omega_t^a}\omega dF_{t-1}(\omega)\left(1+R_t^{k,re}\right)\right] + \gamma^{re}\frac{n_t^{re}}{\pi_t}(1+r_t^E) + w^{e,re}$$

and the external finance premium is determined as

(13) $$P_t^{ext,re} = \frac{\widetilde{K}_t^{re}\, q_t\, \bar{\omega}_{t+1}^a\left(1+R_{t+1}^{k,re}\right)}{q_t\,\widetilde{K}_t^{re} - n_t^{re}} - (1+r_t^E)$$

The no-default lending rate on high-risk firm's debt is

(14) $$R_t^{sb} = \frac{\widetilde{K}_t^{re}\, q_t\, \overline{\omega}_{t+1}^{a} \left(1+R_{t+1}^{k,re}\right)}{q_t\, \widetilde{K}_t^{re} - n_t^{re}}$$

The arbitrage condition for shadow bank savings deposits is

(15) $$r_t^E = \frac{1+r_t^d}{1-\phi_t} - 1 \,.$$

The ex-post default threshold value of high-risk firms is

(16) $$\overline{\omega}_t^b = \overline{\omega}_t^a \,(1+\chi_t^{re})$$

The law of motion of shadow bank's net worth is given by

(17) $$n_t^{sb} = (1-\phi_{t-1})n_{t-1}^{sb} + [1-F_t(\overline{\omega}_t)]R_t^{sb}L_t^{re}$$
$$+(1-\mu)\int_0^{\overline{\omega}_t^b} \omega dF(\omega)\left(1+\ R_t^{k,re}\right)Q_{\widetilde{K},t-1}\,\widetilde{K}_t^{re} - (1+r_t^E)L_t^{re} + w^{sb}$$

The shadow bank's capital ratio is defined as

(18) $$\kappa_t^{sb} = \frac{n_t^{sb}}{L_t^{re}} \,.$$

Finally, the shadow bank's default probability

(19) $$\phi_t = cdf(\kappa_t^{sb}, \sigma^{sb}) \,.$$

Commercial banks and low-risk firms

The law of motion for optimism in the formal banking sector takes the form

(20) $$\chi_t^{rb} = \rho_\chi^{rb}\chi_{t-1}^{rb} - \alpha^{rb}\left(1-\rho_\chi^{rb}\right)(n_t^{se} - \bar{n}^{se})$$

The time-varying interest elasticity due to optimism is

(21) $$\varepsilon_t^{l,op} = \varepsilon^l\left(1+\chi_t^{rb}\right) .$$

The first-order condition with respect to capital utilization of low-risk firms yields

(22) $$r_t^{k,se} = a'(u_t^{se})$$

and the associated rate of return to capital of low-risk firm is

(23) $$1 + R_t^{k,se} = \frac{\pi_t}{q_{t-1}}\left\{[u_t^{se} r_t^{k,se} - a(u_t^{se})] + (1-\delta)q_t\right\}.$$

The low-risk firm chooses capital to maximize profits, so the first-order conditions is

(24) $$R_{t+1}^{rb} - R_{t+1}^{k,se} - 1 + \frac{1}{\beta} = 0 .$$

The associated law of motion of low-risk firm's net worth is

(25) $$n_t^{se} = q_{t-1}\,\widetilde{K}_{t-1}^{re}\,\frac{\gamma^l}{\pi_t}\left(R_t^{k,se} - r_{t-1}^l\right) + \frac{\gamma^l}{\pi_t}\left(1 + r_{t-1}^l\right) n_{t-1}^{se} + w^{e,se} .$$

The relationship between commercial bank's deposits and loans is

(26) $$D_t\,\frac{1-\nu}{\nu} = L_t^{se}.$$

Solving for the commercial bank's deposit rate yields

(27) $$(1 + r_t^d) = \frac{\varepsilon^d}{\varepsilon^d - 1}(1 + R_t^d)$$

The optimal rule for setting the commercial bank's lending rate is[1]

(28) $$1 + r_{t+1}^l = \frac{1}{\varepsilon_{t+1}^{l,op} - 1 + \kappa^l}\left[\varepsilon_{t+1}^{l,op}(1 + R_{t+1}^l) + \kappa^l(1 + r_{t+1}^{l,cb})\right]$$

The relationship between PboC's policy rate and the deposit rate of the wholesale branch of the commercial bank becomes

(29) $$R_t^d = R_t + \frac{c_d}{\bar{Y}}\frac{D_t}{\nu}$$

[1]Since the lending benchmark rate is not necessarily binding, it is important to allow for the possibility that banks set a lending rate lower than the floor determined by the central bank. In order to do that we identify two regimes for equation (28) by means of Guerrieri and Iacoviello's (2014) algorithm:

$$k^l = \begin{cases} x & \text{if} r_t^l < r_t^{l,cb} \\ 0 & \text{if} r_t^l \geq r_t^{l,cb} \end{cases}$$

where x takes on different values depending on the tightness of the lending rate regulation.

Furthermore, the relationship between PboC's policy rate and the lending rate of wholesale branch of the commercial bank

(30) $$R_t^l = R_t + k^w (L_t^{se} - L_t^{cb}) + L_t^{se} \frac{c_l}{\bar{Y}}$$

Households

The first-order condition with respect to time deposits is

(31) $$\left(-\lambda_{n,t}\right) + \frac{\lambda_{n,t+1}\,\beta\,(1+r_t^d)}{\pi_{t+1}} = 0$$

and the first-order condition with respect to consumption yields

(32) $$\lambda_{n,t} - (C_t - b\,C_{t-1})^{(-\sigma_c)} + \beta\,b\,(C_{t+1} - C_t\,b)^{(-\sigma_c)} = 0\ .$$

Aggregate resource constraint

The aggregate resource constraint can be written as

(33) $$C_t + I_t + \eta\mu \int_0^{\bar{\omega}_t^b} \omega dF(\omega)\left(1 + R_t^{k,re}\right)\frac{Q_{\tilde{K},t-1}\tilde{K}_t^{re}}{P_t}$$

$$+\eta a(u_t^{re})\tilde{K}_t^{re} + (1-\eta)a(u_t^{se})\tilde{K}_t^{se} = Y_t\ .$$

First-order conditions associated with Calvo sticky prices and wages

The relevant equations are:

(34) $$\lambda_{n,t}\,Y_t + \beta\,\theta_p\left(\frac{\pi_t^{1-\iota_1}}{\pi_{t+1}}\right)^{\frac{1}{1-\lambda_f}} F_{p,t+1} - F_{p,t} = 0$$

(35) $$\frac{r_t^{k,re}\left(\eta\,(u_t^{re}\,\tilde{K}_{t-1}^{re})^{\rho} + (1-\eta)\,(u_t^{se}\,\tilde{K}_{t-1}^{se})^{\rho}\right)^{1-\frac{1}{\rho}}}{\alpha\,(u_t^{re}\,\tilde{K}_{t-1}^{re})^{\rho-1}\left(\frac{h_t}{\left(\eta\,(u_t^{re}\,\tilde{K}_{t-1}^{re})^{\rho} + (1-\eta)\,(u_t^{se}\,\tilde{K}_{t-1}^{se})^{\rho}\right)^{\frac{1}{\rho}}}\right)^{1-\alpha}}\,\lambda_{n,t}\,Y_t\,\lambda_f$$

$$+\beta\,\theta_p K_{p,t+1}\left(\frac{\pi_t^{1-\iota_1}}{\pi_{t+1}}\right)^{\frac{(-\lambda_f)}{\lambda_f-1}} - K_{p,t} = 0$$

(36)
$$\frac{h_t\left((C_t-b\,C_{t-1})^{(-\sigma_c)}-\beta\,b\,(C_{t+1}-C_t\,b)^{(-\sigma_c)}\right)}{\lambda_w}$$
$$+\frac{\beta\,\theta_w\left(\pi_t^{1-\iota_{w,1}}\right)^{\frac{1}{1-\lambda_w}}}{\pi_{t+1}}\left(\frac{1}{\frac{\pi_{t+1}\,\tilde{w}_{t+1}}{\tilde{w}_t}}\right)^{\frac{\lambda_w}{1-\lambda_w}}F_{w,t+1}-F_{w,t}=0$$

(37)
$$h_t^{1+\sigma_l}+\beta\,\theta_w\left(\frac{\pi_t^{1-\iota_{w,1}}}{\frac{\pi_{t+1}\,\tilde{w}_{t+1}}{\tilde{w}_t}}\right)^{\frac{\lambda_w(1+\sigma_l)}{1-\lambda_w}}K_{w,t+1}-K_{w,t}=0$$

(38)
$$K_{p,t}-F_{p,t}\left(\frac{1-\theta_p\left(\frac{\pi_{t-1}^{1-\iota_1}}{\pi_t}\right)^{\frac{1}{1-\lambda_f}}}{1-\theta_p}\right)^{1-\lambda_f}=0$$

(39)
$$K_{w,t}-\frac{\tilde{w}_t\,F_{w,t}}{\psi_l}\left(\frac{1-\theta_w\left(\frac{\pi_{t-1}^{1-\iota_{w,1}}}{\frac{\tilde{w}_t\,\pi_t}{\tilde{w}_{t-1}}}\right)^{\frac{1}{1-\lambda_w}}}{1-\theta_w}\right)^{1-\lambda_w(1+\sigma_l)}=0$$

Aggregate variables

The expression for aggregate net worth is

(40)
$$n_t^{ag}=n_t^{re}\,\eta+n_t^{se}\,(1-\eta)\,.$$

The total amount of low-risk firm’s loans is given by

(41)
$$L_{t+1}^{se}=q_t\,\tilde{K}_{t+1}^{se}-n_{t+1}^{se}\,,$$

while the total amount of high-risk firm’s loans is

(42)
$$L_{t+1}^{re}=q_t\,\tilde{K}_{t+1}^{re}-n_{t+1}^{re}\,.$$

The leverage of low-risk and high-risk firms is

(43)
$$lev_t^{se}=\frac{q_t\,\tilde{K}_t^{se}}{n_t^{se}}$$

and

(44) $$lev_t^{re} = \frac{q_t \tilde{K}_t^{re}}{n_t^{re}},$$

respectively. Substituting yields average leverage

(45) $$lev_t^{ag} = \eta\ lev_t^{re} + (1-\eta)\ lev_t^{se},$$

and the aggregate loan amount

(46) $$L_t^{ag} = \eta\ L_t^{re} + (1-\eta)\ L_t^{se}.$$

Monetary policy

The PBoC's Taylor-rule for setting the policy rate is

(47) $$R_t = \tilde{\rho}(R_{t-1}) + (1-\tilde{\rho})[\bar{R} + \alpha_\pi(\pi_t - \bar{\pi}) + \alpha_y(Y_t - \bar{Y})] + \varepsilon_t^{MP}.$$

The PBoC's deposit rate ceiling is determined according to

(48) $$r_t^{d,cb} = \bar{r}^d,$$

while the PBoC's lending rate floor is

(49) $$r_t^{l,cb} = \bar{r}^l.$$

Finally, PBoC's window guidance policy follows the Taylor-type rule

(50) $$L_t^{CB} = \phi_l^{cb}(L_{t-1}^{CB}) + (1-\phi_l^{cb})\left(\bar{L}^{se} + \phi_l^{l}[L_t - \bar{L}^{se}] + \phi_l^{\pi}[\pi_t - \bar{\pi}] + \phi_l^{y}[Y_t - \bar{Y}]\right).$$

References

Guerrieri, L. and M. Iacoviello (2014) "OccBin: A Toolkit for Solving Dynamic Models With Occasionally Binding Constraints Easily", *Federal Reserve Board Working Paper No.* 2014–47, Washington.

Verona, F., Martins, M.F. and I. Drumond (2013) "(Un)anticipated Monetary Policy in a DSGE Model with a Shadow Banking System", *International Journal of Central Banking* 9, 73–117.