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Xi Chen and Michael Funke

The dynamics of catch-up and skill  
and technology upgrading in China



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Xi Chen\* and Michael Funke‡

## The dynamics of catch-up and skill and technology upgrading in China

### Abstract

This paper accounts for China's economic growth since 1980 in a unified endogenous growth model in which a sequencing of physical capital accumulation, human capital accumulation and innovation drives the rise in China's aggregate income. The first stage is characterized by physical capital accumulation. The second stage includes both physical and human capital accumulation, and in the final stage innovation is added to the mix. Model calibrations indicate that the growth model can generate a trajectory that accords well with the different stages of development in China.

Keywords: China, economic growth, transitional dynamics

JEL-Classification: D90, O31, O33, O41

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# 1 Introduction

China has achieved a remarkable rate of economic growth over the past three decades. Beginning in 1978, China embarked on a series of institutional changes that have led to a market economy increasingly reliant on markets and price signals for allocating productive resources. Nowadays, China is considered a global powerhouse and one of the fastest-growing emerging market economies in the world.<sup>1</sup> Since the first economic reforms were introduced in 1978, China's real GDP has grown 8-10 percent a year on average. China is already the world's second-largest economy and is striving to create additional multinational firms with 'global-challenger' ambitions. Based on these facts, China's past, current and future economic catch-up and evolution are the subject of intense study.

The economic literature has long recognized that the accumulation of physical capital and knowledge along with R&D-based technological progress are the key drivers of economic growth. The first strand of research was inspired by Solow's (1956) neoclassical growth model. Since technology is exogenous in that model, the emphasis in empirical studies based on this model is usually on physical capital accumulation. The second strand of research, inspired by Uzawa (1965) and Lucas (1988), sees economic growth as the result of human capital accumulation. The third strand of research, founded on the contributions of Romer (1990) and Grossman and Helpman (1991), assumes that technological innovations (R&D) by profit-maximizing agents drive economic development.

In this context it is worthwhile mentioning that Funke and Strulik (2000) have moved a step towards a unified endogenous growth model that integrates these separate lines of research on economic growth.<sup>2</sup> The motivation for the modeling framework is the observation that economic growth appears to progress in different phases. Prior to this contribution, human-capital based growth models and R&D based growth models were studied separately. Funke and Strulik (2000) have suggested that a typical advanced economy goes through three endogenously determined development phases: physical capital accumulation, human capital accumulation, and finally R&D. They provide a theory of development

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<sup>1</sup> Subramanian (2011) has computed a dominance index based on a weighted average of a country's GDP, its status as a creditor, and its trade. According to this index, China has already pulled ahead of the US.

<sup>2</sup> Brock and Durlauf (2001) have used the term 'openendedness' to refer to this set of multiple, overlapping theories emphasizing different growth channels such that the truth of one theory does not imply the falsehood of another.

in which the transitional dynamics of the endogenous growth model reproduces this stylized fact and endogenously determines the two critical junctures.

The paper by Funke and Strulik (2000) has triggered a genuine and vivid line of research. Researchers have attempted to identify other sequences of economic development and have presented extended models that improve the fit of the models to data from various countries. In what follows, we briefly touch on the nature of the model extensions. Gómez (2005) has analysed the equilibrium dynamics of the model. Iacopetta (2010) has presented a unified growth model that can generate a trajectory in which R&D precedes human capital accumulation. In a similar vein, Gómez (2011a,b) has proposed an extended growth model incorporating an externality in the R&D sector. By adding this new feature, the model is also able to generate adjustment dynamics in which the innovative stage precedes the human capital formation. Sequeira (2008) has incorporated a human capital erosion effect in the growth model, claiming that this effect improves the model-data fit. The underlying intuition is that some type of human capital depreciates due to new technologies. Sequeira (2010) has added R&D spillover to the framework and has derived the equilibrium and stability properties of such a model. Finally, Iacopetta (2011) has introduced a unified growth model which matches the long-run US data.

The outline of the paper is as follows. The next section gives the underlying unified growth model. Section 3 analyses the properties of the balanced growth path and the dynamics of the model. Section 4 presents some numerical simulations and discusses the adherence of the model to China's growth trajectory. Section 5 concludes.

## 2 Model ingredients and motivation

To fix ideas, this section explains the cornerstones of the endogenous growth model framework in the spirit of Funke and Strulik (2000) and Gómez (2005, 2011a,b) and derives the model's equilibrium and stability properties. We also add imitation and international R&D spillovers to the growth model set-up. Cutting-edge economies make move ahead by inventing new products and techniques, developing and emerging market economies grow by assimilating know-how from elsewhere via FDI. Starting in the 1980s, China has attempted to obtain advanced technology from advanced economies and to use it to

establish domestic innovation capability. Under its so-called “market for technology” policy, China has evolved as the largest recipient of FDI among the developing countries. The hope is that a single model could incorporate all these features and still be tractable.

## 2.1 Consumption and education

We first describe the household optimization problem. Households with an infinite time horizon maximize the intertemporal utility function

$$\int_0^{\infty} \frac{c^{1-\theta}}{1-\theta} e^{-\rho t} dt, \quad (1)$$

where  $C$  denotes consumption of final goods,  $0 < \frac{1}{\theta} < 1$  defines the intertemporal elasticity of substitution and  $\rho > 0$  denotes the time preference rate. The population size is normalized to unity so that all aggregate quantities are per capita quantities. The total labour supply  $H$  can be devoted to production ( $H_Y$ ), education ( $H_H$ ) and innovation ( $H_n$ ):

$$H = H_Y + H_H + H_n \quad (2)$$

Hence, human capital is divided between working, spending time in education and innovating such that the marginal products of the three activities are equal. Human capital accumulates according to

$$\dot{H} = \xi H_H, \quad (3)$$

where  $H$  is total human capital and  $\xi$  is a productivity parameter. A dot above a variable indicates a time derivative. The budget constraint of the household is given by

$$\dot{A} = rA + w(H - H_H) - C, \quad (4)$$

where  $A$  is wealth,  $w$  is the wage per each of the  $H - H_H$  units of employed human capital, and  $r$  is the rate of return on wealth. Let  $g_x$  denote  $x$ 's growth rate,  $g_x = \frac{\dot{x}}{x}$ . Using the control variables  $C > 0$  and  $H_H \geq 0$ , households maximize their intertemporal utility (1), subject to the human capital accumulation technology (3) and the budget constraint (4). The growth rate of consumption is then

$$g_c = \frac{r - \rho}{\theta} \quad (5)$$

and the growth rate of  $w$  is

$$g_w = r - \xi \text{ and } H_H > 0 \quad (6)$$

in an equilibrium with human capital accumulation, or

$$g_w < r - \xi \text{ and } H_H = 0 \quad (7)$$

in an equilibrium without human capital accumulation.

## 2.2 Production and Innovation

In this section we describe the production side of the economy. The underlying framework follows Romer (1990) and Grossman and Helpman (1991). The market for final goods is perfectly competitive and the final good price is normalized to one. The homogeneous final good ( $Y$ ) is produced using physical capital ( $K$ ), intermediate goods ( $D$ ) and human capital ( $H_Y$ ) according to the constant-returns Cobb-Douglas production function

$$Y = BK^\beta D^\eta H_Y^{1-\beta-\eta}, \quad 0 < \beta, \eta, \beta + \eta < 1, B > 0. \quad (8)$$

There is monopolistic competition in the intermediate-goods sector. The index of differentiated intermediate goods ( $D$ ) is given by the CES technology,



$$D = \left[ \int_0^n x(i)^\alpha di \right]^{\frac{1}{\alpha}} \quad (9)$$

where  $x(i)$  is the amount used for each of the  $i \in [0, n]$  intermediate goods and  $0 < \alpha < 1$  governs the substitutability among the varieties. Consequently, the more the intermediate goods, the higher the productivity of a given level of physical capital.

Physical capital is used only in final good production. For simplicity we neglect depreciation, which results in the resource constraint for the economy:  $\dot{K} = Y - C - \int_0^n x_i di$ . Profit maximization gives the factor demands

$$r = \beta \frac{Y}{K} \quad (10)$$

$$w = (1 - \beta - \eta) \frac{Y}{H_Y} \quad (11)$$

$$P_D = \eta \frac{Y}{D} \quad (12)$$

where  $P_D$  represents the price index for intermediate goods,  $r$  is the interest rate and  $w$  is the wage rate.

The innovation pattern in the model follows Romer (1990). Designs for complementary intermediate goods are developed in an R&D sector and  $n$  is used interchangeably for the number of intermediate goods and the stock of knowledge. The stock of intermediate goods is assumed to develop according to

$$\dot{n} = \delta H_n^\gamma (\bar{n})^\phi \bar{H}_n^{\varepsilon-1}, \quad \delta > 0, 0 < \gamma, \varepsilon \leq 1, \gamma + \varepsilon > 1, 0 \leq \phi < 1 \quad (13)$$

where  $\delta$  is an efficiency parameter. Equation (13) has a number of features that merit discussion. First,  $H_n$  is the human capital employed in R&D. In other words, the accumulation of knowledge results from the intentional efforts of R&D employees searching for new intermediate goods. The parameter restriction  $0 < \gamma < 1$  implies that innovation effort is subject to diminishing returns and R&D activities are present when  $\dot{n} > 0$ .

The second term in equation (13) indicates that the change in  $n$  depends on the number of intermediate goods at the world frontier ( $\bar{n}$ ). This assumption has an intuitively appealing interpretation: the refinement captures the idea that one of China's primary objectives in past decades has been to develop its domestic innovative capacity via trade-

induced learning and by bringing in foreign investment and technology. An emerging market economy far from the technology frontier benefits from international technology spillovers via several spillover channels. (i) Chinese firms can learn about products and technologies brought in by FDI, by means of reverse engineering for example. (ii) Inward FDI has a demonstration effect on local R&D activity. By their mere presence in the domestic markets, foreign high-technology products can inspire and stimulate local innovators to develop new products and processes. This shortens the trial-and-error process of local firms in their search for inventions. (iii) Since the products and technologies that FDI firms bring in have already been tested in foreign markets, the perceived risk of innovating along similar directions is lowered for local firms.<sup>3</sup>

The third term in equation (13) represents research externalities. Following Gómez (2011a,b),  $\bar{H}_n$  represents the average time devoted to innovation. This term thus leads to diminishing returns in R&D caused by duplicative research activities.<sup>4</sup> Finally, we assume an exogenous growth rate of  $\bar{n}$  given by

$$g_{\bar{n}} = \frac{\dot{\bar{n}}}{\bar{n}} = \zeta, \quad \zeta > 0 \quad (14)$$

Actually, this implies that the growth path of technology at the frontier is an exponential function,  $\bar{n} = \bar{n}_0 \exp(\zeta t)$  with the initial value  $\bar{n}_0 > 0$ .

Each firm in the R&D sector owns an infinite patent for selling its variety  $x(i)$ . Firms act under monopolistic competition and maximize operating profits  $\pi(i) = (p(i) - 1)x(i)$ ,  $i \in [0, n]$ . Facing the price elasticity of demand  $\epsilon = \frac{1}{1-\alpha}$ , each firm charges a price  $p = p(i) = \frac{1}{\alpha}$ . With identical technologies and symmetric demand, the quantity supplied is the same for all goods,  $x(i) = x$ ,  $p(i) = p$ . Hence, from  $DP_D = pxn$  and  $D = xn^{\frac{1}{\alpha}}$  we obtain the market-clearing quantity  $X = xn = \alpha\eta Y$ . In turn, profits can be easily re-written as

$$\pi = \frac{(1-\alpha)\eta Y}{n} \quad (15)$$

<sup>3</sup> The interested reader may wish to turn to Blomström and Kokko (1998) and Moran et al. (2005) for reviews of empirical studies examining FDI spillover effects.

<sup>4</sup> The original idea of an erosion effect in human capital appeared in Galor and Moav (2000) and Galor and Weil (2000).

Substituting the expressions  $D = xn^{\frac{1}{\alpha}}$  and  $xn = \alpha\eta Y$  into the production function (8) yields

$$Y^{1-\eta} = B(\alpha\eta)^\eta K^\beta n^{\frac{(1-\alpha)\eta}{\alpha}} H_Y^{1-\beta-\eta} \quad (16)$$

It is evident that  $Y$  is proportional to  $n$ , the number of intermediate good varieties. Consequently, the larger the number of differentiated intermediate goods, the higher the productivity of a given capital stock  $K$ . Another characteristic evident from equation (15) is that the larger the market size for an intermediate variety, the higher the profit and hence the greater the incentive to innovate.

Let  $v$  denote the expected stream of monopoly profits from innovation, defined by  $v = \int_t^\infty e^{-[R(\tau)-R(t)]} \pi(\tau) d\tau$ , where  $R(t) = \int_0^t r(\tau) d\tau$ . Differentiating this expression with respect to time yields the usual no-arbitrage condition:

$$g_v \equiv \frac{\dot{v}}{v} = r - \frac{\pi}{v} \quad (17)$$

Equation (17) implies that the rate of return is given by a dividend rate  $\frac{\pi}{v}$  plus the capital gain  $\frac{\dot{v}}{v}$ . Finally, free entry into the R&D sector requires  $wH_n dt \geq vdn$  in general equilibrium. Given equation (13), this condition can be expressed as

$$w = v\delta H_n^{\gamma-1} (\bar{n})^\phi \bar{H}_n^{\varepsilon-1} \quad (18)$$

for nonnegative innovation, i.e.  $H_n > 0$ . Aggregate wealth in the economy is  $A = K + nv$ . Combining equations (4), (10) - (13), (15) and (17) we obtain the economy-wide resource constraint,

$$\dot{K} = (1 - \alpha\eta)Y - C \quad (19)$$

which gives the growth rate of  $K$  as

$$g_K \equiv \frac{\dot{K}}{K} = (1 - \alpha\eta) \frac{Y}{K} - \frac{C}{K} \quad (20)$$

Let  $\chi \equiv \frac{C}{K}$  represent the consumption-capital ratio. From equations (5), (10) and (20) we obtain

$$g_K = (1 - \alpha\eta) \frac{r}{\beta} - \chi \quad (21)$$

$$g_\chi = \left( \frac{1}{\theta} - \frac{1 - \alpha\eta}{\beta} \right) r + \chi - \frac{\rho}{\theta} \quad (22)$$

Before we proceed with the analysis, we set out some equations that will be useful for establishing the dynamics of different stages of economic development. First, let  $u_Y \equiv \frac{H_Y}{H}$ ,  $u_H \equiv \frac{H_H}{H}$ ,  $u_n \equiv \frac{H_n}{H}$  denote the fraction of labour in production, education and innovation, respectively. Then, taking logarithms and differentiating the expression for  $r$  in equation (10) with respect to time, and given the above relations for  $w$  in equation (11) and  $Y$  in equation (16), we obtain

$$g_r = g_Y - g_K \quad (23)$$

$$g_w = g_Y - g_{H_Y} = g_Y - (g_{u_Y} + g_H) \quad (24)$$

$$(1 - \eta)g_Y = \beta g_K + \frac{(1 - \alpha)\eta}{\alpha} g_n + (1 - \beta - \eta)(g_{u_Y} + g_H) \quad (25)$$

Substituting and simplifying yields

$$g_r = -\frac{1 - \beta - \eta}{\beta} g_w + \frac{\eta}{\beta} \frac{1 - \alpha}{\alpha} g_n \quad (26)$$

$$g_{u_Y} = -\frac{1 - \eta}{\beta} g_w + \frac{\eta}{\beta} \frac{1 - \alpha}{\alpha} g_n + g_K - g_H \quad (27)$$

Finally, log-differentiation of equation (13) yields

$$g_{g_n} = \phi\zeta + (\gamma + \varepsilon - 1)(g_{u_n} + g_H) - g_n \quad (28)$$

To elaborate the model further, we set out the differential equations that describe the dynamics of the economy.

### 3 Solution of the model

We first analyze the dynamics of an advanced industrialized economy with physical capital accumulation ( $u_Y > 0$ ), human capital accumulation ( $u_H > 0$ ) and innovation ( $u_n > 0$ ). After that, we consider the intermediate development phases of an evolving economy.

#### 3.1 The cutting-edge industrialized economy

The dynamics of the advanced industrialized economy can be determined as follows. Substituting equation (6) into equation (26) yields

$$g_r = -\frac{1-\beta-\eta}{\beta}(r-\xi) + \frac{\eta}{\beta}\frac{1-\alpha}{\alpha}g_n \quad (29)$$

From equation (22) we have the consumption-capital ratio  $\chi$

$$g_\chi = \left(\frac{1}{\theta} - \frac{1-\alpha\eta}{\beta}\right)r + \chi - \frac{\rho}{\theta} \quad (30)$$

Defining  $\psi \equiv \frac{H^{\gamma+\varepsilon-1}\bar{n}^\phi}{n}$ , using equation (13) and  $u_n \equiv \frac{H_n}{H}$ , the labour share in the R&D sector is equivalent to

$$u_n = \left(\frac{g_n}{\delta\psi}\right)^{1/(\gamma+\varepsilon-1)} \quad (31)$$

Log-differentiating the expression for  $\psi$ , substituting equation (3) and noting that  $u_Y + u_H + u_n = 1$  yields the growth rate of  $\psi$ :

$$g_\psi = (\gamma + \varepsilon - 1)\xi \left( 1 - u_Y - \left( \frac{g_n}{\delta\psi} \right)^{\frac{1}{\gamma+\varepsilon-1}} \right) - g_n + \phi\zeta \quad (32)$$

From equations (27), (6), (21) and (3) we obtain

$$g_{u_Y} = \frac{(1-\alpha)\eta}{\beta} r - \chi + \frac{\eta}{\beta} \frac{1-\alpha}{\alpha} g_n - \xi \left( 1 - u_Y - \left( \frac{g_n}{\delta\psi} \right)^{1/(\gamma+\varepsilon-1)} \right) + \frac{(1-\eta)\xi}{\beta}. \quad (33)$$

To solve for  $g_n$ , we first log-differentiate the free-entry condition (18) to get  $g_w = g_v + (\gamma + \varepsilon - 2)g_{H_n} + \phi\zeta$ . Substituting  $g_v$  from equation (17),  $\pi$  from equation (15),  $w$  from equation (11) and  $v$  from equation (18) yields

$$g_w = r - \frac{(1-\alpha)\eta u_Y}{(1-\beta-\eta)u_n} g_n + (\gamma + \varepsilon - 2)(g_{u_n} + g_H) + \phi\zeta. \quad (34)$$

Solving for  $g_n$  and using equations (28), (34), (6) and (31), we obtain

$$g_{g_n} = \phi\zeta - \frac{\gamma+\varepsilon-1}{\gamma+\varepsilon-2} (\xi + \phi\zeta) + \frac{(\gamma+\varepsilon-1)(1-\alpha)\eta(\delta\psi)^{\frac{1}{\gamma+\varepsilon-1}} u_Y}{(\gamma+\varepsilon-2)(1-\beta-\eta)} g_n^{1-\frac{1}{\gamma+\varepsilon-1}} - g_n. \quad (35)$$

All in all, we have derived the dynamics of the economy characterized by the growth rates of the variables  $r$ ,  $\chi$ ,  $\psi$ ,  $u_Y$  and  $g_n$ .

### 3.2 The ‘‘Solovian’’ growth phase

In terms of the variables  $r$  and  $\chi$ , the dynamics of the initial neoclassical growth phase characterized by  $u_H = u_n = 0$  is given by the two-dimensional differential equation system

$$g_r = -\frac{1-\beta-\eta}{\beta} \frac{1-\alpha\eta}{1-\eta} r + \frac{1-\beta-\eta}{1-\eta} \chi \quad (36)$$

$$g_\chi = \left( \frac{1}{\theta} - \frac{1-\alpha\eta}{\beta} \right) r + \chi - \frac{\rho}{\theta}. \quad (37)$$

The growth rate of  $r$  results from (29) and (30) and using (21) as well as  $g_{u_Y} = g_n = g_H = 0$ .

### 3.3 The dynamics of the possible intermediate phases

The dynamics of the “education only” intermediate case characterized by  $u_H = 1 - u_Y > 0$  and  $u_n = 0$ ) is described by the three-dimensional differential equation system

$$g_r = -\frac{1-\beta-\eta}{\beta}(r-\xi) \quad (38)$$

$$g_\chi = \left(\frac{1}{\theta} - \frac{1-\alpha\eta}{\beta}\right)r + \chi - \frac{\rho}{\theta} \quad (39)$$

$$g_{u_Y} = \frac{(1-\alpha)\eta}{\beta}r - \chi - \xi(1-u_Y) + \frac{(1-\eta)\xi}{\beta} \quad (40)$$

which is obtained from equations (3), (6), (21) - (22) and (26) - (27). On the other hand, the dynamics of the “innovation only” case characterized by  $u_n = 1 - u_Y > 0$  and  $u_H = 0$ , is described by the four-dimensional differential equation system

$$g_r = -\frac{1-\beta-\eta}{\beta}r + \frac{(1-\alpha)\eta(\alpha u_Y + 1 - u_Y)}{\alpha\beta(1-u_Y)^{2-\gamma-\varepsilon}}\delta\psi - \frac{(1-\beta-\eta)(2-\gamma-\varepsilon)u_Y}{\beta(1-u_Y)}g_{u_Y} - \frac{\phi\zeta(1-\beta-\eta)}{\beta} \quad (41)$$

$$g_\chi = \left(\frac{1}{\theta} - \frac{1-\alpha\eta}{\beta}\right)r + \chi - \frac{\rho}{\theta} \quad (42)$$

$$g_{u_Y} = \frac{(1-\alpha)\eta(1-u_Y)}{\beta(1-u_Y) + (1-\eta)(2-\gamma-\varepsilon)u_Y} \times \left\{ r - \frac{\beta}{(1-\alpha)\eta}\chi + \left[ \frac{(1-\eta)u_Y}{(1-\beta-\eta)(1-u_Y)} + \frac{1}{\alpha} \right] \delta(1-u_Y)^{\gamma+\varepsilon-1}\psi - \frac{(1-\eta)\phi\zeta}{(1-\alpha)\eta} \right\} \quad (43)$$

$$g_\psi = \phi\zeta - [\delta(1-u_Y)^{\gamma+\varepsilon-1}\psi] \quad (44)$$

Here, (41) - (44) result from equations (22), (26) - (27) and (34) using (21) and (13). In the next section we lay down the properties of the balanced growth path.

### 3.4 The balanced growth equilibrium and its stability

This section lays down the existence and the properties of the balanced growth path [see Gomez (2005, 2011a,b)]. The following two propositions give our main results, which characterize the balanced growth path.

**Proposition 1:** Let  $\xi > \rho$ . The economy has a unique positive steady state  $(r^*, \chi^*, g_n^*, \psi^*, u_Y^*)$  given by:

$$r^* = \frac{(M+1)\theta\xi - \rho + \Lambda}{(M+1)\theta - 1} \quad (45)$$

$$\chi^* = \left( \frac{1-\alpha\eta}{\beta} - \frac{1}{\theta} \right) r^* + \frac{\rho}{\theta} \quad (46)$$

$$g_n^* = \frac{WM(\xi - \rho + \Lambda)}{(M+1)\theta - 1} \quad (47)$$

$$\psi^* = \frac{g_n^*}{\delta(u_n^*)^{\gamma+\varepsilon-1}} \quad (48)$$

$$u_Y^* = 1 - u_n^* - \frac{g_H^*}{\xi} \quad (49)$$

where  $u_n^*$  is the steady-state share of labour in R&D sector

$$u_n^* = \left( 1 - \frac{g_H^*}{\xi} \right) \frac{(1-\alpha)\eta g_n^*}{(1-\alpha)\eta g_n^* + [\xi + (\gamma+\varepsilon-2)g_H^* + \phi\zeta](1-\beta-\eta)} \quad (50)$$

The constants  $\Lambda$ ,  $W$  and  $M$  are defined as follows:

$$\Lambda = \frac{\phi\zeta\theta}{\gamma+\varepsilon-1} \quad (51)$$

$$W = \gamma + \varepsilon - 1 \quad (52)$$

$$M = \frac{\alpha(1-\eta-\beta)}{(1-\alpha)\eta W} \quad (53)$$



Furthermore, we obtain the long-run growth rates of human capital ( $g_H^*$ ), income ( $g_Y^*$ ), consumption ( $g_C^*$ ) and physical capital ( $g_K^*$ ):

$$g_H^* = \frac{g_n^* - \phi\zeta}{\gamma + \varepsilon - 1} = \frac{g_n^* - \phi\zeta}{W} \quad (54)$$

$$g_Y^* = g_C^* = g_K^* = \left(1 + \frac{1}{M}\right) g_H^* + \frac{\phi\zeta}{MW} . \quad (55)$$

The system reaches the balanced growth path if and only if the following boundary condition is fulfilled.

$$\theta > \frac{1 + M\left(1 - \frac{\rho}{\xi}\right) + \Omega}{(M+1) + \Omega} \quad \text{with } \Omega = \frac{\phi\zeta}{(\gamma + \varepsilon - 1)\xi} \quad (56)$$

**Proof of Proposition 1:** See Appendix A.

We now analyze stability in the neighborhood of the balanced growth path.

**Proposition 2:** Assume that condition (56) in Proposition 1 holds.

- (i) The steady state equilibrium is either saddle-path stable or unstable.
- (ii) A sufficient condition to rule out instability is

$$\alpha\beta \geq (1 - \alpha)\eta W \quad (57)$$

**Proof of Proposition 2:** See Appendix B.

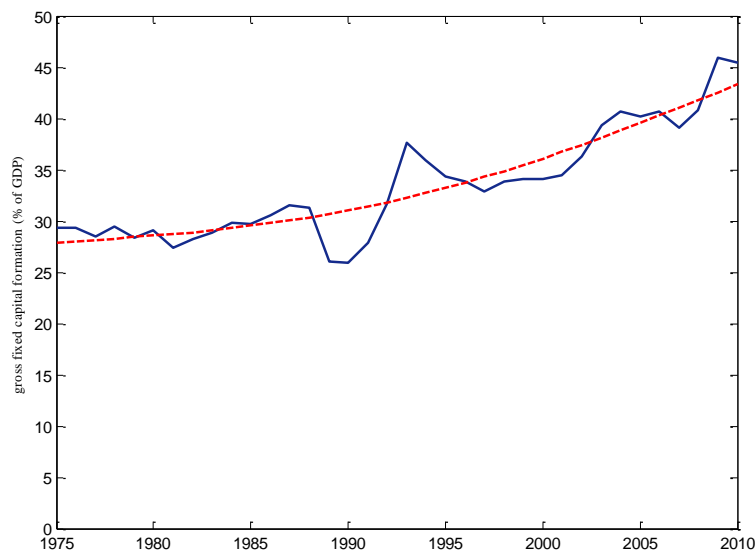
Whether the above growth model is able to explain China's growth performance since 1980 is an empirical question, to which we turn in the next section.

## 4 Calibration and transition dynamics

To examine whether the above modeling results seem to be appropriate for China, we present different indicators for physical capital accumulation, human capital accumulation and innovation in China. Using Chinese data for the years 1980-2010, we attempt to identify the points of transition from the neoclassical phase to the knowledge formation phase and eventually to the fully developed economy with R&D.

Before calibrating, we briefly set out some facts about economic growth in China in the past decades. In 1979, after years of state control of all productive assets, the government embarked on a major program of economic reform.<sup>5</sup> Post-1979 China has witnessed average real growth of 8-10 percent per year. Next we take a brief look at China's growth drivers.

Figure 1 Gross fixed capital formation, % of GDP



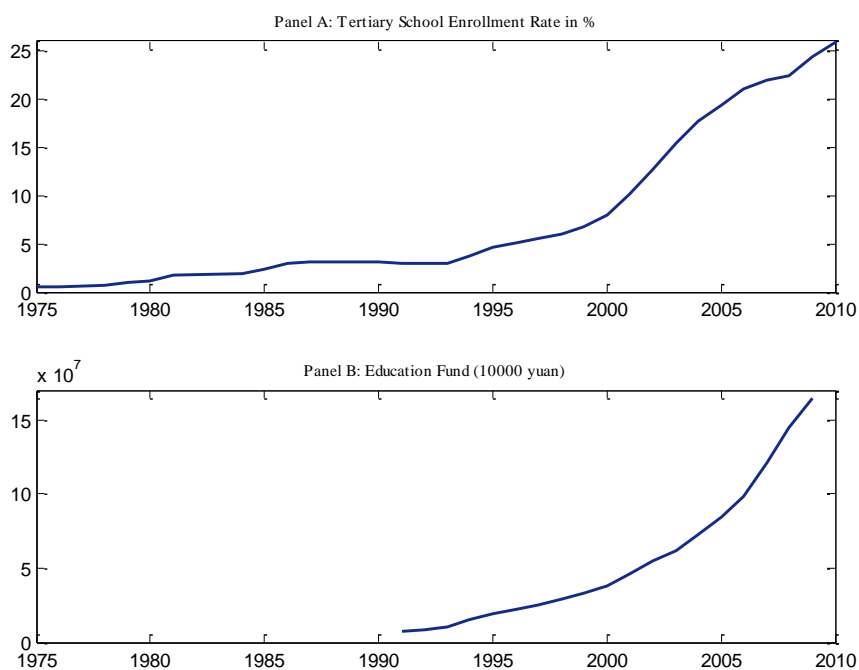
Note: Gross fixed capital formation consists of outlays on additions to the fixed assets of the economy plus net change in inventories. The dashed line shows the trend. Data Source: World Bank WDI Database.

In Figure 1, we report the historical evolution of gross fixed capital formation in percent of GDP. There are two features that draw our attention. First, China already had a very high

<sup>5</sup> Our simulations of the Chinese economy start in 1980 because the model does not explain the radical institutional changes in China's economic institutions at the end of the 1970s. This critical juncture has shaped the incentives to work and invest in technology and physical and human capital [Acemoglu et al. (2001, 2002, 2005)]. The institutional change at the end of the 1970s shows that history is not destiny.

investment rate at the start of the reform process. The average investment share in GDP was already over 25 percent in 1980. The high investment rates, supported by high saving rates, allowed rapid capital accumulation already at the onset of the reform process. The second notable feature is the ever-rising trend, reaching 45 percent by the end of 2010. The overall impression is that China has broadened its production base through massive investment and an enlarged production base since 1980. The figure lends some support to the view that productivity improvements via physical capital accumulation were more important than those via human capital and R&D in the earlier stages of the Chinese transition process. With capital spending subject to decreasing returns, the scope for further growth via capital spending is gradually fading away.<sup>6</sup> Figure 2 shows the time-varying investment in human capital.

Figure 2 China's spending on education and tertiary enrollment rates, 1975-2010



**Note:** Tertiary education, whether or not to the level of advanced research, normally requires for admission at least successful completion of education at the secondary level. The total education funds (10000 RMB) contain five components: the respective government spending for education, funds from private schools, donations and fund-raising for running schools, income from teaching, research and other auxiliary activity and other educational funds. **Data Sources:** World Bank and National Bureau of Statistics and China Statistical Yearbooks.

<sup>6</sup> This is evident from the upward trend in China's incremental capital-output ratio since the 1990s, indicating a decline in investment efficiency. The incremental capital-output ratio is calculated by dividing the ratio of investment to GDP (in %) by the annual increase in GDP (in %). In his cautionary tale of the mid 1990s, Krugman (1994) emphasized that the scope for raising growth through ever-larger injections of physical capital will be rapidly exhausted.

Panel A of Figure 2 presents the tertiary school enrollment rate in percent, which serves as an indicator of human capital. The evidence indicates an expansion in tertiary enrollment only since the mid 1990s and therefore about 15 years after China's catch-up began. This suggests that the relative importance of productivity improvements via education has increased in the 1990s. The evidence is based on data on enrollment rates and not on more subtle measures of human capital. However, Zhang and Zhuang (2011) have convincingly demonstrated that tertiary education has a strong impact on growth although it is not a perfect measure of learning and does not include all relevant effects. A similar picture emerges when considering total education expenses in Panel B of Figure 2.

As mentioned above, another endogenous regime shift takes place in the growth model once R&D becomes profitable for a certain threshold of human capital.<sup>7</sup> Figure 3 therefore presents several measures of innovation. The difficulties of disentangling innovations are well known. Therefore it isn't surprising that contributions differ widely on how innovations are identified. In our study high-technology exports and patent applications are used to proxy technological innovations. It is important to stress that these measures, though crude, are informative.

As experienced by many countries in the process of economic development, the emphasis in China's exports has shifted from primary products to industrial products and among the latter from labor-intensive to capital- and technology-intensive products - as seen in Panel A of Figure 3. As a consequence, there is now a wide assortment of products that have been technologically upgraded and from which Chinese firms can diversify into further higher value-added products.<sup>8</sup>

Panel B indicates that China's R&D expenditures have been on a steep upward trend since 1990. This has increased the production of ideas and laid the ground for technology adaptation, assimilation and incremental innovations.

Panels C and D of Figure 3 chart the numbers of Chinese patent applications since 1985. Patents are an important venue for introducing new products and are widely used as

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<sup>7</sup> It should be stressed that the modelling framework spotlights the close interrelation between human capital accumulation and innovation. A human capital-led growth strategy lays the foundation for an expansion of innovation capacity in the medium and long run.

<sup>8</sup> This change in the Chinese export composition follows the so-called flying geese pattern of industrial development, whereby countries specialize in the export of products in which they enjoy a comparative advantage commensurate with the level of development, while at the same time seeking to upgrade their technology to higher value-added products via inward FDI.

an indicator of innovation and diffusion.<sup>9</sup> Another reason is that most studies find a very strong relationship between R&D and patent applications in cross-sectional data: the median  $R^2$  is around 0.9.<sup>10</sup> China's patent system is evolving fast, and enforcement though lagging has improved over time. China's Patent Law went into effect on 1 April 1985. Generally, any technology to be patented must pass four tests: it must be novel, useful, non-obvious and man-made. The law grants three types of patents: invention, utility model and design patents. Applications for invention patents are more rigorously scrutinized for novelty, and non-obviousness before the patents can be granted. Invention patents receive 20 years of protection, in line with the global standard. On the contrary, the utility and design patents generally cover more incremental innovations and are not subject to examination for novelty and inventiveness. Though the details of the various panels are all important and interesting, the more important lessons come from the big picture, which reveals that China started to climb the technology and export sophistication ladder around the year 2000. This timing coincides with the implementation of the Chinese export promotion strategy "Revitalizing Trade through Science and Technology", which steered the Chinese economy towards trade-induced learning and export-oriented growth.<sup>11</sup>

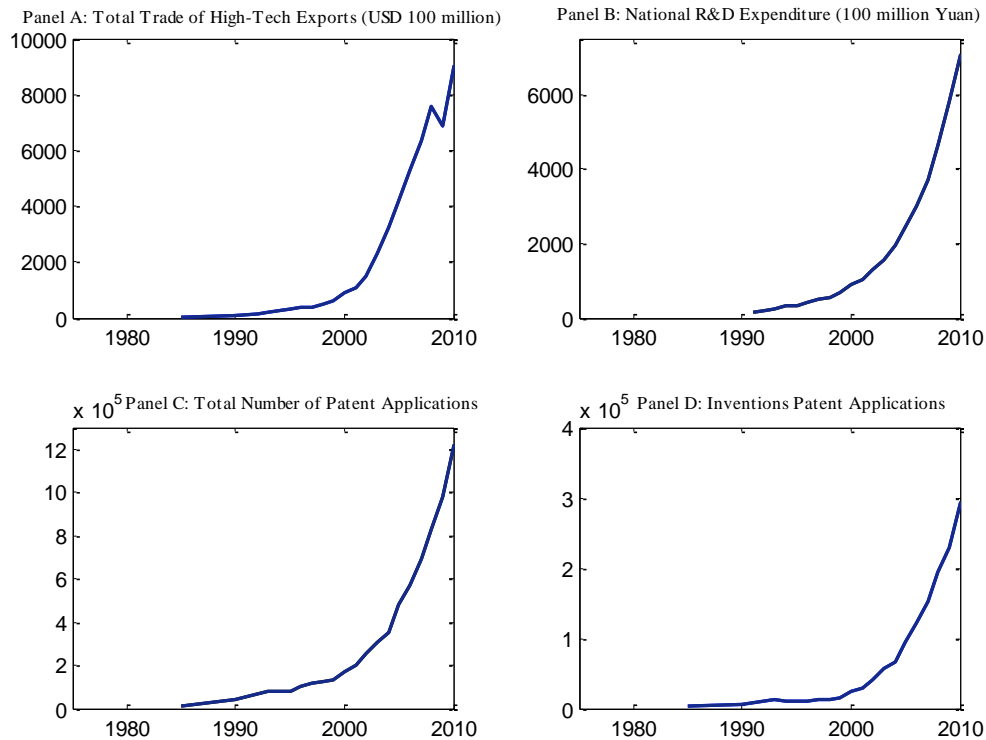
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<sup>9</sup> One might be inclined to think that the number of patent filings is only a sketchy indicator of a country's ability to generate ideas. That is because the usefulness of each patent can vary widely, nor are all invention patents successful commercially. A good reference for the caveats of using patent data as a measure of innovation is Griliches (1990).

<sup>10</sup> See, for example, Griliches (1990) and Hausman et al. (1986) for the international evidence.

<sup>11</sup> A closer inspection of China's export basket since 2000 underscores the higher degree of product sophistication, which creates options for further diversification and innovation. See Felipe et al. (2010, 2012). Rodrik's (2006) estimates reveal that China's export sophistication is exceptionally high. During a transition period, "original" Chinese innovations based on technological breakthroughs may not be as common as second-generation innovations that combine different existing technologies or customize technologies for specific markets. Second generation innovations provide broad scope for advancement because the size of world markets allows for a finer division of labour and greater benefits from specialization. For a sceptical view arguing that current trade statistics are misleading and inflate China's high-technology exports, see Xing (2012).

Figure 3 Chinese innovation activities and successes, 1970-2010



**Note:** Panel A shows Chinese exports of high-tech products from 1985 to 2010. China's statistics in high-technology trade are divided into nine product categories: Computers and telecommunications, life science technologies, electronics, computer-integrated manufacturing, aerospace, optical electronics, biotechnology, materials, and others. Panel B shows Chinese R&D expenditures (in 100 million RMB) from 1991 to 2010. Panel C plots the total numbers of Chinese patent applications between 1985 and 2009. Panel D presents a time series for the evolution of invention patent applications since 1985. **Data Sources:** National Bureau of Statistics, China Statistical Yearbooks and China Science and Technological Statistics.

Figures 1-3 shed light on the broad development patterns unfolding in China. In what follows we calibrate the model to match the Chinese economy. In other words, we will interpret the facts presented above through the lens of the model.

Calibration of a model makes it empirically relevant by ensuring that it captures the key characteristics of the relevant data.<sup>12</sup> Table 1 gives the benchmark parameter values for China as well as the associated steady-state values and eigenvalues.

<sup>12</sup> It should be pointed out that the model simulations are meant to describe actual behavior of the Chinese economy. In other words, they should be considered in a positive sense and need not entail a normative interpretation.

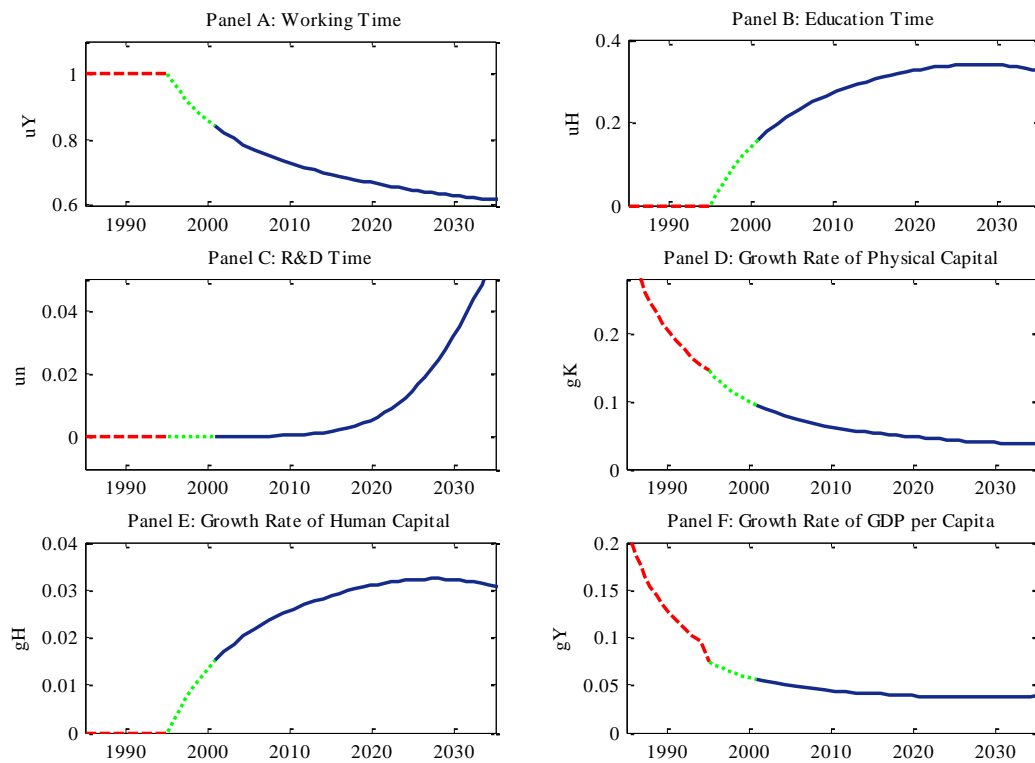
Table 1 Benchmark parameter values, steady-state values and eigenvalues

Parameter Values						
$\alpha$	$\beta$	$\eta$	$\rho$	$\theta$	$\xi$	$\delta$
0.62	0.40	0.36	0.02	2.00	0.095	0.10
$\gamma$	$\varepsilon$	$\phi$	$\zeta$	$n_0$	$B$	
0.90	1.00	0.35	0.021	1.00	1.00	
Steady-State Values						
$u_Y^*$	$u_H^*$	$u_n^*$	$g_Y^*$	$g_H^*$	$g_n^*$	
0.6602	0.2363	0.1036	0.0478	0.0224	0.0276	
Eigenvalues						
0.0726	$0.1897 \pm 0.0733i$	$-0.0965 \pm 0.0593i$				

As shown in Section 3, the dynamic stability of the system depends on the number of negative eigenvalues of the Jacobian matrix. The evolution of our model is described by five variables, two of which are considered to be predetermined. A necessary condition for saddle-path stability is the existence of two stable roots. It is clear from Table 1 that two roots of the Jacobian matrix are negative and therefore the model is saddle-path stable. Given that there are two stable roots in the system, the stable manifold is two-dimensional. Figure 4 maps the transition dynamics of the model for plausible calibration values, computed via the backward integration algorithm.<sup>13</sup> Panels A - C illustrate the resulting time series of the share of labour in production ( $u_Y$ ), education ( $u_H$ ) and innovation ( $u_n$ ). Panels D - E give the growth rates of physical capital ( $g_K$ ), human capital ( $g_H$ ), and GDP ( $g_Y$ ), respectively. The different growth phases are highlighted in color and described in detail below.

<sup>13</sup> Skipping, for the sake of brevity, all technical details, the interested reader is referred to Brunner and Strulik (2002) for an exposition of the backward integration technique.

Figure 4 The calibrated transition dynamics of the Chinese economy



Note: The dashed red line corresponds to the neoclassical “Solovian” economy, the dotted green dashed line to the phase with human capital accumulation, and the solid blue line corresponds to the fully industrialized economy with R&D.

Several points are worth mentioning: First, from 1980 to 1995 the model predicts that physical capital was accumulated as the only growth-generating mechanism. In other words, as long as human capital remained below a certain threshold level, the Chinese economy resembled that of the “Solovian” growth model with physical capital accumulation solely ( $u_H = u_n = 0$ ). Second, an endogenous regime shift was triggered once human capital reached the threshold level. In the mid 1990s the Chinese economy entered the human capital accumulation phase, with education time ( $u_H$ ) increasing steadily. While the level of human capital was low, consumption and the demand for intermediates were low, implying that profits from domestic innovation were low. This depressed the return on domestic R&D, and thus households and firms gained higher rates of return by investing solely in human capital. Consequently, domestic R&D was unprofitable and innovation activities were absent in the second development phase ( $u_n = 0$ ). Third, over time the size of intermediate good markets and thereby the expected returns on R&D investments in-



creased. Finally in 2000, after China entered the ranks of middle-income countries, domestic R&D became profitable and the Chinese economy moved into the third development phase with R&D gaining increasing in significance ( $u_n > 0$ ).<sup>14</sup> From that moment, technology adaption and absorption supplemented by incremental innovation increasingly contributed to economic growth. Finally, the model simulations indicate an initial high-growth take-off of the Chinese economy followed by monotonic growth-slowing to the steady state growth rate  $g_Y^* = 4.78$  percent.<sup>15</sup> The underlying mechanism is that the more an emerging economy resembles the advanced economies, the harder it is to sustain the pace. As the stock of borrowable ideas runs low, the developing economy must begin innovating for itself. The predicted slowing of growth is consistent with the “middle-income trap” suggested in the multi-country review of growth performance by Eichengreen et al. (2011). What emerges from their estimates is a critical threshold: on average, growth slowdowns occur when GDP per capita reaches around USD 16740 at PPP. It is hazardous to extend such analysis to a country as unique as China. Bearing this in mind, China is expected to reach that trigger point in the near future.<sup>16</sup> Eichengreen et al. (2011) project China to grow by about 6 percent per year in the 2011 - 2020 decade and about 5 percent in the subsequent decade 2021 - 2030. Lee and Hong (2010) forecast average Chinese growth over the period 2011 - 2030 at 5.5 percent in their baseline scenario.

Figure 4 shows that despite the presence of complex stable roots, adjustments along the growth trajectory are more or less monotonic. This stems from the rather low magnitude of the imaginary part of the stable roots and agrees with the Chinese data. Last but not least, for the sake of comparison, the actual and simulated series are highlighted in Figures 5 and 6.

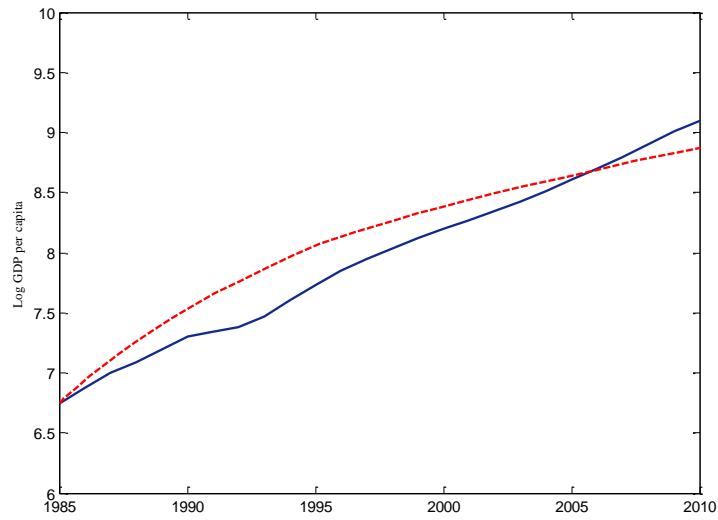
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<sup>14</sup> The idea that market size is important for economic development goes back to Murphy et al. (1989a,b). The underlying framework in these complimentary papers is that producers decide whether to adopt modern increasing returns technologies or to use standard constant-returns-to-scale technologies. The implementation of the advanced technology requires a sufficiently large market. Two outcomes are possible in this modelling framework: an underdeveloped economy with low per capita income or an advanced economy with high per capita income.

<sup>15</sup> According to Worldbank (2012), pp. 85-86, an average annual real GDP growth rate in the range of 4.0 - 6.2 percent is required for China to reach the status of a “high income” country by 2030. The model calibrations indicate that this is achievable.

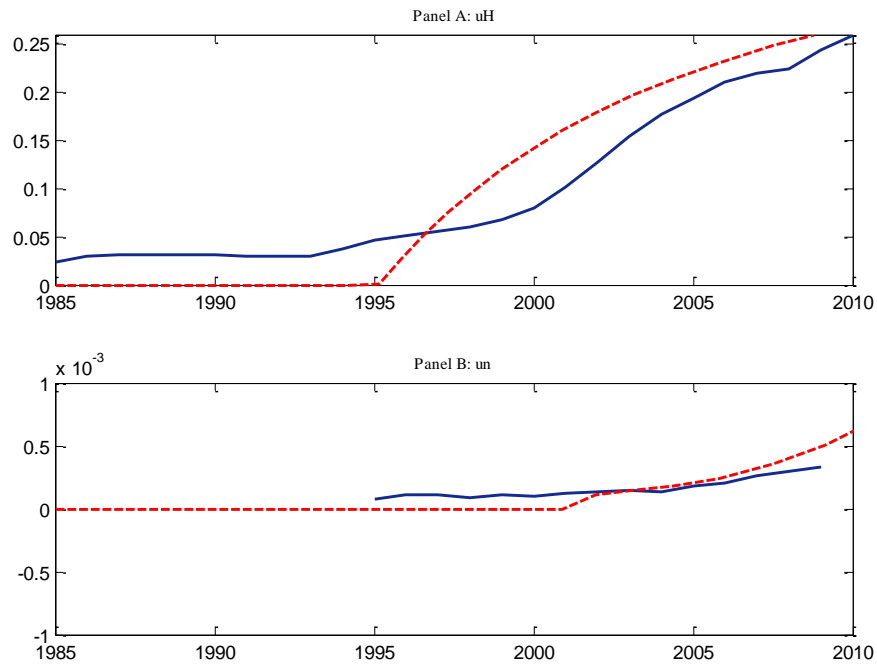
<sup>16</sup> The authors also tested which variables significantly influence the probability of a slowdown. They find that trade openness delays a slowdown, and they attribute the anomalous performance of places like Hong Kong and Singapore to this factor. Factors that bring forward the moment of growth-slowing include a high old-age dependency ratio, an undervalued currency, and a low consumption share in GDP. China suffers from all of these. The latter points will expose China’s future growth to heightened risk.

Figure 6 Actual and calibrated logged Chinese GDP per capita, 1985-2010



Note: The solid blue line gives actual Chinese  $\ln(\text{GDP per Capita})$ , the dashed red line the simulated trajectory.

Figure 5 Actual and calibrated  $u_H$  and  $u_n$ , 1985-2010



Note: The blue lines give the actual  $u_H$  and  $u_n$  series from Figures 2 and 3, respectively. The dashed red lines are the corresponding simulated series.

Overall, the allocation of time from production to education and innovation and the associated regime shifts in Figure 4 accord with the descriptive facts in Figures 1-3. Thus the model matches, in broad brushstrokes, the two trigger points outlined above and thereby provides an intuitive endogenous mechanism to explain the sequencing of the Chinese growth phases.<sup>17</sup> The calibration results also confirm the view that China has reallocated resources along the lines of comparative advantage during transition.

## 5 Conclusions

The Chinese economy has been growing at a rate of about 8-10 percent annually since the launch of reform at the end of the 1970s. Given China's increasing impact on the world economy, due to its sheer size and rapid growth, an in-depth assessment and explanation of China's long-run growth performance is a worthwhile research project in terms of theory and matching empirical data. Furthermore, a better understanding of the process that determines economic growth in China may enable an informed assessment of potential future growth of the Chinese economy, which has received much attention in the literature.

Do the model calibrations support the view that the unified growth model described above can account for the broad Chinese development patterns? Even though the question is still very much open, we consider our numerical results as suggestive evidence that the growth model is a productive avenue for understanding the interaction between physical capital accumulation, human capital accumulation, innovation and economic growth. Despite some caveats, which provide scope for future research, we believe that the fundamental intuition behind our stylized model is plausible and convincing. On the other hand, we would not claim that this model resolves all puzzles about China's remarkable growth trajectory.<sup>18</sup> However, our claim - at least our hope - is that the framework is em-

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<sup>17</sup> Gómez (2011a,b) and Iacopetta (2010) have argued that Britain experienced an innovation-education sequence during the Industrial Revolution in the 19<sup>th</sup> century. They have presented trajectories which accord with the observation that the rise in formal education followed, with a considerable lag, the process of industrialization. It is important to point out that our modelling framework here is also able to replicate this British experience in the mid 19<sup>th</sup> century. Corresponding calibration results are available upon request.

<sup>18</sup> The numerical results make a compelling argument and lend support to the model framework, albeit more needs to be said about the major drivers of Chinese economic growth. What the modelling framework does not deny, though largely ignores, is that institutional and political factors have a considerable impact on the economic growth trajectory. Thus taking into account institutional factors and digging deeper into the micro-

pirically relevant and a productive tool for confronting some key facts of China's growth performance.

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foundations of innovations may produce a more nuanced picture of the forces that have fuelled China's rapid growth.

## Appendix A: Proof of proposition 1

From equation (5) in the text we have  $g_c^* = \frac{r^* - \rho}{\theta}$  and thus  $r$  is a constant on the balanced growth path. Substituting  $g_r^* = 0$  into equation (10) yields  $g_Y^* = g_K^*$ . From equation (30) we have  $g_x^* = 0$ , which implies  $g_c^* = g_K^*$ . Thus we obtain  $g_Y^* = g_c^* = g_K^*$ . Evaluating equations (28), (30) and (3) at the steady state, we obtain equations (54), (46) and (49). Log-differentiating equation (16) with respect to time and using equation (54), we obtain equation (55). Using equations (6) and (29), we obtain  $g_n^* = \frac{\alpha(1-\beta-\eta)}{(1-\alpha)\eta} (r^* - \xi)$ . Taking equations (5), (54) and (55) and collecting terms yields equations (45) and (47). Equations (48) and (50) are derived from equations (13), (6), and (34), respectively. The transversality condition associated with  $H$  is equivalent to  $-\xi + g_H^* < 0$ . From the transversality condition for aggregate wealth  $A$ , we have  $-r^* + g_K^* < 0$ , which can be rewritten as  $(\theta - 1)r^* + \rho > 0$ . Thus

$$\begin{aligned}
 & (\theta - 1)r^* > -\rho \\
 & (\theta - 1) \frac{(M + 1)\theta\xi + \Lambda - \rho}{(M + 1)\theta - 1} > -\rho \\
 & [(M + 1)\xi + \frac{\phi\zeta}{\gamma + \varepsilon - 1}]\theta^2 - \left[ \xi + M\xi - \rho M + \frac{\phi\zeta}{\gamma + \varepsilon - 1} \right] \theta > 0 \\
 & \theta > \frac{\xi + M(\xi - \rho) + \frac{\phi\zeta}{\gamma + \varepsilon - 1}}{(M + 1)\xi + \frac{\phi\zeta}{\gamma + \varepsilon - 1}} \\
 & \theta > \frac{1 + M(1 - \frac{\rho}{\xi}) + \Omega}{(M + 1) + \Omega} \text{ with } \Omega = \frac{\phi\zeta}{(\gamma + \varepsilon - 1)\xi} .
 \end{aligned}$$

For the interior steady state to be feasible, we must have  $u_Y^* > 0$ ,  $u_n^* > 0$ ,  $r^* > 0$ ,  $\chi^* > 0$ ,  $\psi^* > 0$  and  $u_n^* + u_Y^* < 1$ . Equation (49) implies  $0 < u_Y^* + u_n^* = 1 - \frac{g_H^*}{\xi} = \frac{\xi - g_H^*}{\xi} < 1$ , if condition (56) holds. Using equation (56) and assuming  $\xi > \rho$  we have  $r^* > 0$  and  $g_n^* > 0$ . On the other hand, we know that  $g_n^* = \frac{\alpha(1-\eta-\beta)}{(1-\alpha)\eta} (r^* - \xi)$  is positive and thus  $r^* > \xi$ . This implies  $r^* > \xi > \rho$ . This leaves

us with  $g_C^* = \frac{r^* - \rho}{\theta} > 0$  and  $g_C^* = g_Y^* = g_K^* > 0$ . Furthermore,  $u_n^* > 0$  and  $u_Y^* > 0$  are positive because of  $-\xi + g_H^* > 0$ . From equations (48) and (49) we have  $\psi^* > 0$  and  $g_H^* > 0$ . Finally,  $\chi^* = \left(\frac{1-\alpha\eta}{\beta} - \frac{1}{\theta}\right)r^* + \frac{\rho}{\theta} = \frac{1-\alpha\eta}{\beta}r^* - g_K^* > \frac{1-\alpha\eta-\beta}{\beta}r^* > 0$ , if the transversality condition holds. Thus, we have proved that the steady state is feasible.

## Appendix B: Proof of proposition 2

Following Gomez (2005, 2011a,b), we rewrite the dynamic system in terms of the variables  $r, \chi, g_n, z, u_Y$  with  $z \equiv (\delta\psi)^{1/(\gamma+\varepsilon-1)}u_Y$ . Using the fact that  $g_z = g_{u_Y} + \frac{1}{\gamma+\varepsilon-1}g_\varphi$ , leaves us with

$$g_r = -\frac{1-\beta-\eta}{\beta}(r-\xi) + \frac{\eta}{\beta}\frac{1-\alpha}{\alpha}g_n$$

$$g_\chi = \left(\frac{1}{\theta} - \frac{1-\alpha\eta}{\beta}\right)r + \chi - \frac{\rho}{\theta}$$

$$g_{g_n} = \phi\zeta - \frac{\gamma+\varepsilon-1}{\gamma+\varepsilon-2}(\xi + \phi\zeta) + \frac{(\gamma+\varepsilon-1)(1-\alpha)\eta}{(\gamma+\varepsilon-2)(1-\beta-\eta)}zg_n^{1-\frac{1}{\gamma+\varepsilon-1}} - g_n$$

$$g_z = \frac{(1-\alpha)\eta}{\beta}r - \chi + \left[\frac{(1-\alpha)\eta}{\alpha\beta} - \frac{1}{W}\right]g_n + \frac{\phi\zeta}{\gamma+\varepsilon-1} + \frac{(1-\eta)\xi}{\beta}$$

$$g_{u_Y} = \frac{(1-\alpha)\eta}{\beta}r - \chi + \frac{\eta}{\beta}\frac{1-\alpha}{\alpha}g_n - \xi\left[1 - u_Y - \frac{g_n^{\frac{1}{\gamma+\varepsilon-1}}u_Y}{z}\right] + \frac{(1-\eta)\xi}{\beta}$$

where  $z$  and  $r$  are predetermined variables. Hence, once the jump variable  $u_Y$  has settled down on its saddle-path stable trajectory,  $z(0)$  and  $r(0)$  are uniquely determined by the initial values of the predetermined variables  $K, H$  and  $n$ . Now we analyze the dynamics of the model in the neighbourhood of the steady state. Linearizing the differential equations system around the steady state values yields

$$\begin{pmatrix} \dot{r} \\ \dot{\chi} \\ \dot{g}_n \\ \dot{z} \\ \dot{u}_Y \end{pmatrix} = \begin{pmatrix} J_{11} & 0 & J_{13} & 0 & 0 \\ J_{21} & J_{22} & 0 & 0 & 0 \\ 0 & 0 & J_{33} & J_{34} & 0 \\ J_{41} & J_{42} & J_{43} & 0 & 0 \\ J_{51} & J_{52} & J_{53} & J_{54} & J_{55} \end{pmatrix} \begin{pmatrix} r - r^* \\ \chi - \chi^* \\ g_n - g_n^* \\ z - z^* \\ u_Y - u_Y^* \end{pmatrix}$$

where

$$J_{11} = -\frac{1-\beta-\eta}{\beta}r^* < 0$$

$$J_{13} = \frac{(1-\alpha)\eta}{\alpha\beta}r^* > 0$$

$$J_{21} = \left(\frac{1}{\theta} - \frac{1-\alpha\eta}{\beta}\right)\chi^*$$

$$J_{22} = \chi^* > 0$$

$$\begin{aligned}
J_{33} &= -g_n^* + \frac{(1-\alpha)\eta}{(1-\beta-\eta)} z^* g_n^{*1-\frac{1}{\gamma+\varepsilon-1}} & J_{34} &= \frac{(\gamma+\varepsilon-1)(1-\alpha)\eta}{(\gamma+\varepsilon-2)(1-\beta-\eta)} g_n^{*2-\frac{1}{\gamma+\varepsilon-1}} < 0 \\
J_{41} &= \frac{(1-\alpha)\eta}{\beta} z^* > 0 & J_{42} &= -z^* < 0 \\
J_{43} &= \left[ \frac{(1-\alpha)\eta}{\alpha\beta} - \frac{1}{W} \right] z^* & J_{55} &= \xi \left( 1 + \frac{g_n^{*\frac{1}{\gamma+\varepsilon-1}}}{z^*} \right) u_Y^* > 0
\end{aligned}$$

or  $\dot{\mathbf{X}} = J(\mathbf{X} - \mathbf{X}^*)$ , where the Jacobian for the system is  $J$ . Because of the block matrix property, the eigenvalues of  $J$  are the four eigenvalues of its upper left  $4 \times 4$  submatrix,  $\check{J}$  and its last diagonal element,  $J_{55} > 0$ . In the course of the proof we focus on the block matrix  $\check{J}$ . Gomez (2005, 2011a,b) used the Routh-Hurwitz theorem to discuss the number of negative roots in characteristic equation of  $\check{J}$ . In our case, the characteristic equation is

$$p(\lambda) = \lambda^4 - \Delta_3 \lambda^3 + \Delta_2 \lambda^2 - \Delta_1 \lambda + \Delta_0$$

where  $\Delta_0 = \det(\check{J})$ ,  $\Delta_1$  is the sum of all  $3 \times 3$  minors of  $\check{J}$ ,  $\Delta_2$  is the sum of all  $2 \times 2$  minors of  $\check{J}$ ,  $\Delta_3 = \text{tr}(\check{J})$ . Thus, we have

$$\begin{aligned}
\Delta_0 &= \frac{(\gamma+\varepsilon-1)(1-\alpha)\eta(\xi-\rho+\Lambda)}{(2-\gamma-\varepsilon)\beta\theta} r^* \chi^* z^* g_n^{*1-\frac{1}{\gamma+\varepsilon-1}} > 0 \\
\Delta_1 &= J_{11}J_{22}J_{33} + J_{41}J_{13}J_{34} - (J_{11} + J_{22})J_{34}J_{43} \\
\Delta_2 &= J_{11}J_{22} + (J_{11} + J_{22})J_{33} - J_{34}J_{43} \\
\Delta_3 &= J_{11} + J_{22} + J_{33}
\end{aligned}$$

Given the coefficients above, we build the following schema

$$1 \quad \Delta_3 \quad \Gamma \quad \Sigma \quad \Delta_0,$$

where

$$\Gamma = \Delta_2 - \Delta_1 / \text{tr}(\check{J})$$

$$\Sigma = \Delta_1 - [\text{tr}(\check{J}) \det(\check{J}) / \Gamma]$$

The trace of  $\check{J}$  is positive because  $J_{11} + J_{22} = \frac{(1-\alpha)\eta}{\beta} r^* + r^* - g_K^* > 0$  and

$$J_{33} = -g_n^* + \frac{(1-\alpha)\eta}{(1-\beta-\eta)} z^* g_n^{*1-\frac{1}{\gamma+\varepsilon-1}} = \xi - g_H^* > 0$$
 if the transversality condition is satis-



fied. Therefore, there can be at most two variations of sign in our schema. That is to say, the block matrix  $\tilde{J}$  may have 0 or 2 roots with negative real parts. The sufficient condition for saddle-path stability - stated in Gómez (2005) - requires two eigenvalues with negative real parts. This is equivalent to the condition  $J_{43} \leq 0$ , i.e. condition (56). The necessary and sufficient conditions for stability are therefore fulfilled.

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