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Public Funding of Banks and Firms in a Time of Crisis *

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Abstract

We study public funding of banks and non-financial firms in a time of crisis. We find that bank capitalization is more effective in stabilizing the economy than direct funding to firms, but it also creates larger distortions. We show that the optimal, social-welfare-maximizing, structure of a public funding program depends on its size. Small funding programs should target banks while large programs should be directed at non-financial firms.

JEL-codes: E44, G21, G28, G38, H12, H81

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1 Introduction

Governments often provide capital and other direct funding to banks and non-financial firms during economic crises. In the crises that took place between 1970 and 2007, these resolution measures were present in 33 episodes out of 42, and government capitalization of banks averaged around 8 % of GDP (Laeven and Valencia, 2012). During the Great Recession the Federal Reserve System (Fed) and the US Treasury injected capital and direct funding into both banks and non-financial firms, with the sizes of bank capital injection and non-bank financing programs reaching close to 5 and 3 % of GDP, respectively (SIGTARP, 2014; Labonte, 2016).¹ During the COVID-19 crisis the support from the Fed and the US Treasury focused on non-financial firms, with the size of the funding programs exceeding 15 % of GDP.²

In this paper we ask the following question: Suppose that, in a time of crisis, the government has decided to provide public funding to the private sector. How should the money be allocated? Should the government target banks or non-financial firms? We show that the optimal *structure* of the program depends on the *size* of the program. If the program is small or moderate, the government should capitalize banks. But if the program is larger, public funding should be allocated to non-financial firms.

To study the effects of public funding programs on banks' and firms' balance sheets, economic activity, and social welfare, we build on Holmström and Tirole (1997). In their flexible framework (see Tirole, 2006, for applications) entrepreneurs and banks can tap into external funding for leveraging their investments, but this leverage creates moral hazard problems.

¹The US Treasury injected equity capital in the financial sector and direct funding to the automotive industry via the Troubled Asset Relief Program (TARP). The size of TARP was decided prior to the allocation of its funds. The Fed created the Commercial Paper Funding Facility and Term Asset-Backed Securities Loan Facility programs to provide liquidity to the securitization market and to non-financial firms. Similar funding programs were, for example, implemented in the EU and UK (EU Commission, 2014; Grosse-Rueschkamp, Steffen and Streitz, 2019).

²Federal government provided lending through the Federal Reserve, Treasury and Small Business Administration (see, e.g., <https://www.covidmoneytracker.org/explore-data/interactive-table>, accessed Feb 6, 2023). The Fed introduced Primary and Secondary Market Corporate Credit Facilities, and the Main Street Business Lending Program, Paycheck Protection Program Liquidity Facility, and purchased mortgage backed securities (e.g., <https://www.federalreserve.gov/newsevents/pressreleases/monetary20200409a.htm>, accessed May 12, 2020). The size of these support programs was roughly 15 percent of US GDP. Again, similar and even more sizable support programs have been introduced, e.g., in the EU and UK.

Hence sufficiently large banks' and entrepreneurs' own stakes in the investment projects are needed to maintain their incentives. The model provides a convenient environment where both bank and entrepreneurial capital matter for macroeconomic dynamics.

While financial frictions are modeled as in Holmström and Tirole (1997) our characterization of financial intermediation and the real sector also involves other equally important elements. In our model banks have large balance sheets and diversified asset (or firm loan) portfolios whereas non-financial firms are small and specialized. This implies that firms are more vulnerable to idiosyncratic, or firm-specific, shocks while banks are more sensitive to aggregate investment shocks. If there is a negative aggregate shock, more entrepreneurs fail, but limited liability caps the loss at the micro level, while at the macro level specialization of small non-financial firms protects entrepreneurs as a group from spill-overs: successful entrepreneurs are not responsible for the debt that failing — and bankrupt — entrepreneurs cannot pay back. Banks absorb the loan losses, and since they pay in full to their creditors (or depositors), the (negative) macro shock has a levered effect on bank capital.

Because of the *sensitivity* of bank capital to aggregate shocks, bank capital plays a more important role in the shock propagation than entrepreneurial capital. We also show that bank capital tends to be *scarce* in the sense that, compared with the investment- and output-maximizing level, the ratio of bank capital to entrepreneurial capital is too low. The relative scarcity of bank capital implies that a given change in bank capital — and bankers' stakes — has a larger impact on incentives and aggregate investment than a corresponding change in entrepreneurial capital — and entrepreneurs' stakes.

In a time of crisis, public funding can improve social welfare by rendering the financial system more resilient — i.e. less sensitive to (negative) macro shocks. We show that the social welfare benefits depend on the *size* of the program. We measure the size by how much the program reduces the private sector's macro risk exposure. For example, a program of size 0.2 means that the government (or tax payers) takes over 20 % of the macro risk exposure; *ex post*, the program (of size 0.2) reduces bankers' loan losses (due to a macro shock) by 20%. We also show that the fiscal costs of the program depend on its size; that is the fiscal costs are proportional to the macro risk exposure the government assumes.

In our analysis, we take the size of the public funding program as given, e.g. by a political process. The social welfare benefits of public policy also depend on the (negative) macro shock realization (the larger the negative shock realization, the larger the benefits *ex post*). If there is genuine, i.e. Knightian, uncertainty during a crisis, the optimal size of the program would be impossible to determine. Moreover, even if the government could assign a shock distribution, the optimal program size would depend on the specifics of the tax system in place, particularly distortions and welfare losses caused by taxes. However, once the size is set (e.g. by some political process or by an optimizing government that knows the tax system in place) we can (analytically) characterize the optimal structure of the program, even allowing for genuine uncertainty and distortionary taxes.

A program of a given size and given social welfare benefits from improved resilience (contingent on the shock realization) can be constructed by different combinations of bank capitalization and public funding of non-financial firms. We call this the *policy frontier*. Due to the *sensitivity* of bank capital to aggregate shocks, smaller public (ownership) stakes in banks than in non-financial firms are required to achieve a given program size. But we show that this difference becomes less pronounced if the size of the program is large. In this case bank capital is less exposed to macro shocks, since the government has taken over a large part of the macro risk, and strengthening the banks' equity cushions has a smaller relative effect, compared with public firm funding, which reduces banks' exposure to risk from the non-financial sector. In other words, the slope of the policy frontier depends on the size of the program.

Providing public funding to banks and non-financial firms, however, dilutes their existing owners' stakes and incentives. Due to the relative *scarcity* of bank capital, public funds distort incentives and lower social welfare more when placed in banks rather than non-financial firms. This is because capital injections into banks dilute bankers proportionally more than corresponding injections into firms dilute entrepreneurs.

An *optimal structure* of public funding is a combination of bank capitalization and public funding of non-financial firms that minimizes welfare losses from distorted incentives, given the size of the program. The optimal structure of a program depends on its size. A given

stabilization effect can be attained with a smaller public stake but with larger distortions per unit of public funding when the government targets banks rather than firms. Initially, this tradeoff favors bank capitalization. When the desired stabilization effect is larger, there is a smaller difference between the required public stakes. Ultimately, for a sufficiently large program, its larger incentive distortions make bank capitalization inferior to the funding of non-financial firms from the welfare perspective.

Alternatively, we can summarize our main result by saying that we characterize the sets of *dominated* and *undominated* policies. A policy, say A, dominates another policy, say B, if A generates as high welfare as B after any (negative) shock realization, and higher welfare after some shock realizations. A policy is undominated if and only if it is not dominated by any other policy. We show that the mapping from the size of a program to its appropriate structure determines the sets of undominated and dominated policies. Small programs targeting banks, and large programs targeting non-financial firms, belong to the undominated set; if the government implements such a program, there will not be an alternative funding program that could provide as high or higher welfare after every shock realization. By contrast, any small public funding program directed at firms is dominated by a program of the same size but directed at banks, while any large program allocated to banks is dominated by a program of the same size but allocated to firms. Similarly, all mixed programs, simultaneously funding both banks and firms, are dominated by a pure program with the appropriate structure. No matter what the government thinks about the likelihood and intensity of a crisis, it should always select a funding program from the set of undominated options. It should never choose a program which is dominated. Therefore, our results narrow down the menu of potential policy options that a government should consider when planning a crisis funding program in the face of genuine uncertainty.

Our paper contributes to the literature studying the macroeconomic effects of government funding of banks and non-financial firms in a unified framework. Gertler and Kiyotaki (2010) study credit market interventions introduced in the wake of the Great Recession. Without comparing the interventions, they show that the net benefits of these interventions are increasing with the severity of the crisis. Hirakata, Sudo and Ueda (2013, 2017) develop

a credit chain version of the Bernanke, Gertler and Gilchrist (1999) financial accelerator model, where credit constrained firms borrow from credit constrained financial intermediaries which borrow from households. In their model capital injections to banks and firms lower the external finance premium(s) and stimulate the economy, but these policies may also result in more macro volatility. They also find that capital injections to financial intermediaries boost economic activity more than public funding of non-financial firms. Sims and Wu (2020) develop a Gertler-Karadi (2011, 2013) type model of financial intermediation, with an additional constraint that implies that the net issuance of bonds to be absorbed by financial intermediaries depends on the cash flows of non-financial firms. They compare two different Fed asset purchase programs: i) The Fed buys corporate bonds from financial intermediaries (Wall Street QE); and ii) the Fed buys bonds directly from non-financial firms (Main Street QE). They find that if the cash flow constraint does not bind, Wall Street QE and Main Street QE are perfect substitutes — both policies free space in the balance sheets of financial intermediaries. If the constraint binds, financial intermediaries are unwilling to buy corporate bonds, and Wall Street QE becomes ineffective, while Main Street QE can stimulate the economy. Our paper complements the literature by analyzing the socially optimal allocation of public funds between banks and non-financial firms in a framework in which the social costs and benefits of public policies arise from their effects on capital structure decisions and incentive problems in the financial and real sectors. We also differ from the previous literature in our analysis of dominated and undominated funding policies under genuine uncertainty.

There is also a large literature on bank recapitalizations and bailouts. For example, Philippon and Schnabl (2013) study forms of efficient recapitalizations, while Bhattacharya and Nyborg (2013) use the menu of bailout plans as a screening device. In these papers, banks suffer from debtoverhang. Gertler and Karadi (2011, 2013), Curdia and Woodford (2011), and Del Negro et al. (2017) study large scale (private) asset purchases by the central bank, which can be interpreted as direct government funding of non-financial firms, while Bianchi (2016) analyzes the ex post crisis mitigation and the ex ante moral hazard resulting from firm bailouts. Our work differs from the previous literature by emphasizing

the effects of bailouts on intratemporal incentive problems rather than intertemporal ones. Moreover, we analyze public funding of both banks and non-financial firms, and present a simple criterion how to choose between these two.

Finally, there is a growing macro-finance literature applying the Holmström–Tirole (1997) framework. Contributions include Chen (2001), Aikman and Paustian (2006), Meh and Moran (2010), Christensen, Meh and Moran (2011), Chang, Fernández and Gulán (2017), Faia (2018), and Silvo (2019). We extend this macro-finance framework in terms of our modeling of banks: a bank is a balance sheet structure with many bankers. This allows us to combine diversified portfolios at the bank level, and the Holmström–Tirole type incentive problems at the level of individual bankers.³

In the next section we describe the basic model. In Section 3, we explain why bank capital is likely to be scarce in equilibrium. In Section 4, we introduce an aggregate investment shock into the model, and explain why bank capital is more sensitive to these shocks than entrepreneurial capital. In Section 5, we calibrate the model, and further analyze and illustrate the scarcity and sensitivity of bank capital. In Section 6, we analyze injections of public funds in banks and non-financial firms and establish the main result of the paper, i.e. the optimal structure of public funding. Section 7 concludes.

2 The Model

We consider a discrete-time infinite-horizon economy populated by households with three types of members: workers, entrepreneurs, and bankers. On the financial side of the economy, bankers manage financial intermediaries (banks) that obtain deposits from households and finance entrepreneurs. The real economy contains two sectors: i) competitive firms producing final goods from labor supplied by workers and capital supplied by entrepreneurs, and ii) entrepreneurs producing capital goods.

³In the standard Holmström–Tirole based macro framework a bank’s asset portfolio is assumed to be completely correlated. Together with the assumption that unsuccessful firm projects return zero, this renders debt indistinguishable from equity on the liability side of the bank’s balance sheet: either the bank can pay to all stakeholders or it can pay to nobody. In our model with diversified portfolios, the claims of a bank’s creditors and equity holders can be meaningfully distinguished when there are aggregate shocks.

Households own banks and all firms, including those producing capital goods. The production of capital is subject to a dual moral hazard problem in the sense of Holmström and Tirole (1997). Entrepreneurs, who may obtain external finance from households and banks, are tempted to choose less productive projects with higher non-verifiable returns. Bankers can monitor entrepreneurs to mitigate their moral hazard temptations, but since banks use deposits from households to finance entrepreneurs, bankers have an incentive to avoid costly monitoring.

The timing of events in each period is summarized in Table 1, while Appendix A.1 provides a still more detailed description. The key part of the model is Stage 2, where finance and the real economy interact.

Period starts	
Stage 1	Survival probabilities realized and exiting bankers and entrepreneurs give their accumulated assets to households
	Household members separate into their occupations
	Consumption-savings and labor supply decisions are made
	Final goods are produced using capital and labor
Stage 2	Financial contracts are signed, depositors place their funds in banks, and banks finance entrepreneurs
	Bankers choose monitoring intensity
	Entrepreneurs choose the project type
	Successful projects yield capital goods that are sold
	Proceeds are divided according to the contract
Period ends	

Table 1: Timeline of events.

2.1 Households and Final Good Production

Following Gertler and Karadi (2011), we assume that there is a continuum of identical households of measure unity. Within each household, there are three occupations: in every

period t , a fraction of the household members become entrepreneurs, another fraction become bankers, and the rest remain workers. At the beginning of each period, an entrepreneur and a banker exit from their occupations at random according to a Poisson process with constant exit rates $1 - \lambda^e$, $\lambda^e \in (0, 1)$, and $1 - \lambda^b$, $\lambda^b \in (0, 1)$, respectively. The number of household members becoming entrepreneurs and bankers equals the number of exiting entrepreneurs and bankers.

The head of a household decides on behalf of its members how much they will work, consume, and invest in capital. In Section 2.2, we explain in detail how entrepreneurs invest in risky projects to produce capital goods and how bankers provide funding for these investments. In general, entrepreneurs and bankers earn higher returns on their risky investments than workers earn on their deposits. Hence, it is optimal for the household to let its entrepreneurs and bankers keep building up their assets until exiting their occupations. The exiting entrepreneurs and bankers give their accumulated assets to the household which in turn provides new entrepreneurs and bankers with some initial investment capital. Within the household, there is perfect consumption insurance against the risks of entrepreneurs and bankers. Therefore, all household members consume an equal amount in each period.

The problem of a representative household is

$$\max_{\{C_t \geq 0, L_t \geq 0, K_t \geq 0\}_{t=0}^{\infty}} E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{\xi}{1+\phi} L_t^{1+\phi} \right) \right],$$

subject to a budget constraint:

$$C_t + q_t K_{t+1} = W_t L_t + K_t [r_t^K + q_t(1 - \delta)] + T_t. \quad (1)$$

In the households utility function $\sigma > 0$, $\xi > 0$ and $\phi > 0$ are parameters, $\beta \in (0, 1)$ is the time preference discount factor, and C_t and L_t denote consumption and hours worked in period t , respectively. In the budget constraint (1), W_t is the real wage, K_t the stock of physical capital, r_t^K the real rental rate of capital, q_t the price of capital goods, and $\delta \in (0, 1)$ the rate of depreciation of physical capital. Finally, T_t denotes lump-sum transfers (net

payouts from entrepreneurs and bankers) and (possible) taxes.

In equilibrium physical capital stock accumulates according to the law of motion

$$K_{t+1} = (1 - \delta)K_t + p_H R I_t, \quad (2)$$

where I_t is investment in period t . This accumulation equation is standard save for the two parameters of capital good production, $p_H \in (0, 1)$ and $R > 1$, which will be defined more precisely in Section 2.2.

Solving the household's dynamic optimization problem yields the familiar first order conditions for L_t and C_t , respectively: $\xi L_t^\phi C_t^\sigma = W_t$ and

$$1 = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left[\frac{r_{t+1}^K + q_{t+1}(1 - \delta)}{q_t} \right] \right\}. \quad (3)$$

Competitive firms in the final good sector combine capital K_t and labor L_t using the Cobb-Douglas production function $Y_t = K_t^\alpha L_t^{1-\alpha}$, where $\alpha \in (0, 1)$. Profit maximization results in the familiar equations for optimality conditions: $W_t = (1 - \alpha)Y_t/L_t$ and $r_t^K = \alpha Y_t/K_t$.

2.2 Financial Frictions

Capital demanded by firms in the final good sector is produced by entrepreneurs who are endowed with investment projects and some initial wealth. Entrepreneurs can also attempt to leverage their investments by borrowing from bankers and workers. It may be best to think that intermediation of entrepreneurial finance occurs only among households. To clarify how financial intermediation takes place, let us consider three households, A, B, and C. We can think that the workers of household A first deposit their funds with the banks of household B, who then invest the deposits in projects of household C's entrepreneurs along with their own bank capital. The term "deposits" should be interpreted broadly, encompassing both retail deposits and wholesale debt funding of banks. In particular, the marginal unit is always wholesale funding, and not covered by any deposit insurance scheme.

All successful investment projects transform i_t units of final goods into $R i_t$ ($R > 1$)

verifiable units of capital goods, while failed projects yield nothing. The projects differ in their probability of success and in the amount of non-verifiable revenues they create. There is a “good” project that is successful with probability p_H and involves no non-verifiable revenues to the entrepreneur. We adopt the normalization $p_H R = 1$, so that the equilibrium law of motion of capital (2) is the same as in a standard growth model or RBC model.

There is also a continuum of bad projects with a common success probability $p_L = p_H - \Delta p$, where $0 \leq p_L < p_H < 1$ and $\Delta p > 0$, but with differing amounts of non-verifiable revenues $h_t i_t$, $h_t \in (0, \bar{h}]$ attached to them. Non-verifiable revenues are proportional to investment size as in Holmström and Tirole (1997). Departing from Holmström and Tirole (1997), where bad projects generate non-transferable private benefit, we assume, in line with Meh and Moran (2010), Christensen et al. (2011), Faia (2018) and Silvo (2019), that private benefits are divisible and transferable.⁴ In our case, this assumption is only needed to ensure the smoothness of out-of-equilibrium payoffs. If, in an out-of-equilibrium event, an entrepreneur had picked a bad project, her project returns should be transferable and divisible among her household members upon her exit from entrepreneurship. Further, we assume that $q_t p_H R = q_t > q_t p_L R + \bar{h}$ to ensure that the good project is preferable to all bad projects from the household’s point of view.

Bankers are endowed with a variable-scale monitoring technology that enables them to constrain the entrepreneurs’ project choice. Monitoring at the intensity level m_t ($m_t \geq 0$) eliminates all bad projects where $h_t \geq h(m_t)$ from the entrepreneur’s project choice set. The threshold level of non-verifiable revenues $h(m_t)$ is decreasing and convex in monitoring intensity: $h'(m_t) \leq 0$, $h''(m_t) \geq 0$, and $\lim_{m_t \rightarrow \infty} h'(m_t) = 0$. As in Christensen et al. (2011) and Silvo (2019), monitoring involves real costs for the bank: to obtain monitoring intensity m_t , a bank must pay $m_t i_t$ units of final goods to workers.⁵ That is, the more a banker

⁴One interpretation is, reminiscent of Bolton and Scharfstein (1990), that project revenues are verifiable outside a household only up to R , or that only revenues in terms of capital goods are verifiable outside a household. Alternatively, following, e.g. Burkart, Gromb, and Panunzi (1998), we may think that an entrepreneur is able to divert part of her firm’s resources to her own use at an interim stage. As in Burkart et al. (1998), such expropriation of outside investors is costly. Here it is captured by the lower expected project returns if diversion takes place.

⁵Monitoring costs are modeled as a transfer to workers, and they do not enter the real resource constraint of the economy. This assumption is not quantitatively restrictive as the total monitoring cost, while crucial for bankers’ incentives, is very small relative to the size of the real economy. In the baseline calibration of

invests in monitoring, the less his bank can lend to entrepreneurs.

Because of diminishing returns to monitoring investments, the banker will never want to eliminate all bad projects. Therefore, despite monitoring, entrepreneurs must be provided with incentives to choose the good project. In sum, there are two moral hazard problems: one between bankers and entrepreneurs (borrowers), and another between bankers and workers (depositors). The moral hazard problems may be solved by designing a proper financing contract.

2.2.1 The Financing Contract

In each period t , there are three contracting parties: entrepreneurs, bankers, and depositors (workers). Following standard practice, we assume limited liability and inter-period anonymity, and focus on the class of one-period optimal contracts where entrepreneurs invest all their own wealth n_t in their projects. The financial contract then stipulates how much of the required funding of the project of size i_t comes from banks (a_t) and depositors (d_t) and how the project's return R , in case of success, is distributed among the entrepreneur (R_t^e), her bankers (R_t^b), and depositors (R_t^w).

A banker, given his share of project returns, maximizes the bank's profits by choosing monitoring intensity, m_t . Banks behave competitively. As a result, they offer the same contract that would be offered by a single bank, which would maximize the entrepreneur's expected profits. An optimal financing contract therefore solves the following program:

$$\max_{\{i_t, a_t, d_t, R_t^e, R_t^b, R_t^w, m_t\}} q_t p_H R_t^e i_t$$

the model, the steady-state monitoring cost mI equals approximately 0.15 % of total output Y , whereas consumption C is roughly 80% and investment I is roughly 20%. (See the Calibration Appendix C, especially equation C.4.) However, omitting the monitoring cost from the resource constraint allows the analytical characterization of the optimal structure of public funding in Section 6. Technically, this is because the model becomes modular such that frictions in financial intermediation affect the real part of the economy only indirectly through the agency problems.

subject to the entrepreneur's and her banker's incentive constraints,

$$q_t p_H R_t^e i_t \geq q_t p_L R_t^e i_t + h(m_t) i_t, \quad (4a)$$

$$q_t p_H R_t^b i_t \geq q_t p_L R_t^b i_t + (1 + r_t^d) m_t i_t, \quad (4b)$$

the depositors' and banker's participation constraints,

$$q_t p_H R_t^w i_t \geq (1 + r_t^d) d_t, \quad (4c)$$

$$q_t p_H R_t^b i_t \geq (1 + r_t^a) a_t, \quad (4d)$$

and two resource constraints on investment inputs and outputs

$$a_t + d_t - m_t i_t \geq i_t - n_t, \quad (4e)$$

$$R \geq R_t^e + R_t^b + R_t^w. \quad (4f)$$

Equations (4e) and (4f) mean that the aggregate supply of investment funds must satisfy their aggregate demand equation and that the total returns must be enough to cover the total payments, respectively. Variable r_t^a in the banker's participation constraint (4d), denotes the rate of return on bank capital in period t and, similarly, variable r_t^d in the banker's incentive constraint (4b) and in the depositors' participation constraint (4c), is the rate of return on deposits during the capital good production stage of period t , i.e. Stage 2 in Table 1. Since deposits are intra-period, we follow Carlstrom and Fuerst (1997) and set the deposit rate r_t^d to zero, so that the gross rate of return earned by households — or workers — in the capital good production stage is $1 + r_t^d = 1$. One may think that the alternative available for the households is to store the representative consumption good during Stage 2 of the period, to be consumed at the end of Stage 2; the rate of return to storage is 1. This assumption is also in line with simple RBC and growth models, where the representative consumption good is transformed one-to-one into capital goods, and the gross rate of return in this capital good production stage is 1.

Each entrepreneur wants to invest as much as possible without breaking the depositors' and the banker's participation and incentive constraints. Hence, all constraints bind in equilibrium. Using these standard equilibrium properties, we solve the entrepreneur's program in two steps. First, we take the intensity of monitoring m_t and, by implication, the level of private revenues $h(m_t)$ as given and solve for the maximum size of the investment project i_t for a given level of entrepreneurial wealth n_t . Secondly, we solve for the equilibrium level of monitoring m_t .

2.2.2 Investment, Leverage and Monitoring at the Project Level

In the Holmström–Tirole framework, the maximum investment size depends on the amount of funds that can be raised from the outside, which in turn depends on the amount of the project returns that can credibly be pledged to depositors. In Appendix A.2 we show that maximum investment size is

$$i_t = \frac{n_t}{g(r_t^a, q_t, m_t)} \quad (5a)$$

in which

$$g(r_t^a, q_t, m_t) \equiv \frac{p_H}{\Delta p} h(m_t) + \left[1 + \frac{p_H}{\Delta p} \left(1 - \frac{1}{1 + r_t^a} \right) \right] m_t - \rho_t \quad (5b)$$

is the inverse degree of leverage, i.e. the smaller the value of $g(\cdot)$, the larger the size of the investment project i_t for a given level of entrepreneurial wealth n_t . The first term on the right-hand side of equation (5b) shows how agency problems in the non-financial firm reduce leverage by discouraging participation by outside investors. These agency problems can be mitigated through increased monitoring. However, the second term reveals that intense monitoring has two negative effects on leverage: it consumes resources that could otherwise have been invested in the project and makes it harder to satisfy the banker's incentive constraint. These two effects are captured by the first and second terms in square brackets, respectively.⁶ In other words, more extensive monitoring activity worsens the agency problem between a bank and a depositor. To overcome this moral hazard and attract more deposits, a larger share of the investment project must be financed by bank

⁶Note that in equilibrium we must have $r_t^a \geq 0$.

capital. Finally, the term $\rho_t \equiv q_t - 1 > 0$ denotes the net rate of return on the good investment project; the larger the rate of return, the easier it is to attract outside funding.⁷

Given the competitively behaving banking sector, the optimal choice of m_t maximizes the entrepreneur's expected profits $p_H q_t R_t^e i_t$, which may be rewritten, by using equations (4a) and (5a), as $(p_H/\Delta p) h(m_t) n_t/g(r_t^a, q_t, m_t)$. Therefore, the optimal level of monitoring solves the problem

$$\max_{m_t \geq 0} \frac{h(m_t)}{g(r_t^a, q_t, m_t)}. \quad (6)$$

As can be seen from equations (5b) and (6), the effects of monitoring on the entrepreneur's expected payoff are complex. The numerator in the problem (6) shows how a larger scope of extracting private revenues implies a larger equilibrium share of the project returns for the entrepreneur, which dilutes the monitoring incentives. Monitoring incentives are also adversely affected by the negative effects of monitoring costs on leverage (second term in $g(\cdot)$ in equation (5b)). However, smaller agency problems enable larger leverage (first term in $g(\cdot)$ in equation (5b)). This provides an incentive for monitoring.

To derive a tractable analytic solution to problem (6), we specify the following functional form for $h(m_t)$:

$$h(m_t) = \begin{cases} \Gamma m_t^{-\frac{\gamma}{1-\gamma}} & \text{if } m_t > \underline{m} \\ \bar{h} & \text{if } m_t \leq \underline{m}, \end{cases} \quad (7)$$

where $\Gamma > 0$, $\bar{h} > 0$, $\gamma \in (0, 1)$, and $\underline{m} \geq 0$. The first row of equation (7) shows how $h(m_t)$ is differentiable and strictly convex for $m_t > \underline{m}$ and that the monitoring technology is the more efficient, the larger the value of γ or the smaller the Γ . The second row implies that there is a minimum efficient scale for monitoring investments or an upper bound for private revenues. This upper bound ensures that a bad project has a lower rate of return than a good project, even for low levels of m_t .⁸

Under the minimum scale requirement, the entrepreneur may choose a corner solution with no monitoring $m_t = 0$, $h(m_t) = \bar{h}$, or a unique interior solution with $m_t > \underline{m}$. In

⁷See Lian and Ma (2021) for evidence on related cashflow-based financial constraints.

⁸We experimented with many other functional forms besides specification (7), without gaining additional insights or simpler expressions.

Appendix D.3 we determine the conditions under which we can rule out the corner solution. These conditions are met around the steady state, on which we focus on in this paper. After substitution of equations (5b) and (7) we can write the unique interior solution to the entrepreneur's problem (6) as

$$m_t = \frac{\gamma \rho_t}{1 + \frac{p_H}{\Delta p} \left(1 - \frac{1}{1+r_t^a}\right)}. \quad (8)$$

The optimal level of monitoring intensity characterized by equation (8) has intuitive properties. It increases with the elasticity of monitoring technology (directly related to γ) and the rate of return on a good project (ρ_t). Moreover, the larger the negative effects of monitoring on leverage (which are in the denominator), the lower the optimal level of monitoring.

2.2.3 Aggregate Investment, Bank Capital and Firm Capital

We proceed under the assumption that all projects will be monitored with the intensity given by equation (8) and, as a result, all entrepreneurial firms have the same capital structure. That is, for all projects, the ratios a_t/i_t , d_t/i_t , and n_t/i_t are the same.⁹ Given this symmetry, moving from project level to economy-wide level in terms of capital structures is simple. Clearly,

$$\frac{a_t}{i_t} = \frac{A_t}{I_t}, \quad \frac{d_t}{i_t} = \frac{D_t}{I_t}, \quad \text{and} \quad \frac{n_t}{i_t} = \frac{N_t}{I_t}, \quad (9)$$

where capital letters stand for aggregate-level variables.

Combining (9) with the banker's incentive and participation constraints (4b) and (4d) links the equilibrium monitoring intensity m_t to the ratio A_t/I_t and to the rate of return to bank capital: $m_t = (\Delta p/p_H) (A_t/I_t) (1 + r_t^a)$. Since in equilibrium this must be the same as the monitoring intensity chosen at the project level, equation (8), we get

$$1 + r_t^a = \left(1 + \frac{\Delta p}{p_H}\right)^{-1} \left(1 + \gamma \rho_t \frac{I_t}{A_t}\right). \quad (10)$$

For equation (10) to characterize the equilibrium rate of return on bank capital, $1 + r_t^a$ has

⁹But project sizes differ: the larger the entrepreneur's wealth n_t , the larger her investment i_t .

to be greater than 1, the rate of return available for households from deposits, or the storage technology. Near the steady state, the inequality $1 + r_t^a > 1$ holds if $\lambda^b < \beta$ (see Appendix D.2); this is the case with our baseline calibration (see Section 5).

Next, plugging (10) into (8) allows us to write

$$m_t = \left(1 + \frac{p_H}{\Delta p}\right)^{-1} \left(\frac{A_t}{I_t} + \gamma \rho_t\right). \quad (11)$$

The larger (relative) stakes the bankers have in the projects (high A_t/I_t), the greater their incentives to monitor intensively. But by (5a) and (9), inverse firm leverage satisfies the equation $N_t/I_t = g(\cdot)$, where $g(\cdot)$ is given by (5b). Then applying equations (10) and (11) allows us to express the entrepreneurs' maximum incentive compatible non-verifiable revenue from a “bad” project in terms of aggregate variables

$$h_t = \frac{\Delta p}{p_H} \left(\frac{N_t}{I_t} + (1 - \gamma) \rho_t\right). \quad (12)$$

In other words, if the non-verifiable revenue is at or below the threshold value given by equation (12), the entrepreneurs choose the “good” project rather than the “bad” project. The larger the entrepreneurs' (relative) stakes in the projects (high N_t/I_t), the greater their incentives to choose the “good” project even when they are not subject to intense monitoring by bankers (i.e. when h_t is high). In equilibrium both bankers and entrepreneurs must face proper incentives. Using (11) and (12), and noting that by (7) there is a trade-off between moral hazard in banks and firms, yields

$$\left(\frac{A_t}{I_t} + \gamma \rho_t\right)^\gamma \left(\frac{N_t}{I_t} + (1 - \gamma) \rho_t\right)^{1-\gamma} = \left(\Gamma \frac{p_H}{\Delta p}\right)^{1-\gamma} \left(1 + \frac{p_H}{\Delta p}\right)^\gamma. \quad (13)$$

Equation (13) says that in equilibrium the aggregate investment level I_t in the economy depends on both aggregate bank capital A_t and aggregate entrepreneurial capital N_t .

The remaining period t equilibrium conditions are simple. Equations (4e) and (9) imply that aggregate deposits in the banking system are given by $D_t = (1 + m_t) I_t - (A_t + N_t)$. The aggregate investment level is part of a simple aggregate resource constraint $Y_t = C_t + I_t$. Note

that while monitoring involves real costs for banks, it is assumed to consume no aggregate resources. As explained at the beginning of Section 2.2, monitoring involves a transfer of final goods from banks to workers, and is hence included in the lump-sum transfers T_t in the household's budget constraint (1). For more discussion, see also footnote 5.

Finally, we need to determine the evolution of aggregate bank and entrepreneurial capital. At the beginning of the next period $t + 1$, the shares $1 - \lambda^e$ and $1 - \lambda^b$ of entrepreneurs and bankers, respectively, exit their professions and surrender their wealth to the household. More concretely, one may think that in each period the banks and firms pay a constant share of their (gross) revenue as dividends to the households who own them.¹⁰ The surviving entrepreneurs and bankers then have aggregate wealth $\lambda^e q_t p_H R_t^e I_t$ and $\lambda^b q_t p_H R_t^b I_t$, respectively. In Stage 1 of period $t + 1$ they place their funds in the production of the final good, earning the same rate of return as the households, or workers, $[r_{t+1}^K + q_{t+1}(1 - \delta)]/q_t$, — see the right-hand side of the household Euler equation (3). As a result, the aggregate amount of capital held by bankers at the beginning of (the investment) Stage 2 of period $t + 1$ is given by $A_{t+1} = \lambda^b p_H R_t^b I_t [r_{t+1}^K + q_{t+1}(1 - \delta)]$, which can be combined with conditions (4d) and (9) to obtain the following law of motion for the aggregate bank capital:

$$A_{t+1} = A_t (1 + r_t^a) \lambda^b \left(\frac{r_{t+1}^K + (1 - \delta) q_{t+1}}{q_t} \right), \quad (14)$$

where $(1 + r_t^a)$ is given by (10). The law of motion of aggregate entrepreneurial capital is $N_{t+1} = \lambda^e p_H R_t^e I_t [r_{t+1}^K + q_{t+1}(1 - \delta)]$, which we can rewrite as

$$N_{t+1} = N_t (1 + r_t^n) \lambda^e \left(\frac{r_{t+1}^K + (1 - \delta) q_{t+1}}{q_t} \right), \quad (15a)$$

where

$$1 + r_t^n \equiv q_t p_H R_t^e \left(\frac{I_t}{N_t} \right) = 1 + (1 - \gamma) \rho_t \left(\frac{I_t}{N_t} \right) \quad (15b)$$

denotes the rate of return on entrepreneurial capital during Stage 2 of period t . The latter

¹⁰This assumption is standard in much of the macro-finance literature. There is also empirical evidence backing the view that especially banks strive to keep their dividend stream rather stable. One reason may be that dividend payments signal economic strength. See e.g. Floyd, Li, and Skinner (2015).

form of (15b) follows from (4a), (9) and (12).

3 Relative Scarcity of Bank Capital

The equilibrium of the model¹¹ is defined in Appendix D.1, and an analytical solution of the steady state is given in Appendix D.2. If $\max \{\lambda^e, \lambda^b\} < \beta$ financial constraints bind in (and near) the steady state: essentially bank capital and entrepreneurial capital are scarce.¹² As explained in Section 2.2, the production of capital goods is constrained by the availability of A_t and N_t (see equation (13)). The level of investments is suboptimally low in the following sense. Consider a small perturbation where, starting from the steady state, investments rise by a small amount dI in the current period, and the proceeds are consumed in the next period. In the current period, there is less consumption, and this implies a decrease in current period utility equal to $-U_C dI$ (where U_C is marginal utility of consumption in steady state). In the next period, (discounted) utility rises by $\beta U_C (r^K + 1 - \delta) dI$, where $r^K + 1 - \delta$ is the (social) rate of return to investment (in steady state). Hence, the overall change in utility is $\Delta U = [\beta (r^K + 1 - \delta) - 1] U_C dI$. The household Euler equation (3), however, implies that in steady state $\beta (r^K/q + 1 - \delta) - 1 = 0$, where the steady state price of capital $q > 1$, due to financial frictions in capital good production. Then we get $\Delta U = \rho [1 - \beta (1 - \delta)] U_C dI > 0$, where $\rho = q - 1 > 0$ is the steady state net return on the investment project. Hence, an increase in the level of investments raises welfare, and likewise a decrease of investments from the steady-state level lowers welfare.

While both bank capital and firm capital are scarce, also the composition of informed capital — the relative scarcity of A_t and N_t — affects investments in an important way.

¹¹The model variables and equations characterizing the equilibrium are all presented in Section 2. However, some model equations not essential for explaining our main arguments were just mentioned in the text, rather than presented as separate numbered items. To give a mathematical summary of the model, Appendix D.1 provides a list of all the equations needed to characterize the equilibrium.

¹²A similar condition arises in many models with macro-financial linkages. Examples include models applying the Bernanke-Gertler-Gilchrist (1999) financial accelerator (e.g. Christiano, Motto, and Rostagno, 2014; Hiraoka, Sudo, and Ueda, 2013, 2017), the model of financial intermediation developed by Gertler and Karadi (2011, 2013) and recently extended by Sims and Wu (2020, 2021), and macro-finance models using the Holmström-Tirole (1997) approach, cited in the Introduction. Intuitively, the exit rate of entrepreneurs and bankers has to be high enough so that the economy does not outgrow financial constraints.

Let $\nu_t \equiv A_t/N_t$ denote the ratio of bank capital to entrepreneurial capital, and call it the ratio of informed capital. We show in Appendix A.3 that if $\max\{\lambda^e, \lambda^b\} < \beta$, there exists a steady state where the ratio of informed capital (ν) is given by

$$\nu = \frac{\gamma}{1-\gamma} \left(\frac{\frac{\beta}{\lambda^e} - 1}{\frac{\beta}{\lambda^b} \left(1 + \frac{\Delta p}{p_H}\right) - 1} \right). \quad (16)$$

Next, we determine the value of ν_t (denoted by ν^{**}) that would maximize leverage and investments in the economy, and by implication, the economy's output. We show in Appendix A.4 that

$$\nu^{**} = \frac{\gamma}{1-\gamma}. \quad (17)$$

Hence, the investment maximizing ratio of informed capital is equal to the elasticity of monitoring technology. To interpret this result, first recall that in equilibrium both bankers and entrepreneurs channel all their wealth into the investment projects, and the ratio $\nu = A/N$ reflects their relative stakes. Now, suppose that banks have access to an efficient monitoring technology (the elasticity $\gamma/(1-\gamma)$ is large). In such case, an arrangement that maximizes aggregate investments involves intense monitoring. As the entrepreneurs' moral hazard problems are effectively alleviated, more funds for entrepreneurs' investments can be raised from depositors. Ensuring that bankers have incentives to monitor intensively, however, requires sufficiently large banker stakes (i.e. a high ratio $\nu^{**} = A/N$).

In contrast, if the monitoring technology is not efficient (the elasticity $\gamma/(1-\gamma)$ is small), intensive monitoring is less useful. Then, in order to attract funding from depositors, it is better that entrepreneurs, rather than bankers, have large stakes and strong incentives to see that the projects succeed. Hence a low ratio $\nu^{**} = A/N$ maximizes investment scale.

Comparison of equations (16) and (17) yields the following result:

Proposition 1

$$\nu^{**} \begin{matrix} \geq \\ \leq \end{matrix} \nu \quad \text{if} \quad \frac{\lambda^b}{\lambda^e} \begin{matrix} \leq \\ \geq \end{matrix} 1 + \frac{\Delta p}{p_H}.$$

Proposition 1 suggests that the question of whether there is relative *scarcity* of bank or

entrepreneurial capital in a steady state only depends on bankers' and entrepreneurs' exit rates and success probabilities of projects. The scarcity of bank capital prevails in a steady state for a larger range of parameter values than does the scarcity of entrepreneurial capital: Only if the bankers' survival probability is higher than the entrepreneurs' survival probability by a factor strictly larger than one can the bankers accumulate more capital than that needed to maximize investments and output in the economy. In Section 5, we further argue that the relative scarcity of bank capital is the empirically relevant case.

Proposition 1 has the following implication: Differentiating equation (13) around the steady state yields (see Appendix A.5 for details)

$$\left. \frac{dN}{dA} \right|_I = - \frac{1 + \frac{\Delta p}{p_H} - \frac{\lambda^b}{\beta}}{\left(1 + \frac{\Delta p}{p_H}\right) \left(1 - \frac{\lambda^e}{\beta}\right)}. \quad (18)$$

We view $I_t(A_t, N_t)$ as given by equation (13) as the economy's production technology. Then, we may define $|dN/dA|_I \equiv MRTS$ as the absolute value of the steady-state marginal rate of technical substitution of bank and entrepreneurial capital. We state the following result:

Corollary 1

$$MRTS \begin{matrix} \geq \\ \leq \end{matrix} 1 \quad \text{if} \quad \frac{\lambda^b}{\lambda^e} \begin{matrix} \leq \\ \geq \end{matrix} 1 + \frac{\Delta p}{p_H}.$$

If bank capital is scarce, $MRTS$ is greater than one and, as a result, increasing bank capital boosts aggregate investment more than increasing entrepreneurial capital by an equal amount (and vice versa if entrepreneurial capital is scarce).

To better understand the mechanism that leads to the (relative) underprovision of bank capital, we consider the case where $\lambda^e = \lambda^b$. Then, Proposition 1 unambiguously implies that in a steady state bank capital is scarce relative to firm capital. Dividing the law of motion of A_{t+1} by that of N_{t+1} (see the derivation of equations (14) and (15a,b)) shows that in a steady state we have

$$\nu = \frac{R^b}{R^e}.$$

That is, because it is optimal for the household to let its entrepreneurs and bankers retain

and reinvest all their earnings, bankers and entrepreneurs accumulate capital in relation to their conditional project returns in a steady state.

Next note that maximizing leverage is practically equivalent to maximizing the (expected) pledgeable income, $p_H q_t (R_t - R_t^b - R_t^e)$, (i.e. the highest revenue share that can be pledged to depositors without jeopardizing entrepreneurs' and bankers' incentives), minus the cost of monitoring, m_t . But there is a trade-off: an increase in the bank monitoring will increase the entrepreneur's pledgeable income but reduce the banker's pledgeable income and consume funds that could otherwise have been loaned to entrepreneurs. Therefore the investment maximizing amount of bank involvement solves the following program: $\max_{m_t \geq 0} p_H q_t (R - R_t^b - R_t^e) - m_t$ subject to equations (4a), (4b), (7), and $r_t^d = 0$. The first-order condition for this problem may be written as $(R_t^b + m_t / (p_H q_t)) / R_t^e = \gamma / (1 - \gamma)$. Using $\nu^{**} \equiv \gamma / (1 - \gamma)$, a steady state version of this condition can be written as

$$\nu^{**} = \frac{R^b + \frac{m}{p_H q}}{R^e}.$$

This suggests how the aggregate leverage is maximized when bankers' accumulation of capital also takes into account the real costs of monitoring in addition to their revenue share. In a steady state, however, the bankers' capital accumulation only reflects their revenue share. Therefore in a steady state bank capital is scarce.

4 Aggregate Uncertainty and Sensitivity of Bank Capital

Until now we have assumed that investment projects only involve idiosyncratic uncertainty. In this section, we introduce an aggregate shock by assuming that in some period t project success probabilities are given by $\tilde{p}_{\tau t} \equiv p_{\tau}(1 + \varepsilon_t)$, $\tau \in \{H, L\}$, in which $\varepsilon_t \in [\underline{\varepsilon}, 1/p_H - 1]$, with $\underline{\varepsilon} > -1$, is an unanticipated change ("MIT shock") in the success probabilities of all projects in the sense that the financial contracts cannot be written contingent on ε_t . Such an investment shock may be due e.g. to a disruptive technology or due to initial market perceptions (in which case the "shock" is a correction to the initial misperception).

The shock is realized after financing contracts have been signed, monitoring and project choices have been made, and the price of capital goods has been determined. Furthermore, neither the pricing of capital goods nor financial contracts can be made contingent on realization of the shock. While in theory it would be possible to contract on the aggregate level of capital goods produced, in practice such contracts are rare. In essence, we are assuming that capital goods are sold via forward contracts where the price of capital goods is agreed upon simultaneously with the (other) terms of the financing contract, before the delivery of capital goods occurs (see Appendix A.1, for a detailed timing of events). Thus, the price of capital goods in period t , q_t , is unaffected by the shock in period t .

To model the effects of an aggregate shock, we make the distinction between *bankers* and *banks* explicit. In our model, each bank employs a large number of bankers. Funds from the depositors are collected at the bank level and are allocated to individual bankers in such a way that the constraints of the financial contract (equations (4)) are satisfied.¹³ Each banker monitors a single investment project. If the project succeeds, the entrepreneur retains her share of the project returns (R_t^e). The rest of the returns ($R - R_t^e$) are credited to the common account of the bank. If the project fails, neither the entrepreneur nor the bank gets anything. After the returns from all successful projects of the bank are collected, the bank compensates its bankers and refunds depositors according to the financing contract. A banker is paid only if the project that she monitored was successful. In other words, we assume that depositors' claims are senior within a bank; depositors are first paid from the bank's common funds, after which the successful bankers share the remainder.

For brevity, we assume the success probability of the good project is large enough so that a bank never defaults on deposit contracts on the equilibrium path and, hence, in equilibrium deposits are always redeemed at par and the bank's sequential service constraint never binds. As a result, entrepreneurs and depositors always receive their promised share of project returns whereas bankers may get less (in case of a negative shock) or more (in case of a positive shock) than stipulated by the initial financing contract.

¹³This is in the interest of the bank. If an individual banker does not monitor, the remaining bankers within the bank suffer expected losses.

Following an investment shock in period t , the aggregate entrepreneurial capital at the end of period t is given by $\tilde{N}_t(\varepsilon_t) = I_t p_H q_t R_t^e (1 + \varepsilon_t)$. Even though each successful entrepreneur gets her share R_t^e according to the financing contract, the aggregate entrepreneurial capital is reduced (increased) in the aftermath of a negative (positive) investment shock because a smaller (larger) fraction of the entrepreneurs are successful. The evolution of aggregate entrepreneurial capital can be rewritten as

$$\tilde{N}_t(\varepsilon_t) = N_t (1 + r_t^n) (1 + \varepsilon_t), \quad (19)$$

where N_t is (period t) entrepreneurial capital before the investment stage (Stage 2 in Table 1) and $(1 + r_t^n)$ is the expected rate of return to entrepreneurial capital in the investment stage, in the absence of an aggregate investment shock, given by (15b).

In contrast, following an investment shock in period t , the aggregate bank capital at the end of period t is given by $\tilde{A}_t(\varepsilon_t) = I_t p_H q_t [R_t^b + (R - R_t^e) \varepsilon_t]$. Using conditions (4b), (4c) (recalling that $r_t^d = 0$), (4d), (4f), and (9), the evolution of aggregate bank capital can be rewritten as

$$\tilde{A}_t(\varepsilon_t) = A_t (1 + r_t^a) (1 + BL_t \varepsilon_t), \quad (20a)$$

where A_t is (period t) bank capital before the investment stage and $(1 + r_t^a)$ is the rate of return to bank capital in the investment stage, in the absence of an aggregate investment shock, given by (10), while

$$BL_t = 1 + \frac{D_t}{(1 + r_t^a) A_t} \quad (20b)$$

is the *bank leverage accelerator of shocks*. Equation (20b) shows how, compared with the effect of the shock on aggregate entrepreneurial capital, its effect on aggregate bank capital is amplified by the term $D_t / ((1 + r_t^a) A_t)$. For example, in the aftermath of a negative shock, not only do fewer bankers see their projects succeed but each successful banker gets a smaller share of the revenues because of the seniority of depositors' claims. As a result, the higher the bank leverage (the debt-to-equity ratio $D_t / ((1 + r_t^a) A_t)$), the higher the multiplier of the shock.

The different dynamics of entrepreneurial capital and bank capital after an aggregate shock stem from the fact that banks are larger and more diversified than firms: each bank intermediates funding to a large number of firms. The small size of an individual firm protects entrepreneurs as a group against any levered impact of adverse shocks: if an investment project fails, the firm goes bankrupt and the entrepreneur loses her equity, but other entrepreneurs cannot be held accountable for these losses. In contrast, even when a (larger-than-expected) number of investment projects in a bank's portfolio fail, the bank pays its creditors in full and the adverse shocks are absorbed by bankers' equity.

The period $t + 1$ values of entrepreneurial capital and bank capital, before the period $t + 1$ capital good production stage (i.e. after Stage 1, but before Stage 2 of period $t + 1$) are linked to the end-of-period t values by the equations

$$N_{t+1}(\varepsilon_t) = \lambda^e \left(\frac{r_{t+1}^K + (1 - \delta) q_{t+1}}{q_t} \right) \tilde{N}_t(\varepsilon_t) \quad (21)$$

and

$$A_{t+1}(\varepsilon_t) = \lambda^b \left(\frac{r_{t+1}^K + (1 - \delta) q_{t+1}}{q_t} \right) \tilde{A}_t(\varepsilon_t). \quad (22)$$

Here, the only difference in the dynamics of entrepreneurial capital and bank capital derives from the different exit rates of entrepreneurs and bankers (λ^e and λ^b). Although a shock has an asymmetric effect on the sharing of project revenues it does not affect the conditional project returns. Therefore, the effect of the shock on the accumulation of physical capital is again directly related to its effect on project success probability. The aggregate physical capital in period $t + 1$, following an investment shock in period t , is given by

$$K_{t+1}(\varepsilon_t) = (1 - \delta) K_t + I_t (1 + \varepsilon_t).$$

5 Calibration

The principal aim of our calibration exercise is to examine the conditions under which the relative scarcity of bank capital prevails (Proposition 1 and Corollary 1). We follow the RBC

literature in calibrating the parameters of the real block (see Appendix C.1). The upper panel of Table 2 shows the resulting parameters (the period is one year and the parameter values are adjusted accordingly).

Calibration of the parameters of the financial block involves matching the steady state values of the financial variables to empirical moments. Based on the findings in the empirical literature (see Appendix C.1), we set the excess return on entrepreneurial capital (r^n) to 4.5%, non-financial firms' capital-asset ratio (CRF) to 45%, the excess returns on bank capital (r^a) to 12%, banks' capital-asset ratio (CRB) to 8%, and their monitoring cost-asset ratio (MRB) to 1.5%. In Appendix C.2, we show that the parameters of the financial block can be expressed in terms of the matched data moments as follows: $\lambda^e = \frac{\beta}{1+r^n}$, $\lambda^b = \frac{\beta}{1+r^a}$, $\frac{\Delta p}{p_H} = \frac{MRB}{CRB(1+r^a)}$, $\frac{\gamma}{1-\gamma} = \left(\frac{r^a CRB + MRB}{r^n CRF}\right) (1 - CRF)$ and $\Gamma = \left(\frac{1+r^n}{1+r^a}\right) \left(\frac{CRF}{CRB}\right) (1 - CRF)^{\frac{\gamma}{1-\gamma}} MRB^{\frac{1}{1-\gamma}}$.

Parameter	Value	Note
<i>Parameters of the macro block</i>		
β	0.98	discount factor
α	0.33	capital share
δ	0.10	rate of decay of capital
ξ	2	parameter of the disutility of labor
ϕ	0.5	$1/\phi$ Frisch elasticity of labor supply
σ	2	$1/\sigma$ elasticity of intertemporal substitution
<i>Parameters of the financial block</i>		
λ^e	0.9382	survival rate of entrepreneurs
λ^b	0.8754	survival rate of bankers
p_H	0.95	success probability of a good project
$\frac{\Delta p}{p_H}$	0.1674	$\Delta p \equiv p_H - p_L = 0.159$
γ	0.4005	$\frac{\gamma}{1-\gamma}$ elasticity of monitoring function
Γ	0.0032	parameter of monitoring function

Table 2: Calibrated parameter values.

With the calibration based on observed data moments, we can have a new look at the relative scarcity of bank capital. Proposition 1 implies that bank capital is scarce if $\lambda^b/\lambda^e < 1 + \Delta p/p_H$. Our calibration suggests that $\lambda^b/\lambda^e = 0.93$, whereas $1 + \Delta p/p_H = 1.17$. More precisely, equations (C.5) and (C.6) in Appendix C.2 imply that $\lambda^b/\lambda^e = (1 + r^n)/(1 + r^a)$.

Hence, the relative excess returns reflect relative scarcity. Next, using the banker's incentive constraint (4b) and her participation constraint (4d), together with the aggregation equation (9), all evaluated at the steady state, yields $\Delta p/p_H = mI/((1+r^a)A)$. This expression has a natural interpretation. Monitoring costs mI constitute a part of the cost of financial intermediation, and unlike the return to bank capital $(1+r^a)A$, this part of the cost of intermediation does not translate into new banker-owned capital. As argued in Section 3, this is one reason why bank capital is scarce in equilibrium. While these observations are useful for interpreting Proposition 1, equation (C.7) in Appendix C.2 (re)expresses $\Delta p/p_H$ in terms of the data moments we match: $\Delta p/p_H = MRB/((1+r^a)CRB)$. In sum, the relative scarcity of bank capital prevails as long as

$$r^n < r^a + \frac{MRB}{CRB}.$$

This condition, which only includes data moments we match, is likely to hold: for example, Hirtle and Stiroh (2007) and Albertazzi and Gambacorta (2009) estimate the return on bank equity (r^a) to be 12–14% in the US, whereas the average return on firm capital (r^n) is typically estimated to be much lower (see, for example, Fama and French, 2002).

Figure 1 shows the impulse responses of some key real and financial sector variables to a negative investment shock. (The impulse responses of the whole set of variables to an investment shock are given in Appendix C.3.) As a (first-best) benchmark, we show the impulse responses of the macro variables for the standard RBC model.¹⁴ In the RBC model, the shock has small effects. There is a little less physical capital after a negative investment shock, and this slightly lowers the production capacity and output of the economy. Investments increase a little to restore the lower-than-anticipated capital stock.

In our model with banks, investment falls and financial intermediation greatly amplifies the impact of the investment shock on aggregate investment and output. The reason is threefold. First, as shown in Section 2.2, aggregate investment scale depends on bank capital and firm capital. Second, as discussed in Section 3, as well as in the present section, bank

¹⁴Note that equations in Section 2.1, together with $q_t = 1 \forall t$ and $Y_t = C_t + I_t$, provide a full description of the textbook RBC model.

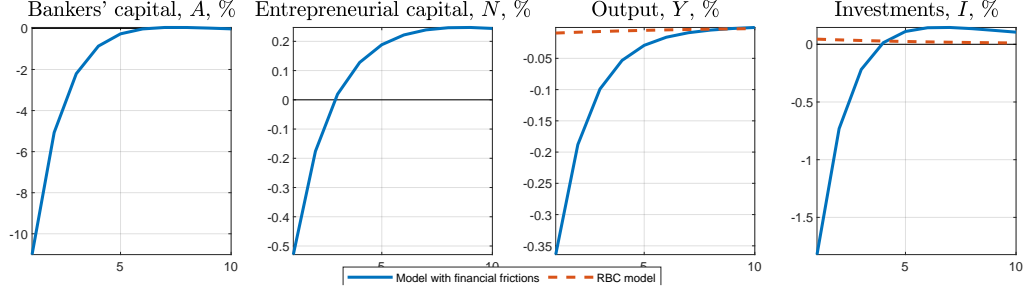


Figure 1: Impulse responses to a negative investment shock (1 percentage point decrease in success probabilities). Horizontal scale refers to years.

capital is scarce relative to entrepreneurial wealth: a change in bank capital has a larger effect on aggregate investment than an equal change in entrepreneurial wealth. Third, as explained in Section 4, an investment shock has a strong effect on bank capital. Also output (the production of the representative good) drops significantly. This is because the capital good producing entrepreneurial firms demand less of the representative good, which they use as an input. Due to lower demand, output declines in equilibrium. Financial frictions amplify the effect of the shock on other macro variables as well (see Figure 5 in Appendix C.3).

6 Public Funding of Banks and Firms

We characterize the optimal allocation of public support between banks and non-financial firms in a time of crisis. In our model, debt and equity are distinct components of banks' capital structure, and we model government support of banks to take place in terms of equity injections. In the case of non-financial firms, debt and (outside) equity are indistinguishable, as in Holmström and Tirole (1997).¹⁵ Hence, both debt and equity interpretations of public funding to non-financial firms are possible.

We assume that the private sector continues to run and monitor the investment projects even in the presence of government support. The government demands for its investments an expected rate of return of $1 + r_t^g$ which includes a penalty relative to the market rate,

¹⁵If the investment project succeeds, the firm can pay both creditors and equity holders. If the project fails the firm can pay neither.

$r_t^g > 0$. Otherwise, banks and firms would want to be funded by the government, resulting in the standard intertemporal moral hazard associated with government bailouts. Also, the need of politicians to signal their toughness to voters may motivate such a penalty rate in practice. Public funding is provided for a single period and injected before the financial contracts are signed. Participation in the government funding program is mandatory for all banks and firms. Although the mandatory participation here is a simplifying assumption, it provides a means to mitigate potential problems arising from the stigma associated with the use of government funds with a penalty rate.¹⁶

The main insights from the analysis of optimal policies are summarized in three remarks and one proposition. Remark 1 characterizes the (relative) incentive distortions and social welfare losses caused by public bank and firm funding. Remark 2 establishes that the social welfare benefits from a more resilient financial system are proportional to the size of the funding program and (ex post) to the size of the shock, but do not depend on its structure. Remark 3 tells how a program of a given size can be constructed with different combinations of public bank and firm funding. These remarks lead to the main result of the paper, Proposition 2 which shows how the optimal structure of a funding program depends on its size.

6.1 Social Costs of Public Funding: Distorted Incentives

We show in Appendix B.1 that the public funding of banks and non-financial firms results in an aggregate investment level (I_t^*) that is implicitly given by the equation

$$\left(\frac{A_t - r_t^g A_t^g}{I_t^*} + \gamma \rho_t \right)^\gamma \left(\frac{N_t - r_t^g N_t^g}{I_t^*} + (1 - \gamma) \rho_t \right)^{1-\gamma} = \left(\Gamma \frac{p_H}{\Delta p} \right)^{1-\gamma} \left(1 + \frac{p_H}{\Delta p} \right)^\gamma, \quad (23)$$

in which $A_t^g \geq 0$ and $N_t^g \geq 0$ denote the aggregate amounts of government funds injected into banks and entrepreneurial firms, respectively.

¹⁶The penalty rate is in line with Bagehot's dictum. As explained by Bernanke (2015, p.148): "...the penalty rate encourages banks to look first to private markets for funding, rather than relying on the Fed. But a side effect of this arrangement was that banks feared they would look weak if it became known that they had borrowed from the Fed — and that would make it even harder from them to attract private funding."

Equation (23) is identical to equation (13) save for the negative terms $-r_t^g A_t^g$ and $-r_t^g N_t^g$ in the numerators of the left-hand side. Thus, public funding of banks and non-financial firms *lowers* the aggregate investment level. Since $r_t^g > 0$, government-owned capital dilutes bankers' and entrepreneurs' stakes in the projects and, consequently, their incentives to monitor and invest. Bankers' weaker monitoring incentives make bank participation costlier for entrepreneurs, further reducing their investment incentive.

Since lower investments imply lower welfare (see Section 3), the government keeps the premium on its funding small to minimize these adverse welfare effects of public funding. Assuming that $r_t^g = dr^g$ is small, and totally differentiating (23) at the steady state, yields

$$\left| \frac{N^g}{A^g} \right|_I = \frac{-\frac{dI}{dA} dr^g}{-\frac{dI}{dN} dr^g} = \left| \frac{dN}{dA} \right|_I = MRTS = \frac{1 - \frac{\lambda^b}{\beta} + \frac{\Delta p}{p_H}}{\left(1 + \frac{\Delta p}{p_H}\right) \left(1 - \frac{\lambda^e}{\beta}\right)},$$

in which we use the definition $MRTS$ familiar from equation (18) and Corollary 1. Here $MRTS$ tells how many units of public funds in non-financial firms corresponds a unit of public funds in banks in terms of welfare losses. Since $MRTS > 1$ is likely to hold (see Sections 3 and 5), public funding invested in banks tends to be more distortionary than when invested in non-financial firms. This property reflects the scarcity of banker-owned capital near the steady state (Proposition 1, Corollary 1): capital injections into banks dilute bankers proportionally more than corresponding injections into firms dilute entrepreneurs. Our baseline calibration yields $MRTS = 5.5$.

Besides its distortionary effects in the current period, public funding, which commands a premium, is costly for bankers and entrepreneurs in terms of lower revenues and wealth in the subsequent periods. We assume that the government grants the entrepreneurs and bankers a lump-sum refund from its premium revenues. Such a refund eliminates the harmful effects of lower insider wealth on future investments, while still keeping public funding unattractive for individual entrepreneurs and bankers, and hence avoiding intertemporal moral hazard – see Appendix B.1.3.

Thanks to this elimination of harmful effects of public funding on the future periods, all welfare losses from public funding are transmitted through its distorting effects on current

investments. We can then apply the result established in equation (18) and Corollary 1 to analytically compare the funding of banks and firms in terms of welfare losses. We summarize the key finding of this subsection by the following remark:

Remark 1 *Public stakes in banks and firms distort incentives which lowers welfare. In terms of the welfare losses, each unit of public funds in banks equals MRTS units of public funds in firms where MRTS is likely to be larger than one.*

6.2 Social Benefits of Public Funding: Enhanced Resilience

Although public funding causes distortions, it also renders the financial system more resilient to negative aggregate shocks. In Appendix B.1.4, we show that when the government provides funding for banks (A_t^g) and non-financial firms (N_t^g), the dynamics of non-government owned bank capital after an investment shock (ε_t) is characterized by equation (22) in which $\tilde{A}_t(\varepsilon_t)$ and BL_t of equations (20a) and (20b), respectively, are replaced by

$$\tilde{A}_t^*(\varepsilon_t) = A_t(1 + r_t^a)(1 + BL_t^*\varepsilon_t) \quad (24a)$$

and

$$BL_t^* = 1 + \frac{D_t - A_t^g - N_t^g}{(1 + r_t^a)A_t + A_t^g}. \quad (24b)$$

Comparison of equations (20b) and (24b) suggests that government funding of banks lowers their leverage because the total bank equity is enhanced, thanks to equity A_t^g purchased by the government (see the denominator of the last term of equation (24b)), and because government-owned capital A_t^g crowds out debt funding from households (the numerator of the last term of equation (24b)).¹⁷ Public funding of non-financial firms N_t^g also lowers bank leverage because it, too, crowds out debt funding from households. However, financing of non-financial firms does not strengthen the equity buffer of the banking system.

¹⁷The expression (24b) holds as an approximation in the sense that $BL(A_t^g, N_t^g, dr_t^g)\varepsilon_t$ (the effect of the shock on banker-owned capital, see equation (24a)) is equal to $BL^*(A_t^g, N_t^g)\varepsilon_t$ plus additional terms which are proportional to the product $dr_t^g \times \varepsilon_t$. We assume that these additional terms are so small that they can be ignored (see Appendix B.1.4).

Public funding reduces the banks' and non-financial firms' possibilities to borrow from households because a part of the banks' and firms' revenues need to be pledged to the government. If the government capitalizes banks, their lending to non-financial firms does not change, but government-owned equity replaces banks' borrowing from households. If the government funds non-financial firms, they borrow less from banks, which in turn borrow less from households.

Equations (24a) and (24b) show how public funding changes the structure of bank balance sheets, making the banking sector more resilient to negative aggregate shocks. Since banks are large and diversified, aggregate shocks have a levered impact on their equity (see Section 4). Public funding lowers the shock accelerator on the bank side. However, public funding has no impact on the vulnerability of the entrepreneurial sector as a whole to the aggregate shock. Entrepreneurial firms are small and non-diversified, and limited liability protects entrepreneurs as a group against negative spillovers. While a negative aggregate investment shock increases entrepreneurial failures, there is no shock accelerator. The loss of aggregate entrepreneurial wealth is proportional to the number of failing projects, and hence to the size of the negative aggregate shock, irrespective of whether the entrepreneurs are funded by banks or by the government. Similarly, public funding leaves the dynamics of capital stock intact, since the accumulation of new capital also depends on the share of failing investment projects, and is therefore directly related to the size of the investment shock.

Since public funding only affects the resilience of banks, the welfare benefits of public funding are focused on the banking sector. We show in Appendix B.2 that the welfare benefits of public funding, in the face of a negative macro shock $\varepsilon_t < 0$, are given by the measure

$$\Delta \tilde{V} = -\tilde{V}_{\tilde{A}} BRE_t S_t \varepsilon_t, \quad (25)$$

in which $\tilde{V}_{\tilde{A}} > 0$ is the steady state value of the derivative of the households' value function with respect to the bankers' end-of-period capital, $BRE_t = (1 + r_t^a) BL_t A_t$ is the *banks' risk*

exposure to a (negative) macro shock under *laissez-faire*, and

$$S_t = 1 - \frac{BL_t^*}{BL_t} \quad (26)$$

measures the share of the risk exposure shifted from banks to the government.

Equation (25) shows how public funding makes banks less vulnerable to a negative aggregate investment shock and therefore reduces the welfare losses due to the shock: $\Delta \tilde{V} > 0$ if $\varepsilon_t < 0$. Government-owned capital absorbs part of the loan losses and, hence, mitigates the bankers' loan losses. After a negative shock hits, banks are better capitalized than they would be under *laissez-faire* and can lend more to non-financial firms which, as a result, can invest more than they could in a *laissez-faire* scenario. Since investments are suboptimally low (see Section 3), this improves welfare, compared to *laissez-faire*.

According to equation (25) the social benefits of the program are proportional to the measure S_t , which can take values between 0 and $1 - 1/BL_t$ (where the maximum S_t corresponds to $BL_t^* = 1$). We can also call S_t the *size* of the public funding program. A public funding program of, say, size $S_t = 0.2$ means that the government takes over 20% of banks' total macro risk exposure — and the program reduces bankers' loan losses by 20% following a negative macro shock.

We show in Appendix B.3 that the fiscal costs of the funding program are given by $FC_t = -BRE_t S_t \varepsilon_t$, which measures the government's losses from a public funding program of size S_t if a negative shock $\varepsilon_t < 0$ hits the economy. If the fiscal costs have to be covered by distortionary taxes, the resulting welfare losses from taxation depend on FC_t . Hence, although modeling distortionary taxes is beyond the scope of this paper, combining this observation with equation (25) implies that the net welfare benefits of the program, net of possible losses from distortionary taxation, depend on the size of the program, S_t .

The following remark summarizes the main result of this subsection:

Remark 2 *A public funding program makes the economy more resilient in the face of a negative macro shock. Welfare gains of the program are proportional to the size of the program and to the size of the realized macro shock.*

Equation (25) also implies that if there is a *positive* aggregate shock ($\varepsilon_t > 0$), public funding *lowers* social welfare: $\Delta\tilde{V} < 0$ if $\varepsilon_t > 0$. Since public funding distorts incentives irrespective of the sign of ε_t (see Section 6.1), it is clear that the government should consider financial support of banks and firms only in times of economic hardship.

6.3 The Policy Frontier

To relate the size of the program S_t to the public funding of banks and firms, we show in Appendix B.3 that

$$BRE_t S_t = BL_t^* A_t^g + N_t^g. \quad (27)$$

Hence, a program of size S_t can be implemented with any combination of A_t^g and N_t^g satisfying equation (27). We call the different combinations of A_t^g and N_t^g satisfying equation (27) the *policy frontier*. A particular combination of A_t^g and N_t^g from the policy frontier defines the *structure* of the program.

In equation (27), BL_t^* acts as (the absolute value of) the slope of the policy frontier: in terms of resilience, each unit of A_t^g corresponds to $BL_t^* \geq 1$ units of N_t^g . The relative efficiency of bank capitalization in stabilization reflects the effects captured by equation (24b). Bank capitalization (A_t^g) makes the banks less vulnerable to a negative aggregate shock both by strengthening their equity cushion and by reducing the amount of outside debt on the liability side of their balance sheets. If the government funds firms, they borrow less from banks which in turn borrow less from households. Hence public firm funding essentially makes banks less vulnerable to aggregate shocks by reducing the asset side of their balance sheets, i.e. the banks' firm loan portfolios. Note that BL_t^* is not only the slope of the policy frontier, it also measures bank leverage when public policy is place. It is intuitive that a policy that essentially works through the asset side of bank balance sheets (firm funding) requires BL_t^* times larger public stakes than a policy which boosts bank equity (bank capitalization). Importantly, the slope of the policy frontier depends on the size of the program: we can rewrite equation (26) as $BL_t^* = BL_t(1 - S_t)$. To summarize, we have the following remark:

Remark 3 *In terms of the resilience of the economy, each unit of public funding in banks equals $BL_t^* = BL_t(1 - S_t)$ units of public funding in non-financial firms.*

6.4 Optimal Structure of Public Funding: Targeting Banks or Firms?

We take the size of a public funding program (S_t) as predetermined, e.g., by a political process, and characterize an optimal structure of the program: How should the public funds be allocated between banks and non-financial firms? Our choice to focus on the optimal structure with a given size is motivated by uncertainty faced by the government in a time of crisis. Genuine uncertainty might even prevent the government from assigning a distribution to a negative macro shock, which would make the determination of an optimal S_t impossible. If we assume away genuine uncertainty, finding the optimal level of S_t would imply balancing the welfare gains from stabilization against the welfare losses from distorted incentives and from distortionary taxes (which we do not model). Alternatively, since the welfare benefits of stabilization and the possible welfare losses due to distortionary taxes are both proportional to the program size (Remark 2) and independent of its structure, we may regard S_t as being fixed at a desired level¹⁸ and seek an optimal structure of the funding program that minimizes its incentive distortions. In Section 6.5 we discuss how the optimal structure of the public funding program relates to the notion of dominated and undominated policies.

Considering an economy near the steady state and taking S as given, we maximize welfare with respect to the structure of the public funding program. Our main result can be stated as follows.

Proposition 2 *Assume that $1 < MRTS < BL$. There exists a threshold value $S^* = 1 - MRTS/BL$ such that $0 < S^* < 1 - 1/BL$. a) If $S < S^*$, it is optimal to capitalize banks, $A^g = S \times BRE/[(1 - S)BL]$ and $N^g = 0$. b) If $S > S^*$, it is optimal to fund non-financial firms, $A^g = 0$ and $N^g = S \times BRE$.*

¹⁸Although modeling distortionary taxes is beyond the scope of this paper, one may think, for example, that the government knows the tax system in place and can assess its distortionary impact. Assuming that there is no genuine uncertainty, the government may find the optimal trade-off between (expected) welfare gains from resilience and (expected) welfare losses from distortionary taxes and distorted incentives imposed on banks, entrepreneurial firms or both.

Proof. See Appendix B.4 ■

Thus, if the size of the funding program is below the threshold value S^* , the government should target banks, while if the size of the program is above the threshold, the government should target non-financial firms. The threshold value S^* is increasing in the *sensitivity* of bank capital to macro shocks under *laissez-faire* (captured by BL) and decreasing in the relative *scarcity* of bank capital (captured by $MRTS$). We show in Appendix B.5 that in any equilibrium (or model calibration) with a meaningful role for financial intermediation, $MRTS/BL < 1$. Intuitively, the opposite case $MRTS/BL > 1$, characterized by scarce bank capital and a banking system struggling to raise outside funding, would correspond to a situation (or model calibration) where financial intermediation is so ineffective and costly that the entrepreneurial firms choose to bypass the banking system and raise funding directly from households.¹⁹ Thus, if the condition stated in Proposition 1 holds and $MRTS > 1$, the optimal policy does indeed consist of two regimes, with a threshold value $S^* \in (0, 1 - 1/BL)$. Our baseline calibration with $MRTS = 5.5$ and $BL = 11.4$ yields $S^* = 0.52$. Hence, the threshold S^* corresponds to the government taking over roughly half of the banks' macro risk exposure. In Appendix B.5, we present equations linking $MRTS$, BL and S^* to the data moments used in the calibration of the model, and conduct robustness analysis with respect to the calibration (see Table 3 in Appendix B.5).

Using equation (26), we may rewrite the condition $S \leq S^*$ as $MRTS \leq BL^*$. To interpret this condition, think of implementing a program of size S by funding either banks or firms. Given Remark 3, BL^* times smaller public ownership stakes are needed if the government targets banks rather than non-financial firms. However, since each unit of public funds creates $MRTS$ times larger distortions in banks (Remark 1), the relative welfare losses from distortions are given by $\widehat{WLD} = WLD^{A^g}/WLD^{N^g} = MRTS/BL^*$, where WLD^{A^g} and WLD^{N^g} denote welfare losses from distorted incentives when the government targets banks and firms, respectively. Evidently, the program should target banks if $\widehat{WLD} < 1$, and non-financial firms if $\widehat{WLD} > 1$. Hence, the choice boils down to comparing $MRTS$

¹⁹This is the corner solution with $m_t = 0$ and $h_t = \bar{h}$.

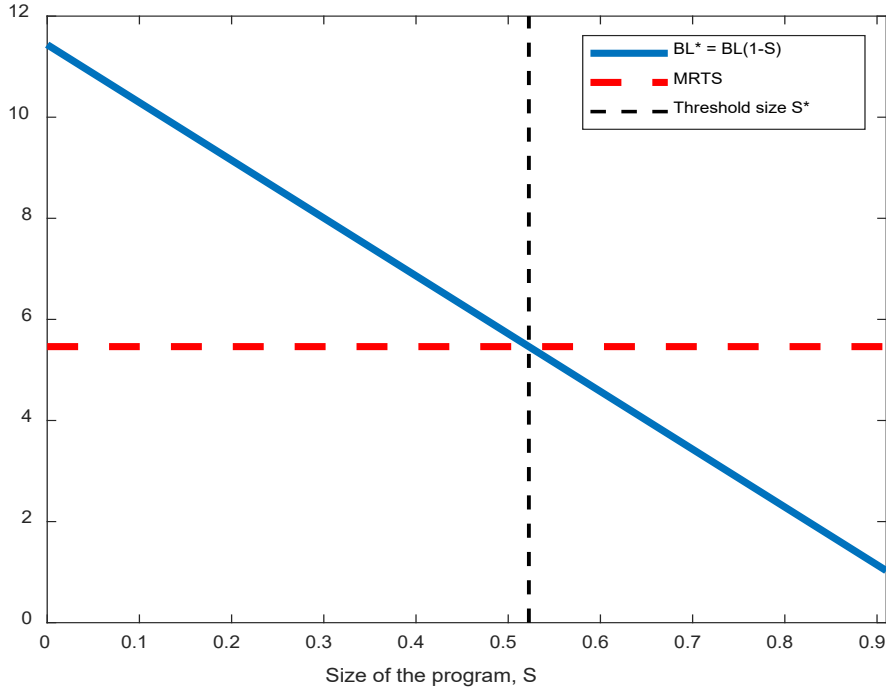


Figure 2: The optimal structure of a program depends on its size. If BL^* is larger (smaller) than $MRTS$ it is optimal to fund banks (firms).

and BL^* . See Figure 2.

The size of the program matters for the optimal structure because (the absolute value of) the slope of the policy frontier equals the target level of the bank leverage accelerator, which in turn decreases with the size of the program $BL^* = BL(1 - S)$ — see Remark 3 and Figure 2. Recall that the government can lower the bank leverage accelerator to the target level BL^* either by i) strengthening the banks' balance sheets through public bank capitalization or i) reducing the size of banks' balance sheets, and the exposure of banks to non-financial firms, by providing public funding directly to non-financial firms. Alternative ii), which essentially works through the asset side of bank balance sheets requires BL^* times larger public (ownership) stakes than alternative i), which works through the equity buffer. This is intuitive as BL^* also measures the assets-to-equity ratio of banks when policy ii) is in place. The more (fewer) assets there are compared to equity, the more (less) effective, in relative terms, the policy that strengthens equity. The banks' assets-to-equity ratio, however, decreases with the size of the program.

6.5 Dominated and Undominated Policies

Proposition 2 may also be interpreted as characterizing the sets of *dominated* and *undominated* policies (for a formal proof, see Appendix B.4). A policy (A^g, N^g, S) dominates an alternative policy $(A^{g'}, N^{g'}, S')$ if (A^g, N^g, S) generates as high social welfare as $(A^{g'}, N^{g'}, S')$ for all shock realizations $\varepsilon_t < 0$, and strictly higher welfare for some shock realizations. A policy (A^g, N^g, S) is undominated, if and only if there is no alternative that dominates it.

The set of undominated policies and the set dominated policies are characterized by the mapping from the size of a program to its appropriate structure, given in Proposition 2. Since i) the distortions and the welfare losses caused by a public funding program depend on its structure while ii) the welfare benefits of the program depend on its size and (ex-post) on the shock realization (the larger the program and the larger the negative shock realization, the larger the benefits), but are independent of the structure, the program can belong to the set of undominated policies if and only if its structure minimizes the welfare losses from distortions, given the size of the program. If not, social welfare (after any shock realization) could be raised by keeping the size of the program fixed but re-optimizing its structure — meaning that the policy is dominated.

Proposition 2 implies that small funding programs ($S < S^*$) targeting banks and large programs ($S > S^*$) targeting firms are undominated. If the government implements one of these programs, there does not exist an alternative program that would generate as high or higher social welfare after all (negative) shock realizations. By contrast, any small public funding program ($S < S^*$) targeting firms is dominated by a program of the same size but targeting banks, while any large program ($S > S^*$) funding banks is dominated by a program of the same size but directed at firms. Similarly, all mixed programs, simultaneously funding both banks and firms, are dominated by pure programs with the appropriate structure described in Proposition 2.

These results are useful for decision making under genuine uncertainty, as they indicate which funding policies should be taken into consideration and which should be avoided. A benevolent, social-welfare-maximizing government should choose a policy that belongs to

the undominated set. It should not implement a dominated policy, no matter what its beliefs about the probability and severity of the crisis are. See Appendix B.6 for an illustrative numerical example.

6.6 Welfare Losses from Non-optimal Structure of Public Funding

Assume that the government fails to follow the prescriptions of Proposition 2 and implements a relatively small funding program ($S < S^*$) for non-financial firms or a relatively large program ($S > S^*$) for banks, or, alternatively, finances both banks and firms simultaneously. How significant are the welfare losses resulting from such non-optimal structures?

We examine this question in Appendix B.7. We find that if the program size S is around the threshold value S^* , it does not matter much whether the government allocates funds to banks, firms, or both. However, the larger the gap between S^* and S , the larger the excess welfare losses from a non-optimal structure (see Figures 3 and 4 in Appendix B.7). From a policy perspective, the result is reassuring. The government may not know for sure the exact value of the threshold S^* , which may make it difficult to identify the optimal structure. However, in situations where the welfare losses from a suboptimal structure are larger, it should be easier to follow the prescriptions of Proposition 2.

Furthermore, we show in Appendix B.7 that funding simultaneously both banks and firms creates a smaller excess welfare loss than funding exclusively either banks or firms with a wrong amount. Thus, if the government is uncertain about the desired size of the program, choosing a mixed program may be a robustly optimal strategy. (Robust optimization here means minimizing the maximum excess welfare loss due to a suboptimal structure.) In Appendix B.8, we study the implementation of such a robust optimal mixed program under two scenarios: i) The government does not know S^* for sure. ii) The government implements a staggered program which can initially be smaller than S^* , but has an option to expand the program so that eventually $S > S^*$. We show that in a robust optimal mixed program the share of funds allocated to non-financial firms relative to banks should be increasing in S , a finding that echoes the results of Proposition 2.

7 Conclusions

In this paper we develop a macro-finance model, in which both banks' and firms' balance sheets matter. We show that in equilibrium, bank capital tends to be scarce, compared to firm capital. Then, a given change in bank capital has a larger impact on aggregate investment than a corresponding change in firm capital. We also show that bank capital is more sensitive to aggregate shocks than firm capital.

Public funding affects the incentives and the balance sheet structures of banks and firms. Our main result links the socially optimal composition of a crisis funding program to its size. Small programs should target banks, while large programs should be directed at non-financial firms. With our baseline calibration, the threshold value between “small” and “large” programs is 0.5, meaning that programs in which the government takes over more (less) than 50% of the macro risk from the private sector should be implemented through firm (bank) funding.

The result reflects the relative scarcity and sensitivity of bank capital. Due to the scarcity of bank capital, public funding distorts incentives more when placed in banks than when placed in non-financial firms. Given the sensitivity of bank capital, however, smaller public stakes are needed in banks than in firms to stabilize the economy. Finally, the relative effectiveness of bank capitalization in stabilizing the economy depends on the size of the program. Initially, capital injections to banks have a large proportional effect on the resilience of the financial system, but this effect diminishes if the government takes over a larger share of the macro risk.

In terms of policy design under genuine uncertainty, a key contribution of this paper is to characterize the menu of crisis funding programs from which the government should choose: which policies should be on the table and which should be off the table. Our results imply that small funding programs targeting banks and large programs targeting firms are on the menu since they belong to the set of undominated policies. The government should pick its policy from this set. Conversely, small funding programs targeting firms and large funding programs targeting banks are off the table, since they are dominated.

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A Model

A.1 Timing of Events

Within each period t there are two main stages. At the beginning of Stage 1, the survival probabilities of bankers and entrepreneurs are realized and exiting bankers and entrepreneurs give their accumulated assets to households. Household members then separate into their occupations, the heads of households make their consumption-savings decisions, and final goods are produced using capital and labor.

The production of capital goods takes place in Stage 2, which is divided into five sub-stages: First, financing contracts among entrepreneurs, bankers and depositors (workers) are signed. These contracts determine whether and how the project is financed, its size, and how eventual revenues are divided. Depositors place their funds in banks, who extend funding to entrepreneurs according to the financing contract. Second, bankers choose their intensity of monitoring. Third, entrepreneurs choose their projects. Fourth, successful projects yield new units of capital goods that are sold. Finally, the proceeds are divided among depositors, bankers and entrepreneurs according to the terms of the financial contract.

While entrepreneurs are assumed to sell the capital goods that they produce, note that our equations in Section 2.1 show that final good firms rent (rather than own) the capital stock that they need in production. This is consistent with the existence of perfectly competitive capital rental firms, fully owned by households. These capital rental firms purchase capital goods from successful entrepreneurs, rent capital services to final goods firms, and refund the rental income to their owners. Note also that, as in Holmström and Tirole (1997), bankers can commit to monitoring before entrepreneurs make their project choice. This sequential timing rules out mixed strategy equilibria.

A.2 Investment Size at Project Level

In this appendix, we derive equations (5a) and (5b). From the entrepreneur's and banker's incentive constraints, (4a) and (4b), we see that the entrepreneur and the banker must get

no less than $R_t^e = \frac{h(m_t)}{q_t \Delta p}$ and $R_t^b = \frac{m_t}{q_t \Delta p}$ respectively, in case of success, as otherwise they will misbehave. Then the return-sharing constraint (4f) shows that depositors can be promised at most

$$R_t^w = R - \frac{m_t + h(m_t)}{q_t \Delta p}. \quad (\text{A.1})$$

Substituting equation (A.1) for the depositor's participation constraint (4c) yields

$$p_H \left\{ q_t R - \frac{m_t + h(m_t)}{\Delta p} \right\} = \frac{d_t}{i_t}. \quad (\text{A.2})$$

Next, we combine the banker's incentive constraint (4b) with his participation constraint (4d) and the input resource constraint (4e) to obtain

$$\frac{d_t}{i_t} = 1 + m_t - \frac{p_H}{\Delta p} \left(\frac{1}{1 + r_t^a} \right) m_t - \frac{n_t}{i_t},$$

which can be then substituted for equation (A.2). Solving the resulting equation for i_t gives equation (5a) and expression (5b).

A.3 Steady State Structure of Informed Capital

In this appendix, we derive equation (16). Substitution of the incentive constraints (4a) and (4b), together with equation (7) and $r^d = 0$ for law of motion of A_{t+1} and that of N_{t+1} (see the derivation of equations (14) and (15a)) gives

$$A_{t+1} = \frac{[r_{t+1}^K + q_{t+1}(1 - \delta)]}{q_t \Delta p} p_H \lambda^b m_t I_t$$

and

$$N_{t+1} = \frac{[r_{t+1}^K + q_{t+1}(1 - \delta)]}{q_t \Delta p} p_H \lambda^e \Gamma m_t^{-\frac{\gamma}{1-\gamma}} I_t.$$

Thus, in a steady state we must have

$$A = \left(\frac{r^K}{q} + 1 - \delta \right) \frac{p_H}{\Delta p} \lambda^b m I \quad (\text{A.3})$$

and

$$N = \left(\frac{r^K}{q} + 1 - \delta \right) \frac{p_H}{\Delta p} \lambda^e \Gamma m^{-\frac{\gamma}{1-\gamma}} I. \quad (\text{A.4})$$

Here and in what follows we denote a steady state of some time-dependent variable X_t by X , i.e., $\lim_{t \rightarrow \infty} X_t = X$. Dividing equation (A.3) by equation (A.4) implies that

$$\nu \equiv \frac{A}{N} = \frac{\lambda^b m^{\frac{1}{1-\gamma}}}{\lambda^e \Gamma}. \quad (\text{A.5})$$

Next, substitution of equation (10) for equation (8) yields, after some algebra, the steady state value of m as

$$m = \frac{\gamma \rho + \frac{A}{I}}{1 + \frac{p_H}{\Delta p}}. \quad (\text{A.6})$$

Equation (13) can be rewritten at a steady state as

$$\frac{\gamma \rho + \frac{A}{I}}{1 + \frac{p_H}{\Delta p}} = \left[\frac{\frac{p_H}{\Delta p} \Gamma}{(1 - \gamma) \rho + \frac{N}{I}} \right]^{\frac{1-\gamma}{\gamma}}. \quad (\text{A.7})$$

Combining equations (A.6) and (A.7) and solving for ρ yields

$$\rho = \frac{1}{1 - \gamma} \left(\frac{p_H}{\Delta p} \Gamma m^{-\frac{\gamma}{1-\gamma}} - \frac{N}{I} \right). \quad (\text{A.8})$$

Inserting equation (A.8) into (A.6) gives

$$m \left(1 + \frac{p_H}{\Delta p} \right) = \frac{\gamma p_H \Gamma}{(1 - \gamma) \Delta p} m^{-\frac{\gamma}{1-\gamma}} + \frac{A}{I} - \frac{\gamma N}{(1 - \gamma) I}.$$

After substituting equations (A.3) and (A.4) for the above formula, we obtain

$$1 + \frac{\Delta p}{p_H} = \Gamma m^{-\frac{1}{1-\gamma}} \left[\frac{\gamma}{1 - \gamma} + \lambda^e \left(\frac{r^K}{q} + 1 - \delta \right) \left(\frac{\lambda^b m^{\frac{1}{1-\gamma}}}{\lambda^e \Gamma} - \frac{\gamma}{1 - \gamma} \right) \right].$$

By using the definition of ν from equation (A.5), this can be rewritten as

$$\nu \frac{\lambda^e}{\lambda^b} \left(1 + \frac{\Delta p}{p_H} \right) = \frac{\gamma}{1 - \gamma} + \lambda^e \left(\frac{r^K}{q} + 1 - \delta \right) \left(\nu - \frac{\gamma}{1 - \gamma} \right).$$

Solving for ν from the above equation gives

$$\nu = \left(\frac{\gamma}{1-\gamma} \right) \left[\frac{\frac{1}{\lambda^e} - \frac{r^K}{q} - 1 + \delta}{\frac{1}{\lambda^b} \left(1 + \frac{\Delta p}{p_H} \right) - \frac{r^K}{q} - 1 + \delta} \right]. \quad (\text{A.9})$$

Finally, from the household's Euler equation (3), we see that in steady state we must have

$$\beta = \frac{q}{r^K + (1-\delta)q}. \quad (\text{A.10})$$

Using equation (A.10), equation (A.9) can be rewritten as

$$\nu = \left(\frac{\gamma}{1-\gamma} \right) \left[\frac{\frac{\beta}{\lambda^e} - 1}{\frac{\beta}{\lambda^b} \left(1 + \frac{\Delta p}{p_H} \right) - 1} \right]. \quad (\text{A.11})$$

It is evident that $\nu > 0$ if the condition

$$\beta > \max \{ \lambda^e, \lambda^b \} \quad (\text{A.12})$$

holds.

A.4 Investment Maximizing Structure of Informed Capital

In this appendix, we derive equation (17). Let $G_t = (A_t + N_t)/I_t$. We study the following (dual) problem. We take the level of aggregate investment I_t as given. We seek the value of ν_t that maximizes the aggregate leverage $1/G_t = I_t/(A_t + N_t)$ and by implication, minimizes the aggregate amount of informed capital $A_t + N_t$ needed for a given level of aggregate investment. Using $A_t/I_t = \nu_t G_t/(1+\nu_t)$ and $N_t/I_t = G_t/(1+\nu_t)$ (and recalling that $r_t^{d*} = 0$), we can rewrite equation (13) — which determines the equilibrium aggregate investment level I_t — as

$$\left(\frac{\nu_t G_t}{1+\nu_t} + \gamma \rho_t \right)^\gamma \left[\frac{G_t}{1+\nu_t} + (1-\gamma) \rho_t \right]^{1-\gamma} = \left(\frac{\Gamma p_H}{\Delta p} \right)^{1-\gamma} \left(1 + \frac{p_H}{\Delta p} \right).$$

Differentiating this equation with respect to G_t^* and ν_t gives

$$\left. \frac{dG_t}{d\nu_t} \right|_{I_t} = \frac{G_t \left\{ 1 - \gamma - \left(\frac{\nu_t G_t}{1 + \nu_t} + \gamma \rho_t \right)^{-1} \left[\frac{G_t}{1 + \nu_t} + (1 - \gamma) \rho_t \right] \gamma \right\}}{(1 + \nu_t) \left\{ \left(\frac{\nu_t G_t}{1 + \nu_t} + \gamma \rho_t \right)^{-1} \left[\frac{G_t}{1 + \nu_t} + (1 - \gamma) \rho_t \right] \gamma \nu_t + 1 - \gamma \right\}}. \quad (\text{A.13})$$

The aggregate leverage is maximized when G_t^* is minimized. A potential minimum is obtained if the term in the curly brackets in the numerator on the right-hand side of equation (A.13) is zero, i.e., if

$$\frac{\frac{\nu_t}{1 + \nu_t} G_t + \gamma \rho_t}{\frac{G_t}{1 + \nu_t} + (1 - \gamma) \rho_t} = \frac{\gamma}{1 - \gamma}.$$

This equation simplifies to

$$\nu_t = \frac{\gamma}{1 - \gamma} \equiv \nu^{**}.$$

From equation (A.13), we then observe that $dG_t/d\nu_t|_{I_t} < 0$ for $\nu_t < \nu^{**}$ and $dG_t/d\nu_t|_{I_t} > 0$ for $\nu_t > \nu^{**}$. Therefore, ν^{**} characterizes the value of ν_t that minimizes G_t and thereby maximizes the aggregate leverage and output.

A.5 Calculation of Marginal Rate of Technical Substitution

Differentiating (13) with respect to A_t and N_t gives

$$\left. \frac{dN_t}{dA_t} \right|_I = - \left(\frac{\gamma}{1 - \gamma} \right) \left[\frac{\frac{N_t}{I_t} + (1 - \gamma) \rho_t}{\frac{A_t}{I_t} + \gamma \rho_t} \right].$$

Evaluating this at a steady state and using equations (A.8) and (A.6) in the numerator and the denominator of the term in the square brackets, respectively, yields, after some algebra,

$$\left. \frac{dN}{dA} \right|_I = - \left(\frac{\gamma}{1 - \gamma} \right) \left(\frac{\Gamma m^{-\frac{1}{1 - \gamma}}}{1 + \frac{\Delta p}{p_H}} \right).$$

Using equation (A.5) to substitute $\lambda^b/(\lambda^e\nu)$ for $\Gamma m^{-\frac{1}{1-\gamma}}$ and equation (A.11) to eliminate $\gamma/[(1-\gamma)\nu]$, we get

$$\left. \frac{dN}{dA} \right|_I = - \frac{\lambda^b}{\left(1 + \frac{\Delta p}{p_H}\right) \lambda^e} \left[\frac{\frac{\beta}{\lambda^b} \left(1 + \frac{\Delta p}{p_H}\right) - 1}{\frac{\beta}{\lambda^e} - 1} \right].$$

This simplifies to

$$\left. \frac{dN}{dA} \right|_I = - \frac{1 + \frac{\Delta p}{p_H} - \frac{\lambda^b}{\beta}}{\left(1 + \frac{\Delta p}{p_H}\right) \left(1 - \frac{\lambda^e}{\beta}\right)}.$$

B Policy

B.1 Public Funding

Assume that the government injects an aggregate amount A_t^g of capital into the banking system and an aggregate amount N_t^g of capital into non-financial corporations. The government demands the rate of return $r_t^g > 0$ for its investments. One may think that the government buys bank equity at the (unit) price $Q_t^b = (1 + r_t^{a*}) / (1 + r_t^g)$ and firm equity at the price $Q_t^e = (1 + r_t^{n*}) / (1 + r_t^g)$, where r_t^{a*} and r_t^{n*} denote the (expected) rate of return of bank capital and firm capital, respectively, when the public funding program is in place.

Let $\omega_t^b \equiv A_t^g/A_t \geq 0$ and $\omega_t^e \equiv N_t^g/N_t \geq 0$. Then $a_t^g = \omega_t^b a_t$ is the quantity of government-owned capital in an individual bank's balance sheet, and $n_t^g = \omega_t^e n_t$ public funding allocated to non-financial firms. Also, $R_t^{gb} = R_t^b \omega_t^b / Q_t^b$ and $R_t^{ge} = R_t^e \omega_t^e / Q_t^e$ are the (expected) shares of the proceeds going to the government in the banking sector and in the non-financial corporate sector, respectively.

B.1.1 Implications for the Financing Contract

With government participation, the optimal financing contract solves the following program:

$$\max_{\{i_t, a_t, a_t^g, n_t^g, d_t, R_t^e, R_t^b, R_t^{gb}, R_t^{ge}, R_t^w, m_t\}} q_t p_H R_t^e i_t$$

subject to the entrepreneur's and her banker's incentive constraints (4a) and (4b), the depositors' and the banker's participation constraints (4c) and (4d), the resource constraints for investment inputs and outputs

$$a_t + a_t^g + d_t - m_t i_t \geq i_t - n_t - n_t^g, \quad (\text{B.1})$$

$$R \geq R_t^e + R_t^b + R_t^{gb} + R_t^{ge} + R_t^w, \quad (\text{B.2})$$

the sizes of government capital injections in banks and non-financial corporations

$$a_t^g = \omega_t^b a_t, \quad (\text{B.3})$$

$$n_t^g = \omega_t^e n_t, \quad (\text{B.4})$$

and the terms of those injections

$$R_t^{gb} = \frac{\omega_t^b}{Q_t^b} R_t^b \quad (\text{B.5})$$

$$R_t^{ge} = \frac{\omega_t^e}{Q_t^e} R_t^e. \quad (\text{B.6})$$

Substitution of $R_t^b = m_t / (q_t \Delta p)$, $R_t^e = h(m_t) / (q_t \Delta p)$, and equations (B.5) and (B.6), into the return-sharing constraint (B.2) shows that depositors can be promised at most

$$R_t^w = R - \frac{\left(1 + \frac{\omega_t^b}{Q_t^b}\right) m_t + \left(1 + \frac{\omega_t^e}{Q_t^e}\right) h(m_t)}{q_t \Delta p}. \quad (\text{B.7})$$

Substituting equation (B.7) for the depositor's participation constraint (4c) yields

$$p_H \left\{ q_t R - \frac{\left[\left(1 + \frac{\omega_t^b}{Q_t^b}\right) m_t + \left(1 + \frac{\omega_t^e}{Q_t^e}\right) h(m_t) \right]}{\Delta p} \right\} = \frac{d_t}{i_t}. \quad (\text{B.8})$$

Next, we combine the banker's incentive constraint (4b) with his participation constraint (4d), the input resource constraint (4e), and the sizes of government capital injections (B.3)

and (B.4) to obtain

$$\frac{d_t}{i_t} = 1 + m_t - (1 + \omega_t^b) \frac{p_H}{\Delta p} \left(\frac{1}{1 + r_t^a} \right) m_t - (1 + \omega_t^e) \frac{n_t}{i_t}. \quad (\text{B.9})$$

Combining equations (B.8) and (B.9), and noting that $Q_t^b = (1 + r_t^a)/(1 + r_t^g)$ and $Q_t^e = (1 + r_t^n)/(1 + r_t^g)$, the program boils down to

$$\max_{m_t \geq 0} \frac{h(m_t)}{\widehat{g}(r_t^a, r_t^g, q_t, m_t)}, \quad (\text{B.10})$$

in which

$$\begin{aligned} \widehat{g}(r_t^a, r_t^g, q_t, m_t) &\equiv (1 + \omega_t^e (1 + r_t^g)) \frac{p_H}{\Delta p} h_t(m_t) \\ &+ \left[1 + \frac{p_H}{\Delta p} \left(1 - \frac{1}{1 + r_t^a} \right) + \omega_t^b \frac{p_H}{\Delta p} \left(\frac{r_t^g}{1 + r_t^a} \right) \right] m_t - \rho_t \end{aligned}$$

is the inverse firm leverage. Given the monitoring technology (7), the unique interior solution to the problem (B.10) is

$$m_t^* = \frac{\gamma \rho_t}{1 + \frac{p_H}{\Delta p} \left(1 - \frac{1}{1 + r_t^a} \right) + \omega_t^b \frac{p_H}{\Delta p} \left(\frac{r_t^g}{1 + r_t^a} \right)}. \quad (\text{B.11})$$

Comparing (B.11) with (8) indicates that public ownership in banks, which dilutes bankers' stakes, lowers monitoring intensity.

B.1.2 Implications for Incentives and Investments

Equation (B.11) characterizes the equilibrium monitoring intensity with government participation. Note that the banker's incentive and participation constraints (4b) and (4d) (together with the aggregation condition (9)) imply that in equilibrium the bankers' monitoring intensity must also be characterized by $m_t = (\Delta p/p_H) (A_t/I_t) (1 + r_t^{a*})$. Combining

these observations, we get the rate of return to banker-owned capital

$$1 + r_t^{a*} = \frac{1 + \gamma \rho_t \frac{I_t^*}{A_t} - \omega_t^b r_t^g}{1 + \frac{\Delta p}{p_H}} = \frac{1 + \gamma \rho_t \frac{I_t^*}{A_t} - r_t^g \frac{A_t^g}{A_t}}{1 + \frac{\Delta p}{p_H}}. \quad (\text{B.12})$$

Comparing (B.12) to (10) indicates that the return to banker-owned capital is lower when public policies are in place. Plugging equation (B.12) into (B.11) then yields

$$\begin{aligned} m_t^* &= \left(1 + \frac{p_H}{\Delta p}\right)^{-1} \left((1 - \omega_t^b r_t^g) \frac{A_t}{I_t^*} + \gamma \rho_t \right) \\ &= \left(1 + \frac{p_H}{\Delta p}\right)^{-1} \left(\frac{A_t - r_t^g A_t^g}{I_t^*} + \gamma \rho_t \right). \end{aligned} \quad (\text{B.13})$$

Comparing equations (B.13) and (11) reveals that public ownership in banks dilutes bankers' stakes. Hence, bankers have weaker incentives to monitor intensively. Next, equations (B.1), (B.3), (B.4) together with the aggregation equation (9) imply that

$$\frac{D_t^*}{I_t^*} = 1 + m_t^* - \frac{(1 + \omega_t^b) A_t + (1 + \omega_t^e) N_t}{I_t^*} = 1 + m_t^* - \frac{A_t + A_t^g + N_t + N_t^g}{I_t^*}. \quad (\text{B.14})$$

Then applying (9) to equation (B.8), and plugging in expressions (B.12), (B.13) and (B.14), allows us to see how the entrepreneurs' maximum incentive compatible non-verifiable revenue from a “bad” project depends on public policies and aggregate variables

$$h_t^* = \frac{\Delta p}{p_H} \left(\frac{N_t - r_t^g N_t^g}{I_t^*} + (1 - \gamma) \rho_t \right). \quad (\text{B.15})$$

Comparing (12) and (B.15) reveals that public stakes in firms dilute the entrepreneurs' stakes. As a result the entrepreneurs have weaker incentives to choose the “good” project. Combining (B.15) with (4a) yields the expected rate of return to entrepreneurial capital

$$1 + r_t^{n*} = 1 + (1 - \gamma) \rho_t \left(\frac{I_t^*}{N_t} \right) - \omega_t^e r_t^g = 1 + (1 - \gamma) \rho_t \left(\frac{I_t^*}{N_t} \right) - r_t^g \left(\frac{N_t^g}{N_t} \right). \quad (\text{B.16})$$

which is defined in the same way as in (15b). Comparing (B.16) with (15b) indicates that the return to entrepreneurial capital is lower, when public policies are in place. But m^*

given by (B.13) and h^* given by (B.15) are linked by the monitoring technology (7). Then, assuming that there is monitoring in equilibrium, we get

$$\left(\frac{A_t - r_t^g A_t^g}{I_t^*} + \gamma \rho_t \right)^\gamma \left(\frac{N_t - r_t^g N_t^g}{I_t^*} + (1 - \gamma) \rho_t \right)^{1-\gamma} = \left(\frac{p_H}{\Delta p} \Gamma \right)^{1-\gamma} \left(1 + \frac{p_H}{\Delta p} \right)^\gamma. \quad (\text{B.17})$$

This is equation (23) of the main text. This equation implicitly determines the aggregate investment level I_t^* in the economy when the government funds banks, non-financial firms, or both.

Taken together, equations (B.12), (B.13), (B.15), (B.16) and (B.17) indicate that public funding dilutes the insiders' stakes in the investment projects, and as a result bankers find it more tempting not to monitor while entrepreneurs face stronger incentives to choose a “bad” project with non-verifiable revenues. Since the severity of these moral hazard problems also increases with the scale of the investment projects, aggregate investments in equilibrium must be lower when the public funding program is in place.

When public policy is introduced, Stage 1 of period t is already behind. Hence, L_t and Y_t are given, and these variables are not affected by policy. Then period t consumption C_t^* is given by the aggregate resource constraint $C_t^* = Y_t - I_t^*$. Therefore, current investment, I_t^* , is a sufficient statistic when evaluating how the distortions from period t public policy affect the current period t .

B.1.3 Public Funding: Dynamic Implications of Distortions

Given equation (B.16), the end-of-period t entrepreneurial capital is given by

$$\tilde{N}_t^* = (1 + r_t^{n*}) N_t = N_t + (1 - \gamma) \rho_t I_t^* - r_t^g N_t^g. \quad (\text{B.18})$$

In particular, the last term $-r_t^g N_t^g$ is the direct effect of public funding. Since government funding commands a premium r_t^g , it raises revenues equal to $r_t^g N_t^g$, and this in turn lowers the revenues accruing to the entrepreneurs.

Given equation (B.12), the end-of-period t banker-owned capital is given by

$$\tilde{A}_t^* = (1 + r_t^{a*}) A_t = \frac{A_t + \gamma \rho_t I_t^*}{1 + \frac{\Delta p}{p_H}} - \frac{r_t^g A_t^g}{1 + \frac{\Delta p}{p_H}}. \quad (\text{B.19})$$

Once again, the last term in (B.19) is the direct effect of public funding. To understand why the direct effect is equal to $-r_t^g A_t^g / (1 + \frac{\Delta p}{p_H})$, rather than $-r_t^g A_t^g$ (as it might seem intuitive) note that, given equation (B.13), public funding lowers banks' monitoring intensity and monitoring costs:

$$m_t^* I_t^* = \frac{A_t + \gamma \rho_t I_t^*}{1 + \frac{p_H}{\Delta p}} - \frac{r_t^g A_t^g}{1 + \frac{p_H}{\Delta p}},$$

where $-r_t^g A_t^g / (1 + \frac{p_H}{\Delta p}) = -\frac{\Delta p}{p_H} r_t^g A_t^g / (1 + \frac{\Delta p}{p_H})$ is the direct effect. Now, while the government earns the amount $r_t^g A_t^g$, only the share

$$r_t^g A_t^g - \left(\frac{\frac{\Delta p}{p_H}}{1 + \frac{\Delta p}{p_H}} \right) r_t^g A_t^g = \frac{r_t^g A_t^g}{1 + \frac{\Delta p}{p_H}}$$

is paid by the bankers while the remaining share $\frac{\Delta p}{p_H} r_t^g A_t^g / (1 + \frac{\Delta p}{p_H})$ is paid by the workers - or households. (Remember that we assume that the banks pay the monitoring costs to the workers.)

To eliminate the harmful direct effects on future bank and firm capital, we assume that the government rebates the revenues from the premium back to the entrepreneurs, bankers, and workers in a lump-sum manner. This minimizes welfare losses, while still keeping public funding unattractive for individual entrepreneurs and bankers, and hence avoiding intertemporal moral hazard. Since entrepreneurs receive a lump-sum rebate $r_t^g N_t^g$ and bankers receive a lump-sum rebate $r_t^g A_t^g / (1 + \frac{\Delta p}{p_H})$, equations (B.18) and (B.19) yield

$$\tilde{N}_t^* = N_t + (1 - \gamma) \rho_t I_t^* = (1 + r_t^n) N_t \quad (\text{B.20})$$

and

$$\tilde{A}_t^* = \frac{A_t + \gamma \rho_t I_t^*}{1 + \frac{\Delta p}{p_H}} = (1 + r_t^a) A_t, \quad (\text{B.21})$$

where $(1 + r_t^n)$ and $(1 + r_t^a)$ are given by equations (15b) and (10), respectively, (with I_t

replaced by I_t^* in these equations) while the (middle-of) period $t + 1$ values are given by

$$N_{t+1}^* = \lambda^e \left(\frac{r_{t+1}^K + (1 - \delta) q_{t+1}}{q_t} \right) \tilde{N}_t^* \quad (\text{B.22})$$

and

$$A_{t+1}^* = \lambda^b \left(\frac{r_{t+1}^K + (1 - \delta) q_{t+1}}{q_t} \right) \tilde{A}_t^*. \quad (\text{B.23})$$

Hence, since the lump-sum rebates eliminate the direct effects, all the distorting effects of period t public funding to future insider wealth run through period t investments I_t^* .

Note that the distorting effects of the program affect also physical capital K_{t+1}^* , the third period $t + 1$ state variable, through period t investments I_t^* . Hence, I_t^* is a sufficient statistic when evaluating how the distortions from period t public policy affect future periods. Finally, since I_t^* is a sufficient statistic for both period t (see Section B.1.2 of this Appendix) and for future periods, I_t^* is also a sufficient statistic for the (current and future) price of capital, q_t .

B.1.4 Implications for Effects of an Investment Shock

In the aftermath of an investment shock, the project success probabilities are given by $\tilde{p}_{\tau t} \equiv p_{\tau}(1 + \varepsilon_t)$, $\tau \in \{H, L\}$. Then, in equilibrium, the aggregate revenue from the projects is $\tilde{p}_{Ht} q_t R I_t^*$. From that revenue, entrepreneurs and the government (which has invested in non-financial firms) are paid $\tilde{p}_{Ht} q_t R_t^e I_t^*$ and $\tilde{p}_{Ht} q_t R_t^{ge} I_t$, respectively. The remaining, stochastic, revenue is left to banks which have to pay the fixed sum D_t^* to depositors, or outside debt holders. What is left is divided between bankers ($\tilde{p}_{Ht} q_t \tilde{R}_t^b I_t$) and the government ($\tilde{p}_{Ht} q_t \tilde{R}_t^{gb} I_t$), where the *ex post* shares \tilde{R}_t^b and \tilde{R}_t^{gb} depend on the shock realization ε_t . Since the bankers' and the government's revenues are proportional to their respective ownership shares, the ratio $\tilde{R}_t^{gb}/\tilde{R}_t^b$ must be the same as the ratio R_t^{gb}/R_t^b , given by equation (B.5). Evidently, bankers and the government absorb the losses due to the aggregate investment shock in proportion to their revenue/ownership shares, implying that the share of losses absorbed by bankers is $R_t^b / (R_t^b + R_t^{gb}) = \left(1 + \frac{\omega_t^b}{Q_t^b}\right)^{-1}$. Then, following an investment

shock in period t , the aggregate bank capital at the end of period t is given by

$$\begin{aligned}\tilde{A}_t^*(\varepsilon_t) &= I_t^* p_H q_t \left[R_t^b + \left(\frac{R_t^b}{R_t^b + R_t^{gb}} \right) (R - R_t^e - R_t^{ge}) \varepsilon_t \right] \\ &= I_t^* p_H q_t \left[R_t^b + \left(\frac{R_t^b}{R_t^b + R_t^{gb}} \right) (R_t^b + R_t^{gb} + R_t^w) \varepsilon_t \right],\end{aligned}\tag{B.24}$$

where the last form follows from equation (B.2). Using (4b), (4c), (4d), (9), (B.5) and (B.11) together with $Q_t^b = (1 + r_t^{a*}) / (1 + r_t^g)$, $\omega_t^b = A_t^g / A_t$, and $r_t^d = 0$, allows us to rewrite (B.24) as follows

$$\tilde{A}_t^*(\varepsilon_t) = A_t (1 + r_t^{a*}) \left[1 + \left\{ 1 + \left(\frac{1}{1 + r_t^g} \right) \left(\frac{D_t^*}{(1 + r_t^{a*}) A_t + A_t^*} \right) \right\} \varepsilon_t \right],\tag{B.25}$$

where $(1 + r_t^{a*})$ is given by (B.12) while (B.14) implies that

$$D_t^* = (1 + m_t^*) I_t^* - (A_t + A_t^g + N_t + N_t^g).$$

Next, assuming that i) bankers receive a lump-sum rebate $r_t^g A_t^g / (1 + \frac{\Delta p}{p_H})$ (see Appendix B.1.3) and ii) $r_t^g = dr_t^g > 0$ is small, we get

$$\tilde{A}_t^*(\varepsilon_t) = A_t (1 + r_t^a) (1 + BL_t^* \varepsilon_t) + O(dr_t^g \varepsilon_t),\tag{B.26}$$

where

$$BL_t^* = 1 + \frac{D_t - A_t^g - N_t^g}{(1 + r_t^a) A_t + A_t^g}.\tag{B.27}$$

Assuming that the terms $O(dr_t^g \varepsilon_t)$, which are of the order $dr_t^g \times \varepsilon_t$, can be disregarded, equation (B.26) yields (24a) in the main text, while (B.27) is (24b).

B.2 Public Funding and the Resilience of the Banking Sector: Welfare Implications

Let us consider the welfare implications of public funding when a (negative) aggregate shock hits the economy. To do so, it is useful to analyze the end-of-period values of bankers'

wealth, $\tilde{A}_t(\varepsilon_t)$, entrepreneurial wealth, $\tilde{N}_t(\varepsilon_t)$, and physical capital, $\tilde{K}_t(\varepsilon_t)$. $\tilde{A}_t(\varepsilon_t)$, $\tilde{N}_t(\varepsilon_t)$ and $\tilde{K}_t(\varepsilon_t)$ are the values of the state variables after the investment stage of period t , when the investment shock is realized, while A_t , N_t and K_t are the corresponding values before the investment stage of period t (but after the production stage of period t). (Also note that A_{t+1} , N_{t+1} and K_{t+1} are the values of the state variables in period $t + 1$, after the production stage but before the investment stage). Under *laissez-faire*, the end-of-period aggregate banker-owned wealth is given by equation (20a). If there is a public funding program in place, the end-of-period bankers' wealth is given by equation (24a). The end-of-period entrepreneurial wealth is given by the equation (19) and the end-of-period physical capital is characterized by

$$\tilde{K}_t(\varepsilon_t) = K_t + I_t(1 + \varepsilon_t)$$

whether or not public policies are in place.²⁰

The welfare of the representative household from the next period ($t + 1$) onwards depends on these end-of-period (t) values of the state variables. Under *laissez-faire*, the value function is $\tilde{V}(\tilde{A}_t(\varepsilon_t), \tilde{N}_t(\varepsilon_t), \tilde{K}_t(\varepsilon_t))$, while if there is a public funding program in place, welfare is given by $\tilde{V}(\tilde{A}_t^*(\varepsilon_t), \tilde{N}_t(\varepsilon_t), \tilde{K}_t(\varepsilon_t))$. Since $\tilde{A}_t^*(\varepsilon_t) > \tilde{A}_t(\varepsilon_t)$, when $\varepsilon_t < 0$, we have $\tilde{V}(\tilde{A}_t^*(\varepsilon_t), \tilde{N}_t(\varepsilon_t), \tilde{K}_t(\varepsilon_t)) > \tilde{V}(\tilde{A}_t(\varepsilon_t), \tilde{N}_t(\varepsilon_t), \tilde{K}_t(\varepsilon_t))$. In particular (if ε_t is small enough), the welfare gain from public funding can be approximated by

$$\begin{aligned} \Delta \tilde{V} &= \tilde{V}(\tilde{A}_t^*(\varepsilon_t), \tilde{N}_t(\varepsilon_t), \tilde{K}_t(\varepsilon_t)) - \tilde{V}(\tilde{A}_t(\varepsilon_t), \tilde{N}_t(\varepsilon_t), \tilde{K}_t(\varepsilon_t)) \\ &\approx \tilde{V}_{\tilde{A}} \left[\tilde{A}_t^*(\varepsilon_t) - \tilde{A}_t(\varepsilon_t) \right] = \tilde{V}_{\tilde{A}} (1 + r_t^a) A_t (BL_t^* - BL_t) \varepsilon_t \\ &= -\tilde{V}_{\tilde{A}} BRE_t S_t \varepsilon_t, \end{aligned}$$

where $\tilde{V}_{\tilde{A}}$ is the derivative of the value function with respect to \tilde{A} , $BRE_t = (1 + r_t^a) BL_t A_t$

²⁰Note that the impact of the shock on the (middle-of-period) $t + 1$ entrepreneurial wealth $\tilde{N}_{t+1}^*(\varepsilon_t)$ depends on public funding. This is because the impact of the shock on the period $t + 1$ price of capital q_{t+1} depends on public policies. However, the connection from (period t) public policies to q_{t+1} and $\tilde{N}_{t+1}^*(\varepsilon_t)$ goes through $\tilde{A}_t^*(\varepsilon_t)$.

is the banks' (macro) risk exposure under *laissez-faire*, and $S_t = (BL_t - BL_t^*)/BL_t$ measures the size of the public funding program. This is equation (25) in the main text.

B.3 The Fiscal Size of the Funding Program and Derivation of the Policy Frontier

If the government funds non-financial firms - say the government buys a diversified (market) portfolio of firm debt - the losses it incurs are directly proportional to the size of the firm funding program N_t^g , and to the size of the (negative shock) ε_t . However, if the government capitalizes banks its losses are amplified. With an equity stake A_t^g in banks, the government stands to lose $A_t^g BL_t^* (A_t^*, N_t^g; r_t^g) \varepsilon_t$ if a negative shock $\varepsilon_t < 0$ hits. Taken together, the losses, or the fiscal costs, of a public funding program (A_t^g, N_t^g) can be expressed as

$$FC(A_t^g, N_t^g; r_t^g, \varepsilon_t) = -A_t^g BL_t^*(A_t^*, N_t^g; r_t^g) \varepsilon_t - N_t^g \varepsilon_t \quad (\text{B.28})$$

Assuming $r_t^g = dr_t^g$ is small, so that terms of the order $dr_t^g \varepsilon_t$ can be disregarded, the equation (B.28) can be rewritten as

$$\begin{aligned} FC(A_t^g, N_t^g; \varepsilon_t) &= -A_t^g \left(1 + \frac{D_t - A_t^g - N_t^g}{Q_t^b A_t + A_t^g} \right) \varepsilon_t - N_t^g \varepsilon_t \\ &= - \left(\frac{Q_t^b A_t}{Q_t^b A_t + A_t^g} (A_t^g + N_t^g) + \frac{A_t^g}{Q_t^b A_t + A_t^g} D_t \right) \varepsilon_t \\ &= -Q_t^b A_t \left(\frac{D_t}{Q_t^b A_t} - \frac{D_t - A_t^g - N_t^g}{Q_t^b A_t + A_t^g} \right) \varepsilon_t \\ &= -(1 + r_t^a) A_t (BL_t - BL_t^*) \varepsilon_t = -BRE_t S_t \varepsilon_t \end{aligned}$$

However, since $FC(A_t^g, N_t^g; r_t^g, \varepsilon_t) = BRE_t S_t \varepsilon_t$, the equation (B.28) yields.

$$BRE_t S_t = BL_t^* A_t^g + N_t^g$$

This is the policy frontier (27).

B.4 Proof of Proposition 2

The aim here is to find the optimal structure of public funding (A_t^g, N_t^g) , and to link the desired structure to the size of the funding program S_t . We also show how the mapping from the size to the appropriate structure defines the sets of undominated and dominated policies.

Assume that the economy is initially in a steady state, and is hit by a negative investment shock ($\varepsilon_t < 0$). We assume that there is genuine uncertainty, and we do not specify a probability distribution over the negative shock realizations. Let us denote the discounted sum of present and future household utility, given a funding program (A_t^g, N_t^g, S_t) , the premium on public funding r_t^g , and a shock realization $\varepsilon_t < 0$ by $V = V(A_t^g, N_t^g, S_t; r_t^g; \varepsilon_t; \cdot)$. Given this environment, our objective is to narrow down the policy options that the government should consider under genuine uncertainty. We aim at characterizing two sets of policies: The undominated set, which includes policies that the government could/should consider; and the dominated set, which includes policies that the government should avoid.

The next step is to bring in more structure into the relationship between the policy program and household welfare, $V(A_t^g, N_t^g, S_t; r_t^g; \varepsilon_t; \cdot)$. We assume that the (excess) return on public funds, $dr_t^g = r_t^g - r_t^d$, is small enough, compared to the other terms, so that its cross terms with the shock, which are proportional to $\varepsilon_t \times dr_t^g$, can be ignored in the analysis. This assumption allows us to decouple, and analyze separately, the welfare benefits of the program (which are proportional to ε_t) and the welfare losses of the program (which are proportional to dr_t^g). This is shown in Appendix B.1, section B.1.4. See in particular equation (B.26), which shows where the cross terms would enter.

On the welfare benefit side of the program, the size of the program S_t , defined in equation (26), is a sufficient statistic. All programs (A_t^g, N_t^g, S_t) which lie on the policy frontier, i.e. satisfy the equation (27), give rise to the same level of stabilization, or enhanced resilience, (S_t) in the face of negative macro shock ε_t . Hence, the welfare benefits of the program, after a negative shock realization $\varepsilon_t < 0$, are proportional to the product $S_t \varepsilon_t$. See equation (25), and the underlying analysis in Appendix B.2.

As shown in Appendix B.1, subsections B.1.2 and B.1.3, the current investments are a sufficient statistic of the social welfare costs of public funding. That is, the distorting effects of the program to all model variables (current and future), are transmitted through current investments, $I_t^*(A_t^g dr_t^g, N_t^g dr_t^g; \cdot)$. Therefore, the harmful effects of the program on social welfare, given by the value function (V), are also transmitted through current investments. If program (A_t^g, N_t^g, S_t) gives rise to lower (higher) current investments than an alternative program $(A_t^{g'}, N_t^{g'}, S_t')$ then program (A_t^g, N_t^g, S_t) creates larger (smaller) welfare losses than program $(A_t^{g'}, N_t^{g'}, S_t')$.

Hence, we can conclude that the mapping from a policy package (A_t^g, N_t^g, S_t) to social welfare (V) has the following structure

$$V = V(I_t^*(A_t^g dr_t^g, N_t^g dr_t^g, \cdot), S_t \varepsilon_t; \varepsilon_t, \cdot),$$

subject to (27). Our task is to find policies (A_t^g, N_t^g, S_t) such that there do not exist alternative policies $(A_t^{g'}, N_t^{g'}, S_t')$ that would dominate (A_t^g, N_t^g, S_t) . We say that a policy $(A_t^{g'}, N_t^{g'}, S_t')$ dominates the policy (A_t^g, N_t^g, S_t) , if $(A_t^{g'}, N_t^{g'}, S_t')$ gives rise to at least as high utility as (A_t^g, N_t^g, S_t) in all states of the world, and higher utility in some states of the world. Here the different states of the world correspond to different (negative) shocks realizations $\varepsilon_t < 0$. More formally, the set of undominated policies (A_t^g, N_t^g, S_t) is defined as follows:

$$\begin{aligned} &\text{Find } A_t^g \geq 0, N_t^g \geq 0 \text{ and } S_t, \text{ such that } \nexists A_t^{g'}, N_t^{g'}, S_t' \\ &V(I_t^*(A_t^{g'} dr_t^{g'}, N_t^{g'} dr_t^{g'}, \cdot), S_t' \varepsilon_t; \varepsilon_t, \cdot) \geq V(I_t^*(A_t^g dr_t^g, N_t^g dr_t^g, \cdot), S_t \varepsilon_t; \varepsilon_t, \cdot), \forall \varepsilon_t < 0 \\ &\text{subject to} \end{aligned}$$

$$\begin{aligned} BRE_t S_t &= N_t^g + BL_t(1 - S_t)A_t^g, \\ BRE_t S_t' &= N_t^{g'} + BL_t(1 - S_t')A_t^{g'}. \end{aligned}$$

Next, recall that S_t is a sufficient statistic of the social welfare benefits of a program (A_t^g, N_t^g, S_t) . Likewise, S_t' is a sufficient statistic of the social welfare benefits of an alternative program $(A_t^{g'}, N_t^{g'}, S_t')$. Then if S_t' is larger (smaller) than S_t , the program $(A_t^{g'}, N_t^{g'}, S_t')$

gives rise to larger (smaller) social welfare benefits than (A_t^g, N_t^g, S_t) after any negative shock realization $\varepsilon_t < 0$. If $S'_t = S_t$, the programs $(A_t^{g'}, N_t^{g'}, S'_t)$ and (A_t^g, N_t^g, S_t) give rise to the same social welfare benefits after any negative shock realization $\varepsilon_t < 0$. We may say that policy $(A_t^{g'}, N_t^{g'}, S'_t)$ dominates policy (A_t^g, N_t^g, S_t) if $S'_t = S_t$ (so that both policies result in the same welfare benefits after any shock realization $\varepsilon_t < 0$) but $(A_t^{g'}, N_t^{g'}, S'_t)$ generates smaller distortions and welfare losses than (A_t^g, N_t^g, S_t) . Since current investments, I_t^* , are a sufficient statistic for welfare losses, we can conclude that $(A_t^{g'}, N_t^{g'}, S'_t)$ dominates (A_t^g, N_t^g, S_t) iff $I_t^*(A_t^{g'} dr_t^g, N_t^{g'} dr_t^g; \cdot) > I_t^*(A_t^g dr_t^g, N_t^g dr_t^g; \cdot)$ and $S'_t = S_t$.

We can find the set of undominated policies in the following way. We choose a target level (S_t) for the stabilization of the financial system, that is the size of the program. Then we find the structure of the program (A_t^g, N_t^g) that minimizes distortions or equivalently, maximizes $I_t^*(A_t^g dr_t^g, N_t^g dr_t^g; \cdot)$, subject to (27). More formally

$$\begin{aligned} & \max_{\{A_t^g, N_t^g\}} I_t^*(A_t^g dr_t^g, N_t^g dr_t^g; \cdot) \\ & \text{subject to} \end{aligned}$$

$$BRE_t S_t = N_t^g + BL_t(1 - S_t)A_t^g, \quad A_t^g \geq 0, \quad N_t^g \geq 0,$$

where S_t , BRE_t and BL_t are taken as given. The corresponding *Lagrangian* is

$$\mathcal{L} = I_t^*(A_t^g dr_t^g, N_t^g dr_t^g; \cdot) + \eta_1 [N_t^g + BL_t(1 - S_t)A_t^g - BRE_t \times S_t] + \eta_2 A_t^g + \eta_3 N_t^g,$$

where η_1 , η_2 and η_3 are Lagrangian multipliers. The first order conditions with respect to A_t^g and N_t^g are

$$\frac{\partial I_t^*(A_t^g dr_t^g, N_t^g dr_t^g; \cdot)}{\partial A_t^g} + \eta_1 BL_t(1 - S_t) + \eta_2 = 0 \quad (\text{B.29})$$

$$\frac{\partial I_t^*(A_t^g dr_t^g, N_t^g dr_t^g; \cdot)}{\partial N_t^g} + \eta_1 + \eta_3 = 0. \quad (\text{B.30})$$

The Kuhn-Tucker conditions associated with the constraints $A_t^g \geq 0$ and $N_t^g \geq 0$ are

$$\begin{aligned}\eta_2 &= 0 \text{ if } A_t^g > 0, & \eta_2 > 0 \text{ if } A_t^g = 0 \\ \eta_3 &= 0 \text{ if } N_t^g > 0, & \eta_3 > 0 \text{ if } N_t^g = 0.\end{aligned}$$

Next note that equation (23) implies that in the neighborhood of the steady state

$$\frac{\frac{\partial I_t^*}{\partial A_t^g}}{\frac{\partial I_t^*}{\partial N_t^g}} = \frac{-\frac{\partial I_t}{\partial A_t} dr_t^g}{-\frac{\partial I_t}{\partial N_t} dr_t^g} = \left| \frac{dN}{dA} \right|_I = MRTS.$$

There are three types of solutions, or three regimes:

i) If $A^g > 0$ and $N_t^g = 0$, we have $\eta_2 = 0$, $\eta_3 > 0$. Then dividing (B.30) by (B.29) gives, after some straightforward algebra

$$\frac{1}{MRTS} = \frac{1}{BL_t(1 - S_t)} + \frac{\eta_3}{\eta_1} \frac{1}{BL_t(1 - S)} \Rightarrow MRTS < BL_t(1 - S_t),$$

where the inequality follows, since $\eta_1 > 0$ and $\eta_3 > 0$.

ii) If $A_t^g = 0$ and $N_t^g > 0$, we have $\eta_2 > 0$, $\eta_3 = 0$. Dividing (B.29) by (B.30) then gives

$$MRTS = BL_t(1 - S_t) + \frac{\eta_2}{\eta_1} \Rightarrow MRTS > BL_t(1 - S_t),$$

where the inequality follows since $\eta_1 > 0$ and $\eta_2 > 0$.

iii) If $A_t^g > 0$ and $N_t^g > 0$, we have $\eta_2 = 0$, $\eta_3 = 0$. Dividing (B.29) by (B.30) then gives

$$MRTS = BL_t(1 - S_t).$$

It is easy to see that this characterization of undominated policies gives us a mapping from the size of the program (S_t) to the optimal structure of the program (A_t^g, N_t^g). Let us define $S_t^* = 1 - MRTS/BL_t$. i) If $S_t < S_t^*$ the optimal structure is $A_t^g = (BRE_t/BL_t)(S_t/(1 - S_t))$, $N_t^g = 0$. ii) If $S_t > S_t^*$ the optimal structure is $A_t^g = 0$, $N_t^g = BRE_t \times S_t$. iii) If $S_t = S_t^*$, any structure $A_t^g \geq 0$, $N_t^g \geq 0$ which satisfies (27) is optimal.

Note that iii) is a knife-edge case, with measure zero.

Finally, all policies (A_t^g, N_t^g, S_t) which do not belong to the undominated set are dominated. ■

B.5 The Threshold Size of a Public Funding Program

In this appendix, we express the threshold size of a public funding program, S^* , with the help of the observable data moments used in the calibration of the model. We also examine the sensitivity of the threshold size S^* to the calibration.

Expressing S^ in terms of data moments.* Remember that $S^* = 1 - MRTS/BL$. Let us first express the bank leverage accelerator (under *laissez-faire*), BL , in terms of the data moments used in calibration (see Section 5 and Appendix C.2). Using (C.2) and (C.3) in Appendix C.2 we get

$$BL = \frac{1 + MRB + r^a CRB}{(1 + r^a) CRB}. \quad (\text{B.31})$$

Note that since $\left(\frac{1+MRB+r^a CRB}{1+r^a}\right)$ tends to be rather close to unity, we get $BL \approx CRB^{-1}$.

Next, plugging equations (C.5), (C.6) and (C.7) in Appendix C.2 into (18) allows us to express $MRTS$ in terms of the data moments we use in calibrating the model

$$MRTS = \frac{(1 + r^n)(r^a CRB + MRB)}{r^n ((1 + r^a) CRB + MRB)}, \quad (\text{B.32})$$

where MRB is a measure of banks' monitoring costs, relative to banks' assets. To gain some further intuition, it is useful to re-express $MRTS$ as

$$MRTS = \left(\frac{1 + \frac{MRB}{r^a CRB}}{1 + \frac{MRB}{(1+r^a)CRB}} \right) \left(\frac{r^a}{r^n} \right) \left(\frac{1 + r^n}{1 + r^a} \right).$$

Note that the terms $1 + \frac{MRB}{r^a CRB}$ and $\left(\frac{r^a}{r^n}\right)$ are potentially quite large (evidently depending on calibration), while the remaining terms $1 + \frac{MRB}{(1+r^a)CRB}$ and $\left(\frac{1+r^n}{1+r^a}\right)$ tend to be close to unity. Then

$$MRTS \approx (r^a + MRB/CRB) / r^n.$$

We now turn to the conditions under which $MRTS < BL$, so that $S^* = 1 - \frac{MRTS}{BL} > 0$. Applying equations (B.31) and (B.32), one can show that this is the case if and only if

$$\frac{1 + r^n}{r^n} < \left(\frac{1 + r^a CRB + MRB}{r^a CRB + MRB} \right) \left(1 + \frac{MRB}{(1 + r^a) CRB} \right). \quad (\text{B.33})$$

In Appendix D.3, however, we show that the model has an equilibrium with a meaningful role for financial intermediation if and only if $r^n > r^a CRB + MRB$ (see condition (D.42)).

This condition can be rewritten as

$$\frac{1 + r^n}{r^n} < \frac{1 + r^a CRB + MRB}{r^a CRB + MRB}. \quad (\text{B.34})$$

Note that (B.34) implies (B.33). Hence, if there is an equilibrium where banks intermediate funding and monitor entrepreneurial firms, there exists a threshold value $S^* > 0$.

Finally, combining (B.31) and (B.32) yields

$$\begin{aligned} S^* &= 1 - \frac{MRTS}{BL} = 1 - \left(\frac{r^a CRB + MRB}{1 + r^a CRB + MRB} \right) \left(\frac{(1 + r^n) CRB}{r^n ((1 + r^a) CRB + MRB)} \right) \\ &= 1 - \left(\frac{r^a + \frac{MRB}{CRB}}{CRB^{-1} + r^a + \frac{MRB}{CRB}} \right) \left(\frac{1 + r^n}{r^n (1 + r^a + \frac{MRB}{CRB})} \right). \end{aligned} \quad (\text{B.35})$$

To gain some further intuition, note that $CRB^{-1} + r^a + \frac{MRB}{CRB} \approx CRB^{-1}$, while the term $\frac{1 + r^n}{1 + r^a + \frac{MRB}{CRB}}$ tends to be rather close to unity (evidently depending on the calibration). Thus, we get

$$S^* \approx 1 - (r^a CRB + MRB) / r^n.$$

These findings are numerically illustrated in Table 3 which reports the threshold size S^* , given by (B.35), for different values of the data moments r^a , CRB and MRB , while $r^n = 4.5\%$ follows the baseline calibration.

$r^a = 10\%$		CRB								
		4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0	12.0
MRB	0.5	0.81	0.79	0.77	0.74	0.72	0.69	0.67	0.65	0.63
	1.0	0.74	0.71	0.68	0.66	0.63	0.61	0.58	0.56	0.54
	1.5	0.68	0.64	0.61	0.58	0.55	0.53	0.50	0.48	0.45
	2.0	0.63	0.58	0.55	0.52	0.48	0.46	0.43	0.40	0.37
	2.5	0.58	0.54	0.49	0.46	0.42	0.39	0.36	0.33	0.30
	3.0	0.55	0.49	0.45	0.40	0.37	0.33	0.30	0.27	0.24
	3.5	0.51	0.45	0.40	0.36	0.32	0.28	0.24	NA	NA
$r^a = 12\%$		CRB								
		4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0	12.0
MRB	0.5	0.80	0.77	0.74	0.71	0.68	0.66	0.63	0.60	0.57
	1.0	0.72	0.69	0.66	0.63	0.60	0.57	0.54	0.51	0.49
	1.5	0.66	0.62	0.59	0.55	0.52	0.49	0.46	0.43	0.40
	2.0	0.61	0.57	0.53	0.49	0.45	0.42	0.39	0.36	0.33
	2.5	0.57	0.52	0.47	0.43	0.39	0.36	0.32	0.29	0.26
	3.0	0.53	0.47	0.42	0.38	0.34	0.30	0.26	0.23	0.19
	3.5	0.50	0.44	0.38	0.33	0.29	NA	NA	NA	NA
$r^a = 14\%$		CRB								
		4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0	12.0
MRB	0.5	0.78	0.75	0.71	0.68	0.65	0.62	0.59	0.55	0.52
	1.0	0.71	0.67	0.63	0.60	0.57	0.53	0.50	0.47	0.44
	1.5	0.65	0.60	0.56	0.53	0.49	0.46	0.42	0.39	0.36
	2.0	0.60	0.55	0.50	0.46	0.42	0.39	0.35	0.32	0.28
	2.5	0.55	0.50	0.45	0.41	0.36	0.32	0.29	0.25	0.21
	3.0	0.52	0.46	0.40	0.35	0.31	0.27	0.23	NA	NA
	3.5	0.49	0.42	0.36	0.31	NA	NA	NA	NA	NA

Table 3: Threshold size S^* with different calibrations of the financial block. The baseline calibration is in bold. *Notes:* CRB is the capital ratio of banks (%), MRB is the ratio of banks' monitoring costs to their assets (%), and r^a is the return on bank equity (%). Following the baseline calibration we set the return on firm equity $r^n = 4.5\%$. NA means that the “no-corner-solution condition” ((D.42) in Appendix D.3) is not satisfied. In these cases the suggested calibration is not valid.

B.6 Dominated and Undominated Policies: Example

We consider the following numerical example. We assume that the utility function involves internal habit persistence, and the subutility function for consumption is $u(C_t, C_{t-1}) = ((C_t - bC_{t-1})^{1-\sigma})/(1 - \sigma)$, where the habit parameter is set to $b = 0.8$. In our simple model, financial frictions introduce a wedge on the consumption/investment margin only. Habit persistence provides a simple way to amplify the welfare effects of the friction while introducing a minimal departure from the basic model.

To keep the illustrative example as simple as possible we consider a case with two possible shock realizations: the small shock is $\varepsilon_t = -0.005$ while the large shock is $\varepsilon_t = -0.03$. For illustrative simplicity, we assume that there are also two possible funding program sizes. The size of a small program is 0.3 and the size of a large program is 0.8. The premium or the penalty rate that the government charges is $r^g = 0.1$. All welfare gains and losses (measured as changes in the household value function) are expressed as a percentage of (per period) steady state consumption.

Welfare gain from stabilization. Note that welfare gain does not depend on the structure of the program.

	small shock	large shock
small program	0.36%	2.15%
large program	0.95%	5.71%

Welfare loss from distortions. Note that welfare loss from distortions does not depend on the shock realization.

	banks	firms
small program	0.25%	0.38%
large program	2.57%	1.01%

Net welfare gain. Net welfare gain is equal to welfare gain from stabilization minus welfare

loss from distortions.

	small program		large program	
	banks	firms	banks	firms
small shock	0.11%	-0.02%	-1.62%	-0.06%
large shock	1.90%	1.77%	3.14%	4.70%

Interpreting the results. We assume that there is genuine uncertainty. When choosing the policy the government does not know which shock (small or large) will realize, and it cannot even assign a probability distribution. However, our results narrow down the options that a benevolent government should consider under genuine uncertainty. A small funding program targeting firms and a large program targeting banks are *dominated*. The government should never choose either of these programs. A small program targeting banks and a large program targeting firms belong to the set of *undominated* policies. The government should choose its policy from this set (in bold).

B.7 Non-optimal Structure of Public Funding and Excess Welfare Losses

How important is it to get the structure of the public funding program right? How significant are the (excess) welfare losses resulting from a non-optimal structure? In this appendix, we address this question from two slightly different, but complementary, angles. First, we benchmark the non-optimally structured program with an optimally structured program with the same size (and the same social benefits from resilience), and compare the welfare losses from distortions. Second, within the set of undominated policies (characterized by Proposition 2) we choose a benchmark program which gives rise to the same level distortions as the non-optimal candidate program, but has a bigger size and hence brings about greater social benefits in terms of enhanced resilience.

B.7.1 Benchmarking to an Optimally Structured Program with the Same Size

If the government chooses to fund non-financial firms, the amount $N^g = BRE \times S$ of public funding is needed, and the welfare losses from distortions (WL) are proportional to $WL(A^g = 0, N^g = BRE \times S) \propto S$. If the government chooses to capitalize banks, fewer funds are needed $A^g = \left(\frac{BRE}{BL}\right) \left(\frac{S}{1-S}\right) \leq BRE \times S$, but since each unit of public funding creates $MRTS$ times more distortions when placed in banks than when placed in firms, the the resulting welfare losses are proportional to $WL(A^g = \left(\frac{BRE}{BL}\right) \left(\frac{S}{1-S}\right), N^g = 0) \propto \left(\frac{S}{1-S}\right) \left(\frac{MRTS}{BL}\right) = S \left(\frac{1-S^*}{1-S}\right)$. It is easy to see that the minimum welfare loss (WL^{\min}), corresponding to the optimal structure of the funding program (see Proposition 2) is (proportional to) $WL^{\min} \propto S \times \min \left\{1, \frac{1-S^*}{1-S}\right\}$, while the maximum welfare loss that arises if the government funds non-financial firms with a small program ($S < S^*$) and banks with a large program ($S > S^*$), is (proportional to) $WL^{\max} \propto S \times \max \left\{1, \frac{1-S^*}{1-S}\right\}$. Denote the *excess welfare loss* resulting from a non-optimal structure of the program by ΔWL . It is easy to conclude that

$$\Delta WL \leq WL^{\max} - WL^{\min} \propto \frac{S}{1-S} |S - S^*|. \quad (\text{B.36})$$

The expression on the right hand side of the inequality (B.36) gives the maximum value of the excess welfare loss. This maximum excess welfare loss is realized if only non-financial firms are funded when bank capitalization would be optimal (and vice versa). If, instead, the government implements some form of mixed program and funds both banks and non-financial firms, the excess welfare loss is given by the expression

$$\Delta WL \propto \frac{M^g}{1-S} |S - S^*| \quad (\text{B.37})$$

where

$$M^g = \begin{cases} N^g/BRE & \text{if } S < S^* \\ BL(1-S)A^g/BRE & \text{if } S > S^* \end{cases} \quad (\text{B.38})$$

denotes the misallocation of public funding.

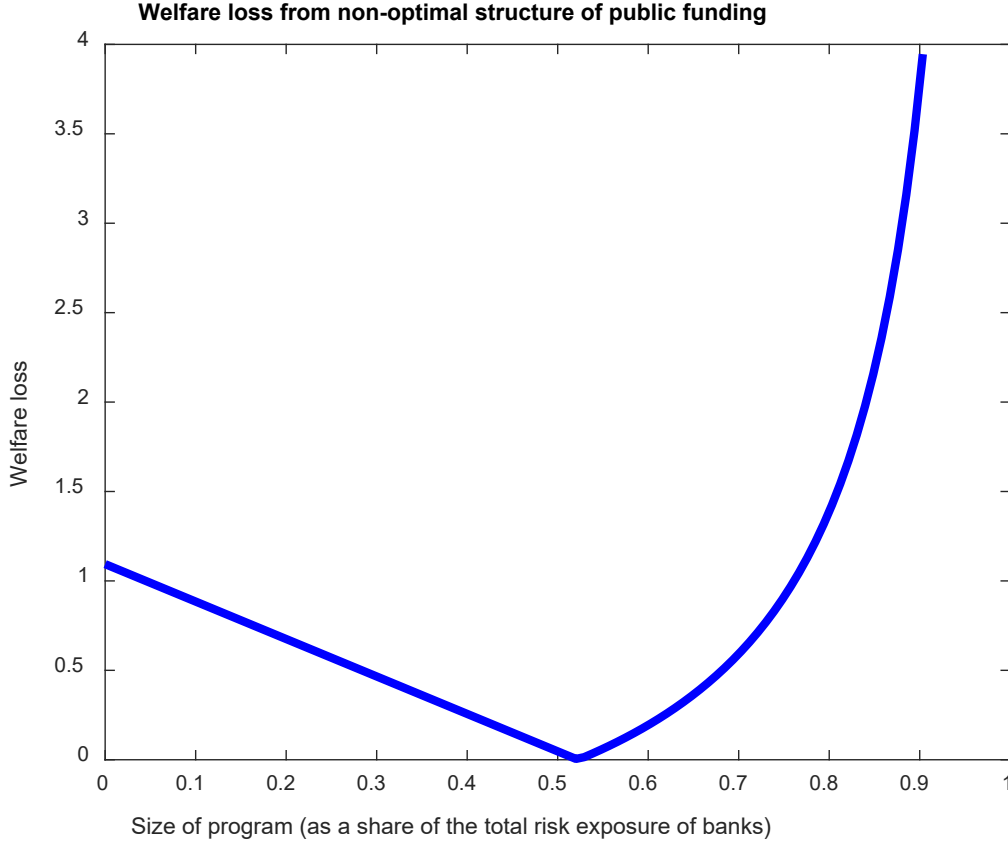


Figure 3: Welfare loss from non-optimal structure of public funding.

In order to further analyze how important the choice of the program structure is, it is useful to compare the excess welfare loss, due to a non-optimal structure, to the minimum welfare loss when the structure is chosen optimally. If the government funds only banks or only firms with the wrong amount, relative excess welfare loss $\Delta\widehat{WL} = \Delta WL / WL^{\min}$ can be expressed as follows

$$\Delta\widehat{WL} = \frac{\Delta WL}{WL^{\min}} = \max \left\{ \frac{S^* - S}{1 - S^*}, \frac{S - S^*}{1 - S} \right\}. \quad (\text{B.39})$$

If the government implements a mixed program,

$$\Delta\widehat{WL} = \frac{M^g}{S} \max \left\{ \frac{S^* - S}{1 - S^*}, \frac{S - S^*}{1 - S} \right\}. \quad (\text{B.40})$$

Figure 3 shows how the (maximum) relative excess welfare loss (the right hand side

of (B.39)) depends on the size of the program with our baseline calibration. The figure indicates that choosing the optimal structure of the program is important if the program is either very small or very large (since in these cases the maximum relative excess welfare loss is large), while it is less important for medium-sized programs. Indeed when the size is the program approaches the threshold value S^* (from either side), the maximum excess relative welfare loss approaches zero.

B.7.2 Benchmarking to an Optimally Structured Program with a Larger Size

Pure programs with the wrong structure

i) Consider a small program, with $S < S^*$. Assume that the program is (non-optimally) implemented by public funding to non-financial firms. Hence $N^g = BRE \times S$, and the welfare losses from distorted investments are (proportional to) $WL \propto S$. Next, assume that the government capitalizes banks instead. Since each unit of public stakes distorts the economy $MRTS$ times more when allocated to banks than when placed in firms, $A^{g'} = \frac{BRE}{MRTS} \times S$ gives rise to the same welfare losses from distortions as the original program. Denote the size of the alternative program, targeting banks, by S' . The size S' can be solved from the equation (27). That is

$$BRE \times S' = BL(1 - S') \frac{BRE}{MRTS} \times S.$$

This gives

$$\frac{S'}{1 - S'} = \frac{S}{1 - S^*}$$

or

$$S' = \frac{S}{1 + S - S^*}. \quad (\text{B.41})$$

Clearly $S' > S$, when $S < S^*$. Also note that $S' < S^*$ when $S < S^*$. Hence, the program of size S' is indeed optimally implemented by bank capitalization.

ii) Consider next a large program, with $S > S^*$. Assume that the program is (non-optimally) implemented by public bank capitalization. Hence, $A^g = \frac{BRE}{BL} \left(\frac{S}{1-S} \right)$, and the

welfare losses from distorted investments are (proportional to) $WL \propto \frac{MRTS}{BL} \frac{S}{1-S} = \left(\frac{1-S^*}{1-S} \right) S$. Next assume that the government funds non-financial firms instead. Since each unit of public stakes distorts the economy $MRTS$ times more when allocated to banks than when placed in firms, $N^{g'} = BRE \left(\frac{MRTS}{BL} \right) \left(\frac{S}{1-S} \right) = BRE \left(\frac{1-S^*}{1-S} \right) S$ gives rise to the same welfare losses from distortions as the original program. Denote the size of the alternative program, targeting firms, by S' . The size S' can be solved from the equation (27). That is

$$BRE \times S' = BRE \left(\frac{1-S^*}{1-S} \right) S.$$

This gives

$$S' = \left(\frac{1-S^*}{1-S} \right) S. \quad (\text{B.42})$$

Clearly $S' > S$, when $S > S^*$.

Note that (B.42) is only valid if $S' \leq S^{\max} = 1 - \frac{1}{BL}$, or $S \leq \frac{S^{\max}}{1+S^{\max}-S^*}$. If $S > \frac{S^{\max}}{1+S^{\max}-S^*}$, we choose as the (optimally structured) benchmark a program with the maximum size. This benchmark, which targets non-financial firms only, gives rise to distortion-related welfare losses which are proportional to

$$WL' \propto S^{\max}.$$

Since $\frac{S}{1-S} > \frac{S^{\max}}{1-S^*}$, it is easy to conclude that $WL' < WL$ (where $WL \propto \left(\frac{1-S^*}{1-S} \right) S$).

Mixed programs

Let us next consider mixed funding programs, with $A^g > 0$ and $N^g > 0$. The welfare loss due to distorted incentives is (proportional to)

$$WL \propto MRTS \times A^g + N^g,$$

while the size of the program is

$$S = \frac{BL \times A^g + N^g}{BRE + BL}. \quad (\text{B.43})$$

(To get this result solve $BRE \times S = BL(1 - S)A^g + N^g$ for S .)

i') First assume that the program is (relatively) small, i.e. $S < S^*$. Further assume that, instead of implementing the (non-optimal) candidate program, the government (only) capitalizes banks. Public stakes $A^{g'}$ placed in banks give rise to the same level of distortions and welfare losses as the candidate program. Hence, $A^{g'}$ can be solved from the equation

$$MRTS \times A^{g'} = MRTS \times A^g + N^g$$

and

$$A^{g'} = A^g + \frac{N^g}{MRTS}. \quad (\text{B.44})$$

The size of the alternative program, S' , can be solved from the equation (27). Hence, we get

$$\frac{S'}{1 - S'} = \frac{BL}{BRE} A^{g'} = \frac{BL}{BRE} \left(A^g + \frac{N^g}{MRTS} \right), \quad (\text{B.45})$$

where the last form is derived using (B.44). Next, it is useful to re-express the equation (B.45) in terms of S (the size of the non-optimal candidate program) and $M^g = N^g/BRE$ (the misallocation of public funding under the non-optimal program). Using (B.43) one can re-express (B.45) as

$$\frac{S'}{1 - S'} = \left(\frac{S}{1 - S} \right) \left[1 + \left(\frac{S^* - S}{1 - S^*} \right) \frac{M^g}{S} \right]. \quad (\text{B.46})$$

Finally, the equation (B.46) can be solved for S'

$$S' = \left(\frac{1 - S^* + (S^* - S) \frac{M^g}{S}}{1 - S^* + S(S^* - S) \frac{M^g}{S}} \right) S. \quad (\text{B.47})$$

Note that when $M^g = S$ (and the government funds only non-financial firms in the non-optimal candidate program), equation (B.47) boils down to (B.41).

ii') Assume now that the program is (relatively) large, i.e. $S > S^*$. Further assume that, instead of implementing the (non-optimal) candidate program, the government (only)

targets non-financial. Public stakes

$$N^{g'} = MRTS \times A^g + N^g$$

in non-financial firms give rise to the same level of welfare losses from distorted incentives as the (non-optimally structured) candidate program. The size of the alternative program, S' , can be solved from the equation (27). Hence, we get

$$S' = \frac{N^{g'}}{BRE} = \frac{MRTS \times A^g + N^g}{BRE}. \quad (\text{B.48})$$

Next, it is useful to re-express the equation (B.48) in terms of S (the size of the non-optimal candidate program) and $M^g = BL(1 - S)A^g/BRE$ (the misallocation of public funding under the non-optimal program). Using (B.43) one can re-express (B.48) as

$$S' = \left(1 + \left(\frac{S - S^*}{1 - S}\right) \frac{M^g}{S}\right) S. \quad (\text{B.49})$$

Note that when $M^g = S$ (and the government funds only banks in the non-optimal candidate program), equation (B.49) boils down to (B.42). Finally, note that the equation (B.49) is only valid if $S' \leq S^{\max} = 1 - \frac{1}{BL}$, or equivalently $S \leq S^{**}$, where

$$S^{**} = \frac{1 + S^{\max} - \widehat{M}S^* - \sqrt{\left(1 + S^{\max} - \widehat{M}S^*\right)^2 - 4\left(1 - \widehat{M}\right)S^{\max}}}{2\left(1 - \widehat{M}\right)} \quad (\text{B.50})$$

and $\widehat{M} = M^g/S$. When $\widehat{M} \rightarrow 1$, $S^{**} \rightarrow \frac{S^{\max}}{1 + S^{\max} - S^*}$. If $S > S^{**}$, we have $S' = S^{\max}$.

Assessing the welfare losses from a non-optimal structure

Remember that the welfare gains from enhanced stability are (approximately) proportional to the size of the program (see equation (25) in the main text; see also Appendix B.2). The difference between the size of the optimally structured benchmark program (S') and the candidate program (S) is a measure of the welfare losses due to the non-optimal

structure

$$\Delta WL^* \propto \Delta S = S' - S \leq \begin{cases} \left(\frac{S^* - S}{1 + S - S^*}\right) S & \text{if } S < S^* \\ \left(\frac{S - S^*}{1 - S}\right) S & \text{if } S^* > S. \end{cases}$$

To further assess the magnitude of these welfare losses, it is useful to divide ΔS by S (since the social welfare benefits from enhanced resilience of the candidate program are proportional to S). If the government funds only banks or only firms with the wrong amount, we obtain

$$\Delta \widehat{WL}^* = \frac{\Delta S}{S} = \begin{cases} \frac{S^* - S}{1 + S - S^*} & \text{if } S < S^* \\ \frac{S - S^*}{1 - S} & \text{if } S > S^*. \end{cases} \quad (\text{B.51})$$

If the government implements a mixed program, we get

$$\Delta \widehat{WL}^* = \frac{\Delta S}{S} = \begin{cases} \frac{(S^* - S)(1 - S) \frac{M^g}{S}}{1 - S^* + S(S^* - S) \frac{M^g}{S}} & \text{if } S < S^* \\ \left(\frac{S - S^*}{1 - S}\right) \frac{M^g}{S} & \text{if } S > S^*, \end{cases} \quad (\text{B.52})$$

where M^g (the misallocation of public funding under a mixed program) is given by (B.38). Note that equations (B.51) and (B.52) are only valid if $S' \leq S^{\max} = 1 - 1/BL$, or equivalently $S \leq S^{**}$, where S^{**} is defined in (B.50). If $S > S^{**}$, we have $\Delta \widehat{WL}^* = \Delta S/S = S^{\max}/S - 1$.

Figure 4 shows how the maximum relative welfare loss (the right-hand side of (B.51)) depends on the size of the program under our baseline calibration for $S \in (0, S^{**})$ where $S^{**} = \frac{S^{\max}}{1 + S^{\max} - S^*}$; under our baseline calibration $S^{**} = 0.66$. The figure essentially indicates that if the program size S is around the threshold value S^* , it does not matter much whether the government allocates funds to banks, firms or both. However, the larger the gap between S^* and S , the larger are the excess welfare losses from a non-optimal structure. Indeed, for (very) small programs, $\Delta \widehat{WL}^* = \Delta S/S \approx 1$, meaning that in this case the (excess) welfare loss from a non-optimal structure (funding firms when one should fund banks) is of the same order of magnitude as the stabilization-related welfare benefits from the candidate program. Overall, we get the same results as above in Appendix B.7.1 when using the welfare loss measure $\Delta \widehat{WL}$ (see expression (B.39) and Figure 3).

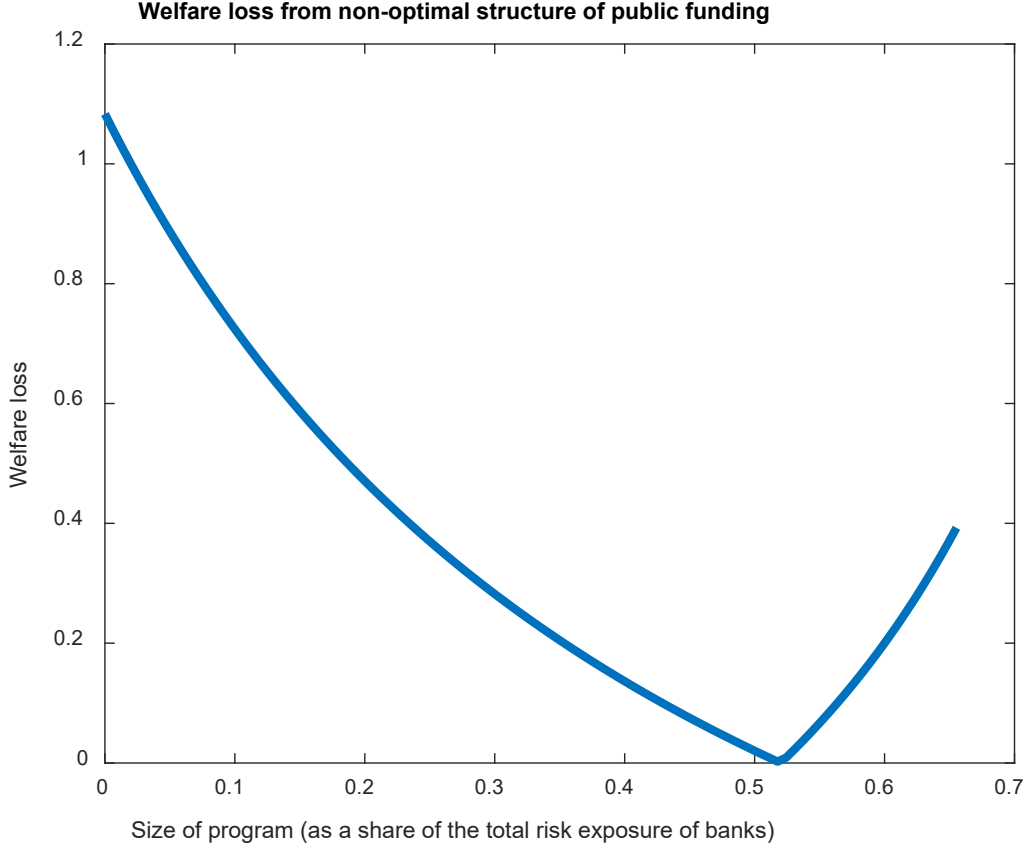


Figure 4: Welfare loss from non-optimal structure of public funding.

B.8 Robustly Optimal Public Funding of Both Banks and Firms

First example: The government does not know the threshold S^ .* According to Proposition 2, the threshold size of public funding is $S^* = 1 - MRTS/BL$. Assume the government does not know $MRTS$ (which tells how much more distorting it is to fund banks rather than non-financial firms), but believes with sufficient certainty that $MRTS$ lies somewhere between $MRTS_1$ and $MRTS_2$, where $MRTS_2 < MRTS_1$. This implies that the threshold size $S^* \in [S_1^*, S_2^*]$, where

$$S_i^* = 1 - \frac{MRTS_i}{BL}, \quad i \in \{1, 2\}.$$

If the size of the program the government wants to implement (S) is either smaller than S_1^* or larger than S_2^* , the situation is unproblematic: the government should capitalize banks (if $S < S_1^*$), or fund non-financial firms (if $S > S_2^*$). However, if $S \in (S_1^*, S_2^*)$, the government faces a dilemma: it may incur an excess welfare loss as large as $\Delta WL \propto$

$S \left(\frac{MRTS_1}{BL(1-S)} - 1 \right) = \frac{S}{1-S} (S - S_1^*)$ if it targets banks (but funding non-financial firms would be the optimal strategy) or as large as $\Delta WL \propto S \left(1 - \frac{MRTS_2}{BL(1-S)} \right) = \frac{S}{1-S} (S_2^* - S)$ if it targets non-financial firms (but capitalizing banks would be the optimal strategy). In such a situation, the government may want to choose, for example, a robust strategy that aims to minimizing the maximum (excess) welfare losses due to the misallocation of public funds. Such a strategy would involve targeting both banks and firms. Note that the welfare losses due to a mixed package are proportional to $WL(A^g, N^g) \propto MRTS \times A^g + N^g$.

A robust min-max strategy solves the following problem:

$$\min_{A^g, N^g} \max_{i \in \{1,2\}} MRTS_i A^g + N^g,$$

subject to $BRE \times S = BL(1-S)A^g + N^g$. One can show that such a strategy would give rise to the mixed policy package

$$N^g = BRE \times S \left(\frac{MRTS_1 - BL(1-S)}{MRTS_1 - MRTS_2} \right) = BRE \times S \left(\frac{S - S_1^*}{S_2^* - S_1^*} \right)$$

and

$$A^g = \left(\frac{BRE \times S}{BL(1-S)} \right) \left(\frac{BL(1-S) - MRTS_2}{MRTS_1 - MRTS_2} \right) = \left(\frac{BRE}{BL} \right) \left(\frac{S}{1-S} \right) \left(\frac{S_2^* - S}{S_2^* - S_1^*} \right),$$

where the government targets both banks and firms as long as $S \in (S_1^*, S_2^*)$. Quite intuitively, the share of funds allocated to non-financial firms (banks) increases (decreases) with the size of the program S .

Second example: Staggered program. Assume that the government knows the threshold value S^* , but it does not know if, in the end, it wants to implement a small program $S_1 < S^*$ or a large program $S_2 > S^*$. Given Proposition 2, the small program S_1 would be optimally implement by capitalizing banks, while the larger program S_2 would be optimally implemented by targeting non-financial firms.

We assume that the government first implements the smaller policy package S_1 , and subsequently decides whether or not it wants to expand the size of the program to S_2 . Let

A_1^g and N_1^g denote the government funding allocated to banks and firms, respectively, under policy package S_1 , while A_2^g and N_2^g denote the government funding allocated to banks and firms, respectively, when policy package S_2 is in place. We further assume that

$$A_2^g \geq A_1^g, \quad N_2^g \geq N_1^g$$

In words, if a certain amount of public funding is allocated to banks (firms) in the initial policy package S_1 , this money cannot be taken back if the program is expanded to S_2 . When designing the (robustly) optimal structure of policy package S_1 , the government takes these constraints into account.

A robustly optimal strategy aims at minimizing the maximum (excess) welfare losses due to the misallocation of public funds. A robustly optimal strategy solves the problem

$$\min_{A_1^g, N_1^g, A_2^g, N_2^g} \max_{i \in \{1, 2\}} MRTS \times A_i^g + N_i^g,$$

subject to

$$BRE \times S_i = BL(1 - S_i) A_i^g + N_i^g \text{ for } i \in \{1, 2\}$$

$$A_2^g \geq A_1^g, \quad N_2^g \geq N_1^g.$$

One can show that the robustly optimal structure of policy package S_1 is given by the equations

$$N_1^g = BRE \times S_1 \left(\frac{\frac{1}{BL_2^*} - \frac{1}{MRTS}}{\frac{1}{BL_2^*} - \frac{1}{BL_1^*}} \right) = BRE \times S_1 \left(\frac{1 - S_1}{1 - S^*} \right) \left(\frac{S_2 - S^*}{S_2 - S_1} \right)$$

and

$$A_1^g = \frac{BRE \times S_1 - N_1^g}{BL_1^*} = \frac{BRE \times S_1^*}{BL_1^*} \left(\frac{\frac{1}{MRTS} - \frac{1}{BL_1^*}}{\frac{1}{BL_2^*} - \frac{1}{BL_1^*}} \right) = \frac{BRE \times S_1}{BL} \left(\frac{1 - S_1}{1 - S^*} \right) \left(\frac{S^* - S_1}{S_2 - S_1} \right).$$

The robustly optimal structure of policy package S_2 is given by the equations

$$A_2^g = A_1^g$$

and

$$\begin{aligned} N_2^g &= BRE \times S_2 - BL_2^* A_2^g = BRE \times \left[S_2 - S_1 \left(\frac{BL_2^*}{BL_1^*} \right) \left(\frac{\frac{1}{MRT} - \frac{1}{BL_1^*}}{\frac{1}{BL_2^*} - \frac{1}{BL_1^*}} \right) \right] \\ &= BRE \times S_2 - \frac{BRE \times S_1}{BL} \left(\frac{1 - S_2}{1 - S^*} \right) \left(\frac{S^* - S_1}{S_2 - S_1} \right). \end{aligned}$$

One can show that the share of public funds allocated to non-financial firms (both N_1^g and N_2^g) increases with S_1 and S_2 , while the share of public funds allocated to banks decreases with S_1 and S_2 .

C Calibration

C.1 Data Moments and Calibration

In calibrating the real sector of the model, we follow the RBC literature. A period is one year. We calibrate the household utility function parameters to involve relatively modest risk aversion and a fairly inelastic labour supply: $\sigma = 2$, $\phi = 0.5$, and $\xi = 2$, and set the discount factor β to 0.98, which approximately corresponds to an annual real interest rate of 2%. We assign the depreciation rate δ to 0.10, a typical value in the literature. The capital share in the final goods sector, α , is set to the often-used value of 1/3. The output shares of investment and consumption are roughly 20% and 80%, respectively.

Calibration of the parameters of the financial block, while less standard, only requires values for excess returns to banks' and entrepreneurial firms' capital, their capital ratios, and bankers' monitoring costs (see Appendix C.2.1). To parametrize the steady state (excess) rate of return on entrepreneurial capital, r^n , we first take the value of 6.5%, commonly used in the RBC literature, as the average return to capital in the economy, and then subtract a riskless rate of 2% from it, yielding $r^n = 4.5\%$ (see, for example, Fama and French, 2002). As to the value for the entrepreneurial firms' steady state capital ratio, N/I , the literature

suggests substantial intertemporal and cross-section variation (e.g. Rajan and Zingales, 1995; de Jong, Kabir and Ngyen, 2008; Graham and Leary, 2011; and Graham, Leary, and Roberts, 2015). We choose the value of 0.45, which is close to the post-1990 estimate for the US by Graham et al. (2015).

The steady state (excess) rate of return on bank capital, r^a , is calibrated based on Albertazzi and Gambacorta (2009), who find the average after-tax return on bank equity in 1999–2003 to vary from 7% in the euro area to 14 – 15% in the UK and the US (Hirtle and Stiroh, 2007, reports similar magnitudes for US retail banks), and on Haldane and Alessandri (2009) who find the pre-tax return on bank equity in the UK to be around 20% on average over the recent decades. We set r^a to 12%, which lies in the mid-range of these estimates.

The bank’s steady state capital ratio is given by $A/(A + D - mI) = A/(I - N)$ (see Appendix D.2). Since the banks in our model have a stake in the projects they fund, the closest empirical counterpart for our bank capital is Tier 1 capital, which includes banks’ common stocks and retained earnings. Typical estimates, (e.g. Acharya and Steffen, 2014) of Tier 1 capital to (non-risk-adjusted) assets vary between 4% and 8%. Our model focuses on firm loans, abstracting from other bank assets. We set $A/(I - N) = 0.08$ to account for the riskiness of corporate lending. This magnitude also corresponds the pre-crisis Tier 1 equity-to-asset ratio in the aggregate bank balance sheet calculated by the FDIC.

Finding an estimate for monitoring costs is not easy. Based on the estimations of Albertazzi and Gambacorta (2009) and Philippon (2015), the unit cost of financial intermediation could be 1 – 4 % of a bank’s total assets. As their unit cost measures include activities in addition to monitoring, that estimate provides an upper bound for the ratio of monitoring costs to assets. However, corporate lending involves more intense monitoring than many other asset classes in a bank’s balance sheet. Furthermore, in our model the total operating costs of a bank are equal to the monitoring costs (since in the model there are no other operating costs). This suggests that the empirical counterpart could include (some) costs of financial intermediation not (directly) related to monitoring. Based on these observations, we choose a monitoring-cost-to-asset ratio ($mI/(I - N)$) of 1.5%.

C.2 Parameters of the Financial Block

C.2.1 Data Moments and Steady State Values of Model Variables

We now define the observable financial data moments in terms of the steady state of the model variables in the financial block (see Appendix D.2). The calibration of the parameters of the financial block of the model is based on the following observables:

- *Excess* rates of return on bank capital r^a , and on entrepreneurial capital r^n . In each period, bankers earn the gross rate of return $(1 + r)(1 + r^a)$ and entrepreneurs earn the rate of return $(1 + r)(1 + r^n)$, where $1 + r = 1/\beta$ is the real interest rate earned by workers in the steady state.
- Non-financial firms' capital ratio

$$CRF = \frac{N}{I}. \quad (\text{C.1})$$

- Banks' capital ratio

$$CRB = \frac{A}{A + D - mI} = \frac{A}{I - N}. \quad (\text{C.2})$$

In the denominator, we subtract the banks' monitoring costs mI from the total amount of funds $A + D$ to obtain the amount of the banks' assets allocated to the investment projects. Note the difference between the balance sheets of non-financial firms and banks. The balance sheets of non-financial firms include funds from bankers and depositors, as well as the entrepreneurs' own capital. The grand total is I . Banks have bankers' own capital and funds from depositors, and the aggregate amount of funds is $I - N$.

- Banks' monitoring costs as a ratio of banks' assets

$$MRB = \frac{mI}{I - N}. \quad (\text{C.3})$$

- Finally, it is useful to express the ratio of monitoring costs to output (mI/Y) with

the help of these data moments, and the investment share of output $\iota = I/Y$

$$\frac{mI}{Y} = MRB (1 - CRF) \iota. \quad (C.4)$$

C.2.2 Equations Linking Data Moments and Parameters of the Financial Block

Based on the equations from the main text, Appendix C.2.1, above, and the analytical solution of the steady state, given in Appendix D.2, we present equations linking the parameters of the financial block to the data moments. The calibrated financial block parameters are:

1. The exit rate of bankers λ^b . Equation (D.11) in Appendix D.2 implies that

$$\lambda^b = \frac{\beta}{1 + r^a} = \frac{1}{(1 + r^a)(1 + r)}. \quad (C.5)$$

2. The exit rate of entrepreneurs λ^e . Equation (D.12) in Appendix D.2 implies that

$$\lambda^e = \frac{\beta}{1 + r^n} = \frac{1}{(1 + r^n)(1 + r)}. \quad (C.6)$$

3. The (relative) difference in the success probabilities of good and bad projects $\Delta p/p_H$. Using the banker's incentive constraint (4b) and her participation constraint (4d), together with the aggregation equation (9), all evaluated at the steady state, yields $\Delta p/p_H = mI/((1 + r^a)A)$. Applying equations (C.2) and (C.3) gives

$$\frac{\Delta p}{p_H} = \frac{MRB}{CRB(1 + r^a)}. \quad (C.7)$$

4. The elasticity of the monitoring function $\gamma/(1 - \gamma)$. Inserting (C.5), (C.6) and (C.7) in (16), and recalling that $\nu = A/N$, yields $\gamma/(1 - \gamma) = A(r^a + MRB/CRB)/(r^n N)$. Then using (C.1) and (C.2) gives

$$\frac{\gamma}{1 - \gamma} = \frac{r^a CRB + MRB}{r^n \frac{CRF}{1 - CRF}}. \quad (C.8)$$

Note that $CRF/(1 - CRF) = N/(I - N)$ is the ratio of entrepreneurial capital to non-entrepreneurial capital in non-financial firms' balance sheets. Applying (C.2) and (C.3), $\gamma/(1 - \gamma)$ can be re-expressed in yet another way

$$\frac{\gamma}{1 - \gamma} = \frac{r^a A + mI}{r^n N} = \frac{\text{banks' profits} + \text{banks' monitoring costs}}{\text{entrepreneurs' profits}}$$

5. Parameter of the monitoring function Γ .

$$\Gamma = \left(\frac{1 + r^n}{1 + r^a} \right) \left(\frac{CRF}{CRB} \right) (1 - CRF)^{\frac{\gamma}{1-\gamma}} MRB^{\frac{1}{1-\gamma}}, \quad (\text{C.9})$$

where γ is given by (C.8). The derivation of (C.9) involves the following steps. In Appendix A.4 we define $G_t = (A_t + N_t)/I_t$. Applying (C.1) and (C.2), the steady state version of this equation can be rewritten as $G = CRB(1 - CRF) + CRF$. Using equations (D.14), (D.24) and (D.25) in Appendix D.2, the measure G can be expressed in terms of ν and the parameters of the financial block (including Γ). Finally, applying (16), (C.5), (C.6) and (C.7), we get (C.9).

C.3 Impulse Responses

Figure 5 portrays the impulse responses of key real and financial sector variables to a negative investment shock. As a benchmark, we also show the real sector impulse responses in a standard RBC model which corresponds to our model with the exception of financial intermediation and associated frictions.

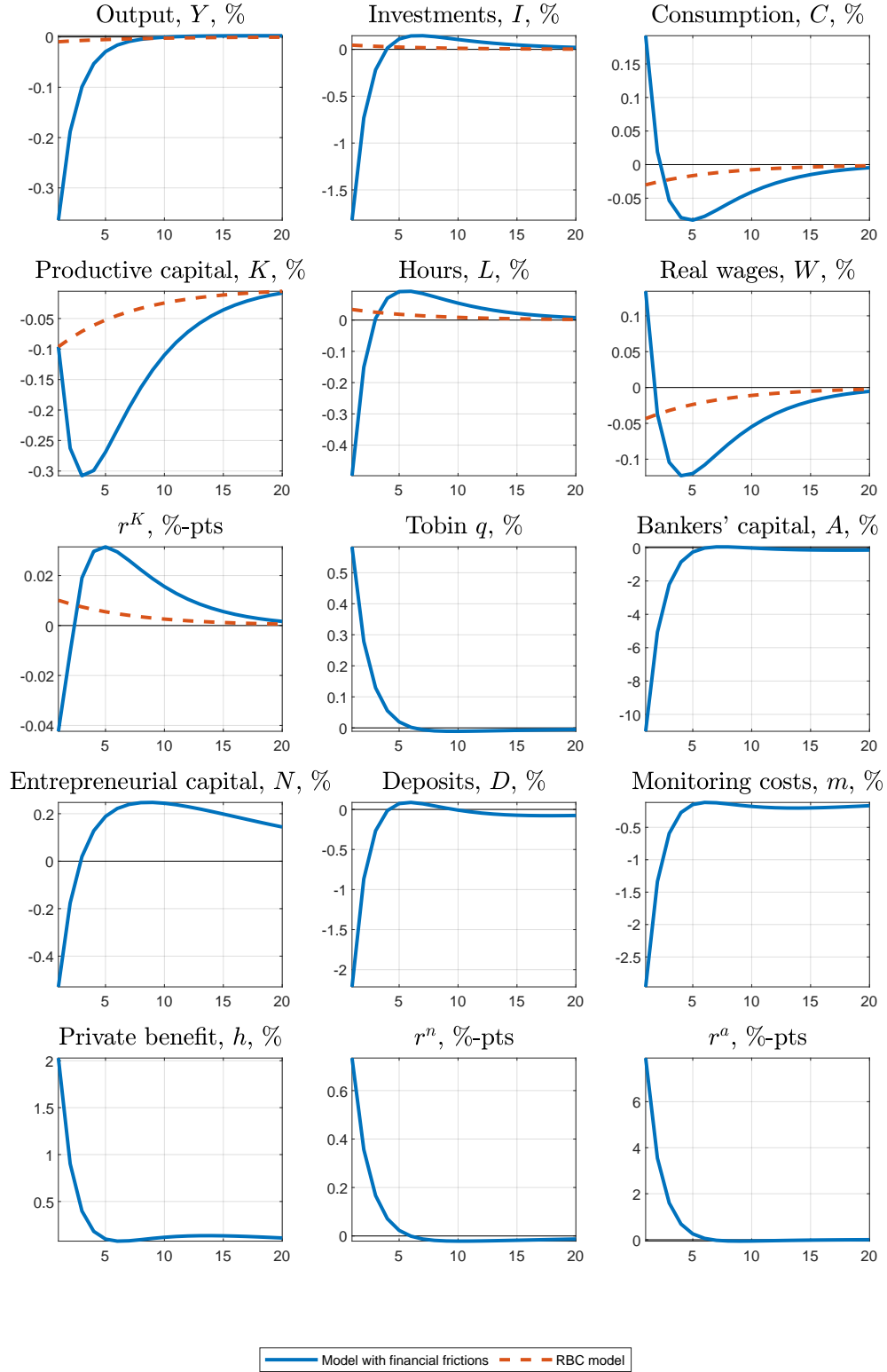


Figure 5: Impulse responses to a negative investment shock (1 percentage point decrease in success probabilities).

D Technical Appendix

D.1 Equilibrium

The equilibrium of the model is a sequence

$$\{K_{t+1}, L_t, Y_t, W_t, r_t^K, C_t, I_t, q_t, \rho_t, r_t^a, r_t^n, m_t, h_t, A_{t+1}, N_{t+1}, D_t\}_{t=0}^{\infty}$$

that satisfies equations (2), (3), (7), (8) (10), (13), (14) and (15a,b), together with

$$\xi L_t^\phi C_t^\sigma = W_t, \tag{D.1}$$

$$Y_t = K_t^\alpha L_t^{1-\alpha}, \tag{D.2}$$

$$W_t = (1 - \alpha) \frac{Y_t}{L_t}, \tag{D.3}$$

$$r_t^K = \alpha \frac{Y_t}{K_t}, \tag{D.4}$$

$$Y_t = C_t + I_t, \tag{D.5}$$

$$D_t = (1 + m_t) I_t - (A_t + N_t), \tag{D.6}$$

$$\rho_t = q_t - 1. \tag{D.7}$$

D.2 Steady State

We present the steady state of the model in three parts. In part A, we solve for the steady state values of the prices and the ratios of quantities (such as $\nu = A/N$) in the the financial block. In addition, we solve for the steady state values of the moral-hazard-related variables (h and m). In part B, we solve the steady state of the real (RBC) block. Finally, in part C, we solve for the steady state values of the quantities (levels) in the financial block.

A) We derive the steady state values of financial variables in four steps.

1. The law of motion of A_t is

$$A_{t+1} = \lambda^b \left(\frac{r_{t+1}^K + (1 - \delta) q_{t+1}}{q_t} \right) p_H q_t R_t^b I_t = \lambda^b \left(\frac{r_{t+1}^K + (1 - \delta) q_{t+1}}{q_t} \right) (1 + r_t^a) A_t \quad (\text{D.8})$$

and the law of motion of N_t is

$$N_{t+1} = \lambda^e \left(\frac{r_{t+1}^K + (1 - \delta) q_{t+1}}{q_t} \right) p_H q_t R_t^e I_t = \lambda^e \left(\frac{r_{t+1}^K + (1 - \delta) q_{t+1}}{q_t} \right) (1 + r_t^n) N_t. \quad (\text{D.9})$$

Since the household Euler equation (3) implies that in steady state

$$\left(\frac{r^K + (1 - \delta) q}{q} \right) = 1/\beta, \quad (\text{D.10})$$

the steady state versions of equations (D.8) and (D.9) immediately yield

$$1 + r^a = \frac{\beta}{\lambda^b} \quad (\text{D.11})$$

and

$$1 + r^n = \frac{\beta}{\lambda^e}. \quad (\text{D.12})$$

Furthermore, the steady state versions of (D.8) and (D.9) give

$$\frac{A}{N} \equiv \nu = \frac{\lambda^b R^b}{\lambda^e R^e} = \frac{\lambda^b m}{\lambda^e h}, \quad (\text{D.13})$$

where the last form follows since $R^b = m/(q\Delta p)$ and $R^e = h/(q\Delta p)$.

2. Denote

$$J_t = A_t + N_t$$

and recall from Appendix A.4 the definition $G_t = (A_t + N_t)/I_t = J_t/I_t$. Combining

these with (D.8) and (D.9), we get

$$J_{t+1} = \left(\frac{r_{t+1}^K + (1 - \delta) q_{t+1}}{q_t} \right) p_H q_t \frac{J_t}{G_t} (\lambda^b R_t^b + \lambda^e R_t^e)$$

(since $I_t = J_t/G_t$). Thus, in steady state

$$1 = \left(\frac{r^K + (1 - \delta) q}{q} \right) p_H q \frac{1}{G} (\lambda^b R^b + \lambda^e R^e).$$

The household Euler equation (3) implies that in steady state

$$1 = \beta \left(\frac{r^K + (1 - \delta) q}{q} \right).$$

Combine

$$R^b = m / (q \Delta p), \quad R^e = h / (q \Delta p),$$

with above to obtain

$$G = \frac{1}{\beta} \frac{p_H}{\Delta p} (\lambda^b m + \lambda^e h). \quad (\text{D.14})$$

3. Use the equilibrium relations

$$m_t = \frac{\frac{\Delta p}{p_H}}{1 + \frac{\Delta p}{p_H}} \left(\gamma \rho_t + \frac{A_t}{I_t} \right) = \frac{\frac{\Delta p}{p_H}}{1 + \frac{\Delta p}{p_H}} (\gamma \rho_t + \mu_t G_t) \quad (\text{D.15})$$

and

$$\begin{aligned} m_t &= \frac{\Delta p}{p_H} \left((1 - \gamma) \rho_t + \frac{N_t}{I_t} \right) \\ &= \frac{\Delta p}{p_H} ((1 - \gamma) \rho_t + (1 - \mu_t) G_t), \end{aligned} \quad (\text{D.16})$$

where

$$\mu_t = \frac{A_t}{A_t + N_t} = \frac{\nu_t}{1 + \nu_t}.$$

Plug (D.14) into (D.15) and (D.16). In steady state we then have

$$m = \frac{\frac{\Delta p}{p_H}}{1 + \frac{\Delta p}{p_H}} \left(\gamma \rho + \frac{\nu}{1 + \nu} \frac{1}{\beta} \frac{p_H}{\Delta p} (\lambda^b m + \lambda^e h) \right) \quad (\text{D.17})$$

and

$$h = \frac{\Delta p}{p_H} \left((1 - \gamma) \rho + \frac{1}{1 + \nu} \frac{1}{\beta} \frac{p_H}{\Delta p} (\lambda^b m + \lambda^e h) \right). \quad (\text{D.18})$$

From (D.13) we get

$$m = \frac{\lambda^e}{\lambda^b} \nu h \quad (\text{D.19})$$

and plugging this into (D.17) and (D.18) yields

$$\frac{\lambda^e}{\lambda^b} \nu h = \frac{\frac{\Delta p}{p_H}}{1 + \frac{\Delta p}{p_H}} \left(\gamma \rho + \nu \frac{1}{\beta} \frac{p_H}{\Delta p} \lambda^e h \right)$$

and

$$h = \frac{\Delta p}{p_H} \left((1 - \gamma) \rho + \frac{1}{\beta} \frac{p_H}{\Delta p} \lambda^e h \right). \quad (\text{D.20})$$

Solving ρ from (D.20) yields

$$\rho = \frac{p_H}{\Delta p} \left(1 - \frac{\lambda^e}{\beta} \right) \left(\frac{h}{1 - \gamma} \right). \quad (\text{D.21})$$

Plugging (D.21) into (D.17) gives

$$\frac{\lambda^e}{\lambda^b} \nu h = \frac{1}{1 + \frac{\Delta p}{p_H}} \left(\left(1 - \frac{\lambda^e}{\beta} \right) \frac{\gamma}{1 - \gamma} + \nu \frac{\lambda^e}{\beta} \right) h. \quad (\text{D.22})$$

Evidently, h cancels out from (D.22), and the equation can be solved for ν

$$\nu = \frac{\lambda^b}{\lambda^e} \left(\frac{1 - \frac{\lambda^e}{\beta}}{1 + \frac{\Delta p}{p_H} - \frac{\lambda^b}{\beta}} \right) \left(\frac{\gamma}{1 - \gamma} \right). \quad (\text{D.23})$$

4. Using the relation (D.19) together with the monitoring technology

$$h = \Gamma m^{-\frac{\gamma}{1-\gamma}} \Leftrightarrow m^\gamma b^{1-\gamma} = \Gamma^{1-\gamma}$$

we get

$$h = \left(\frac{\lambda^b}{\lambda^e} \right)^\gamma \frac{\Gamma^{1-\gamma}}{\nu^\gamma} \quad (\text{D.24})$$

and

$$m = \left(\frac{\lambda^e}{\lambda^b} \right)^{1-\gamma} \Gamma^{1-\gamma} \nu^{1-\gamma}. \quad (\text{D.25})$$

Given steps 1–4, we can express the steady state of prices and ratios as well as moral-hazard-related variables in the financial block in a recursive form. Rate of return to bank capital, equation (D.11):

$$1 + r^a = \frac{\beta}{\lambda^b}$$

Rate of return to entrepreneurial capital, equation (D.12):

$$1 + r^n = \frac{\beta}{\lambda^e}.$$

Ratio of informed capital, equation (D.23):

$$\nu = \frac{\lambda^b}{\lambda^e} \left(\frac{1 - \frac{\lambda^e}{\beta}}{1 + \frac{\Delta p}{p_H} - \frac{\lambda^b}{\beta}} \right) \left(\frac{\gamma}{1 - \gamma} \right).$$

Non-verifiable revenue from a “bad” investment project, equation (D.24):

$$h = \left(\frac{\lambda^b}{\lambda^e} \right)^\gamma \frac{\Gamma^{1-\gamma}}{\nu^\gamma}.$$

Monitoring intensity, equation (D.25):

$$m = \left(\frac{\lambda^e}{\lambda^b} \right)^{1-\gamma} \Gamma^{1-\gamma} \nu^{1-\gamma}.$$

Ratio of informed capital and investment, equation (D.14):

$$G = \frac{1}{\beta} \frac{p_H}{\Delta p} (\lambda^b m + \lambda^e h).$$

The net rate of return to the investment projects, equation (D.21):

$$\rho = \frac{p_H}{\Delta p} \left(1 - \frac{\lambda^e}{\beta} \right) \left(\frac{h}{1 - \gamma} \right).$$

The net rate of return to the investment project, ρ , can be further expressed as

$$\rho = \Gamma \frac{p_H}{\Delta p} \left(\frac{1 - \frac{\lambda^b}{\beta} + \frac{\Delta p}{p_H}}{\gamma} \right)^\gamma \left(\frac{1 - \frac{\lambda^e}{\beta}}{1 - \gamma} \right)^{1-\gamma}. \quad (\text{D.26})$$

B) We now turn to the variables in the real (or RBC) block of the model. The steady state of the real block is linked to the steady state of the financial block (solved in part A above) through the price of capital, q . The steady state version of (D.7) implies that the steady state price of capital is given by

$$q = 1 + \rho. \quad (\text{D.27})$$

Note that the steady state real interest rate is $r = 1/\beta - 1$. Then the steady state version of the household Euler equation (3) implies that in steady state, the rental rate of capital is

$$r^K = q(r + \delta). \quad (\text{D.28})$$

Next, applying the steady state versions of equations (2), (D.1), (D.2) (D.3), (D.4) and (D.5) allows us to solve the steady values of the remaining variables in the real block: Real wage

$$W = (1 - \alpha) \left(\frac{r^K}{\alpha} \right)^{-\frac{\alpha}{1-\alpha}}, \quad (\text{D.29})$$

physical capital stock

$$K = \left[\left(\frac{1 - \alpha}{\xi} \right) \left(\frac{r^K}{\alpha} \right)^{-\frac{\alpha+\phi}{1-\alpha}} \left(\frac{r^K}{\alpha} - \delta \right)^{-\sigma} \right]^{\frac{1}{\phi+\sigma}}, \quad (\text{D.30})$$

hours worked

$$L = K \left(\frac{r^K}{\alpha} \right)^{\frac{1}{1-\alpha}}, \quad (\text{D.31})$$

output

$$Y = \frac{r^K K}{\alpha}, \quad (\text{D.32})$$

investments

$$I = \delta K, \quad (\text{D.33})$$

and consumption

$$C = Y - I. \quad (\text{D.34})$$

C) Finally, we solve for the steady state values of the quantities in the financial block. Using $\nu = A/N$ and $A + N = GI$, gives the steady state values of bank capital

$$A = \frac{\nu}{1 + \nu} GI, \quad (\text{D.35})$$

and entrepreneurial capital

$$N = \frac{1}{1 + \nu} GI. \quad (\text{D.36})$$

Finally, applying the steady state version of (D.6) gives the steady state value of deposits

$$D = (1 + m - G)I. \quad (\text{D.37})$$

D.3 Ruling out the Corner Solution

In this appendix, we study the conditions under which the no-monitoring corner solution, $m_t = 0$, $h(m_t) = \bar{h}$, can be ruled out. Assume that a firm chooses *not* to be monitored: $m_t = 0$. One may for example think that the firm raises outside funding directly from households, without financial intermediation by banks. Then (4a) implies that $R_t^e = \bar{h}/(\Delta p q_t)$. Furthermore, according to equations (5a) and (5b), the maximum leverage, i_t/n_t , the firm

can obtain is given by

$$\frac{i_t}{n_t} = \frac{1}{g(r_t^a, q_t; m_t = 0, h_t = \bar{h})} = \frac{1}{\frac{p_H}{\Delta p} \bar{h} - \rho_t}.$$

Hence, the expected rate of return on entrepreneurial capital, $1 + \hat{r}_t^n$, is given by

$$1 + \hat{r}_t^n = \frac{p_H q_t R_t^e}{g(r_t^a, q_t; m_t = 0, h_t = \bar{h})} = \frac{\frac{p_H}{\Delta p} \bar{h}}{\frac{p_H}{\Delta p} \bar{h} - \rho_t}.$$

To rule out the corner solution, we must have

$$\hat{r}_t^n < r_t^n, \quad (\text{D.38})$$

where r_t^n is the expected rate of return on entrepreneurial capital if the entrepreneur chooses the interior solution that involves monitoring. In particular, the condition (D.38) should apply in the steady state, so that we get the condition

$$\bar{h} > \frac{\Delta p}{p_H} \frac{1 + r^n}{r^n} \rho. \quad (\text{D.39})$$

In addition, we seek a condition that guarantees that it is socially optimal to choose the “good” project rather than the “bad” project with the maximum level of private payoffs \bar{h} . For this condition to hold in the steady state, we must have

$$p_H q R > p_L q R + \bar{h} \Leftrightarrow \bar{h} < \frac{\Delta p}{p_H} q. \quad (\text{D.40})$$

When deriving the latter form of the inequality recall the normalization $p_H R = 1$. It is possible to rule out a corner solution if and only if there exist a value \bar{h} that satisfies both (D.39) and (D.40). Such a value \bar{h} exists if and only if $\left(\frac{1+r^n}{r^n}\right) \rho < q = 1 + \rho$, or equivalently

$$r^n > \rho. \quad (\text{D.41})$$

In Appendix D.4, we show that

$$\rho = r^n CRF + (r^a CRB + MRB)(1 - CRF),$$

where the data moments r^n , r^a , CRF , CRB and MRB are defined in Section 5, or alternatively in Appendix C.2. Then one can rewrite the condition (D.41) in such a way that it only includes data moments we match

$$r^n > r^a CRB + MRB. \quad (\text{D.42})$$

Our baseline calibration satisfies this condition.

D.4 Expressing ρ in Terms of Data Moments

In this appendix, we show that

$$\rho = r^n CRF + (r^a CRB + MRB)(1 - CRF), \quad (\text{D.43})$$

where ρ is the net return on the investment projects in steady state, while the data moments r^n , r^a , CRF , CRB and MRB are defined in Section 5, or alternatively in Appendix C.2. To derive (D.43), first apply equations (16), (D.19) and (D.21) to get

$$\rho = \left(\frac{p_H}{\Delta p} \right) \left(1 + \frac{\Delta p}{p_H} - \frac{\lambda^b}{\beta} \right) \left(\frac{m}{\gamma} \right). \quad (\text{D.44})$$

Next note that

$$m = \left(\frac{mI}{I - N} \right) \left(\frac{I - N}{I} \right) = MRB(1 - CRF), \quad (\text{D.45})$$

where the last form is derived using definitions (C.1) and (C.3) in Appendix C.2. Finally, plugging (C.5), (C.7), (C.8) and (D.45) into (D.44) yields (D.43).

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