Bank of Finland Research Discussion Papers
9 • 2021

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Bank of Finland
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Bank of Finland Research Discussion Papers
Editor-in-Chief Esa Jokivuolle

Bank of Finland Research Discussion Paper 9/2021
9 June 2021

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ISBN 978-952-323-379-9, online
ISSN 1456-6184, online

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Revisiting Intertemporal Elasticity of Substitution in a Sticky Price Model*

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June 8, 2021

Abstract

Macroeconomic models typically assume additively separable preferences where consumption enters the utility function in a logarithmic form. This restriction implies that consumption growth is highly sensitive to movements in real interest rates, which in turn implies an unrealistically steep demand curve and intertemporal trade-off. We re-estimate the stylized New Keynesian Model with US data using King-Plosser-Rebelo (1988) preferences with and without habits and show that the equilibrium real interest rate elasticity of output is in the range of $0.05 - 0.20$ in the US. Such low real interest rate elasticity is better in line with the empirical consumption Euler equation literature and implies relatively weak transmission of monetary policy to output and inflation.

Keywords: Monetary policy, Bayesian estimation, Non-separable utility.

JEL Codes: E32, E52,E21

*This is an updated version of the article which was earlier circulated under the title "Estimating Intertemporal Elasticity of Substitution in a Sticky Price Model". We are indebted to Michael Andrle, Shigeru Fujita, Efrem Castelnuovo, Ivan Jaccard, Mikael Juselius, Thomas Laubach and Ludo Visschers for many useful comments and suggestions. Oskari Vähämaa acknowledges financial support from the OP Financial Group Research Foundation.

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1 Introduction

Preferences must fulfill two important conditions in order for the balanced growth path to exist in the neoclassical growth model. First, the intertemporal elasticity of substitution must be invariant to the scale of consumption and the income and substitution effects associated with sustained growth in labor productivity must not change the labor supply (cf. King-Plosser-Rebelo, 1988). The latter condition states that, in the long run, income and substitution effects must cancel each other out.[1]

In order to fulfill these restrictions, representative agent macroeconomic models typically assume additively separable preferences where consumption enters logarithmically to utility. This restriction implies that the intertemporal elasticity of substitution is one and that the cross-elasticity between consumption and hours worked is zero. The assumption of additive logarithmic preferences is not innocuous for monetary policy transmission. It implies an unrealistically steep demand curve, where consumption growth reacts strongly to movements in real interest rates and thus on monetary policy. Moreover, the zero cross-elasticity of consumption and labor may undermine the transmission of monetary policy through the labor markets.[2] Finally, these assumptions can be challenged on empirical grounds.

First, estimates based on the consumption Euler equation yield consistently low consumption-real interest rate elasticities (see e.g. Hall, 1988, Cambell and Mankiw, 1989; Basu and Kimball, 2002; Fuhrer and Rudebusch, 2004, Yogo, 2004). Second, the zero cross-elasticity between consumption and hours is generally rejected by the empirical literature on labor supply. In particular, the level of consumption tends to fall after retirement or after a person becomes unemployed.[3] The latter evidence is consistent with the complementarity between consumption and work: households like to consume more when they work more. Aguiar et al. (2013) study based on American Time Use Surveys gives also support to macroeconomic models in which consumption and labor are strong complements.

In this paper, we apply the class of preferences similar to King-Plosser-Rebelo (1988, henceforth KPR), that relax the assumption of additive separability between consumption and labor, and estimate, instead of fixing a prior, the equilibrium in-

[1] Consensus from a large number of empirical work on labor supply elasticities also suggests that the income elasticity cannot be much larger than the substitution elasticity. Hence, the preferred estimate of the uncompensated wage elasticity is *weakly* positive.

[2] The recent Heterogenous Agent New Keynesian (HANK) models (see e.g. Kaplan et al. ,2018) suggest that monetary policy is effective because it generates a general equilibrium response of labor demand, and hence of household income, while the intertemporal channel is weak.

[3] See e.g. Banks et al., (1998), Hammermesh (1984), Bernheim et al. (2001), Browning and Crossley (2001), Ameriks et al. (2007), Hurst (2008).

tertemporal elasticity of substitution within the structural New Keynesian model.[4] KPR preferences also allow for a non-zero cross-elasticity between consumption and labor.

Given the structural model, we can make use of the Full Information Bayesian Maximum Likelihood method, instead of relying on the GMM estimation as is widely done in the consumption Euler equation literature. Full-information methods impose more restrictions on the estimated model and thus potentially make more efficient use of the information in the data (see e.g. Magnusson and Mavroeidis, 2010). As shown e.g. in Yogo (2004), Kiley (2010), and Kilponen (2012), the weak instrument problem makes it difficult to identify IES using GMM techniques from macro data.[5] To the extent that the structural model and the restriction on preferences are correct, full-information methods provide more reliable inference than limited information methods such as GMM. This enhances the model and preference validation.

Using the Bayesian Maximum Likelihood Method and US data from the period 1984Q1–2018Q4, we find that the real interest rate elasticity of output is in the range of $0.1-0.2$. While these values contrast starkly with the unitary real interest elasticity of output implied by the logarithmic and additively separable utility, they are well in line with estimates based on the consumption Euler equation. An important implication of our result is that the relatively low elasticity of consumption growth to real interest rate weakens considerably the real interest rate channel of monetary policy. An equal-size monetary policy shock initially delivers a roughly 3 times larger impact on output and about 2 times larger impact on inflation in the model with logarithmic preferences when compared to the model with KPR preferences.

In our extended model with habit persistence, we show that a given equilibrium intertemporal elasticity of substitution is consistent with different combinations of curvature and habit intensity. Smets and Wouters (2007), who estimate a fully fledged DSGE model with KPR preferences, find a low value of curvature and high habit intensity. However, we show that the results are sensitive to the choice of priors of the two key parameters determining the equilibrium IES. We find that the US data weakly support the combination of high curvature and moderate degree of habit persistence. Even though the two specifications lead to similar and low estimated IES, in the range of $0.03 - 0.05$, the strength of the monetary policy transmission is again considerably weaker when the curvature is high.

The rest of the paper is organized as follows. Section 2 describes the model.

[4] Bilbiie (2011) studies the New Keynesian Model with non-separable preferences. His main interest is exploring the implications of non-separable preferences for fiscal policy.

[5] Campbell and Ludvigson (2001) use the US state-level data to evaluate the degree of IES. They find that the IES could take any value between 0 and 1.5.

Section 3 provides the estimation results including robustness analysis. Section 4 concludes.

2 The model

This section develops a stylized sticky price monetary policy model featuring King-Plosser-Rebelo preferences. We follow closely a textbook-type derivation of the sticky price monetary policy model (see e.g. Goodfriend and King, 1997; Walsh, 2010; Woodford, 2003; Gali, 2008).

2.1 Households

The economy is populated by identical infinitely-lived households who solve the following problem

$$
\begin{aligned}
& \max_{C_t(i),N_t} E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \\
& s.t. \\
\int_0^1 P_t(i) C_t(i) di + Q_t B_t &= B_{t-1} + W_t N_t + T_t \\
C_t &\equiv \left(\int C_t(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \\
\lim_{T\to\infty} \mathbb{E}_t(B_T) &\geq 0.
\end{aligned}
$$

where $C_t(i)$ is the quantity of good i consumed by the representative household in period t; $P_t(i)$ is the price of good i; N_t is quantity of labor; W_t is nominal wage, B_t represents purchases of one-period bonds of price Q_t; T_t is the lump-sum component of income, and ϵ is the elasticity of substitution between the differentiated goods.[6] Following King-Plosser-Rebelo (1988), Kimball (1995), Basu and Kimball (2002), we assume that the the additively time-separable felicity function $U(C_t, N_t)$ takes a form

$$
U(C_t, N_t) = \frac{C_t^{1-\gamma}}{1-\gamma} e^{(\gamma-1)v(N_t)}, \tag{1}
$$

where $\gamma > 1$ controls the concavity of the utility function.[7] In this formulation s$\equiv 1/\gamma$ denotes the labor-held-constant intertemporal elasticity of consumption. Note that

[6] ϵ also denotes the absolute value of the own price elasticity of the demand for a good.

[7] Jaimovich and Rebelo (2009) further extend this class of preferences by considering a time-non-separable version. The original KPR preferences arise as a special case of their preferences. Note

γ is (up to scaling) equal to the usual risk-aversion measure only in the special case of exogenously fixed labor as shown by Swanson (2012, corollary 1, p. 1671). That is, the usual measure of risk aversion ignores a household's ability to offset income shocks through adjustment of labor. As discussed further by Swansson (2012), high values of γ (or low values of s) are not ruled out by empirical micro estimates of risk aversion when the labor margin is taken into account. $v(N_t)$ is a strictly increasing function of quantity of labor, representing the disutility from work.[8] Note that in the limiting case where $s \equiv \gamma^{-1} = 1$, the function $U(C_t, N_t) - \frac{1}{1-\gamma}$ converges, by l'Hopital's rule, to $\ln(C_t) - v(N_t)$.

The optimal choice of consumption and labor supply yields the following consumption Euler equation and the labor supply equation

$$Q_t = \beta \mathbb{E}_t \left\{ \frac{U_C(C_{t+1}, N_{t+1})}{U_C(C_t, N_t)} \frac{P_t}{P_{t+1}} \right\} \tag{2}$$

$$\frac{W_t}{P_t} = -\frac{U_N(C_t, N_t)}{U_C(C_t, N_t)} \tag{3}$$

where $U_C(C_{t+j}, N_{t+j}) = C_{t+j}^{-\gamma} e^{(\gamma-1)v(N_{t+j})}$; $U_N(C_t, N_t) = \frac{C_t^{1-\gamma}}{1-\gamma} e^{(\gamma-1)v(N_t)}(\gamma - 1)v'(N_t)$. $\mathbb{E}_t$ is the usual conditional expectation operator. The representative household must also decide on the allocation of her consumption expenditure among the differentiated goods. This gives rise to the familiar demand equations:

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} C_t, \tag{4}$$

where $P_t \equiv \left(\int P_t(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}$ is the aggregate price index.

Focusing on the first-order terms in the Taylor expansion and assuming homoscedasticity of the stochastic processes for $c_t \equiv \ln(C_t)$, $p_t = \ln(P_t)$ and $n_t = \ln N_t$, the optimal consumption and labor supply dynamics can be re-parameterized as

$$c_t = \mathbb{E}_t c_{t+1} - s(i_t - \mathbb{E}_t \pi_{t+1} - \rho) - (1-s)\tau \mathbb{E}_t \Delta n_{t+1} \tag{5}$$

$$w_t - p_t = c_t + \varphi n_t + \iota, \tag{6}$$

that Greenwood, Hercowitz, and Huffman (1988) (GHH) preferences are not consistent with the balanced growth path due to lack of income effect. Hence, a permanent change in productivity would lead into a permanent change in labor supply.

[8] Smets and Wouters (2007) use a similar utility function with the additional assumption of a particular functional form for $v(N_t)$, namely $v(N_t) = (\sigma - 1)/(1 + \xi_l) N_t^{(1+\xi_l)}$, where ξ_l is the labor supply elasticity.

where $\pi_{t+1} \equiv \ln P_{t+1} - \ln P_t$, $i_t \equiv -q_t$, s$\equiv 1/\gamma$, $\rho \equiv -\ln\beta$, $\varphi \equiv \frac{v''(N)/N}{v'(N)}$ and $\iota \equiv \ln\tau - (1+\varphi)n$ and $\tau = WN/PC = v'(N)N$.

According to equation (6), there is a unitary elasticity between the real wage and consumption such that the real wage and the consumption grow at the same rate in the long-run. The elasticity between the real wage and labor depends on the term $\varphi \equiv \frac{v''(N)}{v'(N)}N$. This is an inverse of the consumption-constant elasticity of labor supply.[9]

We can relate this term to Frisch labor supply elasticity ξ, i.e. to the labor supply elasticity that keeps the marginal utility of consumption constant (see Appendix A for detailed derivation) such that

$$\xi = \frac{1}{\varphi + \tau(1-s)}. \tag{7}$$

Equation (7) shows that the Frisch elasticity of labor supply ξ is generally lower than the consumption-constant elasticity of labor supply φ^{-1}. The difference between these two elasticities depends directly on the cross-elasticity of consumption and labor, since $(1-s)\tau = -\frac{U_{CN}}{U_{CC}}\frac{N}{C} = -\frac{dC}{dN}\frac{N}{C}$. Hence, $(1-s)\tau$ parameterizes the elasticity of consumption w.r.t. labor supply. [10]

Non-separability between consumption and labor implies that there is no unique way to measure the willingness of consumers to substitute consumption over time. Under KPR preferences, s is the labor-held-constant intertemporal elasticity of substitution. Allowing the household also to use the labor margin in response to changes in the real interest rate while holding the expected real wage constant gives the following (near steady state approximation for) IES

$$\psi^* = s\varphi\xi. \tag{8}$$

[9] Kimball (1995) argues that the inverse of the consumption-constant labor supply elasticity φ can be calibrated on the basis of marginal expenditure share of leisure being equal to the ratio of marginal expenditure share of consumption to leisure times the wage income consumption share. His preferred value for φ^{-1} is one.

[10] As discussed by Kimball and Shapiro (2008), the consumption-constant labor supply elasticity is most useful for understanding how a permanent change in the real wage impacts the labor supply. The Frisch elasticity gives the impact of a temporary change in the real wage on labor supply. This means that the Frisch elasticity is a more useful concept at the business-cycle frequency. In accordance with this interpretation, it is natural to find that the Frisch elasticity of labor supply is lower than the consumption-constant elasticity. Finally, note that when $s = 1$, these two elasticities coincide.

As in the case of labor elasticities, ψ^* is lower than s when consumption and leisure are complements. In the spirit of Frisch elasticity, we can derive yet another measure of IES by keeping the marginal disutility of labor constant $\psi^{**} = \xi(\varphi s + (1-s)\tau)$. In section 2.3, we introduce the equilibrium IES, ψ, which, in our opinion, is the most relevant elasticity in a macroeconomic context. This elasticity effectively measures the elasticity of consumption growth to the real interest rate after the general equilibrium implications of the model are taken into consideration.

Finally, notice that by letting $s \to 1$, the optimal consumption and labor supply equations given in equations (5)-(6) collapse to

$$c_t = \mathbb{E}_t c_{t+1} - (i_t - \mathbb{E}_t \pi_{t+1} - \rho), \tag{9}$$

$$w_t - p_t = c_t + \varphi n_t + \iota, \tag{10}$$

and where φ can now be interpreted directly as inverse of the Frisch elasticity of labor supply. These equations are also consistent with the balanced growth path but with two important differences. First, employment is no longer a part of the dynamic IS equation. Second, the elasticity of consumption with respect to real interest rate is restricted to unity.

2.2 Firms, optimal price-setting and the inflation equation

Specification of the supply side of the model follows the New Keynesian standard setup. A continuum of firms indexed by $i \in [0,1]$ produce a differentiated goods using homogenous technology. Firms' production possibilities are given by the production function:

$$Y_t(i) = A_t N_t(i)^{1-\alpha}. \tag{11}$$

A_t represents the common stochastic level of technology. All firms face identical isoelastic demand schedule (4), and they take aggregate price and quantities as given.

In order to introduce price rigidity into the model, each firm may re-set its price only with probability $1-\theta$. Thus a measure of $1-\theta$ producers reset their prices in each period. The average duration of price is given by $1/(1-\theta)$. In this framework, combining (log linearized) optimal price-setting rule of the firms with the goods market and the labor market clearing conditions and IS equation (5), delivers a familiar inflation equation[11]

[11] Eichenbaum and Fisher (2007) derive a model in which the elasticity of demand firms face is variable, capital is firm-specific and costly to adjust. This leads to specification of the inflation equation in which the firm specific capital reduces the response of inflation to marginal cost i.e. leads to a smaller slope of the marginal cost term in equation (12).

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \lambda \widehat{mc}_t, \tag{12}$$

where $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}\Theta$, $\Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$ and $\widehat{mc}_t$ is deviation of marginal costs from its flexible price counterpart.

Linking the marginal cost term to the output gap such that

$$\widehat{mc}_t = \frac{(1+\varphi)}{1-\alpha}(y_t - y_t^n), \tag{13}$$

combining (13) with (12), and defining $\tilde{y}_t \equiv (y_t - y_t^n)$, we obtain the New Keynesian Phillips Curve

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \lambda \frac{(1+\varphi)}{1-\alpha}\tilde{y}_t. \tag{14}$$

Note that the marginal cost term (13) depends on labor supply elasticity φ, and therefore indirectly on IES according to equation (7). This dependence, however, is rather weak (see Appendix C).

Contrary to KPR preferences, in a case of additively separable constant relative risk aversion utility function, the relationship between the real marginal costs and output wedge can be written as $\widehat{mc}_t = \left(\gamma + \frac{\xi+\alpha}{1-\alpha}\right)\tilde{y}_t$. This implies that IES and the slope of the Phillips curve are much more tightly linked. A lower intertemporal elasticity of substitution (a higher γ), given other parameter values, implies that inflation is more responsive to fluctuations in the output wedge (see e.g. Gali, 2008).

2.3 IS equation

The final step in the derivations is to express the IS curve in terms of the output wedge $\tilde{y}_t$ and to define the natural rate of interest. Using the approximate log linear production function $y_t = (1-\alpha)n_t + a_t$ and substituting Δn_{t+1} away from the IS curve yields:

$$y_t = \mathbb{E}_t y_{t+1} + \frac{(1-s)\tau}{1-\alpha-(1-s)\tau}\mathbb{E}_t \Delta a_{t+1} - \frac{(1-\alpha)s}{1-\alpha-(1-s)\tau}(i_t - \mathbb{E}_t \pi_{t+1} - \rho) \tag{15}$$

Re-writing above in terms of the output wedge $\tilde{y}_t \equiv (y_t - y_t^n)$ by subtracting y_t^n from both sides, and denoting flexible price output as $y_t^n = a_t + \vartheta_y^n$ we arrive to:

$$\tilde{y}_t = \mathbb{E}_t \tilde{y}_{t+1} - \psi(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n), \tag{16}$$

where r_t^n denotes the natural (real) rate of interest:

$$r_t^n = \rho + \frac{1}{s}\mathbb{E}_t \Delta a_{t+1}. \tag{17}$$

and $\psi \equiv \frac{(1-\alpha)s}{1-\alpha-(1-s)\tau}$. The natural rate of interest given in equation (17) is the equilibrium real rate of return in the flexible-price economy.

Equation (16) takes exactly the same form as in the model with additively separable log preferences, but with the following important difference: The elasticity of output wedge with respect to the real interest rate ψ is different from unity. In this set-up, ψ can be interpreted as (equilibrium) intertemporal elasticity of substitution, i.e. the elasticity of consumption growth to the real interest rate after accounting for the general equilibrium features of the macro model. This elasticity is a function of curvature of utility, cross-elasticity between consumption and labor, and the concavity of the production function.

2.4 Alternative formulations of the IS curve

Another way of formulating the IS equation is to express it in terms of expected growth in labor. This alternative formulation can be achieved by using $y_t = (1-\alpha)n_t + a_t$ to substitute for output in equation (15). This yields

$$\mathbb{E}_t \Delta n_{t+1} = \psi'(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n), \tag{18}$$

where $r_t^n \equiv \rho + \frac{1}{s}\mathbb{E}_t \Delta a_{t+1}$, as defined earlier and $\psi' = \frac{s}{[(1-\alpha)-(1-s)\tau]}$. ψ' now gives the elasticity of labor w.r.t. the real interest rate. It is equal to ψ in the special case where $\alpha = 0$. Otherwise, due to the concavity of the production function, $\psi' > \psi$. Equation (18) shows that labor can be used as an observable in the estimation instead of the output wedge $y_t - y_t^n$, which requires a proxy for the unobservable natural rate of output y_t^n. A clear benefit of using labor is that there is a much less controversy on how to measure labor than how to measure y_t^n, or how to treat the growth component of output in the estimation. Yet another way of writing the IS curve is not to substitute for labor, but simply subtract y_t^n from both sides of (15) and use the fact that $y_t^n = a_t + \vartheta_y^n$. This yields

$$\tilde{y}_t = \mathbb{E}_t \tilde{y}_{t+1} - s(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n) - (1-s)\tau \mathbb{E}_t \Delta n_{t+1}, \tag{19}$$

where $r_t^n \equiv \rho + \frac{1}{s}\mathbb{E}_t \Delta a_{t+1}$.

This representation of the IS curve highlights the usual direct intertemporal channel of monetary policy through the real interest rate and the indirect channel emanating from labor. Note that the direct intertemporal channel is weakened, while the impact through labor is strengthened when IES declines.

3 Estimation

Relaxing the assumption of non-zero cross elasticity between consumption and labor permits a wide range of values for the curvature of utility with respect to consumption without distorting the relationship between output wedge and inflation in the sticky price monetary policy model, while also keeping the model consistent with the long-run labor supply facts. Typical values found in the empirical macro literature for the elasticity of consumption growth to the real interest rate are closer to zero than one.

At the same time, one of the key weaknesses of estimating this elasticity directly from the consumption Euler equation is that some form of instrumental variable estimation needs to be employed (e.g. Hall (1988), Cambell and Mankiw, (1989), Fuhrer and Rudebusch, 2004 and Yogo, 2004). As shown for instance in Yogo (2004), Kiley (2010) and Kilponen (2012), the weak instrument problem makes it difficult to identify this elasticity. In order to rest on more reliable inference, the weak instrument problem is addressed by using weak instrument robust confidence intervals that are typically much wider than the classical ones (see e.g. Yogo, 2004, Stock and Yogo, 2005). But then, large confidence intervals do not allow to statistically discriminate between alternative consumption preference specifications. The Bayesian Maximum Likelihood method applied to the structural model does not suffer from a similar problem. As can be seen later on, credible sets around the point estimates of the equilibrium IES and s are rather tight even when uninformative priors are used.

In this section, we estimate s together with the other key parameters of the model using the structural equilibrium relations given in (32)-(33), and the respective definitions given underneath these equations. For comparison, we also estimate the model with additively separable logarithmic utility. We label these models as $M1$ and $M2$ in what follows. In order to make our estimation exercise more comparable to many other studies, we write the monetary policy rule by allowing interest rate smoothing and assuming that the shocks to the interest rate rule are i.i.d. over time:

$$i_t = \rho_i i_{t-1} + (1-\rho_i)[\rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t] + v_t,\ v_t \sim N(0, \sigma_v^2). \tag{20}$$

We also allow AR(1) shocks to the inflation equation (mark-up shocks) and to productivity shifter a_t as is standard in the literature:

$$\begin{aligned} \epsilon_t^\pi &= \rho_\pi \epsilon_{t-1}^\pi + \varepsilon_t,\ \varepsilon_t \sim N(0, \sigma_\varepsilon^2). & (21) \\ a_t &= \rho_a a_{t-1} + \epsilon_t^a,\ \epsilon_t^a \sim N(0, \sigma_{\epsilon^a}) & (22) \end{aligned}$$

3.1 The data

Our baseline estimation sample is 1984Q1-2018Q4. As observable variables, we use hours worked, inflation, and interest rate. In contrast to many others, our observable vector contains no output gap. A clear benefit of using labor as observable is that there is much less controversy on how to measure labor than how to measure y_t^n, or how to treat the deterministic growth component of output in the estimation.[12] Hours worked are calculated following Smets and Wouters (2007), who also estimate a New Keynesian model with a non-separable utility. Specifically, we compute $n =$ ln((H/L)*(E/100)), where H=average weekly hours worked in non-farm business, E = Employment of 16 years of age and older, and L = members of the population 16 years and older. Our measure of inflation, quarterly log difference of the GDP deflator, is also chosen to be in line with Smets and Wouter (2007). Interest rate is the quarterly federal funds rate. However, given that our stylized model does not take into account the zero lower bound, we use the shadow rates of Wu and Xia (2016) in lieu of the federal funds rates for the periods when the ZLB condition is binding. In section 3.5 where we explore the robustness of our results, we restrict the estimation sample to the period where we can use only the federal funds rate. The corresponding observable variables are shown in Figure 1. Parameters α, τ and ϵ are fixed according to Table 3 in the Appendix C.

3.2 Choice of priors

We rely primarily on the evidence summarized in Hall (2009) when choosing the priors for the key labor market parameters and s. Hall's "priors" for the Frisch elasticity and the interest rate elasticity are as follows: $\xi = 0.7$ and $\psi = 0.5$. These priors translate roughly to following prior values for the IES and the (inverse) consumption-constant labor supply elasticity, $s = 0.20$ and $\varphi = 1.03$ (at given $\tau = 0.5$, $\alpha = 0.33$), respectively. The implied prior for the cross-elasticity between consumption and hours is $\frac{dC}{dN}\frac{N}{C} = (1-s)\tau = 0.4$. Direct empirical evidence on this cross-elasticity is clearly more scarce than on the Frisch elasticity. Hall (2009) provides a brief summary of the empirical literature that attempts to identify this elasticity by looking at what happens to the level of consumption when a person stops working. This means that the cross elasticity is identified from the correlation of consumption and the exogenous movements in the labor supply (due to e.g. unemployment, disability or retirement). Based on this literature, Hall's (2009) preferred value of this cross-elasticity is 0.3. Kimball and Shapiro (2008) use a specific survey

[12] See e.g. Canova (1998).

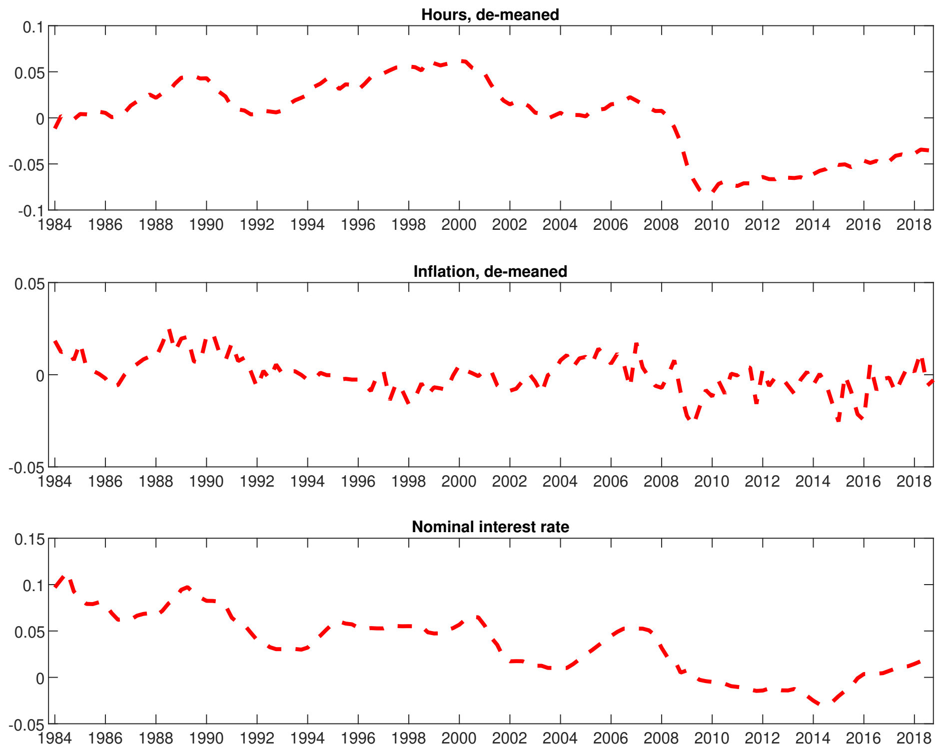

Figure 1: Observable Variables Used in the Estimation

Note: This figure shows the quarterly data from the U.S. Hours is the seasonally adjusted and de-meaned hours worked at non-agricultural industries. Inflation is the annualized quarterly difference of log GDP deflator. Nominal interest rate is the annualized Federal Funds Rate (or the shadow rates of Wu and Xia, 2016, when the ZLB condition is binding). See section 3.1 for more details of the data.

evidence on the response of hours to a large wealth shock to estimate different labor supply elasticities. Unfortunately, they are not able to uncover the cross-elasticity discussed herein. However, their baseline value needed to infer the other labor supply elasticities is also 0.3. Chetty (2006) argues that upper bound of this elasticity is 0.15, considerably lower than the values preferred by Hall (2009) and Kimball and Shapiro (2008).

In comparison to Smets and Wouters (2007), who also use KPR preferences to estimate a more fully specified DSGE model, our prior mean of the curvature of the utility function is quite a bit higher. Otherwise, our priors are rather standard (for comparison, see for instance Del Negro and Schorfheide, 2008 and Smets and Wouters, 2007). Prior densities and estimation results are summarized in Table 1.

3.3 The results

To begin with, the main result for $M1$ is that the data supports a low value for s. Posterior mean of s is as low as 0.05 with a relatively tight 90% credible set, ranging from 0.02 to 0.07. This implies together with the other estimated and calibrated parameters of the model that the posterior mean estimate for the elasticity of output with respect to real interest rate is $\hat{\psi} = 0.16$ $(0.09, 0.24)$.[13] This accords well with the consumption Euler equation based literature. Such a low value of s implies a strong complementarity between consumption and labor and hence a rather large difference between Frisch and consumption-constant elasticity of labor supply.[14] The posterior mean estimate for the Frisch labor supply elasticity is $\hat{\xi} = 0.97$ $(0.50, 1.76)$ while the posterior mean estimate for the consumption-constant elasticity of labor supply $\hat{\varphi}^{-1} = 2.50$ $(0.59, 7.05)$. Our point estimate of Frisch elasticity is close to the mean of micro estimates of aggregate Frisch elasticity, a sum of intensive and extensive elasticities, surveyed by Chetty et al. (2012). However, also in line with many other studies, our confidence interval for Frisch elasticity is relatively wide. This is perhaps to be expected as recent studies have highlighted that aggregate reactions of labor supply depend on the structure of the economy, e.g the wealth distribution, and the business-cycle conditions (see Attanasio et al., 2018). This uncertainty is amplified when a posterior distribution for the consumption constant elasticity is calculated as a nonlinear function of s and ξ^{-1}.

Finally, the posterior mean estimate for the slope of the Phillips curve $\hat{\kappa}$ is low 0.004 $(0.002, 0.006)$, but in line with e.g. Hazel et al. (2020) and to some extent with Mavroeidis et al. (2014). The estimated parameters of the policy rule are $\rho_i = 0.85$, $\hat{\phi}_x = 0.24$, $\hat{\phi}_\pi = 1.61$.

As for the shocks, the monetary policy shock has an (annualized) standard deviation of 52 basis points, while the cost-push shock has an (annualized) standard deviation of 36 basis points and persistence of 0.53. The technology shock is strongly serially correlated and the standard error of innovations is equal to 22 basis points

[13] The numbers in the brackets provide 90% probability sets.

[14] See Chetty and Szeidl (2007) for discussion on how consumption commitments amplify a household's risk aversion with respect to moderate and temporary income shocks and could in principle explain why very high risk aversion is not necessarily inconsistent with consumer behavior.

Table 1: Priors and Summary of Posteriors

	Prior			**Posterior**					
				M1			*M2*		
Parameter	*Density*	*Mean*	*P*	*Mean*	*90% CI*		*Mean*	*90% CI*	
Firms and Households									
θ	$\mathcal{B}$	0.60	0.15	0.93	0.90	0.95	0.92	0.90	0.94
$s^{(*}$	$\mathcal{G}$	0.20	0.10	0.05	0.02	0.07	–	–	–
ξ^{-1}	$\mathcal{G}$	1.40	0.50	1.19	0.49	1.88	1.53	0.70	2.30
β	$\mathcal{B}$	0.99	0.001	0.991	0.990	0.992	0.991	0.990	0.992
Interest rate rule									
ρ_i	$\mathcal{B}$	0.50	0.10	0.85	0.82	0.88	0.78	0.74	0.81
ϕ_x	$\mathcal{G}$	0.20	0.10	0.24	0.19	0.28	0.25	0.21	0.28
ϕ_π	$\mathcal{G}$	1.50	0.25	1.61	1.27	1.94	1.48	1.20	1.76
Exogenous shock processes									
ρ_a	$\mathcal{B}$	0.40	0.10	0.92	0.90	0.95	0.92	0.92	0.92
ρ_π	$\mathcal{B}$	0.40	0.10	0.53	0.44	0.61	0.51	0.42	0.60
σ_a	$\mathcal{G}^{-1}$	0.80	0.50	0.22	0.17	0.26	1.88	1.66	2.09
σ_π	$\mathcal{G}^{-1}$	0.30	0.50	0.09	0.07	0.11	0.10	0.08	0.11
σ_v	$\mathcal{G}^{-1}$	0.20	0.50	0.13	0.12	0.14	0.15	0.13	0.17
LMD					1454.02			1417.52	

Note: $\mathcal{B}, \mathcal{G}$ and $\mathcal{G}^{-1}$ correspond to Beta, Gamma and inverse Gamma distributions. Mean corresponds to mean and P is the standard deviation of the respective prior distribution. Fixed parameters are $\alpha = 0.33$, $\tau = 0.5$, $\epsilon = 6$ as in Table 1. Prior and posterior moments for the standard errors ($\sigma_.$) are in percentage form. *) In $M2$, s is fixed to 1. Hence, prior and posterior moments are irrelevant. LMD is log marginal density. The estimation sample is 1984Q1-2007Q4 and estimations were done using Dynare version 4.6.1. Posterior distribution was obtained by Metropolis–Hastings algorithm.

in quarterly terms. This is somewhat smaller than given by most of the estimates based on Solow residuals.

As for comparison between $M1$ and $M2$, where the intertemporal elasticity of substitution is restricted to unity, the main difference is that the standard error of innovations to technology shocks in $M2$ is almost 9(!) times larger than the respective standard error in $M1$. Furthermore, the data prefers the model $M1$. The ratio of marginal likelihood values (LMDs) between the two models, in favour of $M1$, is equal to 1.026.(see Table 1). As for the other reduced form parameters the slope of the Phillips curve in $M2$ is 0.005 (0.003, 0.008), which is very close to the value estimated for $M1$.

Finally, Figure 2 shows the equilibrium responses to a one standard deviation monetary policy shock in the two models. There is a clear difference in the strength of responses. The initial impact on output and employment is over three times larger in $\mathcal{M}2$ than in $\mathcal{M}1$, while the immediate inflation response is over two times larger. Since the monetary policy shocks in the two models have almost equal standard deviations, differences between the models reflect a stronger amplification in the model with additively separable preferences. Thus, the restrictive assumption that the real interest rate elasticity of output is one clearly has a non-trivial impact on the strength of the monetary policy transmission mechanism.

3.4 Sensitivity to priors and habit persistence

Our results suggest a considerably higher curvature of the utility function due to lower values of s than those obtained by Smets and Wouters (2007). Although the results are not directly comparable due to various different modeling assumptions, we demonstrate that the key reason for the differing results is the choice of priors. Smets and Wouters (2007) impose an informative prior for the curvature parameter γ such that high values of labor-held-constant risk aversion are practically ruled out in their estimation. This is understandable. High values of CRRA are typically ruled out in the standard single good representative agent models, because they imply implausible behavior of consumers over risky choices. High risk aversion means strongly diminishing marginal utility of consumption and translates into very high risk aversion over large stakes. However, as shown by Chetty and Szeidl (2007), when households have consumption commitments, risk aversion can vary with the size of risk. Consumption commitments increase risk aversion over small shocks relative to large shocks as households adjust only a subset of their consumption to small shocks. Chetty and Szeidl (2007) suggest that consumption commitments provide a possible micro-foundation for habit persistence, motivated by technological

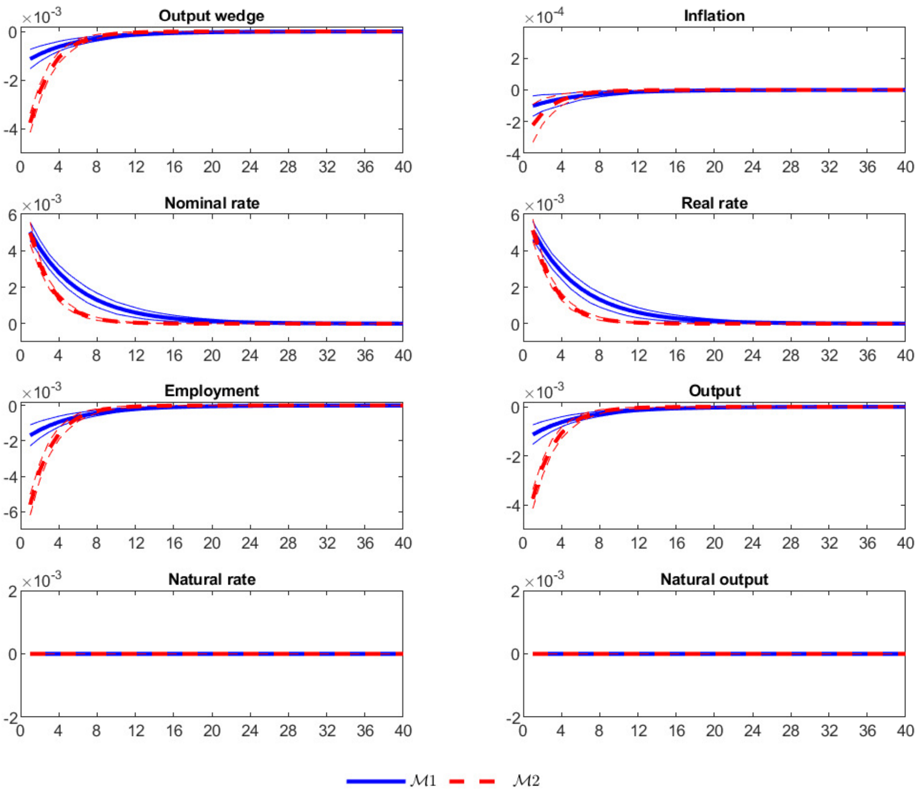

Figure 2: Equilibrium Responses to One Standard Deviation Monetary Policy Shock.

Note: This figure shows the impulse responses of selected variables to a one standard deviation monetary policy shock in the model under KPR preferences, $\mathcal{M}1$ and in the model with additively separable preferences, $\mathcal{M}2$. The bold lines give the equilibrium responses at posterior means, while the thin lines give the 90% HPD intervals for each model.

constraints rather than psychology. Commitments, like habit, magnify the impact of shocks on marginal utility and effectively makes the consumer more risk averse. In the long run, the individual may choose to adjust the committed component as well. The committed portion of consumption thus acts as a state variable that adjusts with a lag, generating lasting impacts on consumption.

In light of this, we extend the model by introducing external habit persistence into consumption, re-estimate the model and study the sensitivity of priors for γ and habit persistence.[15] Habit formation alters the parametrization of the dynamic IS and AS curves. It also further complicates the relationship between equilibrium IES and the curvature of utility.

With habit persistence in consumption, the parameter that governs the equilibrium intertemporal elasticity of substitution, i.e. sensitivity of consumption to the real interest rate, can be expressed as

$$\psi \equiv \frac{(1-b)(1-\alpha)s}{(1+b)(1-\alpha)-(1-s)\tau} \tag{23}$$

where b measures the intensity of external habit persistence. A given real interest rate sensitivity of output is consistent with different combinations of s and habit intensity parameter b. In particular, ψ is decreasing in b and increasing in s. Setting a high prior for the degree of habit persistence, like in Smets and Wouters (2007), makes it more likely that the estimation also produces relatively high values for s. As for the NK Phillips curve, more intense habits increase the dependence of inflation on the output gap difference (see Appendix C).

Smets and Wouters (2007) set the prior mean for habit persistence parameter to 0.70 with a standard error of 0.10, while their prior for γ is 1.50 with standard error of 0.37. Using (23) these priors imply a prior mean of ψ approximately equal to 0.138, given $\tau = 0.50$ and $\alpha = 0.33$ in our setup.

Table 2 shows the estimation results from the extended model with habit persistence.[16] To facilitate comparison with Smets and Wouters (2007) we restrict the estimation sample to 1984Q1–2007Q4, thus excluding the period where the zero lower bound is binding. Column I shows the benchmark results without habit persistence, while columns II–III show the results using the priors of Smets and Wouters (2007). In column II, habit persistence is fixed to zero, while in column III the prior mean for the habit persistence parameter is set equal to 0.7. Following Smets and Wouters (2007) we also set the prior for γ instead of s. Column IV shows the estimation results using our own prior on s (from our benchmark model) and using a lower prior for habit persistence where we rule out high values of habit persistence.

The results show that estimated value for s is sensitive to the choice of priors. This also translates into widely different values for ψ, ranging from 0.03 to 0.82 in different specifications.

The results in column I are comparable to those in Table 1, where we used the longer sample. Allowing for habit persistence leads in general to a lower value of ψ,

15 See Appendix C for description of the model with external habits.

16 Appendix C shows the key equations of the model with habit persistence.

Table 2: Sensitivity of Intertemporal Elasticity of Substitution to Priors

	I	II	III	IV
Prior, s	$\mathcal{B}(0.2, 0.10)$	–	–	$\mathcal{B}(0.2, 0.10)$
Prior, γ	–	$\mathcal{N}(1.5, 0.37)$	$\mathcal{N}(1.5, 0.37)$	–
Prior b	fixed to zero	fixed to zero	$\mathcal{N}[0.7, 0.1]$	$\mathcal{N}[0.45, 0.1]$
Parameter		**Posterior Distribution**		
s	0.11	0.54	0.44	0.11
	(0.05,0.16)	(0.41,0.73)	(0.36,0.55)	(0.05,0.17)
b	–	–	0.87	0.72
			(0.83,0.92)	(0.65,0.80)
ψ	0.31	0.82	0.04	0.03
	(0.20, 0.44)	(0.73, 0.91)	(0.03 0.05)	(0.02, 0.04)
LMD	1012.03	1004.09	1014.13	1039.70

Note: $\mathcal{B}$ and $\mathcal{N}$ correspond to Beta and Normal distributions. Fixed parameters are $\alpha = 0.33$, $\tau = 0.5$, $\epsilon = 6$ as in Table 1. LMD is log marginal density. Estimation sample is 1984Q1-2007Q4 and estimations were done using Dynare version 4.6.1. Posterior distribution was obtained by Metropolis–Hastings algorithm.

but this lower value can be obtained with different values of s. As expected, with the priors from Smets and Wouters, the estimated values of s tend to be higher and with habit persistence the posterior estimates of both s and b are close to their prior contributions (see column III). However, based on log marginal likelihood, the data supports the combination of low s and a moderate degree of habit persistence (see column IV). Even if the implied real interest rate elasticity of output is very similar in both cases, this difference is important. Namely, the combination of low s and moderate habit persistence considerably weakens the transmission of monetary policy in the sticky price model. We demonstrate this in Figure 3, which show the impulse responses to monetary policy shock in the two specifications. The initial reaction of inflation with SW priors is about four times larger than with our priors. Noticeably, inflation rate then overshoots significantly such that the overall impact of contractionary monetary policy shock on price level is positive in the specification with SW priors. The impact on output and employment is about two times larger throughout in the specification with SW priors.

4 Conclusions

One of the most common assumptions of sticky price monetary policy models is the additively separable utility in consumption and labor. In order to make this particular class of utility functions consistent with the balanced growth path, consumption enters the utility function in a logarithmic form, the implications of which are not well supported by empirical evidence. Relaxing the assumption of zero cross-elasticity between consumption and labor along the lines of the KPR-type preferences employed in this paper allows to estimate the IES with full information maximum-likelihood-based methods and yields empirically more plausible results. The Bayesian estimation results suggest that the real interest rate elasticity of output is in the range $0.1-0.2$ in the US during period in $1984-2018$. Restricting the estimation sample to pre-financial crisis years yields only slightly higher estimates for the equilibrium IES. In the model with habit persistence, the data supports the combination of moderate degree of habit persistence and high curvature of utility as opposed to relatively low curvature of utility and high habit persistence as in Smets and Wouters (2007). A combination of high curvature and moderate degree of habit persistence leads to even lower equilibrium elasticity of output to the real interest rate $(0.03-0.05)$, and considerably weakens the direct inter-temporal channel of monetary policy transmission to inflation and output in the sticky price model. At the same time, our estimates suggest a strong complementarity between consumption and labor. The

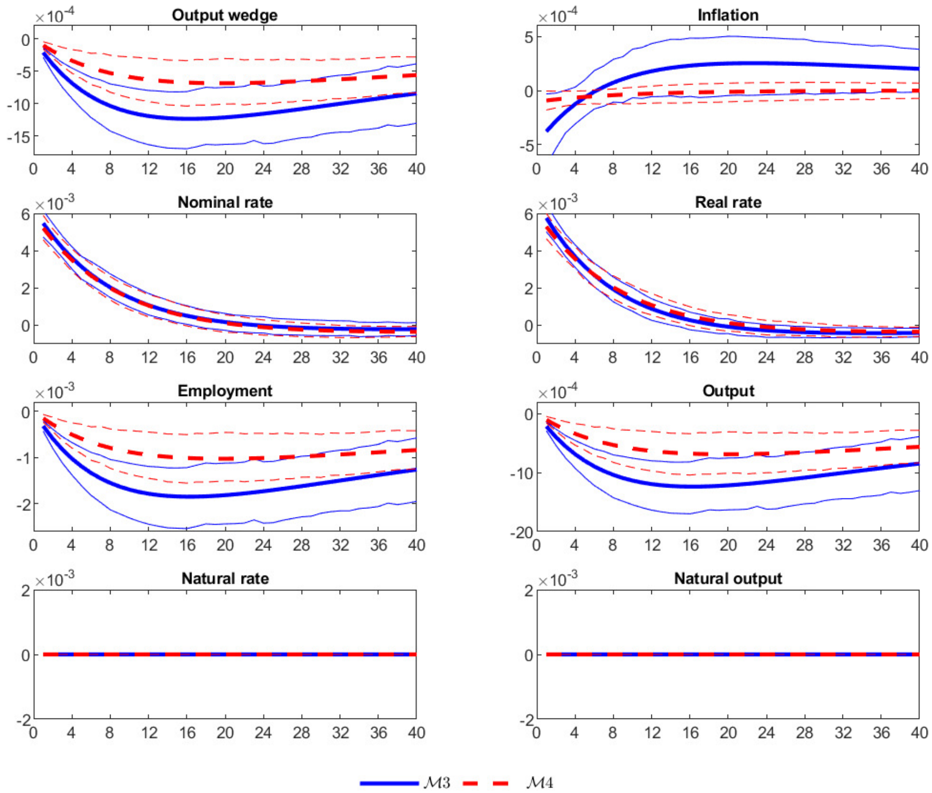

Figure 3: Equilibrium Responses to One Standard Deviation Monetary Policy Shock.

Note: This figure shows the impulse responses of selected variables to a monetary policy shock in the model under KPR preferences and with habit persistence estimated with priors in line with Smets and Wouters (2007) ($\mathcal{M}3$) and with our priors ($\mathcal{M}4$). The bold lines give the equilibrium responses at posterior means, while the thin lines give the 90% HPD intervals for each model. These differences are clearly non-trivial.

study of Aguiar et al. (2013), which is based on the American Time Use Surveys, gives support to macroeconomic models in which consumption and labor are strong complements, but this does not accord with all the micro evidence on labor supply. Hence, further work on testing alternative models of aggregate consumption and labor supply behavior is needed. This future work should address consumption-labor complementarity and the moderate responsiveness of consumption to real interest rates, and be consistent with long-run labor supply facts and the micro evidence on households' intertemporal and intratemporal behavior.

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Appendix

A Frisch elasticity and consumption constant elasticity of labor supply

Frisch elasticity of labor supply is defined as the elasticity of labor supply where the marginal utility of consumption is held fixed. Hence, we must have that

$$\begin{aligned} dU_C(C,N) &= U_{CC}dC + U_{CN}dN = 0 \\ &\Leftrightarrow \qquad (24) \\ \frac{dc}{dn} &\equiv \frac{d\log C}{d\log N} = -\frac{U_{CN}}{U_{CC}}\frac{N}{C} = (1-s)\tau. \qquad (25) \end{aligned}$$

Furthermore, along constant marginal utility of consumption paths

$$c_\omega \equiv \frac{dc}{d\omega} = (1-s)\tau\frac{dn}{d\omega} = (1-s)\tau n_\omega, \qquad (26)$$

where ω denotes the log of the real wage. From the intratemporal condition for labor, we know that

$$n_\omega = \varphi^{-1}(1-c_\omega), \qquad (27)$$

where $\varphi \equiv \frac{v''(N)N}{v'(N)}$. Hence, substituting (26) into (27) and solving for n_ω gives:

$$\begin{aligned} n_\omega &= \varphi^{-1}(1-(1-s)\tau n_\omega) \\ n_\omega &= \frac{1}{\varphi + (1-s)\tau} \qquad (28) \end{aligned}$$

and where $n_\omega \equiv \xi$ is the Frisch elasticity of labor supply.

B Standard model

This appendix replicates the key equations and the parameter definitions of the standard sticky price monetary policy model (adapted from Gali, 2008, ch. 3).

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \tilde{y}_t \quad (29)$$

$$\tilde{y}_t = \mathbb{E}_t \tilde{y}_{t+1} - s(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n) \quad (30)$$

$$i_t = \rho + \phi_\pi \pi_t + \phi_{\tilde{y}} \tilde{y}_t + v_t \quad (31)$$

where $r_t^n \equiv \rho + \frac{1}{s}\psi_{ya}^n \mathbb{E}_t \Delta a_{t+1}$ and

$$\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}\Theta,\ \Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$$

$$\kappa \equiv \lambda\left(\frac{1}{s} + \frac{\xi+\alpha}{1-\alpha}\right),\ \tilde{y}_t \equiv (y_t - y_t^n),$$

$$y_t^n = a_t + \vartheta_y^n,\ \vartheta_y^n \equiv \frac{(1-\alpha)(\ln(1-\alpha)-\mu)}{\frac{1}{s}(1-\alpha)+\xi+\alpha}$$

$$\psi_{ya}^n \equiv \frac{1+\xi}{\frac{1}{s}(1-\alpha)+\xi+\alpha}$$

C Intertemporal substitution and slope of the Phillips curve

In this appendix, we highlight the impact of different values of s on the elasticity of output with respect to the real interest rate ψ, and the slope of the Phillips curve κ in the model with additively separable preferences (standard model), as well as in the model which allows non-zero cross elasticity between consumption and labor (KPR preferences). The first model is referred to as the standard NK model, where $\psi = s$ and $\kappa \equiv \lambda[s^{-1} + \frac{(\xi+\alpha)}{1-\alpha}]$ and where ξ is Frisch elasticity of labor supply (see Appendix B). The AS and IS curves in the model with KPR preferences are given below:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \tilde{y}_t \quad (32)$$

$$\tilde{y}_t = \mathbb{E}_t \tilde{y}_{t+1} - \psi(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n) \quad (33)$$

where $r_t^n \equiv \rho + \frac{1}{s}\mathbb{E}_t \Delta a_{t+1}$ and $\psi \equiv \frac{(1-\alpha)s}{1-\alpha-(1-s)\tau}$, $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}\Theta$, $\Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$, $\kappa = \lambda\frac{(1+\varphi)}{1-\alpha}$, $\tilde{y}_t \equiv (y_t - y_t^n)$, $y_t^n = a_t + \vartheta_y^n$, $\vartheta_y^n \equiv \frac{(1-\alpha)(\ln(1-\alpha)-\iota-\mu)}{(1+\varphi)}$, $\iota \equiv \tau - (1+\varphi)n$.

Table 3: Parameterization

ξ	α	θ	β	ϵ	τ
1	0.33	0.67	0.99	6	$0.5^{(a)}$

Note: Except for τ, these calibrated parameters are taken from Gali (2008, Ch. 3, p. 52). (a) This value is chosen to reflect roughly the narrow measure of the (after tax) labor share in the US.[18]

Table 4: Comparisons

	Additively Separable Preferences				KPR Preferences			
s	1	1/2	1/5	1/10	1	1/2	1/5	1/10
ψ	1	0.50	0.20	0.1	1	0.80	0.50	0.30
κ	0.13	0.19	0.38	0.69	0.13	0.11	0.10	0.09
φ^{-1}	1	1	1	1	1	1.33	1.67	1.82

Note: This table compares dependencies of the key parameters in the standard model with additively separable preferences and in the model with KPR preferences when s varies from unity to 1/10. Otherwise, the parameter values are chosen according to Table 3.

Baseline calibration is shown in Table 3 and the results from comparisons are reported in Table 4. We want to highlight the following two results. First, the equilibrium intertemporal elasticity of substitution ψ is generally higher than the labor held constant intertemporal elasticity s under KPR preferences, since the former captures the adjustment through labor. As the labor-held-constant intertemporal elasticity of substitution falls from unity to 1/10, the equilibrium interest rate elasticity of output ψ only falls from unity to roughly 0.30.

The second result is that the standard NK model with additively separable preferences yields (in an empirical sense) an implausibly steep Phillips curve at low values of s (and, of course, is also inconsistent with the balanced growth path requirement). On the contrary, the slope of the Phillips curve is practically invariant to different values of s[17] in the model with KPR preferences.

Because the slope of the Phillips curve is not particularly sensitive to different

[17] The variability of κ is due to the fact that we have fixed Frisch elasticity of labor supply equal to unity, and instead let φ vary in accordance with equation (7).

values of s, this allows us to identify it primarily from the relationship between ex-ante real interest rate and output or employment, as is done in the consumption Euler equation estimations by GMM methods. Conversely, when using the standard preference specification, low values of s should be associated with an unrealistically high slope of the Phillips curve from an empirical standpoint.

However, the advantage is that we have a structural model for inflation and interest rates, permitting us to use full information maximum likelihood based methods, instead of GMM methods. GMM methods applied to estimate s from aggregate data typically suffer from weak instrument problems and lack of identification. Furthermore, if one would be willing to assign prior directly on the consumption-constant elasticity of labor supply, the slope of the Phillips curve and IES would be structurally independent in the case of KPR preferences (see equation (14)).

D Model with habit persistence

This appendix shows the key log linearized equations and the parameter definitions of the sticky price monetary policy model with external habit persistence and King-Plosser-Rebelo preferences. Detailed derivation of the model is available by request from the authors.

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \kappa\tilde{y}_t + \lambda\frac{b}{1-b}\Delta\widetilde{y}_t \tag{34}$$

$$\tilde{y}_t = \omega_1\tilde{y}_{t-1} + \omega_2\mathbb{E}_t\tilde{y}_{t+1} - \psi(i_t - \mathbb{E}_t\pi_{t+1} - r_t^n) \tag{35}$$

$$r_t^n = \rho + \frac{1}{s}\left(\begin{array}{c} \frac{(1-s)\tau}{(1-b)(1-\alpha)}E_t\left(\Delta a_{t+1} - \Delta y_{t+1}^n\right) \\ +\frac{1}{(1-b)}E_t\Delta y_{t+1}^n - \frac{b}{(1-b)}\Delta y_t^n \end{array}\right) \tag{36}$$

$$y_t^n = \frac{(1-\alpha)\,b}{(1+\varphi)\,(1-b)+(1-\alpha)\,b}y_{t-1}^n + \frac{(1+\varphi)\,(1-b)}{(1+\varphi)\,(1-b)+(1-\alpha)\,b}a_t \tag{37}$$

$$i_t = \rho + \phi_\pi\pi_t + \phi_{\tilde{y}}\tilde{y}_t + v_t \tag{38}$$

where

$$\lambda \equiv \frac{(1-\theta)\,(1-\beta\theta)}{\theta}\Theta,\ \Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$$

$$\kappa \equiv \lambda\frac{(1+\varphi)}{1-\alpha},\ \omega_1 = \frac{b(1-\alpha)}{(1+b)(1-\alpha)-(1-s)\tau}$$

$$\omega_2 = \frac{(1-\alpha)-(1-s)\tau}{(1+b)(1-\alpha)-(1-s)\tau},\ \psi = \frac{(1-b)(1-\alpha)}{(1+b)(1-\alpha)-(1-s)\tau}$$

$$\tilde{y}_t \equiv (y_t - y_t^n)$$

Furthermore, it can easily be shown that the relationship between Frisch elasticity of labor supply, ξ_h, and consumption constant elasticity of labor supply in the presence of external habits is given by $\xi_h = 1/(\varphi + \frac{1-s}{1-b}\tau)$.

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