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wavelet analysis



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# The Evolution of US and UK GDP components in the Time-Frequency Domain: A Continuous Wavelet Analysis

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## Abstract

Understanding the relationship between national income GDP components is an essential part of macroeconomics. This study investigates quarterly real GDP component data for the U.S. and the U.K. and applies continuous wavelet analysis on cross comparisons of the data, from both within and between the two datasets. The results show that the cyclical interactions between consumption and investment are the most complex and most substantial at several different frequencies. The relationship of exports with other macroeconomic variables has also developed over time, likely due to the evolution of an international business cycle.

**Keywords:** Business cycles, growth cycles, time-frequency domain, macroeconomic synchronization, economic policy.

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# 1 Introduction

The relationship between national income component variables lies at the heart of the study of the macroeconomy, and has given rise to theories of consumption, the accelerator theory and numerous other hypothesized relationships. Although the econometrics studies using macro variables are extensive, there is little in the way of time-frequency analysis in this area. The early studies by Granger (such as Granger and Hatanka (1964) and Granger (1966)) and the follow up studies by Levy and Dezhbakhsh (2003b) and Levy and Dezhbakhsh (2003a) are the most notable. These studies now appear dated<sup>1</sup> given the more recently developed time-frequency domain techniques such as wavelet analysis and empirical mode decomposition. Also, with the lengthening of data sets that inevitably comes with the course of time, there is now scope to use these new and improved techniques to evaluate the macroeconomic cyclical interactions across a wider frequency range and in greater depth.

This paper is the culmination of a series of papers we have undertaken to decompose the growth in US and UK national income components. Most of this analysis has taken place using discrete wavelet analysis, which is the counterpart to the approach taken here. Our research sequence first decomposed the cyclical features of GDP growth and its components (Crowley and Hughes Hallett (2015)); we then used these decompositions to analyze the cross correlation features of U.S. and U.K. GDP growth components (Crowley and Hughes Hallett (2016)). This first part of this paper is therefore the continuous wavelet analysis companion paper to Crowley and Hughes Hallett (2016), as it studies the coherence and phasing of cyclical features of U.S. and U.K. GDP components from the late 1960s to the present. The second part extends that analysis to look at the coherence and phasing of GDP and its components between the U.S. and the U.K.

Apart from being the first paper to use continuous wavelet analysis to study the time-frequency interaction between GDP growth components, the paper provides several key results. First it is noted that (as with discrete wavelet analysis) various different cycles are apparent at different frequencies between GDP components, and some of these cycles appear to wax and wane over time and mostly do not accord with traditional notions of business cycles. The second key result is that the largest amount of cyclical co-movement appears to occur between C and I for both the U.S. and the U.K. The third key result is that an international business cycle appears to be at work at a frequency somewhere between 8 and 14 years, and that this began to emerge in the early 1980s, and has continued throughout the remainder of the time period under study.

Section 2 of this paper describes continuous wavelet analysis, while section 3 presents the cross-spectral analysis for the U.S. and the U.K. separately, and then section 4 does an international comparison of the

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<sup>1</sup>In some cases these studies are also executed incorrectly, due to the application of spectral analysis to non-stationary variables. See Crowley (2010) for a comparison between different time-frequency approaches.

cyclical features of U.S. and U.K. GDP and its components. Section 5 is devoted to a commentary on our results, while section 6 then concludes.

## 2 Continuous wavelet (CWT) analysis

One of the problems with discrete wavelet analysis is that there is no technique currently available that allows time-varying analysis of correlations and phase shifts, although Crowley and Lee (2005) and Chaplyguin V. and Richter (2006) show how approximate results can be obtained by applying dynamic correlation analysis to analyze how the crystal wavelet correlations change over time with European growth data. To fill that gap, we show in this paper how how continuous wavelet techniques (CWT) can be used to produce spectral-type measures of association so as to show how these relationships change over time, and not just within a frequency band, but over the entire spectrum.

Continuous wavelet transforms (CWTs), rather than looking at a range of frequencies to increase the time resolution, have the ability to focus in on greater frequency resolution ( - more continuous cyclical decompositions). This is equivalent to temporal narrow-band filtering. Perhaps the best introduction into the theoretical CWT literature can be found in Lau and Weng (1995), Holschneider (1995) and Chiann and Morettin (1998), while Torrence and Compo (1998) provides the most illuminating examples of empirical applications to time series from meteorology and the atmospheric sciences.

In brief, a representation of a covariance stationary process in terms of its frequency components can be made using Cramer's representation as follows:

$$x_t = \mu + \int_{-\pi}^{\pi} e^{i\omega t} z(\omega) d\omega \quad (1)$$

where  $i = \sqrt{-1}$ ,  $\mu$  is the mean of the process,  $\omega$  is measured in radians and  $z(\omega)d\omega$  represents a complex orthogonal increment processes with variance  $f_x(\omega)$ , where it can be shown that:

$$f_x(\omega) = \frac{1}{2\pi} \left( \gamma(0) + 2 \sum_{\tau=1}^{\infty} \gamma(\tau) \cos(\omega\tau) \right) \quad (2)$$

where  $\gamma(\tau)$  is the autocorrelation function. Here  $f_x(\omega)$  is known as the spectrum of the series as it defines a series of orthogonal periodic functions which represent a decomposition of an empirical variance into an infinite sum of waves of different frequencies. Given a large value of  $f_x(\omega_i)$ , say at particular values of  $\omega_i$ ,  $\hat{\omega}_i$ , this implies that frequency  $\hat{\omega}_i$  is a particularly important, and hopefully statistically important component of the series.

Given a time series  $x(t)$  and an analysing wavelet function  $\psi(\theta)$ , then the continuous wavelet transformation (CWT) is given by:

$$W(t, s) = \int_{-\infty}^{\infty} \frac{d\tau}{s^{\frac{1}{2}}} \psi^* \left( \frac{\tau - t}{s} \right) x(\tau) \quad (3)$$

For an easier computation making use of FFT algorithms this can be rewritten in Fourier space. For a discrete numerical evaluation we get:

$$W_k(s) = \sum_{k=0}^N s^{\frac{1}{2}} \widehat{x}_t \widehat{\psi}^*(s\omega_k) e^{i\omega_k t \partial t} \quad (4)$$

where  $\widehat{x}_k$  is the discrete Fourier transform of  $x_t$ :

$$\widehat{x}_k = \frac{1}{(N+1)} \sum_{k=0}^N x_t \exp\left\{\frac{-2\pi i k t}{N+1}\right\} \quad (5)$$

Here we use a Morlet wavelet, which is defined as:

$$\psi(\theta) = e^{i\omega\pi} e^{-\frac{\pi^2}{2}} \quad (6)$$

This is a symmetric wavelet, and is widely used in CWT analysis in the wavelet literature. Given our analysis above, it is also possible to calculate conventional spectral measures, such as the spectral power:

$$WPS(t, s) = E\{W(t, s)W(t, s)^*\} \quad (7)$$

where  $*$  represents the complex conjugate. The wavelet power spectra measures the strength of cycles at various frequencies - it is the analogue measure of energy for a discrete wavelet transform or a variance decomposition in the context of time series analysis<sup>2</sup>.

With two variables,  $x$  and  $y$ , it is also possible to derive and empirically estimate the cross wavelet power spectrum:

$$WCS^{xy}(t, s) = E\{W^x(t, s)W^y(t, s)^*\} \quad (8)$$

This gives rise to other multivariate spectral measures such as the coherence (which essentially normalizes the cross wavelet spectrum):

$$WCO^{xy}(t, s) = \frac{|WCS^{xy}(t, s)|}{[WPS^x(t, s)WPS^y(t, s)^*]^{\frac{1}{2}}} \quad (9)$$

which can also be measured as the magnitude of the squared coherence, being simply  $[WCO^{xy}(t, s)]^2$ . As wavelet analysis in effect identifies cycles in the data, if such cycles are detected, then the phasing,  $\Phi(s)$  between those cycles can be calculated from:

$$WCS^{xy}(t, s) = |WCS^{xy}(t, s)| e^{i\Phi(s)} \quad (10)$$

When plotting the coherence and phase, the arch drawn in the plots shows the "cone of influence"<sup>3</sup>, so points outside the cone are to be interpreted as being less reliable statistically than those placed within the cone (because of the problem of fitting wavelets to fluctuations at each end of the series).

<sup>2</sup>These are not shown in this paper, but the plots can be obtained from the corresponding author upon request.

<sup>3</sup>This indicates the central area of the graph where the full length wavelets are applied to the data, so are free of any biases resulting from the use of boundary coefficients to enable wavelet application.

Continuous wavelet analysis has been used quite extensively in macroeconomics, with Crowley, Maraun, and Mayes (2006) and Rua (2012b) contributing the first attempts to use this approach in the macroeconomics literature. Useful methodological background articles on the application of continuous wavelet analysis can be found in Aguiar-Conraria and Soares (2010) and Aguiar-Conraria and Soares (2014), and then subsequent notable recent macroeconomics contributions using continuous wavelet analysis have been made by Gallegati, Gallegati, Ramsey, and Semmler (2011), Rua and da Silva Lopes (2012), Rua (2012a), Rua (2013), Aguiar-Conraria, Martins, and Soares (2018) with also several contributions also contained in Gallegati and Semmler (2014). Some financial economics contributions using this approach can also be found in Mandler and Schnargl (2014), Kilponen and Verona (2016), Verona (2016), Mandler and Schnargl (2019) and Verona (2019).

### 3 Cross spectral analysis

#### 3.1 U.S.

Here we plot magnitude squared coherence and phase (equations (9) and (10)) for aggregate demand components in the US. These measures will likely vary from the static correlations obtained from discrete wavelet transform for several reasons. In particular: i) the wavelet function used in this exercise is symmetric rather than the asymmetric wavelet function used in the discrete wavelet transform exercise; ii) the measure of association used here is coherence, which is different from (but equivalent to) a correlation measure; and iii) the continuous wavelet transform is not limited to a specific range of frequencies, but rather ranges over all frequencies, and hence may only register significant coherence at one point rather than over a specified interval of frequencies, in which case it will not be highlighted as significant for that interval.

In Figure 1 coherence is plotted using a jet spectrum, where blue indicates low levels of coherence; green, moderate levels; and yellow and red are high and very high levels respectively. The areas of 90 percent significant (or higher) are also plotted on both the coherence and phase plots, as is the cone of influence. The level of significance here denotes a point significance test performed against a background of independent (unrelated) spectra. In other words, the null hypothesis is specified as a series of seemingly unrelated economic processes. In addition, the vertical scale in this figure is measured in years, so that a time-varying plot can be drawn for frequencies of roughly 1 year cycles through to 24 year cycles. In the phase plot in particular, a key is provided which shows the approximate phase of the strongest (linear) linkage between the series. Here there is significant coherence between consumption and private investment between 3 and 8 years from around 1950 through 1993 and then from 1999 through to the present, with a phasing that is roughly contemporaneous. A longer 16 year cycle is also significantly coherent between C and I from around 1950 to 1968, which then remains strongly coherent thereafter, although the phasing is such that C actually lags I.

If we now move to the cross-spectral relationship between C and G in figure 2, the only formally significant areas of coherence are in the early 1950s. But there are high levels of coherence at a four to six year cycle length from around 1980 to 1998 and then from around 2000 to the present; and a small area of significance in the latest data at a 7 year periodicity which has exactly a half cycle difference. This suggests that the usual Keynesian pump priming of the economy has had rather little impact in practice, except in rebuilding the economy post-war, during the Korean War and during the oil price hike recessions of the 1980s/1990s.

That said, the phasing of the non-significant areas is such that C lags G by around a quarter to a third of a cycle. High coherence is also observed at longer cycles with 10 to 14 years periodicity beginning in 1985 and continuing through to the present, although the components of this cycle are much more in phase, with G only leading C slightly. The shorter cycles here are likely to correspond to counter-cyclical fiscal policy, and the coherent longer cycles to increases in G in reaction to the increases in revenue which had got ahead of normal revenue streams and led to temporary fiscal deficits in the 1987-92 period.

In figure 3 the coherence and phase plots for C and X are shown, with some significant areas apparent at business cycle frequencies (of 4 to 6 years) from 1947 through to 1967. Cycles are coherent again from 1974 to 1979 at roughly a 3 to 4 year frequency, and then again from 1980 to 1992 at 5 to 7 year cycles. Beyond 1992 there are still high levels of coherence at the 5 to 7 year cycle frequency up until 1995 (with one "spur" going to 2000) then it abruptly stops. The phasing is always such that C leads X by about a quarter cycle. This fully corroborates the static results obtained with cross-correlations in Crowley and Hughes Hallett (2016), where the contemporaneous correlations were all close to zero. A strong level of coherence is also evident at a 16 year cycle.

Next, figure 4 shows that in general there is very little significant coherence between I and G, except in the 1950s and 1960s where it appeared in 3 different frequency groups (2-4 years, 7 years, and over 16 year cycles). For the shorter cycles I led G by roughly a quarter cycle; while at longer cycles they were nearly a-cyclical, which suggests that a crowding out effect was in play at the longer cycles. More recently there appears to be high coherence with small patches of significance from 1975 to around 1998 with a shorter (but short-lived) cycle of just over 2 years where I lags G by over a quarter cycle, and another between 4 and 6 years where I leads G by nearly a full half cycle.

Meanwhile, the plots for I vs X in Figure 5 are interesting. Here a long cycle at around 16 years appears again, which is significant until 1992 but then continues further with weaker but insignificant coherence, and has phasing of nearly a half cycle (which is picked up by the  $d6$  cycle in the previous section). Mirroring the results of the discrete wavelet analysis, there are also shorter cycles at a roughly 7 year periodicity that are significant, but only for two short periods (- in the early 1960s and from 1980 to 1990) and where I shows a short lead over X. More recently even shorter cycles of 2 to 4 years have become coherent and



significant and roughly in phase.

Lastly, the coherence and phase plots are presented for G and X in Figure 6. As might be expected from the discrete wavelet analysis, nearly all the longer cycles between G and X are roughly a half cycle out of phase and the only significant coherences are found in the early part of the period under study. Partly in contrast, the shorter cycles are significant with periodicities between 2 and 4 years, and only for short periods, notably in the mid-1980s and the mid-2000s. These shorter cycles have phases that are also nearly a half-cycle different. Only one period in the early 1960s appears to have a significant coherence between 5 year cycles where G lags X, but only slightly.

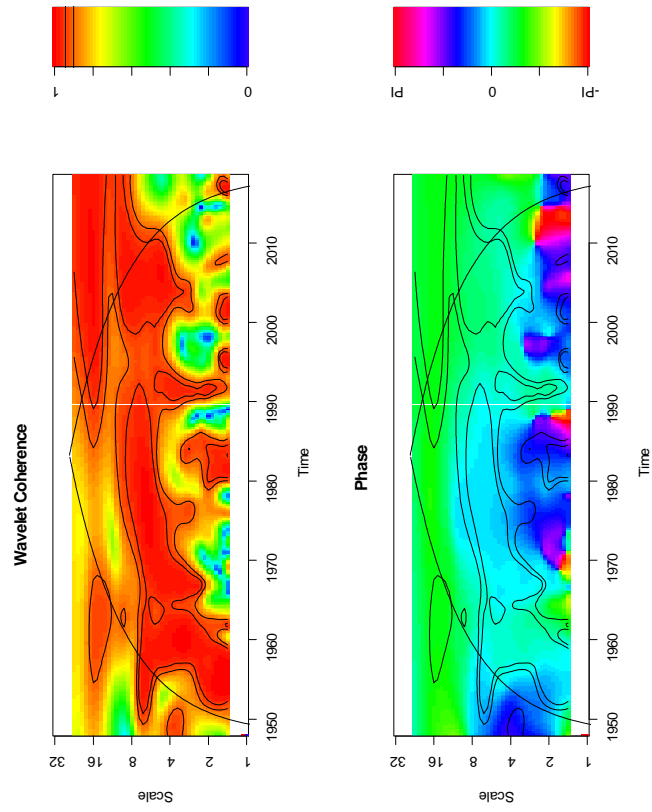


Figure 1: Coherence and phase for US C vs I

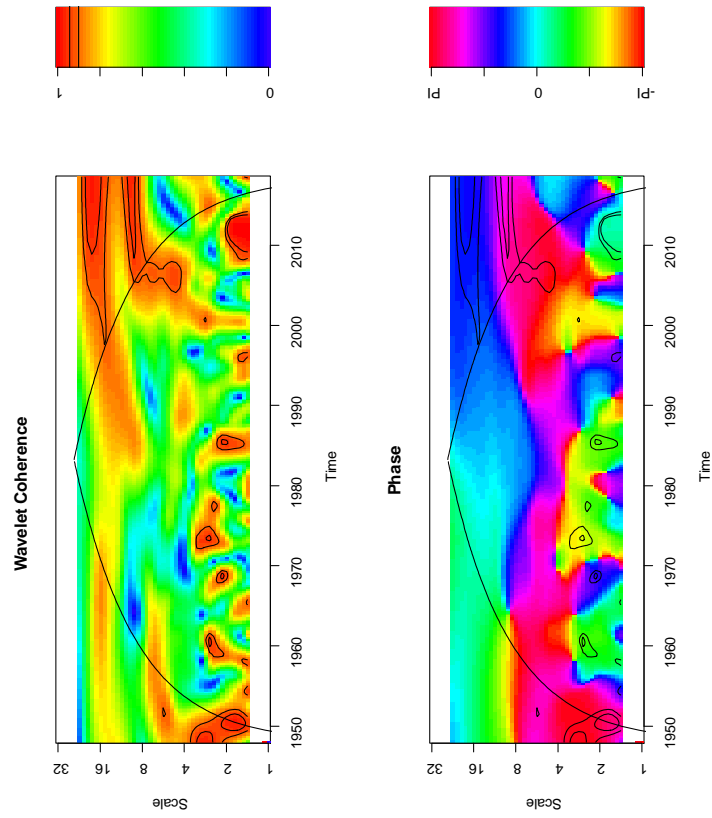


Figure 2: Coherence and Phase for US C vs G

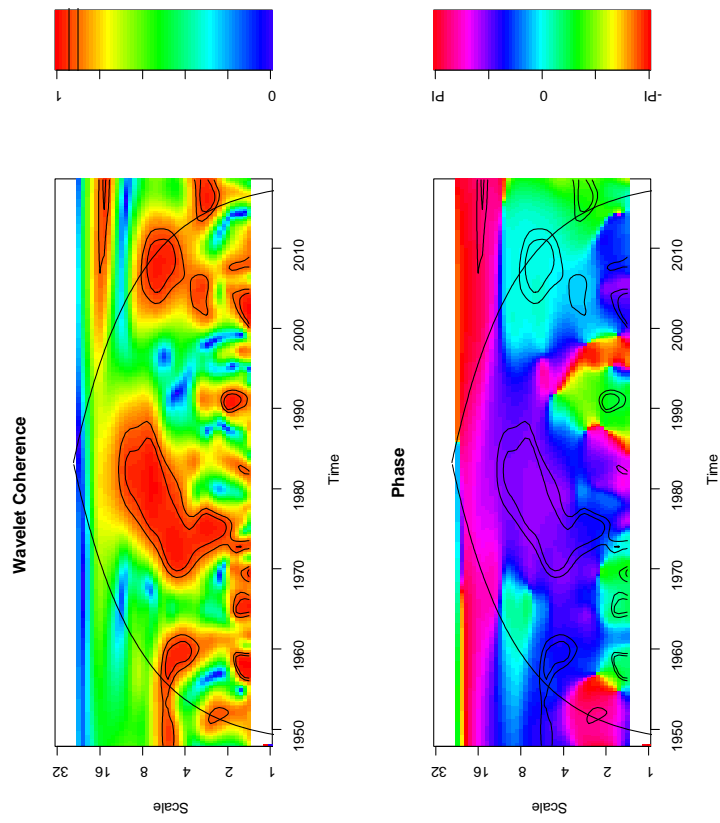


Figure 3: Coherence and Phase for US C vs X

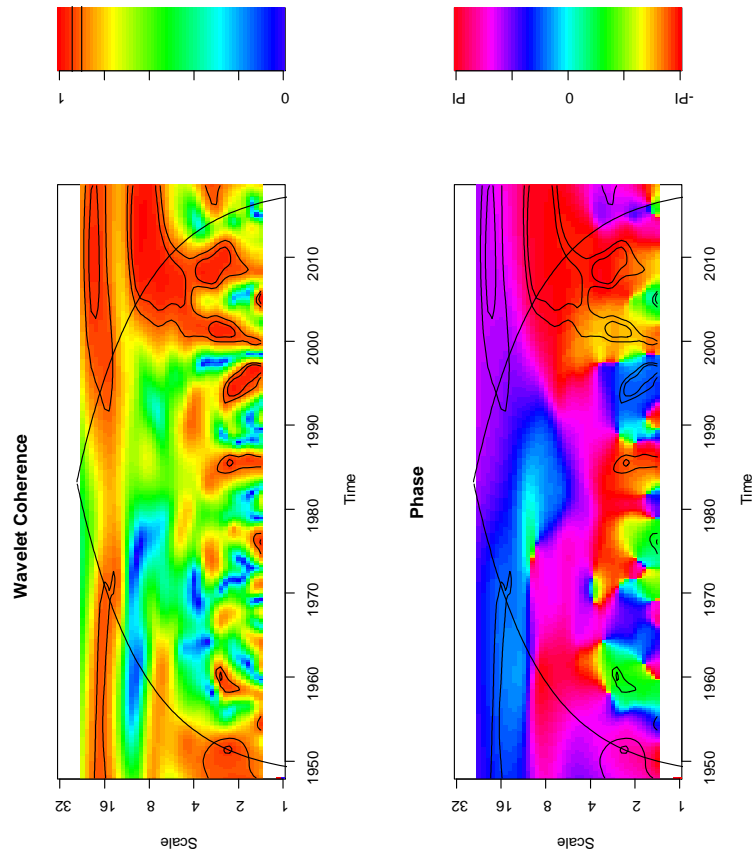


Figure 4: Coherence and Phase for US I vs G

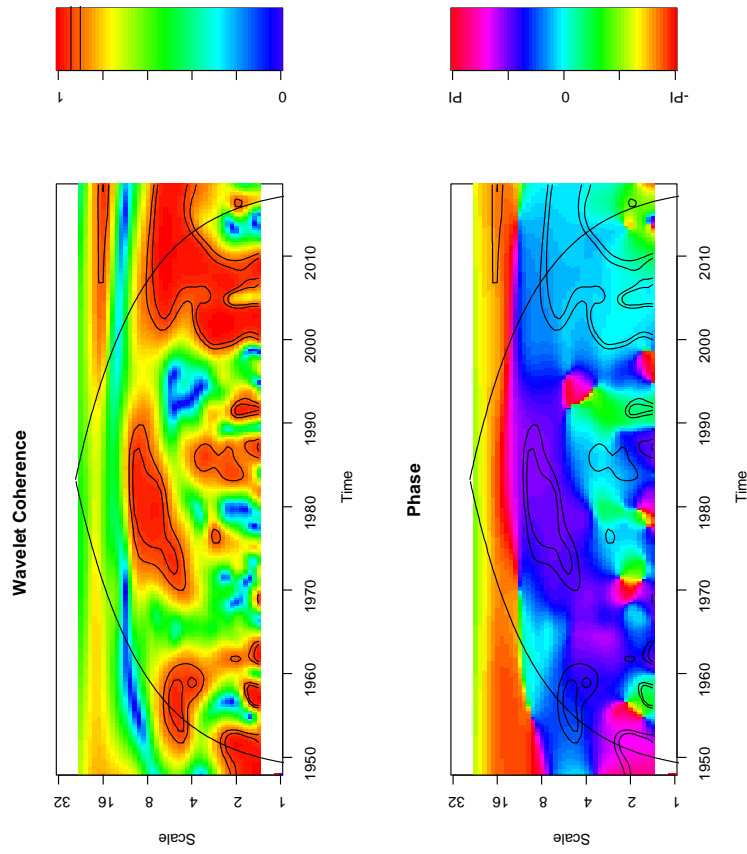


Figure 5: Coherence and Phase for US I vs X

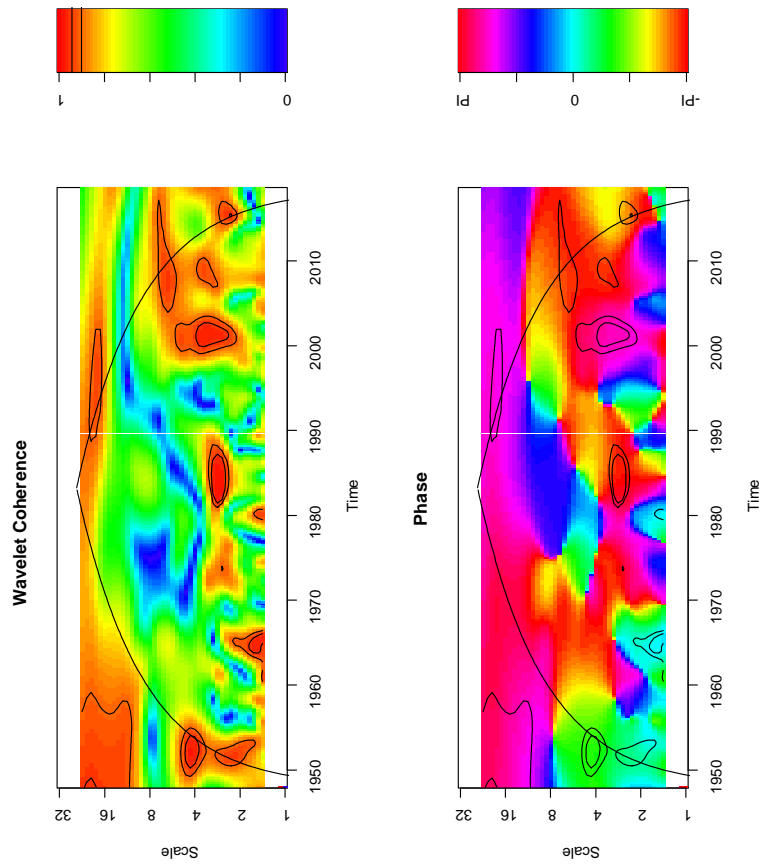


Figure 6: Coherence and Phase for US G vs X

### 3.2 The UK

For the UK, we repeat the same exercise, and obtain somewhat different results when compared to the US. In Figure 7, UK consumption and private investment expenditure cross spectral plots are shown. The coherence is significant throughout the time period, but three specific episodes of coherence stand out at different frequencies. First there are 2 coherent cycles in the 1955 to 1970 period, one at around a 3 year cycle and the other at approximately a 12 year cycle. Second, from 1970 to 1982 there is a coherent cycle at around a 6 year frequency, and third there is broad cyclical coherence over roughly 6 to 16 year cycles from around 1982 through to the end of the period. Oddly, there is a small area of significance apparent from 1984 to 1992, but at just over a 2 year cyclical frequency. As with the U.S., these two variables are roughly in phase, although at shorter frequencies C tends to lag I slightly.

Figure 8 shows little in the way of significant coherence for consumption vs government spending, although there is high and significant coherence from 1982 through until 2005 at a 16 year cycle with C leading G by about 4 years. The only other significant patches of high coherence are in the early 1990s and around the time of the great recession. These cycles all contain a half cycle lag in G signifying countercyclical fiscal policy is at work and probably effectively so. What is also noticeable in the phase plot is that, at low frequencies C leads G, but at high frequencies C lags G. The former observation suggests an anticipation or consumer confidence effect, while the latter looks like standard Keynesian stabilization spending.

Figure 9 shows the coherence and phase plots for consumption expenditures against exports. There are not many significantly coherent areas, but C lags X by a quarter cycle at a 6 to 8 year cyclical frequency from 1980 until 2015. This would imply the emergence of a more globalized world economy where the stabilizing stimulus is longer term and comes largely from outside the UK.

UK private investment vs government expenditure coherence and phase are plotted in Figure 10. There are small areas of insignificant coherence at around the 8-12 year frequency from 1960 to 1975 and then again from 1982 to 1995, but this time the coherence is significant. In both cases I leads G.

In Figure 11, which shows coherence and phase plots for private investment and exports, there are few significant areas of coherence. There is a cycle that emerges around 1970 at a 10 year cycle until about 1995, but then this cycle shortens to a significant 4 to 8 year cycle from 2008 to 2015. In all cases I tends to lag X suggesting, once again, that the emergence of globalization had become a driving force in economic growth (at least for the UK).

Lastly, in Figure 12, coherence and phase plots for government spending vs exports are shown, but this time there is very little significant coherence to see; possibly some in the mid to late 1970s of an anti-cyclical nature, but virtually none after that. For the 1950s G led X by a small amount, suggesting that fiscal stimulus was important. In the 1970s that was reversed: G led X by nearly a half cycle, suggesting



that counter-cyclical policy was at play.

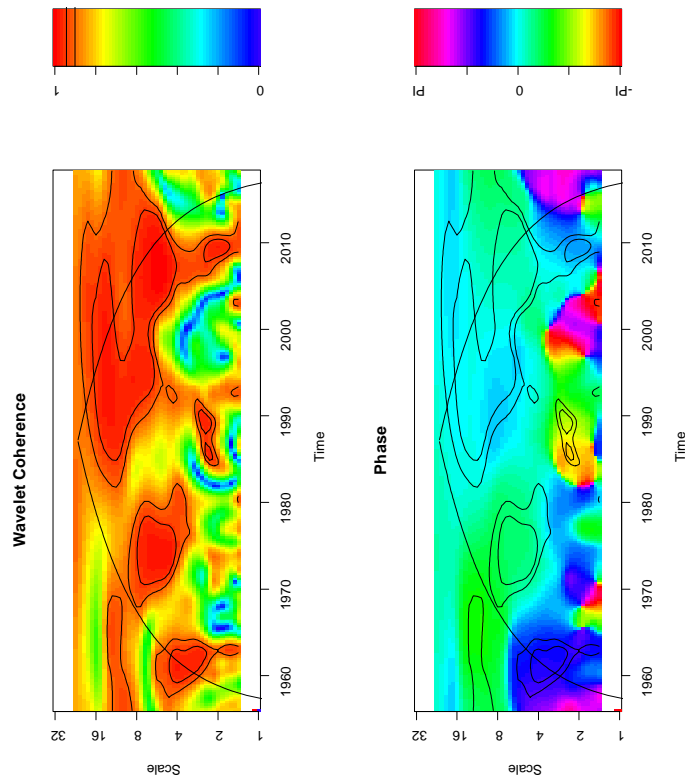


Figure 7: Coherence and Phase for UK C vs I

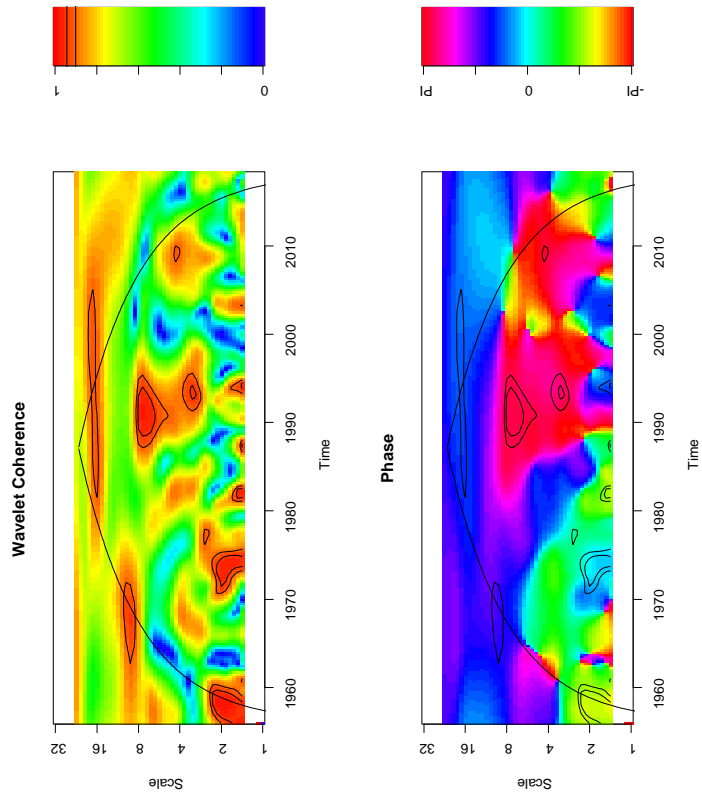


Figure 8: Coherence and Phase for UK C vs G

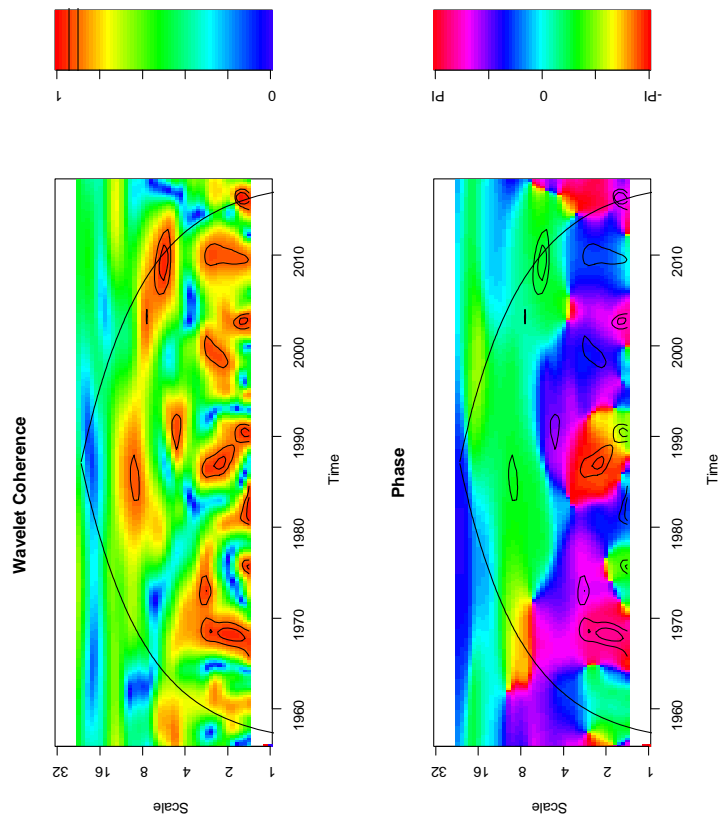


Figure 9: Coherence and Phase for UK C vs X

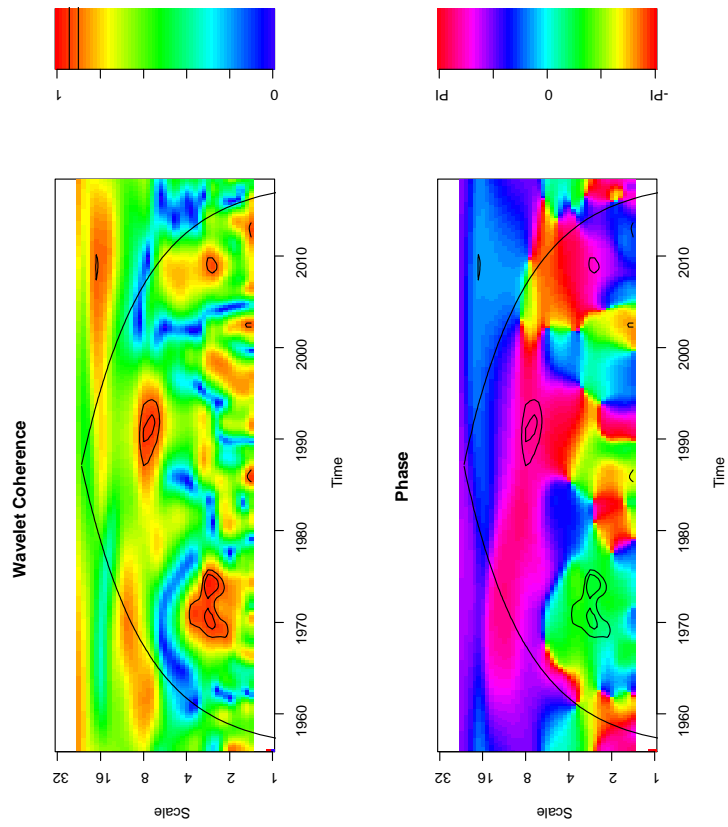


Figure 10: Coherence and Phase for UK I vs G

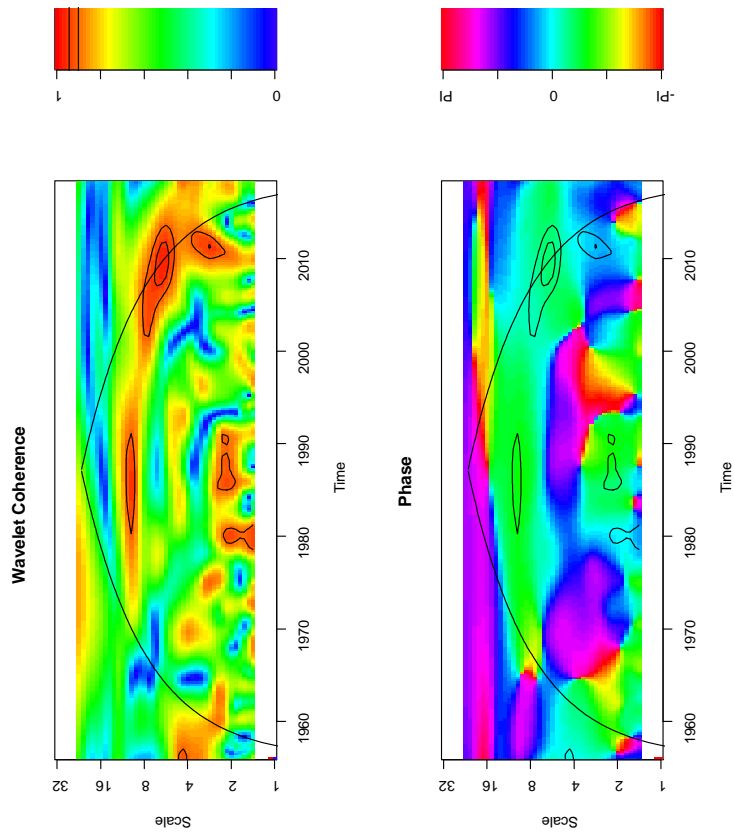


Figure 11: Coherence and Phase for UK I vs X

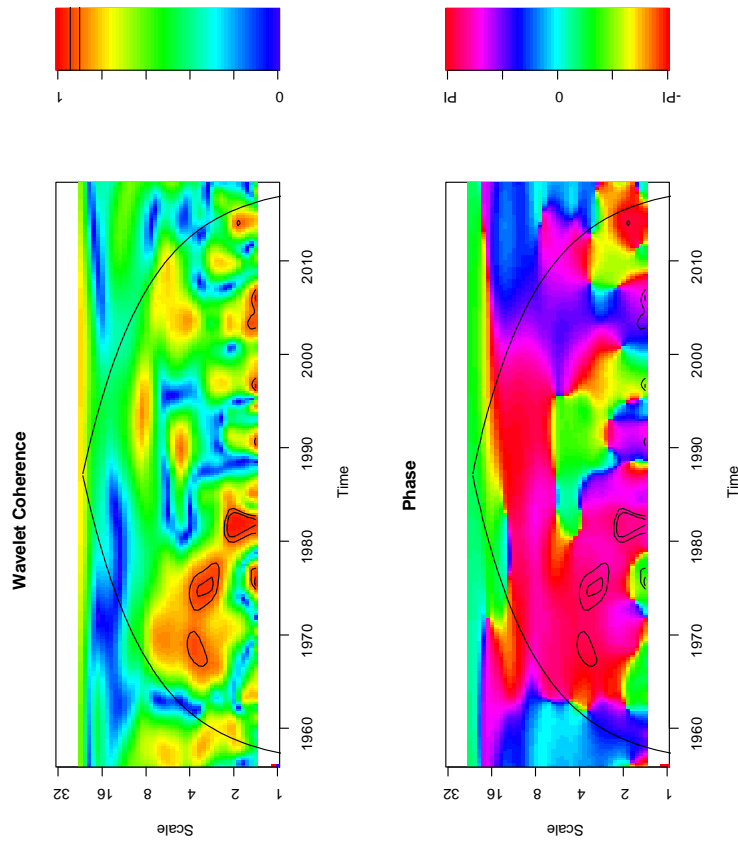


Figure 12: Coherence and Phase for UK G vs X

## 4 International Comparisons of GDP and components

In this section we look at the relationship between the GDP in the U.S. and the U.K. and the relationship of each component of GDP to its transatlantic equivalent. This exercise therefore identifies common cycles between U.S. and U.K. macroeconomic variables. This is useful for 2 reasons: first, it allows identification of international business cycles and if they can be said to propagate between the two countries. That would allow us to test, for example, whether the U.S. downturn during the "great recession" was propagated to the U.K. through the financial sector. And second, it allows for an assessment of real business cycle theory, as this theory predicts that productivity shocks should be synchronized across developed countries, as outlined by Baxter and Stockman (1989), Kydland and Prescott (1990), Backus, Kehoe, and Kydland (1992) and others.

In figure 13 the coherence and phase shifts between U.S. GDP and U.K. GDP growth are shown. The coherence chart suggests that there has been a long cycle (of over 24 years) at work between the two countries, but the only similar business cycle movements at additional frequencies appears to have been during the early 1970s recession and the "great recession". Between 1982 and 2004 there also appears to have been a strong common cycle with frequency of between 10 and 18 years which continues in a weaker form through until the end of that period. All significant coherent cycles show that they are in phase, or the U.S. has a slight lead over the U.K.

Figure 14 shows the coherence and phase shifts between U.S. consumption and U.K. consumption growth. Now, there only appears to be coherence from around 1980 through until 2010 at cycles of between 10 and 18 years, and these cycles are all in phase.

For investment, figure 15 shows that in the "great recession" there was definitely concordance in investment cycles between the U.S. and the U.K., with the downturn in investment over cycles up to about 15 years, but elsewhere the pattern of common cycles was more complex. From the mid-1960s to mid-70s there were 2 sets of significant cycles, one at about a 3 year periodicity and another from around 6-8 years in periodicity, and then from 1975 to 1995 one at a 10 to 16 year periodicity. Interestingly, all these significantly coherent cycles were in phase, except for the long 1975 to 1995 cycle, where the U.S. led the U.K., which tends to suggest that this particular cycle was technology induced.

As fiscal policy was directed at specific circumstances in each country, looking at common cycles between the U.S. and the U.K. should not, *a priori*, yield any strong results, and this is borne out by the coherence and phase plot for government expenditures shows in figure 16. This figure shows that there were no common cycles in government spending beyond 1986. Before this time the cycles were likely induced by efforts to offset oil price shocks common to both countries, with the frequency centred around a 3 year cycle with the U.S. leading the U.K. in both instances.

Lastly, in figure 17 US export and UK export growth coherence and phase plots are shown. Interestingly



there appears to have been a cycle or cycles between 8 and 14 years in length that have been consistent up until roughly 2012. The phasing of this cycle is also interesting, with the U.S. lagging the U.K., which might suggest that the U.K. is more vulnerable to international business cycles than the U.S., perhaps because of the increased openness of the U.K. economy to international trade at that time. There are also a few high frequency cycles that are similar, but the only one that lasts for any significant period of time starts in 1988 and ends in 2000 and is at roughly the 1 to 3 year frequency. In this case the U.S. leads the U.K. cycle.

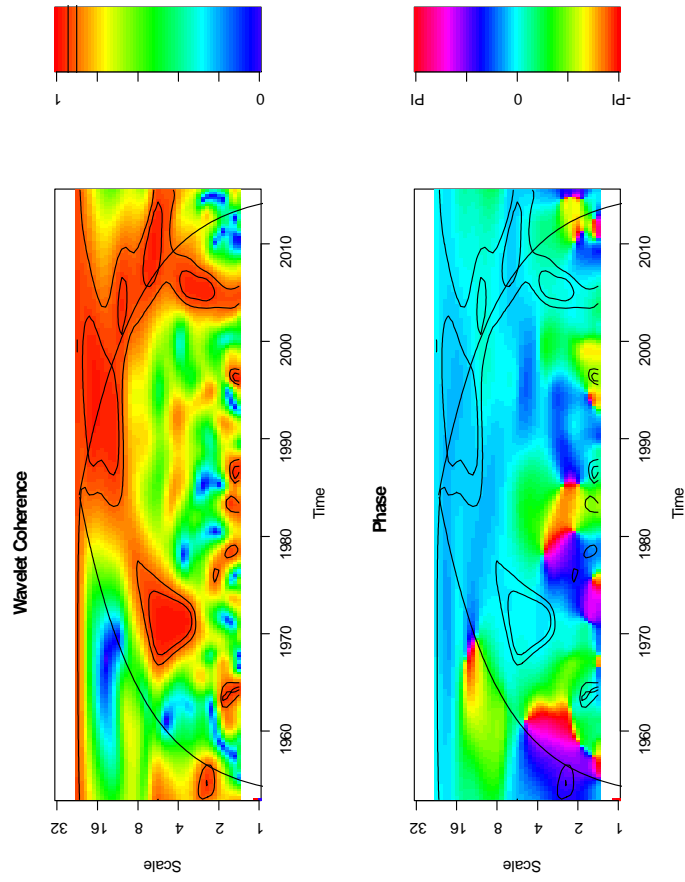


Figure 13: Coherence and phase for US GDP vs UK GDP growth

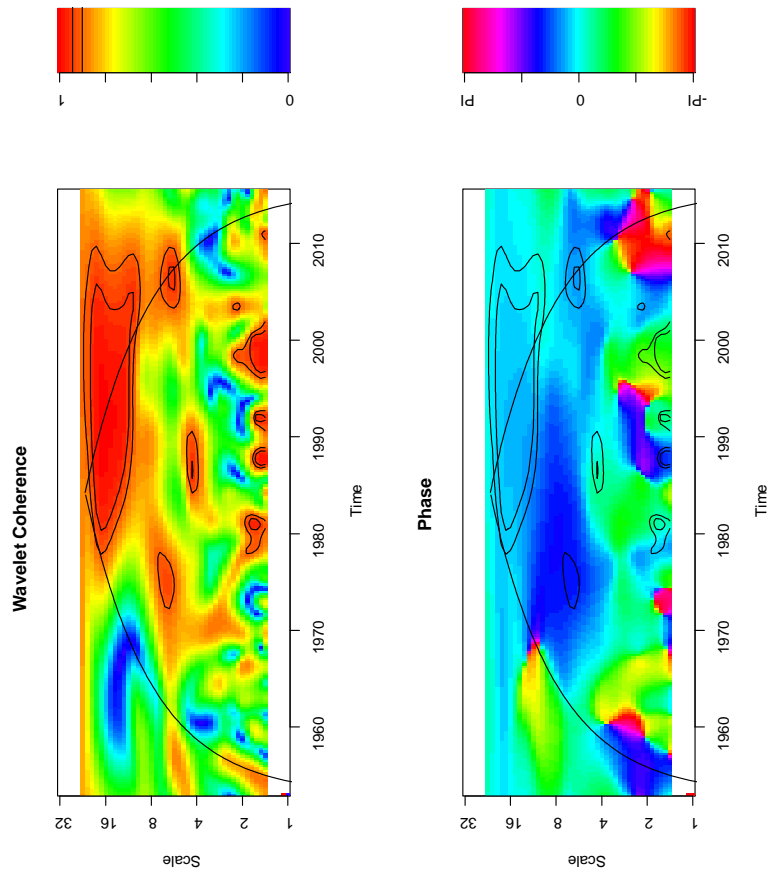


Figure 14: Coherence and Phase for US C vs UK C

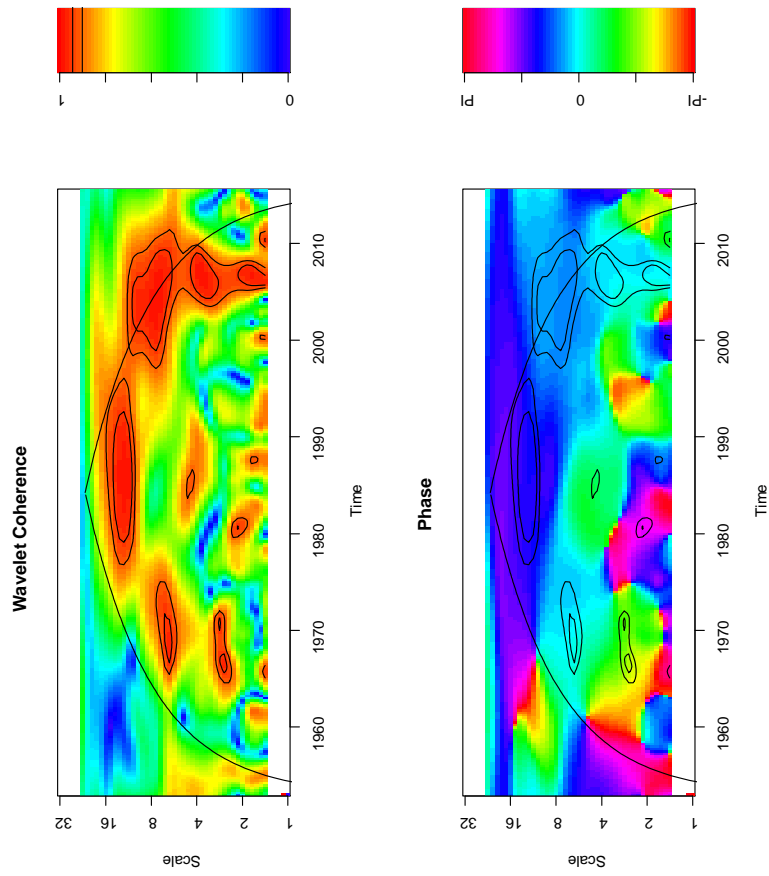


Figure 15: Coherence and Phase for US I vs UK I

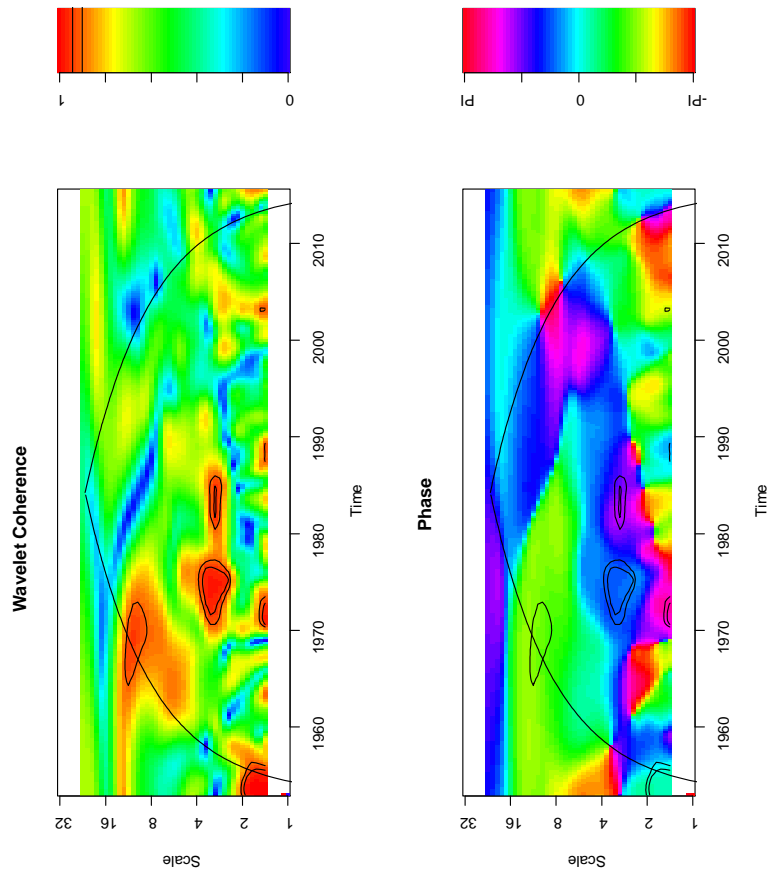


Figure 16: Coherence and Phase for US G vs UK G

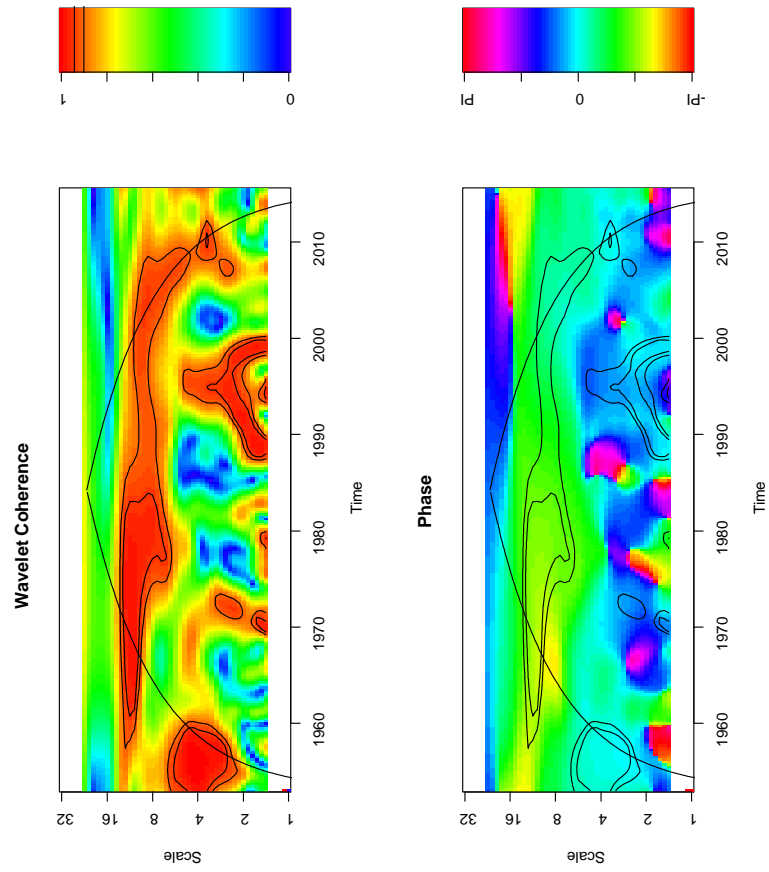


Figure 17: Coherence and Phase for US X vs UK X

## 5 Commentary: What have we learned?

### 5.1 General Comments

The numerical results generated in sections 2 and 3 contain sufficient variation in coherences, phase shifts and between cycle lengths (cyclical bands) to make it clear that it would be naïve to expect any systematic pattern to emerge in the correlations between GDP internationally, or between the components of GDP in any one economy as theory might lead us to suppose.

The expected (hoped for) correlations may still be there of course. But if the analysis is focused on a series of specific time intervals across the sample, it is quite possible that we observe strong and significant correlations between a pair of GDP components (or between a pair of national economies) in one period, but reduced and insignificant correlations in other periods. The upshot then may well be weak or insignificant correlations over the sample as a whole – and therefore for the standard coherence measures we have used as well. The advantage of our time-varying cyclical decompositions is that they show you exactly when the coherences are significant, or at least strong, and when they are not. This avoids concluding there is no meaningful coherence (correlation) between two components on average; and that theory is not supported by evidence when in fact it is supported – but only significantly so, at certain time periods and policy episodes, or at certain cycle lengths.

It is important not to throw the baby out with the bathwater in this type of research. We fear that this is what may have happened in testing RBC models, giving them a bad press.

A second observation is that the same thing can easily happen with phase shifts. If two GDP components were normally thought, on the basis of theory or reasoning from first principles, to have a high (positive) coherence between them, then it would only take a quarter cycle phase shift between them to reduce that coherence to something smaller and insignificant because only one component is contributing any power to the coherence at the point of observation. And it would only take a half-cycle shift to negate their coherence/correlation because the power contributed by that component is offset (if incompletely) by the power contributed by the other. So again it will appear as if there is no coherence/correlation between the two components, whereas they would have the expected coherence were it not for the phase shifts. But as above, we will never see that unless the analysis is made to focus on a time-varying cyclical decomposition of the data; and the theory underlying our standard models will get unfairly rejected (thrown out with the bathwater).

A third and very likely possibility is the power of the different cycles that constitute the GDP component pairs that we are looking at can vary over time depending on; the type and timing of shocks that hit the economy; on the parameters that control the dynamics of that economy; and most importantly on any changes in the policy rules that get applied to that economy. In earlier work (Crowley and Hughes Hallett (2018)) we have given a number of worked examples to show how easily that can happen in our standard

models. Fairly obviously, if events do conspire to weaken the power of certain cycles, or to extend them to a different length, then the coherence of a GDP component principally composed of that cycle will be reduced, numerically, with other components whose cycles (power, length) are not affected or equally affected.

## 5.2 Specific Comments

In terms of the actual empirical results, there are 3 major results that we can draw from the analysis above. First, and most important, this analysis shows that various different cycles are apparent at different frequencies between GDP components, and although these are not always consistent between the U.S. and the U.K., the point needs to be made that some of the strongest cycles are outside of the traditional business cycle frequency range of between 3 and 8 years. A second noteworthy result is that the largest amount of cyclical co-movement appears to occur between C and I for both the U.S. and the U.K., but the co-movement tends to move between different frequencies and can operate simultaneously at more than one frequency (see figures 1 and 7). The third key result is that an international business cycle appears to be at work at a frequency somewhere between 8 and 14 years, and that this began to emerge in the early 1980s, and has continued throughout the remainder of the time period under study is particularly apparent in figure 17 when comparing X for the U.S. and the U.K. Lastly, there is apparently a consistent 8 to 14 year cycle between I and G in the U.S., but this relationship does not carry over to the U.K.

## 6 Conclusions

This paper shows that the cyclical components of aggregate demand are not negligible, as might be presumed by the proponents of current macroeconomic thinking. Indeed from the accelerator theory of Keynesian theory, the closest association lies between consumption and investment and, although this occurs at business cycle frequencies, the evidence provided by this paper shows that there are significant and coherent cycles linking the GDP components at other frequencies, as well.

One of the major results of this paper is that the cyclical relationships between the macroeconomic components of GDP clearly differ by country, and there is a much stronger relationship between GDP components in the US than in the UK, and in particular between consumption and investment.

Another result found in this paper is that mere contemporaneous correlations do not capture the rich dynamics of the interactions between the GDP components, particularly in terms of the phasing of relationships between them.

Further research could focus on i) the interaction between individual GDP components across other countries; and ii) the flow of causation between the GDP components across frequencies. The former would clearly be a simple extension of the results presented here, and the latter would entail using causality analysis



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