BANK OF FINLAND DISCUSSION PAPERS **16/93**

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30.11.1993

Risky Debt, Bad Bank and Government

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ISBN 951-686-382-5
ISSN 0785-3572

Suomen Pankin monistuskeskus
Helsinki 1993

Abstract

The purpose of this paper is to put forward a valuation framework for interest rate sensitive claims. We concentrate on secured loans. The value of the secured loan depends upon the coupon rate, the maturity, the term structure of interest rates and the value of the collateral as well as the probability of default. We follow Schwartz and Torous (1992) and assume that borrower's conditional probability of default is given by a hazards function. Furthermore, we value guarantees, junior secured debt and unemployment insurance.

Tiivistelmä

Työssä johdetaan hinnoittelumalli korkoherkille rahoitusvaateille. Tarkastelemme erityisesti vakuudellisten lainojen hinnoittelua. Vakuudellisen lainan hinta riippuu mm. lainan korosta, juoksuajasta, korkorakenteesta, vakuuden arvosta sekä konkurssiriskistä. Työssä lainaajan ehdollista konkurssitodennäköisyyttä mallitetaan hazardifunktion avulla. Työssä hinnoitellaan myös takauksia, verovelkaa ja työttömyysturvaa. Lisäksi hinnoittelumallin avulla tarkastellaan omaisuudenhoitoyhtiöihin liittyviä kysymyksiä.

Contents

1 Introduction

This paper analyses the valuation of risky debt. We present the model, which incorporates certain stylized facts regarding default behaviour during the current Finnish banking crisis. More specifically, we follow Schwartz and Torous (1992) and use a hazards function to characterize default behaviour in valuing secured debt. We also value different risk-sharing schemes and other interest-rate sensitive claims.

This valuation framework is used to discuss some of the issues related to the current banking crises in the Nordic countries. In most of the Nordic countries, financial deregulation triggered excessive credit expansion and economic boom, which were followed by an unusually deep recession, falling real estate prices and severe banking crises. In particular, we discuss the valuation of risky debt in the context of good bank/bad bank transactions, which is one of the measures used by the authorities and banks in coping with the banking crises.[1]

In the good bank/bad bank transaction some of the bank's problem assets are transferred from the bank's balance sheet to the bad bank. The bad bank can be owned directly or indirectly by the government, by the good bank or by a third party.

There can be several rationales for a creation of a bad bank. First, an independent bad bank can improve a good bank's ability to raise capital and fund itself. Second, when a crisis occurs bank managers are tied to work mainly with the problem assets. The danger is that they do not have the time or resources to take care of the healthy business. After a good bank/bad bank transaction, bank managers can be freed to concentrate on healthy assets. Third, the bad bank is often needed if the good bank is sold to a third party. Without the bad bank, the value of the bank can be negative and very difficult to estimate.

However, there are at least two important questions to be answered prior to a good bank/bad bank transaction. The most obvious question is how to value assets which are transferred to the bad bank. The valuation problem is by no means trivial in practice, especially because the number of different loans which should be valued can be quite large. There are typically no market prices to loans for businesses and households. Furthermore, the value of the collateral

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[1] Among the Nordic countries bad banks have thus far been used in Finland and Sweden. In Finland, the first bad bank was formed as a part of the STS/Kansallispankki transaction. STS Bank was a small commercial bank, which was merged to the Kansallispankki. In the transaction, STS Bank's bad assets were left to the STS Bank, which became bad bank (now Bridgebank). The second bad bank was formed in a autumn 1993 when Savings Bank of Finland (SBF) was sold in parts to other major Finnish banking groups and its bad assets were transferred to a bad bank. The SBF was the second largest bank (or the bank group) in Finland when measured by deposits.

can be difficult to estimate when the liquidity of the real estate markets is low. Thus, there is a need for a valuation framework.[2]

The second question is how to share risks. Moral hazard problems can be very severe when risks are moved from the bank's balance sheet to the bad bank. Moral hazard problems can arise in two instances. First, when the bad assets are transferred from the bank to the bad bank, the bank can have incentives to remove "hidden" problems, which are not easily discovered and/or valued. Second, moral hazard problems can arise if the loans owned by the bad bank are still managed by the good bank. The problem is how to ensure that the management of the good bank properly monitors and manages loans. One approach is to use risk-sharing schemes. Risk-sharing can also be used when there is great uncertainty as to the quality of assets to be sold. We demonstrate how to use our valuation framework to apply to different risk-sharing schemes.

The plan of this paper is as follows: In the next section we present our framework for the valuation of secured loans and for the valuation of performing and performing non-performing loans. In section 3 we discuss the transactions with guarantees. In section 4 we value tax debt and unemployment insurance and discuss whether the good bank and the government can have different incentives. In this section we also demonstrate that in certain situations the creditor is better off if some of the nominal debt is written off. Finally, in section 5 we present conclusions and policy implications.

2 The valuation framework

2.1 Valuation of secured loans

In this section, we first present the framework used throughout this paper for the valuation of interest rate sensitive claims. We start by modelling the value of the secured loans.

The traditional approach to modelling default risk, as pioneered by Merton (1974) and extended by Black and Cox (1976), assumes that the market value of the issuing firm follows an exogenously specific stochastic process and the term structure is deterministic. We assume that the term structure of interest rates is stochastic and concentrate on the value of the collateral. We choose to concentrate on the value of the collateral because in most default situations the collateral dictates the amount of default losses. Analytically, we could easily substitute the value of the firm for the value of the collateral.

Thus, the price of the loan at time t is assumed to be a function of the instantaneous interest rate, r, and the value of the collateral, C, i.e.,

[2] Note that the need for valuation is not removed when the transaction is done on criteria other than "fair price" as, for example, book value. There is still the need to calculate the amount of government aid implied by the difference between transaction price and fair price. Authorities, tax payers and competitors should know the amount of aid given to the good bank.

$P = P(r, C, t)$. We assume that the loan collateral is real estate and that the borrower does not have other assets for use as collateral.[3]

The dynamics of the instantaneous interest rate are assumed to be given by the following stochastic differential equation:

$$dr = \kappa(m - r)dt + \sigma_r r^{1/2} dW_r, \tag{1}$$

where κ is the speed of adjustment coefficient, m is the long-term mean instantaneous rate and $\sigma_r^2 r$ is the instantaneous variance of changes in r. Equation (1) is a familiar mean-reverting square-root process used by Cox, Ingersoll and Ross (1985) and many authors since.

The value of the collateral evolves over time according to

$$dC = (\mu - b)C dt + \sigma_c C dW_c \tag{2}$$

where μ, b, and σ_c^2 are the instantaneous expected rate of return, payout rate and instantaneous variance of return, respectively, of the real estate.

Unanticipated changes in the value of real estate are assumed to be correlated with unanticipated changes in the instantaneous risk-free interest rate; $dW_r dW_C = \rho dt$, where ρ is the instantaneous correlation coefficient.

Default risk

We do not impose an optimal default policy as used, for example, by Titman and Torous (1989).[4] They assume that a borrower will default on a mortgage if at any time prior to maturity the value of the mortgaged building falls below of the mortgage. However in Finland, for example, the vast majority of loans which were granted in the credit boom years have collateral values below loan values. Optimal default policy would imply that these loans should have been defaulted. This is clearly not the case. The borrower's default decision depends upon many other factors in addition to the value of the underlying collateral value relative to the value of the loan. For example, significant costs due to reputation losses in case of default ensure that the most borrowers are not willing to default loans.

Instead of using optimal default policy we assume that the borrower's conditional probability of default is given by a hazards function as specified below. The hazards function captures some stylized facts regarding the behaviour of defaults during the banking crises. The default risk is the highest among lenders that were indebted during the credit boom years, when expected

[3] For the case of valuation when the borrower has both secured and unsecured debt outstanding, see e.g. Stulz and Johnson (1985).

[4] In the Black and Cox (1976) and Merton (1974) models, a loan is defaulted at maturity whenever the value of the firm is less than the promised payment to the debtholders. Black and Cox (1976) also consider safety covenants which give the bondholder the right to bankrupt the firm if the firm's value hits a specific absorbing barrier.

returns on the investments and value of real estate were at exceptionally, and many cases unrealisticly, high levels (see Murto, 1994).

More specifically, we use the following hazards function:

$$\delta(C,t) = \pi_0 \exp\{\beta[F(t) - C(t)]/C(t)\}, \text{ when } C(t) < F(t), \text{ 0 elsewhere.} \tag{3}$$

Equation (3) is a proportional hazards function, where π_0 is the baseline hazard. F(t) is the nominal amount of debt. The parameter β (>0) determines the speed of default.

The hazards function gives the rate per unit time that the loan is defaulted conditional on the fact that default has not yet occurred. The probability of default is a function of the term [F(t) - C(t)]/C(t), which increases as the collateral value decreases with respect to the nominal loan amount. As C(t) approaches zero, the value of $\delta(C,t)$ approaches infinity for a given value of F(t) (>0). Correspondingly, when F(t) decreases as the maturity date comes closer, the likelihood of default decreases for a given value of C. In figure A1 in the Appendix we illustrate the behaviour of the hazards function.

The valuation equation

Standard arbitrage arguments imply that the value of the loan must satisfy the following partial differential equation:[5]

$$1/2\sigma_r^2 rP_{rr} + 1/2\sigma_c^2 C^2 P_{cc} + \sigma_r \sigma_c r^{1/2} C\rho P_{rc} + (\kappa(m-r) + \lambda r)P_r + C(r-b)P_c - rP + \xi = -P_t \tag{4}$$

where the subscripts on P denote partial derivates and λ is the market price of risk. The payout rate, ξ, will be defined below.[6]

The above equation corresponds to the partial differential equation given by Schwartz and Torous (1992) to value mortgage pass-through securities. The differences between the mortgage pass-through securities and secured loans discussed in this study are captured in the payout rate ξ and the boundary conditions.

The term ξ is defined as follows:

$$\xi(r,C,t) = I + \delta(C,t)[C(t) - P(r,C,t)]. \tag{5}$$

[5] More specifically, we can form the risk-free portfolio by selling and buying the following three assets: the default-free bond, the real estate and the secured debt. Given that the values of the these assets are determined as functions of two state variables only, they can be combined into a portfolio that is instantaneously risk-free.

[6] We can already see from equation (4) that the price of the loan does not depend on the expected return of the collateral. This is, of course, a standard result from contingent claims models. One does not need to estimate the expected return for the collateral. It is enough that one has estimates for the current real estate prices, volatility and payout ratio.

I is the loan's total payout rate including the coupon payment. The precise nature of the security determines the payout rate I. The last term reflects the fact that with probability δ default occurs and the market value of the collateral is received.

The coupon payment will depend among other things on whether the loan is a performing, a performing non-performing or a non-performing asset. When the loan is a performing the coupon rate is normal and the value of the collateral is higher than the value of the loan. The loan is a performing non-performing loan when it is current on payments of principal and interest but the collateral value has dropped below the value of the loan. The loan is non-performing when the loan is not current on payments of principal and interest. In the following, we concentrate on performing and, especially, on performing non-performing loans.

Payout also depends on whether or not the loan is full amortized. In the application that follows we value fully amortizing, fixed-rate loans. As a result, the total payout rate is

$$I = iF(0)/[1 - \exp(-iT)],$$

with principal outstanding at time t, F(t), given by

$$F(t) = F(0)[1 - \exp(-i(T-t))]/[1 - \exp(-iT)]. \tag{7}$$

i is the rate of the continuously paid coupon on the debt.

Since the loan is fully amortizing, the following terminal condition must be satisfied:

$$P(r,C,T) = 0,$$

where T denotes the maturity date. In addition to the equation (8) we need four additional boundary conditions.

We use the following boundary conditions: For sufficiently large values of r and C, we have P = 0 and P_C = 0, respectively. For r = 0 we impose P_r = 0 and for C = 0 we impose P = 0. The last boundary condition reflects the fact that when the value of the collateral approaches to zero the probability of default approaches one.

2.2 Parameter values

In this section we present valuation results using the model presented above. We consider a 10-year fully amortizing loan with a fixed continuously compounded coupon rate of 11 per cent. We assume the face value of the loan at pricing to be 100.

In order to solve equation (4) numerically, we have to fix parameter values.

The parameters for the interest rate process are $\sigma_r = 0.10$, $\kappa = 0.5$ and $m = 0.08$. These values are higher than estimates acquired, for example, by Barone, Cuoco and Zautzik (1991) or values used by Schwartz and Torous (1992), but lower than estimated by Murto (1992).

The value of λ is obtained from the following formula:

$$\lambda = k[1 - (m/r_L)] + \sigma_r^2 r_L/2km. \tag{9}$$

Equation (9) gives a value of λ corresponding the Cox, Ingersoll and Ross (1985) term structure model with a specified value for the long-term interest rate, r_L. The value of λ is calculated assuming that the long run interest rate, r_L is 10 % per year.

The real estate payout rate is assumed to be 0.06 and the real estate return volatility 0.2. We assume that unanticipated increments to the instantaneous riskless rate of interest rate are uncorrelated with unanticipated increments to real estate returns; i.e. $\rho = 0$. The results are not sensitive to the last assumption according to the sensitity analysis.

We start by assuming that $\beta = 0.1$ and the baseline hazard = 0.15. This is consistent with annualized default probability of 0.165 when collateral value is half of F(0). The low value of β implies that the default risk is insensitive to changes in the value of C unless the value of the collateral is very low. On the other hand, the high value of π_0 implies that the default risk is high whenever C < F.[7]

2.3 Results

We present our first valuation results in table 1a. For a given short-term interest rate, we provide corresponding loan values for different values of the collateral.

As expected, the loan values are sensitive to prevailing interest rates. As the interest rate increases, the value of the fixed-rate loan decreases. For example, when the value of the collateral is 60, the loan value decreases from 95.20 to 84.92 as the instantaneous interest rate increases from 0.06 to 0.16.

[7] The values of β and π_0 can be estimated given that there exists sufficient data on defaults. The value of β has great importance in determining the values of collateral where the default risk starts increasing. In case the data is not sufficient to estimate parameters, i.e. under great uncertainty on the interaction of default and collateral values, we recommend the use of low values for β. Low values imply that the default risk is insensitive to changes in the collateral values and the default risk is dominated by the level of π_0.

Table 1a. **The value of the loan as a function of the short-term interest rate and the value of the collateral, when $\pi_0 = 0.15$**

	C					
r	40	60	80	100	120	140
0.05	85.34	96.26	104.28	109.85	112.27	113.02
0.06	84.44	95.20	102.97	108.17	110.46	111.11
0.07	83.53	94.12	101.72	106.66	108.67	109.25
0.08	82.66	93.07	100.45	105.04	106.89	107.40
0.09	81.77	92.02	99.20	103.53	105.16	105.60
0.10	80.93	90.98	97.95	101.96	103.42	103.82
0.11	80.06	89.95	96.71	100.47	101.75	102.08
0.12	79.25	88.93	95.49	98.94	100.06	100.36
0.13	78.39	87.91	94.25	97.46	98.43	98.68
0.14	77.59	86.91	93.06	95.98	96.79	97.03
0.15	76.72	85.91	91.82	94.54	95.23	95.41
0.16	75.91	84.92	90.62	93.09	93.68	93.82
0.17	75.16	83.93	89.45	91.66	92.13	92.26

The long-term interest rate is 10 %. The baseline hazard is 0.15 and $\beta = 0.1$. The coupon payment is 11 %.

Table 1b. **The value of the loan as a function of the short-term interest rate and the value of the collateral, when $\pi_0 = 0.30$**

	C					
r	40	60	80	100	120	140
0.05	71.12	86.64	98.75	107.59	111.50	112.74
0.06	70.65	86.01	97.81	106.09	109.79	110.87
0.07	70.17	85.35	96.95	104.83	108.08	109.04
0.08	69.72	84.71	96.03	103.39	106.38	107.22
0.09	69.25	84.06	95.13	102.09	104.72	105.44
0.10	68.81	83.42	94.23	100.68	103.04	103.69
0.11	68.33	82.76	93.30	99.36	101.42	101.96
0.12	67.91	82.12	92.40	97.98	99.77	100.26
0.13	67.42	81.46	91.45	96.64	98.20	98.60
0.14	67.00	80.82	90.56	95.30	96.57	96.96
0.15	66.50	80.15	89.56	93.97	95.07	95.35
0.16	66.05	79.50	88.60	92.62	93.54	93.77
0.17	65.64	78.84	87.69	91.30	92.00	92.21

The long-term interest rate is 10 %. The baseline hazard is 0.3 and $\beta = 0.1$. The coupon payment is 11 %.

For sufficiently high values of the collateral, the value of the loan is not sensitive to changes in the collateral value. This is due the fact that the value of the loan approaches the value of the riskless debt as the value of the collateral goes to infinity. On the other hand, when the loan is a performing non-performing loan, the loan value is sensitive to the changes in the value of the collateral. As the value of the collateral decreases, the value of the performing non-performing loan also decreases. The value of the performing non-performing loan is between the value of the riskless debt and the value of the collateral.

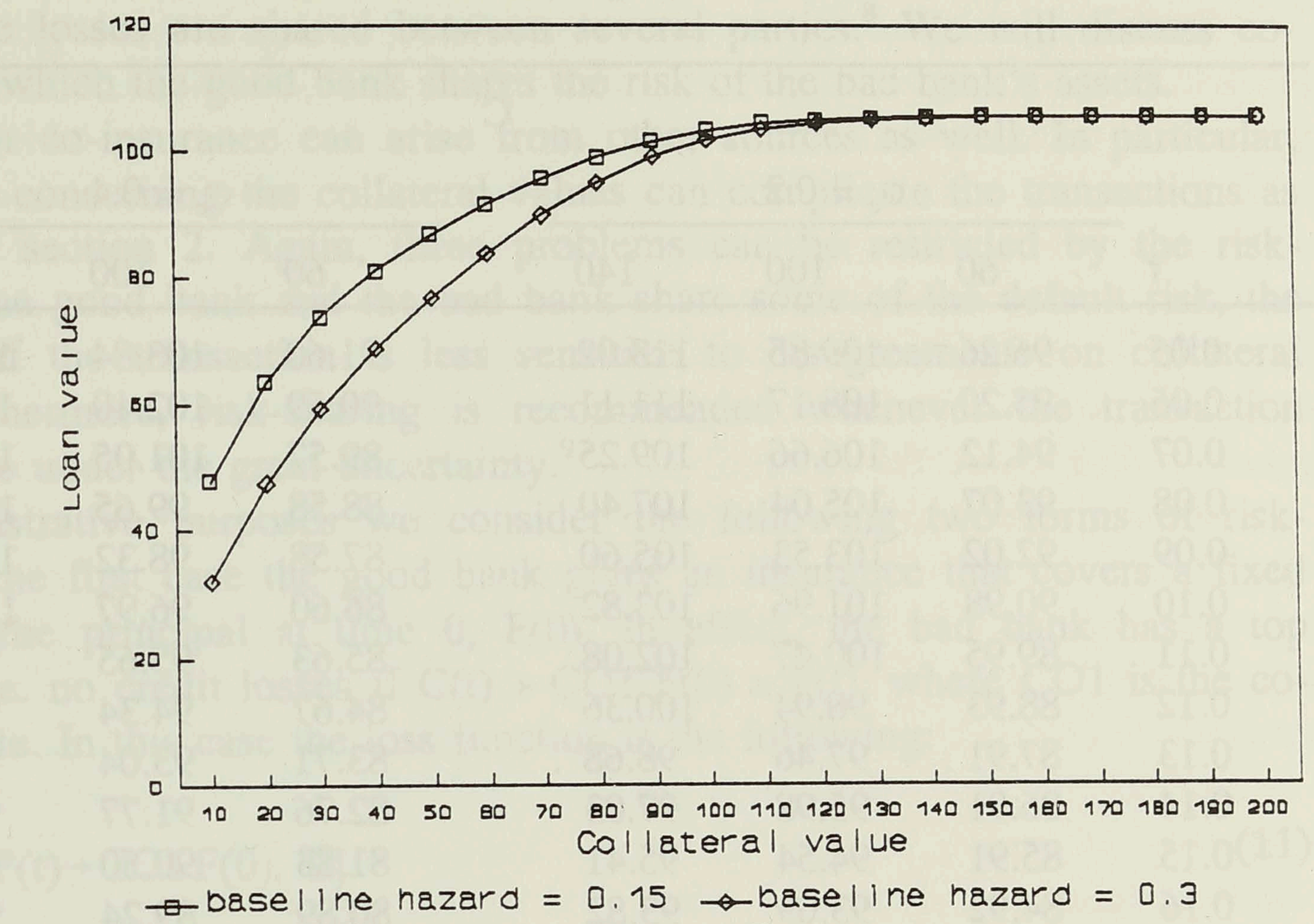
In table 1b we present results for a baseline hazard of 0.30. Comparing tables 1a and 1b, we can examine how the loan values changes as the base default risk is doubled. The impact on the values is clear when C is under 100. For r = 12 and C = 40 the value of the performing non-performing loan is about 79 and 68 per cent of the (relatively riskless) performing loan with C = 140, when the baseline hazard is 0.15 and 0.3, respectively. Figure 1 graphically summarizes these results for r = 0.09.

An increase in default risk decreases the values of performing non-performing loans. As the default risk increases, the value of the performing non-performing loan approaches the collater value. However, the impact is much less significant when collateral values are sufficiently high. For example, when C = 140, doubling the default risk has only a marginal effect on loan values. This is because the probability that default will cause losses is low with high collateral values.

The above results facilate our discussion of some of the difficulties associated good bank/bad bank transactions. During the banking crisis a large share of the loans in banks portfolios consists of performing non-performing loans. Furthermore, there is a considerable default risk among borrowers. Taken together, these facts make the transferrable loan values very sensitive to changes in underlying collateral values, as demonstrated above. In order to value loans correctly, one must get good estimates of the collateral values. This can be difficult.

In such a broad-based recession, liquidity in the real estate market, as well as in markets for other collateral, is dramatically worsened. This makes it difficult to estimate collateral values. True sale could be arranged only after a substantial search for potential buyers. As Schleifer and Vishny (1992) have noted, liquid assets are better collateral than illiquid assets. In the next section we discuss some of the possible ways to circumvent these problems.

Figure 1. **Loan values**



If banks or authorities are interested in the market value of bank loans, they should keep track of the collateral values. However, the results can give a possible reason why the banks did not typically have a good record on collateral values. Whenever the collateral values exceeds the nominal loan values, the loan values are not sensitive to the changes in the collateral values. In these circumstances there is no urgent need to keep a record of collateral values. However, whenever the loan becomes a performing non-performing loan the situation is changed. Then banks should know the collateral values.

Our next concern is to simulate the reactions of values to changes in the volatility of collateral return. Table 2 documents the sensitivities of loan values to changes in the collateral return volatility. We tabulate the loan values for two different values of σ_C. On the left are values corresponding to the same parameter values as in table 1a. On the right are values using the values of table 1a, but with a volatility of collateral of 40 per cent per annum instead of 20 per cent per annum.

We observe that the value decreases as the volatility increases. An increase in the variance rate increases the dispersion of possible values of the collateral during the time to maturity. Since there is a maximum payment the lender can receive, a mean-preserving increase in the dispersion reduced the expected repayment amount, lowering the value of the debt.

Table 2. **Volatility of the value of the collateral and the value of the loan.**

	C					
	$\sigma_c = 0.2$			$\sigma_c = 0.4$		
r	60	100	140	60	100	140
0.05	96.26	109.85	113.02	91.60	103.84	108.75
0.06	95.20	108.17	111.11	90.59	102.40	107.10
0.07	94.12	106.66	109.25	89.57	101.05	105.47
0.08	93.07	105.04	107.40	88.58	99.65	103.86
0.09	92.02	103.53	105.60	87.58	98.32	102.27
0.10	90.98	101.96	103.82	86.60	96.97	100.70
0.11	89.95	100.47	102.08	85.63	95.65	99.16
0.12	88.93	98.94	100.36	84.67	94.34	97.64
0.13	87.91	97.46	98.68	83.71	93.04	96.14
0.14	86.91	95.98	97.03	82.76	91.77	94.66
0.15	85.91	94.54	95.41	81.83	90.50	93.20
0.16	84.92	93.09	93.82	80.89	89.24	91.77
0.17	83.93	91.66	92.26	79.97	88.01	90.35

The long-term interest rate is 10 %. The baseline hazard is 0.15 and $\beta = 0.1$. The coupon payment is 11 %.

3 Valuing guarantees

The purpose of this section is to show how we can use our framework to analyse different kinds of risk-sharing schemes. As above, we discuss risk-sharing schemes in particular in the context of good bank/bad bank transactions. Previous contingent-claims models of loan guarantees include Merton (1977) and Jones and Mason (1980), among others.

The good bank/bad bank transaction requires case by case investigation of loans and other assets. A careful investigation would ideally reveal the information required to value transferrable loans correctly. However, for several reasons moral hazard and adverse selection problems can arise, as well as problems due to the great uncertainty regarding loan quality.

Banks have typically gained access to private information about their customers as a result of an ongoing business relationship with them over the years. Thus the good bank has superior information on the borrower's debt servicing capacity and business prospects as well as on the borrower's character, which are not necessarily fully revealed in an investigation done by outsiders. The asymmetric information, combined by the incentives to avoid losses, can create severe moral hazard problems, especially if the ownership of the bad bank is outside the good bank. The good bank can have incentives not to reveal all relevant information as well as to transfer the loans where the asymmetric information problem is most severe and mispricing is probable.

One solution to the moral hazard and adverse selection problems that has arisen in different markets is risk-sharing through co-insurance. With co-insurance the losses are shared between several parties.[8] We will discuss co-insurance in which the good bank shares the risk of the bad bank's assets.

Need for co-insurance can arise from other sources as well. In particular, uncertainties concerning the collateral values can complicate the transactions as discussed in section 2. Again, these problems can be restricted by the risk-sharing. If the good bank and the bad bank share some of the default risk, the realization of the transaction is less sensitive to disagreements on collateral values. Furthermore, risk-sharing is recommended whenever the transaction must be done under the great uncertainty.[9]

For illustrative purposes we consider the following two forms of risk-sharing. In the first case the good bank gives an insurance that covers a fixed fraction of the principal at time 0, F(0). In effect, the bad bank has a top insurance, i.e. no credit losses if $C(t) + CO1\,F(0) \geq P(t)$, where CO1 is the co-insurance rate. In this case the loss function is the following:

$$MIN[C(t) - P(t) + CO1\,F(0),\ 0]. \tag{11}$$

When CO1 = 0, there is no risk-sharing.

The above example can be generalized straightforwardly to the case where the good bank gives an insurance on the remaining principal F(t).

In the second case we assume that the good bank gives an insurance such that whenever the loan is defaulted, the good bank pays a fixed percentage of the difference between F(t), the remaining principal, and C(t), the value of the collateral, given that the difference F(t)-C(t) is positive. This implies that the loss function can be written as:

$$C(t) - P(t) + CO2\,MAX[F(t) - C(t),\ 0], \tag{12}$$

where CO2 determines the level of the co-insurance. Again, if CO2 = 0, there is no risk-sharing.[10]

We emphasize that the above forms of co-insurance are only examples. There are many other types of risk-sharing, which can be used to restrict moral

[8] Explanations of co-insurance have been given by Marshall (1976), Holmström (1979) and Shavell (1979), among others.

[9] For example, in the liquidation of the Savings Bank of Finland, the buyers were not able to investigate the loans beforehand.

[10] The lower boundary conditions with respect to C need to be modified in the case of co-insurance. For C = 0, we impose P = CO1 F(0) or P = CO2 F(t) depending the nature of the guarantee.

hazard problems and resolve problems associated with the great uncertainty regarding loan quality.[11]

Table 3 gives the results in the case where the good bank guarantees that the bad bank can have 20 per cent of the F(0) in case of default. Table 3 shows the results when π_0 is 0.15 and 0.30 respectively. Furthermore, table 4 gives the results for the case where the good bank guarantees 20 per cent of the difference between the remaining principal and collateral value. The value of the guaranteed debt is the sum of the value of the debt without a guarantee and the value of the guarantee. Since the values of the debt without a guarantee are already reported in table 1a and 1b, we can concentrate on the value of the guaranteed debt and the value of the guarantee.

Not surprisingly, the guarantee has the greatest impact on the loan value when the collateral value is low. The guarantee is the most valuable when the loss due to default – if it occurs – is high. These results confirm previous results: when the value of the collateral is sufficiently high the loan value is not sensitive to changes in default risk or inclusion or exclusion of guarantees.

Increase in the default risk reduces the value of the guaranteed debt but increases the value of the guarantee. The increase in the volatility of the collateral return has the same effect (for brevity we do not report those results here). Note also that the value of the guarantee is interest-rate sensitive. As the interest rates increases the value of the guarantee decreases. We can also see from the results reported in tables 3 and 4 that the first guarantee scheme is more valuable than the latter.

Table 3. **The effect of co-insurance on the value of the loan when the good bank guarantees 20 per cent of F(0).**

	C					
	π_0 = 0.15			π_0 = 0.30		
r	60	100	140	60	100	140
0.05	104.77	112.68	113.38	99.37	112.18	113.34
0.06	103.45	110.86	111.43	98.43	110.45	111.40
0.07	102.15	109.02	109.52	97.49	108.66	109.49
0.08	100.85	107.22	107.64	96.44	106.92	107.62
0.09	99.56	105.45	105.80	95.59	105.20	105.78
0.10	98.29	103.70	104.00	94.63	103.49	103.98
0.11	97.02	101.99	102.23	93.67	101.82	102.22
0.12	95.76	100.29	100.49	92.70	100.14	100.48
0.13	94.50	98.63	98.79	91.73	98.50	98.79
0.14	93.26	96.98	97.13	90.75	96.88	97.12
0.15	92.02	95.37	95.49	89.76	95.29	95.49
0.16	90.79	93.79	93.89	88.78	93.72	93.88
0.17	89.57	92.24	92.32	87.78	92.18	92.31

The long-term interest rate is 10 %. β is 0.1. The coupon payment is 11 %.

[11] In particular, one can use options. For example, in the case of SBF's liquidation, the buyers get an option to return some of the purchased assets to the bad bank. This resembles the FDIC's practice in P&A transactions.

Table 4. **The effect of co-insurance on the value of the loan when the good bank pays 20 per cent of (F(t)-C(t)), when default occurs.**

	C					
	π_0 = 0.15			π_0 = 0.30		
r	60	100	140	60	100	140
0.05	99.04	110.16	113.06	90.91	108.07	112.81
0.06	97.87	108.60	111.16	90.13	106.77	110.94
0.07	96.72	106.96	109.28	89.38	105.32	109.09
0.08	95.57	105.40	107.44	88.62	103.94	107.27
0.09	94.44	103.82	105.63	87.86	102.56	105.49
0.10	93.32	102.27	103.85	87.10	101.18	103.73
0.11	92.21	100.76	102.10	86.35	99.84	102.00
0.12	91.12	99.22	100.38	85.60	98.45	100.30
0.13	90.03	97.74	98.70	84.85	97.10	98.63
0.14	88.96	96.24	97.05	84.10	95.73	96.99
0.15	87.90	94.78	95.43	83.36	94.39	95.38
0.16	86.84	93.33	93.83	82.62	93.05	93.79
0.17	85.80	91.90	92.27	81.88	91.72	92.24

The long-term interest rate is 10 %. β is 0.1. The coupon payment is 11 %.

4 Government, tax debt and unemployment insurance

4.1 Tax debt and unemployment insurance

The valuation framework presented in section 2 can be easily applied to value different kind of interest-rate sensitive claims. In this section we use our valuation framework to value three claims: bank debt, tax debt and unemployment insurance. By considering tax debt and unemployment insurance we introduce the government into our analysis.

The inclusion of the government in our models is of interest for two reasons. First, in most cases the government is the owner of the bad bank. For example, in Finland the owner of a bad bank can only be the Goverment Guarantee Fund or the goverment. Second, given that the government owns the bad bank, it is interesting to ask whether the incentives of the government and the good bank differ. We study this issue in the context of debt relief. We show that in certain circumstances the incentivies of the good bank and the goverment can differ.

In the following, we assume that the borrower has a tax debt to the government. The tax debt is assumed to be a junior secured debt. This implies that the loss function is of the following form:

$$\mathrm{MIN}[FT(t),\ \mathrm{MAX}(C(t) - F(t),\ 0)] - T(t), \tag{13}$$

where FT(t) is the nominal tax debt and T(t) is the corresponding market value of the tax debt.

The introduction of the tax debt alters the default and hazards functions for bank debt. First, whereas the hazards function had previously had a positive value only for F(t) greater than C(t), the condition is now F(t) + FT(t) greater than C(t). This, of course, is also true for the tax debt. Second, the loss function for bank debt in case of bankruptcy is written as (min(F(t), C(t)) - P(t)). Thus we take into account the fact that the maximum value of the collateral that the lender can have in the event of default is the remaining nominal principal of the loan. Finally, the introduction of the tax debt increases the indebtedness of the firm for a given value of bank debt, which in turn increases the probability of default.

We assume that the tax debt has an infinite maturity. It corresponds, as a first approximation, to the common practice where the borrower can roll over the tax debt.[12]

The second new claim, which we introduce here, is the unemployment insurance. The government has promised to pay unemployment benefits for the employees in case the company is in bankruptcy and the employees became unemployed. As a first approximation we assume that the bankruptcy triggers a lump-sump payment to the employees by the writer of the unemployment insurance, i.e. the government.[13]

Notice that this liability is written to the employees, not to the company. This implies, among other things, that the default probability does not depend on the nominal amount of the unemployment insurance. We assume that the unemployment insurance has an infinite maturity.[14]

Table 5 reports values for the bank loan, tax debt and unemployment insurance. We assume that the face value of the tax debt is 25 and the lump-sum unemployment benefit is 50. We also assume that the coupon rates for the bank and the tax debt are the same.

The inclusion of the tax debt decreases the value of the bank debt as the indebtedness of the borrower increases, which in turn increases the default risk. The impact is, however, quite small. This is because the bank debt is senior to the tax debt.

The value of the tax debt deviates much more rapidly from the riskless debt value than the value of the bank debt. The holder of the tax debt looses everything in the case of default whenever C(t) is less than or equal to F(t), whereas the loss function for the senior debt is much smoother. Since the tax

[12] The boundary conditions used are same as in the case of bank debt. Furthermore, we make the auxilarily assumption that the tax debt is amortized so that F(oo) = 0.

[13] The generalization to the case where the default triggers a continuous payment with stochastic maturity is straightforward.

[14] The corresponding boundary conditions are as follows: for sufficiently high values of C we impose U = 0 and for C = 0 we impose U = FU, where FU is the nominal unemployment benefit. Correspondingly, for high value of r, U = 0, and for r = 0, $U_r = 0$.

debt is junior debt, its value reacts differently to some of the changes in parameter values than does the value of the bank loan. Especially in certain cases, the value of the tax debt increases as the volatility of the collateral increases, whereas the value of the bank loan decreases.

The value of the unemployment insurance decreases as the value of the collateral increases. Insurance is most valuable when the default risk is highest. It is clear that the value of the insurance increases as the baseline hazard increases. Furthermore, the value of the unemployment insurance increases as the volatility of the collateral increases (these results are not presented here).

Table 5. **Values of the bank debt (B), tax debt (T) and unemployment insurance (U)**

	C								
	B			T			U		
r	60	100	140	60	100	140	60	100	140
0,05	95,60	108,60	112,50	15,37	21,65	28,15	32,24	27,37	16,28
0,06	94,58	107,14	110,68	15,23	21,45	27,74	31,60	26,72	15,45
0,07	93,53	105,75	108,89	15,09	21,31	27,33	31,00	25,84	14,65
0,08	92,54	104,32	107,12	14,96	21,14	26,92	30,39	25,11	13,87
0,09	91,51	102,96	105,38	14,83	20,96	26,51	29,80	24,39	13,12
0,10	90,53	101,55	103,66	14,70	20,78	26,10	29,22	23,66	12,40
0,11	89,52	100,21	101,97	14,57	20,58	25,69	28,65	22,95	11,70
0,12	88,55	98,83	100,30	14,44	20,41	25,28	28,10	22,24	11,04
0,13	87,57	97,50	98,66	14,31	20,20	24,87	27,53	21,61	10,40
0,14	86,61	96,16	97,04	14,19	20,02	24,46	27,03	20,88	9,79
0,15	85,64	94,85	95,45	14,06	19,82	24,05	26,48	20,26	9,21
0,16	84,70	93,54	93,89	13,94	19,63	23,65	26,01	19,55	8,64
0,17	83,75	92,25	92,35	13,82	19,42	23,25	25,48	18,95	8,11

The long-term interest rate is 10 %. The baseline hazard is 0.3 and β = 0.1. The coupon payment is 11 % for the bank debt and the tax debt.

Table 6. **Change in the market value of bank debt (B), tax debt (T) and unemployment insurance (U) given a write-off in the bank debt**

C	B	T	U	SUM
Case a. Relief 25 % i.e. F(0) = 75.				
40	−4.07	0.47	0.44	−3.16
60	−5.97	1.81	0.82	−3.34
80	−12.71	7.10	3.97	−1.64
100	−20.17	11.20	11.98	3.01
120	−23.24	6.47	17.23	0.64
140	−26.06	3.17	10.18	−12.71
160	−27.31	1.56	5.68	−20.07
Case b. Relief 50 % i.e. F(0) = 50				
40	−8.51	1.95	0.91	−5.65
60	−21.17	11.19	5.08	−4.90
80	−35.32	19.41	22.08	6.17
100	−45.45	17.18	31.23	2.96
120	−50.35	9.20	26.18	−14.97
140	−53.90	4.52	14.47	−34.91
160	−55.42	2.28	7.79	−45.35
Case c. Relief 75 % i.e. F(0) = 25				
40	−27.74	15.11	5.09	−7.54
60	−45.69	23.64	29.60	7.55
80	−61.94	23.89	38.44	0.39
100	−73.26	18.90	37.75	−16.61
120	−78.56	9.94	28.90	−39.72
140	−82.21	4.87	15.67	−61.67
160	−83.77	2.46	8.36	−72.95

The long-term interest rate is 10 % and the instantanous interest rate 5 %. The baseline hazard is 0.75 and $\beta = 0.1$. The coupon payment is 11 % for the bank debt and the tax debt.

4.2 Debt relief

We now turn our attention to the different incentives that the government and the good bank can have. To analyse this issue, we study the simplified case where the holder of the bank debt is given an option to give debt relief to the borrower. We ask whether or not it is optimal to exercise it. For tractability, we consider the case where the option holder can exercise the option only once, i.e. now.

Table 6 reports changes in asset values resulting from the nominal bank loan write-off.[15] The parameter values are the same as in table 5 with the exception that $\pi_0 = 0.75$. Thus, we are looking at the case where the probability of default is very high.

It is clear that the market value of the bank debt decreases as the nominal value of the debt is lowered. Note, however, that the drop in market values is much smaller with low collateral values than with high collateral values. On the other hand, the market value of the tax debt is increased. This is due to two factors. First, the probability of default decreases as the borrower's indebtness decreases. Secondly, the expected credit losses for a given value of the collateral is decreased with low values of collateral. The market value of the unemployment insurance debt decreases as the probability of default increases.

It is obvious that the holder of the tax debt and the writer of the unemployment insurance are better off if the bank debt is written off. It is, however, more interesting to study how the portfolio of bank debt, tax debt and unemployment insurance behaves.

The final column in table 6 shows the change in the market value of the portfolio consisting of bank debt, tax debt and unemployment insurance. In most of the cases the holder of the portfolio is worse off because of the write-off. Note, however, that there are cases where it would be optimal to exercise the relief option. The drop in the market value of the bank debt is more than set-off by the increase in the value of junior debt and the decrease in the value of the unemployment insurance.

The outcome from the write-off is conditional on the collateral values. With sufficiently high values of collateral the write-off does not increase the wealth of the portfolio holder. As the collateral values increase the market values of tax debt and unemployment insurance become less sensitive to the changes in nominal bank debt. On the other hand, as the collateral value decreases the portfolio holder has incentives to give greater debt relief. The above results imply that it is worthwhile to give a nominal debt relief to the bad borrowers, but not to the good borrowers, which have sufficiently high collateral to loans ratios.

Note that our results rely on the assumption that the employees with unemployment insurance have not hedged against changes in the firm's capital structure. Thus the government can exercise an option that lowers the value of

[15] In this section we assume that the unemployment insurance and other claims are insured against further changes in the firm's capital structure. Thus we can calculate the impact of the debt relief by simply calculating the difference between the value of the claim with and without the debt relief.

the unemployment insurance.[16] With this respect our case corresponds to the analysis of Longstaff (1990), who investigates the case where bondholders have an option to extend the maturity date of the debt. Bondholders can benefit themselves by extending the maturity of the debt by expropriating the legal fees, i.e. the extension can be done in the costs of lawyers.

The above results facilitate our discussion of the different incentives that the government and the good bank can have. It is clear that if the option is given to the good bank, it does not in general have incentives to exercise it. The benefits would go to the government. On the other hand, if the government (as an owner of the bad bank) owns the bank loan, it can have incentives to give some relief, at least in certain circumstances. Taken together, the bad bank can make a difference from the borrower's point of view at least in some cases, if the government takes into account all the liabilities and assets it has.

5 Policy discussion

In this paper we have used the contingent-claim valuation approach to investigate the valuation of secured loans and to illustrate some of the aspects of the good bank/bad bank transactions. We used the hazards function to characterize the default behaviour and valued performing and performing non-performing loans, guarantees on these loans, as well as tax debt and unemployment insurance.

We can derive several policy implications. First, it is obvious that one should be very cautious about using book values in the good bank/bad banks transactions. Loan values are sensitive to collateral values, market interest rates, loan coupon rates and volatility of the collateral values. In general, book values do not reflect these.

Second, in valuation one must place special emphasis on the real estate appraisal, given that the value of the performing non-performing loan depends crucially on the collateral value and real estate prices are not easily observed from depressed real estate markets. Our results also suggest that banks should pay more attention to the collateral values as the value of the performing non-performing loans depends crucially on collateral values.

We also discussed the valuation of guarantees. The risk-sharing is recommended given that there can be moral hazard and adverse selection problems as well as great uncertainty regarding the loan quality in good bank/bad bank transactions. Risk-sharing can reduce these problems.

Finally, we demostrated that it can be beneficial to give nominal debt relief to the borrower. In our specific example, the loss of bank loan value can be more than set-off by the increase in junior debt value and the decrease in the unemployment insurance value. This implies, among other things, that the incentives of the good bank and the bad bank can differ. The moral lesson of our analysis is interesting: One shoud give debt relief to the bad borrowers, which have low collateral to loan ratios, rather than to the good borrowers, which have high collateral to loan ratios.

[16] It is, however, quite probable that the employees prefer not to exercise unemployment insurance.

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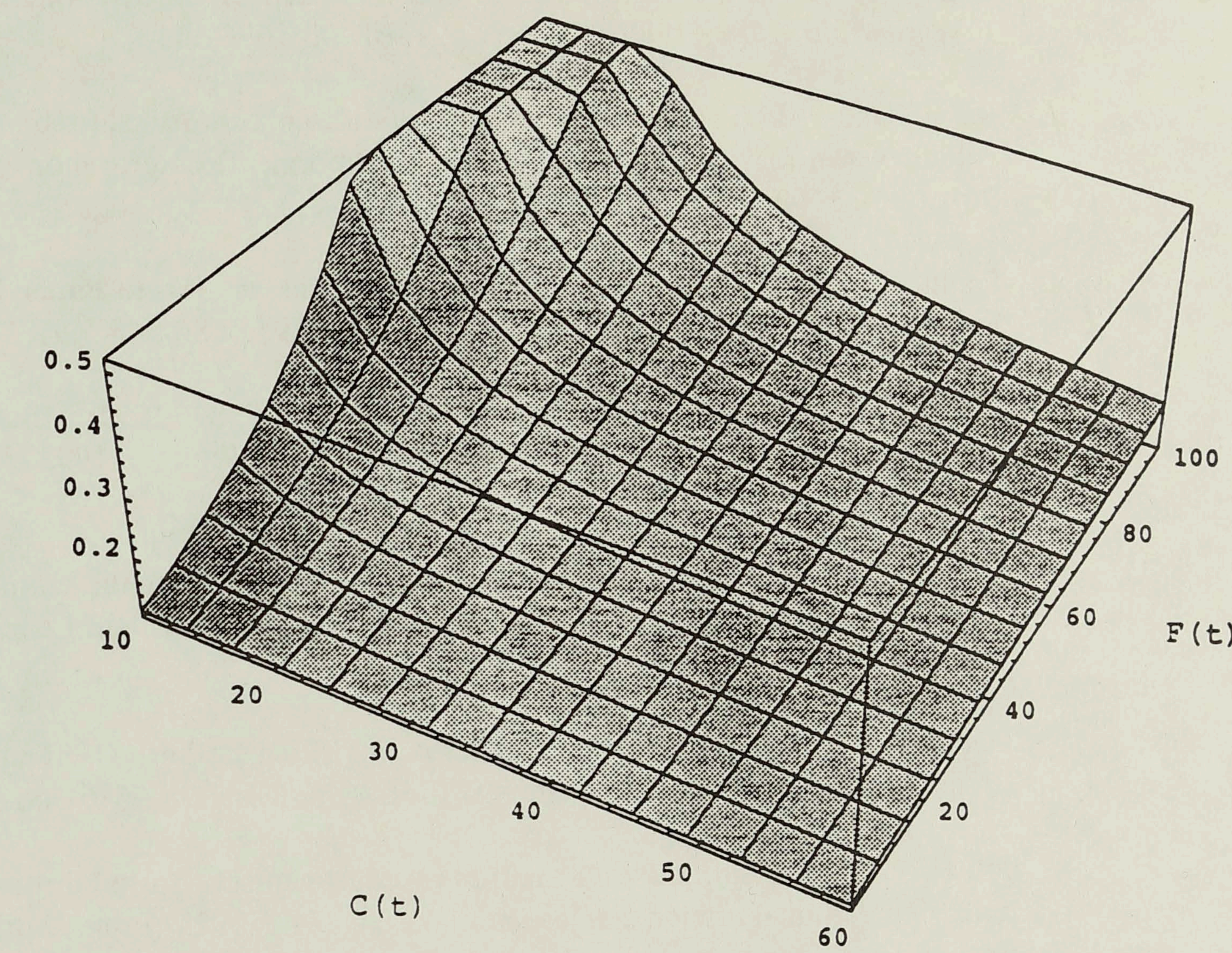
Appendix

Figure A1. **Values of the hazards function, when $\pi_0 = 0.15$ and $\beta = 0.3$**



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