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U.S. Macroeconomic Policy Evaluation in an Open Economy Context using Wavelet Decomposed Optimal Control Methods

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Abstract. It is widely recognized that the policy objectives of fiscal and monetary policymakers usually have different time horizons, and this feature may not be captured by traditional econometric techniques. In this paper, we first decompose U.S macroeconomic data using a time-frequency domain technique, namely discrete wavelet analysis. We then model the behavior of the U.S. economy over each wavelet frequency range and use our estimated parameters to construct a tracking model. To illustrate the usefulness of this approach, we simulate jointly optimal fiscal and monetary policy with different short-term targets: an inflation target, a money growth target, an interest rate target, and a real exchange rate target. The results determine the reaction in fiscal and monetary policy that is required to achieve an inflation target in a low inflation environment, and when both fiscal and monetary policy are concerned with meeting certain economic growth objectives. The combination of wavelet decomposition in an optimal control framework can also provide a new approach to macroeconomic forecasting.

Keywords: Discrete Wavelet Analysis; Fiscal Policy; Monetary Policy; Optimal Control

JEL classifications C61 . C63 . C88 . E52 . E61 . F47

1. Introduction

Macroeconomic policy in the United States is based around a general tendency for fiscal policy to be expansionary, but coupled with a largely independent monetary policy and a freely floating exchange rate policy. Also the Federal Reserve's monetary policy is widely recognized as following a modified Taylor rule, so clearly inflation and the output gap are also policy-relevant. But macroeconomic policy also operates on different time horizons, so for example government spending can be for long term projects, or for short term current expenditure purposes, and also monetary policy can also be targeted to be effective over various time horizons. The purpose of this paper then is to introduce a framework to establish a model that will operate at specific frequency bands so as to differentiate the macroeconomic behavioral and policy responses over different time horizons. To this end, we utilize a wavelet-based optimal control model that is based on an accelerator framework for the U.S. macroeconomy.

This paper is a further extension of the wavelet-based optimal control models that we have recently been using as part of a series of papers to simulate economic policy for both the U.S., euro area, and South African economies. In our first contribution, namely Crowley and Hudgins (2015), we obtained the time-frequency domain cyclical decomposition of quarterly U.S. GDP component data, and then simulated optimal fiscal policy. Next, Crowley and Hudgins (2017, 2018b) and Crowley and Hudgins (2018) expanded the wavelet-based control model to simulate jointly optimal fiscal and monetary policy within a closed economy framework.[1] The first open-economy wavelet-based control models were developed as partial accelerator models by Crowley and Hudgins (2018) and Hudgins and Crowley (2019), which analyzed various policy simulations using South African data as a developing country example. This paper extends that analysis by including money supply as a operational monetary policy variable, and utilizes data from the US, which illustrates the application of the open economy version of the model to a developed economy.

The wavelet-decomposed optimal control model can improve upon the policies derived solely from aggregate models. For example, Crowley and Hudgins (2015) found that in a recessionary period, the optimal expansionary government spending in the aggregate model was consistently above the fiscal growth target, whereas optimal government spending in the wavelet-based model was more active and had more flexibility with lower deficits than in the aggregate model. This paper aims to illustrate the usefulness of our approach in terms of economic policy formulation, as the wavelet decomposed variables allow a richer set of dynamics to play out in the simulations, which in aggregate provide better simulations of different policy frameworks.

While our approach offers considerable insight, it is not a fully calibrated large scale model, so it is more illustrative than prescriptive. Wavelet analysis is appropriate for determining cycles, but cannot provide policy forecasts. Traditional optimal control models can produce policy prescriptions in the aggregate, but cannot target cycles operational within the macroeconomic variables. Thus, the aggregate model may not capture the distribution of policy intensity over different time horizons, thus for causing for example larger fiscal deficits and more expansionary money policy than is necessary. The wavelet-based control model is in principle therefore able to utilize the

[1] This research followed Kendrick and Shoukry (2014), who developed an optimal control accelerator model without wavelet decomposition to analyze optimal fiscal policy.

benefits of the information gained from wavelet analysis to improve upon the policy prescriptions obtained from traditional aggregate optimal control models.

Section 2 examines the wavelet decomposition of the data in the time-frequency domain, over the period 1973 – 2018. Section 3 expands the methods employed by Crowley and Hudgins (2015, 2017, 2018a,b) and Crowley and Hudgins (2019) to develop an open economy time-frequency optimal tracking model that generates feedback rules for monetary and fiscal policy.

Section 4 then presents the results of simulating the model first using a baseline jointly optimal fiscal and monetary policy and then extending the exercise to consider using various emphases on achieving short-term objectives, with differing priorities given to an inflation target, a money growth target, an interest rate target, and a real exchange rate target. Section 5 concludes.

2. MODWT Wavelet Analysis

Discrete wavelet analysis is a time-frequency domain method that has the ability to extract cyclical information from time series. As in Crowley and Hudgins (2015, 2018a), the value of a variable x at time instant k, x_k, can be expressed using Mallat's pyramid algorithm and multiresolutional analysis, as

$$x_k \approx S_{J,k} + d_{J,k} + d_{J-1,k} + ... + d_{1,k} \tag{1}$$

The $d_{j,k}$ terms are wavelet detail "crystals", $j = 1,\ldots, J$; $S_{J,k}$ is a trend component, called the wavelet "smooth", and J represents the number of scales (frequency bands). There are many different wavelet filter functions that are used in discrete wavelet analysis to direct a filtering process that utilizes pairs of low pass and high pass filters, including the Biorthogonal, Coiflet, Daubechies, Discrete Meyer, Haar, and Symlet. For this paper, we utilize the asymmetric Daubechies 4-tap (D4) wavelet function, and employ the MODWT as our method of time-frequency decomposition. Using the MODWT avoids the dyadic data requirements and non-shift invariant shortcomings of the DWT (Crowley, 2007).

Wavelet analysis is already widely accepted in the physical and medical sciences, but now wavelet analysis is becoming much more widely used in economics, with numerous papers appearing in economics journals exploring a variety of existing issues in the time-frequency domain. Examples include Aguiar-Conraria and Soares (2011), Gallegati et al. (2011), Dar, Samantaraya, and Shah (2014), Tiwari et al. (2015), Chen (2016), Crowley and Hughes Hallett (2016), Verona (2016), and Crowley and Hughes Hallett (2018) and Lubik, Matthes and Verona (2019).

2.1 MODWT Wavelet Decomposition Analysis

Following Crowley and Hudgins (2015, 2018b), for the US national income data, as well as the OECD data, we apply the MODWT to the data using a two-step procedure that extracts the crystals and the smooth (trend and any residual cycles) at frequencies $j = 1, \ldots, 5$.[2] First, the wavelet decomposition was undertaken in terms of annual differences of the absolute values of each series – this is done to remove any

[2] We use a Daubechies 4-tap wavelet for MODWT, with periodic boundary conditions.

seasonal variation in the underlying series[3]. Once the crystals are obtained, they are then summed sequentially to create level equivalents.[4] To ensure consistency in terms of the decomposition for each series, a residual was calculated to create a *modified smooth* (*S*), so that the sum of the level equivalent crystals and the *modified smooth* equals the actual observation.[5]

Table 1

The time intervals associated with each of the frequencies

J	*Time interval in quarters*	*Time interval in years*
1	2 to 4 quarters	6 months to 1 year
2	4 – 8 quarters	1 – 2 years
3	8 – 16 quarters	2 – 4 years
4	16 – 32 quarters	4 – 8 years
5	32 – 64 quarters	8 – 16 years

Table 1 defines the time-frequency ranges for all of the wavelet decompositions. U.S. nominal and real interest rates, the foreign (G6=G7 minus U.S.) GDP weighted nominal interest rate, and U.S. inflation rate[6] are plotted in Figure 1.

[3] This is in contrast to Lubick, Matthes and Verona (2019), who use quarter on quarter changes in U.S. real GDP. In addition to the seasonal adjustment issue, there is also the issue of the variance decomposition of the series and the results are somewhat different too in terms of much greater emphasis on high frequency cyclical movement than we see by using the annual change data.

[4] The two-stage procedure ensures that business and growth cycles are properly identified, as these cycles are most apparent in the first-differenced series. This mirrors conventional macroeconomic practice, where real GDP growth is identified either by log differencing or first differencing the original level time series. If discrete wavelet analysis is applied to the level series, volatility in the cycles is not fully identified, as the trend tends to dominate the cyclical composition of the series.

[5] The *modified smooth* is calculated as a residual. First, the first difference wavelet crystals are summed sequentially, observation by observation, to convert to level equivalent cycles. Then, these separate level equivalent cycles are then summed and the residual (or *modified smooth*) is calculated as the difference between the actual series and the sum of these level equivalent cycles. This ensures that the level equivalent cycles and the modified smooth sums to the original series (by construction).

[6] G6 interest rates are sourced from the OECD and US rates are sourced from the Federal Reserve. The G6 rates use real GDP in US$ weights sourced from either the IMF or OECD.

Figure 1

Short term US and G5 Interest Rates and US Inflation

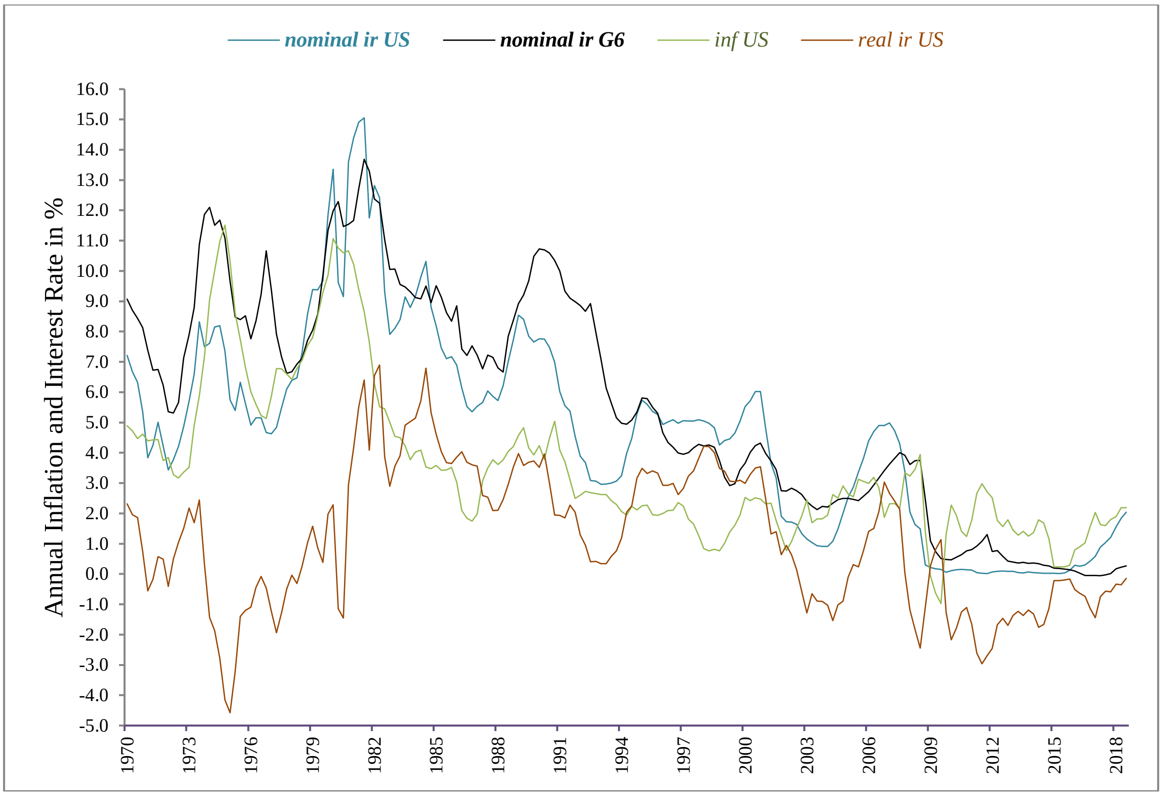


Figure 2 panels (a) – (f) plot the wavelet decompositions for the U.S. GDP component data. The U.S. quarterly national income data are chain-weighted, seasonally adjusted and in 2012 prices. This data is sourced from the BEA database.[7] Figure 3 panels (a) – (c) plot the wavelet decompositions for the US interest rate, the foreign interest rate, and the real exchange rate (*RER*)[8].

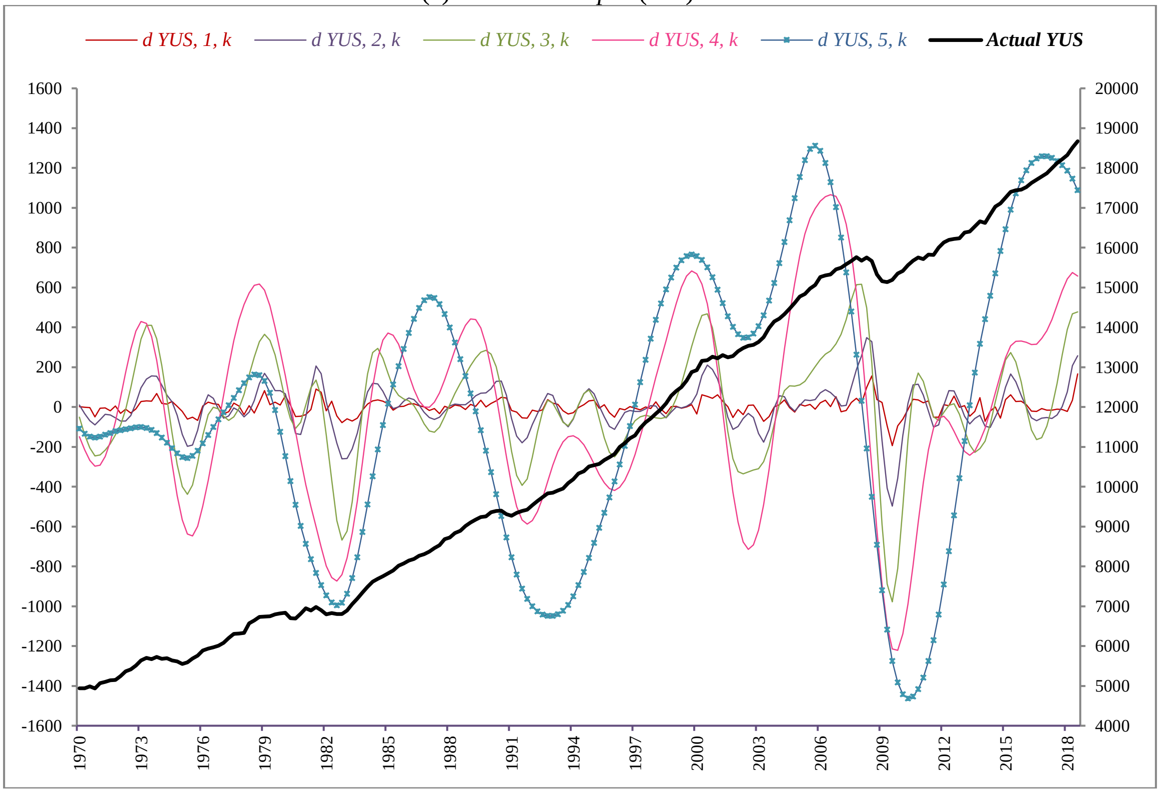
[7] See http://www.bea.gov

[8] The RER was sourced from FRED and is based on wholesale prices.

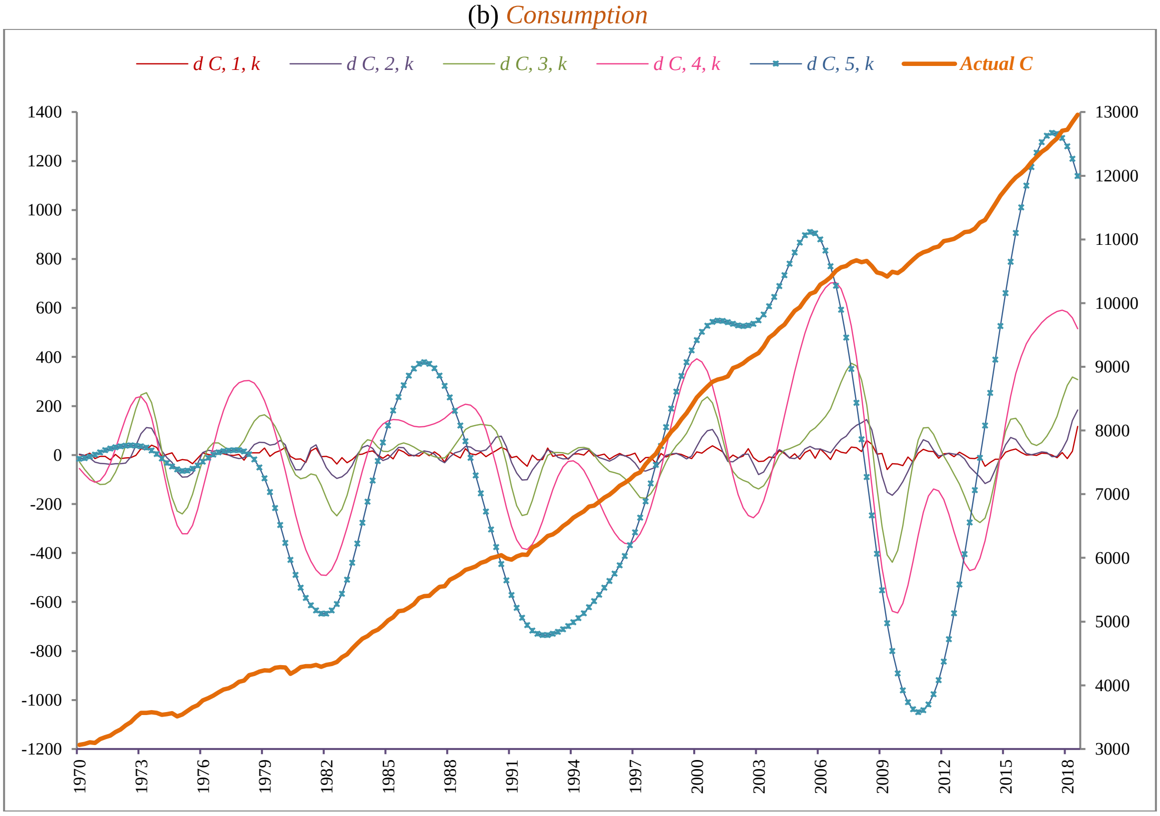
Figure 2

United States Wavelet Decomposition for GDP Component Data:

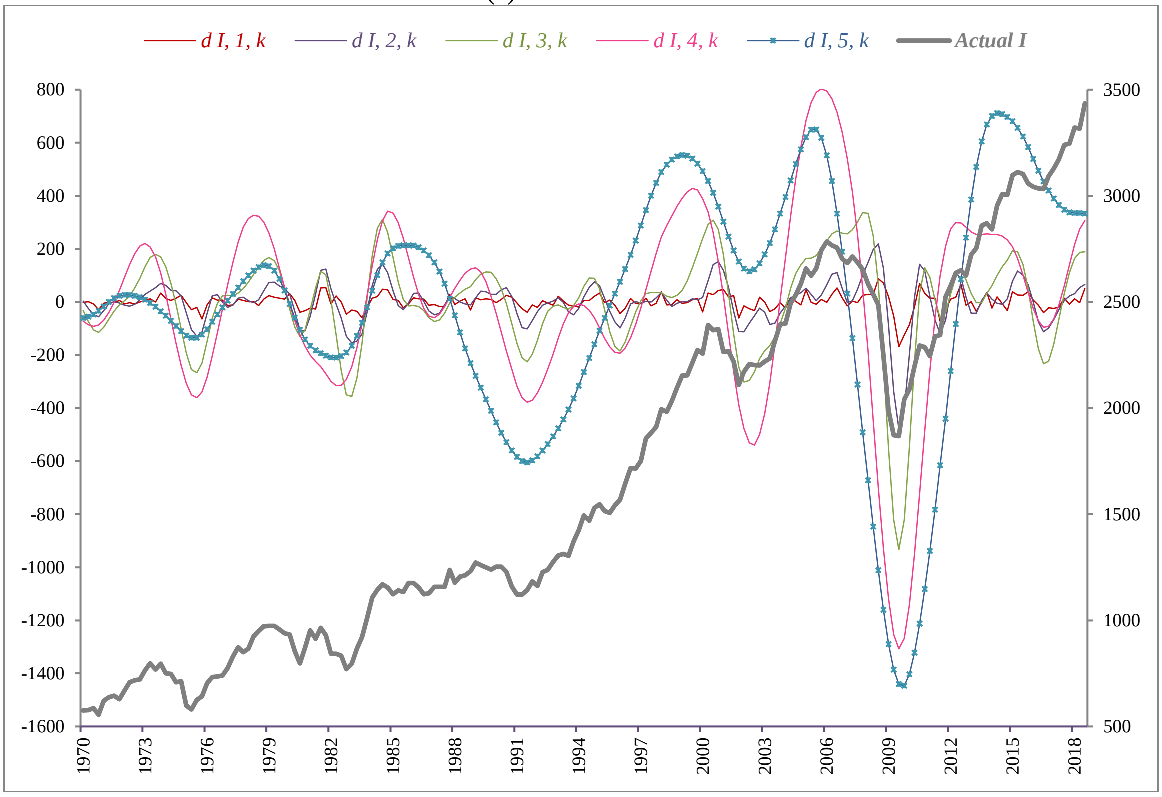
(a) *National Output* (Y^{US})



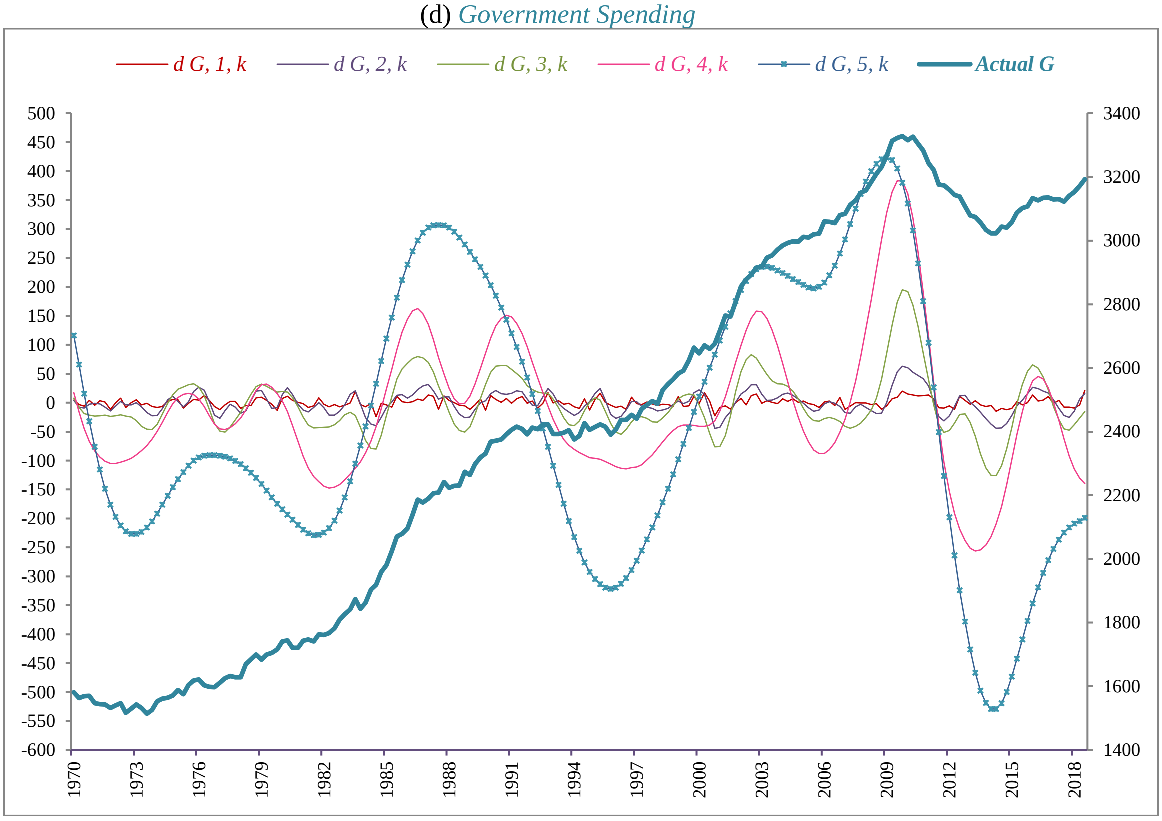
(b) *Consumption*



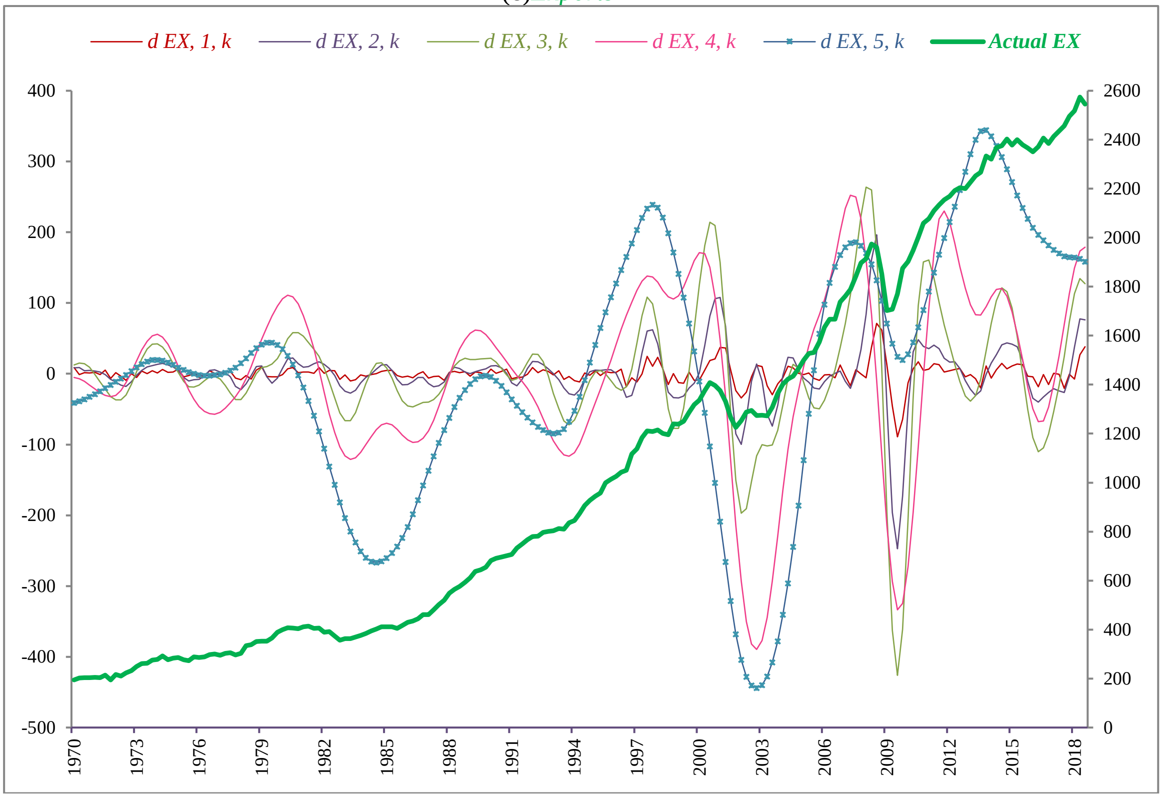
(c) *Investment*



(d) *Government Spending*



(e)*Exports*



(f) *Imports* (***IM***)

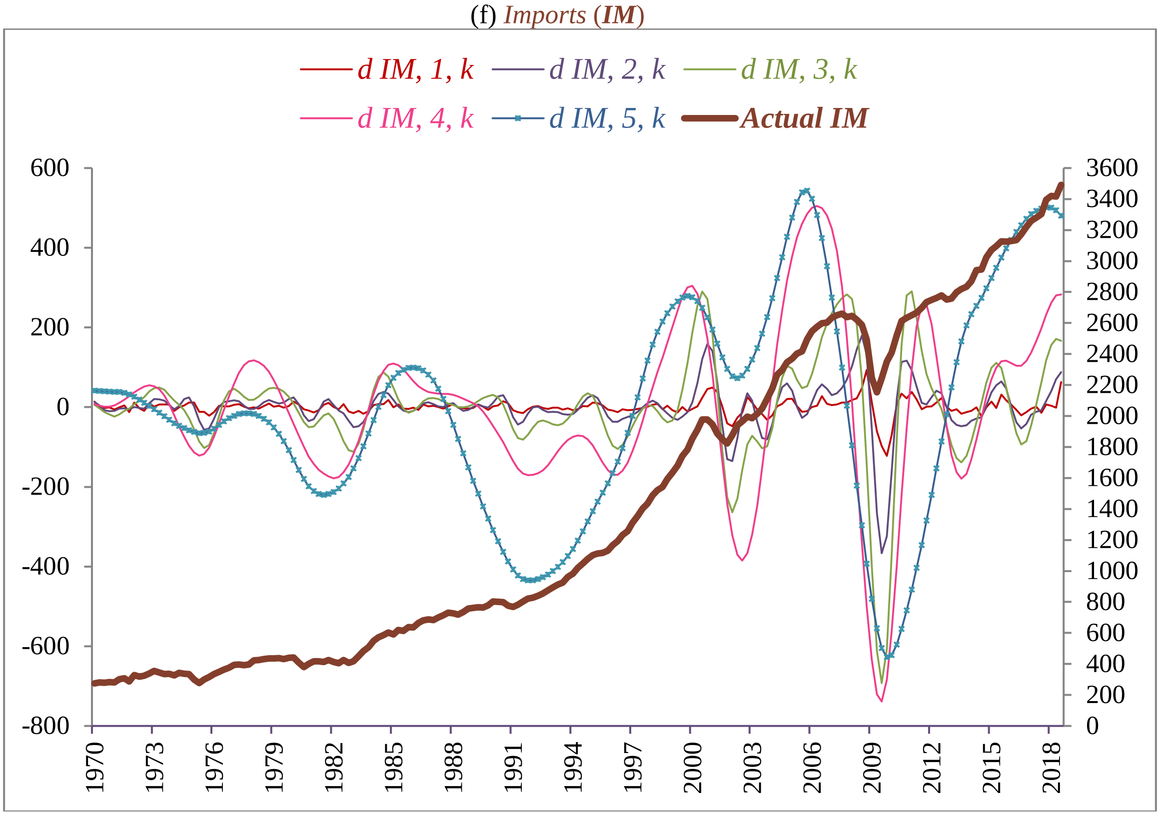
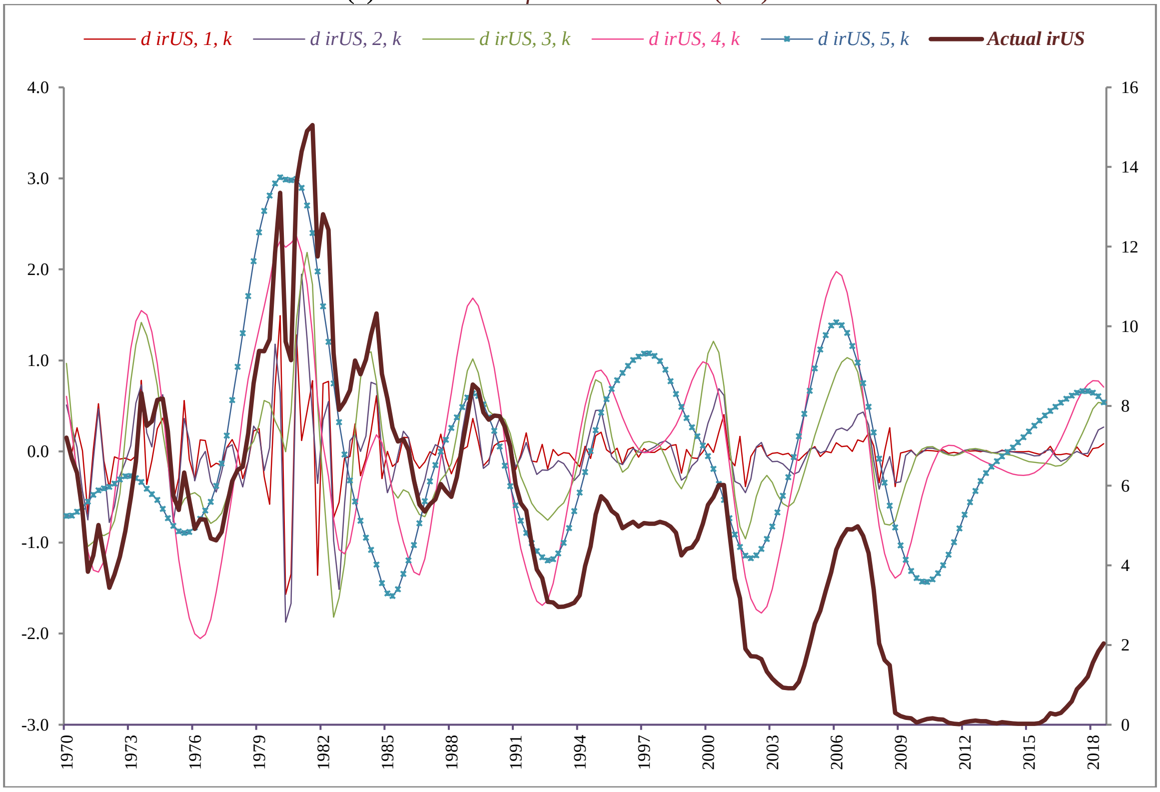


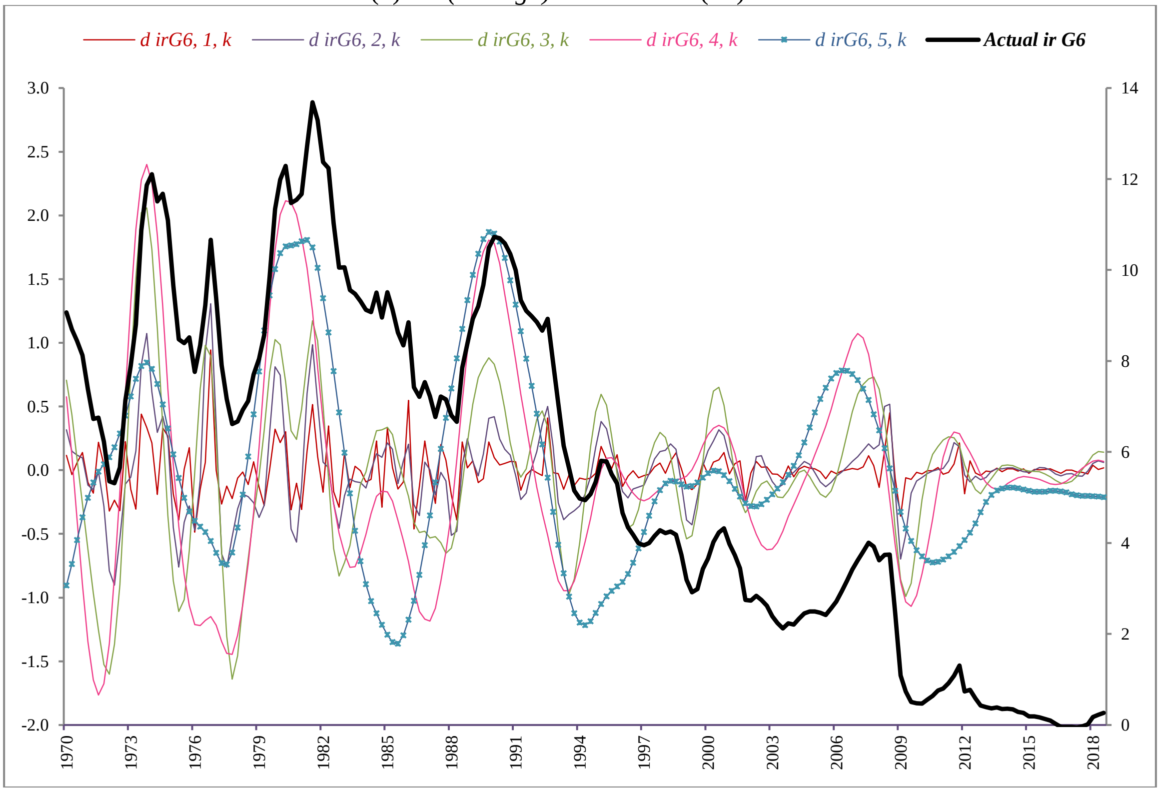
Figure 3

Wavelet Decomposition for Financial Data:

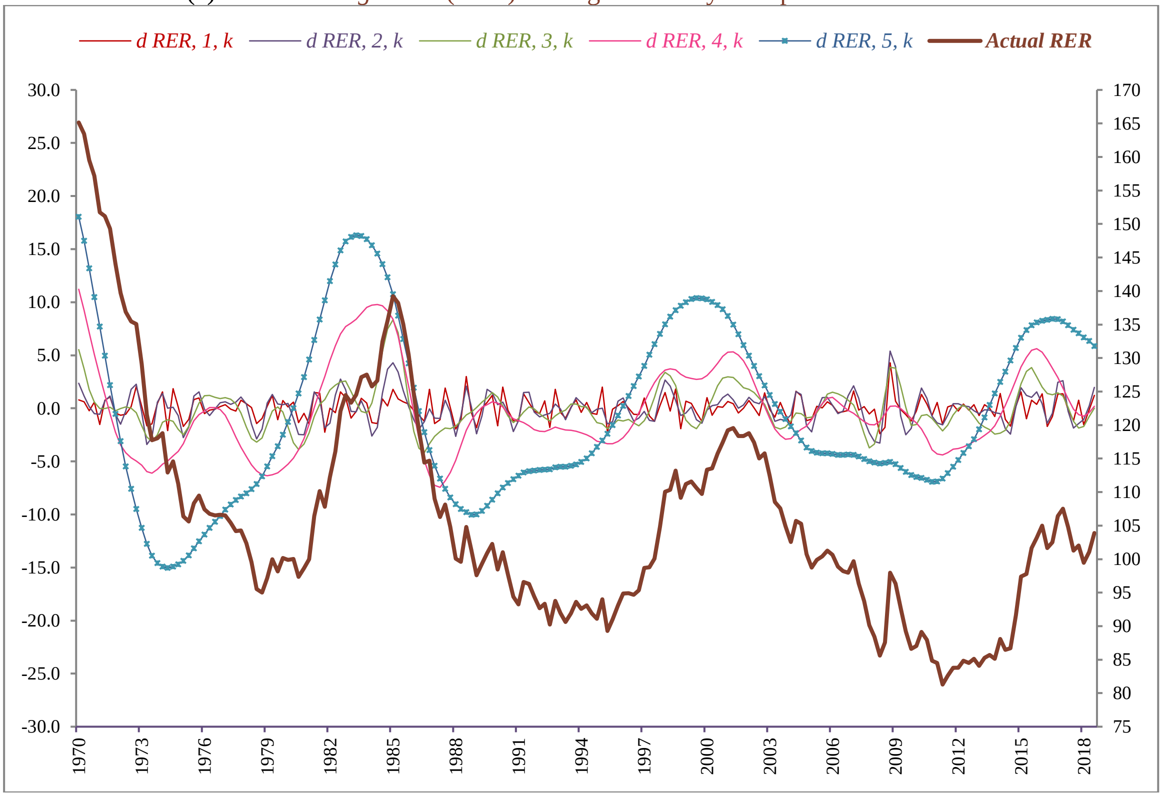
(a) *Interest Rate for United States* (ir^{US})



(b) *G6 (Foreign)* Interest Rate ($\boldsymbol{ir}^{f}$)



(c) *Real Exchange Rate* (***RER***): Foreign Currency Unit per US Dollar



In Figure 2(a) cyclical activity is clearly evident in U.S. GDP growth, with most of the activity located in the lower frequency cycles. It is interesting to note that any sudden large downturns manifest themselves across frequencies with all cycles eventually turning down, but with short cycles responding first. It is also interesting to note that since the last downturn the lowest frequency cycle appears to have peaked, although other cycles so far have not. This suggests an overall slowing in economic growth in the U.S., with the *d4* crystal also just peaking even though higher frequency growth cycles are still in an expansionary phase. In figure 2b) real consumption follows a similar pattern to real GDP except that the downturn in longer cycles in recent quarters is now more pronounced. As expected due to smoothing, consumption also appears to exhibit more regular cyclical activity. Figure 2c) shows that real investment is dominated by the downturn during the great recession, with cycles at all frequencies exhibiting a deep downward trough, and this is particularly apparent in the lower frequency cycles. Of course these cyclical downturns in investment are most pronounced during recessions, but what is surprising is that in 2016 wavelet analysis shows that there was a significant downturn in investment, but only at higher frequencies (- this is related to oil-price-induced slump in investment in the onshore oil and gas industry). Figure 2d) shows real U.S. government expenditures over the span of over nearly 50 years. The counter-cyclical nature of the rise in expenditures throughout the period is clear, but there are also some notable features of this decomposition: firstly, there is obviously some "blurring" between wavelet frequency ranges as can clearly be seen in the 1980s at medium cycle frequencies such that the fluctuations are not captured by a single wavelet, but across two wavelets (*d3* and *d4*); and second, between 2009 and 2014, real governnment expenditure fell sharply, which is reflected in the longer cycles, rather than short term cycles. This fall in government spending at longer cycles in likely due to the fall in military expenditures relating to the Iraq and Afghanistan wars. The fluctuations in real exports in Figure 2e) have once again been mostly been contained in lower frequency cycles, but interestingly there have been 3 periods where real exports fell – between 2000 and 2004, in 2008-09 and a slight fall in 2016. The wavelet decomposition picks up these sudden decreases at every frequency except for the last downturn when the lowest frequency wavelet did not respond. Lastly in Figure 2f) for imports, the downturn during the great recession is the most prominent dip in the series, but there are also quite large fluctuations in the lower frequency cycles throughout the series.

In figure 3(a), U.S. 3-month interest rates tend to be more volatile at somewhat shorter frequencies than GDP component variables, with the *d4* crystal picking up more of the cyclical features of the series, but clearly there is still a large amount of the variation in rates that is tied to lower business cycle frequencies. It is noteworthy that the most recent datapoints imply a halt in the rise in interest rates and that has led to the wavelet crystals indicating a peak in the tightening cycle, particularly at the lower frequencies. The GDP weighted G6 3-month interest rates is shown in figure 3b), and this has similar cyclical characteristics, except that the recent uptick in rates seen in the U.S. is hardly noticeable for the G6 rates. In Figure 3(c) the U.S. RER has only one prominent low frequency cycle, and the rest of the cyclical activity is concentrated in smaller higher frequency fluctuations. Despite the recent RER appreciation of the U.S. dollar, it is noteworthy that overall over the last 50 years the RER exhibits a depreciating trend.

3. Macroeconomic Model Derivation and Estimation

We next proceed to construct a frequency-range defined macroeconomic model based on the wavelet decomposition above. The GDP components of domestic output (Y) are nested in the following blocks: personal consumption (C_j); private domestic investment (I_j); government expenditure (G_j); exports (EX_j); and imports (IM_j). At each frequency range, the wavelet-based GDP components remove the effects at all other four frequency ranges, so that each component only includes the crystal (d) at that frequency range and the modified smooth base-level trend (S). The wavelet-based components for any variable are therefore defined in equation (2) as follows:

$$X_{j,k} = d_{X,j,k} + S_{X,j,k} \qquad j = 1, \ldots, 5; \quad k = 1, \ldots, K \tag{2}$$

Equation (2) provides for a cyclical analysis of the level series over different frequency ranges by allowing economic cycle fluctuations to be superimposed onto the wavelet smooth. Thus, the level series can be analyzed by incorporating the separate cycles inherent within the pre-determined frequency ranges from discrete wavelet analysis.

The reduced form model in equations (3) through (8) expands the linear accelerator reduced-form block component matrix system of Crowley and Hudgins (2018a,b) and Hudgins and Crowley (2019) for each frequency range, j = 1, …, 5, as defined in table 1, where the $\beta_{j;0}$ coefficients are constants and the number of lags for any given variable is denoted by $L_{(.)}$. The ir^{US}_j and ir^f_j blocks represent the wavelet decomposition of the short-term domestic (US) and foreign (G6) interest rates, respectively. Block RER_j is the wavelet decomposed real exchange rate (index of foreign currency unit per US dollar), and the $\omega_{(.),j}$ terms represent blocks of random disturbance errors. Equation (3) specifies the consumption block as linearized functions of lag structures of consumption, government spending, and the real exchange rate. Equation (5) specifies the investment block as linearized functions of the domestic GDP and the domestic interest rate.

$$C_{j,k} = \beta_{C,j,0} + f_{C,j}(C_{j,k-1},\dots, C_{j,k-L_C}, G_{j,k-1},\dots, G_{j,k-L_G}, RER_{j,k-1},\dots, RER_{j,k-L_{RER}}) + \omega_{C,j,k-1} \tag{3}$$

$$I_{j,k} = \beta_{I\,j,0} + f_{I,j}(Y^{US}{}_{j,k-1},\dots, Y^{US}{}_{j,k-L_C}, ir^{US}{}_{j,k-1},\dots, ir^{US}{}_{j,k-L_{irUS}}) + \omega_{I,j,k-1} \tag{4}$$

$$G_{j,k} = \rho_j G_{j,k-1} + \omega_{G,j,k-1} \tag{5}$$

Equation (5) extracts the current trend in the government spending block (G_j). Future government spending will be determined by an optimal control system in section 4, which simulates the optimal policy forecasts during the period starting in 2018 quarter 3. The estimated government spending autocorrelation coefficients (ρ_j) are all about 1.0034 at all frequency ranges.

Equation (6) gives the net export equation block, where net exports are a function of the lag structures of net exports, domestic GDP (Y^{US}), foreign GDP (Y^f), and the real exchange rate at each frequency range.[9]

$$NX_{j,k} = \beta_{NX,j,0} + f_{NX,j}(NX_{j,k-1},\dots, NX_{j,k-L_{NX}}, Y^{US}{}_{j,k-1},\dots, Y^{US}{}_{j,k-L_{Y^{US}}}, Y^f{}_{j,k-1},\dots, Y^f{}_{j,k-L_{Y^f}}, RER_{j,k-1},\dots, RER_{j,k-L_{RER}}) + \omega_{NX,j,k-1} \tag{6}$$

The real exchange rate is given by equation (7), where the real exchange rate is determined by the lagged structures the domestic interest rate, the foreign interest rate, and the real exchange rate.[10] Equation (7) captures interest rate parity influences from domestic and foreign interest rates.

$$RER_{j,k} = \beta_{RER,j,0} + f_j^{(5)}(ir^{US}{}_{j,k-1},\dots, ir^{US}{}_{j,k-L_{irUS}}, ir^f{}_{j,k-1},\dots, ir^f{}_{j,k-L_{irf}}, RER_{j,k-1},\dots, RER_{j,k-L_{RER}}) + \omega_{RER,j,k-1} \tag{7}$$

Tables A1 – A5 show the OLS regression coefficient estimates for with *p-values* (in parentheses) for the MODWT decomposition accelerator system in equations (3), (4), (6), (7), respectively, using the data for the post-Bretton Woods period 1973 quarter 3 – 2018 quarter 2. For all of the estimated consumption equations in table A1, R^2 is 0.99, suggesting a good fit over each frequency range. The government spending

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[9] Crowley and Hudgins (2018a) utilize separate equations for exports and imports in South Africa. We have also used different specifications that estimate exports and imports separately for the U.S.. Nevertheless, the net export specifications obtained here fits much better than did equations that modeled exports and imports separately.

[10] Uncovered interest parity generally includes a default risk term. The country risk for financial assets in the US, however, is in relative terms one of the lowest in the world, hence the risk term is omitted. The authors have tested some measures of risk as an explanatory variable, and did not find a good fit with any of the risk-proxy variables.

coefficients ($\beta_{C,j,2}$) all have the expected positive signs, and are statistically significant. The one lag real exchange rate coefficients ($\beta_{C;j,4}$) are positive at the three highest frequencies, and negative at the lowest two frequencies, whereas the two lag real exchange rate coefficients ($\beta_{C;j,5}$) all have the opposite signs. The dominant exchange rate effect is positive at all frequency ranges suggesting that real exchange rate appreciation has a positive effect on consumption.

The estimated investment equations are given in Table A2, where the R^2 values are all greater than 0.64. The national output coefficients ($\beta_{I,\,j,\,1}$) are positive for all frequency ranges, suggesting a crowding-in effect of income on investment. The interest rate coefficients ($\beta_{I,\,j,\,2}$) all have the expected negative signs at all frequencies, which means that higher interest rate increases have a negative effect on investment.

Table A3 shows the estimates for the net export equations. As expected, net exports vary negatively with domestic output, and vary positively with foreign output at all frequency ranges. Net exports decrease at all frequencies when the *RER* appreciates, since the net sum of the lagged *RER* coefficients is always negative.

Table A4 estimates determines the real exchange rate cycle at each frequency range based on the influence of interest rate parity. The positive domestic interest rate coefficients ($\beta_{RER,\,j,\,1}$) at each frequency range give the expected exchange rate appreciation as the domestic interest rate increases. The negative coefficients ($\beta_{RER,\,j,\,2}$) on the foreign interest rate imply exchange rate depreciation as the foreign interest rate increases.

Equation (8) models the modified smooth trend processes for national output, consumption, investment, government spending, net exports, the interest rate, and the real exchange rate as first-order difference equations, where the ω_S terms represent random disturbances in each equation, thereby satisfying the standard assumptions.

$$S_k = s_1 S_{k-1} + s_2 X_{k-1} + \omega_{S,k-1} \tag{8}$$

In equation (8), the coefficients on the lagged modified smooth trend variables (S), and the coefficients on the lagged component variables (X), produce a weighted average growth contribution toward the current trend values of each component series.

Table A5 gives the estimates for the coefficients of the modified smooth trends (with *p-values* in parentheses) for all of the series, as specified in equation (8). Summing the two coefficients in each of the equations produces a weighted average trend growth rate. In the GDP trend series equation, the coefficient on the lagged smooth value of the series is $s_{Y,1} = 0.9822$, which is much larger than coefficient on the lagged value of aggregate output, given by $s_{Y,2} = 0.0221$. This pattern holds for all of the other modified smooth trend series, where the coefficients on the lagged value of each trend series exceeds 0.87, while the coefficients on the lagged aggregate variable of the series are less than 0.12. All four equations achieve a good fit, with statistically significant coefficients and $R^2 > 0.98$ in each equation.

Following Crowley and Hudgins (2015, 2018b), the current national debt level influences consumption and investment through changes in expected national output, due to rational expectations. Define the following variables:

$DEBT_k$	=	the total stock of government debt in quarter k
$\hat{G}^{d}_{j,k}$	=	the trend government obligations at frequency range j in quarter k
$G^{e}_{j,k}$	=	expected contribution of government spending to national output

Equation (9) defines the trend process for government spending at each frequency range, where the current trend value depends on the lagged value of the actual level of government purchases, where ρ_j is the growth coefficient, estimated by equation (5).

$$\hat{G}^{\,d}_{j,k} = \rho_j \, G_{j,k-1} + \omega_{G,\,j,\,k-1} \qquad j = 1, \ldots, 5 \tag{9}$$

In equation (10), the expected value of government spending in any period k is determined based on a weighted average of the lagged actual spending and the lagged trend value within the frequency range.

$$G^{e}_{j,k} = \phi_{j,k}\left[G_{j,k-1} - \pi_{j,k}(DEBT_{k-1} - DEBT_0)\right] + (1-\phi_{j,k})\hat{G}^{\,d}_{j,k-1} \tag{10}$$
$$0 < \phi_j < 1; \; j = 1,\ldots,5$$

Government spending only affects the economy through expected national output, so that all government spending changes have a limited impact. The effectiveness of fiscal policy at any given frequency range increases with the value of ϕ. This formulation permits rational expectations behavior. Any new fiscal initiative pulses the current cycle at each frequency range, but the current contribution of government spending toward national output production is crowded-out by any national debt stock that exceeds its initial value.

We substitute the variable G^e from equations (9) and (10) into equation (3) so that it replaces G, and then augment the system with the government debt and government trend spending to obtain the reduced form equation for determining consumption at each frequency. Based on the estimates in Tables A1 and the rational expectation parameters, the consumption equation is given as follows:

$$\begin{aligned} C_{j;k} = {} & \delta_{j,0} + \delta_{j,1} C_{j,k-1} + \delta_{j,2} G_{j,k-1} + \delta_{j,3} \hat{G}^{\,d}_{j,k} + \delta_{j,4} C_{j,k-2} \\ & + \delta_{j,5} RER_{j,k-1} + \delta_{j,6} RER_{j,k-2} + \delta_{j,7} DEBT_{k-1} + \omega_{C,\,j,\,k-1} \end{aligned} \tag{11}$$

We similarly derive the linearized investment, export, import, and interest parity equations by assuming that the current coefficients at each frequency range are determined by combining rational expectations parameters that give some weight to the expected value of the variables (x^e) into equations (4) – (7).

Equation (12) expresses an expanded and modified Phillip's curve type of accelerator equation that determines inflation (*inf*).

$$\begin{aligned} inf_k = {} & \beta_{inf,0} + \beta_{inf,1}\, inf_{j,k-1} + \beta_{inf,2}\,(Y_{k-1} - Y^*_{k-1}) + \beta_{inf,3}\, RER_{k-1} \\ & + \beta_{inf,4}\, MS_{k-1} + \beta_{inf,5}\, inf_{k-2} + \omega_{inf,\,k-1} \end{aligned} \tag{12}$$

Inflation is influenced by the lagged inflation, the national output gap, monetary policy, and the *RER*. Relatively expansionary monetary policy results in increased money growth that puts upward pressure on prices ($\beta_{inf,4} > 0$), and also lower domestic interest rates that cause investment and aggregate demand to increase relative to the trend level, thus exerting upward pressure on inflation ($\beta_{inf,2} > 0$), a lower (depreciated) *RER*, and generally stimulates aggregate demand and inflation through a larger trade balance and exchange rate pass-through ($\beta_{inf,3} < 0$). The Fed monetary policy specifies that the US short-run target inflation rate is 2%.[11]

Since the Fed focuses primarily on setting an interest rate target, the money supply is specified so that it adjusts to accommodate the interest target and economic cycle. Equation (13) determines the real money growth as it adjusts to the real interest rate, the output gap, and 1 and 2 lags of the real money growth.

$$\begin{aligned} MS_k - inf_k = {} & \beta_{MS,0} + \beta_{MS,1}(ir^{US}_{k-1} - inf_{k-1}) + \beta_{MS,2}(Y_{k-1} - Y^*_{k-1}) \\ & + \beta_{MS,3}(MS_{k-1} - inf_{k-1}) + \beta_{MS,4}(MS_{k-2} - inf_{k-2}) \\ & + \omega_{inf,k-1} \end{aligned} \tag{13}$$

In equation (13), equilibrium is maintained in the money and asset markets, where the coefficient estimates for quarterly real money growth (as measured by the difference between M2 money growth and inflation) are provided in appendix table A8. When the central bank decides to increase the real interest rate, it achieves this by maintaining a tighter monetary stance that leads to a decrease in growth of the real money stock, so that $\beta_{MS,1} < 0$. When real output increases relative to its trend, the demand for real money balances also increases. Thus, in order to maintain a constant interest rate, the money stock in circulation must also increase to accommodate the output increase, thus yielding $\beta_{MS,2} > 0$. Equation (13) also assumes that real money growth is persistent by making it a function of its previous two lagged values.

The model is closed by equations (14) through (17). Equation (14) contains the national income identity.

$$Y_k = C_k + I_k + G_k + NX_k \tag{14}$$

Equation (15) defines net taxes (T_k), as the total government tax and income minus total government transfer payments in quarter k, which to be generated as a constant percentage (τ) of national output.

$$T_k = \tau Y_k \tag{15}$$

Following Kendrick and Shoukry (2014) and Crowley and Hudgins (2018b), we limit the active fiscal policy to government spending at each frequency range, and compute government tax income and transfer payments as passively determined variables. This is consistent with Kliem and Kriwoluzky (2014), where there is limited empirical evidence for the typical simple fiscal policy rules when tax rates respond to output that is derived in Dynamic Stochastic General Equilibrium (DSGE) models.

Equation (16) calculates the resulting government budget deficit (or surplus, when the value is negative) in quarter k, which is given by DEF_k :

$$DEF_k = G_k - T_k \tag{16}$$

$$DEBT_k = 0.25\,DEF_k + (1 + i_k)\,DEBT_{k-1} \tag{17}$$

[11] The FOMC noted in its statement that the Committee judges that inflation at the rate of 2 percent (as measured by the annual change in the price index for personal consumption expenditures, or PCE) is most consistent over the longer run with the Federal Reserve's statutory mandate." Dec 19, 2018, Federal Reserve.

The national debt ($DEBT_k$) in equation (17) is the sum of the current budget deficit (converted from annualized rates to quarterly levels) and the previous period debt stock, which grows at the quarterly interest rate of i_k.

This model in equations (3) through (17) can be specified with either constant coefficients, as in this paper, or with time varying coefficients. The model derives from the widely accepted macroeconomic accelerator framework that has been employed by Kendrick (1981), Kendrick and Shoukry (2014), Crowley and Hudgins (2015, 2016, 2018a,b). It includes money growth, the interest rate, and the real exchange rate as outlets for the transmission of monetary policy, and also adjusts fiscal policy effectiveness and the national income components for rational expectations based on current variable expectations and the current government debt stock.

This research is meant to be exploratory, and its purpose is to illustrate how to employ optimal open economy monetary and fiscal and policy through tracking control in the time-frequency domain, so as to evaluate different policy objectives under selected specific scenarios. Our MODWT wavelet-based accelerator framework can be employed to generate deterministic, stochastic, and robust optimal feedback control designs, as in Hudgins and Crowley (2018), and thus offers considerable insight.

4. Optimal Tracking Control

The objective of the LQ tracking problem is as follows. Given the linear state equations given by (3) – (17), the fiscal policymakers choose the level of government spending at each of the five frequency ranges, while the monetary authorities choose the short-term interest rate in order to minimize the expected value of a quadratic performance index consisting of the weighted tracking errors for the variables of the model. This will determine the optimal simulated forecast paths for money growth, inflation, the *RER*, the GDP components, and all the other macroeconomic variables in the model.

The linearized equations given by equations (3) – (17) can be combined, augmented with target variables, and rearranged in order to obtain the 137-equation matrix state-space equation system given by (18).

$$x_{k+1} = A_k x_k + B_k u_k + D_k \omega_k \quad ; \qquad x(1) = \boldsymbol{x}_1 \tag{18}$$

$dim\, x = (137, 1)$ $\quad dim\, u = (10, 1)$ $\quad dim\, \omega = (38, 1)$
$dim\, A = (137, 137)$ $\quad dim\, B = (137, 10)$ $\quad dim\, D = (137, 38)$

Equation (18) contains a state vector (x) that embeds the reduced form equation constants, wavelet decomposed variables, aggregate variables, and target variables. It also contains the control vector (u) and the disturbance vector (ω) that includes all of the disturbance terms.

Define the (*) as the target for any given variable. In the model, policymakers select the optimal targets for the state and control variables that grow at distinct quarterly target rates of $g(.)$, which results in annual growth rates of $\{[1 + g(.)]^4 - 1\}$ per year. The growth rate is $g(.) = 0$ for variables with constant targets. Equation (19) defines target variable equation for quarterly growth for each of these respective series.

$$x^*_{k+1} = [1 + g(.)_k] x^*_k \tag{19}$$

Let the superscript (^{T}) represent the matrix transpose. The objective is to minimize the quadratic tracking index in expression (20), given equations (18) and (19).

$$\min_u E[J(u)] = (x_{K+1} - x^*_{K+1})^T Q_f (x_{K+1} - x^*_{K+1}) \tag{20}$$

$$+ \sum_{k=1}^{K} \left[(x_k - x^*_k)^T Q_k (x_k - x^*_k) + (u_k - u^*_k)^T R_k (u_k - u^*_k) \right]$$

The index given by (20) provides three terms that penalize the policymakers for the tracking errors in the final state vector, the state vector in each period, and the control vector, respectively. The non-negative definite matrices Q_f and Q_k are the penalty weighting matrices for the final period state tracking errors, and current state tracking errors, respectively. The positive definite matrix R_k is the penalty weighting matrix for the control variable tracking errors. As in Crowley and Hudgins (2015, 2017), the LQ tracking problem can be transformed into a LQ regulator problem by redefining the state and control vectors so that they embed the tracking errors, thus leading to penalty weighting matrices that have the following sizes:

$dim\ Q_f = (137, 137)$ $\quad$ $dim\ Q_k = (137, 137)$ $\quad$ $dim\ R_k = (10, 10)$

One additional aspect of the system that aids in policy assessment and implementation is that the model penalizes policymakers for large changes between policy variables between periods, as in Crowley and Hudgins (2018a,b). The government is penalized for large changes in spending between periods, which reflects the reality of traditional incremental budgeting, rather than zero-based budgeting. Additionally, the model penalizes the central bank for large changes in the interest rates between periods (i.e., interest rate instability). Incorporating these instability penalties for policymakers increases the size of the state-space model, since the state vector must be augmented to include the lags and lagged differences of government expenditure and interest rates.

To complete the transformation from the LQ tracking problem into the LQ regulator problem, the control vector elements are defined as the tracking errors between the actual and targeted level of the fiscal and monetary variables at each frequency:

$$u_k = \left[u_{1,k} \ ; \ u_{2,k} \ ; \ \dots \ ; \ u_{10,k} \right]^T \tag{21}$$

$$u_{m,k} = G_{m,k} - G^*_{m,k} \text{ for } m = 1, \dots, 5;$$

$$u_{m,k} = ir^{US}_{m-5,k} - ir^*_{m-5,k} \text{ ; } m = 6, \dots, 10$$

The ten control variables ($u_{m,k}$) contain the subtracted targeted levels of government spending and the interest rate at each frequency. These target variables must therefore be added back to state equations for consumption, investment, exports, imports, and the *RER* in the state-space specification. The state vector also adds the target level of government spending and the interest rate to $u_{m,k}$ over each frequency range, thus retrieving the component values for the simulations.

Given the definition in expression (21), the index in (20) can be rewritten so that the objective is to minimize the following performance index:

$$\min_{u} E[J(u)] = x_{K+1}^{T} Q_f x_{K+1} + \sum_{k=1}^{K} \left[x_k^T Q_k x_k + u_k^T R_k u_k \right] \tag{22}$$

This analysis only considers the deterministic LQ-regulator problem, which sets the disturbance vector to be the null vector (ω_k = 0), or alternatively, defines the disturbance coefficient matrix to be zero (D_k = 0). The solution is computed by solving the recursive equations (23) and (24) offline in retrograde time, as in Crowley and Hudgins (2017).

$$F_k = \left[B_k^T P_{k+1} B_k + R_k \right]^{-1} B_k^T P_{k+1} A_k \tag{23}$$

$$P_k = Q_k + A_k^T P_{k+1} \left[A_k - B_k F_k \right]; \qquad \boldsymbol{P_{k+1} = Q_f} \tag{24}$$

Utilizing the matrices from the recursive Riccati[12] equations (23) and (24), equation (25) generates the unique optimal closed-loop feedback control policy in forward time.

$$u_k^{Optimal} = -F_k x_k \tag{25}$$

When the disturbance terms in equation (18) fluctuate, then the model can be simulated as a stochastic linear-quadratic Gaussian (LQG) design, as in Chow (1975), Kendrick (1981), and Kendrick and Shoukry (2014), or as a worst-case robust design as in Hudgins and Na (2016), or as a mixed robust /stochastic LQG design (Hudgins and Crowley, 2018).

4. Simulation Analysis

The simulations consider a 4-year (16-quarter) planning horizon. The state variables are assigned their initial values at period k = 1. The fiscal authorities choose the optimal level of government spending, and the monetary authorities determine the optimal market interest rate at each frequency range, j = 1, …, 5, starting in period k = 1. At the end of the planning horizon, the optimal government spending and interest rate in quarter K = 16 determines the levels of consumption, investment, and the other state variables in period K + 1 = 17.

The estimated equations were derived for the post-Bretton-Woods period of 1973 quarter 3 to 2018 quarter 2, so the initial values for the simulations set the state variables in period 1 to correspond to the US and foreign data in 2018, quarter 2. The annual target growth rates for all real GDP component variables, both aggregate and at each frequency range, are set at 2.5%. The target inflation rate is set at the Federal Reserve (Fed) constant target of 2% annually, which, combined with the targeted real GDP growth, leads to a targeted 4.5% annual nominal GDP growth. To be consistent

[12] A Riccati equation is any first-order ordinary differential or difference equation that is quadratic in the unknown function.

with a stable income velocity of money, this leads to a target money growth rate of 4.5%.

These targets are consistent with a target real short-term interest rate of 2%, and a target nominal interest rate of 4%.[13] Since the initial short-term nominal interest rate is only 2%, the simulations specify that the Federal Reserve policy follows a “liftoff” strategy, as in Crowley and Hudgins (2018b). In these simulations, the target annual interest rate is initially 2%, and then it steadily increases over the horizon, where it achieves a final value of 4%.[14]

The initial stock of government debt is set at $DEBT_0$ = 0, since only the discrepancy between the current value and initial value has an impact on the state equations and the tracking errors. Following Kendrick and Shoukry (2014) and Crowley and Hudgins (2018b), the US net tax rate as a percentage of national income is fixed at $\tau_0 = 0.16$. The quarterly interest rate on the government debt is set at i_0 = .0025, which is 1% per year.

The fiscal and monetary policymakers could use the wavelet decompositions to place relatively more importance on consumption and investment performance by placing the most weight on achieving targets at the desired frequency ranges, as explored by Crowley and Hudgins (2015, 2017b). The simulations in this analysis all assume political cycle targeting, where frequency ranges 3 and 4 get the most weight, since the primary US political cycle is 4 years. The political cycle target incorporates the political and economic motivations of the policymakers, with primary emphasis being on the cycles between 2 and 8 years. In terms of the real exchange rate (*RER*), given that the U.S. is generally recognized as having a benign U.S. dollar exchange rate policy, there is no specific objective for the *RER*, but some kind of objective is required in each of the simulations we consider. Therefore in all but one of the simulations we have set a constant *RER* objective so as to focus attention on other key U.S. macroeconomic variables – a constant *RER* objective should therefore not be confused with or imply a constant *RER* policy target. It is, however, consistent with a mild preference for the stability of the terms of trade.

The simulations consider five cases, where the emphasis on inflation and money growth is varied:

(1) balanced policy emphasis with constant *RER* objective;
(2) emphasis on inflation with constant *RER* objective;
(3) emphasis on money growth target with constant *RER* objective;
(4) emphasis on interest rate target with constant *RER* objective;
(5) emphasis on *RER* target with depreciating *RER* objective.

It is important to stress that this analysis is exploratory, and is the first analysis to build a large-scale wavelet-based control model that includes fiscal policy, monetary policy that contains interest rate, money growth, real exchange rate and inflation targets. The main purpose of the simulations is to analyze the relative changes in the optimal macroeconomic forecast trajectories that occur when the emphasis of the policy changes in these cases, as opposed to focusing on the absolute levels of the forecast trajectories. These simulations illustrate how the wavelet-based model can be utilized

[13] This balances a real interest rate of 2% with a productivity growth of 2%. Given an annual population growth of 0.5%, this is consistent with an annual real GDP target growth of 2.5%.

[14] Thus, the target interest rate is growing at a constant quarterly compounded growth rate of 0.04729. This would represent an approximated interest rate response in the short-term bond market to series of eight semi-annual Fed discount rate increases by 25 basis points over the four-year horizon.

to analyze policy effects, and this framework can be used to augment the conclusions of existing policy forecasting models.

4.1 Balanced Policy Emphasis with Constant RER Objective [Case 1]

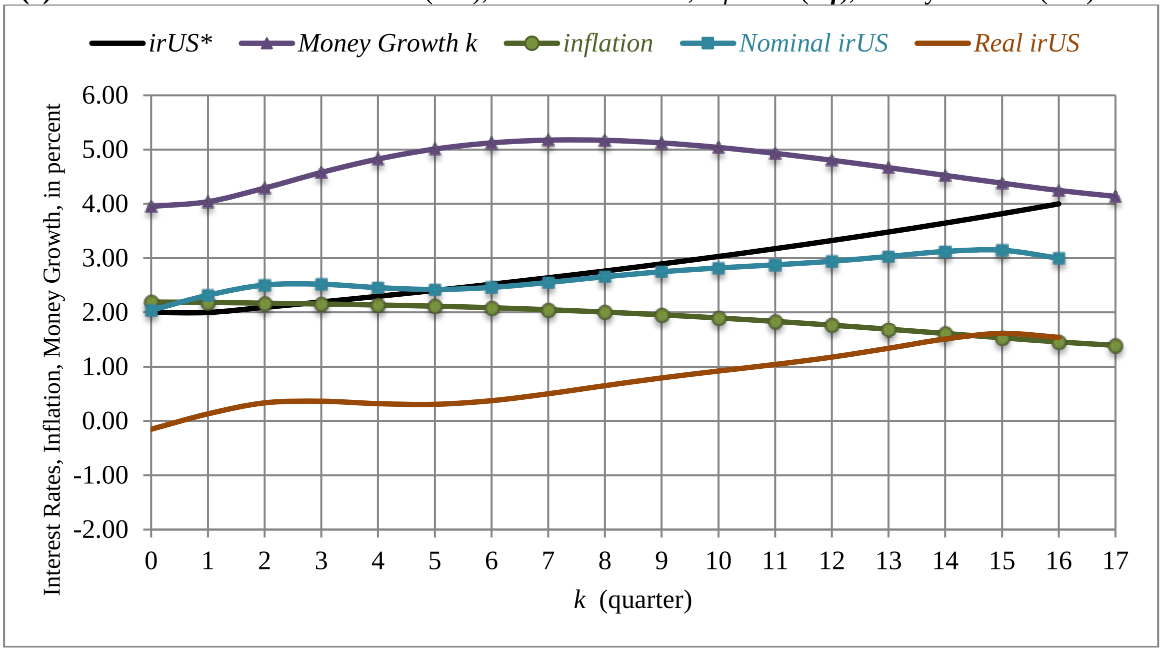
In Case (1), both fiscal and monetary policymakers actively track consumption and investment, and each policymaker also assigns a sizeable weight to tracking its respective policy growth targets. The policymakers desire real exchange rate stability, and thus the growth rate target in the *RER* is 0. The relative weight on the deviations from the constant *RER* objective are given some weight, as are the deviations of inflation, money growth, and the other macroeconomic variables. Figure 4(a) shows the forecast trajectories for the US short-term interest rates, inflation, and money growth. Panels (b) through (f) show the *RER*, net exports, investment, government spending, and consumption, respectively.

Figure 4

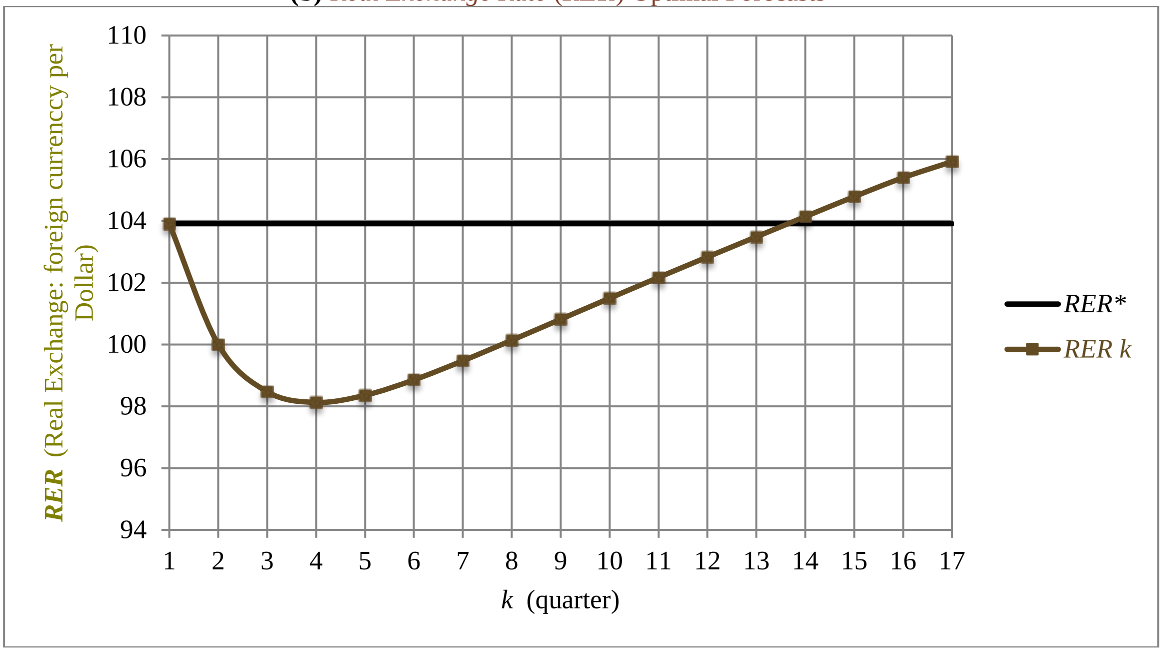
Balanced Policy Objectives: Small Weight on *Inflation* Tracking Error

RER Growth target of 0% per quarter

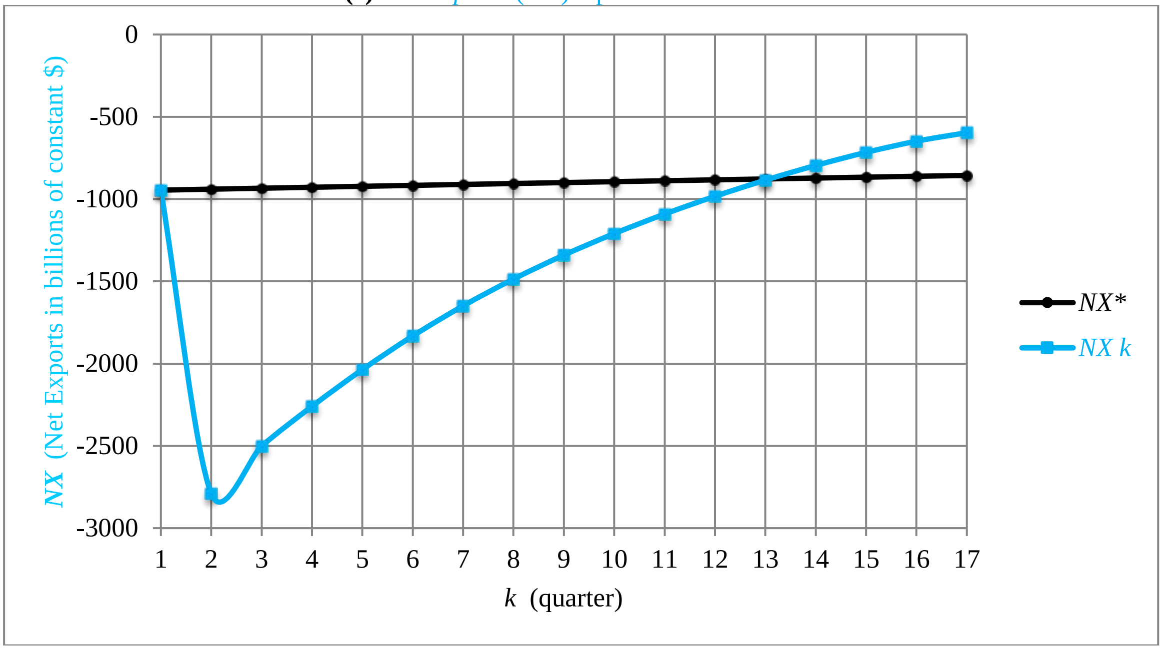
(a) *Short-term Nominal Interest Rate* (***ir***^{US}), *Real Interest Rate, Inflation* (***inf***), Money Growth (***MS***)



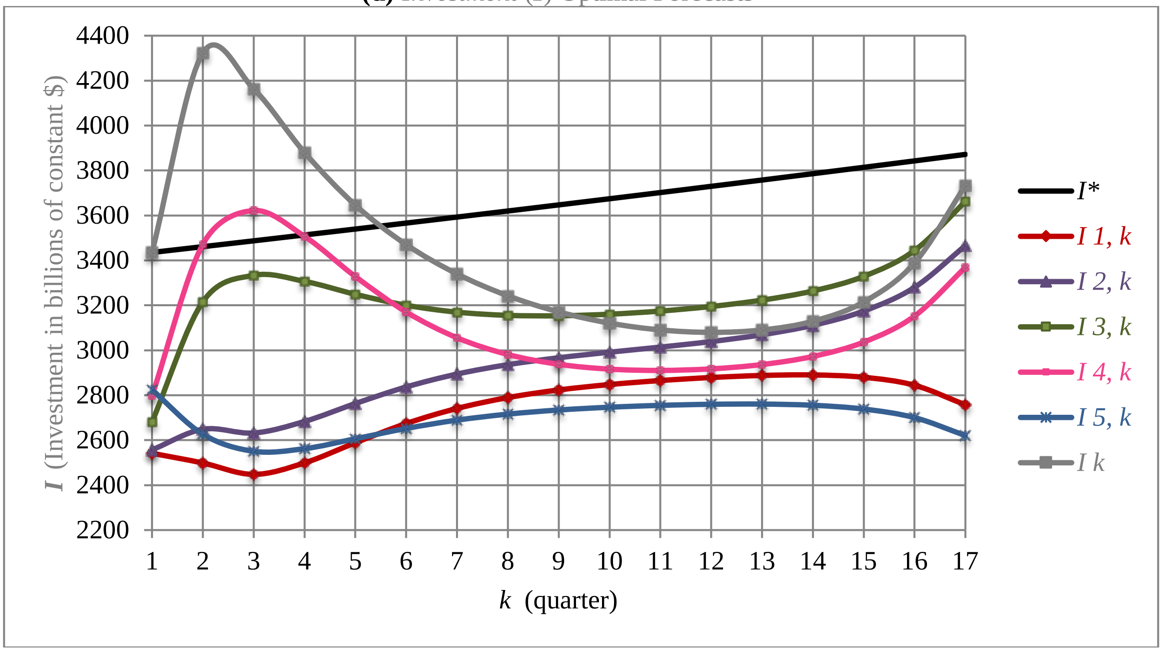
(b) *Real Exchange Rate* (***RER***) Optimal Forecasts



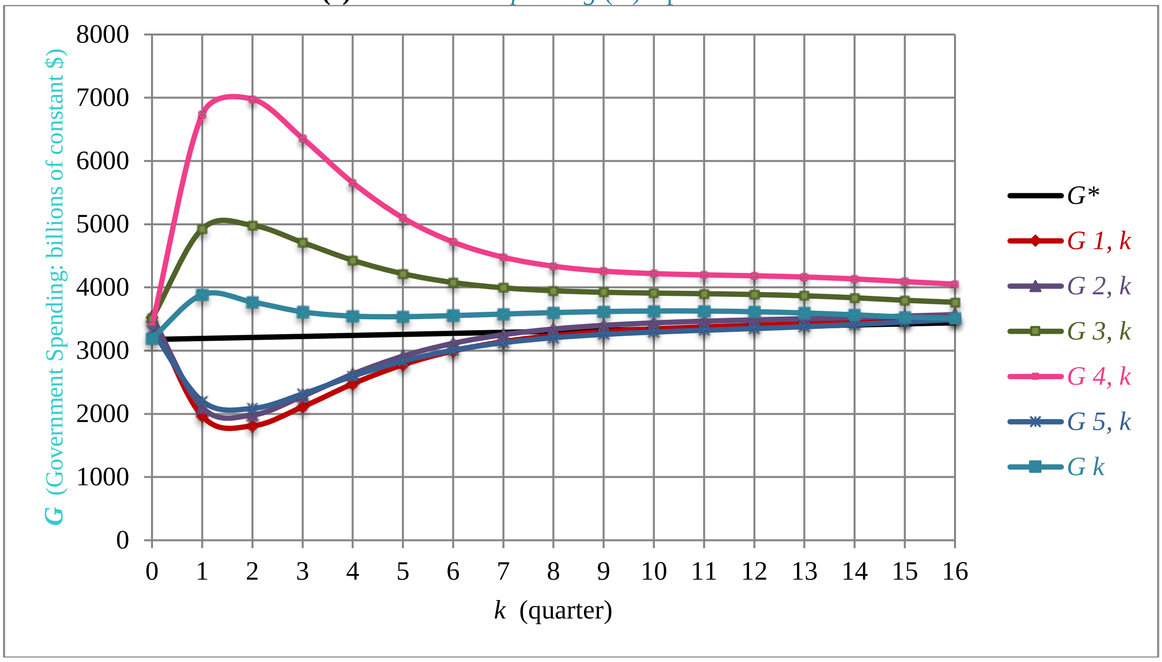
(c) *Net Exports* (***NX***) Optimal Forecasts



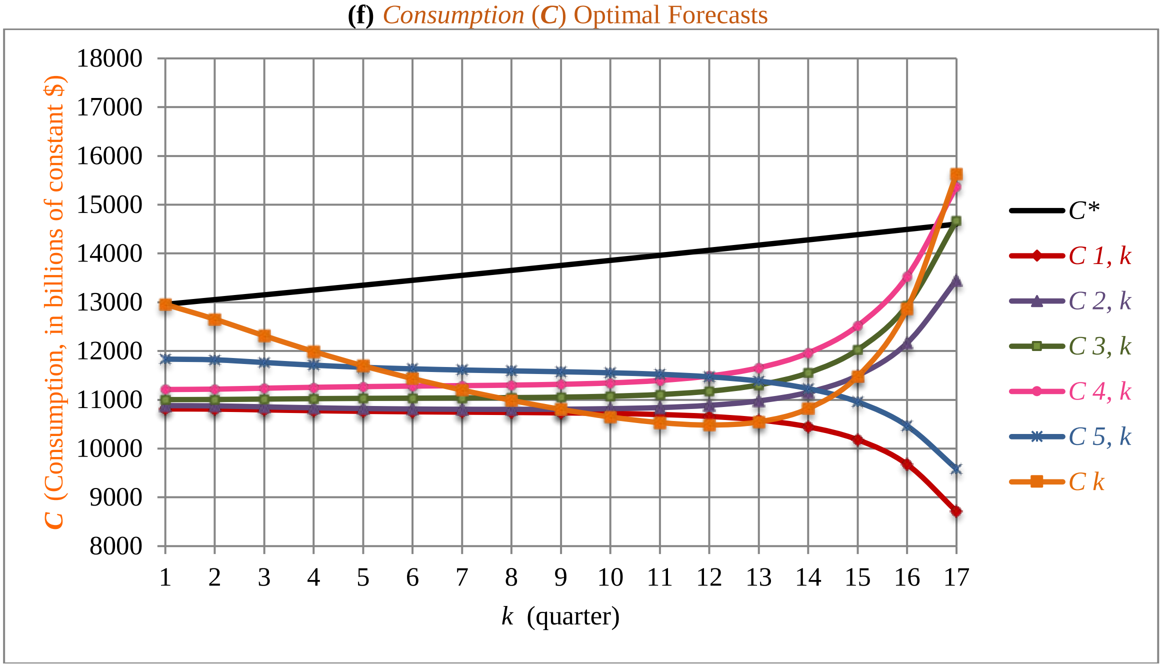
(d) *Investment* (***I***) Optimal Forecasts



(e) *Government Spending* (***G***) Optimal Forecasts



(f) *Consumption* (***C***) Optimal Forecasts



In panel (a), the nominal interest rate increases to 2.5% in quarters 2 and 3, and then falls in quarters 4 and 5, and actually matches the target path in quarter 5. It then steadily increases at a slow rate that is below the target increase. The aggregate interest rate ends the horizon at 3%, which is 1% below its target. Thus, the central bank is employing a slightly expansionary stance throughout most of the horizon by keeping the interest rate below its target trajectory.

The inflation rate begins the horizon at 2.19%, and remains relatively steady throughout the first half of the horizon. It then begins a slight steady decrease to end the horizon at 1.4%. The real interest rate is initially around 0%, and then follows an undulating trajectory with an increasing trend. The final real interest rate is 1.54%, which is below the 2% target. Money growth is initially 4%, and it steadily increases over the first seven quarters to reach 5.17% in quarter 7. Thereafter, it steadily decreases before ending the horizon at 4.14%. Money growth exceeds its target throughout the middle of the horizon (quarters 3 through 14), which is consistent with an expansionary stance that depresses the interest rate slightly below its target.

The *RER* depreciates slightly by about 6 index points during the first three quarters, which results from the initial state where the real exchange rates at each of the frequency ranges are all initially above the smooth trend value. As the US nominal interest rate increases and the money growth rate decreases, the decomposed *RER* trajectories steadily increase over the remainder of the horizon, in order to move towards interest rate parity. Although the objective is constant, the *RER* ends the horizon at about 2 index points above the target.

Net exports (NX) follow a J-curve pattern, as shown in panel (c). After a noticeable initial decrease, net exports begin a steady recovery. The net export trajectory reaches the target level by quarter 13, and finishes the horizon above the target.

Panel (d) shows that aggregate investment falls during the middle of the horizon, before recovering and almost reaching the target at the end of the horizon. Investment is the lowest at the highest and lowest frequency ranges. Since the central bank is keeping the interest rate below the target during the middle and end of the horizon, this increases the growth of output and investment toward the end of the horizon.

Panel (e) shows that government spending at the heavily weighted political cycle frequency ranges 2 to 8 years initially increases before steadily declining toward the target at the end of the horizon, while spending at the other frequency ranges initially decreases before steadily increasing the target of the remaining quarters. In these simulations, the tracking error on fiscal spending is relatively heavily weighted, so that the monetary policy effects are somewhat more pronounced than they would be if fiscal policy were more active. Aggregate government spending is above the target, but tracks the target closely toward the end of the horizon.

Figure 4(f) shows that consumption lies mostly below the target, which helps to facilitate higher consumption in the future through larger current investment. Consumption is the highest at the most heavily weighted frequency ranges 3 and 4. At the beginning of the horizon, consumption is above the smooth trend at all frequency ranges. Since the initial increase in government spending is not enough to increase the consumption components relative to the trend, aggregate consumption initially falls, and then it begins to increase, and ends the horizon slightly above the target level.

4.2 Large Tracking Weight on Inflation with Constant RER Objective [Case 2]

Case (2) explores the scenario where policymakers place a relatively high importance on achieving a 2% inflation target. To simulate case (2), the relative weights in the performance index are the same as in case (1), except that the tracking error on the inflation rate is much more heavily weighted. Since the GDP components are tracking targets above their current values, the economy does not begin with an inflationary gap. As seen in case (1), the inflation rate falls below 2% over the latter part of the horizon, and ends at 1.4%. In case (2), fiscal and monetary are generally more expansionary in an attempt to increase the inflation rate, especially across the middle of the horizon.

The case (2) results are shown in Figure 5. When comparing case (1) and case (2), the inflation rate is about the same in the first half of the simulation period, although it tracks the target slightly more closely in case (2). The inflation rate is noticeably higher in case (2) across the latter half of the horizon, however, and ends with a value of 1.88%.

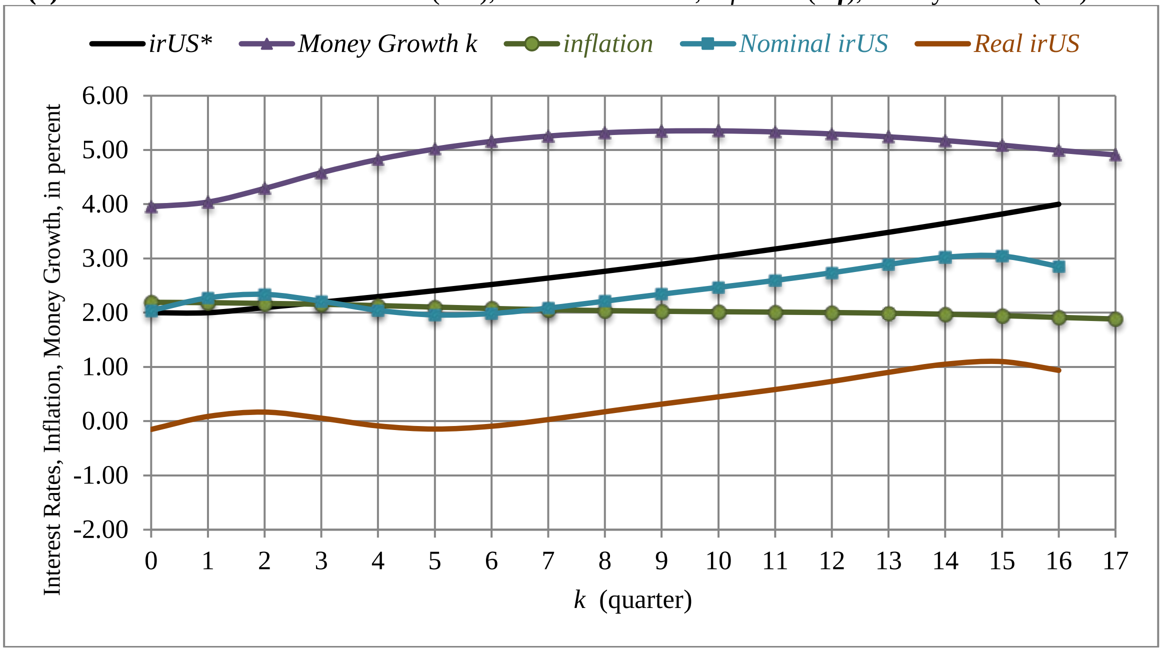
The aggregate nominal interest rate trajectory is lower in case (2) in order to provide additional economic stimulus, and the interest rate reaches a low value of 1.96 in period 5, as opposed to the lowest value in case (1) of 2.42 in period 5. The interest rate ends the horizon at 2.85%, whereas the final period interest rate is 3% in case (1). The lower nominal interest rates and higher inflation rates in case (2) cause the case (2) real interest rate trajectory to fall below that in case (1). In case (2), the real interest rate is negative in periods 4 through 6, and only reaches as high as 1.1% in quarter 15. Conversely, the real interest rate in case (1) is above 1% from period 11 onward, and reaches 1.6% at near the end of the horizon.

The money growth trajectory is also higher and more expansionary in case (2) than in case (1). In case (2), money growth reaches a high of 5.35% in quarter 9, whereas it reaches a high of only 5.17% in period 7 in case (1). In case (2) the money growth trajectory remains above 5% until the last two quarters, and ends the horizon at a value of 4.91%, which is substantially above the case (1) final value of 4.14%.

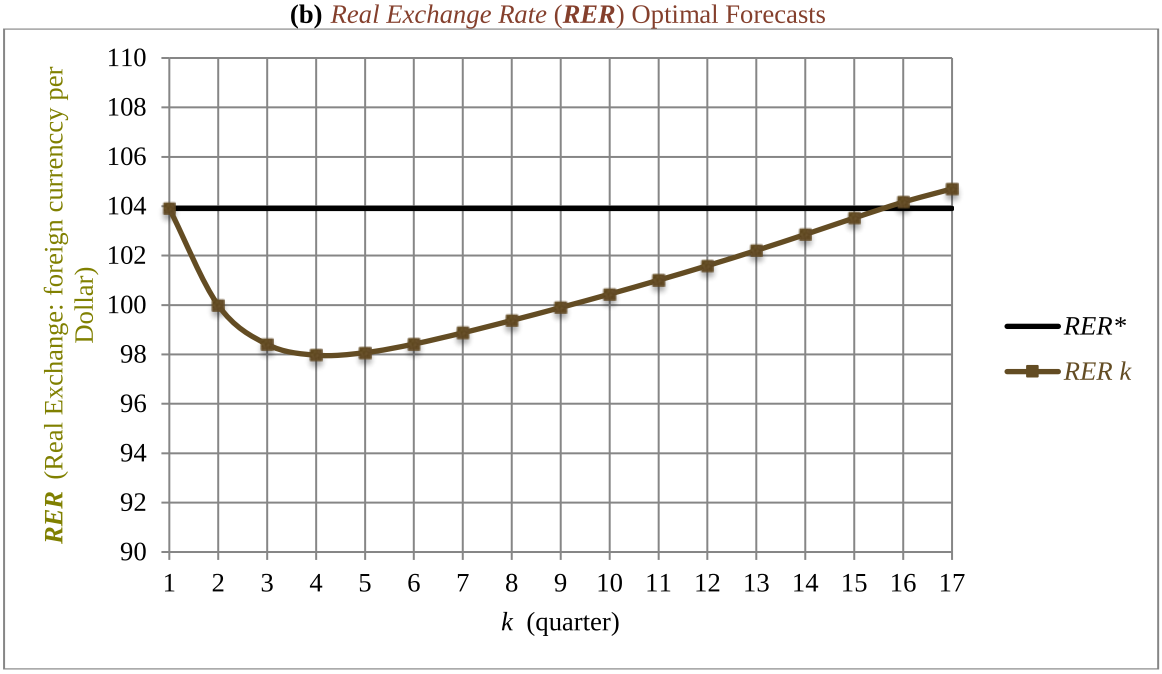
The lower interest rate and higher money growth in case (2) leads to a more depreciated *RER*, as contrasted with case (1). As shown in panel (b), the *RER* finishes the horizon less than 1 point above the target in case (2), whereas the *RER* achieved a final index value of 2 points above the target in case (1).

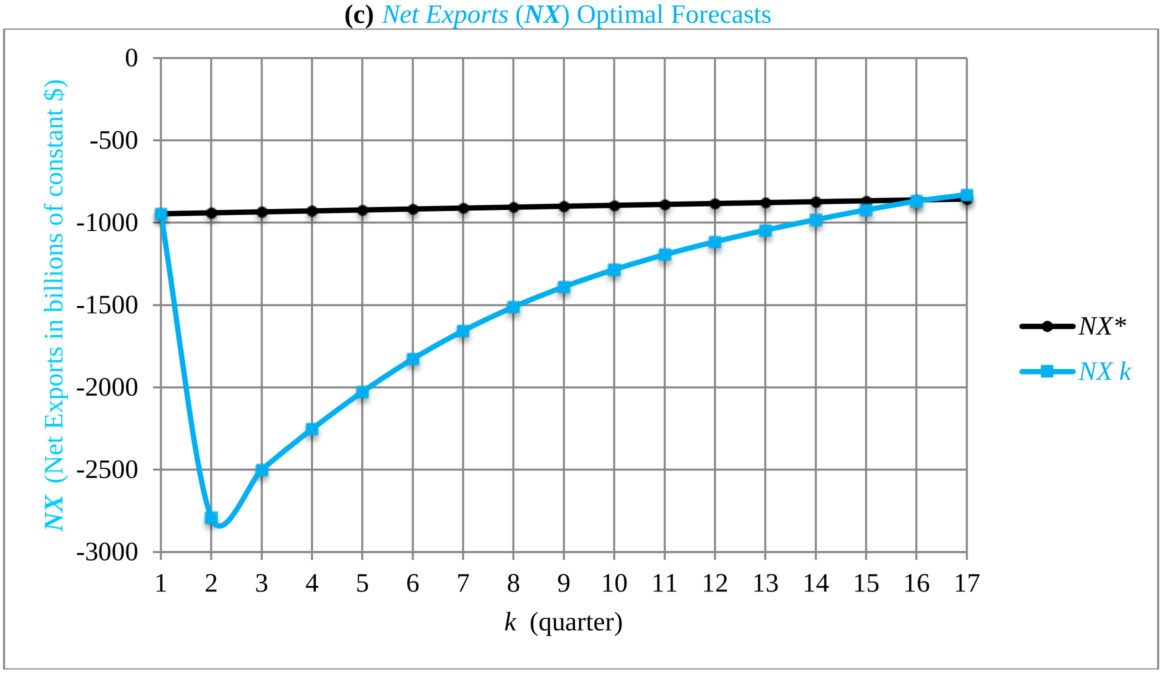
Figure 5
Large Weight on *Inflation* Tracking Error
RER Growth target of 0% per quarter

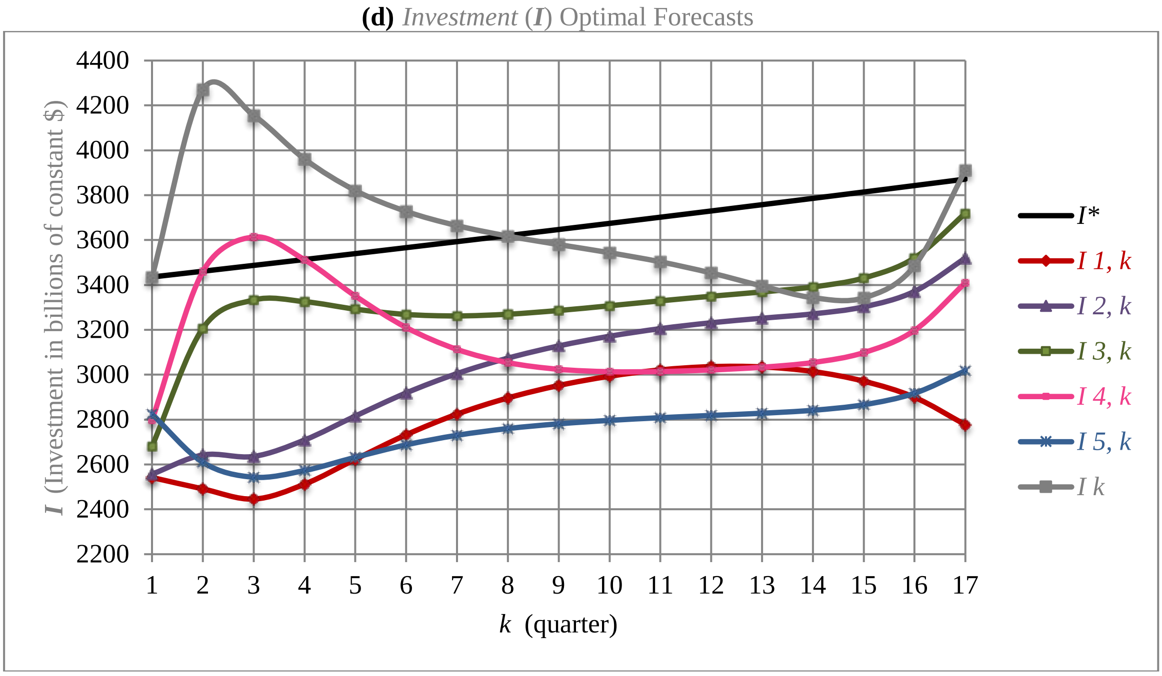
(a) *Short-term Nominal Interest Rate* (***ir***^{US}), *Real Interest Rate*, *Inflation* (***inf***), Money Growth (***MS***)



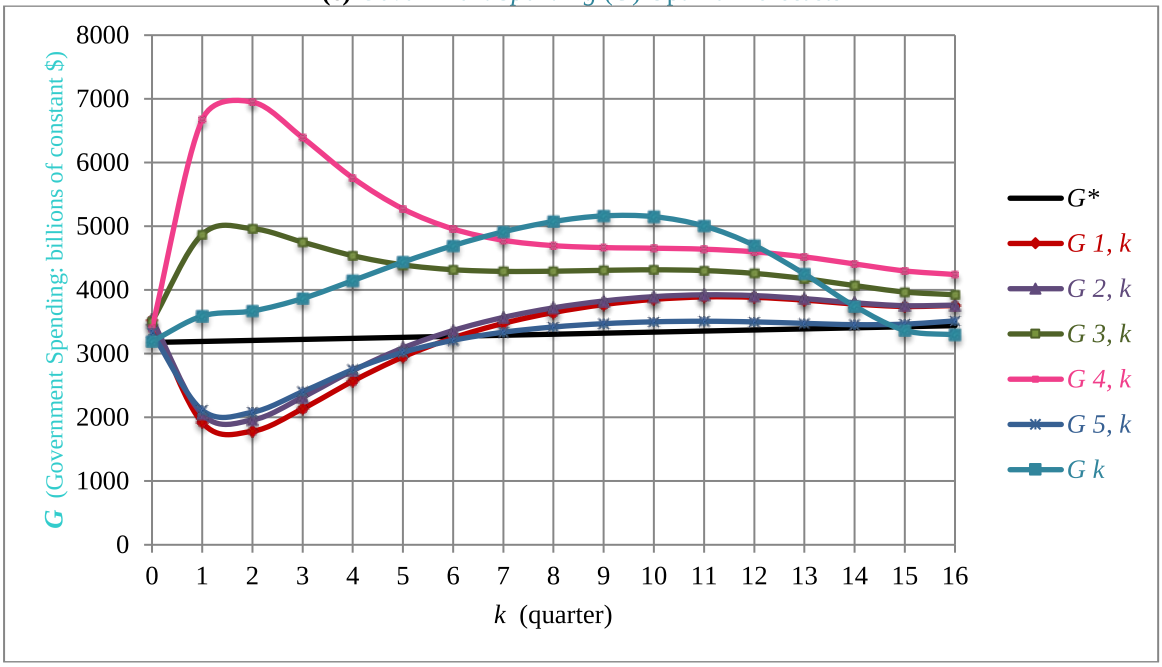
(b) *Real Exchange Rate* (***RER***) Optimal Forecasts



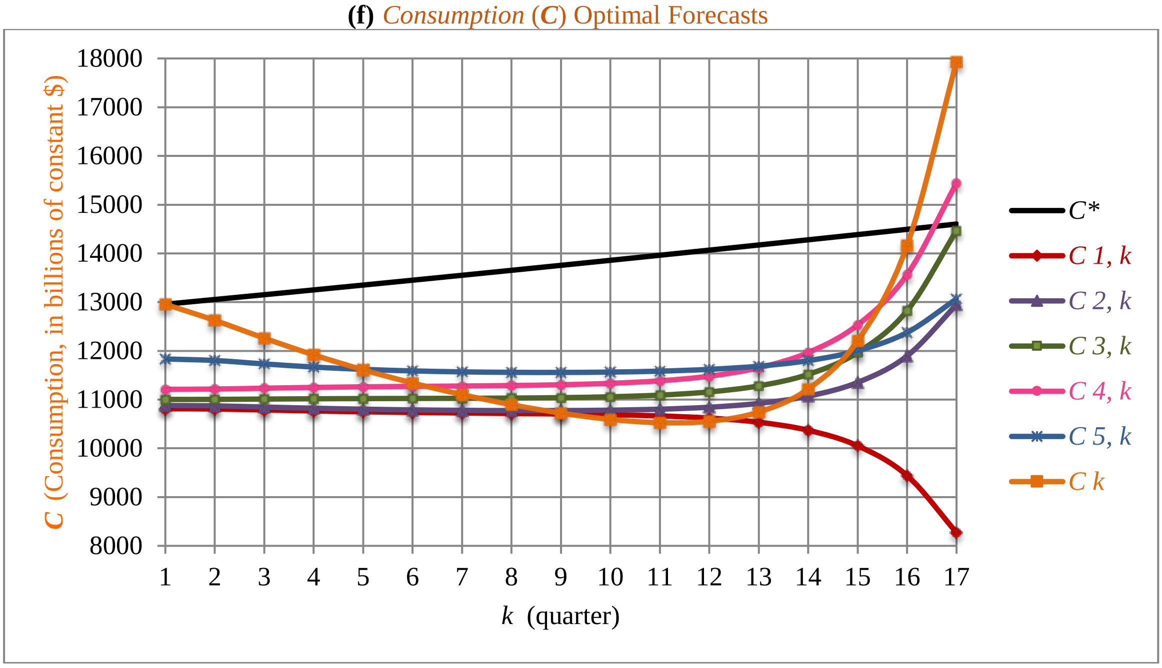

(c) Net Exports (NX) Optimal Forecasts
NX (Net Exports in billions of constant $)
0
-500
-1000
-1500
-2000
-2500
-3000
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17
k (quarter)
NX*
NX k


(d) Investment (I) Optimal Forecasts
I (Investment in billions of constant $)
4400
4200
4000
3800
3600
3400
3200
3000
2800
2600
2400
2200
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17
k (quarter)
I*
I 1, k
I 2, k
I 3, k
I 4, k
I 5, k
I k

(e) *Government Spending* (***G***) Optimal Forecasts



(f) *Consumption* (***C***) Optimal Forecasts



In case (2), the relatively smaller (depreciated) *RER* has a negative impact on the trade balance. As shown in panel (c), the aggregate net exports trajectory shows a similar initial drop in cases (1) and (2). However, the remaining trajectory is lower in case (2) compared to case (1). The cumulative value of net exports across the entire horizon is 5.8% lower in case (2) than in case (1).

In case (2), the lower interest rate clearly stimulates investment, as shown in Figure 5(d). In case (2), cumulative aggregate investment is 6.31% larger than it was in case (1). The lowest value for aggregate investment is \$3342.3 in case (2), as contrasted with a low value of \$3079.9 billion in case (1). Longer-term investment (frequency range 5) is clearly improved by the consistently looser monetary policy required to keep the inflation rate higher and the *RER* lower. The positive influence on investment associated with the depreciated *RER* trajectory is consistent with the findings of Ng and Souare (2014) and Crowley and Hudgins (2018a).

Fiscal policy is considerably more expansionary in case (2) than in case (1), Government spending is much larger in periods 4 through 14 in case (2) than in case (1), as shown in panel (e). Cumulative government expenditure is 19.6% higher over the entire horizon in case (2) compared to case (1).

Panel (f) shows that the consumption trajectories in case (2) are somewhat similar to those in case (1). The cumulative aggregate consumption is about 2.2% larger in case (2) than in case (1). In case (2), aggregate consumption also reaches the somewhat larger value of \$17,933 billion, as opposed the smaller case (1) final value of \$15,629 billion.

4.3 Large Tracking Weight on Money Growth with Constant RER Objective [Case 3]

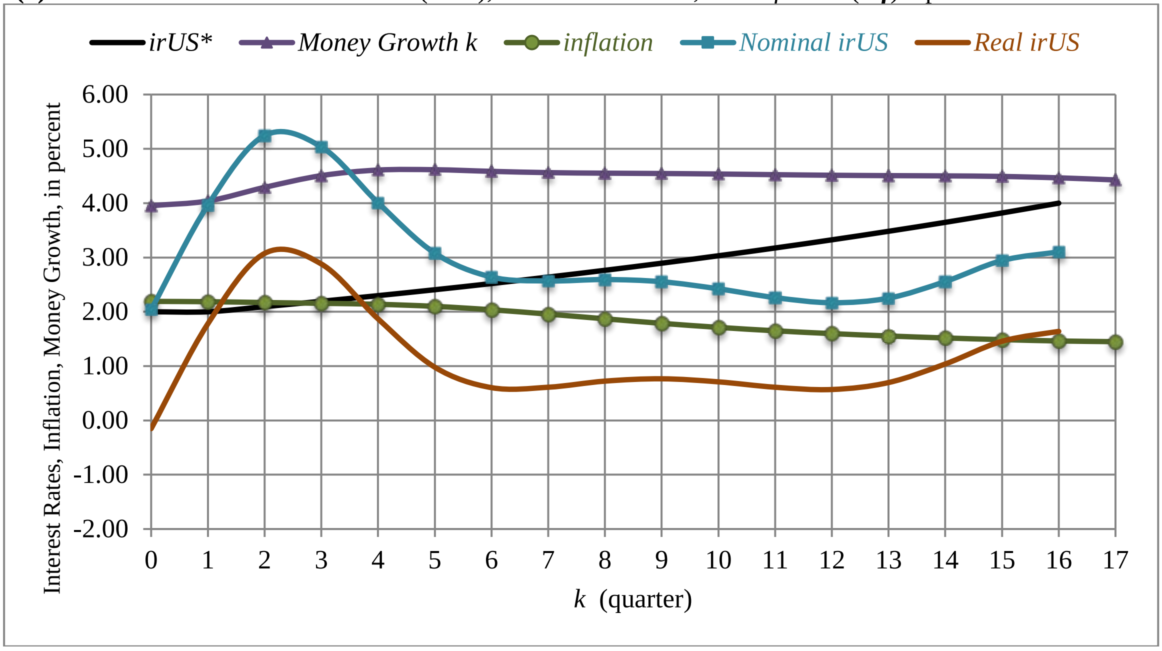
In case (3), the central bank's emphasis shifts more toward monetary growth as a primary operating target, also it still pursues the interest rate as a secondary target. Our analysis is the first paper to include money growth within a wavelet-based control model, and hence it represents a substantial contribution to the literature. Figure 6(a) shows that the money growth rate reaches 4.61% by quarter 4, and then falls slightly until it tracks its 4.5% target closely during quarters 7 through 15 before achieving a final value of 4.43%.

This monetary growth trajectory is lower in case (3) than in cases (1) and (2) in the first part of the horizon. Thus, the initially tighter monetary stance in case (3) is associated with a much larger nominal interest rate increase in quarters 2 through 5, where the nominal interest rate reaches a maximum of 5.25% in quarter 2. However, in periods 6 – 12, the declining nominal interest trajectory is slightly above the declining inflation rate trajectory, until the nominal interest rate increases to an ending value of just above 3%. The nominal interest rate trajectory in case (3) is thus lower than the trajectory in case (1) from quarters 8 through 15.

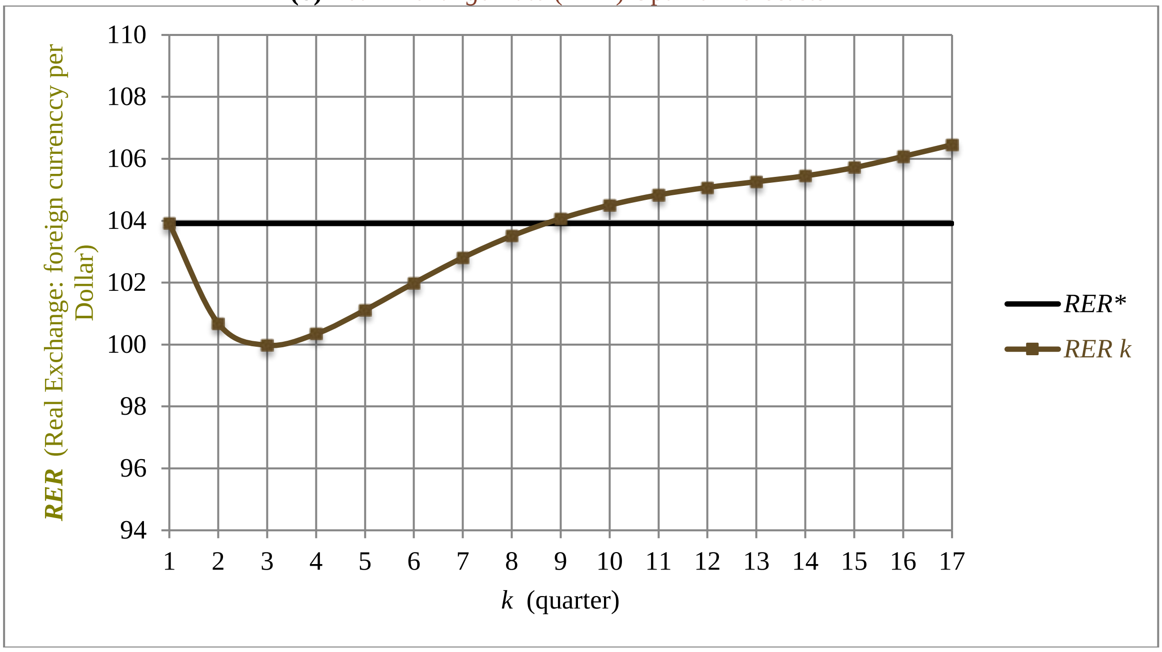
Case (3) has interesting implications for the real interest rate. The case (1) real interest rate is near 0 for the first 6 quarters and then steadily increases to reach about 1.54% at the end of the horizon. Conversely, in case (3), the real interest rate quickly increase to 3.07% in quarter 2, and then begins to decline, where it remains between 0.6% and 0.7% from quarters 6 through 12. Case (3) real interest rate then begins to increase, and finishes the horizon slightly above the case (1) real interest rate.

Figure 6
Large Weight on Money Growth Tracking Error
RER Growth target of 0% per quarter

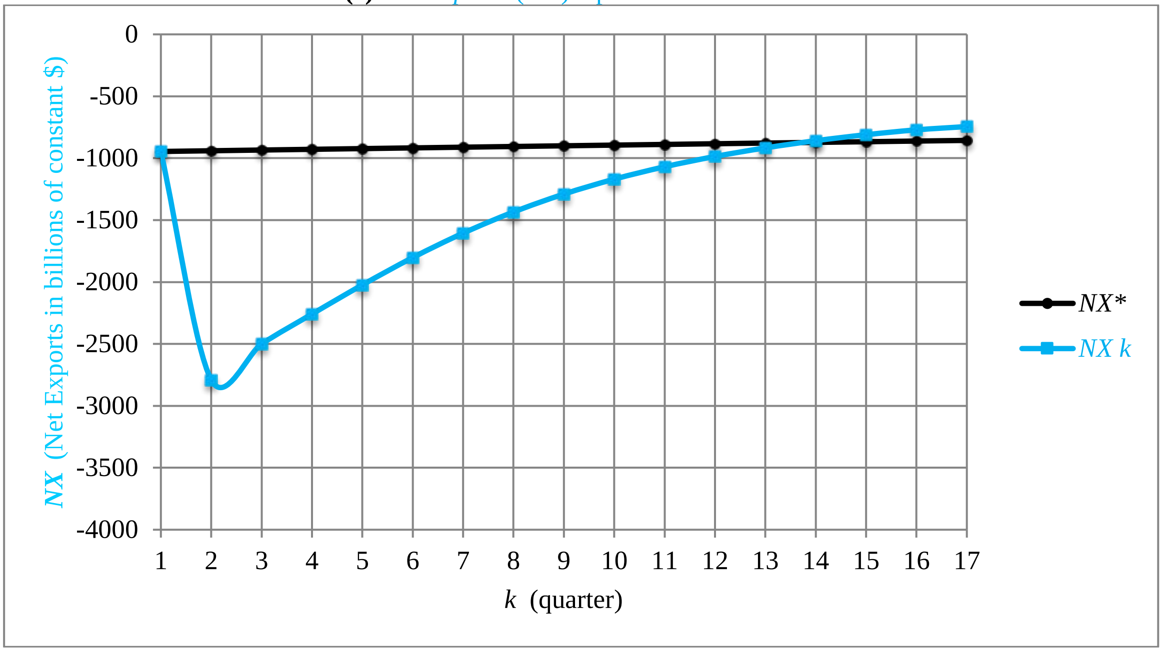
(a) *Short-term Nominal Interest Rate* (***irSA***), *Real Interest Rate*, and *Inflation* (***inf***) Optimal Forecasts



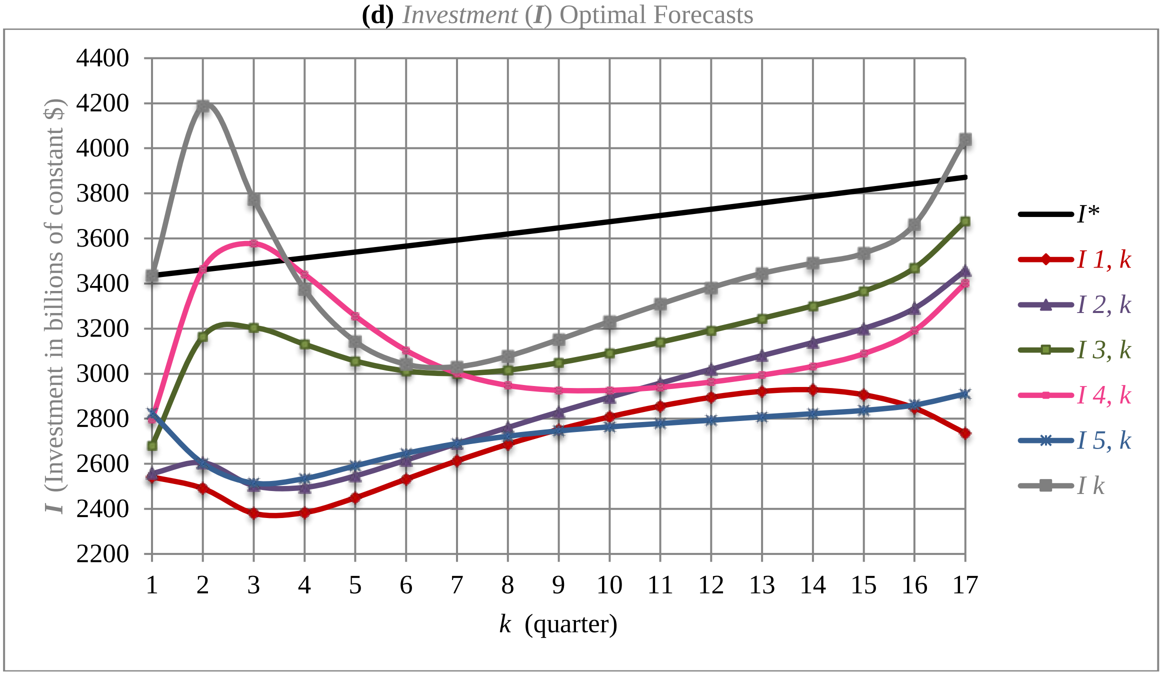
(b) *Real Exchange Rate* (***RER***) Optimal Forecasts

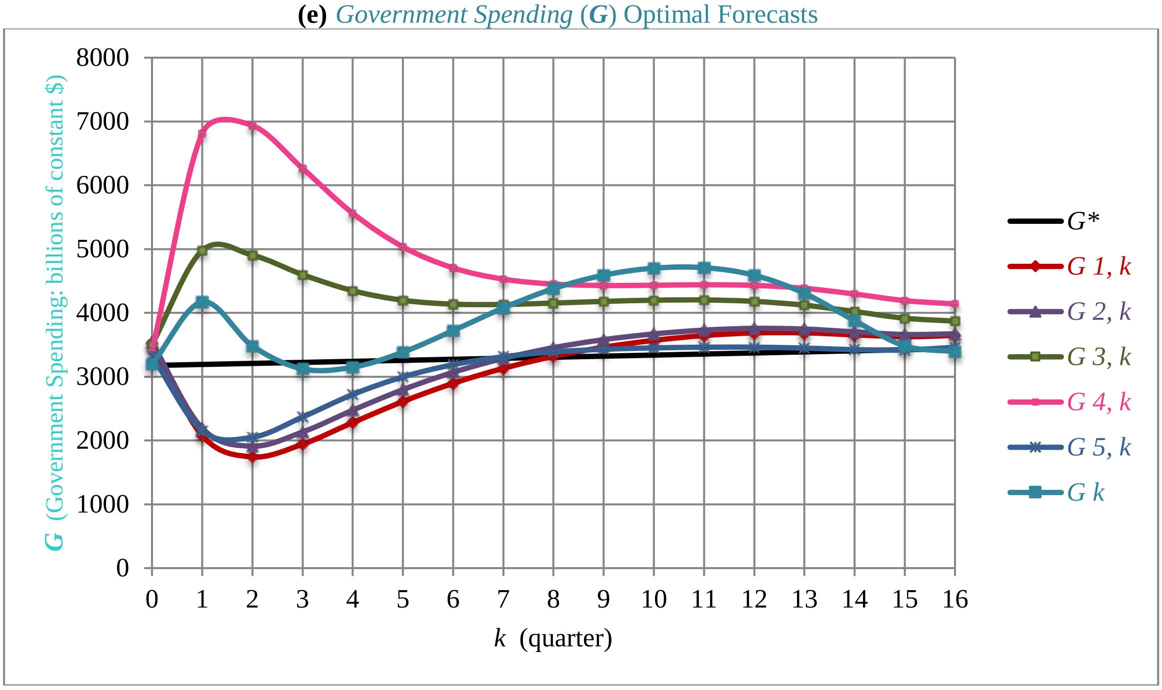


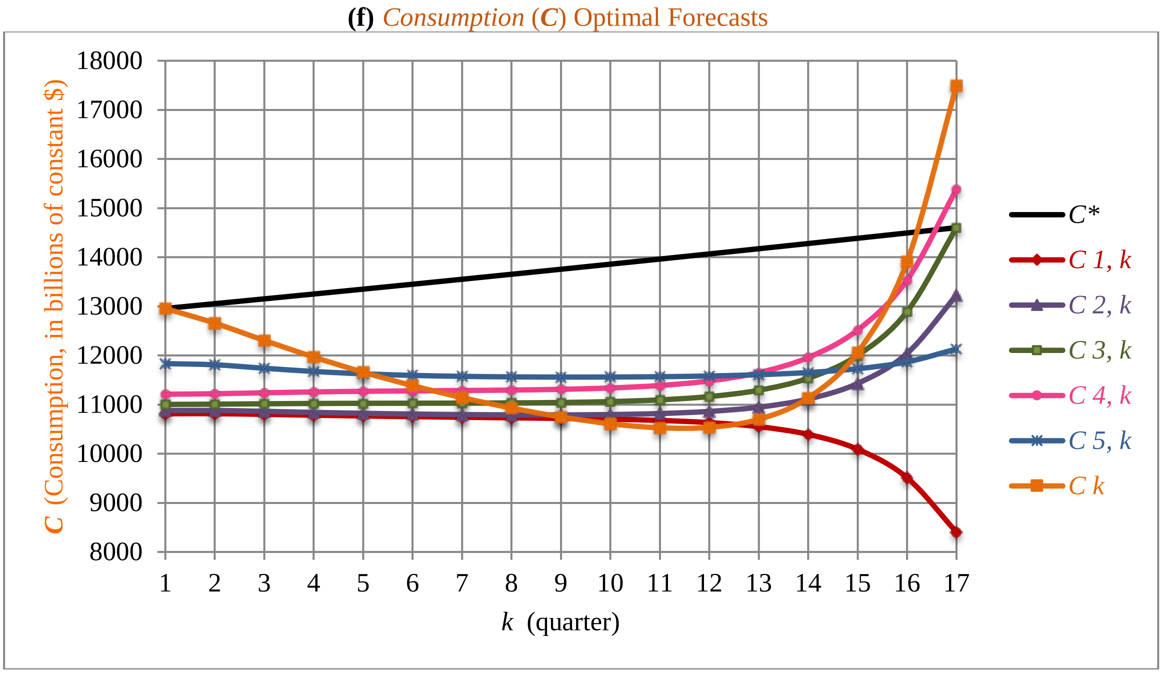
(c) *Net Exports* (***NX***) Optimal Forecasts



(d) *Investment* (***I***) Optimal Forecasts







The *RER* in case (3) tracks its target much more closely in case (3) than in cases (1) and (2), as shown in figure 6(b). This is expected, since the penalty weight on the money growth tracking error was increased, while the penalty weights for all other tracking errors was held constant. The model is now therefore emphasizing monetary policy more heavily relative to the other variables. The *RER* thus initially drops only by 4 index points in case (3), as opposed to 6 points in case (1). Then, the case (3) *RER* closely tracks its constant target in quarters 6 through 16, before ending the horizon slightly below its value in case (1). The higher (appreciated) *RER* trajectory in case (3) also causes the net export trajectory to be lower than that in case (1), as shown in panel (c). The cumulative value of net exports is about 0.88% lower in case (3) than in case (1).

In case (3), the investment trajectory reaches its minimum in quarter 7 before increasing thereafter to end the horizon at value slightly above the target, as shown in panel (d). This can be contrasted with case (1), where the investment trajectory does not reach its minimum until quarter 12, and finishes the horizon with a value slightly below the target.

In panel (e), the case (3) government spending trajectories follow a similar pattern to that in case (2), but in case (3) the aggregate government spending trajectory stays much closer to the target than in case (2). In case (3), cumulative government spending is 9.30% larger than in case (1), but case (3) cumulative government spending is 8.6% smaller than case (2) cumulative spending. The case (3) consumption trajectories are similar, but slightly lower, than those in case (2). Aggregate consumption reaches a final quarter value of $17,495 billion in case (3), as opposed to $17,933 billion in case (2). Cumulative consumption is about 0.32% lower in case (3) than in case (2).

4.4 Large Tracking Weight on Interest Rate with Constant RER Objective [Case 4]

In case (4), the central bank places a relatively larger emphasis on achieving its "liftoff" strategy of having the nominal interest closely track a target that begins at 2% and steadily increases until it reaches 4% in quarter 16. As shown in figure 7(a), the nominal interest rate case (4) closely tracks this target, and ends the horizon at 4.12%. While in case (1) the nominal interest rate slightly exceeds the case (4) interest rate in quarters 1 through 4, the case (4) interest rate is larger than in case (1) thereafter.

The larger nominal interest rate in case (4) reflects a more contractionary monetary policy, and is also associated with a lower money growth trajectory than in case (1). Whereas the money growth trajectories in cases (1) and (4) are similar for the first 8 quarters, the money growth trajectory in case (4) is lower thereafter. The case (4) money growth rate is only 3.93% in the last quarter, as opposed being 4.14% in case (1).

The inflation trajectories are similar in cases (1) and (4), but the inflation trajectory is slightly lower in the latter periods in case (4). The case (4) real interest rate is smaller than the case (1) real interest rate in quarters 1 through 5, but the case (4) real interest rate begins to steadily increase, and exceeds the case (1) real interest rate in all the remaining quarters. In case (4) the real interest rate reaches a final value of 2.68% in quarter 16.

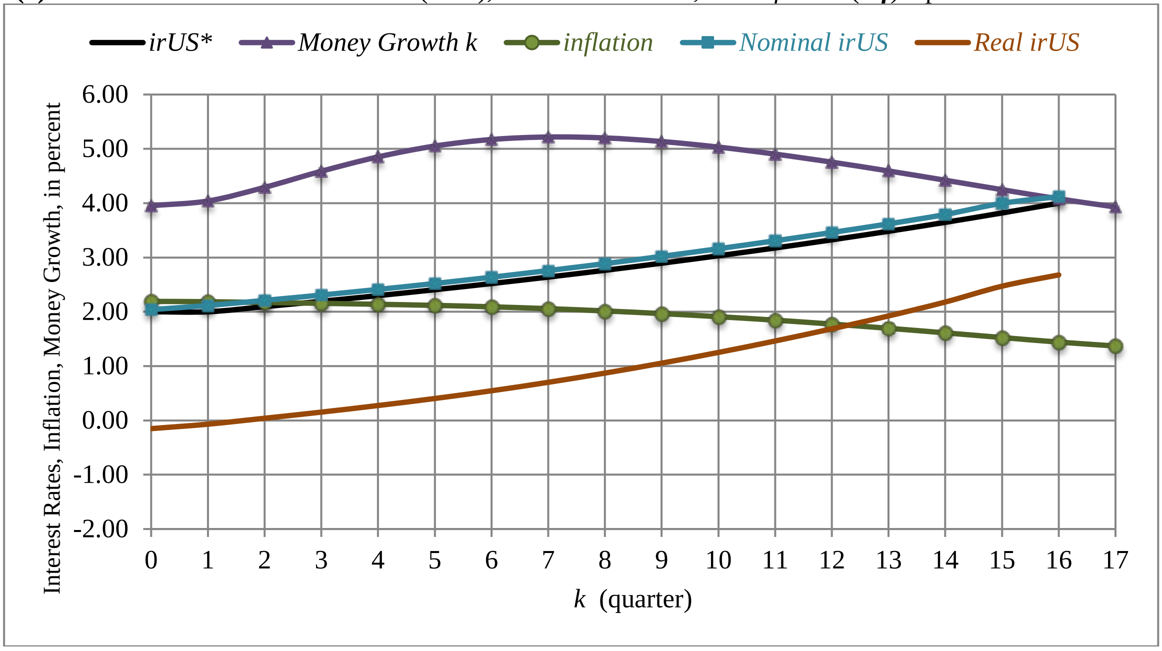
Figure 7(b) shows that the *RER* trajectory is higher in case (4) than in case (1). This is due to the interest rate parity adjustments. Since the domestic nominal interest rate is relatively higher, investors are shifting financial investment away from foreign currency assets toward domestic currency assets. The case (4) *RER* trajectory ends the horizon about 2 index points larger than its final period value in case (1).

Figure 7

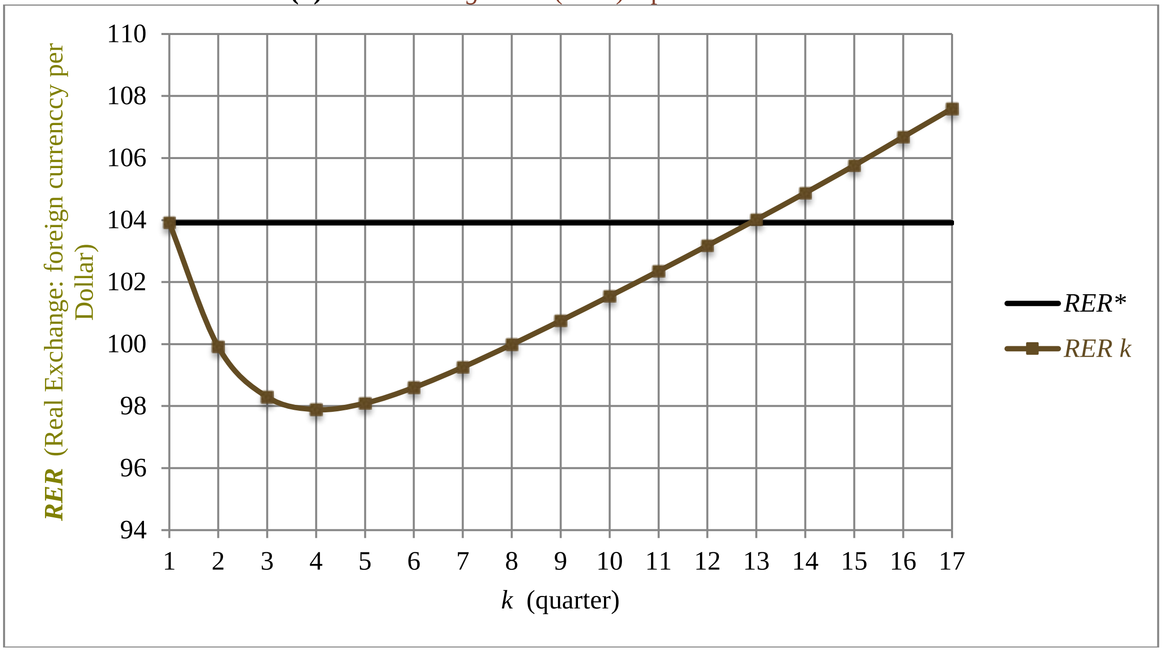
Large Weight on Interest Rate Tracking Error

RER Growth target of 0% per quarter

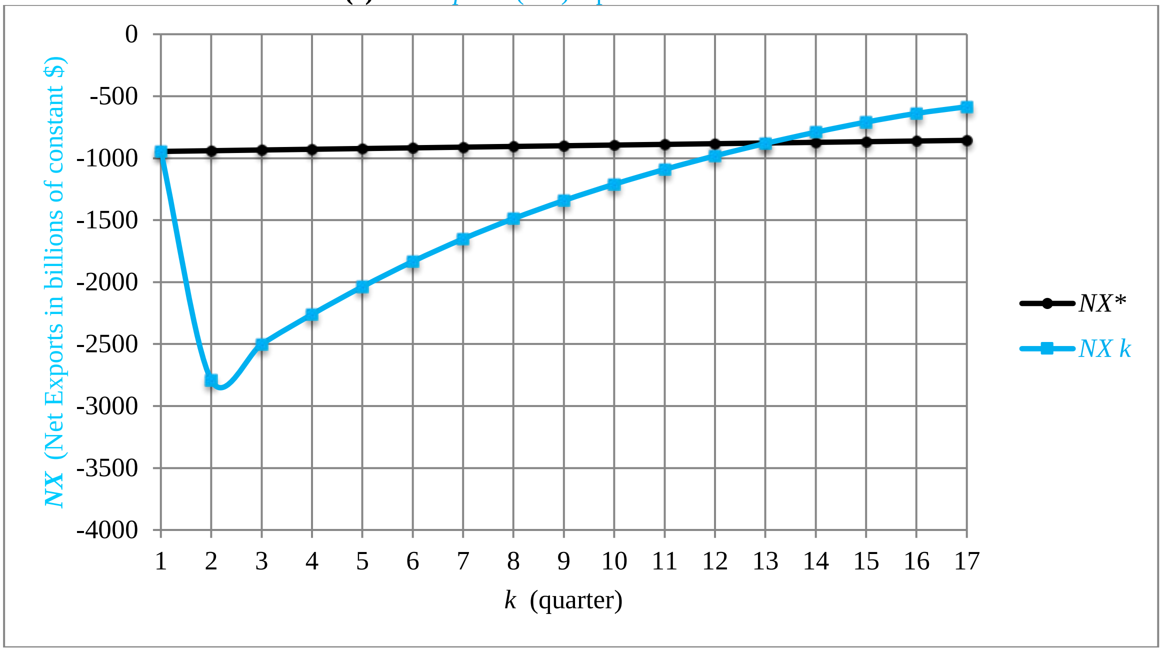
(a) *Short-term Nominal Interest Rate* (***irSA***), *Real Interest Rate*, and *Inflation* (***inf***) Optimal Forecasts



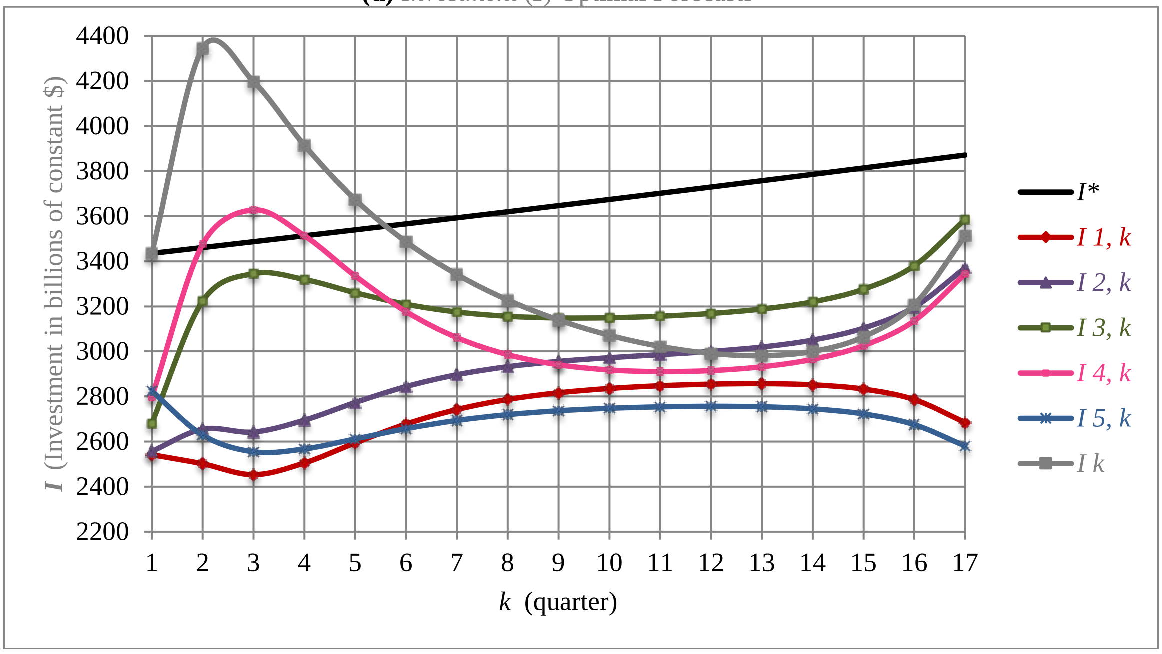
(b) *Real Exchange Rate* (***RER***) Optimal Forecasts



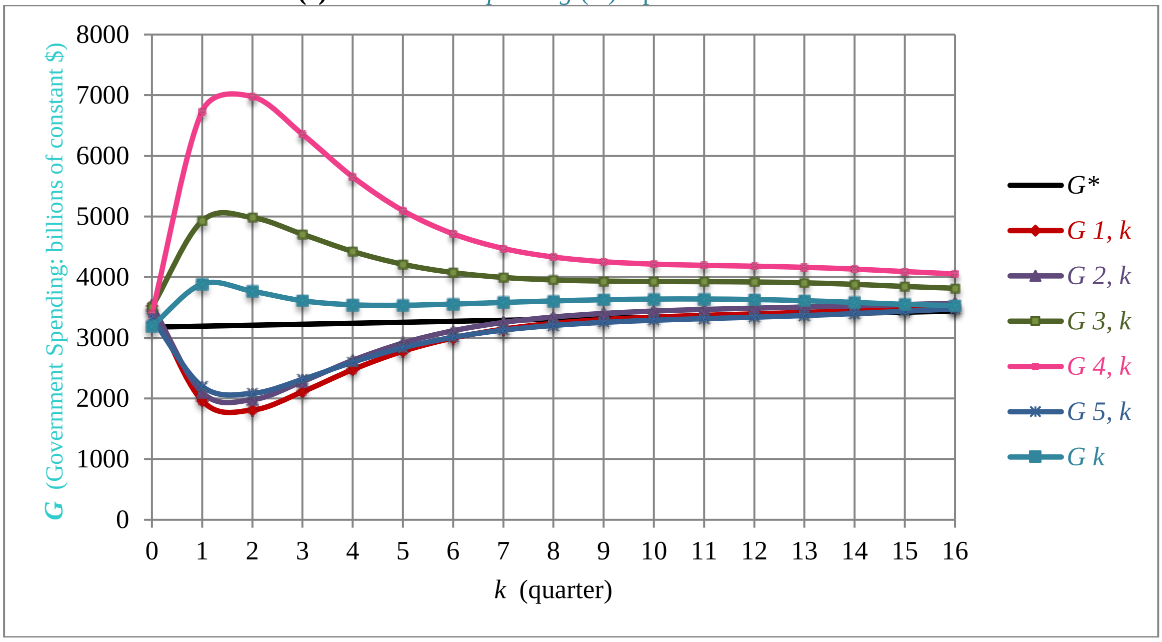
(c) *Net Exports* (***NX***) Optimal Forecasts



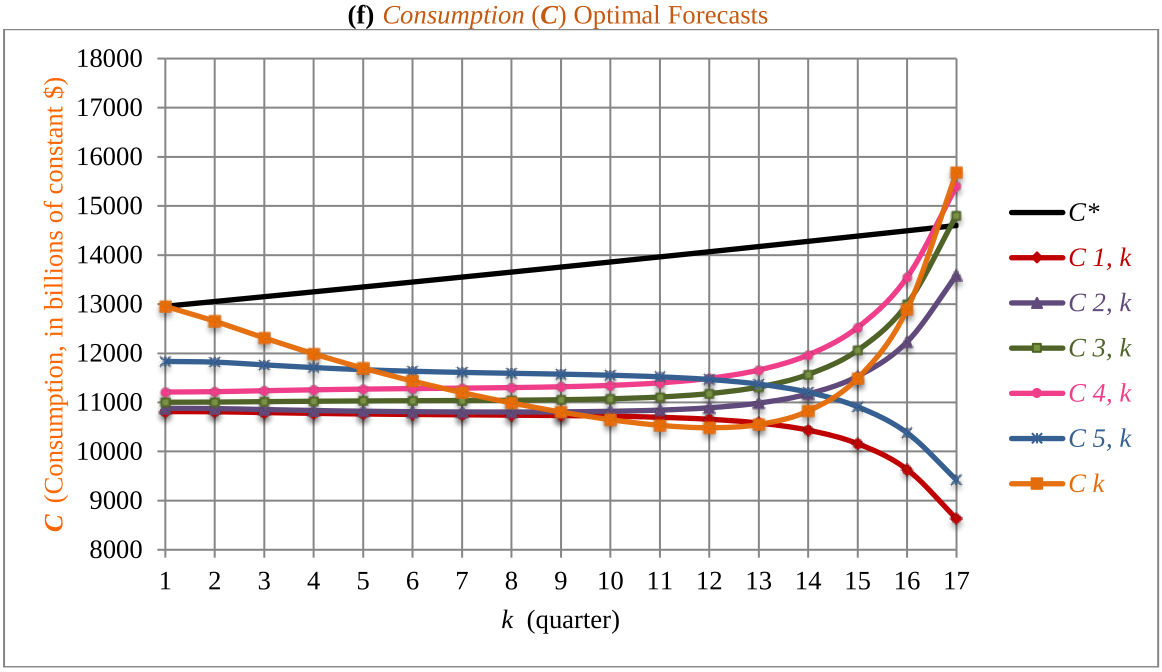
(d) *Investment* (***I***) Optimal Forecasts



(e) *Government Spending* (***G***) Optimal Forecasts



(f) *Consumption* (***C***) Optimal Forecasts



The net export trajectories, as shown in panel (c), are very similar in cases (1) and (4), despite the slightly higher *RER* in case (4). However, the investment trajectories in case (4) are somewhat lower than they are in case (1), as shown in panel (d). Due to the relatively contractionary monetary stance in case (4), cumulative aggregate investment is about 1.54% less in case (4) than in case (1). In case (4), investment only achieves a value of $3,513.4 billion in the last quarter, which is somewhat less than the case (1) last quarter value of $3,733.3 billion.

The government spending trajectories in panel (e) are similar in cases (1) and (4), but the relatively contractionary monetary policy (4) results in a more aggressive fiscal policy that partly compensates for higher interest rates. Cumulative aggregate government spending is about $149 billion dollars (which is about 0.26%) larger in case (4) than in case (1). The consumption trajectories are similar in cases (1) and (4), as shown in panel (f). Cumulative consumption is almost identical in cases (1) and (4), as there is a shift away from investment toward short-term consumption.

4.5 Emphasis on RER Target with Depreciating RER Objective [Case 5]

In Case (5), the *RER* objective is a depreciation of about 10% lower over the horizon, which means that the *RER* target depreciates annually by about 2.355% (0.00539% per quarter). In order to simulate this case, the tracking on the *RER* target was increased to reflect the additional importance on tracking the *RER* depreciating target, while all other tracking error penalty weights are held constant. The case (5) results are shown in figure 8.

The case (5) depreciating *RER* objective allows the Fed to pursue a more expansionary interest rate policy than in case (1). The lower nominal interest rate trajectory in case (4) maintains the interest rate parity adjustments, since the objective is an *RER* deprecation.

This is an interesting case for the US, because it is consistent with an attempted liftoff strategy that is abandoned in favor of a period of reversion to low market interest rates, as shown in figure 8 panel (a). The nominal interest rate increases in the first two quarters, and then declines over the next four quarters until it reaches an annual rate of about 0.5%. The nominal rate remains at that low level until quarter 13, where it begins a slow steady increase to end the horizon at about 1.5%.

The lower nominal interest rates in case (5) are accompanied by noticeably higher money growth trajectory, as well as a higher inflation trajectory than in case (1). Since the inflation rate exceeds the nominal interest rate over the middle and end of the horizon, the real interest rate is negative in quarters 4 through 16, reaching a low of about 1.57% in quarters 6 through 8.

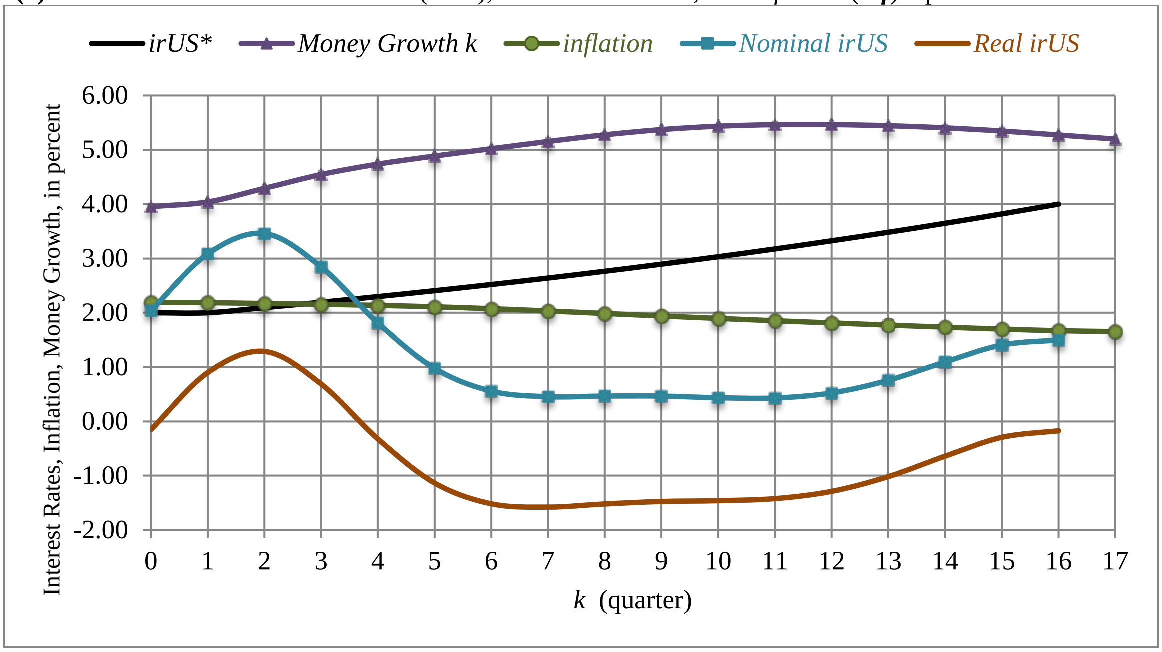
The *RER* in case (5) includes a similar drop to that in case (1), but due to the declining target trajectory, the *RER* in case (5) only briefly levels off before beginning another steady decline. The final *RER* value of 98 is just over 3 points above the target. Due to the negative impact value effect of the *RER* depreciation, and the expansionary effect on domestic income that causes increases in the volume of imports, the net export trajectory is slightly lower in case (5) than in case (1), as shown in panel (c). Cumulative net exports are about 1.13% lower in case (5) than in case (1).

Figure 8

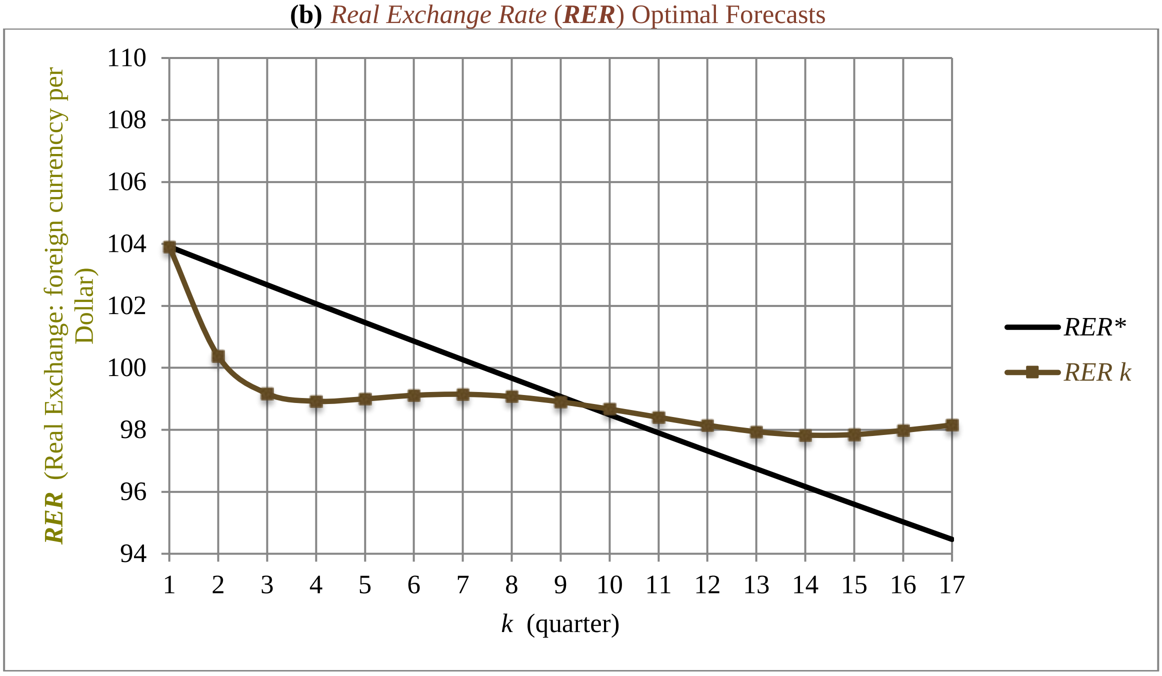
RER Target Emphasis

RER Growth target of – 0.75% per quarter

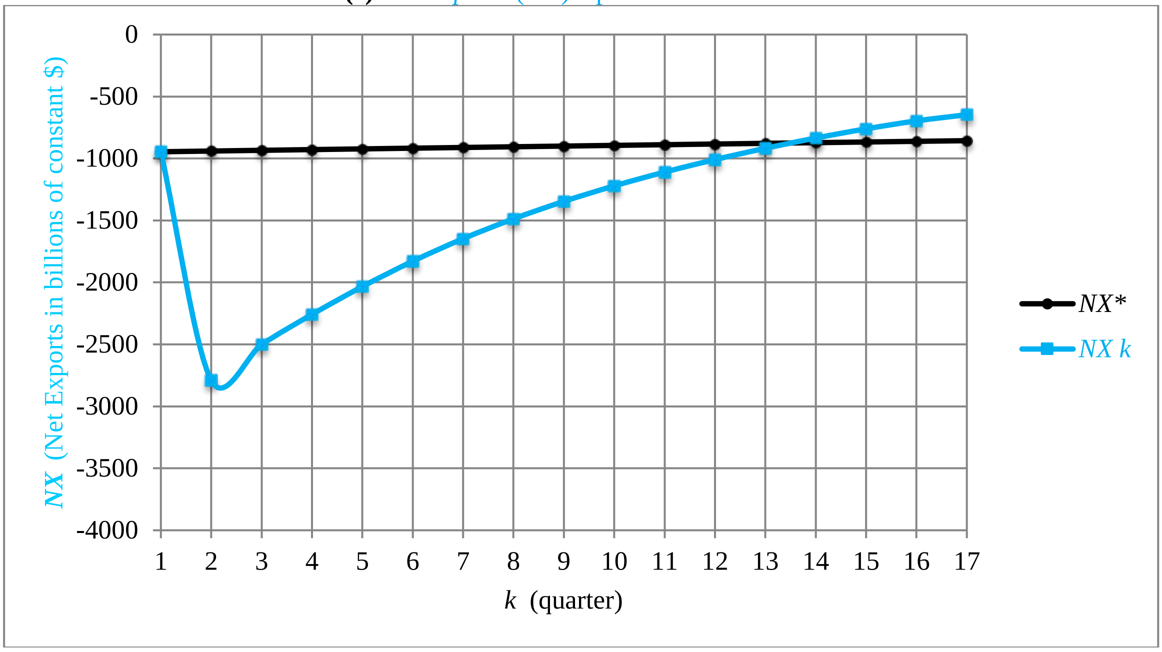
(a) *Short-term Nominal Interest Rate* (***irSA***), *Real Interest Rate*, and *Inflation* (***inf***) Optimal Forecasts



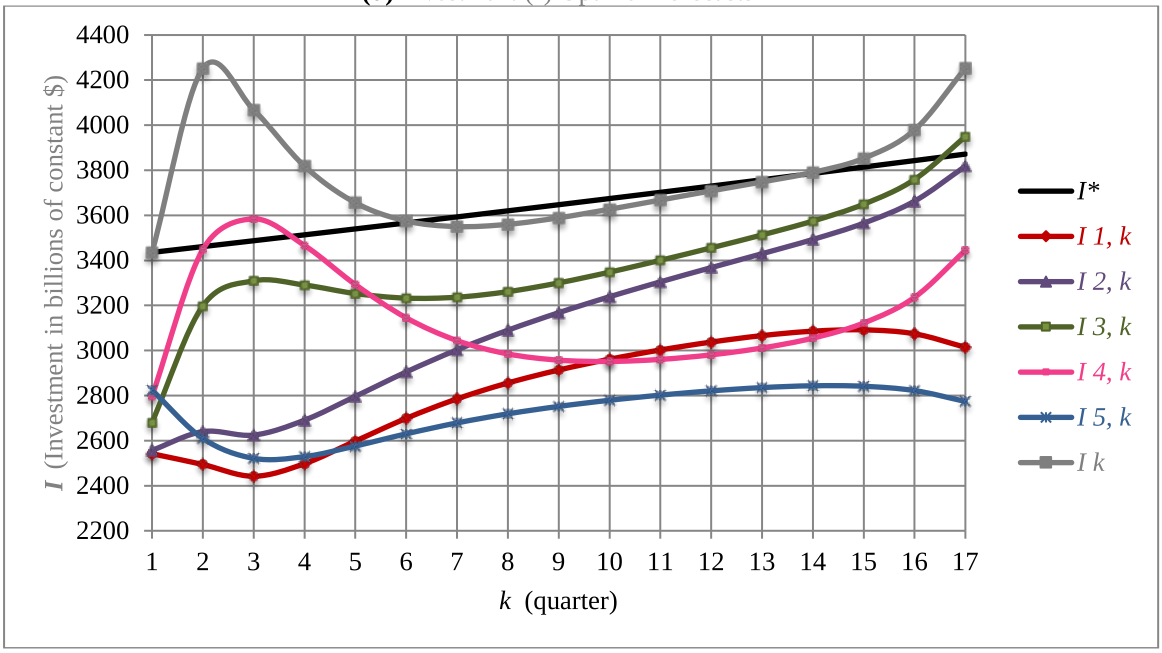
(b) *Real Exchange Rate* (***RER***) Optimal Forecasts



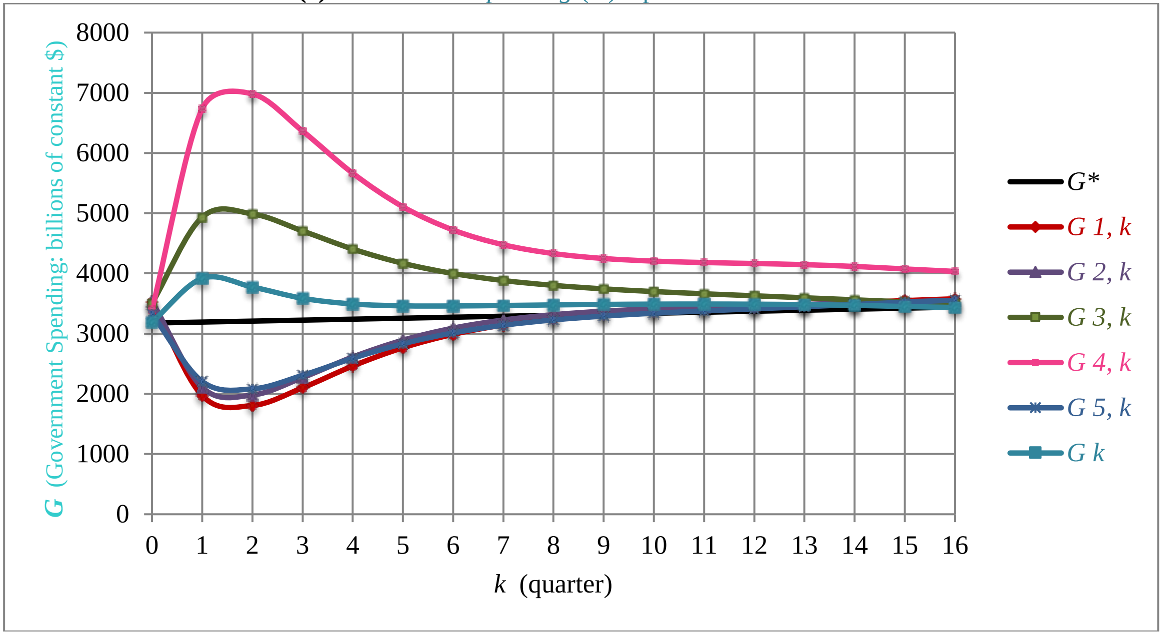
(c) *Net Exports* (***NX***) Optimal Forecasts



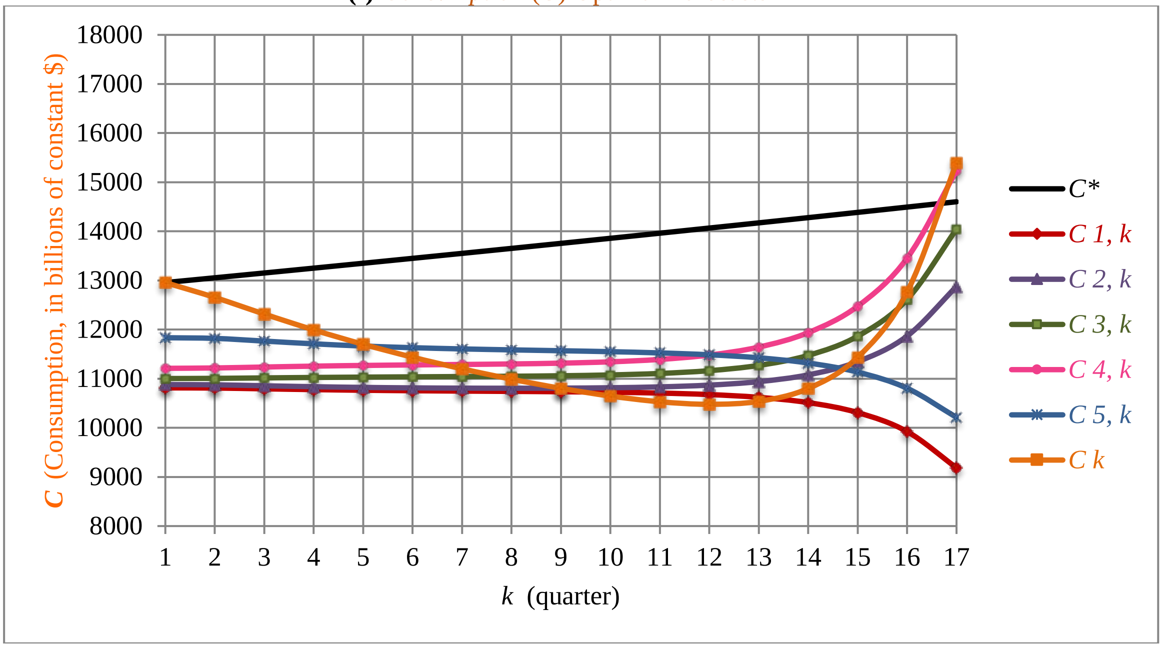
(d) *Investment* (***I***) Optimal Forecasts



(e) *Government Spending* (***G***) Optimal Forecasts



(f) *Consumption* (***C***) Optimal Forecasts



The lower interest rates in case (5) cause the investment trajectory in panel (d) to be considerably higher in case (5) when compared to case (1). Case (5) cumulative aggregate investment exceeds that in case (1) by about 9.6%. Aggregate investment has a value of $4,253 billion in the last quarter in case (5), compared with a value of only $3,733 billion in case (1).

In case (5), looser monetary policy is met with a slightly tighter fiscal policy. And although case (1) and (5) government spending trajectories are similar, cumulative aggregate government spending is 2.3% lower in case (5). Aggregate government spending ends the horizon in case (5) slightly below the target, whereas the final value is slightly above the target in case (1). Consumption, however, is slightly lower in case (5) than in case (1), with case (5) cumulative aggregate consumption being about 0.2% lower in case (5).

4.6 *Summary*

If we take the baseline simulation [case (1)] as our benchmark, then one way to summarize the different simulation results is to calculate the cumulative changes in each of the real GDP components over the forecast horizon and compare with the benchmark cumulative changes. In table 2 we show this summary assessment of the 4 other simulation cases [cases (2) to (5)] compared with the baseline simulation.

Table 2

Comparison of Cumulative Differences (in %) by Real GDP component by Simulation

Case	**2**	**3**	**4**	**5**
C	2.2	1.9	0.0	-0.2
I	6.3	-0.4	-1.5	9.6
G	19.6	9.3	0.3	-2.3
NX	5.7	0.9	-0.1	1.2
Y	6.2	3.0	-0.2	1.2
infl	0.15	-0.08	0.00	0.04

In terms of output growth, the inflation target simulation delivers most output growth, but this mostly comes from fiscal stimulus which in the current U.S. economic environment is unlikely. The inflation target simulation does deliver higher consumption and investment too as well as a significant improvement in the trade balance. By targeting money supply, economic growth does outperform the baseline by 3% over the forecast horizon, but a sizeable fiscal stimulus is still apparent, but there is very little movement in the other GDP aggregates apart from a mild increase in consumption. Targeting interest rates is the most restrictive in terms of growth, with a fall in investment and a worsening of the external account only being partially compensated for by a slight increase in government spending. Lastly case (4), which involves the depreciation of the *RER* gives the biggest cumulative boost to investment, but actually causes a contraction in fiscal policy and this feeds through to a small reduction in consumption as well. In case (4) though, economic growth is only modestly higher than the baseline.

5. Conclusions

In this paper we demonstrate that the wavelet-based optimal control model bestows benefits from analyzing the forecasts of different policy objectives at different frequency ranges under different underlying policy scenarios, not only to better understand the different cyclical activity embedded in key macro variables, but also for the reconstruction of the aggregate variable simulation trajectories. This is the first paper to model the U.S. in an open economy context using this approach, but also the first paper to incorporate the money supply as a key intermediate target for monetary policy.

We first decompose the key macroeconomic variables in the time-frequency domain using discrete wavelet techniques, and then estimate an optimal control model using conventional regression techniques. In order to better understand the key features of our estimated model, we simulate the key macro policy variables from 2018 quarter 2 over a 16 quarter horizon using five different economic policy scenarios: a dual emphasis on fiscal and monetary policy, emphasis on meeting an inflation target, a money growth target, an interest rate target and lastly a real exchange rate depreciation target.

Given the low inflation rate environment that the U.S. is currently experiencing, the main impact of the 5 scenarios is on both real interest rates and the external balance. We use the dual emphasis case as our baseline and then compare and contrast placing emphasis on the different policy variables in terms of the outcome for growth, net exports, the real exchange rate, and real interest rates. Targeting inflation rates leads eventually to positive real interest rates, but an immediate decline in the real exchange rate, whereas targeting money supply leads to higher nominal and real rates and targeting nominal interest rates leads to a steadily growing real interest rate but lower growth. The last case we investigate is where a depreciating real exchange rate target is adopted, and this leads to negative real interest rates for most of the forecast horizon with consequent enhanced investment but lower consumption, presumably due to exchange rate pass through.

Our approach is illustrative of the benefits for policymakers that can be gained by modelling using a time-frequency approach and then aggregating the results. The macroeconomic behaviors and policy responses over different time horizons can be incorporated into the model structure, thereby producing a more rich and systematic analysis of the macroeconomic dynamics at play.

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Appendix

Table A1

Consumption coefficient estimates from equation (3), with *(p-values)*

$$C_{j;k} = \beta_{C,j,0} + \beta_{C,j,1} C_{j,k-1} + \beta_{C,j,2} G_{j,k-1} + \beta_{C,j,3} C_{j,k-2} + \beta_{C,j,4} RER_{j,k-1} + \beta_{C,j,5} RER_{j,k-2} + \omega_{C,j,k-1}$$

j	*Quarters*	$\beta_{C,j,0}$	$\beta_{C,j,1}$	$\beta_{C,j,2}$	$\beta_{C,j,3}$	$\beta_{C,j,4}$	$\beta_{C,j,5}$	R^2
1	2 to 4	-111.08 *(0.1618)*	1.9374 *(0.0000)*	0.0498 *(0.0011)*	-0.9478 *(0.0000)*	1.2518 *(0.5886)*	-0.6029 *(0.7839)*	0.9996
2	4 to 8	-102.32 *(0.1283)*	1.9594 *(0.0000)*	0.0657 *(0.0004)*	-0.9690 *(0.0000)*	1.8540 *(0.4695)*	-1.2730 *(0.6014)*	0.9997
3	8 to 16	-94.07 *(0.1453)*	1.9424 *(0.0000)*	0.0325 *(0.0182)*	-0.9488 *(0.0000)*	1.8555 *(0.4022)*	-1.2478 *(0.5550)*	0.9998
4	16 to 32	-77.19 *(0.3218)*	1.8454 *(0.0000)*	0.0269 *(0.1918)*	-0.8506 *(0.0000)*	-2.3360 *(0.2984)*	2.8651 *(0.1885)*	0.9900
5	32 to 64	-110.23 *(0.1041)*	1.8445 *(0.0000)*	0.1059 *(0.0000)*	-0.8660 *(0.0000)*	-1.4095 *(0.5161)*	1.5700 *(0.4607)*	0.9900

Table A2

Investment coefficient estimates from equation (4), with *(p-values)*

$$I_{j;k} = \beta_{I,j,0} + \beta_{I,j,1} Y_{j,k-1} + \beta_{I,j,2} d_{irUS,j,k-1} + \omega_{I,j,k-1}$$

j	*Quarters*	$\beta_{I,j,0}$	$\beta_{I,j,1}$	$\beta_{I,j,2}$	R^2
1	2 to 4	-721.02 *(0.0000)*	0.2209 *(0.0000)*	-75.2372 *(0.6180)*	0.64
2	4 to 8	-678.68 *(0.0000)*	0.2172 *(0.0000)*	-143.8609 *(0.2010)*	0.65
3	8 to 16	-584.73 *(0.0000)*	0.2090 *(0.0000)*	-148.9494 *(0.0204)*	0.70
4	16 to 32	-485.83 *(0.0000)*	0.1999 *(0.0000)*	-97.1338 *(0.0003)*	0.81
5	32 to 64	-503.90 *(0.0000)*	0.2018 *(0.0000)*	-63.7730 *(0.0164)*	0.83

Table A3

Net Export coefficient estimates from equation (6), with *(p-values)*

$$NX_{j;k} = \beta_{NX,j,0} + \beta_{NX,j,1}\, NX_{j,k-1} + \beta_{NX,j,2}\, Y^{US}_{j,k-1} + \beta_{EX,j,3}\, Y^{f}_{j,k-1} + \beta_{NX,j,4}\, RER_{j,k-1} + \beta_{NX,j,5}\, RER_{j;k-2} + \omega_{EX\ j,k-1}$$

j	*Quarters*	$\beta_{NX,j,0}$	$\beta_{NX,j,1}$	$\beta_{NX,j,2}$	$\beta_{NX,j,3}$	$\beta_{NX,j,4}$	$\beta_{NX,j,5}$	R^2
1	2 to 4	69.02 *(0.4762)*	0.9716 *(0.0000)*	-0.0229 *(0.0052)*	0.02 *(0.0071)*	1.02 *(0.7415)*	-2.1956 *(0.4663)*	0.977
2	4 to 8	72.22 *(0.4259)*	0.9732 *(0.0000)*	-0.0199 *(0.0119)*	0.02 *(0.0186)*	3.83 *(0.3395)*	-4.9196 *(0.2068)*	0.980
3	8 to 16	139.56 *(0.1001)*	0.9830 *(0.0000)*	-0.0128 *(0.1067)*	0.01 *(0.1895)*	4.17 *(0.1959)*	-5.5679 *(0.0752)*	0.984
4	16 to 32	60.89 *(0.4817)*	0.9717 *(0.0000)*	-0.0191 *(0.0268)*	0.02 *(0.0496)*	0.74 *(0.7744)*	-1.6289 *(0.5134)*	0.980
5	32 to 64	-27.35 *(0.7130)*	0.9282 *(0.0000)*	-0.0252 *(0.0007)*	0.03 *(0.0014)*	0.58 *(0.8215)*	-0.9000 *(0.7212)*	0.965

Table A4

Real Exchange Rate coefficient estimates from equation (7), with *(p-values)*

$$RER_{j;k} = \beta_{RER,j,0} + \beta_{RER,j,1}\, ir^{US}_{j,k-1} + \beta_{RER,j,2}\, ir^{f}_{j,k-1} + \beta_{RER,j,3}\, RER_{j;k-1} + \omega_{RER\ j,k-1}$$

j	*Quarters*	$\beta_{RER,j,0}$	$\beta_{RER,j,1}$	$\beta_{RER,j,2}$	$\beta_{RER,j,3}$	R^2
1	2 to 4	5.3773 *(0.0000)*	0.3226 *(0.0035)*	-0.1664 *(0.1026)*	0.9402 *(0.0000)*	0.976
2	4 to 8	4.4362 *(0.0000)*	0.3598 *(0.0000)*	-0.2201 *(0.0048)*	0.9505 *(0.0000)*	0.987
3	8 to 16	5.0791 *(0.0000)*	0.4059 *(0.0000)*	-0.2447 *(0.0058)*	0.9434 *(0.0000)*	0.985
4	16 to 32	5.7034 *(0.0000)*	0.5318 *(0.0000)*	-0.3428 *(0.0006)*	0.9371 *(0.0000)*	0.980
5	32 to 64	6.3675 *(0.0000)*	0.5775 *(0.0000)*	-0.4015 *(0.0001)*	0.9318 *(0.0000)*	0.972

Table A5

Empirical Estimation of the *Modified Smooth* Trend Series

	GDP		*Consumption*		*Investment*		*Government Spending*	
	$s_{Y,1}$	$s_{Y,2}$	$s_{C,1}$	$s_{C,2}$	$s_{I,1}$	$s_{I,2}$	$s_{G,1}$	$s_{G,2}$
Coefficient	0.9822	0.0221	0.9929	0.0115	0.9721	0.0328	0.9803	0.0238
(*p-value*)	*(0.0000)*	*(0.1230)*	*(0.0000)*	*(0.2936)*	*(0.0000)*	*(0.0487)*	*(0.0000)*	*(0.0260)*

	Net Exports		*RER*		*Interest rate* (ir^{US})	
	$s_{NX,1}$	$s_{NX,2}$	$s_{RER,1}$	$s_{RER,2}$	$s_{irUS,1}$	$s_{irUS,2}$
Coefficient	0.9821	0.0263	0.9024	0.0961	0.8752	0.1272
(*p-value*)	*(0.0000)*	*(0.0818)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*

Table A6

Inflation coefficient estimates from equation (12), with *(p-values)*

$$inf_k = \beta_{inf,0} + \beta_{inf,1}\, inf_{j,k-1} + \beta_{inf,2}\,(Y_{k-1} - Y^*_{k-1}) + \beta_{inf,3}\, RER_{k-1} + \beta_{inf,4}\, MS_{k-1} + \beta_{inf,5}\, inf_{k-2} + \omega_{inf,k-1}$$

	$\beta_{inf,0}$	$\beta_{inf,1}$	$\beta_{inf,2}$	$\beta_{inf,3}$	$\beta_{inf,4}$	$\beta_{inf,5}$	R^2
Coefficient	0.002614	1.454326	0.000017	-0.000117	0.023288	-0.498588	0.96
(p-value)	*(0.9938)*	*(0.0000)*	*(0.7940)*	*(0.9743)*	*(0.1613)*	*(0.0000)*	

Table A7

Real Money Growth coefficient estimates from equation (13), with *(p-values)*

$$MS_k - inf_k = \beta_{MS,0} + \beta_{MS,1}\,(ir^{US}_{k-1} - inf_{k-1}) + \beta_{MS,2}\,(Y_{k-1} - Y^*_{k-1}) + \beta_{MS,3}\,(MS_{k-1} - inf_{k-1}) + \beta_{MS,4}\,(MS_{k-2} - inf_{k-2}) + \omega_{inf,k-1}$$

	$\beta_{MS,0}$	$\beta_{MS,1}$	$\beta_{MS,2}$	$\beta_{MS,3}$	$\beta_{MS,4}$	R^2
Coefficient	0.458430	-0.043637	0.000016	1.374078	-0.507528	0.87
(p-value)	*(0.0004)*	*(0.2548)*	*(0.9121)*	*(0.0000)*	*(0.0000)*	

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