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Tax Progression is Good for Employment in Popular Models of Trade Union Behaviour

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Summary

It is a widely held popular belief that the more progressive is the tax system, the greater is the disincentive to work effort. This belief can be justified within the context of conventional labour supply analysis. Increased progression with unchanged tax revenues decreases work effort and is thus bad for employment But does it hold in unionized economies, where trade unions play their role in wage and employment determination? Using three popular models of trade union behaviour – the monopoly union, the 'right-to-manage' and the efficient bargain model – as the framework for analysis this paper provides an unambiguously negative answer; under plausible assumptions an increased tax progression lowers wages and is good for employment in all three popular models of trade union behaviour. This means that effects of taxation appear to be very sensitive to the structure of labour markets.

Tiivistelmä

Melko yleisesti omaksutun käsityksen mukaan – joka voidaan perustella kuluttajan valintateorian avulla – verotuksen progression kasvu annetulla verotulokertymällä vähentää työnteon kannustimia ja työllisyyttä. Pitääkö tämä käsitys paikkaansa talouksissa, joissa palkat eivät määräydy kysynnän ja tarjonnan mukaan, vaan ne joko asetetaan tai niistä neuvotellaan AY-liikkeen ja yritysten välillä? Paperissa osoitetaan käyttäen analyysikehikkoina kolmea tunnettua AY-liikemallia – monopoliunioni-, "right-to-manage"- ja tehokkaiden sopimusten mallit – ettei näin ole asianlaita; luontevilla oletuksilla verotulokertymän säilyttävä veroprogression lisäys alentaa palkkoja ja parantaa työllisyyttä! Verotuksen vaikutukset näyttävät siis olevan herkkiä työmarkkinoiden rakenteesta tehtäville oletuksille.

Contents

1 Introduction

It is a widely held popular belief that the more progressive is the tax system, the greater is the disincentive to work effort. The question of how progressivity affects work effort is a complex one and can be formulated in a number of different ways depending on the definition of progression and the basis on which alternative tax systems are compared. In isolating the effect of increased progressivity as such it is desirable to assume that the average tax rate could in some sense be held constant. One alternative is to assume that progression is increased subject to the constraint that the "real income" of workers does not change. An alternative standard would be that of constant tax revenue. Under both standards and plausible assumptions increased tax progression does in fact decrease work effort (see e.g. Sandmo (1983)).[1] In both cases the qualitative effect of increased progressivity depends on the negative substitution effect of the marginal tax rate on labour supply.

But one can argue that in unionized economies it is a bad approximation to assume that wages are determined by equality of demand for and supply of labour, instead they are subject to bargaining. What are the wage and employment effects of increased progressivity under these circumstances? Is it still true that increased progression is bad for employment? This paper addresses the question by using three popular models of trade union behaviour as the framework for analysis. Perhaps surprisingly, the answer turns out to be totally different from the conventional one. Under plausible assumptions a strong result can be obtained; increased progression unambiguously leads to lower wage rate and to higher employment in all cases! Effects of taxation appear to be very sensitive to the structure of labour markets.

The paper is organized as follows: Section 2 presents three popular models of trade union behaviour, the monopoly union, the "right-to-manage" and the efficient bargaining models of wage and employment determination. The wage and employment effects of increased tax progression are developed in section 3.

2 Wages, employment and taxes in popular models of trade union behaviour

In the recent trade union literature three approaches have dominated the debate on trade union behaviour, namely the monopoly union, "right-to-manage" and efficient bargain models (see Oswald (1985) and Manning (1987) for an introductory survey and some elaborations of the literature respectively). This section presents these models and incorporates the tax system together with relevant comparative statics.

[1] Sandmo (1983) has extended a representative worker model to that of many workers and shown that in order for increased progression to reduce labour supply the negative substitution effects must dominate the income effects resulting from redistribution of income over workers. For this to be true does not seem to require any peculiar assumptions about preferences.

2.1 The monopoly union model

To keep the model simple, let the trade union fix the wage and assume that firms set employment unilaterally. Employment is determined by maximizing $\Pi = f(L) - wL$ in terms of L. With strictly concave revenue function ($f' > 0$, $f'' < 0$) this gives the labour demand $L = L(w)$ with $L' < 0$, where w is the wage rate. The revenue function summarizes the technology of the firm and the demand function for the product jointly. If the technology is of the form g(L) with $g' > 0$, $g'' < 0$ and the demand function of the constant elasticity type $D = Ap^{-\varepsilon}$ with $\varepsilon > 1$, then $f(L) = pD = g(L)^{1-1/\varepsilon}A^{1/\varepsilon}$. The revenue function is concave because $f' = Fg' > 0$ and $f'' = F(g'' - (g')^2(\varepsilon g)^{-1}) < 0$ where $f = (1 - 1/\varepsilon)g^{-1/\varepsilon}A^{1/\varepsilon}$.

Assume that the trade union utility function is of the utilitarian form

$$V = (u(w(1-t) + ta) - u^0)L \tag{1}$$

where $u'(.) > 0$, $u''(.) < 0$, t = the constant marginal tax rate, a = level of tax exemption and $u^0 = u(b)$ = valuation of leisure or outside option. Under this regime the tax revenues of government are

$$T = t(w-a)L \quad \text{if } w-a>0 \tag{2}$$

The monopoly union chooses the wage rate so as to maximize (1) subject to labour demand constraint $\Pi_L = 0$. This gives

$$V_w = 0 = (1-t)u'(.)L + (u(.)-u^0)L_w \tag{3}$$

Provided that the second-order condition $V_{ww} < 0$ holds, the first-order condition (3) implicitly defines the wage rate as a function of tax parameters so that $w = w(t,a)$. Substituting the wage function for w in (1) gives the indirect utility function in terms of tax parameters, $V^*(t,a) = v$. Using the envelope theorem one gets

$$\begin{aligned} &\text{(i)} \quad V^*_t = -u'(.)(w-a)L < 0 \\ &\text{(ii)} \quad V^*_a = u'(.)tL > 0 \end{aligned} \tag{4}$$

Not surprisingly, marginal tax rate will affect negatively and tax exemption positively the maximum utility. Inverting the indirect utility function for a in terms of t and v gives $a = g(t,v)$ and substituting this function for a in the indirect utility function yields

$$V^*(t, g(t,v)) = v \tag{5}$$

This compensated indirect utility function (see e.g. Diamond and Yaari (1972)) answers the following question: If the marginal tax rate is increased, how much tax exemption has to be changed so as to keep the "real income" of the trade union unchanged? By differentiating (5) with respect to t gives $V_t^* + V_a^* g_t = 0$ so that

$$g_t = -V_t^*/V_a^* = (w-a)/t > 0 \tag{6}$$

Moreover, it is known that

$$w(t, g(t, v)) = w^c(t, v) \tag{7}$$

where w^c is the compensated wage function, which gives the minimum wage rate to achieve a given level of utility v at the marginal tax rate t. Differentiating the identity (7) with respect to the marginal tax rate gives $w_t^c = w_t + w_a g_t$ so that we have

$$w_t = w_t^c - w_a(w-a)/t \tag{8}$$

This is the Slutsky equation for the wage rate, according to which the total effect of the marginal tax rate can be decomposed into the substitution effect and income effect. It is straightforward to show that the income effect $(-w_a(w-a)/t)$ is positive, while the substitution effect w_t^c is negative so that the total effect is ambiguous a priori. Under fairly reasonable assumptions, however, the total effect is positive.[2]

2.2 The "right-to-manage" model

Like in the monopoly union case employment is determined unilaterally by firms, but now the wage is assumed to be determined in a bargain between the trade union and the firms. If we represent the outcome of this bargaining by an asymmetric Nash bargaining with β representing the power of the trade union, and abstract from the threat point of firm, the "right-to-manage" model is

$$\max_w U = (V - V^0)^\beta \Pi^{1-\beta} \qquad \text{subject to } \Pi_L = 0 \tag{9}$$

where $V - V^0 = (u(w(1-t)+ta) - u^0))L$. For simplicity it is assumed that the threat point of the firms is zero. The first-order condition for the maximization is

[2] Roughly this means that the average tax rate has a positive effect on the wage rate, which lies in conformity with empirical evidence. A complete set of results is available from the authors upon request. This result, though in a slightly different context, has been derived earlier by Hersoug (1984).

$$U_w = \beta \Pi V_w + (1-\beta)(V-V^0)\Pi_w = 0 \tag{10}$$

Given that the second-order condition $U_{ww} < 0$ holds, (10) defines implicitly the wage function in terms of tax parameters and bargaining power so that $w = w(t,a;\beta)$. Like earlier one can define the indirect Nash maximand $U^*(t,a;\beta)$ and use the envelope theorem to give $U_t^* = BV_t^* < 0$ and $U_a^* = BV_a^* > 0$, where V_t^* and V_a^* have been defined in section 2.1 and where $B = \beta(V-V^0)^{\beta-1}\Pi^{1-\beta} > 0$. Given $U_a^* > 0$, the indirect Nash maximand can be inverted for a so that we have $a = h(t,v_0;\beta)$, where v_0 denotes the maximum value for U. If we substitute this for a in the indirect Nash maximand we get the compensated indirect Nash maximand. It is easy to show, following the arguments presented in section 2.1, that the total effect can be decomposed into the Slutsky equation as follows $w_t = w_t^c - w_a(w-a)/t$.

As for the comparative statics of the components of the Slutsky equation we have $w_a = (-U_{ww})^{-1}U_{wa}$, where $U_{wa} = \beta\Pi V_{wa} + (1-\beta)\Pi_w V_a < 0$ because $V_{wa} < 0$, $V_a > 0$ and $\Pi_w < 0$. Thus also in the "right-to-manage" model a rise in tax exemption decreases the wage rate. The compensated effect of a change in the marginal tax rate is in turn $w_t^c = (-U_{ww})^{-1}(-\beta\Pi u'(.)L) < 0$ so that the substitution effect of the marginal tax rate is negative. Though the "right-to-manage" model looks more realistic than the monopoly union model, its comparative statics is qualitatively similar to that of the monopoly union model. Thus this might be preferred to the "right-to-manage" model on the grounds of Occam's Razor.

2.3 The efficient bargain model

The monopoly union and "right-to-manage" models have been critisized due to the inefficiency of equilibrium. The outcome does not lie on the bargaining contract curve; moving from the earlier derived solutions, both parties can gain. This is a starting point of the efficient bargain model of trade union behaviour, in which the trade union can bargain about employment as well as the wage. A typical efficient bargain model would be

$$\max_{(w,L)} U = (V-V^0)^\beta \Pi^{1-\beta} \tag{11}$$

The first-order conditions can be expressed as follows

$$\begin{aligned} &\text{(i)} \quad U_w = \beta B V_w + (1-\beta)C\Pi_w = 0 \\ &\text{(ii)} \quad U_L = \beta B V_L + (1-\beta)C\Pi_L = 0 \end{aligned} \tag{12}$$

where $B = (V-V^0)^{\beta-1}\Pi^{1-\beta} > 0$ and $C = (V-V^0)^\beta\Pi^{-\beta} > 0$. The first-order conditions can be combined to give the contract curve in the (w,L) space in the following form

$$(u(.)-u^0)/(1-t)u'(.) = w - f'(L) \tag{13}$$

Along the contract curve w–f'(L) > 0 so that firms are off the labour demand curve. Like in the symmetric information version of the implicit contract theory (Azariadis (1975)), the firms are being induced to employ more workers than they would like at the agreed-upon wage.[3] The slope of the contract curve can be obtained by differentiating (13)

$$(dw/dL) = -f''(L)/(1-t)A(.)(w-f'(L)) > 0 \tag{14}$$

where $A(.) = -u''(.)/u'(.) > 0$ is the Arrow-Pratt absolute risk aversion measure. Thus in the case of a utilitarian union the contract curve is upward-sloping in the (w,L) space. It can be seen from the equation (14) that if the marginal utility of the income of the trade union were constant ($A(.) = 0$), then the contract curve would be vertical. In that case the opportunity cost of labour is unaffected by the wage rate. This special case is analyzed in Hall and Lilien (1979).

Given that the second-order conditions hold the first-order conditions (12) define implicitly the efficient wage rate and employment in terms of tax parameters and bargaining power so that $w^* = w^*(t,a;\beta)$ and $L^* = L^*(t,a;\beta)$. Substituting these into the maximand U gives the indirect Nash maximand $U^* = U^*(t,a;\beta) = u^*$. Again the envelope theorem implies that $U^*_t = BV^*_t < 0$ and $U^*_a = BV^*_a > 0$, where the terms have been presented in section 2.2. Because of $U^*_a > 0$ the indirect Nash maximand can be inverted for a so that $a = j(t,u^*;\beta)$ and we get the compensated indirect Nash maximand $U^*(t,j(t,u^*;\beta);\beta) = u^*$. It is known that the uncompensated wage rate and employment functions are related to the compensated ones as follows

$$\begin{aligned} &\text{(i)} \quad w^*(t,j(t,u^*;\beta)) = w^{*c}(t,u^*) \\ &\text{(ii)} \quad L^*(t,j(t,u^*;\beta)) = L^{*c}(t,u^*) \end{aligned} \tag{15}$$

By differentiating these equations with respect to t and utilizing the envelope results according to which $j_t = (w-a)/t$ yields the Slutsky equations for the wage

[3] The outcome of the efficient bargain model that firm's are off their labour demand curve has been criticized on various grounds: (i) efficient bargain model is not incentive compatible; firms always have an incentive, once the wage rate is fixed, to renege by jumping to the labour demand curve. (ii) if layoffs are not random, but by seniority ("last in, first out"), then more seniority workers become indifferent to the total level of employment and the efficient solution lies on the demand curve after all (Oswald (1985)). Recently Oswald (1993) has argued convincingly in favour of this kind of seniority model of trade union on empirical grounds. If the seniority model is accepted, then we are back in the "right-to-manage" model. And Occam's Razor brings us back to the monopoly union model after all! (iii) it has been argued that with labour turnover the situation eventually converges to the one, where the trade union is no longer concerned about employment; firms hire enough workers to make up for those who leave and the efficient bargain is on the labour demand curve again (see Layard and Nickell and Jackman (1991), p. 112–118).

rate and employment in the efficient bargain context. They are of the following form

$$\text{(i)} \quad w_t = w_t^c - w_a(w-a)t^{-1} \qquad (16)$$

$$\text{(ii)} \quad L_t = L_t^c - L_a(w-a)t^{-1}$$

These Slutsky equations decompose the total effect into the substitution and income effects. Though simple, they contain elements both from trade union and firm behaviour and are complex.[4] One can show after a considerable, though straightforward, manipulations the following results (see the appendix for the details), which are new to our knowledge:

First, the effect of tax exemption has a negative effect both on the wage rate and on employment (w_a, $L_a < 0$). This similar sign is due to the upward sloping contract curve. E.g. a rise in tax exemption works like as an income effect by shifting the indifference curve of the trade union inwards. This is illustrated in Figure 1, where L^d describes the labour demand, b outside option for the trade union, Π_i the iso-profit curve of the firms and I_i the indifference curve for the trade union. CC denote the contract curve.

Second, the substitution effect of the tax rate is negative for wages ($w_t^c < 0$) and positive for employment ($L_t^c > 0$), even though the contract curve is upward-sloping. This is due to the fact that e.g. a compensated rise in the tax rate will change the slope of the indifference curve of the trade union in the clockwise direction. Hence, the wage rate falls and employment increases (see Figure 2). The substitution effect is obtained by keeping the Nash maximand U and the trade union objective function $V - V^0$ as given. Hence, the new indifference curve I^1 is targential to the original iso-profit curve Π^0 at B.

Third, and finally, the total effect of the tax rate on wages consists of the negative substitution and positive income effect ($-w_a(w-a)t^{-1} > 0$), which run counter to each other, while in the case of employment the substitution and income effect ($-L_a(w-a)t^{-1} > 0$) reinforce each other. This, in the efficient bargain model a rise in the marginal tax rate increases employment, despite the fact that its effect on the wage rate is a priori ambiguous. Under plausible assumptions, however, the wage rate rises.[5] Geometrically this is illustrated in Figures 3 and 4 respectively. E.g. a rise in the tax rate both shifts the indifference curve outward and will change its slope in the clockwise direction. What happens to the level of profits of the firm is unclear. In figures 3 and 4 it has been assumed that it increases.

[4] Comparative statics of the efficient bargain model is not very much studied. See however McDonald and Solow (1981), who studied how the contract curve is affected by an improvement in the firm's product market.

[5] A complete set of results is available from the authors upon request.

Figure 1. **A rise in tax exemption**

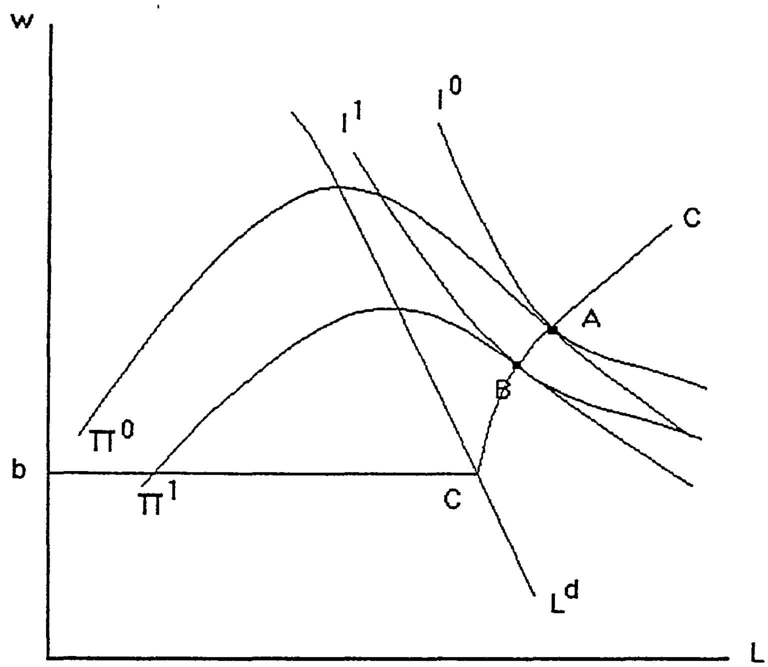


Figure 2. **Substitution effect of a rise in the marginal tax rate**

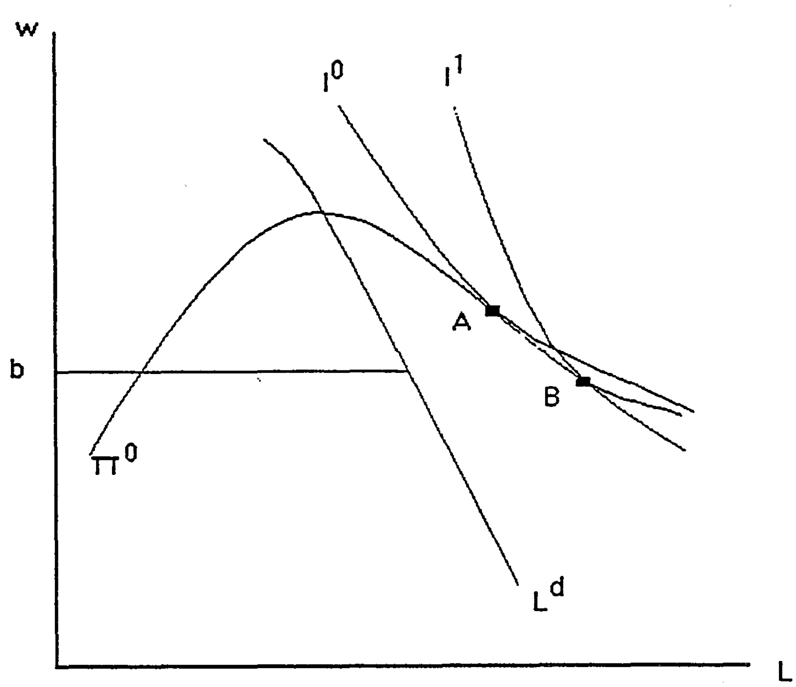


Figure 3. **Uncompensated effect of a rise in the marginal tax rate. The wage rate increases**

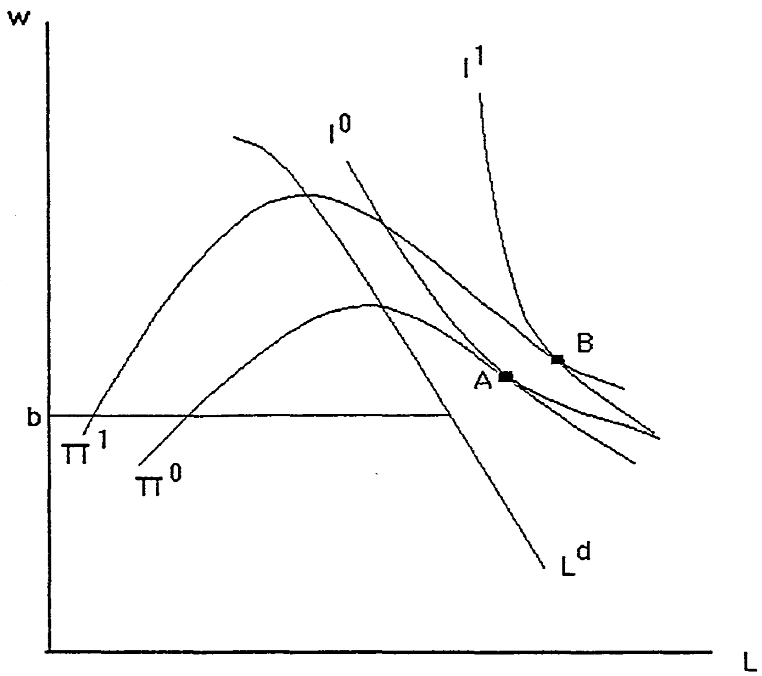
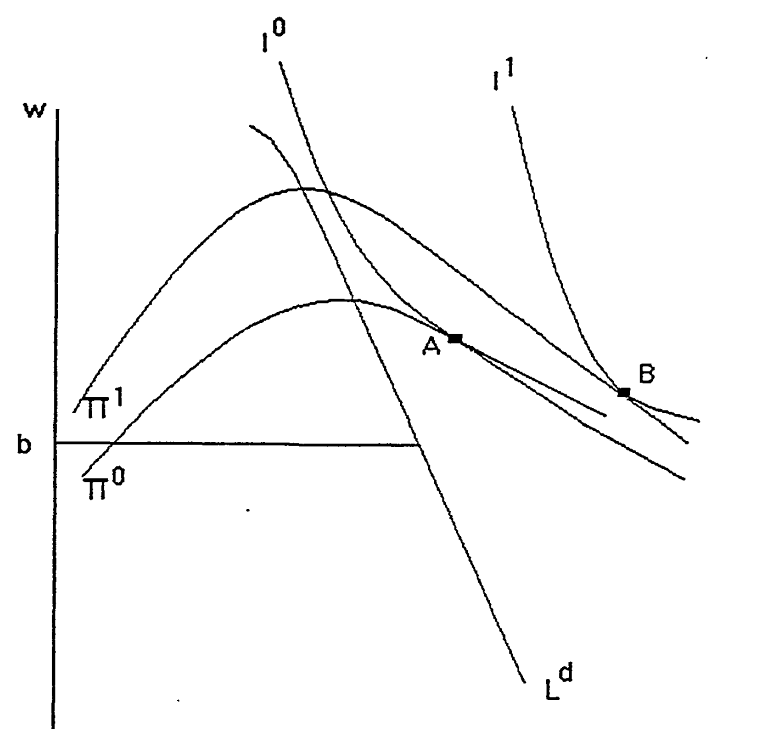


Figure 4. **Uncompensated effect of a rise in the marginal tax rate. The wage rate decreases**



3 Increased tax progression, wages and employment

Let us now turn to develop the implications of a rise in progressivity for the wage rate and employment in the earlier presented models of trade union behaviour. Before proceeding one should fix the standard for changes in tax progressivity. It might be tempting, but wrong, to argue that the Slutsky equations for the wage rate (and employment in a efficient bargain) conveys everything that one has to say about the effects of increased progression. Namely, the Slutsky equations are results that apply to a simultaneous increase in the marginal as well as average rate of tax. In isolating the effects of increased progressivity as such that the average tax rate should in some sense be held constant. More specifically, the marginal tax rate is increased and it is compensated by a change in the tax exemption a so that the tax revenue $T = t(w-a)L$ does not change.[6] It can be seen that simultaneous increases in t and a make the tax schedule more progressive.[7] This can be regarded as the pure change in progressivity <u>in the ex post sense</u>.

In the case of the <u>monopoly union</u> the tax parameters have no direct effect on L so that we have $dT = 0 = (w-a)Ldt - tLda + tLdw$. This can be written as $da = (w-a)t^{-1}dt + dw$ as $dT = 0$. On the other hand, $dw = w_t dt + w_a da$. Substituting the r.h.s. of the da-expression for da in the total differential for the wage rate gives

$$(dw/dt)_{dT=0} = (1 - w_a)^{-1} w_t^c < 0 \qquad (17)$$

Hence, an increased progresion decreases the wage rate and increases employment via the labour demand. A smaller share of increase in the wage rate will be kept by the workers. That makes it beneficial for the trade union to want more employment for the lower after-tax wage rate. Tax progression is good for employment![8]

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[6] It is easy to check that this tax schedule, given positive values of t and a, is progressive according to either of the following three definitions of progressivity, suggested by Musgrave and Thin (1948)): (i) the average tax rate is increasing with the wage rate, (ii) the elasticity of the tax function with respect to income before tax is greater than one, (iii) the elasticity of income after tax with respect to income before tax is less than one.

[7] In the earlier analyses the tax parameters have been taken as given and it has been assumed that the trade unions do not perceive any connection between the taxes and benefits. The analysis is changed in details if there is a perception of the link between taxes paid and benefits received (for an interesting study along these lines, see Summers and Gruber and Vergara (1993)).

[8] Hersoug (1984) has also studied the wage effect of a "pure" change in progressivity in the monopoly union model from ex ante viewpoint. He has shown with a non-linear tax function a more limited result; a "pure" rise in progression so as to keep the average tax rate unchanged at the initial wage rate will decrease the wage rate. This is the same as the pure change in progressivity <u>in the ex ante sense</u>, i.e. the substitution effect of the marginal tax rate. Namely, the combination of changes in t and c, which keeps T uncharged for given w and L is $da = (w-a)t^{-1}dt$ so that $(dw/dt) = wt + wa(w-a)t^{-1} = w_t^c < 0$, as $dT = 0$ for <u>given</u> w and L.

In the "right-to-manage" model the wage rate is determined in a bargain between the trade union and the firms, while employment is still determined unilaterally by firms. The compensated tax rate change is similar than in the monopoly union case. Hence, the expression (17) is relevant in this case as well; an increased progression by decreasing the wage rate raises for employment.

The analysis of the effects of increased progression in the efficient bargain model is much more complicated, because tax parameters have direct effects not only on the wage rate but also on employment as well. The total effects of a change in the marginal tax rate and tax exemption on the wage rate and employment are $dw = w_t dt + w_a da$ and $dL = L_t dt + L_a da$ respectively. In order to get the change of tax parameters, which will keep the tax revenue $T = t(w-a)L$ constant, we differentiate totally T with respect to L,w,t and a given $dT = 0$. This yields

$$da = (w-a)t^{-1}dt + dw + (w-a)L^{-1}dL \qquad \text{as } dT=0 \tag{18}$$

Now one can substitute the r.h.s. of (18) for da in the total differentials for dw and dL. Doing this and utilizing the Slutsky decompositions (16) gives a two-equation system for wages and employment

$$\begin{aligned} &\text{(i)} \quad (1-w_a)dw - w_a(w-a)L^{-1}dL = w_t^c dt \\ &\text{(ii)} \quad -L_a dw + (1-L_a(w-a)L^{-1})dL = L_t^c dt \end{aligned} \tag{19}$$

Solving this gives the effects of a compensated change in the tax rate on the wage rate and employment. The total tax revenue-neutral effects can be decomposed as follows

$$(dw/dt)_{dT=0} = \Psi^{-1}(w_t^c(1-L_a(w-a)L^{-1}) + L_t^c w_a(w-a)L^{-1}) \tag{20}$$

and

$$(dL/dt)_{dT=0} = \Psi^{-1}(L_t^c(1-w_a) + w_t^c L_a) \tag{21}$$

where $\Psi = 1-w_a - L_a(w-a)L^{-1}$.

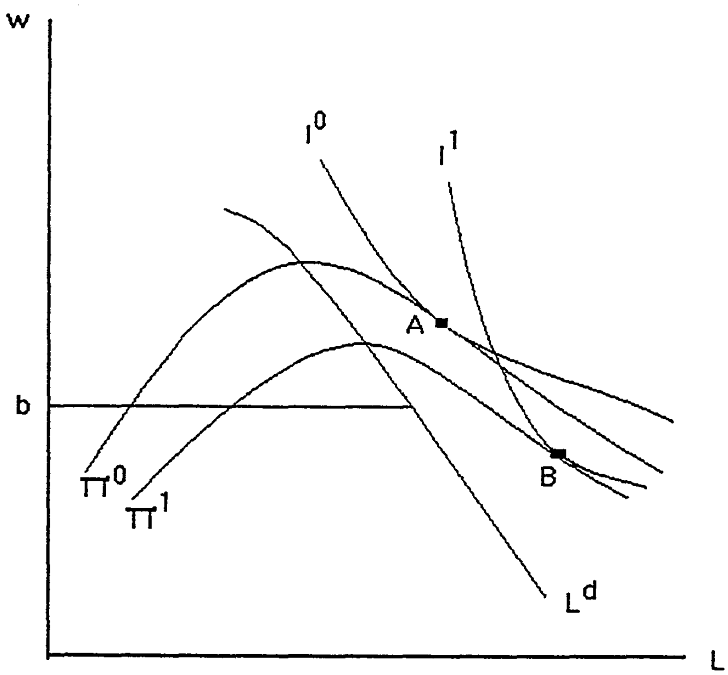
Signing expressions (20) and (21) requires signing their common denominator Ψ, which depends on the relationship between tax revenues T and tax exemption parameter a. The relationship between tax revenues and tax rates is sometimes called in the literature the Dupuit-Laffer curve. Tax parameters affect tax revenues both directly and indirectly via behavioural responses. If the direct effects dominate, then the relationship between tax revenues and taxes is positive so that the Dupuit-Laffer curve is upward-sloping. We proceed by that assumption.[9] This implies that tax revenues are negatively related to tax exemption parameter a and more importantly that $\Psi > 0$. Proof is the following; take first the total differential of $T = t(w-a)L$ with respect to w,L and a, which yields $(dT/da) = -tL + t(w-a)(dL/da) + tL(dw/da)$. Since we are interested in the relationship between T and a, we substitute partial derivatives for total differentials so as to get

$$T_a = -tL\Psi < 0 \tag{22}$$

so that $\Psi > 0$ if the Dupuit-Laffer curve is upward-sloping.

We are now in the position to fix the wage and employment effects of the tax revenue-neutral change in t, i.e. the signs of the expressions (20) and (21). With their positive denominator ($\Psi > 0$), the signs depend on the numerator terms. As for wages the first r.h.s. term $w_t^c(1-L_a(w-a)L^{-1})$ is negative as well as the second r.h.s term $L_t^c(w_a(w-a)L^{-1})$. Thus the wage rate falls. As for the employment term the first r.h.s. term $L_t^c(1-w_a)$ is positive as well as the second term $w_t^c L_a$. Thus employment increases. This is illustrated in Figure 5. Arise in the tax rate will shift both the indifference curve of the trade union outwards and change its slope in the clockwise direction. This is offsetted by a rise in tax exemption, which shifts trade union indifference curve inward. What happens to $V - V_0$ and Π is unclear. It is assumed in Figure 5 that Π increases.

Figure 5. **A tax revenue-neutral rise in progression**



[9] Empirically this is a good assumption, for a historical survey of the literature about the relationship between tax rates and government tax revenues and an empirical analysis with U.S. data, see Fullerton (1982).

We have established also in the context of the efficient bargain model that the increased tax progression decreases the wage rate and is favourable for employment. This result is valid under utilitarian trade union and strictly concave revenue function and thus does not require any peculiar assumptions.

Appendix

Comparative statics of the efficient bargaining model

The Nash maximand is reproduced here for convenience

$$U=(V-V^0)^{\beta}\Pi^{1-\beta} \tag{A1}$$

where

$$V-V^0=\{u[w(1-t)+ta]-u^0\}L,\ u^0=u(b)$$

$$\Pi=f(L)-wL,\ \ f'>0, f''<0$$

The necessary first order conditions for the (unique) maximum of (A1), $U_W = 0$ and $U_L = 0$ respectively, are

$$\beta\Pi V_w+(1-\beta)(V-V^0)\Pi_w=0 \tag{23}$$

$$\beta\Pi V_L+(1-\beta)(V-V^0)\Pi_L=0 \tag{24}$$

which, together, amount to the familiar equation for the contract curve

$$\frac{\Pi_w}{\Pi_L}=\frac{V_w}{V_L}\quad \text{i.e.}\quad \Pi_w V_L=\Pi_L V_w \tag{25}$$

Now, note that (25) implies

$$[V_{La}da+V_{Lt}dt]\Pi_w=[V_{wa}da+V_{wt}dt]\Pi_L \tag{26}$$

In order to analyze the effects of a and t on w and L in an efficient bargaining framework, we use Cramer's rule. First of all we have

$$
\begin{bmatrix} \beta\Pi V_{WW}+V_W\Pi_W & \beta\Pi V_{WL}+\Pi_W V_L+(1-\beta)(V-V^0)\Pi_{WL} \\ \beta\Pi V_{WL}+\Pi_W V_L+(1-\beta)(V-V^0)\Pi_{WL} & V_L\Pi_L+(1-\beta)(V-V^0)\Pi_{LL} \end{bmatrix} \begin{bmatrix} dw \\ dL \end{bmatrix} =
$$

$$
-\begin{bmatrix} \beta\Pi V_{Wa}+(1-\beta)V_a\Pi_W & \beta\Pi V_{Wt}+(1-\beta)V_t\Pi_W \\ (\frac{\Pi_L}{\Pi_W})(\beta\Pi V_{Wa}+(1-\beta)V_a\Pi_W) & (\frac{\Pi_L}{\Pi_W})(\beta\Pi V_{Wt}+(1-\beta)V_t\Pi_W) \end{bmatrix} \begin{bmatrix} da \\ dt \end{bmatrix} \tag{27}
$$

where use has been made of equation (26) in deriving the r.h.s. matrix. By the second-order sufficient conditions for maximum, the determinant of the l.h.s. matrix is positive. Now, $B\Pi V_{WL} + \Pi_W V_L + (1-\beta)\Pi_{WL} = \beta\Pi(1-t)u' - (2-\beta)(u-b)L$, and by the f.o.c. $U_W = 0$ we have $\beta\Pi(1-t)u' = (1-\beta)(V-V^0)$, i.e. $\beta\Pi(1-t)u' = (1-\beta)(u-u^0)L$. Thus $\beta\Pi(1-t)u' - (2-\beta)(u-u^0)L = -(u-u^0)L$. Solving (27) gives

$$
w_a = \frac{\partial w}{\partial a} = -|\Omega|^{-1}(\beta\Pi V_{wa} + V_a\Pi_w)(1-\beta)(u-u^0)Lf'' < 0 \tag{28}
$$

$$
L_a = \frac{\partial L}{\partial a} = -|\Omega|^{-1}(\beta\Pi V_{wa} + V_a\Pi_W)\beta\Pi V_{ww} < 0 \tag{29}
$$

and where Ω denotes the matrix on the l.h.s. of equation (27) and $|.|$ signifies its determinant. So increases in the tax exemption parameter a reduces both wages and employment. Turning to the effects of the marginal tax rate note first that

$$
\begin{aligned} U_{wt} &= U^c_{wt} - U_{wa}(w-a)t^{-1} \\ U_{Lt} &= U^c_{Lt} - U_{La}(w-a)t^{-1} \end{aligned} \tag{30}
$$

where the compensated tax terms, i.e. the tax terms which has no income effects, are

$$
\begin{aligned} U^c_{wt} &= \beta\Pi V^c_{wt} = -\beta\Pi u'L < 0 \\ U^c_{Lt} &= 0 \end{aligned} \tag{31}
$$

Equation (31) implies that the compensated tax effects on wages and employment are

$$w_t^c = |\Omega|^{-1}[u-u^0][(f'-w)+(1-\beta)f''L]\beta\Pi u'L<0$$

$$L_t^c = |\Omega|^{-1}(u-u^0)\beta\Pi u'L^2>0 \tag{32}$$

This is equivalent to

$$L_t^c = L_t + L_a(w-a)t^{-1}>0$$

$$w_t^c = w_t + w_a(w-a)t^{-1}<0 \tag{33}$$

So $L_t > 0$, but w_t is ambiguous a priori.

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