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Conditional Betas and the Price of Risk in a Thin Asset Market: A Sensitivity Analysis

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Abstract

This paper examines the sensitivity of tests of the Sharpe-Lintner Capital Asset Pricing Model (CAPM) to different estimation methods and asset return samples in a thin European asset market, i.e. the Finnish asset market. A time-varying-parameter model is introduced as an alternative to the static market model. We run a regression over a pooled data set in addition to the second-pass Fama-McBeth regressions. Our tests are carried out with four asset specific samples. In every case, cross-sectional OLS estimation of the betas leads to the rejection of the mean-variance efficiency of the market index. The price of market risk is statistically significant, but negative. Our tests on the time-varying betas indicate just the opposite. We are not able to reject the mean-variance efficiency of the market index in any of the samples. The price of market risk is positive and statistically significant for the stock return data set that most closely resemble the normal distribution.

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Contents

1 Introduction

Recently, a number of empirical papers have focused on the time variation of conditional expected returns, variances and covariances in tests of the Sharpe-Lintner CAPM. Almost all conditional tests of the CAPM employ U.S. data. Some empirical papers support the single-period CAPM, whereas others reject it. Most of the tests are conducted using the generalized methods of moments (GMM)[1] or different versions of autoregressive conditional heteroscedasticity (ARCH)[2] models. However, these methods rely on strong assumptions. Nelson (1991) discusses the drawbacks of ARCH models. GMM involves expected parameters conditioned on the true market information set, which is unobservable. We, instead, employ a subset of observable instrumental variables. Further, GMM tests assume that a linear function relates conditional expectations to the information set.

This paper examines the Sharpe-Lintner CAPM in which a time-varying-parameter model is an alternative to the traditional static market model. We demonstrate the crucial role of the time-varying market risk in the thin Finnish asset market. Our cross-sectional regressions on the static OLS betas reject for all asset samples the mean-variance efficiency of the stock market index and imply that the price of market risk would be significantly negative. Our tests on the time-varying betas indicate just the opposite. We are not able to reject the mean-variance efficiency of the market index in any of the samples. The market risk is priced in the manner of the CAPM, most evidently in the stock return data set that most closely resemble the normal distribution. The risk-return relationship is notably weaker in the most traded stocks data set. This finding does not support the prior understanding that the errors-in-variables problem due to the nonsynchronous trading most seriously violates tests of the CAPM.

Prior evidence from unconditional tests of the CAPM generally leads to rejection of the model for all data sets. The unconditional market risk is usually rewarded in U.S. studies but is not rewarded in studies using data from other asset markets. The latter is also the case for studies of the Finnish stock market (see e.g. Korhonen (1977) and Berglund (1986)).

The poor performance of the CAPM in thin markets may be largely a result of serious errors-in-variables problems. Berglund, Liljeblom and Löflund (1989) and Martikainen (1991) show that OLS-betas for thinly traded stocks tend to be downward biased and are little improved by the use of several correction procedures. The same authors and Knif (1989) provide evidence that Finnish firms' betas are not stable. Knif applies the Kalman filter technique in order to model time variation in the market risk. He shows that Finnish common stock betas usually follow a stationary autoregressive process.

Malkamäki (1992) examines the Sharpe-Lintner CAPM in which time-varying-parameter models are alternative to the static market model. The monthly data employed covers all Finnish common stocks listed throughout the period 1972–1989. He computes two alternative rolling beta estimate series, assuming that the betas are constant over a period of three or five years. As an alternative

[1] See e.g. Harvey (1989).

[2] See e.g. Bollerslev, Engle and Wooldridge (1988), Bodurtha and Mark (1991) and Ng (1991).

approach, he applies dynamic OLS and maximum likelihood (ML) Kalman filter techniques that account for the time variation in the market risks. He computes a pooled regression over the return and estimated beta series instead of Fama-McBeth (1973) second-pass regressions in order to increase the power of the tests and avoid the problems implied in the univariate tests. Malkamäki finds that every analysis on the OLS betas rejects the mean-variance efficiency of the market index and gives a negative price for the market risk. The regression over the forecasted ML beta series does not reject the mean-variance efficiency of the market index. The price of market risk takes a positive sign but is not statistically significant. He tests the CAPM also in the restricted form and finds that the regression over the ML betas gives the highest risk premium and corresponding test statistic for significance.

Berglund and Knif (1992) compute Fama-McBeth tests using Finnish common stock data from 1970–1988. They analyse the changes in test statistics for the risk premium of the CAPM in quarterly returns when time-varying betas are used instead of constant betas. They find that the risk premium is negative and not significant in the constant beta regression and positive but not significant in the time-varying beta regression. Berglund and Knif run also cross-sectional regressions of monthly, bi-monthly and quarterly stock returns over the predicted time-varying beta series and find in each case a positive average risk premium that is not statistically significant. However, a weighted least squares correction that gives less weight to betas that have high prediction variance improves their results considerably and the monthly risk premium turns out to be statistically significant. Further, they find a non-linear relationship between ex post risk premiums and returns.

The purpose of this paper is to test the robustness of the findings in Malkamäki (1992) and Berglund and Knif (1992) and to provide further analysis of the risk-return relationship implied by the Sharpe-Lintner CAPM and mean-variance efficiency of the stock market index. We compute a modified version of the Fama-McBeth univariate tests and the pooled regression introduced in Malkamäki (1992) in four asset samples. We estimate static estimates for the beta series through five year OLS regressions and time-varying estimates for the betas through the ML Kalman filter procedure as in Berglund and Knif (1992) and Malkamäki (1992). All tests for the risk premium are also carried out on the subperiods that were employed in Malkamäki (1992). The data covers all listed Finnish common stock excess return series and an index for corporate bond returns. In the first phase, we take the CAPM as a true model and compute the time series and cross-section regressions without a constant in order to test the significance of the risk premium implied by the CAPM. We then add the constant and repeat all the tests over alternative asset samples and subperiods.

The remainder of this paper is organised as follows. Section two discusses the methodological problems in estimating the betas and risk premiums in thin asset markets. The next section describes the data. The empirical results are presented in section four, and finally, section five concludes with the key findings of the paper.

2 The Model and Methodological Considerations

The capital asset model states that expected returns on an asset are linearly related to the systematic risk, which is measured by the asset's beta. The Sharpe-Lintner version of the model in the excess return form is:

$$E(r_i) = \beta_i E(r_m), \tag{1}$$

where $E(r_i)$ = Expected excess return for security i

$\beta_i = \dfrac{\mathrm{cov}(r_i, r_m)}{\mathrm{var}(r_m)}$

$E(r_m)$ = Expected excess return for the market.

Actually, the CAPM is not testable as stated in Roll (1977), because the true market portfolio is not observable. Therefore, the CAPM is just a statement about the mean-variance efficiency of a given market portfolio. Thus, in our empirical analysis, we test whether the observed stock market portfolio is mean-variance efficient. The test is then a joint test of whether the given market portfolio is mean-variance efficient and whether the CAPM is the correct model.

Unfortunately, we don't observe the true beta coefficient β_i implied by the CAPM. The beta is usually estimated, under the assumption of constant market risk, by computing an OLS regression over Sharpe's well-known time series (TSR) market model

$$r_{it} = \alpha_i + \beta_i r_{mt} + \varepsilon_{it}, \tag{2}$$

where r_{it} = The excess return on asset i at time t,

α_i = The intercept term,

β_i = The beta coefficient of asset i,

r_{mt} = The excess return on the stock market portfolio at time t,

ε_{it} = A random error term.

Fama and McBeth (1973) in their seminal paper introduce an iterative technique to test the CAPM. They revise the TSR each month in order to get a series of "rolling" beta estimates for each asset and compute the following second-pass cross-sectional regression (CSR) each month:

$$r_{it} = a_t + \lambda_t \hat{\beta}_{it-1} + e_{it}, \tag{3}$$

where r_{it} = The expected excess return implied by the CAPM on asset i for period t (here monthly/quarterly return),

a_t = The intercept term (= 0 according to the CAPM),

λ_t = The risk premium at time t,

β_{it-1} = The beta coefficient estimated for the previous period,

e_{it} = A random error term.

In the literature, we find that the betas are estimated, using a rule of thumb, over a five year period of time prior to each CSR. The final Fama-McBeth estimates for the intercept and risk premium are computed as the sample means from the time series of these coefficients. Computation of the standard errors is based on the assumption that the time series of estimates are independent and identically distributed with the means of final estimates. However, we know that the independence assumption is not strictly satisfied due to the measurement error in the beta estimates. We also introduce an errors-in-variables (EIV) problem in the second-pass regression by regressing returns on betas that are subject to measurement error. Due to the EIV problem, our CSR estimates are biased and inconsistent in small samples (for a review of EIV problems, see e.g. Shanken (1991) and for thin markets, Malkamäki (1992).

We can avoid the critisism regarding the standard deviations of the univariate tests by computing just one regression over a pooled data series, as in Malkamäki (1992). This is done by constructing a single composite return vector for all return series and a corresponding beta vector for the entire period analysed. Alltogether, we have 3875 observations in these two vectors in the monthly analysis for the time period 1977:2–1989:12. This implies that our tests of the risk premium have extremely high degrees of freedom, i.e. the tests are powerfull. Model 3 is now rewritten as

$$r_{it} = a_t + \lambda\hat{\beta}_{it-1} + e_{it}$$

Note that the lamda now has no time subscript. The pooled regression has a nice feature in that it gives greater weight to those observations that have high correlations with each other as compared to the corresponding behavior of the standard univariate tests.

We estimate the Market Model (2) also by applying the dynamic Kalman filter estimation procedure, which accounts for time variation in the betas. The Market Model is now rewritten in state space form as

$$r_{it} = X_t'\theta_t + \varepsilon_t, \qquad (5)$$

where $X_t' = [1, r_{mt}]$

$\theta_t' = [\alpha_{it}, \beta_{it}]$

ε_t = A random error with variance v_t.

According to Knif (1989), the parameter vector θ_t is assumed to vary according to the stationary first order autoregressive (AR1) model

$$\theta_t - \bar{\theta} = F(\theta_{t-1} - \bar{\theta}) + u_t \tag{6}$$

where $\bar{\theta}$ = The mean vector of the parameters
F = The weights for the AR1 and mean parameters
u_t = A random error with covariance matrix M_t.

The state space representation for the Market Model is now

$$r_{it} = \begin{bmatrix} X'_t & X'_t \end{bmatrix} \begin{bmatrix} \bar{\theta}_t \\ \theta_t - \bar{\theta}_t \end{bmatrix} + \varepsilon_t \tag{7}$$

$$= B'_t \gamma_t + \varepsilon_t$$

and for the parameter vector as

$$\gamma_t = \begin{bmatrix} \bar{\theta}_t \\ \theta_t - \bar{\theta}_t \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & F \end{bmatrix} \begin{bmatrix} \bar{\theta}_{t-1} \\ \theta_{t-1} - \bar{\theta}_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ u_t \end{bmatrix} \tag{8}$$

$$= A\gamma_{t-1} + e_t$$

where F = Diag $[\omega_1, \omega_2]$
θ_1 = θ_{t-1} for all t
e_t = A random error with covariance matrix N_t

The random errors ε_t and e_t are independent of each other. The corresponding variance v_t and covariance matrix N_t are assumed to be known, i.e. we estimate them. We employ the ML method to estimate minimum mean square values for γ_{t-1} and its covariance matrix Σ_{t-1}. We update the estimates for the Σ_t and γ_t, given r_{it} and X_t, at each time t through the Kalman filter updating equations that are given in Appendix one.

Our Kalman filter technique is actually a three step procedure.[3] First, we compute a maximum likelihood solution for the parameter vector using the above forward recursive Kalman equations, which use past and current information. Next, we use information from the whole sample period and find another set of ML estimators by applying the backward recursions of the Kalman smoother. As a final step we employ the AR(1) model to estimate the forecasted beta series. These beta series are employed in the CSR and pooled data analyses. The EIV problem is reduced at least to some extent in the case of our mean-reverting AR1 model when we use forecasted betas as the independent variable in the second-pass regressions. This is the case assuming that the changing residual variance of the Market Model is dependent on the time-variation of beta.

[3] For details concerning the maximization algorithms, see Goodrich (1989).

3 Description of the Data

The data employed in this study cover the period 1972–1989. The availability of short term interest rate data prior to January 1972 was the limiting factor. The analyses are carried out on monthly and, to some extent, on quarterly returns. The stock market data consist of end-of-month returns on all the common stocks listed on the Helsinki Stock Exchange. The HSE general index, which is used here, is value weighted (see Berglund–Wahlroos–Grandell (1983)). In the index, prices are corrected for cash dividends, splits, stock dividends and new issues. The correction is based on the principle that all income from a stock is reinvested in the stock with no transaction cost. No portfolios are formed for the analysis as is usually done in the U.S. studies. This is because of the extremely limited number of actively traded shares. Instead, four asset samples are included (Table 1). The first sample includes all 25 restricted ordinary stock series listed throughout the period analysed. The second sample includes the 16 most traded restricted stocks for the period. The third sample includes the 15 return series that most closely resemble the normal distribution. Sample 1 is also enlarged by introducing a corporate bond return index into the analysis.

Table 1. **Asset samples employed in the study**[a]

Sample	Assets
Sample 1	All (25) common stocks listed throughout the whole period of 1972–1989.
Sample 2	the 16 most traded common and preferred stocks
Sample 3	the 15 return series that most closely resemble a normal distribution
Sample 4	Sample 1 and a corporate bond return index

[a] See appendices 2.1–2.3 for firm names.

Asset pricing tests are convenient to run in excess returns form. To compute excess returns, we use the one month return for the three month Eurorate on the Finnish markka. The interest rate series is introduced in Malkamäki (1992). Figure one illustrates the corresponding nominal and excess return series employed in the analysis.

The corporate bond return index is based on the corporate bond yield series described in Alhonsuo, Söderlund and Tarkka (1989). The yield series includes corporate bonds maturing in three to six years. The return index was computed by approximating the average maturity of bonds to be 4.5 years. The duration for the assumed bond was computed by using the corresponding average bond yield over 1972–1989. This enables us to compute the one-month-holding-period return for the corporate bonds (Figure 2).

Figure 1. **Monthly return series for the Helsinki stock exchange**

Nominal returns 72M02 89M12 ————
Excess returns 72M02 89M12 – – – – –

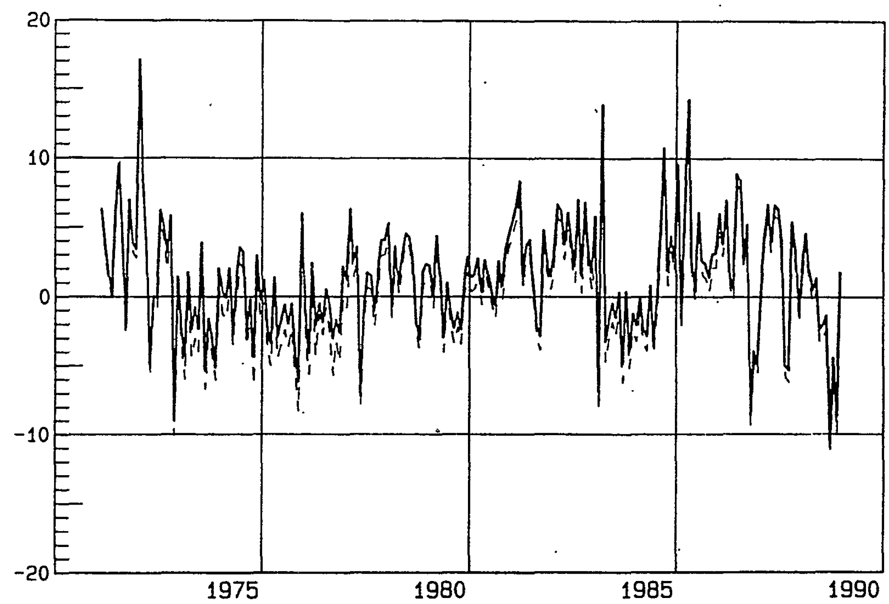

Figure 2. **Monthly interest and return series for the corporate bonds**

Monthly rate of interest 72M02 89M12 ————
Monthly rate of return 72M02 89M12 – – – – –

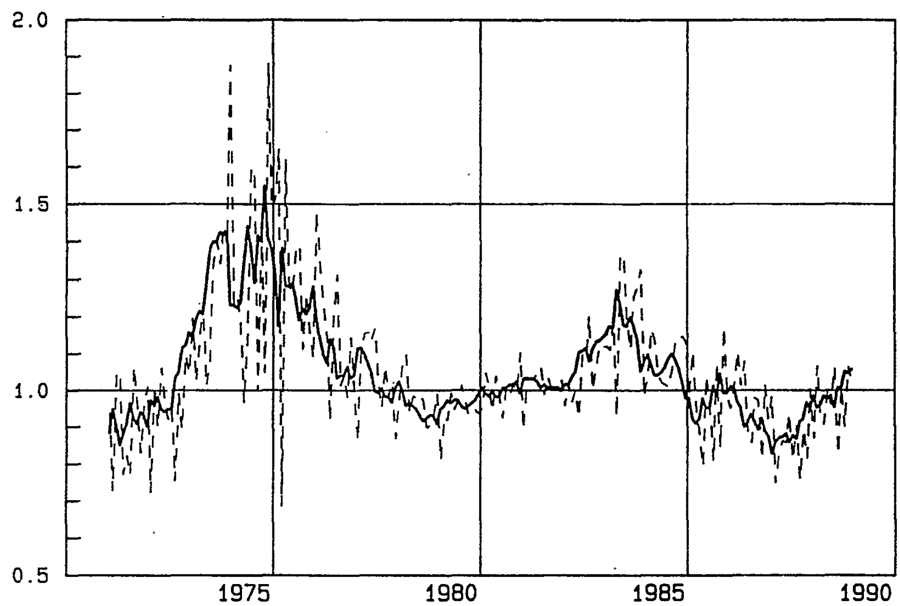

Table 2. **Summary statistics for asset excess returns (in percent per month) for 1972:2–1989:12 (215 observations)**

Asset	Mean	Std.	Skewness	Kurtosis
AB	0.746	10.683	1.332	16.702
EFFO	0.423	8.204	0.348	1.448
ENSOA	-0.234	7.922	0.645	3.114
FISKK	1.292	7.252	0.296	1.959
HUHTK	0.782	6.610	1.056	2.655
INSTA	1.156	7.118	0.607	3.206
KEMI	-0.360	10.646	-0.694	4.457
KESK	0.658	5.147	1.060	2.481
KONE	0.432	7.084	1.462	4.969
KOP	0.104	6.644	0.751	4.821
KYMI	-0.002	6.410	0.597	1.908
LASS	1.298	9.240	1.336	7.143
LOHJA	0.930	7.333	0.141	0.295
NOKIK	-0.009	6.910	0.159	0.684
OTAVK	1.234	9.496	1.773	10.212
PART	0.522	6.594	0.242	0.555
RAUM	0.034	6.741	1.042	2.112
SOKEI	0.694	8.094	0.762	2.001
STOCA	0.895	6.645	0.521	2.684
SYPA	0.433	6.083	1.230	4.425
TAMF	0.717	9.835	-0.486	6.468
TAMP	0.051	7.788	1.276	5.215
TAOK	1.866	9.520	-0.031	1.863
TRIK	0.136	11.697	0.255	6.330
WARTI	0.613	7.480	0.764	1.155
YHTYK	0.707	7.459	0.380	0.934
BONDS[a]	-0.054	0.683	-2.71	15.047
VWI[b]	0.254	4.230	0.265	0.976

[a] Corporate bond index return.
[b] Value weighted stock market index return.

Summary statistics of the monthly real returns on 27 assets and the stock market general index are shown in Table 2. Statistics are also given for three subperiods; 1972:2–1978:2, 1978:3–1984:3 and 1984:4–1989:12; see appendices 3.1–3.3. The first period ends with a devaluation of the Finnish markka. The second period ends in 1984:3 for two reasons: (1) Unrestricted shares, i.e. shares that foreign investors are allowed to buy, have been listed separately on the Helsinki Stock Exchange since January 1, 1984. Another major change was initiated by the Bank of Finland, which gave the right to central bank funding to foreign banks from April 1, 1984. This meant in practice that short term money markets started to function freely in Finland for the first time. For a closer description, see Malkamäki (1992).

The summary statistics tell us that the mean of returns change significantly over the periods. The monthly return distribution is somewhat skewed to the right and leptokurtic. If we include only the most traded share series, i.e. sample 2 (not reported separately), it turns out that the skewness is unchanged but the leptokurtosis is slightly negative. The quarterly return distribution (not reported here) for sample 1 is also positively skewed and negatively leptokurtic. The standard deviation of nominal and excess returns is almost the same as would be expected based on Figure 1.

4 Empirical Results

4.1 Beta Estimation

Our first hypothesis is that the market risk of individual stocks is constant over time. We carry out, according to this hypothesis, the well-known "rolling" beta estimation procedure introduced in Fama and McBeth (1973). Our alternative hypothesis is that the market risk of individual stocks is changing over time. We apply the Kalman filter technique to allow for time variation in the beta coefficients; see equation 4 and appendix 1. We estimate three additional sets of OLS beta series to provide additional sensitivity analysis. We estimate the market risk coefficients from monthly nominal and quarterly real returns and compute the market model without the constant term.

Table 3. **Correlation matrix of estimated beta series for sample 1 (3875 observations per series)**

Variable	B5Y	B5YNC	B5YN	BKFAR1
B5Y	1.000			
B5YNC	0.968	1.000		
B5YN	0.997	0.972	1.000	
BKFAR1	0.430	0.426	0.439	1.000

Variables in the cross-moment matrix:

B5Y = Five-year beta estimation period.
B5YNC = Five-year beta estimation period, no constant in the TSR.
B5YN = Five-year beta estimation period, nominal returns.
BKFAR1 = Kalman filter (AR1) beta. This series contains forecasted betas ($\beta_t = \omega\beta_{t-1} + (1-\omega)\bar{\beta}$)

All the beta estimations are revised monthly. The first estimation period for all the five-year OLS regressions is 1972:2–1977:1. We proceed by dropping and adding one observation and computing everything again. The last estimation period is 1984:12–1989:11. The outcome of the Kalman filter beta estimations is available in Appendix 4. Table 3 provides us with a correlation matrix of pooled monthly beta series for sample 1. All the five-year OLS beta series have very high correlations with each other, at least 0.968. The Kalman filtered beta series employs forecasted beta values, which are employed again in the second-pass regressions. The correlations between the OLS and dynamic beta series are approximately 0.43.

4.2 Tests of the CAPM and Risk Premiums

In the first phase, we carry out a modified version of the Fama-McBeth univariate tests and in the second phase, a pooled regression test for four asset samples. We compute the CSR of equation 3 iteratively each month. The final estimates of the intercept and risk premium are computed as the sample means of the time series of these coefficients.[4] Table 4.1 presents the outcome of these tests for the whole sample period. We find in the table that our analysis using the static OLS beta series always leads to rejection of the mean-variance efficiency of the market and implies a statistically significant negative risk-return relationship. Our tests using the time-varying betas do not enable us to reject the mean-variance

Table 4. **Monthly average risk premiums (in percent per month) associated with the stock market general index for 1977:2–1989:12 (155 CSR's)**

	a	λ	$R^{2\,a}$	SSR(%)[b]	P(Q)[c]
Sample 1					
OLS	1.787	-1.058	5.7	15.5	0.0
	(3.67)	(-2.61)			
KFAR1	1.128	-0.281	6.3	18.7	94.4
	(1.85)	(-0.40)			
Sample 2					
OLS	1.664	-0.932	7.8	14.2	21.6
	(3.17)	(-1.98)			
KFAR1	1.375	-0.006	6.9	19.4	26.2
	(1.74)	(-0.68)			
Sample 3					
OLS	0.158	-0.066	8.0	12.3	0.4
	(2.90)	(-1.45)			
KFAR1	0.788	0.219	9.3	25.8	94.7
	(1.22)	(0.30)			
Sample 4					
OLS	1.375	-0.648	4.8	18.1	0.0
	(3.99)	(-2.16)			
KFAR1	0.533	0.357	4.8	14.8	46.0
	(1.62)	(0.76)			

[a] Mean of the rates of determination in the CSR's.
[b] Percentage of statistically significant R^2s (at 10 % level of risk) in the CSR's.
[c] Significance level (based on Ljung-Box test statistics Q(36) for the risk that there is no autocorrelation in the risk premium series.

[4] The methodological problems involved and our attepmts to control for them are discussed above.

efficiency of the market index. The price of risk is not significant on the average.[5]

The above univariate tests are not necessarily very robust and they tell us almost nothing about the pricing of market risk if the premiums vary through time. Therefore, we compute another test statistic (SSR(%) in the table), which gives us the percentage of statistically significant rate of determinations at the 10 % level of risk (the rate of determination is bigger than 11.84 %) in the cross-sectional regressions. These statistics show that there are a lot more significant CSR's that could be found randomly, i.e. market risk may be priced in the Finnish asset market. The risk-return relationship appears to be positive and relatively strong in sample 3, where almost 26 % of the cross-sectional regressions are statistically significant at the 10 % level of risk.

Finally, we compute the Ljung-Box statistic to test whether the estimated risk premium time series are autocorrelated, i.e. whether risk premiums change systematically over time conditional on their own time series information. The results are quite straight forward. The OLS beta risk-premium series tend to be autocorrelated while the Kalman filter beta risk-premium series are not autocorrelated.

The above results support the findings of Malkamäki (1992) and show that the puzzling results with the OLS beta series are robust over the asset samples and that the time-varying market risk may be rewarded in the subsample where the return series most closely resemble a normal distribution.

Table 5 contains more analysis with the OLS betas in sample 1. The first two restricted regressions suggest that our proxy for the riskless return is reasonably accurate.[6] Strictly speaking, we accept the CAPM in the restricted version of the model and estimate the risk premium implied by it. Consequently, the unrestricted regression is a test for validity of the CAPM or a test for the risk premium implied by one-factor capital asset pricing model, where the prespecified factor is the market index. The third regression on the monthly nominal returns shows that the outcome is puzzling whether we employ excess returns or not. The fourth regression indicates that the price of market risk is negative also for quarterly returns.

Appendices 5.1–5.3 provide a closer look at the results for the subperiods introduced in section 4. The negative price of the OLS market risk is especially clear for the last subperiod. The positive risk-return relationship is most evident for the Kalman filtered market risk exposures in sample 3.

The time series for the estimated risk premiums are presented in Appendix 6. The above analysis did not support the hypothesis that there would be time variation in the quarterly risk premiums. The corresponding figure in Appendix 2 shows, however, that there is a clear break point in the data at the end of March 1984. This break point was found to be very significant also for monthly data in Malkamäki (1992).

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[5] The market risk coefficient of the bond return index (RDK36 in Appendix 4) is not significant, especially in the 1980s. One should be aware of this when interpreting the outcome of sample 4.

[6] The mean of rate of determinations in the cross-sectional regressions is not reported for the restricted versions, as it is not comparable to the corresponding R^2s in the unrestricted model.

Table 5. **Monthly and quarterly average risk premiums (in percent per month) associated with the stock market general index for 1977:2–1989:12 (155 CSR's)**

	a	λ	$R^{2\,a}$	SSR(%)[b]	P(Q)[c]
Sample 1, OLS					
Monthly Excess Returns					
– Five-year betas without constant in the CSR	-	0.703 (2.21)	-	-	0.0
– Five-year betas without constant in the TSR and CSR	-	0.701 (2.21)	-	-	0.0
Monthly Nominal Returns					
– Five-year betas	2.850 (5.99)	-1.062 (-2.67)	5.8	15.5	0.0
Quarterly Excess Returns					
– Five-year betas	4.748 (3.17)	-2.221 (-2.12)	5.3	15.7	45.4

[a] Mean of the rates of determination in the CSR's.
[b] Percentage of statistically significant R^2s (at 10 % level of risk) in the CSR's.
[c] Significance level (based on Ljung-Box test statistics Q(36) for the risk that there is no autocorrelation in the risk premium series.

Table 6 gives the correlation matrix for the time series of estimated monthly risk premiums. The estimation methodology for the market risk exposure appears to divide the risk premium series into two parts. Risk premiums computed with static/dynamic beta estimates tend to have highest correlations with each other. One additional finding concerns the risk premiums that are computed from the conditional beta series. The risk premium time series of samples 1 and 4 are highly correlated with the corresponding series of the sample 3, whereas the correletion between the corresponding series and sample 2 is considerably lower. The major difference between samples 2 and 3 is that banks are excluded from sample 3. We could suggest that the omission of the banks' return series enhances the risk-return relationship in sample 3 but we do not have clear evidence that this is the case because sample 3 includes five additional firms that are not included in sample 2. However, the financial markets were liberalized in Finland beginning in 1983. The rapid expansion in all relevant financial market aggregates suggests that the nature of banking business changed considerably (see Malkamäki and Virén (1990)), which implies that there could have been a sift in banks' market risk coefficients, which our mean reverting AR1 model is unable to account for.

Table 6. **Correlation matrix for estimated risk premiums**

Variable	5Y25	KF25	5Y16	KF16	5Y15	KF15	5Y26	KF26
5Y25	1.000							
KF25	0.303	1.000						
5Y16	0.463	0.170	1.000					
KF16	0.263	0.527	0.405	1.000				
5Y15	0.738	0.358	0.433	0.306	1.000			
KF15	0.156	0.909	0.187	0.466	0.331	1.000		
5Y26	0.929	0.412	0.476	0.353	0.680	0.265	1.000	
KF26	0.239	0.948	0.162	0.565	0.297	0.869	0.430	1.000

Monthly risk premiums in the cross-moment matrix:

5Y25 = Five-year beta estimation period, all (25) common stocks.
5Y26 = Five-year beta estimation period, 26 assets.
5Y16 = Five-year beta estimation period, 16 most traded stocks.
5Y15 = Five-year beta estimation period, 15 most normally distributed asset return series.
KF25 = Kalman filter (AR1) forecasted beta*, all (25) common stocks.
KF16 = Kalman filter (AR1) forecasted beta*, 26 assets.
KF15 = Kalman filter (AR1) forecasted beta*, 16 most traded stocks.
KF26 = Kalman filter (AR1) forecasted beta*, 15 most traded stocks.

* These series contain forecasted betas ($\beta_t = \omega\beta_{t-1} + (1-\omega)\bar{\beta}$)

We can increase the power of the above univariate tests by applying a pooled regression method. We compute, as usual, monthly returns over the estimated beta series. This is done by constructing a single composite vector of firms' returns and a corresponding beta vector for the period analysed. Our tests then have extremely high degrees of freedom (3873 for the entire period), i.e. the tests are powerfull. The pooled regression also has the nice feature that it gives higher weight to those observations that have high correlations with each other compared to the corresponding univariate tests. Furthermore, this method avoids the criticism of the univariate tests' standard deviations computed from the second-pass time-series estimates (for details, see Malkamäki (1992).[7]

Table 7 gives the pooled regression results for sample 3, where the risk-return relationship turned out to be the strongest in the above analysis. The first regression is computed over the whole sample period. The constant is relatively small and not significant. The risk-premium coefficient is more than twice as big as in Table 4, but the t-value is not significant. The second regression excludes extraordinary periods in the Finnish economy, the periods before 1978:3 and after 1988:12. The Finnish markka was devaluated three times in the excluded period

[7] Regression over the pooled data implies an assumption that the cross-sectional and time-series variability (error variance) is the same. If this does not hold, our standard deviations are too big, i.e. our t-values are biased downwards.

Table 7. **Monthly average risk premiums associated with the stock market general index (in percent per month) in the pooled data regression[a] (for sample 3)**

Period	a^b	λ^b	δ^{bc}
1977:2–1989:12	0.241 (0.37)	0.823 (1.19)	7.810
1978:3–1988:12	0.076 (0.11)	1.635 (2.19)	7.778

[a] Model estimated: $r_{it} = \lambda\beta_{it-1} + e_{it}$, where $\beta_{it-1} = \omega\beta_{it-1} + (1-\omega)\bar{\beta}$

[b] All coefficients are multiplied by 100.

[c] Standard error of estimate.

prior to 1978:3 and the Finnish economy turned sharply downward in early 1989. The exclusion of these periods has a major inpact for the risk-return relationship. The size of the risk premium is now statistically significant and in accordance with what would be expected based on the average excess returns shown in Table 2.[8]

[8] The exclusion of the data after 1989 turned out to have a bigger impact in favor of the CAPM. This implies that our betas are not able to account for the dramatic stock price drops within that year. One explanation for this phenomenon is reported in Berglund and Knif (1992). They find a non-linear relationship between ex post risk-premiums and returns for Finnish stock market data. However, they excluded the data for 1989.

5 Conclusions

This paper examines the Sharpe-Lintner CAPM in which a time-varying-parameter model serves as an alternative to the traditional static market model. We test the models for the thin Finnish asset market. Prior unconditional tests of the CAPM usually led to rejection of mean-variance efficiency of the Helsinki Stock Exchange index and found that market risk is not priced or, as in recent studies, the price of market risk is negative. We carry out the traditional Fama-McBeth univariate tests and reject the mean-variance efficiency of the market index and again find the negative risk-return relationship. We also demonstrate that this result is very robust, recurring in every one of our four asset samples. The first sample includes all 25 restricted ordinary share series listed throughout the whole period analysed. The second sample includes the 16 most traded restricted stocks during the period. The third sample includes the 15 return series that most closely resemble the normal distribution. Sample 1 is also enlarged by introducing a corporate bond return index into the analysis.

Two recent papers, Malkamäki (1992) and Berglund and Knif (1992), suggest that the market index may be mean-variance efficient and the time-varying market risk may be rewarded in the Finnish stock market. The market risk coefficients were allowed to vary over time according to a mean-reverting AR1 model in both of these studies. We examine the robustness of these results in four asset samples and in every case are unable to reject the mean-variance efficiency of the market index. The risk premium coefficient is not significant in our standard Fama-McBeth univariate analysis. However, the number of statistically significant cross-sectional regressions is clearly highest, and statistically significant, in sample 3, i.e. among the 15 return series that most closely resemble the normal distribution. The pooled regression analysis on sample 3 suggests that the risk premium is clearly positive and statistically significant if we exclude the two extraordinary periods for the Finnish economy.

The risk-return relationship is notably weaker in the sample of most traded stocks. This finding does not support the prior understanding that the poor performance of the CAPM for the data from a thin stock markets is mainly due to non-synchronous trading. Our evidence suggests that the nonnormality of the return series may be an even bigger problem in tests of asset pricing models. The major difference in samples 2 and 3 is that banks are excluded from sample 3. We leave it for subsequent research to determine whether the omission of the banks' return series explains the enhanced risk-return relationship in sample 3.

We also showed that the risk-return relationship is sensitive to the time period considered. In particular, our conditional betas are not able to account for the return behavior of stocks under such drastic expectations as in the Finnish economy in 1989. More research is needed. One could, for example, let the risk premium vary over time and see if it is possible to explain the return behaviour in 1989.

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Appendix 1

The forward updating Kalman equations are:

$$\Sigma p = A_t \Sigma_{t-1} A'_t + N_t,$$

where Σp = One-step ahead prediction based on the prior information for the covariance matrix of the new parameter vector.

$$F = X_t' \Sigma p X_t + v_t,$$

where F = One-step -ahead prediction for the variance of the new parameter vector.

$$K_t = \Sigma p X_t F^{-1},$$

where K_t = Kalman gain, i.e. the correction weight based on the one-step ahead prediction for the covariance matrix Σp and variance F.

$$\tilde{r}_{it} = r_{it} - X_t A_t \gamma_{t-1},$$

where $\tilde{r}_{it}$ = One-step-ahead prediction error.

$$\Sigma_t = \Sigma p - \Sigma p X_t F^{-1} X_t' \Sigma p,$$

where Σ_t = Updated estimate for the covariance matrix of new parameter vector.

$$\gamma_t = A_t \gamma_{t-1} + K_t \tilde{r}_{it},$$

where r_t = Updated estimate of the parameter vector.

Appendix 2.1

Firms included in the analysis. Sample 1. This sample includes all restricted ordinary stocks listed throughout the whole period 1972:2–1989.

Firm	Designation
Bank of Åland Ltd K	AB
Effoa-Finland Steamsip Co Ltd K	EFFO
Enso-Gutzeit Ltd A	ENSOA
Fiskars Corporation	FISKK
Huhtamäki Corporation K	HUHTK
Instrumentarium Corporation	INSTA
Kemi Corporation	KEMI
Kesko Corporation	KESK
KANSALLIS-OSAKE-PANKKI	KOP
Kymmene Corporation	KYMI
Lassila & Tikanoja Ltd	LASS
Lohja Corporation A	LOHJA
Nokia Corporation	NOKIK
Otava Publishing Company Ltd	OTAVK
Partek Corporation	PART
Rauma-Repola Corporation	RAUM
Finnish Sugar Co Ltd I	SOKEI
Stockman A	STOCA
Union Bank of Finland Ltd A	SYPA
Tamfelt Group K	TAMF
Tampella Ltd	TAMP
Talous-Osakekauppa Co	TAOK
Suomen Trikoo Corp. A	TRIK
Wärtsilä Co I	WARTI
United Paper Mills Ltd K	YHTYK

Appendix 2.2

Firms included in the analysis. Sample 2. This sample includes the 16 most traded restricted stocks.

Firm	Designation
Enso-Gutzeit Ltd A	EFFO
Fiskars Corporation	FISKK
Instrumentarium Corporation A	INSTA
Kesko Corporation	KESK
Kone Corporation B (preference share)	KONE
KANSALLIS-OSAKE-PANKKI	KOP
Kymmene Corporation	KYMI
Lohja Corporation A	LOHJA
Nokia Corporation	NOKIK
Partek Corporation	PART
Rauma-Repola Corporation	RAUM
Finnish Sugar Co Ltd I	SOKEI
Union Bank of Finland Ltd A	SYPA
Tampella Ltd	TAMP
Wärtsilä Co I	WARTI
United Paper Mills Ltd K	YHTYK

Appendix 2.3

Firms included in the analysis. Sample 3. This sample includes the 15 stocks that most closely resemble the normal distribution

Firm	Designation
Effoa-Finland Steamsip Co Ltd K	EFFO
Fiskars Corporation	FISKK
Huhtamäki Corporation K	HUHTK
Kesko Corporation	KESK
Kymmene Corporation	KYMI
Lohja Corporation A	LOHJA
Nokia Corporation	NOKIK
Partek Corporation	PART
Finnish Sugar Co Ltd I	SOKEI
Stockman A	STOCA
Tamfelt Group K	TAMF
Talous-Osakekauppa Co	TAOK
Suomen Trikoo Corp. A	TRIK
Wärtsilä Co I	WARTI
United Paper Mills Ltd K	YHTYK

Appendix 3.1

Summary statistics for the asset excess returns, (in percent per month) for 1972:2–1978:2 (73 observations)

Asset	Mean	Std.	Skewness	Kurtosis
AB	-0.234	11.341	-0.891	5.441
EFFO	-0.758	6.695	-0.243	1.797
ENSOA	-1.732	8.098	-0.027	3.707
FISKK	0.199	6.573	-0.103	0.044
HUHTK	-0.157	5.375	2.353	8.174
INSTA	0.623	7.574	0.343	0.517
KEMI	-1.191	8.916	0.162	1.704
KESK	-0.434	5.058	1.772	4.584
KONE	-0.807	6.761	1.766	10.257
KOP	-0.857	6.867	0.063	0.404
KYMI	-0.926	6.521	0.231	2.054
LASS	-0.069	8.549	2.660	13.570
LOHJA	-0.215	7.519	0.648	0.988
NOKIK	-0.738	6.290	0.180	0.742
OTAVK	0.967	11.330	2.331	13.188
PART	-0.829	6.709	-0.007	0.096
RAUM	-0.881	7.280	1.131	1.873
SOKEI	-0.375	7.859	0.849	1.956
STOCA	-1.082	5.825	0.650	1.750
SYPA	-0.077	6.822	1.143	2.630
TAMF	-0.913	12.530	-0.756	4.601
TAMP	-2.466	6.961	0.071	0.501
TAOK	2.786	8.880	0.760	2.176
TRIK	-0.739	8.292	1.057	3.638
WARTI	-1.414	7.328	0.700	1.242
YHTYK	-0.483	7.854	0.421	0.466
BONDS[a]	-0.149	1.084	-1.559	4.574
VWI[b]	-0.721	4.406	0.942	2.167

[a] Corporate bond index return.
[b] Value-weighted stock market index return.

Appendix 3.2

Summary statistics for the asset excess returns (in percent per month) for 1978:3–1984:3 (73 observations)

Asset	Mean	Std.	Skewness	Kurtosis
AB	1.263	7.092	0.067	2.436
EFFOA	0.655	6.433	-0.338	2.210
ENSOA	0.307	5.710	0.983	2.273
FISKK	0.931	5.492	0.108	1.176
HUHTK	2.584	7.120	0.992	3.180
INSTA	2.318	5.577	1.938	5.857
KEMI	-1.538	12.402	-1.469	5.919
KESK	0.748	3.368	-0.118	0.329
KONE	2.426	7.357	2.021	4.749
KOP	1.538	6.459	2.683	13.159
KYMI	0.178	5.365	0.512	2.742
LASS	1.993	6.848	0.029	1.367
LOHJA	2.052	6.423	-0.183	0.790
NOKIK	0.645	6.016	0.112	2.406
OTAVK	1.459	5.017	-0.090	1.482
PART	1.311	4.980	1.299	3.060
RAUM	0.680	6.502	1.264	2.163
SOKEI	2.189	7.682	0.820	1.867
STOCA	2.333	6.608	0.502	6.768
SYPA	1.005	5.551	2.423	13.827
TAMF	1.853	6.335	1.338	3.601
TAMP	0.659	6.517	0.695	3.246
TAOK	2.388	9.657	-0.332	3.069
TRIK	1.421	12.346	-1.345	10.440
WARTI	2.813	7.230	0.681	0.538
YHTYK	1.672	7.087	0.512	1.865
BONDS[a]	-0.030	0.408	-4.826	33.708
VWI[b]	1.172	3.278	0.000	2.024

[a] Corporate bond index return.
[b] Value-weighted stock market index return.

Appendix 3.3

Summary statistics for the asset excess returns (in percent per month) for 1984:4–1989:12 (69 observations)

Asset	Mean	Std.	Skewness	Kurtosis
AB	1.236	12.885	3.124	21.096
EFFO	1.427	10.860	0.442	0.010
ENSOA	0.776	9.476	0.982	4.491
FISKK	2.830	9.166	0.212	1.523
HUHTK	-0.131	6.926	0.400	-0.257
INSTA	0.490	7.974	0.590	4.397
KEMI	1.766	10.143	0.274	0.629
KESK	1.718	6.468	0.654	0.815
KONE	-0.413	6.739	0.555	0.377
KOP	-0.397	6.431	-0.251	0.953
KYMI	0.784	7.228	0.896	1.039
LASS	2.008	11.774	0.937	4.501
LOHJA	0.954	7.859	-0.021	-0.151
NOKIK	-0.211	8.309	0.198	-0.181
OTAVK	1.280	11.003	1.028	1.963
PART	1.118	7.740	0.176	-0.319
RAUM	0.321	6.380	0.876	3.277
SOKEI	0.245	8.629	0.770	2.655
STOCA	1.466	7.073	0.373	0.642
SYPA	0.367	5.822	0.457	-0.182
TAMF	1.241	9.549	0.316	4.861
TAMP	2.073	9.125	1.986	5.879
TAOK	0.340	9.973	-0.261	0.593
TRIK	-0.298	13.915	1.176	3.198
WARTI	0.429	7.363	1.175	2.586
YHTYK	0.944	7.350	0.342	1.260
BONDS[a]	0.018	0.213	-0.907	1.116
VWI[b]	0.314	4.737	0.042	0.241

[a] Corporate bond index return.
[b] Value-weighted stock market index return.

Appendix 4

Maximum likelihood estimation results for the Kalman filter AR1 specification (estimation period 1972:2–1989:12)

Asset	ω	$\bar{\beta}$	q	σ^2	α	R^2	P(F)
AB	-.0902	.7483*	.0016	.0104	.0049	.0954	.9995
EFFO	-.0227	.8885*	.4523	.0046	.0035	.1844	1.0
ENSOA	.0172	1.0106*	.6726	.0035	-.0052	.2575	1.0
FISKK	.0322	.7953*	.2926	.0035	.0126*	.2258	1.0
HUHTK	.4469*	.9676*	.4967	.0019	.0020	.2981	1.0
INSTA	-.0346	.8680*	.0989	.0036	.0097*	.2580	1.0
KEMI	-.0797	.9139*	1.1405	.0079	-.0057	.1317	1.0
KESK	-.2138	.6707*	.1303	.0016	.0055*	.2955	1.0
KONEB	-.0294	.7427	.2776	.0035	.0021	.1999	1.0
KOP	.0027	1.0664*	.3458	.0016	-.0031	.3859	1.0
KYMI	.0694	.9342*	.2063	.0022	-.0013	.3590	1.0
LASS	.2163	.7103*	.1765	.0074	.0120*	.0938	.9994
LOHJA	-.0785	1.2017*	.0221	.0026	.0073*	.4318	1.0
NOKIK	-.0526	1.0748*	.1426	.0025	-.0036	.4354	1.0
OTAVK	.1064	.6889	2.8235	.0035	.0116*	.1357	1.0
PART	-.0104	1.0863*	.2200	.0021	.0031	.4179	1.0
RAUM	-.0472	.9927*	.3495	.0022	-.0038	.3930	1.0
SOKEI	.0329	.9927*	.5949	.0039	.0050	.2550	1.0
STOCA	-.0626	.8699	.0788	.0030	.0072	.2844	1.0
SYPA	.0431	1.0106*	.5339	.0011	.0011	.5006	1.0
TAMF	.3272	.9094*	.9270	.0062	.0079	.1870	1.0
TAMP	.7896*	.9484	.0262	.0045	.0006	.2410	1.0
TAOK	.0383	.4740	.1097	.0085	.0176*	.0419	.8982
TRIK	-.2379	1.0390*	4.5252	.0050	.00005	.1222	1.0
WARTI	.2530	1.001*	.0009	.0038	.0044	.3001	1.0
YHTYK	.0587	1.1527*	.3089	.0026	.0052	.4426	1.0
RDK36	-.0294	.7427	.2776	.0035	.0021	.1999	1.0

Estimated model: $r_{it} = \alpha_t + \beta_t r_{mt} + \varepsilon_t$,

where α_t = constant $var(\varepsilon_t) = \sigma^2$

$\beta_t = \omega\beta_{t-1} + (1-\omega)\bar{\beta} + v_t$ $var(v_t) = q$

Appendix 5.1

Monthly average risk premiums (in percent per month) associated with the stock market general index for 1977:2–1984:3 (86 CSR's)

	a	λ	$R^{2\,a}$	SSR(%)[b]	P(Q)[c]
Sample 1					
OLS	1.032	-0.197	5.3	16.3	30.5
	(2.44)	(-0.52)			
KFAR1	0.965	-0.120	6.6	18.6	35.7
	(1.31)	(-0.13)			
Sample 2					
OLS	0.755	0.070	7.3	11.6	57.2
	(1.56)	(0.14)			
KFAR1	0.889	-0.045	6.6	19.8	14.3
	(0.98)	(-0.04)			
Sample 3					
OLS	0.056	0.053	8.6	16.3	82.8
	(1.10)	(1.16)			
KFAR1	0.722	0.411	9.8	27.9	95.2
	(0.86)	(0.43)			

[a] Mean of the rates of determination in the CSR's.
[b] Percentage of statistically significant R^2s (at 10 % level of risk) in the CSR's.
[c] Significance level (based on Ljung-Box test statistics Q(27)) for the risk that there is no autocorrelation in the risk premiums.

Appendix 5.2

Monthly average risk premiums (in percent per month) associated with the stock market general index for 1978:3—1984:3 (73 CSR's)

	a	λ	$R^{2\,a}$	SSR(%)[b]	P(Q)[c]
Sample 1					
OLS	1.506	-0.018	5.4	16.4	31.5
	(3.43)	(-0.48)			
KFAR1	1.041	0.329	6.4	17.8	75.2
	(1.29)	(0.34)			
Sample 2					
OLS	0.997	0.338	7.2	17.8	36.4
	(2.01)	(0.66)			
KFAR1	0.840	0.005	6.7	21.9	23.4
	(0.84)	(0.43)			
Sample 3					
OLS	0.079	0.083	8.9	16.4	72.4
	(1.42)	(1.76)			
KFAR1	0.838	0.848	9.7	30.1	99.7
	(0.89)	(0.80)			

[a] Mean of the rates of determination in the CSR's.
[b] Percentage of statistically significant R^2s (at 10 % level of risk) in the CSR's.
[c] Significance level (based on Ljung-Box test statistics Q(24)) for the risk that there is no autocorrelation in the risk premiums.

Appendix 5.3

Monthly average risk premiums (in percent per month) associated with the stock market general index for 1984:4–1989:12 (69 CSR's)

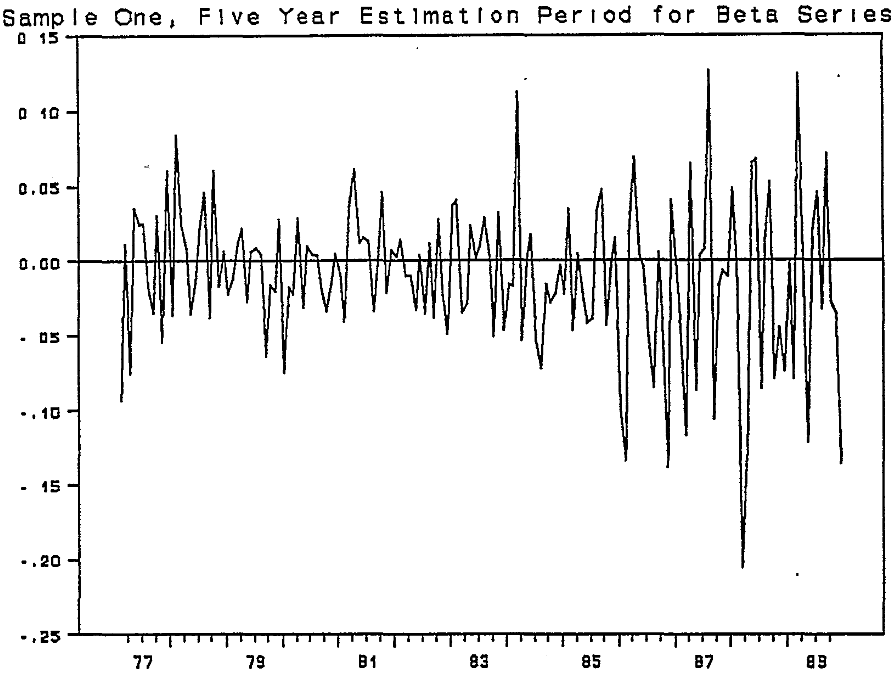
	a	λ	R^2 [a]	SSR(%)[b]	P(Q)[c]
Sample 1					
OLS	2.729	-2.132	6.2	14.5	0.0
	(2.86)	(-2.80)			
KFAR1	1.331	-0.481	5.9	18.8	81.7
	(1.30)	(-0.43)			
Sample 2					
OLS	2.796	-2.181	8.4	17.4	63.4
	(2.80)	(-2.63)			
KFAR1	1.981	-0.013	7.2	10.1	78.6
	(1.43)	(-0.89)			
Sample 3					
OLS	0.285	-0.214	8.5	7.2	0.0
	(2.76)	(-2.64)			
KFAR1	0.870	-0.0002	8.7	23.2	85.8
	(0.85)	(-0.02)			

[a] Mean of the rates of determination in the CSR's.
[b] Percentage of statistically significant R^2s (at 10 % level of risk) in the CSR's.
[c] Significance level (based on Ljung-Box test statistics Q(24)) for the risk that there is no autocorrelation in the risk premiums.

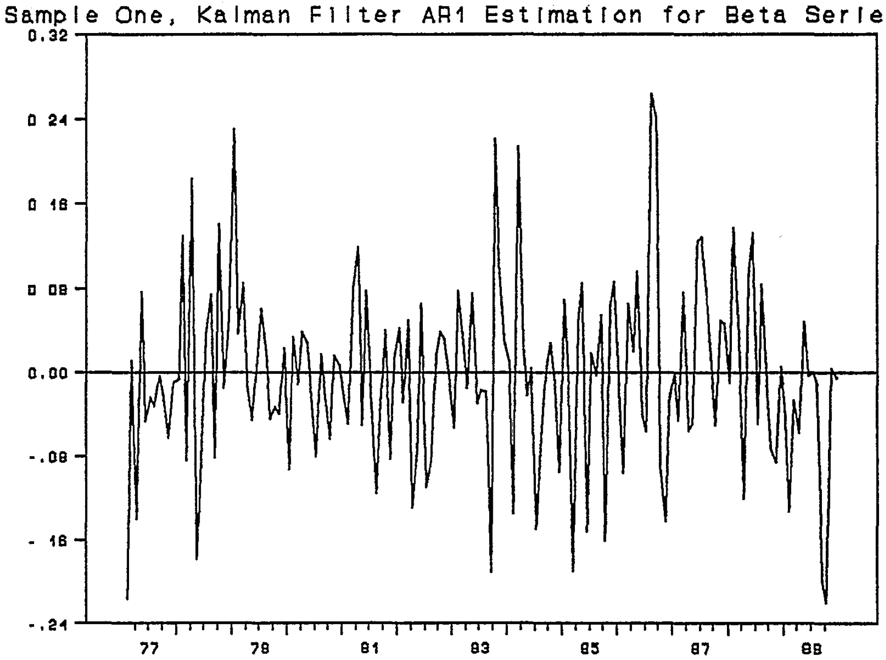
Appendix 6

Estimated time series for the risk premiums: alternative samples of assets, estimation methods for the betas and time aggregation of the returns

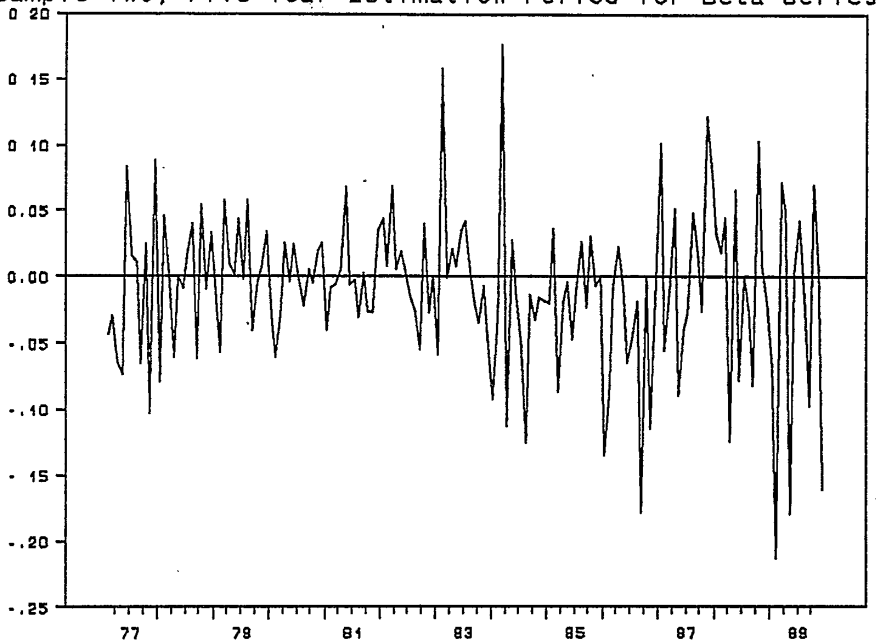
Sample One, Five Year Estimation Period for Beta Series



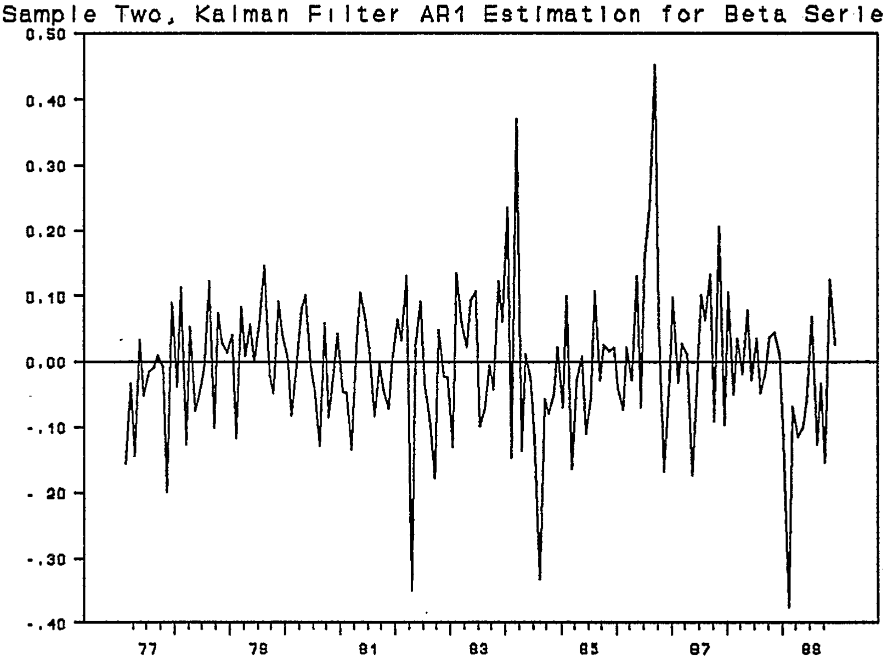
Sample One, Kalman Filter AR1 Estimation for Beta Series



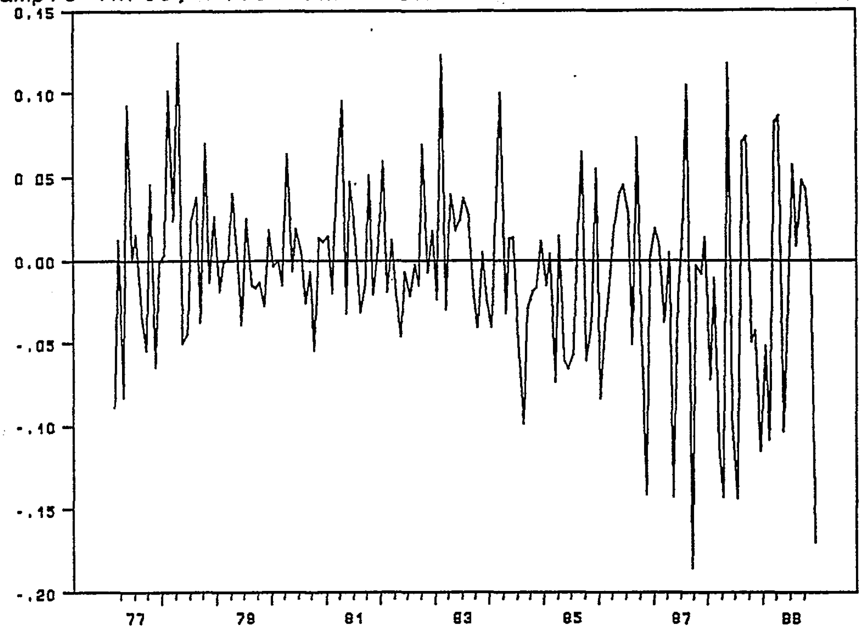
Sample Two, Five Year Estimation Period for Beta Series



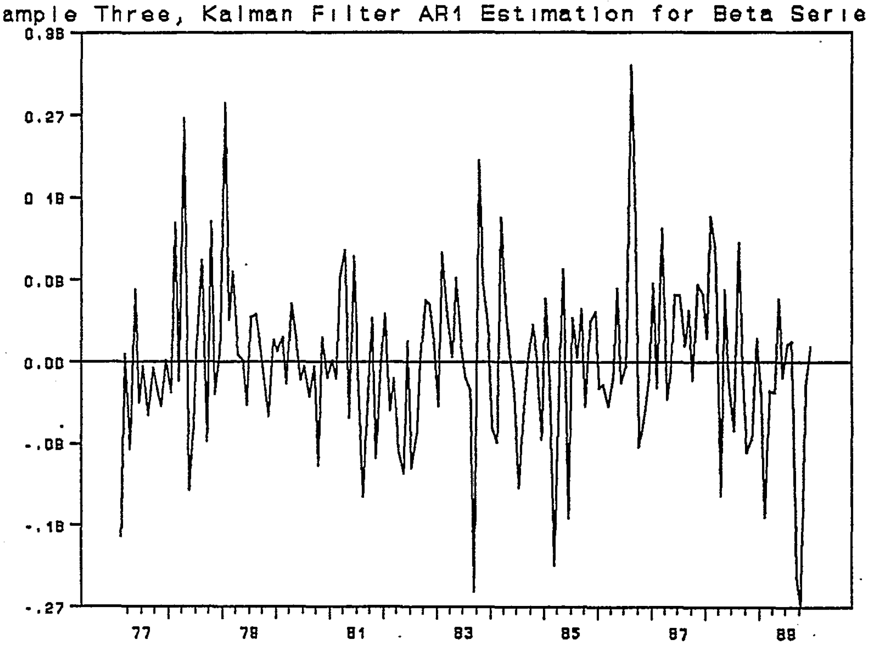
Sample Two, Kalman Filter AR1 Estimation for Beta Series



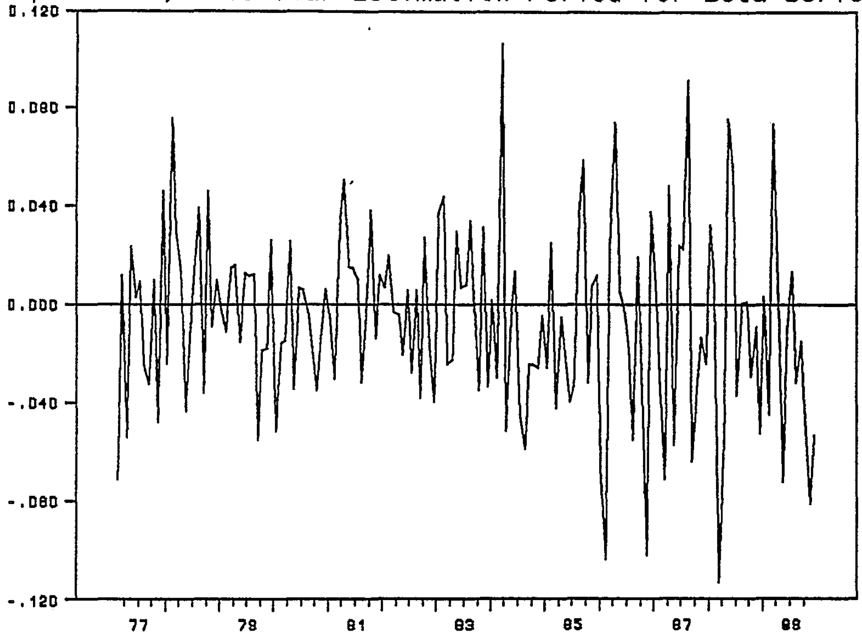
Sample Three, Five Year Estimation Period for Beta Series



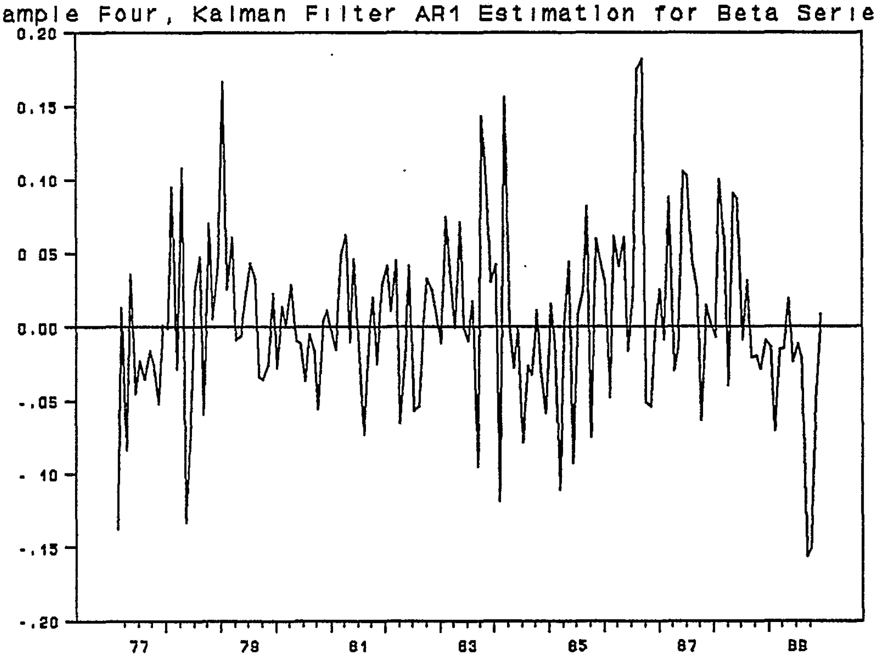
Sample Three, Kalman Filter AR1 Estimation for Beta Series



Sample Four, Five Year Estimation Period for Beta Series



Sample Four, Kalman Filter AR1 Estimation for Beta Series



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