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# Estimating Conditional Betas and the Price of Risk for a Thin Stock Market

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Abstract

This paper examines the Sharpe-Lintner Capital Asset Pricing Model (CAPM) in which time-varying-parameter models are alternative to the static market model. Prior evidence does not support the CAPM and suggests that market risk is not priced or the price of the beta risk is significantly negative for a thin European stock market, e.g. the Finnish stock market. We show that this phenomenom is due to static ordinary least squares beta estimates that are spurious. We reduce the errors-in-variables problem by estimating the firm-specific betas using the Kalman filter technique and employ the forecasted beta values in cross-sectional analysis. It turns out that in our analysis of pooled data the sign for coefficient of the price of risk becomes positive and we are no longer able to reject the mean-variance efficiency of the market index. The data covers all Finnish common stocks listed on the Helsinki Stock Exchange throughout the years 1972–1989.

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# Contents

# 1 Introduction

The CAPM states that expected return on an asset is positively and linearly related to its systematic risk, which is measured by the asset's beta. The Sharpe (1964)–Lintner (1965) version of the model states that

$$E(R_i) = R_f + \beta_i E(R_M - R_f), \tag{1}$$

where $E(R_i)$ = Expected return on security i
$R_f$ = Risk-free rate of interest
$\beta_i$ = Systematic risk coefficient (beta) for security i
$E(R_M\text{-}R_f)$ = Expected excess return on the market over the risk-free rate.

Actually, the CAPM is not testable as stated in Roll (1977), because the true market portfolio is not observable. Therefore, the CAPM is just a statement about the mean-variance efficiency of a given market portfolio. Thus, we test in empirical analysis whether the observed stock market portfolio is mean-variance efficient. The test is, of course, a joint test that a given portfolio is mean-variance efficient and that the CAPM is the correct model.

Unfortunately, we don't observe the true beta coefficient $\beta_i$ implied by the CAPM. In the traditional two-pass approach, the beta is estimated by applying Sharpe's well known time-series regression (TSR) model, i.e. the market model, which is expressed below in terms of excess returns:

$$r_{it} = \alpha_i + \beta_i r_{mt} + \varepsilon_{it}, \tag{2}$$

where $r_{it}$ = the excess return on asset i at time t,
$\alpha_i$ = The intercept term,
$\beta_i$ = Beta coefficient of asset i,
$r_{mt}$ = The excess return on the stock market portfolio at time t,
$\varepsilon_{it}$ = A random error term.

In their seminal paper, Fama and McBeth (1973) introduce an iterative technique for estimating the second-pass cross-sectional regression (CSR). They revise the TSR each month in order to get a series of "rolling" beta estimates for each asset and compute the following CSR on the beta estimates for each month:

$$r_{it} = a + \lambda_t \hat{\beta}_{it-1} + e_{it}, \tag{3}$$

where $r_{it}$ = The expected excess return implied by the CAPM on asset i for period t (here monthly return over the entire period analysed),
a = The intercept term ($H_0$: a = 0 according to the CAPM),
$\lambda_t$ = The risk premium,
$\beta_{it-1}$ = The beta coefficient estimated at the previous period,
$e_{it}$ = A random error term.

In the literature, the betas are generally estimated from a five year period of time prior to each CSR. The final estimates for the intercept and risk premium are computed as the sample means of the time series of these coefficients. Computation of the standard errors is based on the assumption that the time series of estimates are independent and distributed identically to the means of final estimates. However, we know that the independence assumption is not strictly satisfied due to the measurement error in the beta estimates. We also introduce an errors-in-variables (EIV) problem in the second-pass regression by regressing returns on the betas that are measured with error. Due to the EIV problem, our CSR estimates are biased and inconsistent in small samples (for a review of EIV problems, see e.g. Shanken (1991)).

Most prior studies of Finnish stock market data follow the above approach. These studies do not support the existence of a robust positive risk-return relationship as implied by the CAPM. The most puzzling result is obtained in Malkamäki (1991) with monthly and quarterly stock market data on excess returns. He runs Fama-McBeth OLS regressions in a multifactor context, as in Ferson and Harvey (1991), and finds that the price of market risk is negative and statistically significant.

This paper examines the risk-return relationship implied in the Sharpe-Lintner CAPM using data from the thin Finnish stock market. We estimate the CAPM's systematic risk by employing the traditional OLS rolling beta procedure and, as alternatives, two different kinds of time-varying-parameter models. The time series for the beta coefficient of each stock are estimated by applying Sharpe's (1964) market model. We estimate the rolling OLS betas over five and three-year periods of time. We compute the dynamic beta estimates by applying OLS and maximum likelyhood (ML) Kalman filter techniques.

We proceed by testing whether the market risk, either static or time-varying, is priced by the market. We regress, as usual, expected monthly returns over the estimated beta series. However, we run this regression with pooled return and beta series. This is done by constructing only one composite return vector for all firms' return series and one corresponding beta vector for the entire period analysed here.[1] Therefore, we have in the monthly analysis 3875 observations in these two vectors for the time period 1977:2—1989:12. This implies that our tests for the risk premium have extremely high degrees of freedom, i.e. the tests are powerfull. Our pooled data estimation procedure avoids the above criticism regarding the standard deviations in the univariate tests. The pooled regression also has the nice feature that it gives higher weight to those observations that are highly correlated with each other, as compared to the standard univariate tests. As a final topic in our analysis, we examine whether our assumption of a stable risk premium should be relaxed.

This paper aims to do four things. First, the short term interest rate is computed from the Eurofutures market for the Finnish markka. This enables us to analyse excess returns of this kind for the first time with Finnish stock returns. Second, we compute static and dynamic monthly estimates for the beta time series. Third, we pool the data in order to test the pricing of risk and the potential variation in it. Fourth, the CAPM is tested in its restricted and unrestricted forms

[1] Regression over the pooled data implies an assumption that the cross-sectional and time-series variability (error variance) is the same.

to analyse the acuracy of our beta estimates obtained using different estimation methods.

The remainder of this paper is organised as follows. Section 2 discusses the methodological problems of estimating constant and time-varying betas and the risk premium. Section 3 describes the Kalman filtering technique. The next section describes the data and economic conditions in Finland during the period studied. Empirical results are presented in section 5 and, finally, we present the key findings and draw some conclusions in the section 6.

# 2 Methodological Aspects of Beta Estimation

## 2.1 Constant Beta Models

Let us assume that a firm's true beta is constant over time. Now, there are two ways to reduce the measurement error in the TSR. First, we could increase the time series period employed in the beta estimation. On the other hand, we know that betas are non-stationary which implies that five years is already a reasonably long period with respect to beta estimation. This is also a relevant criticism of contemporaneous multivariate tests of the CAPM (see e.g. Gibbons, Shanken and Ross 1989). These tests are statistically efficient but do not allow for time variation in the beta.

The second possibility is to minimize variance of the error term in the TSR. This is most commonly achieved by grouping securities into portfolios if the errors are not perfectly correlated cross-sectionally. The problem is that we should find a sorting variable that is highly correlated with the true betas and uncorrelated with the estimation error. However, Ball and Kothari (1989) and Shanken and Weinstein (1990) show that the grouping of securities may easily lead to spurious results as well. The security grouping approach is actually not available on thin stock markets because there are only a limited number of listed stocks available and different kinds of stock series are involved (e.g. common and preference shares and restricted and unrestricted stock series on the Helsinki Stock Exchange). The potential diversification effect in portfolio formation would be very limited. If we do not form portfolios, we have to face another statistical problem: non-normality of certain individual asset returns.

There are two additional problems when applying the two-pass regression approach in the analysis. These are the possible existence of autocorrelation in the TSR residuals and heteroscedasticity in the TSR and CSR residuals. The problem with autocorrelation in TSR residuals is usually reduced by measuring the returns over one month intervals, instead of using shorter intervals. If the residual variance turns out to be changing, one common solution is to use the weighted least squares regression method. Another solution is to apply conditional asset pricing models.

## 2.2 Time Varying Beta Models

Time variation of the market risk has been documented recently by a number of reserchers. We have, in fact, at least three relevant estimation procedures available when modeling the time variation in the betas. Bollerslev, Engle and Wooldridge (1988) and Ng (1991) employ different versions of the multivariate generalized autoregressive conditional heteroskedasticity (GARCH) method in modelling the conditional covariances as a function of past conditional covariances. However, Nelson (1991) states that there are at least three major drawbacks involved in the GARCH models and developes a univariate exponential ARCH model that does not suffer from these drawbacks. A multivariate version of his univariate model or some other satisfactory improvement on the ARCH models is still needed to avoid these problems when applying the models in asset pricing applications.

Harvey (1989) applies the generalized method of moments (GMM) method to allow conditional covariances to vary in a test of the CAPM. This procedure involves expected returns conditional on the true market information set. A problem here is that we do not observe the true market information set. Instead, we find a subset of observable variables that are called instrumental variables. Further, we assume in the GMM applications that a linear function relates conditional expectations to the information set.

The time-varying parameter (TVP) models are the third possibility to control for the time-variation in the betas. We can estimate TVP models by applying the Kalman filter technique. This technique gives us an insight into how a rational investor would revise his beta estimates in a Bayesian fashion when new information is available. A driving economic force behind the time-varying beta coefficient could be, for example, a change in leverage or riskiness of investment projects of a firm. Knif (1989) applies the Kalman filter technique to model time variation of the firm specific market risk in Finnish common stock data. He finds that the betas are clearly time varying. However, the most betas of Finnish common stocks follow a stationary autoregressive process.

# 3 The Kalman Filter

We apply four different estimation procedures to compute the betas. We run the OLS regressions by applying the market model of equation three in order to get the rolling beta estimates. We compute the betas from five and three year time periods prior to each cross-sectional regression. We estimate the market model also by applying the dynamic Kalman filter OLS and ML estimation procedures that account for time variation in the betas. Our first Kalman filter application is the OLS random walk (for the OLS version, see e.g. Doan, Litterman and Sims (1984), Cuthbertson (1988) and Knif (1989)). In this approach, only ex ante and current information are used in evaluating the initial values for the filtering and in updating the parameter vector and its covariance matrix in the Kalman equations. This is in accordance with the reality, where investors try to evaluate estimates for a beta conditional on the information available at the time. The market model is now conveniently written in state space form as

$$r_{it} = X_t' \theta_t + \varepsilon_t, \tag{4}$$

where $X_t' = [1, r_{mt}]$

$\theta_t' = [\alpha_{it}, \beta_{it}]$

$\varepsilon_t$ = A random error with variance $v_t$.

The parameter vector $\theta_t$ is assumed to vary over time according to random walk transition equation

$$\theta_t = \theta_{t-1} + u_t, \tag{5}$$

where $u_t$ = A random error with covariance matrix $M_t$

Random errors $\varepsilon_t$ and $u_t$ are independent of each other. The corresponding variance $v_t$ and covariance matrix $M_t$ are assumed to be known, i.e. we estimate them. If we also have initial values for $\theta_{t-1}$ and its covariance matrix $\Sigma_{t-1}$, then the updated estimates for the $\Sigma_t$ and $\theta_t$, given $r_{it}$ and $X_t$, are:

$$\Sigma p = \Sigma_{t-1} + M_t, \tag{6}$$

where $\Sigma p$ = One-step-ahead prediction based on the prior information for the covariance matrix of the new parameter vector.

$$F = X_t' \Sigma p X_t + v_t, \tag{7}$$

where F = One-period prediction for the variance of the new parameter vector.

$$K_t = \Sigma p X_t F^{-1}, \tag{8}$$

where $K_t$ = Kalman gain, i.e. the correction weight based on the one-step-ahead prediction for the covariance matrix $\Sigma p$ and variance F.

$$\tilde{r}_{it} = r_{it} - X_t'\theta_{t-1}, \tag{9}$$

where $\tilde{r}_{it}$ = One-step-ahead prediction error.

$$\Sigma_t = \Sigma p - \Sigma p X_t F^{-1} X_t' \Sigma p, \tag{10}$$

where $\Sigma_t$ = Updated estimate of the covariance matrix of the new parameter vector.

$$\theta_t = \theta_{t-1} + K_t \tilde{r}_{it}, \tag{11}$$

where $\theta_t$ = Updated estimate for the parameter vector.

The initial values for $V_t$, $M_t$, $\theta_{t-1}$ and $\Sigma_{t-1}$, as well as parameters for tightness in the Kalman filter estimation, were obtained by the method suggested in Doan, Litterman and Sims (1984). We estimated the initial values through an ordinary least squares regression for the market model over the period 1972:2-1975:1. The initial estimate for the variance $V_t$ was weighted by 0.9 and the relative tightness on time variation for the parameter vector was assumed to be 0.1. The initial covariance matrix was assumed to be that of the OLS estimation, which implies that overall tightness was assumed to be one. The Kalman filter estimation covered the period of 1975:2–1989:12. As one observes, we use only ex ante and current information in this Kalman filter technique. This is in accordance with the real situation, where investors evaluate the time-variation in the betas.

Our maximum likelihood Kalman filter procedure is based on the study of Knif (1989); see also Goodrich (1989) and Berglund and Knif (1991). Knif found that Finnish common stock betas change according to a stationary first order autoregressive (AR1) process with a constant coefficient. The parameter vector is now assumed to vary according to the AR1 model, that is,

$$\theta_t - \bar{\theta} = F(\theta_{t-1} - \bar{\theta}) + u_t \tag{12}$$

where $\bar{\theta}$ = The mean vector of the parameters
F = The weights for the AR1 and mean parameters
$u_t$ = A random error with covariance matrix $M_t$.

The state space representation for the market model is now

$$r_{it} = \begin{bmatrix} x'_t & x'_t \end{bmatrix} \begin{bmatrix} \bar{\theta}_t \\ \theta_t - \bar{\theta}_t \end{bmatrix} + \varepsilon_t \tag{13}$$

$$= B'_t \gamma_t + \varepsilon_t$$

and for the parameter vector

$$\gamma_t = \begin{bmatrix} \bar{\theta} \\ \theta_t - \bar{\theta}_t \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & F \end{bmatrix} \begin{bmatrix} \bar{\theta}_{t-1} \\ \theta_{t-1} - \bar{\theta}_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ u_t \end{bmatrix} \tag{14}$$

$$= A\gamma_{t-1} + e_t$$

where $\underline{F}$ = $\underline{D}$iag $[\omega_1, \omega_2]$
$\theta_1 = \theta_{t-1}$ for all t
$e_t$ = A random error with covariance matrix $N_t$

We need to revise only the following updating Kalman equations

$$\Sigma_p = A_t \Sigma_{t-1} A'_t + N_t \tag{15}$$

where $\Sigma_{t-1}$ = The covariance matrix of $\gamma_{t-1}$.

$$\tilde{r}_{it} = r_{it} - X_t A_t \gamma_{t-1} \tag{16}$$

$$\gamma_t = A_t \gamma_{t-1} + K_t \tilde{r}_{it} \tag{17}$$

Our AR1 model collapses into the random walk model if the A matrix is equal to one. However, the estimation technique is now quite different. In the first phase, we compute a maximum likelyhood solution for the parameter vector through the above forward recursive Kalman equations that use only past and current information. Next, we use information from the whole sample period and find another set of ML estimators by applying the backward recursions of the Kalman smoother, see Goodrich (1989) and for details. As a final stage, we employ the mean reverting AR(1) model to compute the forecasted beta series. We run the second-pass regression over the forecasted betas in order to reduce the EIV problem, at least to some extent. This will be the case if the changing residual variance of the market model is dependent on the time-variation in the betas.

# 4 The Data

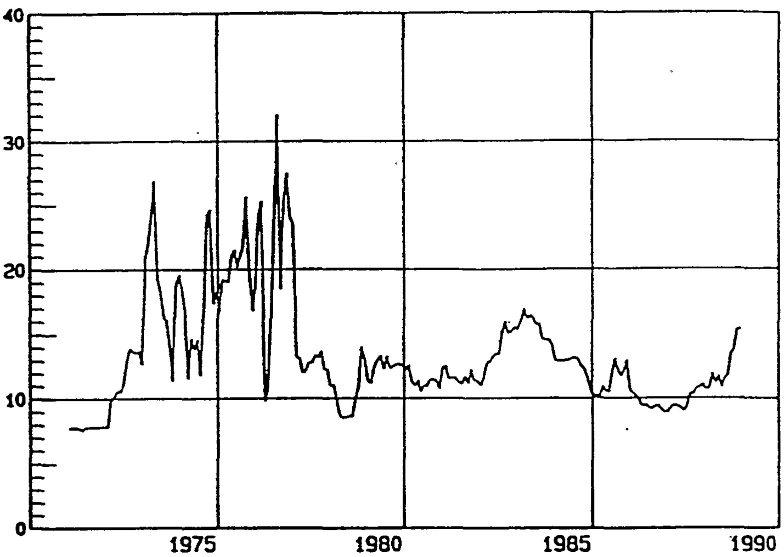
The stock market data employed here consist of end-of-month excess returns for all the common stocks listed on the Helsinki Stock Exchange (HSE) for the whole period covered, 1972:2–1989:12 (Table 1). The availability of short term interest rate data prior to 1972:1 was the limiting factor. The HSE general index used here is value weighted and described in Berglund – Wahlroos – Grandell (1983). Prices are corrected for cash dividends, splits, stock dividends and new issues. The correction is based on the principle that all income from a stock is reinvested in the stock with no transaction costs. In this study, returns are measured as changes in logarithmic indices.

Table 1 **Firms included in the analysis. This sample includes all restricted ordinary share series listed during the whole period of 1972:2–1989.**

| Firm | Designation |
|---|---|
| Bank of Åland Ltd K | AB |
| Effoa-Finland Steamsip Co Ltd K | EFFO |
| Enso-Gutzeit Ltd A | ENSOA |
| Fiskars Corporation | FISKK |
| Huhtamäki Corporation K | HUHTK |
| Instrumentarium Corporation | INSTA |
| Kemi Corporation | KEMI |
| Kesko Corporation | KESK |
| KANSALLIS-OSAKE-PANKKI | KOP |
| Kymmene Corporation | KYMI |
| Lassila & Tikanoja Ltd | LASS |
| Lohja Corporation A | LOHJA |
| Nokia Corporation | NOKIK |
| Otava Publishing Company Ltd | OTAVK |
| Partek Corporation | PART |
| Rauma-Repola Corporation | RAUM |
| Finnish Sugar Co Ltd I | SOKEI |
| Stockman A | STOCA |
| Suomen Trikoo Corp. A | TRIK |
| Union Bank of Finland Ltd A | SYPA |
| Tamfelt Group K | TAMF |
| Tampella Ltd | TAMP |
| Talous-Osakekauppa Co | TAOK |
| Wärtsilä Co I | WARTI |
| United Paper Mills Ltd K | YHTYK |

It is convenient to run asset pricing tests in the excess returns form. To do this, we constructed a short term interest rate which we were able to employ for the first time in studies of Finnish asset market returns (Figure 1). The Bank of Finland created the market for US dollar forwards in January 1972. Since then, the shortest maturity continously traded monthly is a three month forward contract. To compute the end-of-month time series for a three month interest rate, we collected end-of-month data on three month currency forward prices, currency spot rates and US dollar interest rates. The three month interest rate for the Finnish markka was then computed by using the standard formulas. However, in the monthly analysis we still had to use the one-month return in the three-month Euromarket return for the Finnish markka, i.e. we assume that interest rate yield curve is flat between the one and three month maturities.

Figure 1. **Approximated three month interest rate on the Finnish markka, 1972–1989**




As seen in figure 1, the short term interest rate was extremely volatile in the 1970s after the first oil crisis. There was a recession in Finland after the crisis, and on several occasions speculation occurred concerning a possible devaluation of the Finnish markka. The Bank of Finland devaluated the markka two times in 1977 and once again in February 1978, which reduced considerably the volatility of short term interest rates. The end of February 1978 is also the first point at which we apply Chow's (1983) test to examine whether the assumption of constant price of risk should be relaxed. Our hypothesis is that major changes in economic conditions are likely to change the pricing of market risk. However, we do not argue that the risk premium is constant over any period.

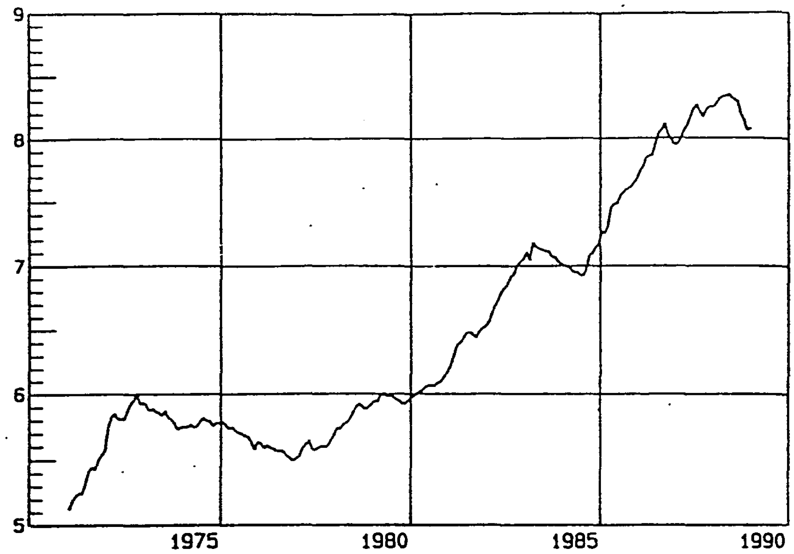
Table 2. **Summary statistics for the excess returns (in percentages per month) for 1972:2–1989:12 (215 observations)**

| Asset | Mean | $\sigma_m$ | Skewness | Kurtosis |
|---|---|---|---|---|
| AB | 0.746 | 10.683 | 1.332 | 16.702 |
| EFFO | 0.423 | 8.204 | 0.348 | 1.448 |
| ENSOA | -0.234 | 7.922 | 0.645 | 3.114 |
| FISKK | 1.292 | 7.252 | 0.296 | 1.959 |
| HUHTK | 0.782 | 6.610 | 1.056 | 2.655 |
| INSTA | 1.156 | 7.118 | 0.607 | 3.206 |
| KEMI | -0.360 | 10.646 | -0.694 | 4.457 |
| KESK | 0.658 | 5.147 | 1.060 | 2.481 |
| KOP | 0.104 | 6.644 | 0.751 | 4.821 |
| KYMI | -0.002 | 6.410 | 0.597 | 1.908 |
| LASS | 1.298 | 9.240 | 1.336 | 7.143 |
| LOHJA | 0.930 | 7.333 | 0.141 | 0.295 |
| NOKIK | -0.009 | 6.910 | 0.159 | 0.684 |
| OTAVK | 1.234 | 9.496 | 1.773 | 10.212 |
| PART | 0.522 | 6.594 | 0.242 | 0.555 |
| RAUM | 0.034 | 6.741 | 1.042 | 2.112 |
| SOKEI | 0.694 | 8.094 | 0.762 | 2.001 |
| STOCA | 0.895 | 6.645 | 0.521 | 2.684 |
| SYPA | 0.433 | 6.083 | 1.230 | 4.425 |
| TAMF | 0.717 | 9.835 | -0.486 | 6.468 |
| TAMP | 0.051 | 7.788 | 1.276 | 5.215 |
| TAOK | 1.866 | 9.520 | -0.031 | 1.863 |
| TRIK | 0.136 | 11.697 | 0.255 | 6.330 |
| WARTI | 0.613 | 7.480 | 0.764 | 1.155 |
| YHTYK | 0.707 | 7.459 | 0.380 | 0.934 |
| VWI[a] | 0.254 | 4.230 | 0.265 | 0.976 |

[a] The stock market index return.

Summary statistics of monthly excess returns for 25 firms and for the market index are shown in Table 2. Statistics are reported also for three subperiods according our structural analysis: 1972:2–1978:2, 1978:3–1984:3 and 1984:4–1989:12 (see Appendix 1). The statistics show us that the mean varied greatly as between periods. The monthly return distribution is, on the average, always somewhat skewed to the right and leptokurtic, as is usual. The first period ends with the third devaluation of the markka descriped above. The second period ends in 1984:3 for two reasons: First, unrestricted shares, i.e. shares that foreign investors are allowed to buy, have been listed separately on the Helsinki Stock Exchange since January; 1984. This may have changed the pricing of Finnish stocks. Another major change was made by the Bank of Finland, which first gave foreign banks access to central bank financing April; 1984. This meant in practice that competition was greatly enhanced in the Finnish money market.

Figure 2. **The Helsinki Stock Exchange general index, 1972–1989**




We selected five additional economic changes that could affect the risk return relationship. The first of them occurred at the end of December 1981 when the stock market index started its rapid raise (Figure 2). Another break point occurs at the end of October 1982. The Finnish markka was devaluated twice in that month. The third change was associated with a partial abolition of the regulation of average rates on bank lending to the public. This change took place at the end of April 1983 and increased banks' activity in the money market. The next potential break point, after March 1984, is at the end of December 1986, when the market for certificates of deposit were introduced in Finland. The last stability test is computed because of the crash of October 1987.

# 5 Empirical Results

## 5.1 Market Risk Estimates

We carry out here the well known "rolling" beta estimation method introduced in Fama and McBeth (1973). We estimate the betas with the traditional market model described in the equation 2. Since the market risk is known to be non-stationary, we compute the OLS betas for three-year periods as well as for the traditional five year period. Furthermore, we also employ the OLS and ML Kalman filter techniques, which allow for conditional time variation in the betas.

Table 3. **Summary statistics for estimated beta series (155 observations per series)**

| Firm | 5 year beta Mean | $\sigma_m$ | 3 year beta Mean | $\sigma_m$ | KF RW Mean | $\sigma_m$ | KF AR1 Mean | $\sigma_m$ |
|---|---|---|---|---|---|---|---|---|
| AB | 0.655 | 0.146 | 0.627 | 0.143 | 1.014 | 0.127 | 0.748 | 0.000 |
| EFFO | 0.614 | 0.175 | 0.553 | 0.250 | 1.254 | 0.248 | 0.888 | 0.005 |
| ENSOA | 0.758 | 0.258 | 0.767 | 0.406 | 1.377 | 0.186 | 1.010 | 0.005 |
| FISKK | 0.657 | 0.194 | 0.630 | 0.274 | 0.759 | 0.055 | 0.795 | 0.005 |
| HUHTK | 0.948 | 0.436 | 0.918 | 0.495 | 0.577 | 0.195 | 0.495 | 0.018 |
| INSTA | 0.872 | 0.106 | 0.910 | 0.164 | 0.416 | 0.205 | 0.868 | 0.002 |
| KEMI | 1.171 | 0.598 | 1.134 | 0.821 | 0.875 | 0.074 | 0.913 | 0.037 |
| KESK | 0.556 | 0.139 | 0.576 | 0.202 | 0.424 | 0.174 | 0.670 | 0.024 |
| KOP | 1.136 | 0.223 | 1.083 | 0.325 | 1.193 | 0.074 | 1.069 | 0.001 |
| KYMI | 1.008 | 0.137 | 1.088 | 0.241 | 0.284 | 0.344 | 0.935 | 0.009 |
| LASS | 0.754 | 0.194 | 0.751 | 0.279 | 0.351 | 0.132 | 0.711 | 0.021 |
| LOHJA | 1.231 | 0.131 | 1.210 | 0.241 | 0.979 | 0.052 | 1.202 | 0.010 |
| NOKIK | 1.112 | 0.224 | 1.157 | 0.293 | 0.851 | 0.128 | 1.074 | 0.006 |
| OTAVK | 0.543 | 0.392 | 0.479 | 0.297 | 1.421 | 0.010 | 0.684 | 0.102 |
| PART | 1.020 | 0.194 | 0.989 | 0.309 | 0.838 | 0.096 | 1.086 | 0.002 |
| RAUM | 1.103 | 0.208 | 1.071 | 0.304 | 1.377 | 0.201 | 0.993 | 0.011 |
| SOKEI | 0.932 | 0.145 | 0.888 | 0.225 | 1.173 | 0.152 | 0.928 | 0.010 |
| STOCA | 0.879 | 0.194 | 0.921 | 0.346 | 0.572 | 0.088 | 0.870 | 0.003 |
| SYPA | 0.940 | 0.091 | 0.912 | 0.135 | 1.398 | 0.216 | 1.010 | 0.018 |
| TAMF | 0.967 | 0.520 | 0.971 | 0.720 | 0.781 | 0.063 | 0.908 | 0.101 |
| TAMP | 0.935 | 0.462 | 0.935 | 0.553 | 0.710 | 0.040 | 0.935 | 0.063 |
| TAOK | 0.471 | 0.326 | 0.607 | 0.599 | -0.124 | 0.397 | 0.474 | 0.002 |
| TRIK | 1.005 | 0.212 | 0.901 | 0.556 | 1.184 | 0.178 | 1.042 | 0.388 |
| WARTI | 1.307 | 0.329 | 1.300 | 0.460 | 0.572 | 0.131 | 1.001 | 0.000 |
| YHTYK | 1.278 | 0.186 | 1.247 | 0.304 | 0.963 | 0.108 | 1.153 | 0.011 |
| MEAN | 0.914 | 0.249 | 0.905 | 0.358 | 0.849 | 0.147 | 0.899 | 0.034 |
| St.dev. | 0.237 | 0.135 | 0.231 | 0.178 | 0.402 | 0.092 | 0.186 | 0.079 |

We revise all the beta estimations monthly. The first five-year period for beta estimation is 1972:2–1977:1 and the corresponding three year period is 1974:2–1977:1. We then proceed by dropping and adding one observation and repeating the computation. Table 3 presents summary statistics for all the beta series. The time series for three-year betas are more volatile than those for five-year betas. Surprisingly, both the Kalman filtered beta series are a lot less volatile, specially the ML betas, than the static estimates for the betas. Correlation between the monthly beta series is highest for the static betas and lowest between the static betas and the OLS (RW) Kalman filter betas (Table 4).

Table 4. **Correlation matrix for estimated beta series (3875 observations)**

| Variable | B5Y | B3Y | BKFRW | BKFAR1 |
|---|---|---|---|---|
| B5Y | 1.000 | | | |
| B3Y | 0.756 | 1.000 | | |
| BKFRW | 0.023 | -0.050 | 1.000 | |
| BKFAR1 | 0.430 | 0.311 | 0.417 | 1.000 |

Variables in the cross-moment matrix:

| | | |
|---|---|---|
| B5Y | = | Five-year beta estimation period. |
| B3Y | = | Three-year beta estimation period. |
| BKFRW | = | Kalman filter (RW) beta. |
| BKFAR1 | = | Kalman filter (AR1) beta. This series contains forecasted betas ($\beta_t = \omega\beta_{t-1} + (1-\omega)\bar{\beta}$) |

A closer look at the estimated beta series was found to be very interesting. For this purpose, we took three beta series for a firm in the same figure and reported the mean value for the AR1 beta estimates (see Appendix 2). By looking at these figures one sees immediately that the magnitude of variation in the statically estimated beta series is extremely high. These beta estimates typically vary between 0.3 and 2.0, five-year betas having a slightly lower range of variation. It is somewhat perplexing that an ongoing firm's beta could vary this much. Furthermore, shocks to the beta estimates seem to persist over time. This is evident in the beta estimates for the forest industry companies (Kemi, Kymi, Tamf, Tamp and Yhtyk). The Finnish markka was devaluated in April 1977. This shock caused a dramatic drop in the betas after three or five years, depending on the length of the estimation period.

The series for both the RW (OLS) and AR1 (ML) Kalman filter betas are more stable by nature, as was, of course, expected for the AR1 beta series that are mean reverting. The AR1 maximum likelihood estimation results are available in Appendix 3. Our ML beta estimates are even more mean reverting than those of Knif (1989:154). The beta values for a firm typically vary within the range of 0.2. However, the Random Walk OLS beta series do display dramatic jumps in the beta values in the late 1980s and particularly in 1989.[2] This indicates that there has been a rapid change in investors' expectations regarding the future earnings of certain companies. There are several reasonable explanations for this

[2] Note that the scale in the figures is extremely large for the Kalman filter betas. This creates the impression that the Kalman filter series hardly varies at all before the mid 1980s.

phenomenom: First, the collapse of trade with the Soviet Union, which had a drastic impact on only certain companies. Second, the suggestion of the EEC to start negotiations with the EFTA countries in order to create a large European Economic Area. Third, the revaluation of the Finnish markka in March 1989. Fourth, the Bank of Finland raised the cash reserve requirement several times, which reduced banks' earnings.

## 5.2 Pricing of Risk

The Sharpe-Lintner version of the CAPM is usually tested by iterating the CSR described in equation 4. We computed this regression over all periods t in one regression on pooled data instead of iterating monthly. We estimated the model with and without the constant. Strictly speaking, we assume that the CAPM holds in the restricted version of the model and obtain an estimate of the risk premium implied by the CAPM. Consequently, the unrestricted regression is, strictly speaking, a test of the mean-variance efficiency of the market index or a test of the risk premium implied by one-factor capital asset pricing model, where the prespecified factor is the market index.

Table 5 gives the outcome of these regressions. We find that the estimated risk premium of the restricted regressions is positive and highly significant. The regressions on the Kalman filter beta series have the highest t-values. However, the ML beta forecasts capture clearly better the variation of the excess returns. Note also that the annual market index excess return for the whole period is about 25 %.

The three first unrestricted regressions imply that the CAPM is not valid or the index is not mean-variance efficient because the constant has significant t-values. Surprisingly, the risk premium is negative and highly significant for the regression on five year beta estimates. This supports the results of Malkamäki (1991). These betas have been commonly employed in the CSR's for about fifteen years to test the validity of the CAPM for thin markets. However, the three-year and OLS Kalman filter beta regressions do not give significant estimates for the negative risk premium.

The fourth regression on the forecasted ML betas gave quite different results. The constant is not significant i.e. we do not reject the mean-variance efficiency of the market index. Furthermore, the coefficient of the price of risk is now positive, as in the CAPM, but not statistically significant.

We assumed in the above tests that the risk-return relationship does not change over time. However, it is well known that this is not the case. Our pooled data enables us to test whether some prespecified structural break points in Finnish economy have affected the pricing of risk. We described the potential breaks to be tested in Chapter 4. We carried out the well-known Chow tests for structural stability (Chow 1983). The model here is always estimated with a constant. A test for a break always covers two years of data before and after the break. This implies that we have 1196 degrees of freedom in the F-value analysis. The outcome of these tests is presented in Table 6. The evidence is very straight forward. The risk premium is not stable over time. We have at least five clear breaks in the risk-return relationship for our entire period. The outcome was almost exactly the same regardless of the beta series employed.

Table 5. **Monthly average risk premiums associated with the stock market general index (in percent per month) for 1977:2–1989:12 (3875 observations for 25 ordinary shares)**

| Model | $a^c$ | $\lambda^c$ | $\sigma^{cd}$ |
|---|---|---|---|
| **Restricted**[a] | | | |
| 5 year beta | - | 0.696 (5.35) | 7.975 |
| 3 year beta | - | 0.684 (5.41) | 7.975 |
| Kalman filter RW Ex ante | - | 0.768 (5.71) | 7.971 |
| Kalman filter AR1 ML | - | 0.949 (6.84) | 7.957 |
| **Unrestricted**[b] | | | |
| 5 year beta | 1.791 (5.20) | -0.994 (-2.84) | 7.949 |
| 3 year beta | 1.295 (4.57) | -0.456 (-1.63) | 7.954 |
| Kalman filter RW Ex Ante | 1.128 (3.99) | -0.290 (-0.98) | 7.956 |
| Kalman filter AR1 ML | 0.615 (1.05) | 0.297 (0.468) | 7.957 |

a. Model estimated: $r_{it} = \lambda\beta_{it-1} + e_{it}$ $\quad i = 1, ..., 25$
$t = 1, ..., 155$

b. Model estimated: $r_{it} = a + \lambda\beta_{it-1} + e_{it}$ $\quad i = 1, ..., 25$
$t = 1, ..., 155$

c. All the coefficients are multiplied by 100.

d. Standard error of estimate

Table 6. **Chow tests for structural breaks in the risk return relationship**

| Suggested break at the end of | | F-value | P-value |
|---|---|---|---|
| February | , 1978 | 33.05 | 0.00 |
| December | , 1981 | 15.96 | 0.00 |
| October | , 1982 | 0.02 | 0.98 |
| April | , 1983 | 13.40 | 0.00 |
| March | , 1984 | 11.10 | 0.00 |
| December | , 1986 | 0.36 | 0.70 |
| September | , 1987 | 32.50 | 0.00 |

Model estimated: $r_{it} = a + \lambda\beta_{it-1} + e_{it}$.

The model is estimated for a two year period before and after a tested break point and for the combined four year period. The F-tests have 1196 degrees of freedom.

Next, we did the same regression analyses as above but for subperiods (see Table 7). The periods are based on the empirical finding in Malkamäki (1991) that the risk premium varied around zero between 1978:3 and 1984:3 and were clearly negative after 1984:3. The first period is quite short. It was included because the break at the end, 1978:2, is extremely significant. All regressions gave the same result for the middle period: the risk premium is not significant. However, the regression on ML betas has a positive sign for the price of risk. The period after 1984:3 is interesting. The price of risk was negative and clearly significant in the regressions computed over the static estimates for betas. The risk premium has a negative sign in the Kalman filter beta regressions but the statistics show no significance. Our last time period studied excludes the periods before 1978:3 and after 1988:12. The latter period is excluded according to drastic slowdown of the Finnish economy that started in the early 1989. The risk-return relationship implied by the CAPM is strongest for this period. The regression on ML beta series has a t-value of 1.60 for the risk premium coefficient and does not reject the mean-variance efficency of the market index.

Table 7. **Monthly average risk premiums associated with the stock market general index (in percent per month) for a set of subperiods**

| Period | a[a] | λ[a] | σ[ab] |
|---|---|---|---|
| **77:2–78:2** | | | |
| 5 year beta | -2.416<br>(-2.03) | 0.614<br>(0.51) | 5.904 |
| 3 year beta | -2.244<br>(-3.11) | 0.401<br>(0.64) | 5.912 |
| Kalman filter RW | -1.765<br>(-1.70) | -0.074<br>(-0.07) | 5.916 |
| Kalman filter AR1 | 0.475<br>(0.304) | -2.550<br>(-1.51) | 5.895 |
| **78:3–84:3** | | | |
| 5 year beta | 1.861<br>(4.43) | -0.54<br>(-1.35) | 7.093 |
| 3 year beta | 1.357<br>(4.02) | -0.020<br>(-0.65) | 7.097 |
| Kalman filter RW | 1.910<br>(3.90) | -0.663<br>(-1.24) | 7.094 |
| Kalman filter AR1 | 0.258<br>(0.34) | 1.205<br>(1.45) | 7.092 |
| **84:4–89:12** | | | |
| 5 year beta | 2.772<br>(4.37) | -2.185<br>(-3.12) | 8.967 |
| 3 year beta | 2.232<br>(3.81) | -1.559<br>(-2.42) | 8.977 |
| Kalman filter RW | 1.034<br>(2.62) | -0.149<br>(-0.37) | 8.992 |
| Kalman filter AR1 | 0.962<br>(0.98) | -0.056<br>(-0.05) | 8.993 |
| **78:3–88:12** | | | |
| 5 year beta | 2.428<br>(6.72) | -1.053<br>(-2.89) | 7.986 |
| 3 year beta | 1.829<br>(5.96) | -0.399<br>(-1.31) | 7.998 |
| Kalman filter RW | 1.446<br>(4.33) | 0.024<br>(0.07) | 7.997 |
| Kalman filter AR1 | 0.472<br>(0.74) | 1.11<br>(1.60) | 7.993 |

Model estimated: $r_{it} = a + \lambda \beta_{it-1} + e_{it}$

a. All the coefficients are multiplied by 100.
b. Standard error of estimate.

# 6 Conclusions

We did an empirical analysis of the Sharpe-Lintner version of the CAPM for the thin Finnish stock market. Two unconditional and conditional sets of the market risk parameters of firms were computed. The unconditional beta of a firm was computed by the traditional OLS rolling beta technique, assuming that the beta is constant over a period of five/three years. As an alternative approach we applied dynamic OLS and ML Kalman filter techniques that account for time variation in the market risk. We found that the mean of returns, the systematic risk and pricing of the systematic risk vary through time. We also found again the puzzling prior result that the regression on the static OLS betas give us a negative and statistically significant coefficient for the risk premium. We show that this phenomenom is strongest after April 1984. Our ML Kalman filter betas also have extreme difficulties in explaining anything for that period. More research is needed to explain this phenomenom. However, our ML Kalman filter beta estimates are clearly able to capture best the excess returns on Finnish shares. In the restricted regressions, the regression ran over these betas had the largest risk premium, as well as the highest t-value. In the unrestricted regressions, the regression on the ML betas was the only one with a positive coefficient for the risk premium and contained also relatively high significance when we excluded extraordinary periods of the Finnish economy from the analysis. Furthermore, this regression does not reject the mean-variance efficiency of the market index, as the others do. In further research, it would be useful to examine whether our results are robust for other samples of firms. It is also quite evident that the constant risk premium assumption should be relaxed.

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# Appendix 1.1

**Summary statistics for the excess returns, (in percentages per month) for 1972:2–1978:2 (73 observations)**

| Asset | Mean | $\sigma_m$ | Skewness | Kurtosis |
|---|---|---|---|---|
| AB | -0.234 | 11.341 | -0.891 | 5.441 |
| EFFO | -0.758 | 6.695 | -0.243 | 1.797 |
| ENSOA | -1.732 | 8.098 | -0.027 | 3.707 |
| FISKK | 0.199 | 6.573 | -0.103 | 0.044 |
| HUHTK | -0.157 | 5.375 | 2.353 | 8.174 |
| INSTA | 0.623 | 7.574 | 0.343 | 0.517 |
| KEMI | -1.191 | 8.916 | 0.162 | 1.704 |
| KESK | -0.434 | 5.058 | 1.772 | 4.584 |
| KOP | -0.857 | 6.867 | 0.063 | 0.404 |
| KYMI | -0.926 | 6.521 | 0.231 | 2.054 |
| LASS | -0.069 | 8.549 | 2.660 | 13.570 |
| LOHJA | -0.215 | 7.519 | 0.648 | 0.988 |
| NOKIK | -0.738 | 6.290 | 0.180 | 0.742 |
| OTAVK | 0.967 | 11.330 | 2.331 | 13.188 |
| PART | -0.829 | 6.709 | -0.007 | 0.096 |
| RAUM | -0.881 | 7.280 | 1.131 | 1.873 |
| SOKEI | -0.375 | 7.859 | 0.849 | 1.956 |
| STOCA | -1.082 | 5.825 | 0.650 | 1.750 |
| SYPA | -0.077 | 6.822 | 1.143 | 2.630 |
| TAMF | -0.913 | 12.530 | -0.756 | 4.601 |
| TAMP | -2.466 | 6.961 | 0.071 | 0.501 |
| TAOK | 2.786 | 8.880 | 0.760 | 2.176 |
| TRIK | -0.739 | 8.292 | 1.057 | 3.638 |
| WARTI | -1.414 | 7.328 | 0.700 | 1.242 |
| YHTYK | -0.483 | 7.854 | 0.421 | 0.466 |
| VWI[a] | -0.721 | 4.406 | 0.942 | 2.167 |

[a] The stock market index return.

# Appendix 1.2

**Summary statistics for the excess returns (in percentages per month) for 1978:3—1984:3 (73 observations)**

| Asset | Mean | Std. | Skewness | Kurtosis |
|---|---|---|---|---|
| AB | 1.263 | 7.092 | 0.067 | 2.436 |
| EFFOA | 0.655 | 6.433 | -0.338 | 2.210 |
| ENSOA | 0.307 | 5.710 | 0.983 | 2.273 |
| FISKK | 0.931 | 5.492 | 0.108 | 1.176 |
| HUHTK | 2.584 | 7.120 | 0.992 | 3.180 |
| INSTA | 2.318 | 5.577 | 1.938 | 5.857 |
| KEMI | -1.538 | 12.402 | -1.469 | 5.919 |
| KESK | 0.748 | 3.368 | -0.118 | 0.329 |
| KOP | 1.538 | 6.459 | 2.683 | 13.159 |
| KYMI | 0.178 | 5.365 | 0.512 | 2.742 |
| LASS | 1.993 | 6.848 | 0.029 | 1.367 |
| LOHJA | 2.052 | 6.423 | -0.183 | 0.790 |
| NOKIK | 0.645 | 6.016 | 0.112 | 2.406 |
| OTAVK | 1.459 | 5.017 | -0.090 | 1.482 |
| PART | 1.311 | 4.980 | 1.299 | 3.060 |
| RAUM | 0.680 | 6.502 | 1.264 | 2.163 |
| SOKEI | 2.189 | 7.682 | 0.820 | 1.867 |
| STOCA | 2.333 | 6.608 | 0.502 | 6.768 |
| SYPA | 1.005 | 5.551 | 2.423 | 13.827 |
| TAMF | 1.853 | 6.335 | 1.338 | 3.601 |
| TAMP | 0.659 | 6.517 | 0.695 | 3.246 |
| TAOK | 2.388 | 9.657 | -0.332 | 3.069 |
| TRIK | 1.421 | 12.346 | -1.345 | 10.440 |
| WARTI | 2.813 | 7.230 | 0.681 | 0.538 |
| YHTYK | 1.672 | 7.087 | 0.512 | 1.865 |
| VWI[a] | 1.172 | 3.278 | 0.000 | 2.024 |

[a] The stock market index return.

# Appendix 1.3

**Summary statistics for the excess returns (in percentages per month) for 1984:4–1989:12 (69 observations)**

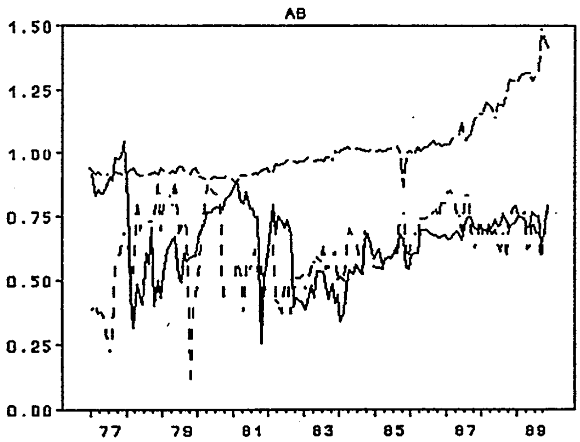
| Asset | Mean | Std. | Skewness | Kurtosis |
|---|---|---|---|---|
| AB | 1.236 | 12.885 | 3.124 | 21.096 |
| EFFO | 1.427 | 10.860 | 0.442 | 0.010 |
| ENSOA | 0.776 | 9.476 | 0.982 | 4.491 |
| FISKK | 2.830 | 9.166 | 0.212 | 1.523 |
| HUHTK | -0.131 | 6.926 | 0.400 | -0.257 |
| INSTA | 0.490 | 7.974 | 0.590 | 4.397 |
| KEMI | 1.766 | 10.143 | 0.274 | 0.629 |
| KESK | 1.718 | 6.468 | 0.654 | 0.815 |
| KOP | -0.397 | 6.431 | -0.251 | 0.953 |
| KYMI | 0.784 | 7.228 | 0.896 | 1.039 |
| LASS | 2.008 | 11.774 | 0.937 | 4.501 |
| LOHJA | 0.954 | 7.859 | -0.021 | -0.151 |
| NOKIK | -0.211 | 8.309 | 0.198 | -0.181 |
| OTAVK | 1.280 | 11.003 | 1.028 | 1.963 |
| PART | 1.118 | 7.740 | 0.176 | -0.319 |
| RAUM | 0.321 | 6.380 | 0.876 | 3.277 |
| SOKEI | 0.245 | 8.629 | 0.770 | 2.655 |
| STOCA | 1.466 | 7.073 | 0.373 | 0.642 |
| SYPA | 0.367 | 5.822 | 0.457 | -0.182 |
| TAMF | 1.241 | 9.549 | 0.316 | 4.861 |
| TAMP | 2.073 | 9.125 | 1.986 | 5.879 |
| TAOK | 0.340 | 9.973 | -0.261 | 0.593 |
| TRIK | -0.298 | 13.915 | 1.176 | 3.198 |
| WARTI | 0.429 | 7.363 | 1.175 | 2.586 |
| YHTYK | 0.944 | 7.350 | 0.342 | 1.260 |
| VWI[a] | 0.314 | 4.737 | 0.042 | 0.241 |

[a] The stock market index return.

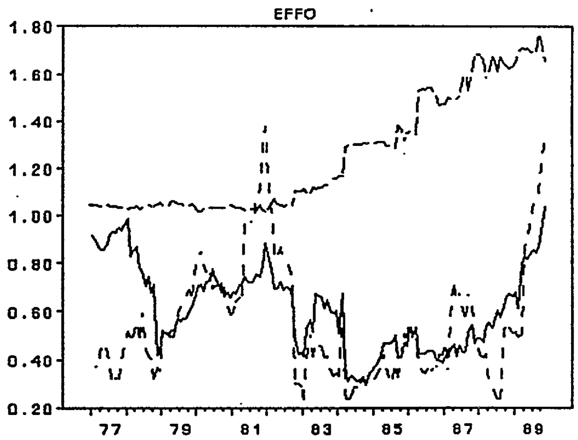
Appendix 2

**Monthly Beta Series for 25 Firms**

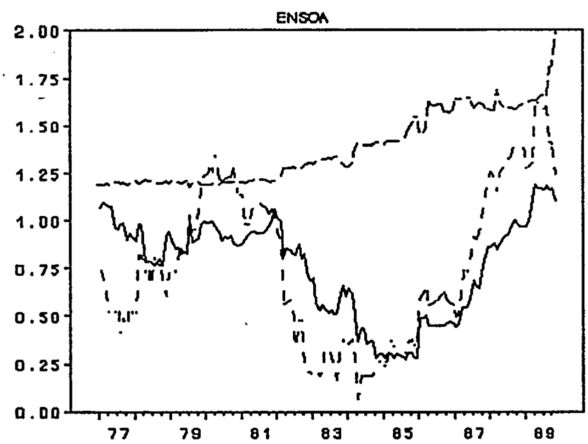
——— 5 year estimation period for a beta
----- 3 year estimation period for a beta
-.-.-.- Kalman filter estimation for a beta
$\bar{\beta}$ = The mean beta parameter in the AR1 parameter equation



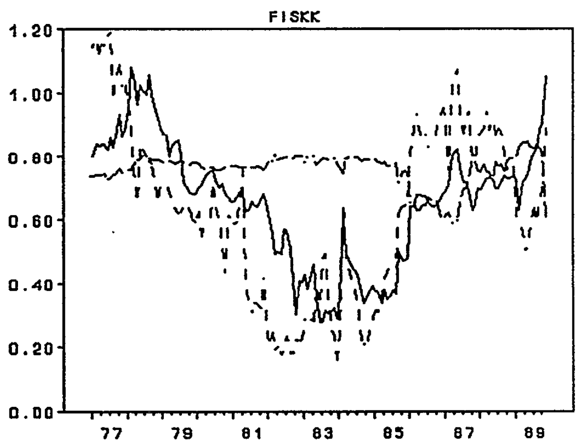

$\bar{\beta} = .748$



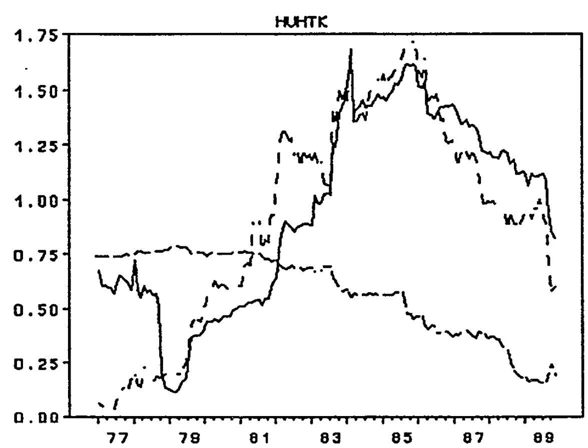

$\bar{\beta} = .889$



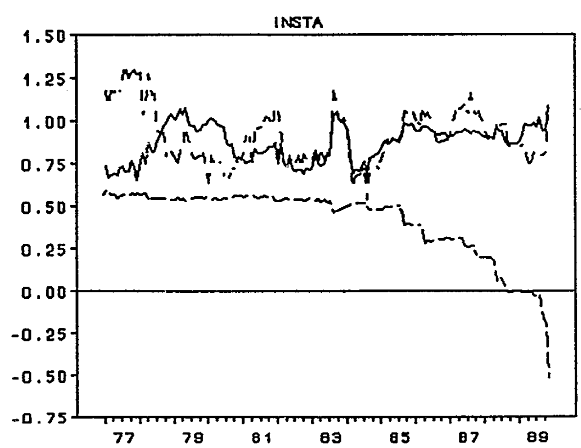

$\bar{\beta} = 1.011$



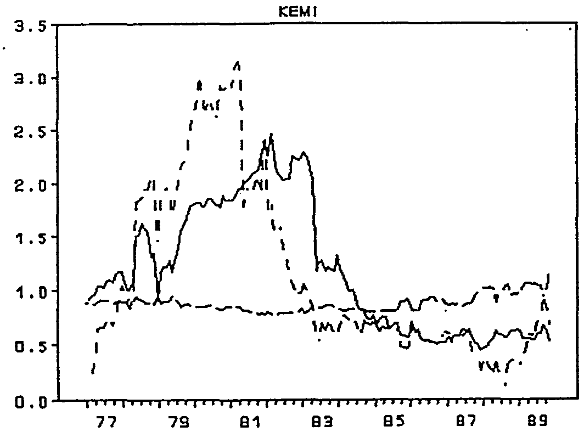

$\bar{\beta} = .795$



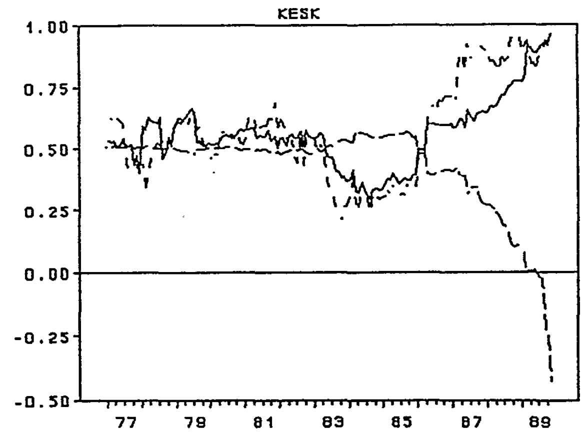

$\bar{\beta} = .968$



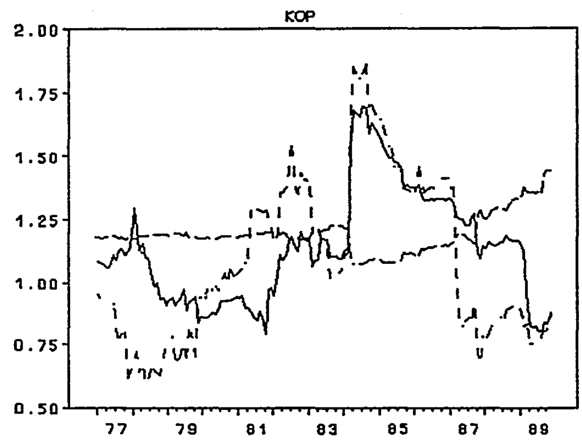

$\bar{\beta} = .868$



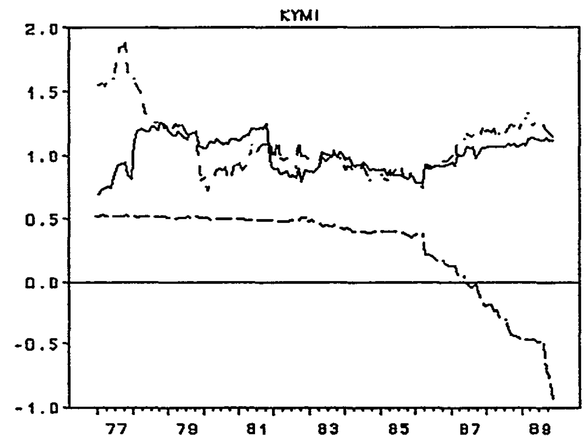

$\overline{\beta}$ = .914



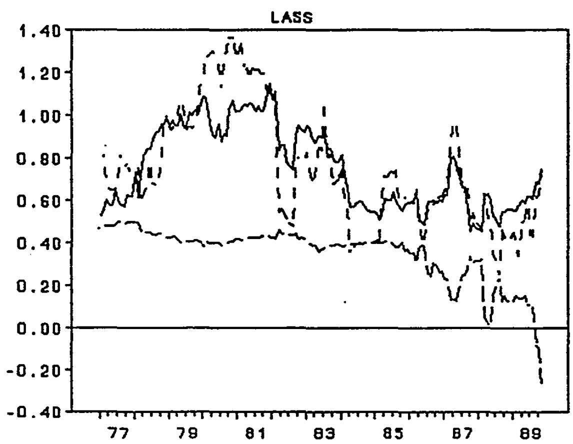

$\overline{\beta}$ = .671



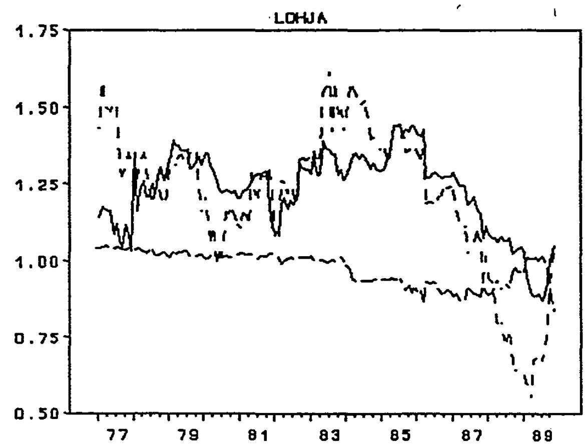

$\overline{\beta}$ = 1.067



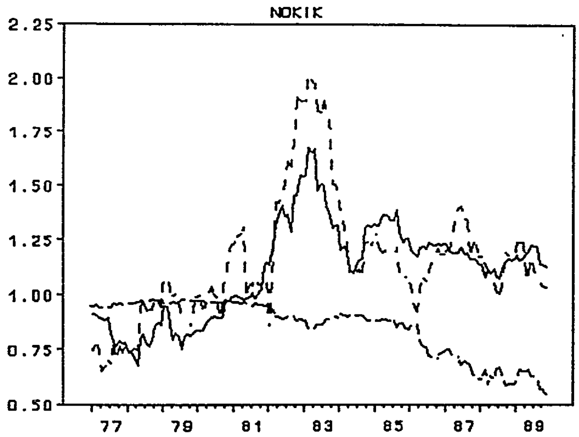

$\overline{\beta}$ = .934



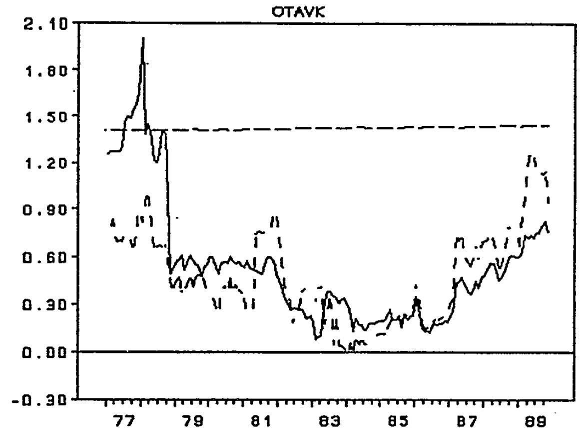

$\overline{\beta}$ = .710



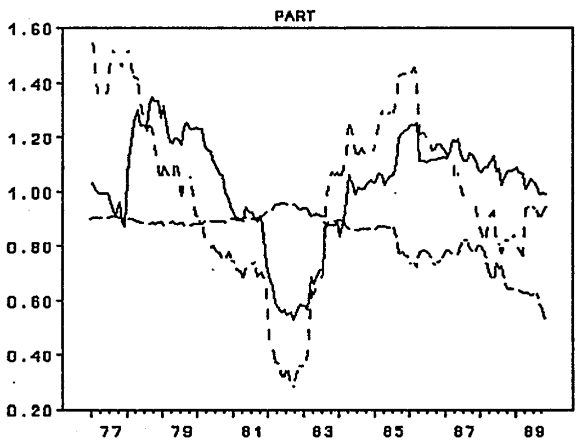

$\overline{\beta}$ = 1.202



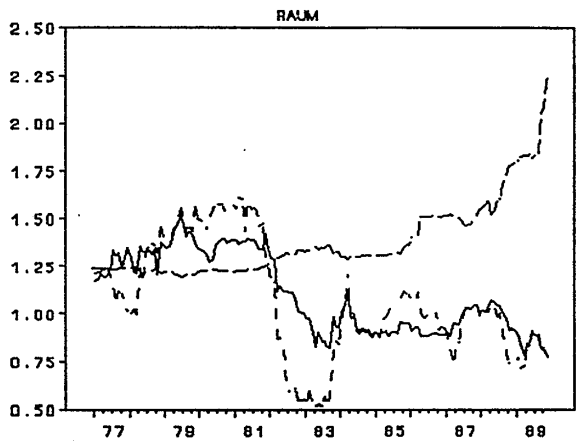

$\overline{\beta} = 1.075$



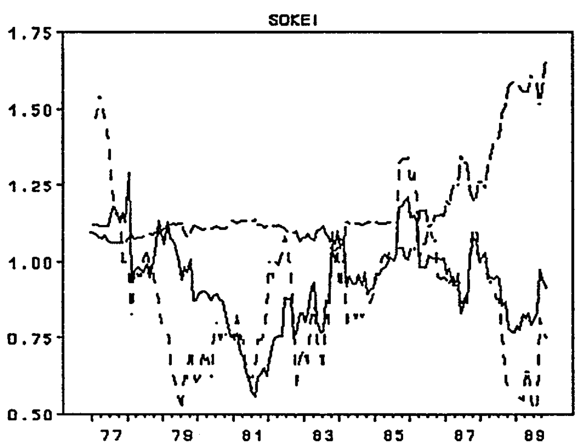

$\overline{\beta} = .689$



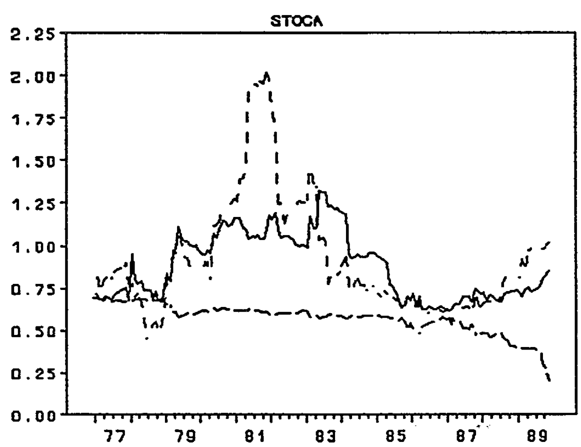

$\overline{\beta} = 1.086$



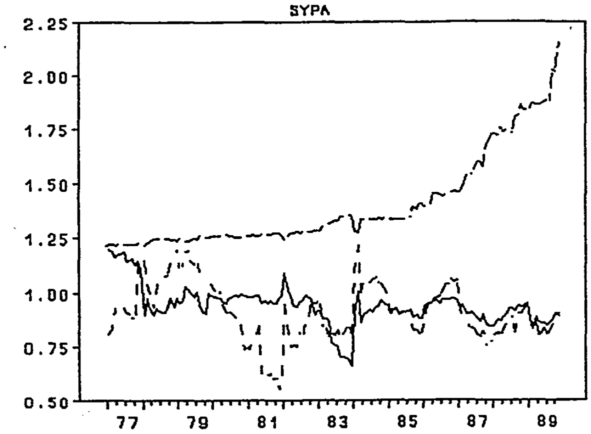

$\overline{\beta} = .993$



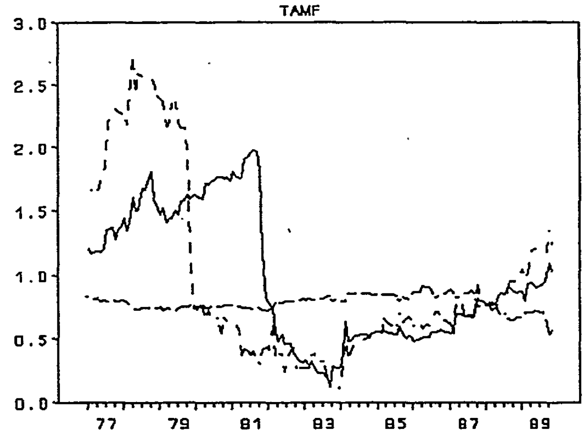

$\overline{\beta} = .993$



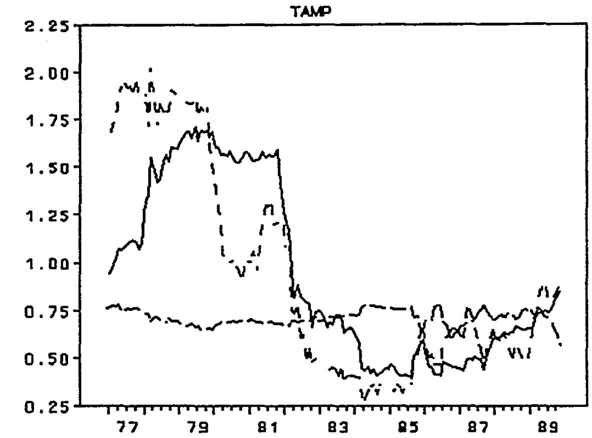

$\overline{\beta} = .870$



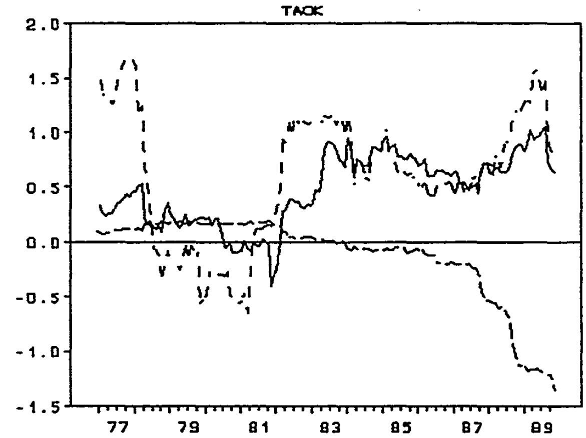

$\overline{\beta} = 1.011$



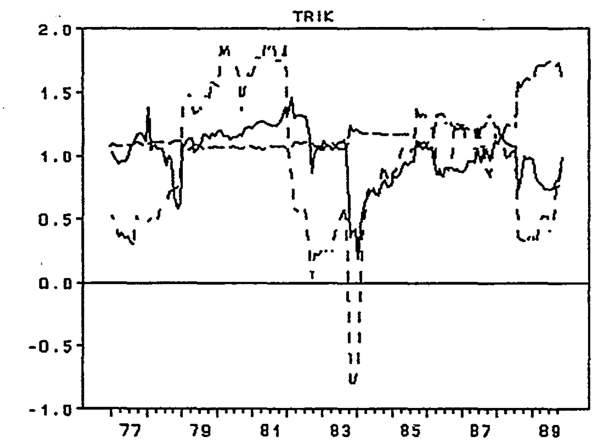

$\overline{\beta} = .909$



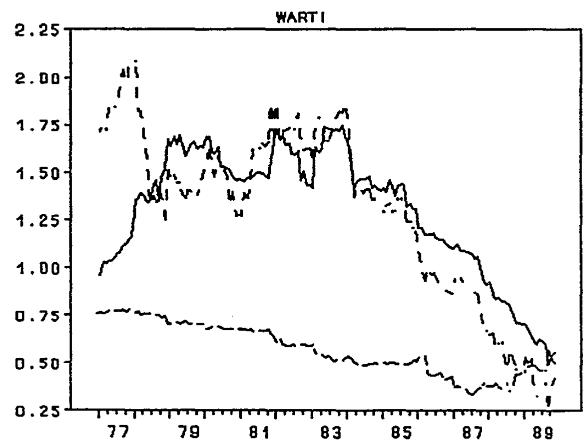

$\overline{\beta} = .948$



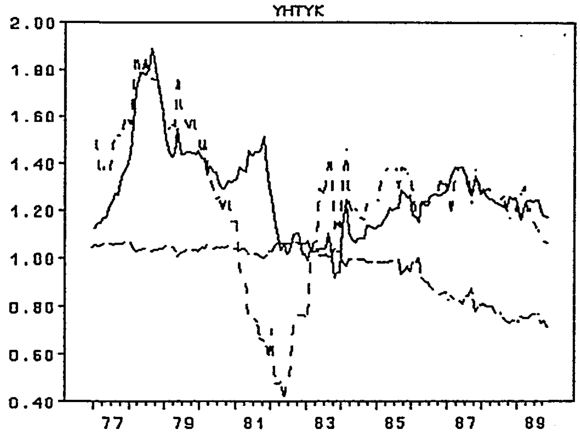

$\overline{\beta} = .474$




$\overline{\beta} = 1.039$




$\overline{\beta} = 1.001$




$\overline{\beta} = 1.153$

# Appendix 3

**Maximum likelihood estimation results for the Kalman filter AR1 specification (estimation period 1972:2–1989:12)**

| STOCK | $\omega$ | $\bar{\beta}$ | q | $\sigma^2$ | $\alpha$ | $R^2$ | P(F) |
|---|---|---|---|---|---|---|---|
| AB | -.0902 | .7483* | .0016 | .0104 | .0049 | .0954 | .9995 |
| EFFO | -.0227 | .8885* | .4523 | .0046 | .0035 | .1844 | 1.0 |
| ENSOA | .0172 | 1.0106* | .6726 | .0035 | -.0052 | .2575 | 1.0 |
| FISKK | .0322 | .7953* | .2926 | .0035 | .0126* | .2258 | 1.0 |
| HUHTK | .4469* | .9676* | .4967 | .0019 | .0020 | .2981 | 1.0 |
| INSTA | -.0346 | .8680* | .0989 | .0036 | .0097* | .2580 | 1.0 |
| KEMI | -.0797 | .9139* | 1.1405 | .0079 | -.0057 | .1317 | 1.0 |
| KESK | -.2138 | .6707* | .1303 | .0016 | .0055* | .2955 | 1.0 |
| KOP | .0027 | 1.0664* | .3458 | .0016 | -.0031 | .3859 | 1.0 |
| KYMI | .0694 | .9342* | .2063 | .0022 | -.0013 | .3590 | 1.0 |
| LASS | .2163 | .7103* | .1765 | .0074 | .0120* | .0938 | .9994 |
| LOHJA | -.0785 | 1.2017* | .0221 | .0026 | .0073* | .4318 | 1.0 |
| NOKIK | -.0526 | 1.0748* | .1426 | .0025 | -.0036 | .4354 | 1.0 |
| OTAVK | .1064 | .6889 | 2.8235 | .0035 | .0116* | .1357 | 1.0 |
| PART | -.0104 | 1.0863* | .2200 | .0021 | .0031 | .4179 | 1.0 |
| RAUM | -.0472 | .9927* | .3495 | .0022 | -.0038 | .3930 | 1.0 |
| SOKEI | .0329 | .9927* | .5949 | .0039 | .0050 | .2550 | 1.0 |
| STOCA | -.0626 | .8699 | .0788 | .0030 | .0072 | .2844 | 1.0 |
| SYPA | .0431 | 1.0106* | .5339 | .0011 | .0011 | .5006 | 1.0 |
| TAMF | .3272 | .9094* | .9270 | .0062 | .0079 | .1870 | 1.0 |
| TAMP | .7896* | .9484 | .0262 | .0045 | .0006 | .2410 | 1.0 |
| TAOK | .0383 | .4740 | .1097 | .0085 | .0176* | .0419 | .8982 |
| TRIK | -.2379 | 1.0390* | 4.5252 | .0050 | .00005 | .1222 | 1.0 |
| WARTI | .2530 | 1.001* | .0009 | .0038 | .0044 | .3001 | 1.0 |
| YHTYK | .0587 | 1.1527* | .3089 | .0026 | .0052 | .4426 | 1.0 |

Estimated model: $r_{it} = \alpha_t + \beta_t r_{mt} + \varepsilon_t$,

where $\alpha_t$ = constant $\quad$ $\text{var}(\varepsilon_t) = \sigma^2$

$\beta_t = \omega\beta_{t-1} + (1-\omega)\bar{\beta} + v_t$ $\quad$ $\text{var}(v_t) = q$

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