Paavo Peisa*
Bank of Finland Research Department
29.10.1988

26/88

FIRM GROWTH: ADJUSTMENT AND FLUCTUATIONS

*Bank of Finland Economics Department

Suomen Pankin monistuskeskus
Helsinki 1988
ISBN 951-686-173-3
ISSN 0785-3572

ABSTRACT

Firm growth is analysed using panel data for a sample of Finnish firms over the period 1978 - 1985. The paper focuses on the relationship between firm characteristics and growth, paying attention to both latent and measured characteristics.

Estimation results indicate that small firms grew significantly faster than large firms during the sample period. Small businesses expanded during the first years of the sample period, whereas later on there were no differences in performance between different size groups. These conclusions are based on a simple econometric model but they survived a study of several potential causes of biased statistical inference, including measurement errors, improperly truncated lag structures and heteroskedasticity. In addition to size, other firm characteristics such as industrial branch, location and age were considered. Age was significant as an explanatory variable, and growth appeared to be especially rapid during the first three years of the firm's life-cycle. However, in quantitative terms, knowledge of these characteristics does not appear to much improve the accuracy of predicting the growth of individual firms. Nor are economy-wide changes in aggregate demand or the price level very important. Common trends or aggregate time series variables capture at most ten per cent of the total variance of firm growth.

The typical growth pattern changed markedly during the sample period. Initially, growth was persistent in the sense that exceptionally rapidly growing firms were the same from year to year. In the latter part of the period growth was no longer positively autocorrelated and even showed signs of enhanced short-period fluctuations.

Overall, firm growth appears to be quite random from year to year, as assumed in the simple random growth model. A special version of the model predicting that growth is smooth can be rejected, however.

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1 INTRODUCTION

In the period following the oil crises economic growth has faltered in most European countries. As a result, employment in Europe is at the same level as it was in the early 1970's, and about 10 per cent of the labour force is now unemployed compared to only 2 per cent in 1970. It is quite widely believed that the growth record reflects structural problems rather than demand weakness. The need for structural adjustment has arisen mainly from mismatches between production capacity and demand but a reinforcing factor may have been developments in financial markets, at least in countries where major steps have been taken towards the deregulation of domestic interest rates and foreign credit flows.

The bulk of aggregate output and employment growth derives from the growth of existing firms. In mature economies, the share of old and large firms in total supply tends to be larger than in newly industrialized economies. A sclerosis - a diminution of either growth capacity or growth flexibility - at the firm level implies then a relative stagnation, especially in conditions where drastic adjustment is required and circumstances are not favourable for successful entry of new firms. Are firms in Europe capable of adjusting to the new economic environment and of restoring the growth of economic activity and employment, or are they already too old to regenerate themselves? Do Finnish firms necessarily face the same fate as their competitors in those countries where both industrialization and financial deregulation started earlier than here, or can they maintain their favourable growth performance in the future?

This paper is the first step in a disaggregated investigation of growth and fluctuations in the Finnish business sector during the recent past, the ultimate goal being to assess the effect of recent changes in financial markets on firms' production and investment decisions. Panel data for a sample of Finnish firms is analyzed over the period 1978 - 1985 and an attempt is made to answer some versions of questions such as "Do firm characteristics affect grow?" and in particular, "Have patterns of firm growth changed during the sample period?"

In the literature, several studies have investigated the relationship between firm characteristics and firm growth. The particular characteristic which has stimulated most research is firm size. Early studies found no relation between the size of firms and their growth rates (see Hart and Prais, 1952 or Simon and Bonini, 1958). Later on, models of firm growth have shown that firms should grow in proportion to their size, provided that some simplifying assumptions such as constant returns to scale hold (see, in particular, Lucas, 1967 and 1978). Thus there is both theoretical and empirical evidence showing that firm growth is independent of firm size. However, recent studies tend to find that small firms grow faster than large firms. It is well known that growth decreases with size for smaller firms, but there has been a tendency to dismiss this finding because smaller firms also display more variable growth and are thus less likely to survive: the inverse relationship may be an artifact of sample censoring (see Mansfield, 1962). Nevertheless, Evans, 1987 and Hall, 1987 showed that firm growth decreases with firm size even after controlling for sample censoring. Evans also presented evidence showing that in all size groups, firm growth decreases with firm age. (For a theoretical explanation consistent with these findings, see Jovanovic, 1982.)

In this paper we focus on growth dynamics, trying to shed some light on the failure of size-growth independence as well as on the flexibility of firm growth and size structure. A natural consequence of the adopted approach is that in addition to the average growth performance, attention is devoted to the variability of growth, not only in the cross-section direction but also over time. Recent advances in the analysis of panel data (see Chamberlain, 1984) offer an avenue, little utilized thus far, to investigate parameter changes with a relatively short sample periods. With the limited number of variables analyzed in this paper, the challenge is to ascertain whether the growth patterns of the representative or average firm have changed recently in Finland and to indicate the direction and some of the possible causes of the change, if changes are detected.

2 GIBRAT'S LAW

Gibrat's law, or the independence of size and growth, has usually been tested by regressing the size variable against its lagged value using ordinary least squares or by otherwise using size information for a sample of firms at two moments of time. If S denotes the logarithm of firm size Y, the basic model of firm growth can be written as

(1) $S - S_{-1} = a + b\, S_{-1} + e.$

where S_{-1} is the lagged logarithmic size variable, a and b are constant coefficients and e is an error term. The independence of size and growth is usually deduced from the cross-section estimate of b, significant departures from 0 being considered as evidence against the law.

We consider two ways of justifying Gibrat's law. First, it may be assumed that size increases smoothly over time in a random fashion. The simplest version of this model is specified in continuous time by the random differential

$$dY = Y\,(a\,dt + \sigma\,dw)$$

where a and σ are constants. Then the stochastic process $S=\ln(Y)$ is a simple Brownian motion with independent, identically distributed normal increments. In this <u>continuous random growth model</u>, the average growth

$$S - S_{-1}$$

has a normal distribution with expected value and variance equal to $g*T$ and σ^2*T, respectively, for any length of the period T. Moreover, the conditional distribution of $S-S_{-1}$ given S_{-1} is normal with mean $S(1+g*T)$ and variance σ^2*T.

If the assumptions of the model hold, the error term in (1) is an i.i.d. normal random variable with zero mean and variance $\sigma^2 * T$, for any length of the unit period T. In a cross section of firms, if logarithmic sizes are explained by their lagged values, the estimate of b should be approximately equal to 0, provided that the sample size is large enough to allow for reasonable statistical inferences.

The continuous growth model takes Gibrat's law as a simple model for growth dynamics, stating that firm growth cannot be predicted, at least on the basis of knowing nothing else but the firm size. An alternative interpretation asserts that the determinants of firm size are time-invariant.

The alternative model starts from the assumption that, except for a common trend factor, logarithmic firm size is determined additively by a vector of constant size characteristics. Introducing the indices i and t for the firm and for the moment of time, this <u>permanent size model</u>, considered by Leonard (1986) for plant size, can be written as

$$(2) \quad S_{i,t} = A_t + X_i B + u_{i,t},$$

where A_t is a scalar and B a vector constant. $u_{i,t}$ denotes an i.i.d. normal error term with zero mean. Except for a common trend term A_t, the permanent size model embodies a time invariant distribution and a transient random error for the firm size. Following the process in equation (2), the logarithm of firm size is normally distributed with mean $A_t + X_i B$ and variance σ^2. The variance of the error term, σ^2, is now interpreted either as a measure of our ignorance of the determinants of firm size or as an error of measuring the "true" or permanent firm size. In part, it may be due to random shocks in product demand or to tipping in product market share in response to unobserved technological innovations. If some factors of production are specific to individual firms, or if economies or diseconomies of scale are not negligible, the optimal scale is not fully determined by the observable X's.

The variance of S is equal to the sum of the variance of the transient size component $u_{i,t}$ and the variance of the permanent size component X_iB, denoted by

$$s^2 = E\{(X_iB - EX_iB)^2\}.$$

The constant A_t captures the effects of common trends, such as a secular increase in firm size or an increase in the price level if size is measured in monetary units, on the size of firms.

If equation (2) is lagged and deducted from itself, we obtain

$$(3) \qquad S_{i,t} - S_{i,t-1} = g_t + e_{i,t}$$

where $g_t = A_t - A_{t-1}$ is a constant.

Differencing is widely used in panel data models to "wash out" firm-specific fixed effects. In (2) there are no time-varying explanatory variables and thus all explanatory variables drop out in differencing. The consequence is that (3) tells us nothing about factors affecting permanent firm size. However, it is not entirely useless. With the knowledge of the $S_{i,t}$'s only, it can be used to evaluate the common deterministic growth trend component g_t. Moreover, (3) holds irrespective of whether the Xi's are observable or not. Even if some of the components of Xi are observable, (3) provides a test of the constancy of their effects on firm size. Finally, the permanent size model provides a specification for the error term which is very different from the continuous random growth version of Gibrat's law.

The permanent size model closes the specification of Gibrat's law in (1) with the following set of assumptions:

Given Xi, $S_{i,t}$ and $e_{i,t}$ have a joint normal distribution with

$$E\{ S_{i,t} \mid X_i \} = E\{ e_{i,t} \} = 0,$$

$E\{ S_{i,t} e_{i,t} \mid X_i \} = -\sigma^2$
$e_{i,t}$ is distributed independently of X,

and

$$E\{e_{i,t} e_{i,t'}\} = E\{(u_{i,t} - u_{i,t-1})(u_{i,t'} - u_{i,t'-1})\}$$

$$= 0 \text{ if } |t - t'| > 1$$
$$= -\sigma^2 \text{ if } |t - t'| = 1$$
$$= 2\sigma^2 \text{ if } |t - t'| = 0.$$

In (3), growth is independent of the permanent size $S_i = B\,Xi$. However, it is not independent of the actual size because $S_{i,t-1}$ and $e_{i,t}$ are correlated. Actually, the assumptions above imply that given $S_{i,t-T}$, the distribution of $S_{i,t} - S_{i,t-T}$ is normal with mean

$$g^*(T-1) + (\sigma^2 /(s^2 + \sigma^2)) (S_{i,t-1} - Si)$$

and variance σ^2. Moreover, the model predicts that $e_{i,t}$ and $e_{i,t-1}$ are negatively correlated.

Both the permanent size hypothesis and the continuous random growth hypothesis seem to be legitimate formalizations of Gibrat's law and, except for the specification of the error term, they are identical. However, the long-term growth implications are quite different. In the continuous random growth model shocks to size are permanent. Hence growth patterns are differ widely in different firms, but in an unpredictable manner so that firm characteristics - i.e. size - cannot be utilized to predict growth. The permanent size hypothesis maintains that firm characteristics are unimportant because ultimately all firms grow at the same rate, i.e., apart from the common trend, there is nothing to predict.

To see the difference in the long-run growth patterns between the two models, note that under the permanent size hypothesis the variance of average firm growth over a period of length T-1 is inversely proportional to the length of the period, i.e.

$$E\{ [S_{i,t} - S_{i,t-T}) - E\,(S_{i,t} - S_{i,t-T})]/(T-1)\}^2$$
$$= 2\ \sigma^2/(T-1)$$

and thus the average growth rate converges to a fixed limit when T increases. Under the continuous random growth model,

$$E\{ [S_{i,t} - S_{i,t-T}) - E\,(S_{i,t} - S_{i,t-T})]/(T-1)\}^2$$
$$= \sigma^2 * (T-1).$$

The long-run growth rates vary in a random manner over firms, the degree of uncertainty about growth performance increasing pari passu with the length of the time period considered.

3 A FIRST LOOK AT THE EMPIRICAL EVIDENCE

The data utilized in this paper were originally collected by Teollistamisrahasto Oy (Industrialization Fund of Finland Ltd.) as background material for its loan decisions. In addition to manufacturing firms, Teollistamisrahasto Oy also finances restaurants and hotels. Special attention is devoted to developing new firms. Loans are granted at fixed interest rates for 7-12 years. In 1983, the basic annual interest rate was 11 per cent, but for smaller loans (up to 400 000 markkas), the interest rate was 1/2 - 1 percentage point lower. Larger loans (in 1983, more than 1 million markkas in all) are granted partly in foreign currency, the share varying between 30 and 60 per cent. In 1985, the total number of firms with an outstanding loan from Teollistamisrahasto Oy was about 1 500.

The panel selected for this study consisted of all firms with data on annaul sales from 1978 to 1985. The sample contains information on 526 small and large firms. Excluding hotels and restauraunts as well as conglomerates, the remaining 459 manufacturing firms in the sample cover approximately 20 per cent of total sales of manufacturing firms in Finland, excluding the conglomerates.

In processing its loan decisions, Teollistamisrahasto Oy requires each applicant to provide a compendium of information, including its profit and loss statement and balance sheet for the three preceding accounting years. The firm is also requested to submit its accounts to Teollistamisrahasto Oy until the debt is paid back. Thus, the firms included in the original files but excluded from the sample had either become clients of Teollistamisrahasto Oy after 1981 or ceased to be clients before 1985.

In this study, size is measured by the amount of sales, measured in thousands of markkas. Suppressing the subscripts, we denote the natural logarithms of the sales variables and its lagged value by S and S_{-1}. The data allow for the choice of the length of the unit period, in years, any integer between one and seven. If the shortest period is adopted, the maximum number of observations is obtained by pooling sales data over 1979-1985 for S and over 1978-1984 for S_{-1}. The number of observations is then 7*526 = 3682.

Part A of Table 1 contains some descriptive statistics for pooled variables. As noted several times before in the literature, starting with Hart and Prais, the logarithm of the firm size, however measured, is almost normally distributed, although there is some skewness to the right. The average logarithmic firm size 9.6 in the sample - i.e. the mean of $(S + S_{-1})/2$ - corresponds to a yearly sales figure of almost 15 million markkas in 1981-1982. The variance of the logarithmic size is approximately 4, which indicates that size differences are very large. The sample variance of the S_{-1} variable is somewhat larger than the variance of the S variable. This summary information suggest that size differences have decreased, on average, over the period of investigation.

Before proceeding to the discussion of the estimation results proper, it may be appropriate to note that both the continuous random growth model and the permanent size model are consistent with the log-normal size distribution in the sense that they always preserve it. The continuous random growth model also explains why the log-normal distribution arises in the first place. The simplest version of the argument assumes that all firms have started at the same time 0 from the same seed size $S_{i,0}$; then the size distribution is normal for any $t > 0$. Alternatively, it can be assumed that $S_{i,0}$ is a random variable with a given (non-normal) distribution and the same conclusion is arrived at by letting t grow without limit. (For more elaborated versions of the argument leading to the log-normal distribution or to other stable distributions, such as Pareto or Yule distribution, see Simon 1955 and Simon and Bonini, 1958).

The positive skewness of the size distribution implies that there are too many large firms relative to the normal distribution. The existence of these "superstars" is not easily explained by chance factors alone but is consistent with the evolutionary theories of firm growth, such as the one proposed by Jovanovic. Note, moreover, that the skewness has increased over time, as shown in Table 1. Although small firms have, on average, grown faster than large firms during the estimation period, the growth record of the largest firms cannot have been very poor either.

The ordinary least squares estimates of model (1) using pooled data with one year taken as the unit period are presented in the first column of Table 2. The estimate .977 for b differs significantly from 1. This also indicates that, during the estimation period, small firms have grown faster than large firms. The size effect is quite pronounced, given the differences in firm sizes. The estimate implies that doubling the size of the firm, decreases the expected yearly growth rate by 1.6 per cent.

Although both S and S_{-1} are almost normally distributed, both their difference and the residual of the simple regression equation are far from being normally distributed. They are skew to the left, more strongly than levels are to the right, and they also deviate from the normal distribution by being much more peaked. That is, the residual contains a disproportionate amount of relative failures and, in addition, "outliers" in both directions.

The ordinary least squares estimates of the model are unbiased if the continuous random growth model is adopted, but not under the permanent size model. In fact, the latter model provides a standard measurement error model explanation for the statistical failure of Gibrat's law. According to this explanation, the estimate of the coefficient of the lagged sales variable is biased because the explanatory variable is correlated with the error term. Instead of the permanent size, we observe only its erroneous measure, the actual size. Firms which have transitorily low size in the initial period seem to grow faster than those with transitorily high size, although all firms have the same permanent growth rate.

In model (3) the expected value of the bias is equal to the ratio of the measurement error component to the total variance of the sales variable. Assuming that the error of the short regression is entirely due to measurement error as specified in Section 2, we obtain an upper-bound estimate for the variance of the error term by dividing the variance of the error term by 2. Hence, from Table 1 we obtain an estimate 0.03 for the variance of the measurement error. As the variance of the logarithmic sales variable itself is over 4, the bias due to measurement error cannot be larger than 0.03/4 = .75 per cent. Thus, in this version, estimation bias can explain at most one third of the measured departure from Gibrat's law.

To consider the actual importance of transient size changes, an instrumental variables estimation of the model was conducted, with $S_{i,85}$ and $S_{i,84}$ as instruments for $S_{i,78}$ and $S_{i,79}$, and $S_{i,t-2}$ for the other $S_{i,t}$ variables. In the simple permanent size model these are valid instruments, i.e. they are correlated with the explanatory variable but not with the error term. The use of instruments increased the value of the estimate, but only slightly (see column II in Table 2). We conclude that the actual bias due to short-lived measurement errors is little and that independent measurement errors of about one year's duration cannot be the explanation for the observed failure of the law. In the present context, the error in measuring the permanent size introduces very little bias because it is swamped by the large variance in (permanent) size across the firms.

4 ARE LARGE FIRMS MORE RISKY THAN SMALL FIRMS?

Continuing with the analysis, we consider two versions of a rather subtle statistical explanation for the failure of Gibrat's law. According to this explanation, the preciseness of the estimates is illusory, because the standard errors of the regression coefficient are biased towards zero. In this chapter, we focus on heteroskedasticity as a source of biased standard errors. There is some reason to expect that <u>the variability</u> of growth depends on size, and thus, the rejection of the law may be a reflection of nothing more fundamental than random sampling error.

Hymer and Pashigian (1962) have argued that the variance of the growth rate should <u>decrease</u> as the firm size increases: "Let us assume that there is a certain critical minimum size after which unit costs are constant. And let us make a second crucial assumption: we suppose a large firm to be merely a collection of independent small firms of the critical minimum size. In other words, we assume that a large firm is essentially a holding company operating independent divisions. (...) Because large firms are able to diversify, their growth rates will have less variability (smaller standard deviation) than do the growth rate of small firms. Not only can we conclude this, but by an elementary theorem in statistics we can predict exactly the decline in the standard deviation that will result. The large firm is now a large sample of small firms. The standard deviation of the mean of large samples is $1/\sqrt{n}$ times the standard deviation of the population where n is the size of the sample."

The continuous time growth model considered in Section 2 can be written in a more general form as follows:

$$dY = f(t,Y)\ dt + \sigma(t,Y)\ dw$$

where $f(t,Y)$ and $\sigma(t,Y)$ are constant functions and dw denotes Brownian motion. The geometric Brownian motion assumption in Section 2 was

$$\sigma(t,Y) = v(t)\ Y$$

whereas Hymer and Pashigian assume that

$$\sigma(t,Y) = v(t)\ Y^{1/2}.$$

The model

$$(4) \quad \sigma(t,Y) = v(t)\ Y^{\alpha}.$$

cover these as special cases. The constant α measures the degree of growth diversification. It is expected to be between 0 and 1/2, as the cases considered above are the somewhat extreme cases of no diversification and perfect diversification, respectively.

A consistent estimate for 2α is obtained by regressing the logarithms of the squared residuals against a constant term and S (see Harvey, 1976) . The estimates from this regression are presented in Table 3. The first column residual is taken from the pooled yearly regression reported in the first column of Table 2, and similarly for the other columns. The column I estimate of α is about 12-13 per cent. A minimum distance estimation procedure for the exponential function with the same data yielded a similar estimate. Although these estimate differ significantly from zero, indicating that firms diversify their growth, it is nearer to the zero value of no diversification than to the value of 1/2 for perfect diversification.

Why do large firms fail to diversify? Or how are they able to survive without diversifying? The explanation advanced by Hymer and Pashigian is that either there are returns to scale, so that the large firm cannot diversify but obtains a lower cost level than a sample of small firms, or that large firms are able to compensate for their variability by offering larger profits, obtained by exploiting a monopoly position or other imperfections on the demand side. A special version of the productivity argument is provided by the model of Lucas (1978), where the size distribution of firms is determined by the population's entrepreneurial ability distribution. The empirical results of this chapter imply that in terms of the Lucas model, variations in the amount or efficiency of managerial input are the main source of variations in the growth performance.

The argument advanced by Hymer and Pashigian apparently presupposes a well-functioning capital market in the sense that the pooling of small firm risks is considered as a relevant alternative. If capital markets operate imperfectly, large relatively unprofitable and risky firms may also survive, if they have access to the stock market. They may even be priced at a premium because of the liquidity service or insurance they provide, if only partially, in pooling risks.

Aron, 1988, develops a model of diversification that is based on the agency problem between the firm's managers and owners. The model predicts that optimal firm size, degree of diversification and size of the production units are all positively correlated. If we assume that the risks of the production units are independent, Aron's results imply in terms of model (4) that $0 < \alpha < 1/2$. Thus the empirical results considered in this section are consistent with his analysis.

Heteroskedasticity calls for a weighting of the variables, in order to obtain unbiased standard errors and, perhaps, more efficient parameter estimates. The weighting scheme suggested by the results of the above regressions discounts smaller firms. The use of estimated variance in correcting for heteroskedasticity led, practically speaking, to the same set of estimates than earlier. Moreover, instead of adopting a specific model for the variance term, we computed White's covariance matrix estimates. These do not depend on a formal model of the structure of the heteroskedasticity. This exercise suggested that the bias in the usual standard error of the sales coefficient is of the order of 5 per cent in the pooled regression. Taken together, the evidence presented in this section suggests that biased standard errors are not a major explanation for the empirical failure of Gibrat's law.

5 TIME EFFECTS

An alternative explanation for biased standard errors is related to the correlations of growth differences. If different observations are not independent, pooling exaggerates the degrees of freedom actually available. Growth correlations may arise from different sources. One possibility is that the residuals are highly correlated within the years. If all variation in the data comes from differences between the years, the effective number of degrees of freedom is equal to the number of years only. In this case, aggregate time series contain all the sample information for a variable.

Inappropriate pooling of variables over years also provides one explanation for the failure of Gibrat's law. Pooling presupposes that all years are similar, and thus excludes both endogenous and exogenous changes in firm behavior. One implication is that fluctuations in aggregate sales are assumed to be due to entry and exit of firms, which is very restrictive. In more realistic terms, it is likely that, in a growing economy, the average firm size is, at least in the sample, relatively small at the outset and large at the end of the investigation period, especially in nominal terms. A general deceleration of nominal aggregate growth - in either prices or quantities - during the estimation period may be captured by the size variable, leading to the erroneous conclusion that small firms grow more rapidly than large firms.

The importance of time effects can be investigated by introducing constant year dummies into equation (1). The results of this regression are presented in column IV of Table 2. The coefficients of the dummy variables reflect, at least to some extent, changes in aggregate nominal growth.

The constant terms are higher during the initial phase of estimation, notably in 1979, reflecting rapid inflation and an upturn in aggregate economic activity. The moderate aggregate fluctuations after 1981 do not show up at all in the coefficient estimates.

The estimate of b increases by about .5 per cent if year dummies are included in the regression. Further introduction of year-specific b-constants does not improve the fit of the model very much, although the contrasts between years are significant, but yields otherwise interesting results. During the last half of the estimation period, the estimate of b is almost exactly equal to 1. The departure from Gibrat's law occurs at the outset. In 1979, the estimate of b deviates from 1 by more than 4 per cent. This evidence suggests that pooled variables consist of heterogeneous components.

There are two explanations for time-variant coefficients: either aggregate conditions have changed during the estimation period in such a way that the growth of small firms has been hampered or firms in the sample have changed so that their growth patterns have changed. According to the first interpretation, some significant changes have occured in the Finnish economy during the estimation period, detrimental to the growth of small enterprises; according to the second interpretation, the results are due to the particular pattern by which the sample is selected, and imply nothing about the growth of firms outside the sample.

Although year dummies are significant in the statistical sense, they are not overwhelmingly important in explaining variations in firm growth. The introduction of year dummies into the equation reduces the sum of residual squares by about 8 per cent. The implication is that aggregate effects cannot be very important in explaining the growth of individual firms, even if we assume that all time effects are entirely due to aggregate effects only. The limited amount of within-period correlation in the residuals also implies that the standard error estimate of the b-coefficient remains almost unaffected by the introduction of the year dummies.

6 GROWTH DYNAMICS: ADJUSTMENT OR FLUCTUATIONS?

This section deals with the persistence of growth differences and other aspects of growth dynamics. For reasons to be discussed shortly, persistent growth differences may explain why Gibrat's law fails. They also reduce the effective number of observations. In the extreme case where individual error terms in model (1) are perfectly correlated over time, the rank of their covariance-variance matrix is equal to the number of firms, instead of the number of observations.

In the context of panel data, the simple model

$$S_{i,t} = S_{i,t-1} + g_i + u_{i,t}$$

provides a useful point of departure for the treatment of growth dynamics. In this model, the individual trend coefficient g_i is assumed to be constant over time but to vary over firms.

The versions of Gibrat's law considered above allow for the existence of individual trends only if $S_{i,t-1}$ and g_i are independent random variables, so that g_i can be dissipated into the error term. If $S_{i,t-1}$ and g_i are correlated, the coefficient of $S_{i,t-1}$ receives biased estimates. For example, young firms tend to be small. If they grow faster than old firms and the sample contains firms of different ages, the omitted age variable results in an estimate of b which is less than one. Even if the age - growth curve itself is independent of (initial) size, the extent of the bias depends on the proportion of young firms in the sample.

Individual trends show up in positively correlated error terms in consecutive yearly regressions, and they can be eliminated by further differencing. This leads to the model

$$S_{i,t} - S_{i,t-1} = S_{i,t-1} - S_{i,t-2} + u_{i,t} - u_{i,t-1}$$

or

$$S_{i,t} = 2\, S_{i,t-1} - S_{i,t-2} + u_{i,t} - u_{i,t-1}.$$

In this model Gibrat's law holds in the sense that the sum of the b-coefficients is equal to one. Note that the elimination of firm-specific trends introduces a moving average component into the error term, even though the original model is of the random growth variety.

A somewhat similar case but with less persistent trends arises from the model

$$S_{i,t} = X_i B + \delta^*(S_{i,t-1} - X_i B) + e_{i,t}$$

which augments the permanent size model with a gradual adjustment towards the permanent size. The adjustment coefficient δ is expected to be between zero and one: the two models considered in Section 2 are included as special cases $B=0$, $\delta=1$ and $B \neq 0$, $\delta=0$. "In the long run" the logarithm of firm size is distributed normally with mean $X_i B/(1-\delta)$ and variance $\sigma^2/(1-\delta^2)$. Here σ^2 denotes the variance of conditional size distribution, given the size one period earlier.

Taking differences in the partial adjustment model leads to

$$S_{i,t} = (1 + \delta)S_{i,t-1} - \delta\, S_{i,t-2} + e_{i,t} - e_{i,t-1}.$$

In this model, too, growth is ultimately independent of size. However, at any given time, firms below their optimum are overrepresented among the group of smaller firms, and firms above optimum are overrepresented among the group of larger firms, just as in the permanent size model in Section 2. Growth differences are more persistent than in the static model, however, and hence the instruments used in Section 2 are not valid for this model.

The third example combines the two models in Section 2:

$$X_{i,t} = a_t + b_t X_{i,t-1} + u_{i,t}$$
$$S_{i,t} = X_{i,t} + w_{i,t}$$

where a_t and b_t are constants for a given t and $u_{i,t}$ and $w_{i,t}$ are independent random variables with possibly time-varying variances. In this model $w_{i,t}$ is the error in measuring actual size $S_{i,t}$ instead of the "proper" size $X_{i,t}$. In contrast to the model in Section 2, however, the "proper" size here is not constant or permanent but grows in a random fashion. The combined model is a special case of the standard ARMA(1,1) model with time-varying coefficients

$$(1 - b_t L) S_{i,t} = (1 - \beta_t L) \quad e_{i,t},$$

where L is the lag operator in t, $e_{i,t}$ is white noise and b_t and β_t are coefficients for each t.

As an alternative derivation of the ARMA-model, consider instead of (2) the model

$$(5) \qquad S_{i,t} = \phi_t A_i + A_t + u_{i,t},$$

which allows a limited amount of interaction between firm-specific and growth factors. Specifically, it is assumed that, except for a time-dependent scale factor, size factors affect future size in the same way as they have affected past size, and that the scale factor is the same for every factor.

Assuming that ϕ_t is not equal to 0, we can solve A_i from (5) for any t and hence

$$S_{i,t} = (\phi_t / \phi_s) S_{i,s} + A_t - (\phi_t / \phi_s) A_s$$
$$+ u_{i,t} - (\phi_t / \phi_s) u_{i,s}.$$

In the same way the autoregressive model

$$s_{i,t} = \alpha_t + \beta_{t,1}s_{i,t-1} + \beta_{t,2}s_{i,t-2} + \ldots + \beta_{t,M}s_{i,t-M} + \phi_t\alpha_i + u_{i,t}$$

reduces after quasi-differencing to

$$s_{i,t} = a_t + b_{t,1}s_{i,t-1} + b_{t,2}s_{i,t-2} + \ldots + b_{t,M+1}s_{i,t-M-1} + e_{i,t}$$

with

$$a_t = \alpha_t - f_t\,\alpha_{t-1}$$
$$b_{t,1} = \beta_{1,t} + f_t$$
$$b_{t,j} = \beta_{t,j} - f_t\,\beta_{t-1,j-1} \text{ for } 1 < j < M+1$$

and

$$b_{t,M+1} = - f_t\,\beta_{t-1,M},$$

where for shortness,

$$f_t = (1 + \phi_t)/(1 + \phi_{t-1}).$$

The error term is given by

$$e_{i,t} = u_{i,t} - f_t u_{i,t-1}.$$

The simple models analyzed in this section suggest that, instead of (1) with, perhaps, a moving average presentation for the error term, one should estimate a model in which the autoregressive part of the model contains two lags. Even longer lags cannot be excluded on a priori grounds. Because the successive observations for each firm are highly multicollinear, it is unlikely that all autoregression coefficients can be estimated precisely in a regression where levels are used. However, these regressions should be helpful in identifying the proper lag length and in deciding whether the failure of Gibrat's law is due to improperly specified growth dynamics.

In estimating autoregressive models of type (6), we utilize the framework proposed by Chamberlain for dynamic panel data models. This allows for non-stationary individual effects, and a fairly general error term specification, including arbitrary error structures between periods as well as heteroskedastic errors within periods. Essentially, the method is to apply a multivariate GLS instrumental variable estimator to the set of yearly equations, using an estimate of the covariance matrix which is obtained from the residuals of a consistent but inefficient preliminary estimator. The resulting GLS estimator is efficient in the class of linear instrumental variable estimators (see Holz-Eakin et al, 1985, for more details).

The steps in the estimation procedure were as follows:

(1) An autoregressive model was estimated for each period by two stage least squares using lagged sales values as instruments. In period t, observations up to period t-2 were accepted as instruments, i.e. the moving average part of the process was restricted to MA(1). The maximum number of autoregressive components allowed by the data, given the identifying restrictions, was included in each initial regression. Thus, for example in 1980 an AR(1) process was estimated, thereafter the maximum lag length was increased by one each year, and finally, in 1985, an AR(6) process was estimated.

(2) The residuals from the preliminary estimates were used to estimate the covariance-variance matrix M of the disturbances, allowing for heteroskedasticity and between-periods correlations. The autoregression parameters were thereafter estimated using the GLS estimator with weighting matrix M.

(3) Finally, the lag length was squeezed stepwise, using the residuals from the GLS estimation to test restrictions on the lag lengths. Let N denote the number of cross-section observations, Q the unrestricted residual sum of squares and QR the restricted sum of squares, obtained by shortening the maximum lag length by one year. It has been proved by Holz-Eakin et al. that the $L = Q/N - QR/N$ has a chi-squared

distribution with degrees of freedom equal to the number of restrictions on the lag structure.

The results from this exercise are presented in Table 4, beginning from the lag length of 3 years. The data easily accept a reduction from AR(3) to AR(2). The value of the test statistic L = 3.60 = 12.81 - 9.21 exceeds the critical value of the chi-squared distribution with three degrees of freedom only at about the 30 per cent significance level. A further shortening of the lag length by one period is significant only at the 20 per cent level. In view of these results, the simple model (1), with a possible MA(1) presentation for the error term appears to be acceptable for the years 1981 - 1985. Moreover, the efficient parameter estimates of the simple model (1) are very close to the OLS or instrumental variables estimates, as, too, is the sum of the coefficients of the lagged variables in more complicated models.

It may be concluded that Gibrat's law does not fail because of inappropriately truncated lag structure. However, the value of this conclusion is somewhat lessened by the fact that during the main part of the present estimation period - namely 1983 - 1985 - the OLS estimates are consistent with Gibrat's law. Individual trends and partial adjustment may very well have been important during the initial phase falling outside the estimation period if we use Chamberlain's methods.

It is not difficult to present evidence pointing to a change in the time series properties of the representative or average firm's growth patterns. Table 5 presents correlation matrices for logarithmic levels and differences of the sales variable. Differences (as well as the yearly residuals from regression analyses) are initially strongly positively autocorrelated, but after 1981 the first order autocorrelation becomes mildly negative.

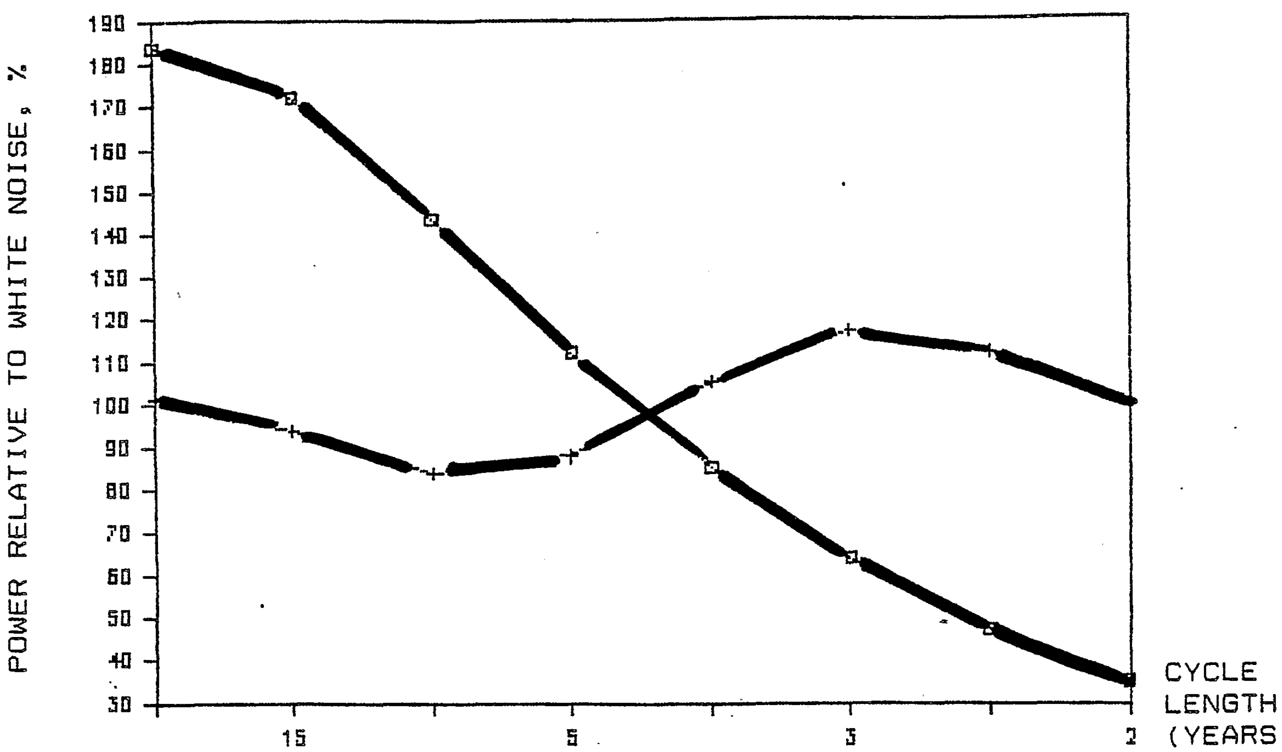
Figure 1 gives a frequency domain summary of the changes in the firm growth dynamics during our data period, depicting the power spectrum for the representative firm's sales growth for the initial and end phases of the period.

The analysis in frequency domain focuses on the contributions of various periodic components to the total variation of a time series. Any stationary time series can be thought of as a sum of an infinite number of uncorrelated periodic components, each associated with different periodicities or frequencies. The spectrum of a completely random series, white noise, is characterized by a horizontal line. A time series generated by random noise contains no cyclical features: all periodicities contribute to the power at the same force. On the other hand, a trend is monotonic and therefore nonrepeating. It is characterized by a near - infinite period and a spectrum with main mass near the origin. Low-frequency components of a time-series can be associated with long-run time intervals, and high frequency components with short-term fluctuations.

In the present case, we are interested in the (unobserved) growth of the representative firm. This is thought of as a stationary time series, and each particular growth path as its different realization. Systematic time effects on growth are taken into account by the year dummies; otherwise, it is assumed that individual growth experiences at the firm level are independent of the others.

In Figure 1, the horizontal axis measures the length of growth cycles in years. The two spectra are computed from the autocorrelations reported in Table 4. The spectrum for the initial ("by 1980") and end ("by 1985") period are obtained from the information presented in columns 1979 and 1985, respectively, of the Table, using a weighted covariance estimator with a rectangular lag window of three years. Both spectra are normalized to have the same total power.

FIGURE 1. CHANGE IN THE GROWTH PATTERN DURING THE ESTIMATION PERIOD.



□ SPECTRUM BY 1980 + SPECTRUM BY 1985

At the end of the investigation period, the growth of the representative firm, as presented in Figure 1, consists almost entirely of pure random fluctuations. It is as if there were no systematic or predictable forces behind the growth performance, although there is some short term power in the series. In contrast, during the initial phase power was concentrated on lower frequencies, very much as in a typical aggregate output growth series.

Summarizing the evidence presented in this section, it may be concluded that some firms, including a disproportionate number of small ones, grew persistently faster than other firms during the first few sample years. Thereafter there did not exist any systematic differences between firms in growth performance. There is some evidence indicating that the extremely simple lag structure specified in the two basic models in Section 2 may - but need not - be inappropriate initially. However, after 1981 the lag structure in the basic models is fully sufficient to capture the growth dynamics of the representative firm.

If we disregard average growth differences between the firms, which one of the two versions of Gibrat's law considered in Section 2 is more consistent with the empirical evidence? This problem can be resolved quite conclusively by varying the length of the unit period in the analyses. If the longest period available, seven years, is adopted for the unit period, the period of investigation is confined to 1985 only, with 526 observations, and lagged sales values are derived from 1978.

The third column of Table 2 shows the results obtained from an ordinary least squares estimation of model (1) using the long difference. This regression does not assume that different years are identical, and even allows for some persistence over time in the error terms. The estimate of b is .883, with a standard deviation .013. It differs significantly from 1 and is in fact quite near the pooled year-rate estimate, provided that the latter is appropriately compounded. However, the estimate from the pooled data is somewhat lower, corresponding to a compounded value of .850. Even if we compound the 5 per cent upper bound for the yearly estimate, the value remains slightly lower than the estimate from the long difference.

Table 1 B contains descriptive statistics for the variables in the long regression. As far as the levels S and S_{-1} are concerned, the information in part B of the Table is quite similar to that one obtained using pooled year differences in part A. The most important item in the Table is the variance of the error term. In the long regression this is exactly seven times the variance of the error term in the yearly difference regression. The evidence is consistent with the random growth model but not with the permanent size model. As a description of firm growth, the random growth model is clearly superior to the permanent size model.

7 FIRM CHARACTERISTICS, SIZE AND GROWTH

The permanent size model (2) prompts one to consider the basic determinants of firm size. Our data set contains three time-invariant pieces of information for the firm, namely the year of foundation, the SIC-industrial branch code and the location of operations. Of these, the first variable is of particular interest in the present context because it provides a glimpse at the long-run growth record.

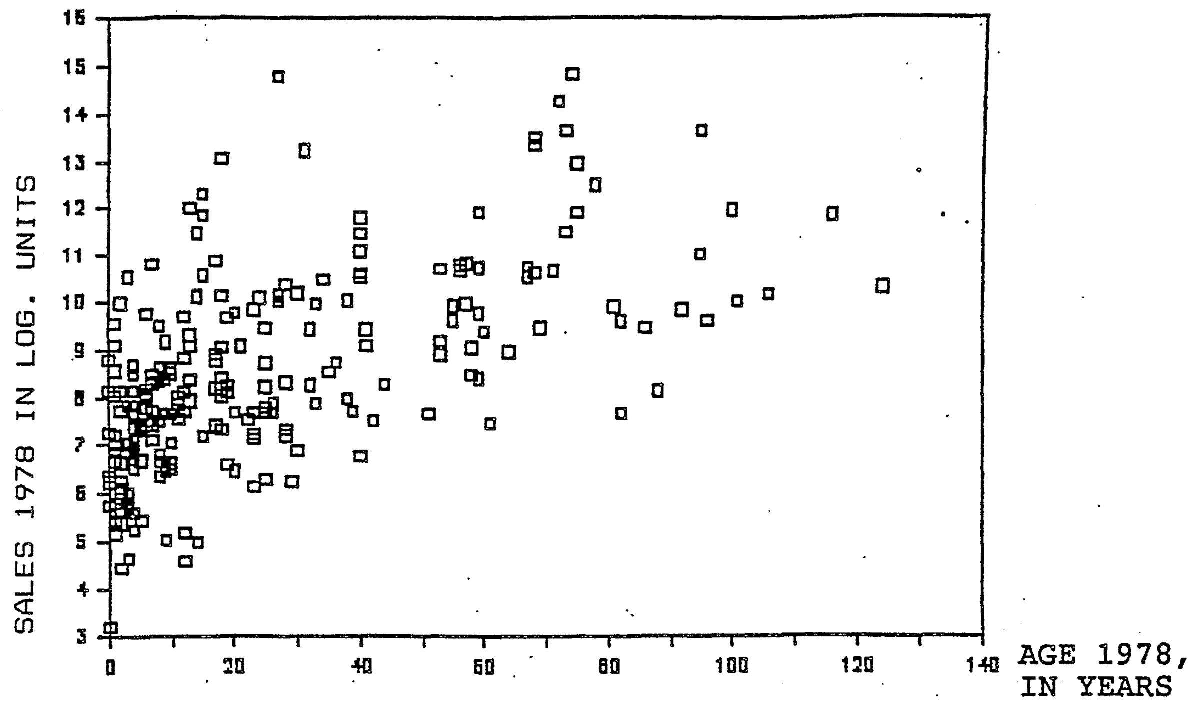
In the sample, there are many firms which are more than hundred years old. The oldest reported age exceeds 300 years, but it is clearly an exception in the age distribution. The average age was 27.5 years in 1978 but there is some evidence, to be presented shortly, that firms with missing age information are older on average than others, and hence the proper mean age of all firms in the sample is likely to be somewhat higher than the above estimate.

Parts A and B of Figure 2 provide scatter diagrams of the relationship between age and size in 1978 and in 1985. Old firms tend to be larger than young firms, and the growth of firms also appears to continue in old age. However, the bulk of largest sizes is found in the age range 70-80 years.

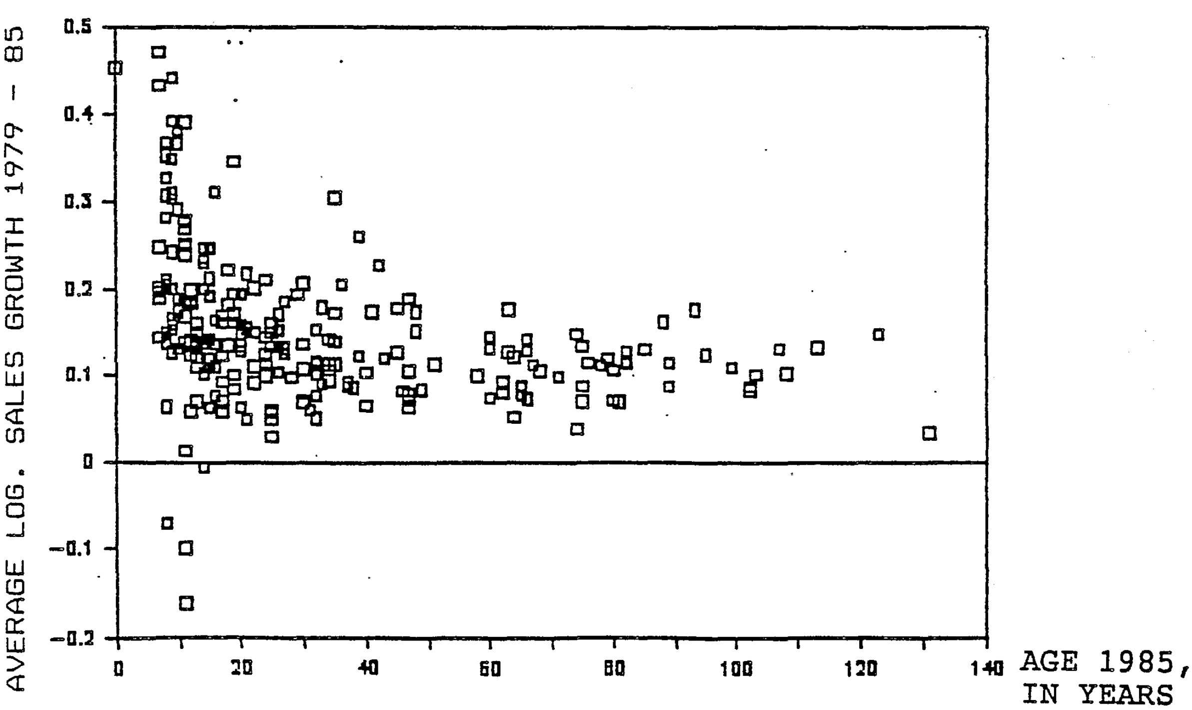
Parts A and B of Table 6 contain results from regressions where the logarithmic size in 1978 and 1985, respectively, is explained by the age at the

FIGURE 2.

A. FIRM AGE AND SIZE



B. FIRM AGE AND SALES GROWTH



time of regression, the AGEDUMMY variable for observations with missing age information and the DUMMY31...DUMMY9 variables for different SIC-industries. In the age variable, missing observations were replaced by the average of observed firm ages.

The coefficient of the age variable gives an estimate of the past average growth rate during the firm's lifetime. Growth here means real growth in the sense that firm sizes are measured in terms of the same year's markkas. The estimates of the coefficient of the age variable imply that, other things being equal, the representative firm had, during its lifetime, grown in real terms at the rate of almost 3 per cent in 1978 and at the rate of 2 per cent in 1985.

In a cross section analysis like this it is hard to distinguish between age effects proper, for example that young firms grow faster than old firms, and aggregate effects, for example that firms grow faster when aggregate demand is brisk: i.e. it makes no difference whether firms in the sample grow in their early days because demand happens then to be brisk or because young firms always grow rapidly - both cases enhance the importance of the age variable in the same manner.

One explanation for the decline in the value of the coefficient of the age variable is based on changes in the economic environment of the firms. Aggregate growth was slower during the estimation period than during the earlier lifetime of most firms in the sample. However, in order to reduce the lifetime average growth rate by one percentage point, it would be necessary for aggregate growth to be much slower during the seven year period than earlier. If the sample period covers about 20 per cent of the total lifetime of the average firm in 1985, then the aggregate impulse on firm growth ought to have ceased altogether during the sample period in order to induce the required reduction in the average lifetime growth estimate. This is an unrealistic assumption. An alternative explanation for the reduction in the lifetime growth estimate, referring to a non-linear relation between age and size as well as to sample selection, will be presented shortly.

The coefficient of the AGEDUMMY variable was .43 in the 1978 regression and .23 in the 1985 regression. The estimates suggest that the firms which did not report age information were on average about 10-20 years older than others.

Viewed by industry, size differences are quite marked. According to the results of Part A of Table 5, other things being equal, the sales of an average unclassified firm (SIC 9 also includes conglomerates) is 20-30 times larger than the sales of an average furniture firm (SIC 332). The average firm size is also high in food and kindred products (SIC 31), the textile, wearing apparel and leather industry (SIC 32), the manufacture of paper and pulp products (SIC 341) and the basic metal industries (SIC 37), whereas in the sample small average sizes are found not only in the manufacture of furniture and fixtures but also in the manufacture of wood and wood products (SIC331), printing and publishing (SIC 342), the manufacture of chemicals (SIC 35), non-metallic mineral products (SIC 36), the manufacture of fabricated metal products, (SIC 38) other manufacturing industries (SIC 39) and trade, restaurants and hotels (SIC 6).

An attempt was also made to assessthe relation between size and location. Dummy variables for location in the Uudenmaa, Turku, Ahvenanmaa, Häme and Kymi areas, respectively, did not differ significantly from zero whereas a dummy for operations in many locations obtained a rather large positive value. As is to be expected, the last mentioned variable is positively correlated with firm age and thus its introduction into the model reduced the estimate of the age effect. As the age variable is also correlated with the multi-branch dummy variable DUMMY9, the exogeneity of the explanatory variables in our analysis can be questioned. If the time horizon is very long both the location and the branch, as well as perhaps the age, should be ideally treated, at least to some extent, as endogenously determined variables.

Much of the evidence presented in this section and earlier points to a dissimilarity between the early and latter parts of the investigation period. The sample selection argument claims that, in the sample, there is initially a disproportional number of small and rapidly growing firms, and that the growth impetus of these firms is not permanent, but wanes during the estimation period. If new entrants are excluded from the sample, the sample becomes censored during the passage of time. The sample also suffers from sample selection in the other direction, i.e. from the lack of failing firms. This problem is probably more important for the earlier estimation period, as the survival rate is known to increase sharply with age and size. Because quitting firms are censored from the sample, the growth prospects of young and small firms may appear overoptimistic in the sample.

Adding size in 1978 to the set of explanatory variables in the 1985 regression provides an analysis of <u>growth</u> between 1978 and 1985. Regression analysis reveals that the average growth performance did not differ much across different branches, in the sense that the differences between the branch dummies were insignificant. This part of the evidence is consistent with the permanent size model (2). On the other hand, the coefficients of the age and age dummy variables -.003 and -.16 were significant, at least when standard errors were computed in the usual way.

As age and size are positively correlated, the coefficient of the lagged size variable is reduced by the introduction of the age variable. Nevertheless, estimates indicate that size is a deterrent to growth, even after controlling for other firm characteristics. In order to capture possible non-linearities in the early part of the growth curve and to somehow take the sample selection argument into account, we supplemented the age variable with separate dummy variables for ages 1 - 10 years in 1978. The third column in Table 5 reports the results from this regression. It turns out that firms which had started operations during 1975-1978 grew especially rapidly during the estimation period whereas increasing age depressed growth. The coefficient of the age variable was -.002, indicating that each 70 years in age reduces yearly growth rate by 2 per cent.

The coefficient of the 1978 size variable is .910 whereas the corresponding estimate of the simple model in Section 3 was .882. The difference is not very large. Experiments with yearly estimates also yielded results which closely parallel those of the simple model.

How should the results from the growth-characteristics regression be interpreted? Formally, the model is identical to the partial adjustment model presented in Section 5. If the partial adjustment model is adopted, we can impute "long-run" elasticities from the estimates. As the estimated speed of adjustment is very low and the estimates biased, this interpretation is, perhaps, too ambitious. Alternatively, we can consider regressions in this section as one kind of sensitivity analysis assessing the stability of the simple results in Section 3. If this interpretation is adopted, the results suggest that it is not difficult to find characteristics such as age which are significant in explaining firm growth. However, the analysis also gives the impression that, although firm characteristics may be significant, they are not quantitatively important in explaining firm growth. This is a proposition which cannot be proved generally for all characteristics, but it holds for the set of characteristics considered here.

8 SUMMARY AND CONCLUSION

Gibrat's law, or the assertion that firm growth is independent of size, can be interpreted in different ways. In this paper, the analysis starts from two basic versions of the law. The random growth model takes Gibrat's law as a simple model for growth dynamics, contending that firm growth cannot be predicted, at least on the basis of knowing nothing else but the size of the firm. The alternative permanent size model assumes that each firm has a well-determined optimum size, whose determinants are time-invariant except for a common trend factor. Both models are consistent with Gibrat's law but they have different long-run growth predictions for a sample of firms.

In the empirical analysis, panel data for a sample of 526 Finnish firms were examined over the period 1978 - 1985. The analysis focused on the empirical validity of Gibrat's law and its different versions, using (nominal) sales as the size measure. An attempt is made to take latent variables into account, which gives some generality to the otherwise oversimplified models analyzed in this paper.

Some of the evidence can be summarized as follows:

(1) Size varies greatly across firms, so that as a first approximation the size distribution is well described by a log-normal distribution. Size differences are rather permanent. Older firms tend to be larger than younger firms and there are large differences in average size between industrial branches.

(2) In a cross-section analysis, where size is explained by its lagged value using data pooled over the whole period, the ordinary least square coefficient estimate of the lagged size

variable is significantly less than one. The estimate implies that, in our sample, small firms have, on average, grown faster than large firms.

(3) The permanent size model provides one explanation for the empirical failure of Gibrat's law, predicting that the ordinary least squares estimates of the lagged size are biased towards zero. However, it turns out that random year-to-year size variations play very little role in explaining the observed departure from Gibrat's law.

(4) The length of the unit period does not affect the above conclusions very much: broadly speaking, "long" estimates replicate the results of combined "short" estimates. However, the effect of size becomes slightly less pronounced if the length of the unit period is increased.

(5) Common time effects such as those induced by changes in the price level or aggregate demand appear to be statistically significant in explaining firm growth. From the point of view of an individual firm, however, they are not very important. The year dummies, reflecting at least to some extent changes in aggregate nominal growth, capture at most ten per cent of the total variance of firm growth. The coefficients of year dummies are higher during the initial phase of estimation, notably in 1979, reflecting rapid inflation and an upturn in aggregate economic activity. The moderate aggregate fluctuations after 1981 do not show up at all in the coefficient estimates.

(6) The firm characteristics considered in addition to size were age, industrial branch and location. Firm age obtained a significant coefficient and growth, in particular, appeared to be exceptionally rapid during the first three years of the firm's life-cycle. However, in quantitative terms, knowledge of firm characteristics does not much improve the accuracy of predicting the growth of individual firms.

(7) There is some evidence indicating that firms grow more by expanding existing product lines than by diversifying. A

large firm shows more sales variability than an artificial conglomerate of small firms, when both are of the same size.

(8) The estimation and testing of dynamic models using panel data requires several years of data. Hence, the use of these methods dictates a curtailing of the estimation period. An investigation throughout the period 1981-85 revealed that Gibrat's law did not fail because of improperly truncated lag structure. However, the value of this conclusion is somewhat diminished by the fact that during the period the ordinary least squares estimates were also consistent with Gibrat's law. Individual trends and partial adjustment captured by longer lag structures may have been important during the initial phase falling outside the sample in this exercise.

Overall, the evidence gives the impression that firm growth is quite random from year to year, as assumed in the simple random growth model. However, in many instances it is in variance with the specific version of the model predicting that firm growth is continuous. Growth rates are not normally distributed, and moreover, the size distribution is skew to the right, suggesting that in the long run growth rates are positively autocorrelated. The last conclusion evidently implies that large firms grow ultimately more rapidly than small firms.

The typical growth pattern changed markedly during the sample period. Initially, growth was not only rapid on average but also persistent in the sense that same firms grew exceptionally rapidly in consecutive years. Towards the end of the period growth was slower and more random, even showing signs of enhanced short-period fluctuations.

The results in this paper are likely to reflect, at least to some extent, behaviour which is typical to the firms in the sample. However, further generalizations may be hampered by sample selection problems, such as changing age structure in the sample as well as the disproportionate amount of succesful firms in the sample, at least during the earlier sample period. Macroeconomic conclusions based on the sample used in this paper require further study, including a careful evaluation of the role of sample selection problems.

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TABLES:

TABLE 1. DESCRIPTIVE STATISTICS

A. POOLED YEARLY DATA

	S_{-1}	S	$S-S_{-1}$	RES
Mean	9.53	9.69	0.16	0.00
Variance	4.23	4.11	0.06	0.06
Minimum	3.18	4.73	-4.22	-4.36
Maximum	15.69	15.76	2.54	2.29
Skewness	0.48	0.51	-1.23	-1.52
Kurtosis	-0.00	-0.01	36.83	38.37
Median	9.28	9.41	0.14	-0.00

B. LONG DIFFERENCE DATA

	S_{-1}	S	$S-S_{-1}$	RES
Mean	8.93	10.02	1.09	0.00
Variance	4.61	4.00	0.48	0.41
Minimum	3.18	5.51	-3.72	-4.69
Maximum	14.85	15.76	3.76	2.30
Skewness	0.45	0.53	0.07	-0.52
Kurtosis	-0.04	-0.00	6.30	6.50
Median	8.67	9.76	1.02	0.02

Explanations to Table 1.

A. Variables S and S_{-1} are obtained by pooling logarithmic sales over the periods 1979 - 1985 and 1978 -1984, respectively, and $S-S_{-1}$ is their difference. RES is the ordinary least squares residual obtained from regressing S on S-1, reported in column I of Table 2.

B. Variables S and S_{-1} are logarithmic sales in 1985 and 1978, respectively, and $S-S_{-1}$ is their difference. RES is the ordinary least squares residual obtained from regressing S on S-1 (reported in column II of Table 2).

TABLE 2. ESTIMATES OF SOME SIMPLE GROWTH MODELS

	I Pooled	II Long	III Instr.	IV Pooled	V Instr.	VI Yearly
C	.372 (.019)	2.139 (.120)	.340 (.019)	.230 (.022)	.230 (.022)	.142 (.052)
D79				.185 (.015)	.179 (.015)	.548 (.068)
D80				.155 (.015)	.145 (.015)	.338 (.070)
D81				.069 (.015)	.066 (.015)	.144 (.071)
D82				.020 (.015)	.015 (.015)	.135 (.072)
D83				.015 (.015)	.014 (.015)	.029 (.073)
D84				.015 (.015)	.018 (.015)	-.013 (.073)
S_{-1}	.977 (.002)	.883 (.013)	.981 (.002)	.983 (.002)	.986 (.002)	.994 (.005)
S_{-1}*D79						-.039 (.007)
S_{-1}*D80						-.019 (.007)
S_{-1}*D81						-.007 (.007)
S_{-1}*D82						-.012 (.007)
S_{-1}*D83						-.001 (.007)
S_{-1}*D84						-.013 (.007)
RSS	222.7	217.2	222.9	205.6	205.6	202.4
R^2	0.985	0.897	0.985	0.986	0.986	0.987
NOBS	3682	526	3682	3682	3682	3682

TABLE 3. GENERALIZED LEAST SQUARES ESTIMATES FOR AUTOREGRESSIVE MODELS

	1980	1981	1982	1983	1984	1985
C	.51	.30	.28	.18	.11	.14
	(.05)	(.04)	(.15)	(.08)	(.15)	(.05)
S_{-1}	.972	.985	.982	.992	1.00	.995
	(.005)	(.005)	(.016)	(.007)	(.014)	(.004)

Q/N=20.27

	1980	1981	1982	1983	1984	1985
C	.50	.29	.16	.41	.02	.14
	(.05)	(.05)	(.84)	(.21)	(.23)	(.05)
S_{-1}	.973	.975	1.535	.154	1.406	1.000
	(.005)	(.005)	(2.792)	(.677)	(1.085)	(.243)
S_{-2}	..	.012	-.55	.823	-.402	-.004
		(.049)	(2.752)	(.677)	(1.079)	(.243)

Q/N=12.81

	1980	1981	1982	1983	1984	1985
C	.51	.28	-.51	.46	-.25	.03
	(.06)	(.10)	(3.92)	(.24)	(.29)	(.08)
S_{-1}	.972	1.006	4.080	.20	1.631	1.688
	(.006)	(.193)	(15.291)	(.773)	(1.345)	(.526)
S_{-2}	..	-.019	-3.285	.922	-.604	-.452
		(.189)	(16.402)	(.749)	(1.287)	(.396)
S_{-3}	..	..	.223	.032	-.022	-.239
			(1.324)	(.085)	(.164)	(.159)

Q/N=9.21

The test statistics Q/N is explained in the text.

TABLE 4. CORRELATION MATRICES: LOGARITHMIC SALES, CORRELATIONS OVER TIME.

A. LEVELS

	1978	1979	1980	1981	1982	1983	1984	1985
1978	1.000							
1979	.992	1.000						
1980	.982	.994	1.000					
1981	.975	.987	.994	1.000				
1982	.968	.980	.989	.994	1.000			
1983	.962	.974	.983	.989	.994	1.000		
1984	.958	.972	.980	.985	.990	.993	1.000	
1985	.947	.960	.969	.977	.983	.986	.991	1.000

B. YEARLY DIFFERENCES

	1979	1980	1981	1982	1983	1984	1985
1979	1.000						
1980	.355	1.000					
1981	.043	.109	1.000				
1982	.021	.113	-.037	1.000			
1983	.059	.001	.102	-.051	1.000		
1984	.093	-.020	-.101	-.001	-.265	1.000	
1985	.006	.066	.184	.058	.023	-.129	1.000

TABLE 5. SIZE AND CHARACTERISTICS

	Dependent variable:		
	S_{1978}	S_{1985}	$S_{1985}-S_{1978}$
S_{1978}			-.090
			(.016)
AGE_{1978}	.027	.020	-.002
	(.004)	(.003)	(.001)
AGEDUMMY	.377	.173	-.112
	(.163)	(.158)	(.068)
DUMMY31	9.141	10.251	1.986
	(.278)	(.281)	(.190)
DUMMY32	8.223	9.291	1.861
	(.304)	(.304)	(.185)
DUMMY331	7.536	8.970	2.142
	(.315)	(.314)	(.181)
DUMMY332	6.988	8.132	1.794
	(.388)	(.383)	(.191)
DUMMY341	8.018	9.596	2.332
	(.548)	(.537)	(.245)
DUMMY342	7.465	8.795	2.064
	(.290)	(.292)	(.177)
DUMMY35	7.552	8.753	1.950
	(.305)	(.305)	(.177)
DUMMY36	7.440	8.689	1.924
	(.489)	(.480)	(.225)
DUMMY37	8.137	9.329	2.033
	(.615)	(.601)	(.267)
DUMMY38	7.364	8.716	2.069
	(.183)	(.190)	(.153)
DUMMY39	7.337	8.560	1.876
	(.531)	(.520)	(.240)
DUMMY6	7.775	8.590	1.647
	(.478)	(.468)	(.238)
DUMMY9	10.278	11.278	1.971
	(.302)	(.305)	(.213)
AGE1			.487
			(.207)
AGE2			.421
			(.236)
AGE3			.710
			(.236)
AGE4			.064
			(.188)
AGE5			-.131
			(.249)
AGE6			-.207
			(.267)
AGE7			.186
			(.247)
AGE8			-.197
			(.248)
AGE9			-.041
			(.280)
AGE10			-.465
			(.270)

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