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DEVALUATION EXPECTATIONS AND SPECULATIVE ATTACKS ON THE CURRENCY*

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ABSTRACT

In this paper balance-of-payments crises are studied in a framework in which investors do not know the threshold level of foreign reserves, the attainment of which implies that the central bank abandons its fixed exchange target. Investors are alternatively risk neutral or risk averters. It is shown that, depending on whether the threshold level is stochastic or fixed but unknown to investors, currency speculation reveals itself as, respectively, a speculative outflow distributed over a longer time period or a sudden speculative attack on the currency.

CONTENTS

1 INTRODUCTION

There is general agreement that a regime of fixed exchange rates can be sustained only if domestic economic policy is consistent with it. An excessively expansive monetary policy, for instance, leads to the depletion of foreign reserves, eventually forcing the central bank to either devalue the currency or allow it to float. However, changes in the foreign exchange target have typically been preceded by speculative attacks on the currency.

It has been shown in the literature on balance-of-payments crises that speculative attacks on the currency are not in contradiction with the assumption of rational behaviour and that the timing of such attacks is foreseeable.[1] These results, however, are derived under the assumption that there is some binding threshold level of foreign reserves, known by everyone, below which foreign reserves are not allowed to be depleted. As long as foreign reserves are above that level the central bank adheres with certainty to its fixed exchange rate target but after reserves have been depleted to the critical threshold level the central bank either devalues the currency or allows it to float.

One can, however, doubt whether any such binding minimum level of foreign reserves exists. A central bank facing a perfect capital market can, at least in principle, create foreign reserves by borrowing. Thus negative foreign reserves are also feasible.[2] However, even if such a binding threshold level of foreign reserves did exist and was known by everyone, it is very likely that the exchange rate target would be changed well before reserves had been depleted to the binding minimum level of reseves. This implies that the actual threshold level can be much higher than the binding minimum level.[3] Moreover, its size is not known by the public.

In this paper speculative behaviour associated with devaluation expectations is studied in a framework in which the threshold level of foreign reserves is unknown to public. Its value is determined in three alternative ways: a new value is drawn from a known probability distribution at the end of each period, a value for the threshold level is drawn from the distribution only once and, as a combination of these two, a new value for the threshold level is drawn with a probability greater than zero but smaller than one at the end of each period. Investors are alternatively risk neutral or risk averse. The foreign exchange risk is one-sided, i.e. there is only risk of a devaluation.

The paper is organized as follows. In section 2 an equation for the domestic interest rate incorporating the risk premium is derived. In section 3 the processes determining the probability of devaluation are defined and in section 4 a simple macromodel is specified. Finally, in section 5 the speculative behaviour arising from devaluation expectations is studied. Because of the highly nonlinear nature of the model, this is done by utilizing a numerical simulation technique.

2 THE DETERMINATION OF THE DOMESTIC INTEREST RATE

Assume a perfect international bond market in which domestic and foreign bonds differ only in one respect: their currency denomination. The foreign exchange risk is the only source of uncertainty affecting the portfolio allocation between domestic and foreign bonds. The utility function possessed by investors is assumed to be of the form

$$u(W_{t+1}) = u_0 - u_1 \exp(-aW_{t+1}); \qquad u_0,\ u_1,\ a > 0 \tag{1}$$

where W_{t+1} is the non-monetary wealth at the beginning of the period t+1, and a is the measure of absolute risk aversion. The larger is the value of a, the more risk averse investors are. Wealth at the beginning of period t+1 is

$$W_{t+1} = (1+r_t)H_t + (1+r_t^* + \Delta_{t+1})F_t \tag{2}$$

where r_t and r_t^* denote interest rates on domestic and foreign bonds, respectively. H_t is the amount of domestic bonds and F_t the amount of foreign bonds in the portfolio of a domestic investor during period t. Both of them are expressed in domestic currency. Δ_{t+1} denotes the percentage change in the exchange rate between periods t and t+1.[4] Substitute (2) into (1) to obtain

$$u(W_{t+1}) = u_0 - u_1 \exp[-a((1+r_t)H_t + (1 + r_t^* + \Delta_{t+1})F_t)] \tag{3}$$

The maximizing problem for expected utility in period t is now

$$\max_{H,F} E_t\{u_0 - u_1 \exp[-a((1+r_t)H_t + (1+r_t^*+\Delta_{t+1})F_t)]\} \tag{4}$$

where E_t refers to expectations formed at the beginning of period t. The first order condition of (4) implies

$$E_t[(1+r_t^*+\Delta_{t+1})\exp(-aW_{t+1})] = E_t[(1+r_t)\exp(-aW_{t+1})] \tag{5}$$

Typically, under a regime of fixed exchange rates, the foreign exchange risk is not symmetric. If foreign reserves are continuously depleting and are expected to do so in the future as well, then devaluation will be much more likely than revaluation. In the following we assume that the probability of revaluation is practically zero and hence we specify

$$\Delta_{t+1} = \begin{cases} \delta \text{ with probability } \pi_t \\ 0 \text{ with probability } 1-\pi_t \end{cases} \tag{6}$$

where $0 \leqslant \pi_t \leqslant 1$ and δ is the devaluation percentage, if the central bank decides to devalue.

With perfect information about r_t and r_t^*, equations (5) and (6) imply

$$\pi_t(r_t^*+\delta-r_t)\exp\{-a[(1+r_t)W_t + (r_t^*+\delta-r)F_t]\} = (1-\pi_t)(r_t-r_t^*)\exp\{-a[(1+r_t)W_t + (r_t^*-r_t)F_t]\} \tag{7}$$

which gives for F_t

$$F_t = (1/a\delta)\{\log[\pi_t(r_t^*+\delta-r_t)] - \log[(1-\pi_t)(r_t-r_t^*)]\} \tag{8}$$

We see that F_t is determined only with values of r_t such that $r_t^* < r_t < r_t^* + \delta$. In the case where $r_t < r_t^*$ foreign bonds strictly dominate domestic bonds, and in the case where $r_t > r_t^* + \delta$ the situation is, of course, reversed. In both of these cases the risk relevant from the point of view of portfolio diversification is eliminated and hence only the values of r_t in the interval $[r_t^*, r^* + \delta]$ are admissible.

For the purposes of the rest of this paper we prefer equation (8) to be normalized with respect to the domestic interest rate. We obtain

$$r_t = r_t^* + \delta\pi_t - Z_t \tag{9}$$

where

$$Z_t = \pi_t\delta[\exp(a\delta F_t)-1]/[\exp(a\delta F_t) + \pi_t/(1-\pi_t)] \tag{10}$$

Z_t is the risk premium. It is easy to see that Z_t can obtain values only in the interval $[0, \delta\pi_t]$. In the risk neutral case $(a \to 0)$ $Z_t \to 0$ and hence equation (9) implies uncovered interest parity. We also see that if $\pi_t \to 0$ or $\pi_t \to 1$ then $Z_t \to 0$. Hence, equation (9) also collapses to the equation of uncovered interest parity in the limiting case of perfect foresight (i.e. π_t can only obtain the values 0 or 1).

3 PROBABILITY OF DEVALUATION

In the literature on balance-of-payments crises, it is normally assumed that there is a fixed threshold level of foreign reserves known by everybody below which the central bank does not allow the reserves to be depleted. The attainment of this threshold level implies either devaluation or a permanent shift from a fixed exchange rate regime to a floating rate regime.[5]

We preserve the assumption of a threshold level of foreign reserves. However, to take into account the fact that investors are uncertain about how much of its potential reserves the central bank is willing to use to defend its fixed exchange rate target, we assume that the threshold level of foreign reserves is drawn from a probability distribution. Investors know the probability distribution and its moments but they do not know the size of the threshold level drawn from that distribution. They only know that once the foreign reserves have been depleted below the threshold level the central bank will devalue the currency by δ per cent at the beginning of the next period.

We study the speculative behaviour of investors in three informationally different cases. In the first case the central bank does not possess any informational advantage over the public: it makes a draw from the same probability distribution at the end of each period. If the threshold level drawn is greater than or equal to the level of the foreign reserves attained in the period in question, then there is a devaluation of the currency at the beginning of the next period. Otherwise, there is a new draw at the end of the next period. The process continues until the threshold level drawn is below the level of reserves attained.

Assume that the probability distribution from which draws are made is truncated so that the threshold level can obtain values only in the interval $[-\infty, R^u]$. This implies that during period t the probability attached to the occurrence of devaluation at the beginning of period t+1 is

$$\pi_t = \begin{cases} 0, & \text{if } R_t > R^u \\ 1-G(R_t)/G(R^u), & \text{if } -\infty < R_t \leq R^u \end{cases} \tag{11}$$

where G refers to the distribution function and R_t is the level of the foreign reserves in period t. With $R_t < R^u$ we see that the closer R_t is to R^u the closer is π_t to zero, and the further below R^u is R_t the closer π_t is to unity. In the case $R^u \to \infty$ equation (11) reduces to $\pi_t = 1 - G(R_t)$.

In the second case it is assumed that a threshold level is drawn only once (i.e. at the beginning of the game) from a known probability function. With reserves diminishing and without the occurrence of devaluation, this allows investors to learn that the threshold level drawn by the central bank is below the lowest level of the foreign reserves attained up till the beginning of the present period. From the point of view of investors, the situation is the same as if, at the end of each period, the central bank made a new draw from a probability distribution in which the truncation point changes with respect to time so that

$$R_t^u = \min(R_{t-1}, R_{t-1}^u) \tag{12}$$

Equation (12) states that, if during the previous period, the foreign reserves have been depleted below the level corresponding to the truncation point at the beginning of the previous period, then in the present period the truncation point equals the level of foreign reserves at the end of the previous period. Otherwise, the truncation point is the same as in the previous period.

The probability that the currency will be devalued at the beginning of the next period is now

$$\pi_t = \begin{cases} 0, & \text{if } R_t > R_t^u \\ 1 - G(R_t)/G(R_t^u), & \text{if } -\infty < R_t \leq R_t^u \end{cases} \tag{13}$$

where R_t^u is determined by (12).

There is an important difference between these two cases. In the first case the threshold level is stochastic and hence depletion of foreign reserves does not give investors any information about the size of the threshold level. In the second case, changes in foreign reserves supply this kind of information. This results from the fact that now investors know that the threshold level is a fixed figure although, as in the first case, its actual size is unknown to them.

Perhaps more realistic than either of these two cases is the case in which investors do not know with certainty if the threshold level adopted by the central bank is fixed or reconsidered at the end of each period. This case is examined more closely in section 6.4.

4 THE MODEL

In the literature on balance-of-payments crises the model presented by Flood and Garber (1984) is widely used. Our starting point is this same single-good, full-employment small open economy model. We extend the model in two directions: investors are allowed to be risk averse, and the threshold level of foreign reserves, which triggers devaluation, is unknown to investors.[6] Now, the model can be written as

$$M_t/p_t = b_0 - b_1 r_t \qquad ; b_0, b_1 > 0 \tag{14}$$

$$M_t = R_t + D_t \tag{15}$$

$$D_t = D_0 + \mu t \qquad ; \mu > 0 \tag{16}$$

$$p_t = p_t^* s_t \tag{17}$$

$$r_t = r_t^* + E_t(s_{t+1}/s_t - 1) - Z_t \tag{18}$$

$$E_t(s_{t+1}/s_t - 1) = \pi_t\delta \qquad ; \delta > 0 \tag{19}$$

$$Z_t = \pi_t\delta[\exp(a\delta F_t) - 1]/[\exp(a\delta F_t) + \pi_t/(1-\pi_t)] \quad ; a \geq 0 \tag{20}$$

$$R_t = R_{t-1} + p_t T_t - (F_t - F_{t-1}) \tag{21}$$

$$T_t = c_0 + c_1[r_t - E_t(p_{t+1}/p_t - 1)] \quad ; \quad c_1 > 0 \tag{22}$$

$$\pi_t = \begin{cases} 0, & \text{if } R_t > R_t^u \\ 1 - G(R_t)/G(R_t^u), & \text{if } -\infty < R_t \leq R_t^u \end{cases} \tag{23}$$

$$R_t^u = R^u \tag{24a}$$

or

$$R_t^u = \min(R_{t-1}, R_{t-1}^u) \tag{24b}$$

where M is the domestic money stock, r the domestic nominal interest rate, p the domestic price level, R the stock of foreign exchange reserves, D domestic credit, p* the foreign price level, s the spot exchange rate, r* the foreign nominal interest rate, Z the risk premium, π the probability that the currency will be devalued at the beginning of the next period. F the stock of foreign assets held by domestic residents and T the trade balance in real terms.

Equation (14) defines the demand for money and equation (15) the supply of money. Equation (16) states that domestic credit always grows at the positive constant rate μ and (17) defines purchasing power parity. Equations (18) and (20) are the interest rate and risk premium equations derived in section 2 and equation (19) defines the unconditional expected rate of devaluation. The balance-of-payments identity defines the change in the foreign assets as the difference between the trade balance surplus and the change in the stock of foreign assets. Equation (22) defines the trade balance as a function of the real interest rate and equation (23) defines the probability of devaluation with the truncation point of the distribution function G determined alternatively by equation (24a) or (24b).

We assume that p^* and r^* are constant and that as long as the central bank does not devalue $s_t = \bar{s}$. By solving R_t from equations (14) - (19), we obtain

$$R_t = \beta\bar{s} - D_0 - \alpha\delta\bar{s}\pi_t + \alpha\bar{s}Z_t - \mu t \tag{25}$$

where $\beta = b_0p^* - b_1p^*r^*$ and $\alpha = b_1p^*$. We assume that both β and α are positive. Equations (17) - (19) and (21) - (22) imply the following relation for the stock of foreign assets

$$F_t = \gamma\bar{s} - \eta\bar{s}Z_t - (R_t-R_{t-1}) + F_{t-1} \tag{26}$$

where $\gamma = c_0p^* + c_1p^*r^*$ and $\eta = c_1p^*$. The behaviour of the model in the fixed exchange rate regime $s_t = \bar{s}$ is now determined by equations (25), (26), (20), (23) and (24a) or (24b).

5 SPECULATIVE BEHAVIOUR WITH AN UNKNOWN THRESHOLD LEVEL OF FOREIGN RESERVES

In this section we study the speculative behaviour of investors when the threshold level of foreign reserves is assumed to be unknown to investors, i.e. it is drawn from a probability distribution. The distribution function is assumed to be normal with parameters m and σ^2. Because of nonlinearities in the model, it can only be solved numerically. In our numerical simulation experiments, the normal distribution is approximated by Hastings' best approximation formula.[7] Hence, we define $G(R_t) = \phi[(R_t - m)/\sigma]$, where ϕ refers to the normal distribution function with mean m and standard error σ. In the fixed exchange rate regime $s_t = \bar{s} = 1$ the model (25), (26), (20), (23) and (24a or b) can be rewritten as

$$R_t = \beta - D_0 + \alpha\delta\pi_t + \alpha Z_t + \mu t \tag{27}$$

$$F_t - F_{t-1} = \gamma - \eta Z_t - (R_t - R_{t-1}) \tag{28}$$

$$Z_t = \delta\pi_t[\exp(a\delta F_t) - 1]/[\exp(a\delta F_t) - \pi_t/(1-\pi_t)]; \quad a > 0 \tag{29}$$

$$\pi_t = \begin{cases} 0, & \text{if } R_t > R_t^u \\ 1 - \phi[(R_t - m)/\sigma]/\phi[(R_t^u - m)/\sigma], & \text{if } -\infty < R_t \leq R_t^u \end{cases} \tag{30}$$

$$R_t^u = R^u \tag{31a}$$

$$R_t^u = \min(R_{t-1}, R_{t-1}^u) \tag{31b}$$

In all of our simulation experiments, we assumed that $\beta - D_0 = 110$, $\alpha = 500$, $\delta = 0.1$ and $\mu = 1$. The trend variable t obtained values 1, 2, 3, etc. (except in section 6.4 where it obtained values 2, 3, 4, etc.).

In choosing the value of m we use as a benchmark the case of perfect foresight, i.e. all the probability mass of the distribution function ϕ is concentrated on the point m. This point must be equal to the level of reserves at the end of the period preceding the attack, as it is the period when the probability of devaluation jumps from zero to one. If, in the perfect foresight case, the threshold level were zero, is then equation (27) would imply that the pre-attack level of reserves is $\alpha\delta + \mu$. Hence, we set $m = \alpha\delta + \mu = 51$.

We start our analysis by assuming first that investors are risk neutral. In section 5.1 the threshold level is stochastic and in section 5.2 fixed but unknown to investors. The speculative behaviour of risk averse investors is examined in section 5.3. In section 5.4 the analysis is extended so that investors are uncertain if the new threshold level is drawn from a known normal distribution or if the threshold level in force at the beginning of the previous period is still in force.

5.1 The Case of a Stochastic Threshold Level and Risk Neutral Investors

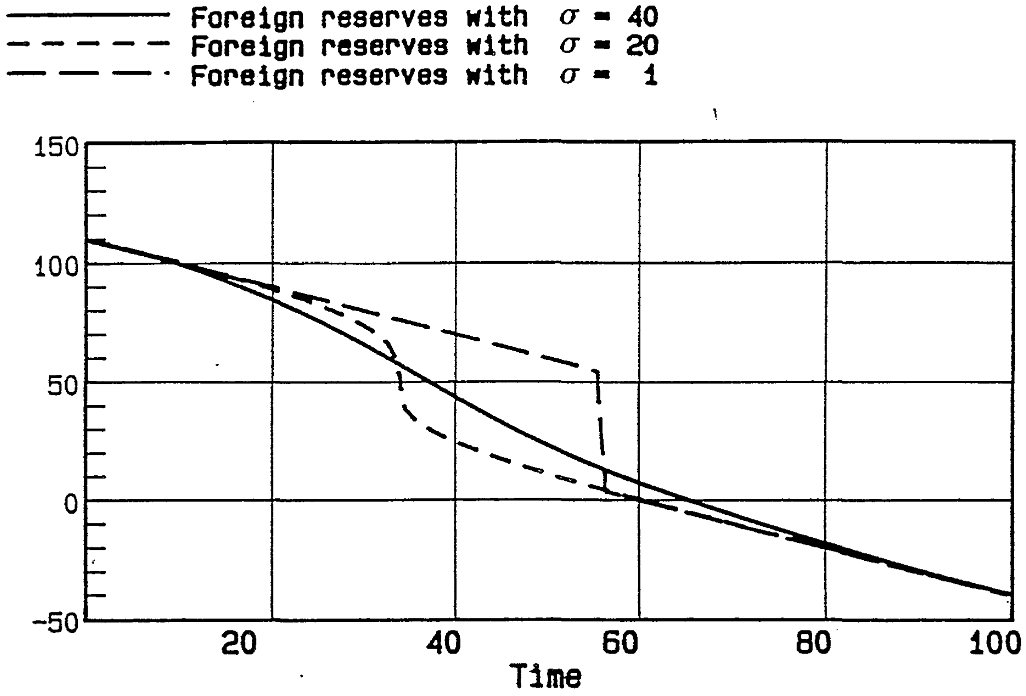
In this section we assume that investors are risk neutral (the risk premium $Z_t = 0$) and that the central bank draws a new threshold level of foreign reserves from a known normal distribution at the end of each period. In this case the behaviour of the model in the fixed exchange regime $s_t = 1$ is determined by equations (27), (30) and (31a).

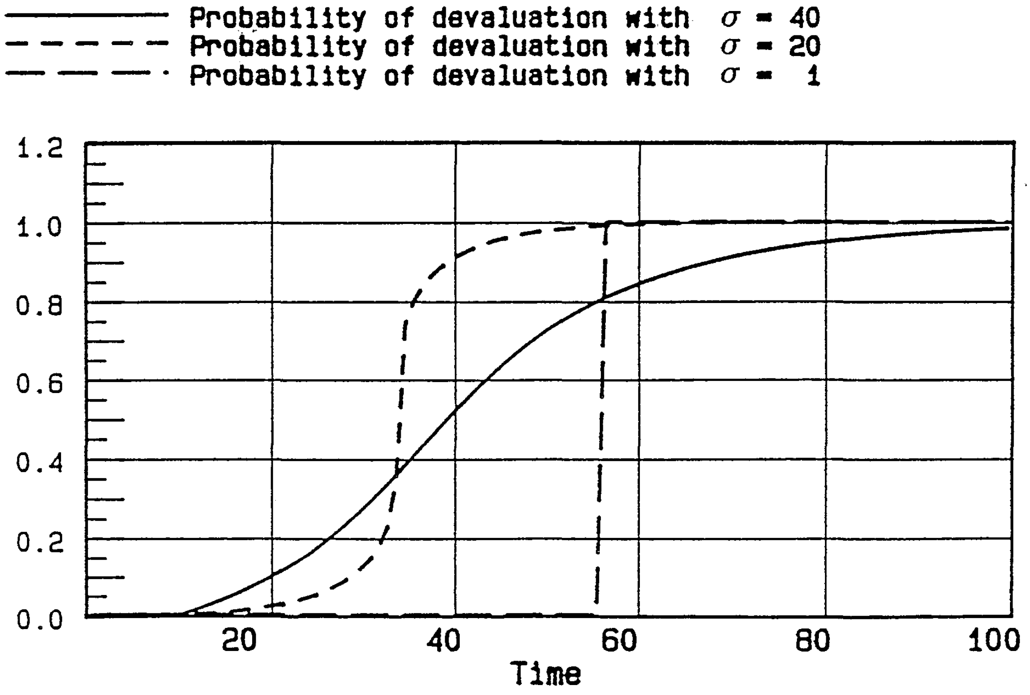
The model was simulated with alternative values of σ, i.e. with $\sigma = 40$, $\sigma = 20$ and $\sigma = 1$. R^u was set to equal 100, implying that as long as foreign reserves are above that level the probability of devaluation is zero. We see from figure 1 that, independently of the size of σ, the cumulative capital outflow caused by devaluation expectations is of equal size, which in our examples is 50. However, the smaller is σ the shorter is the period over which speculative behaviour is concentrated. In the case where $\sigma = 40$, the speculative outflow is distributed quite evenly over a long time period.

Table 1. The maximum size of the depletion of the foreign reserves and its timing with different values of σ

σ	period	the size of the capital outflow
40	38	2.3
20	35	21.2
1	57	50.9
0	60	51

Figure 1.
Currency speculation with risk neutral investors
(stochastic threshold level)





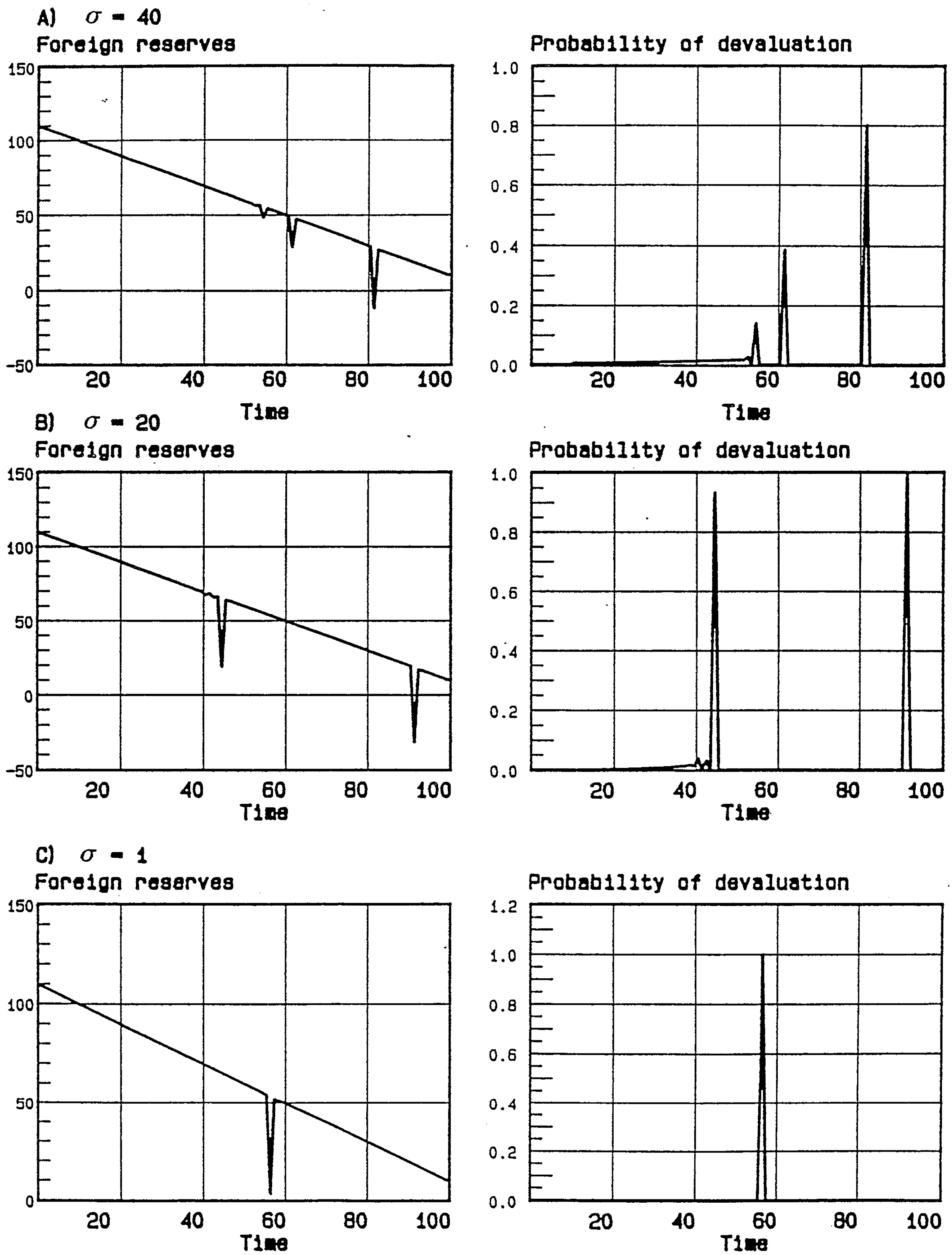
From table 1 it can be seen that when $\sigma = 20$ almost half of, and in the case when $\sigma = 1$, practically all of the speculative capital outflow is concentrated in a single period. The speculative capital outflow is greatest in the period in which the increase in the probability of devaluation is greatest. This can be seen from figure 1b, which shows the time paths of the probability of devaluation with alternative values of σ. As investors are risk neutral, figure 1b also shows the time paths of the interest rate differential $r_t - r_t^*$, so that with $\pi_t = 0$ also $r_t - r_t^* = 0$ and with $\pi_t \to 1$ the differential $r_t - r_t^* \to \delta$.

5.2 The Case of a Fixed but Unknown Threshold Level and Risk Neutral Investors

In this section, we assume that, instead of being stochastic, the threshold level of foreign reserves is fixed. Its value is drawn by the central bank from a known truncated normal distribution before the foreign reserves have been depleted below the truncation point of that distribution. Hence, investors know that the threshold level is fixed but they do not know its values. If investors are risk neutral, equations (27), (30) and (31b) determine the behaviour of the model. In equation (27) the risk premium $z = 0$.

Simulation experiments were made with the same values of σ as in the previous section i.e. with $\sigma = 40$, $\sigma = 20$ and $\sigma = 1$. The initial value of R^u_{t-1} was set to equal 100. The results are shown in figure 2. Unlike in the case of the stochastic threshold level, the probability of devaluation does not grow continuously as reserves diminish. Rather, there is a series of speculative attacks the duration of which is one period. Between the successive attacks, the probability of devaluation is zero. Only before the first attack does the probability of devaluation deviate from zero for a longer time period. However, during this period, its size is quite small (in our examples less than 0.1 at its maximum) and is the more clearly observable the greater is the standard error σ. It can also be seen that the sizes of the successive attacks and the probabilities of

Figure 2.
Currency speculation with risk neutral investors
(fixed but unknown threshold level)



that the sizes of the successive attacks and the probabilities of devaluation associated with their occurrence grow as foreign reserves diminish. The maximum size of the attack is $\alpha\delta$, which equals 50 in our examples. In addition the time interval between successive attacks grows as the size of the attack grows. This is due to the fact that the following attack occurs after the period in which the foreign reserves have attained the level $R_t < R_t^u + \mu$ for the first time. The attack does not occur earlier because investors know that the threshold level of reserves is below R_t^u, i.e. the level to which reserves were depleted during the preceding attack. The increase in the magnitude of the successive attacks is due to the fact that the lower the truncation point of the probability distribution, the more concentrated the probability mass is in the neighbourhood of the truncation point. Figure 2 also shows that the smaller is σ the earlier speculative attacks attain a size very close to their maximum size $\alpha\delta$.

5.3 Speculative Behaviour with Risk Aversion

In this section the assumption of risk neutrality is relaxed. This implies that the risk premium equation (29) and equation (28) determining the stock of foreign assets held by domestic residents are needed.

R_t^u is determined by equation (31a) if the threshold level of foreign reserves is stochastic and by equation (31b) if it is fixed but unknown to investors. As above we assume that R^u in (31a) and the initial value of R_{t-1}^u in equation (31b) equal 100. In the simulation experiments shown in figures 3 and 4, parameters γ, η and σ were set to equal 0, 1 and 20, respectively. The initial value of F_{t-1} was set to equal 110 and the risk aversion parameter a obtained, alternatively, values of 0.05 and 0.1. Simulations were also run with different values of η, the size of which depends positively on the size of the interest sensitivity of the trade balance. Our simulation results showed that the size of η is significant for the timing of speculative behaviour. The greater was η, the earlier speculative capital movements started. As in other respects speculative behaviour resembled that presented in figures 3 and 4, these simulations are not presented here.

Figure 4 shows the case with a stochastic threshold level of foreign reserves. We see that the more risk averse investors are (i.e. the greater is a), the later the speculative capital outflow occurs. Also, the shape of the time paths associated with the probability of devaluation deviates from that of the risk neutral case (a = 0).

The lowest panel of figure 3 shows developments in the risk premium divided by the size of devaluation δ. As expected, we see that the greater is parameter a, the greater is the risk premium. It can also be seen that the risk premium grows at first but, after attaining its maximum value, it suddenly decreases and starts to converge towards zero. This kind of time pattern is due to the fact that, with probabilities of devaluation close to zero and close to unity, uncertantities concerning the occurrence of devaluation are small. In our examples the maximum values of the risk premium are associated with probabilities of devaluation greater than 0.5. With a = 0.05 and a = 0.1 these probabilities are 0.62 and 0.73, respectively.

As regards the domestic interest rate this kind of time pattern for the risk premium implies that, up to the point the risk premium attains its maximum value, the interest rate differential between domestic and foreign interest rates grows more slowly than the probability of devaluation but thereafter it approaches the size of the expected devaluation $\delta\pi_t$ fairly rapidly.

It can be seen from figure 4 that, in the case of a fixed but uncertain threshold level of foreign reserves, an increase in risk aversion delays the occurrence of the first speculative attack on reserves. In our examples, when a = 0 (see figure 3) the first speculative attack occurs in period 40, while when a = 0.05 and a = 0.1 the first attack occurs in periods 58 and 101, respectively. We also see that with low probabilities of devaluation the time pattern of the risk premium is quite similar to that of unconditional

Figure 3.
Currency speculation with risk averse investors (stochastic threshold level)

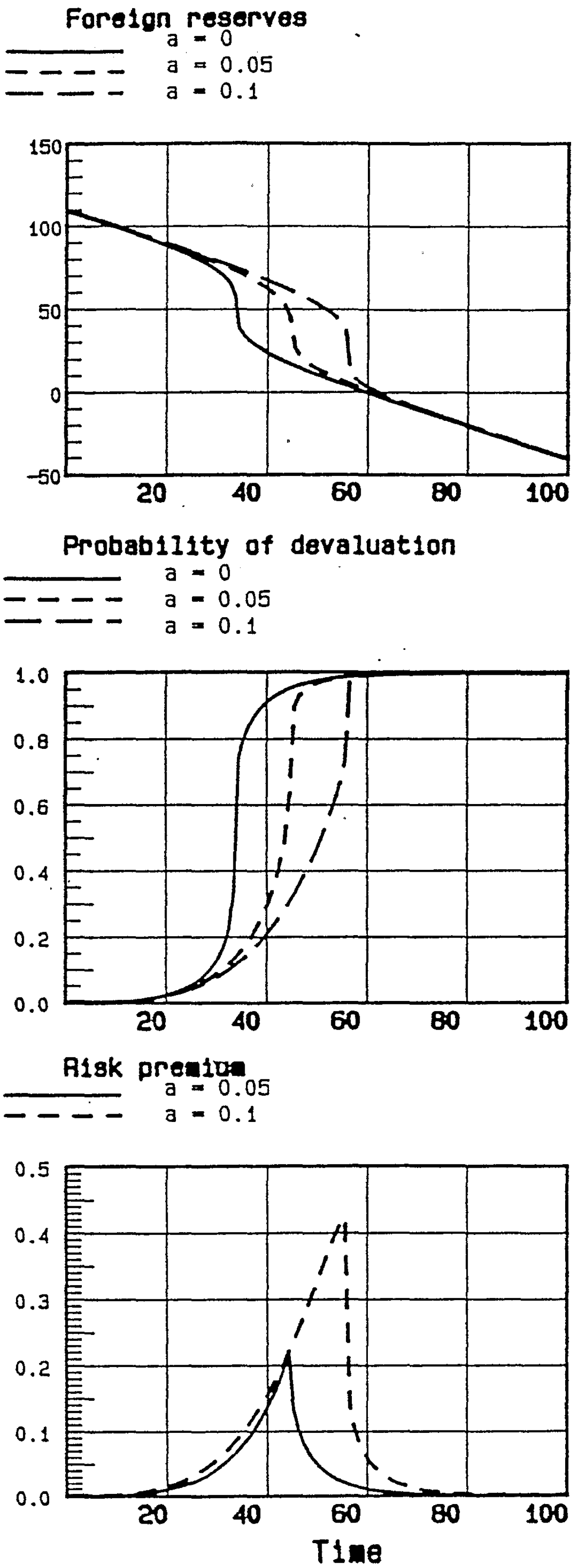
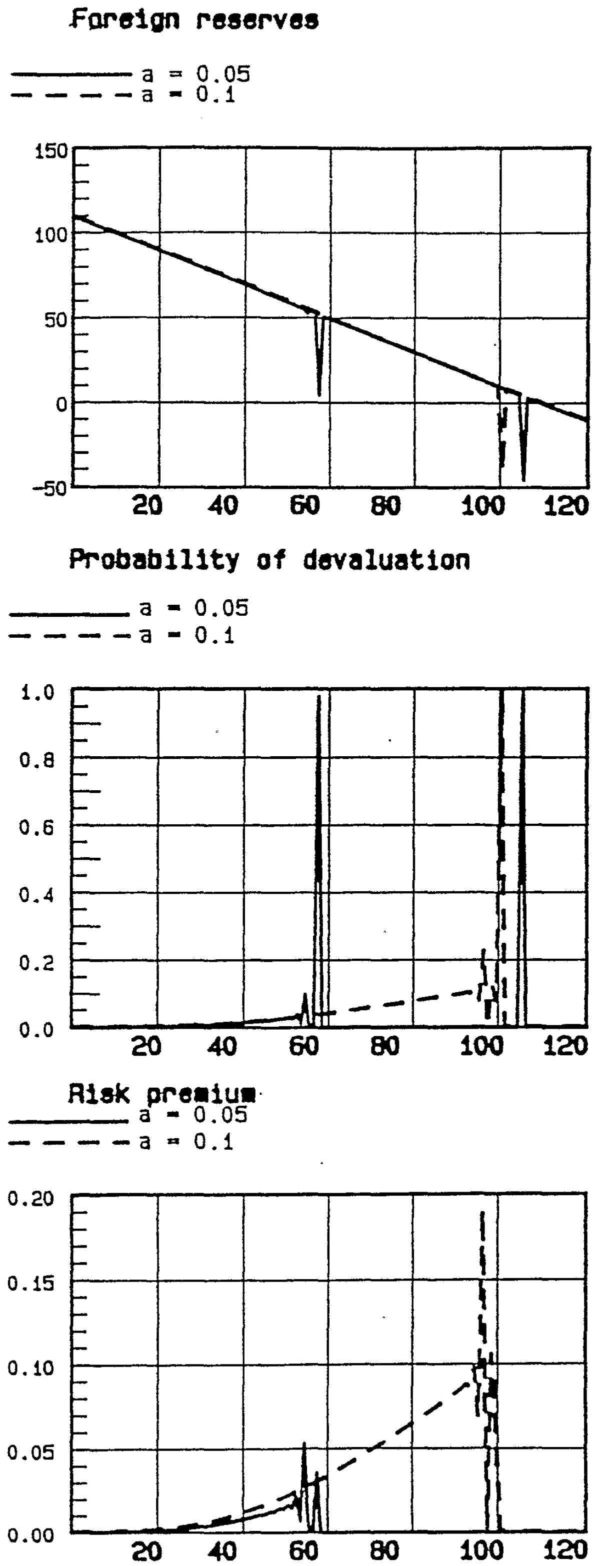


Figure 4.
Currency speculation with risk averse investors (fixed but unknown threshold level)



expected devaluation $\pi_t\delta$. However, the risk premium drops permanently to zero in our examples in connection with the first speculative attack. This is due to the fact that, after the first attack, the probability of devaluation drops to zero and hence there is no uncertainty in the model. After foreign reserves have, without speculative capital movements, diminished to the level which they attained when the first attack occurred, the probability of devaluation suddenly jumps close to unity. In that case, there is practically no uncertainty in the model either, and hence the risk premium stays at zero.

5.4 The Combined Case

So far, speculative behaviour associated with one-sided foreign exchange risk has been studied in two polar cases, i.e. in the case in which the threshold level of foreign reserves is a stochastic variable and in the case in which the threshold level is fixed but unknown to investors. In this section, we combine these two cases by assuming that, with probability 1-k, the central bank chooses a new threshold level from a known normal distribution at the end of each period and that, with probability k, the threshold level is same as at the end of the preceding period. In this combined case developments in the probability of devaluation are determined by a fairly complicated formula. This formula and its derivation are presented in the appendix.

In the fixed exchange rate regime the behaviour of the model is determined by equations (27) - (29) and (A3) - (A5).

In simulations of the model the parameter k was set to equal 0.9, implying that a new threshold level is chosen with a probability of 0.1 at the end of each period and that the threshold level is the same as in the preceding period with a probability of 0.9. σ was set to equal 40 and the initial value of F_{t-1} was set to equal zero.

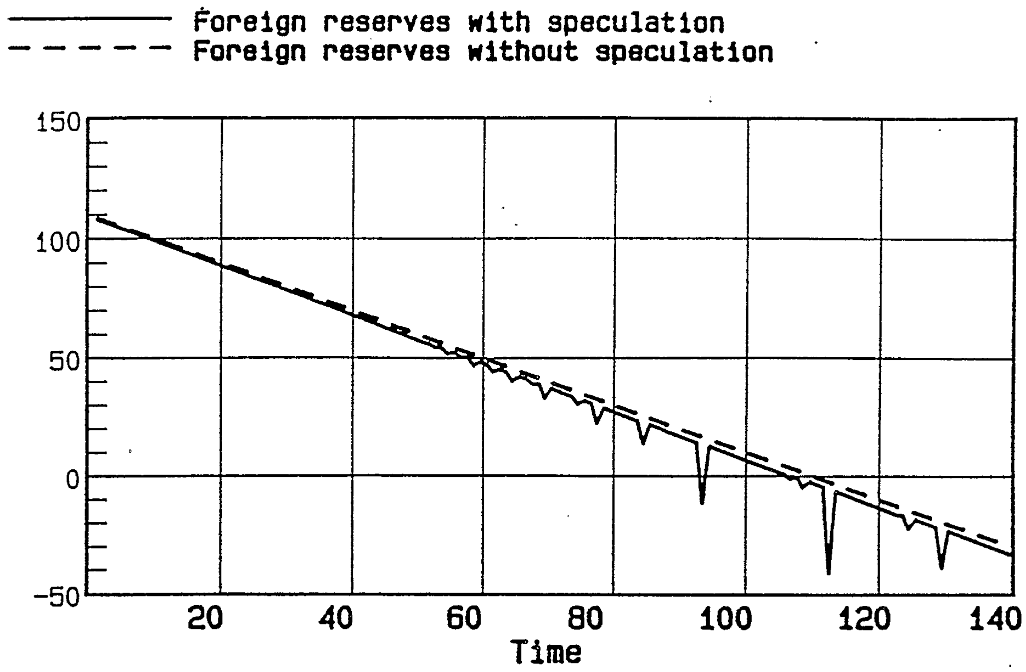
As in the case of a fixed but unknown threshold level, there are speculative attacks on the currency the duration of which is one period (see figure 5). After a speculative attack foreign reserves do not, however, return to the level corresponding to the paths of zero devaluation expectations. This is due to the fact that in equation (A3) the term $(1-k)\pi(R_t)$ is always greater than zero.

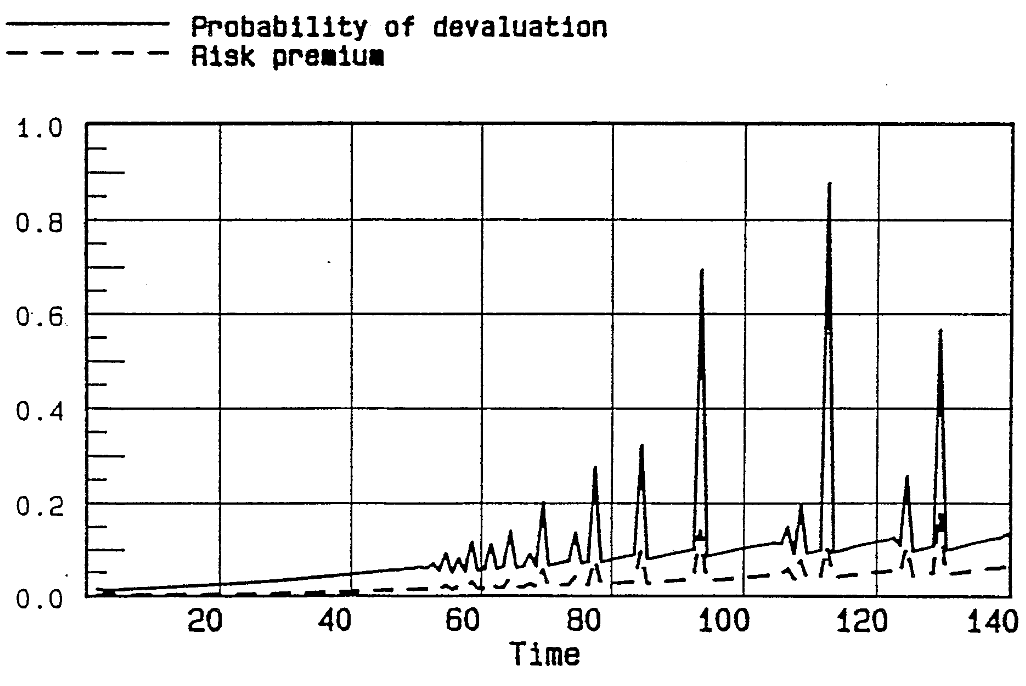
The sizes of successive attacks do not grow steadily. Between bigger attacks there can be smaller attacks. As a result, the timing of successive attacks is less regular than in the case of a fixed but unknown threshold level. When a speculative attack occurs, the pre-attack level of reserves can be well above the level which reserves attained during the previous attack. This is due to the fact that investors cannot be sure that the threshold level of reserves has not been changed since the reserves attained their previous absolute or local minimum level.

We can also see that the behaviour of the risk premium is quite different from what it was in the cases studied in the previous section. In our present example the risk premium does not converge or drop to zero with low values of foreign reserves.

In the previous section we found that values of the probability of devaluation close to zero or close to unity were associated with values of practically zero for the risk premium. In the present case, the probability of devaluation never drops to zero and it can obtain values close to unity only during periods of speculative attack. This explains why the risk premium in figure 5 is above zero all the time. It can be seen that the risk premium also rises during the periods of speculative attack. This, in turn, reflects the fact that probabilities of devaluation associated with these attacks are not close enough to unity.

Figure 5.
Currency speculation with risk averse investors (the combined case)





6 SUMMARY AND CONCLUDING REMARKS

Previous studies of balance-of-payments crises have shown that in a fixed exchange rate regime speculative attacks on the currency can result from rational behaviour. This result, however, was derived in a framework in which investors have perfect information about the threshold level of foreign reserves, the attainment of which implies that a central bank abandons with certainty its fixed exchange rate target and devalues the currency or, alternatively, allows it to float. Owing to this perfect information assumption, a speculative capital movement occurs only at the moment (or in the period) immediately preceding the shift in the exchange rate regime. Hence, it is not able to explain the empirical fact that speculative attacks on the currency quite often occur in anticipation of a devaluation which does not materialize or materializes much later.

In this paper we relaxed the assumption of perfect information about the threshold level of foreign reserves which triggers devaluation of the currency. Speculative behaviour was studied under alternative assumptions about the process which determines the threshold level of foreign reserves. In all cases investors were allowed to be risk averters. Hence, we were also able to study changes in the risk premium associated with speculative capital movements.

In the first case the level was assumed to be stochastic, i.e. a new value for the threshold level was obtained from the known probability distribution in each period. In this case the central bank did not possess any informational advantage over the public. In the second case the threshold level of foreign reserves remained fixed over time. However, investors did not know its value. All they knew, was the probability distribution which had generated the threshold level. The third case was a combination of the first two cases. With a certain probability a new value for the threshold level of reserves was generated at the end of each period. If the new value was not drawn from the known probability distribution, then investors knew that it remained the same as at the beginning of the preceding period.

There were drastic differences in speculative behaviour between the first two cases. In the case of the stochastic threshold level of foreign reserves, steadily growing domestic credit expansion resulted in a speculative outflow of capital distributed over several periods of time, whereas in the case of a fixed but unknown threshold level there were repeated single period attacks on the currency with zero probability of devaluation between the attacks. Hence, in the latter case, if the devaluation did not materialize, reserves were rebuilt in the period following the attack. No restrictive policy measures by the central bank were needed. This kind of behaviour results from the fact that, as there is no devaluation, investors know that the threshold level must be lower than the level to which the attack depleted reserves. The probability of devaluation drops to zero and stays at that level until the reserves have depleted to the level attained during the preceding attack. Then there is a new attack, the size of which is greater than the size of the preceding attack.

In the third case, which is a combination of the previous two, there were also successive attacks on the currency but now the probability of devaluation did not drop to zero between attacks. Successive attacks did not steadily increase in size, unlike in the second case, and new attacks could also occur well above the level to which foreign reserves were depleted by the previous attack.

How did risk aversion affect speculative behaviour? We found that the greater is risk aversion the longer it took - i.e. the lower was the level to which reserves fell - before speculative capital movements started to play major role. We also found that the risk premium converged towards zero the closer to zero or unity the probability of devaluation approached.

Our main conclusion is that speculative behaviour is quite sensitive to the specification of the process which produces the critical lower level of foreign reserves. This may be stochastic, fixed but unknown to investors, or any combination of these two. One area for further research, which would reduce our ignorance about this matter, would be to include the utility function of the central bank in the analysis and to study the problem in a genuine game-theoretic framework.

APPENDIX

THE DETERMINATION OF THE PROBABILITY OF DEVALUATION IN THE COMBINED CASE

Denote the probability of devaluation by $\pi(R_t)$ if the threshold level is drawn from a known normal distribution at the end of each period and by $\pi(R_t, R_t^u)$ if the threshold level is drawn from a known truncated normal distribution with R_t^u as the truncation point.

Assume that under the fixed exchange rate regime $s_t = \bar{s}$ there has not yet been any speculative attack on the currency. This implies that R_{t-1} is the minimum value of R until the beginning of period t. The probability that the currency will be devalued at the beginning of the next period is now simply

$$\pi(t) = (1-k)\pi(R_t) + k\pi(R_t; R_{t-1}) \tag{A1}$$

After the first attack the situation becomes more complicated. Assume that there was an attack in the period t-1-n and that R_{t-1} is the minimum value R in the interval [t-n, t-1]. n is an integer such that $n \geq 0$. Now, with probability k^n, the threshold level is the same in period t as in period t-n and, with probability $(1-k^n)$, a new threshold level has been drawn. If the new value has been drawn, then at the beginning of period t investors know that its value is smaller than R_{t-1}. The probability that the currency will be devalued at the beginning of the next period is now

$$\pi(t) = (1-k)\pi(R_t) + k[k^{n_t}\pi(R_t, R_t^u) + (1-k^{n_t})\pi(R_t, R_{t-1})]$$

with

$$n_t = \begin{cases} n_{t-1} + 1, & \text{if } R^u_{t-1} < R_{t-1} \\ 0 & \text{otherwise} \end{cases}$$

$$R^u_t = \min(R^u_{t-1}, R_{t-1}) \tag{A2}$$

Formula (A2) is not general, however, because it does not take into account the possibility that, between the time interval t-n-1 and t-1, there may have occurred smaller attacks on the currency associated with a level of reserves lower than R_{t-1} but greater than R^u_t. Denote by the absolute minimum level of foreign reserves by $R_t^{u_0}$ and all the successive local minima by $R_t^{u_i}$ such that $R_t^{u_{i-1}} < R_t^{u_i} < R_{t-1}$ (i = 1, ..., q). This condition states that an earlier local minimum contains information which is not included in later local minima only as long as it is smaller than all later local minima until period t. If we denote by $n_{0,t}$ and $n_{i,t}$, the numbers of the periods elapsed since absolute and local minimum values were equal to the level of reserves at the beginning of the period, then the probability of devaluation can be written in general form as

$$\pi(t) = (1-k)\pi(R_t) + k \Big[k^{n_{0,t}}\pi(R_t;\ R_t^{u_0}) + \sum_{i=1}^{q} k^{n_{i,t}}\pi(R_t;\ R_t^{u_i})$$

$$\prod_{j=1}^{i} (1-k^{n_{i-j,t}}) + \pi(R_t,\ R_{t-1}) \prod_{i=0}^{q} (1-k^{n_{q-i,t}})\Big]$$

with

$$R_t^{u_0} = \min(R_{t-1}^{u_0},\ R_{t-1})$$

$$R_t^{u_i} = \begin{cases} \min(R_{t-1}^{u_i},\ R_{t-1}), & \text{if } R_{t-1} < R_{t-2} \text{ or } R_{t-1}^{u_i} \neq R_t^{u_{i-1}} \\ R_{t-1}, & \text{if } R_{t-1} > R_{t-2} \text{ and } R_{t-1}^{u_i} = R_{t-1}^{u_{i-1}} \end{cases}$$

$$n_{0,t} = \begin{cases} n_{0,t-1} + 1, & \text{if } R_{t-1}^{u_0} \leq R_{t-1} \\ 0, & \text{if } R_{t-1}^{u_0} > R_{t-1} \end{cases}$$

$$n_{i,t} = \begin{cases} n_{i,t-1} + 1, & \text{if } R_{t-1}^{u_i} \leq R_{t-1} \\ 0, & \text{if } R_{t-1}^{u_i} > R_{t-1} \end{cases} \tag{A3}$$

and q is any positive integer which is great enough to take into account the number of all possible successive local minimum values of reserves.

It is easy to see that with $R_t^{u_0} = R_t^{u_i} = R_{t-1}$ relation (A3) reduces to (A2) and that with $R_t^{u_0} < R^{u_i} = R_{t-1}$ it reduces to (A1).

After defining

$$\pi(R_t) = 1 - \phi[(R_t - m)/\sigma] \tag{A4}$$

$$\pi(R_t;\ R_t^{u_i}) = \begin{cases} 0, & \text{if } R_t < R_t^{u_i} \\ 1 - \phi[(R_t - m)/\sigma]/\phi[(R_t^{u_i} - m)/\sigma], & \end{cases}$$

$$\text{if } -\infty < R_t \leq R_t^{u_i};\quad i = 0, 1, \ldots, q \tag{A5}$$

equations (A3)-(A5) determine the probability of devaluation in the combined case. With k = 0 and k = 1, the model reduces to the case of the stochastic threshold level and the case of the fixed but unknown threshold level, respectively.

FOOTNOTES

1 See e.g. Krugman (1979), Flood and Garber (1984), Grilli (1986), Obstfeld (1984, 1986a,b), Wyplosz (1986), Buiter (1987) and Calvo (1987).

2 See the discussion by Obstfeld (1986a).

3 Krugman (1979) and Wyplosz (1986) discuss, in a heuristic way the case in which there is a series of possible threshold levels of foreign reserves with probabilities associated with each of them. The analysis in this paper can be thought of as a formalization and extension of their discussion.

4 To be precise Δ_{t+1} is $(1+r_t^*)$times the devaluation percentage.

5 Willman (1987a and b) studies balance-of-payments crises by taking into account the possibility that the consistency between the fixed exchange rate target and economic policy is restored through a change in monetary policy.

6 The model is, of course, open to the criticism that it has not been derived from choice-theoretic foundations with intertemporal budget constraints. Recently, however, the balance-of-payments crises literature has also been developed in that direction [see e.g. papers by Calvo (1987), Drazen and Helpman (1987a and b) and Wijnbergen (1987)].

7 If $x \sim N(0,1)$ and $y = |x|$, then Hastings' best approximation formula for the distribution function of normal distribution is

$$\phi(x) = \begin{cases} 1-F(y) & \text{if } x \geq 0 \\ F(y) & \text{if } x < 0 \end{cases}$$

where $F(y) = (1/\sqrt{2\pi})\exp[-y^2(0.31938153t - 0.356563782t^2 + 1.78147937t^3 - 1.821255978t^4 + 1.330274429t^5)/2]$ and $t = 1/(1+0.2316419y)$.

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