Suvi Vasama

On moral hazard and persistent private information

Bank of Finland Research
Discussion Paper
15 • 2017

On Moral Hazard and Persistent Private Information *

Suvi Vasama

April 18, 2017

Abstract

I examine a simple model of dynamic moral hazard in which the agent has persistent private information. I show that despite the complexity of the framework, the problem has a simple solution that can be found using standard methods. The incentives at the optimal contract can be captured using two state variables: the agent's continuation value and his information rent. The optimal contract uses a combination of nonnegative payments and inefficient liquidation threat to provide the agent incentives. In the beginning, the inefficient liquidation threat is severe, but the expected length of the relationship long, such that the agent's information rent is high. Over time, the information rent decays and continuation value increases as function of the past outcomes. Depending on the past performance, these two processes meet and liquidation at a fixed threshold becomes optimal. In particular, early weak performance leads to a permanent distortion that cannot be undone by performing well in the future.

Keywords: Dynamic moral hazard, persistent private information, real options

JEL Classification: D82, D86

1 Introduction

An extensive literature examines dynamic moral hazard with persistent private information. Persistent private information arises in situations in which an eco-

*Humboldt-Universität zu Berlin, Institute for Economic Theory, suvi.vasama@hu-berlin.de.
I am grateful to Yuliy Sannikov for many helpful discussions. I gratefully acknowledge the financial support by the DFG (German Science Foundation) under SFB 649. A part of the paper was written while I was visiting the Bank of Finland.

nomic variable, that is relevant for the contracting problem, is serially correlated over time. This is widely consistent with empirical evidence: consumption data and firm returns exhibit strong positive correlation over time. However, the contracting problem in such an environment tends to become complex, which may limit the usefulness of the models in practice.

The contribution of this paper is to develop a tractable dynamic principal-agent model and show how to solve it using standard dynamic programming tools. In particular, I develop and solve a model in which (*i*) the firm cash-flow is positively correlated over time, (*ii*) the agent has persistent private information, and (*iii*) that is simple enough such that it can be used for applications in real options and finance. The optimal contract is characterized by a strong form of hysteresis: low outcomes at early stages of the history lead to permanently tight covenants of the contract that cannot be relaxed by performing well in the future.

Specifically, an agent observes the firm cash-flow in continuous time and decides if he delivers it to the principal or diverts part of it for his private benefits. The cash-flow changes according to a geometric Brownian motion and probability distribution over the future cash-flow depends on today's outcome. If the agent diverts cash-flows, the players' expectations about the future diverge. The agent has persistent private information.

Both players are risk-neutral, but the agent is protected by limited liability. In the beginning of the relationship, the players agree on a contract that determines payments from the principal to the agent and a liquidation policy as a function of the past cash-flows that the principal receives from the agent. If the agent diverts cash-flows, he is able to turn a constant share of it for a private benefit. To prevent the agent from diverting cash-flows, the principal has to compensate him for the private benefit - both instantaneously and for the entire expected length of the future relationship.

I characterize the optimal contract using three state variables: the firm cash-flow, the agent's continuation value and the agent's information rent. The cash-flow is a sufficient statistic for the effect of the past on the future cash-flows and, therefore, the firm value. The agent's continuation value is a sufficient statistic for the agent's incentives not to depart from the equilibrium path. The agent's information rent is a book-keeping constraint for past incentives and a sufficient statistic for the promises made to the agent in the past.

The optimal contract relies on an inefficient termination threat to provide the agent incentives, which relaxes over time following good past performance. In the beginning of the relationship, the agent faces a severe liquidation threat, but the promised compensation for delivering higher cash-flows is high since the expected length of the relationship increases sharply with good performance. The agent's continuation value is low, but the information rent high. Over time, the liquidation threat is relaxed, but the information rent increases less strongly for delivering higher cash-flows.

The optimal contract entails a permanent distortion following early weak performance. The distortion is necessary to save on incentive cost to the agent following a history of good performance. The contract enters a regime, in which payments to the agent are initiated and the liquidation policy is independent of the performance but characterized by a fixed threshold policy for the cash-flow. To be more concrete, the contract is terminated in this fixed liquidation regime if the cash-flow falls below a certain cutoff. The particular cutoff, that is implemented, depends on the performance in earlier stages of the relationship.

One of the big challenges in the literature has been to characterize conditions that are both necessary and sufficient for incentive compatibility. I propose a simple model, in which the agent's private benefit is a constant share of the diverted cash-flow. As a consequence, the gain of diversion is the same on and off the equilibrium path and the agent's information rent is independent of the past. I propose a simple method to verify sufficient conditions that allows me simply to check that the agent's global incentive constraint binds.

The optimal contract can be solved from the principal's optimization problem that depends on three state variables. I show that the problem can be written in a form which one handle using standard methods. The method that I propose provides a significantly simpler solution concept than previously introduced in the literature.

The seminal papers that derive the optimal contract in a dynamic cash-flow diversion framework include DeMarzo and Sannikov (2006); DeMarzo and Fishman (2007); Biais et al. (2007).[1] My framework extends these models by taking into account the empirical fact that returns are correlated over time. The optimal contract entails a permanent distortion in liquidation policy that is absent in the model with independently distributed cash-flows. In the same time, my model relates more directly to the standard real options problem that has been very successful in finance. Combing the insights of the two branches of literature is a necessary step towards models that can be used as a basic for empirical research.

My solution methods borrow from a recent paper by DeMarzo and Sannikov (2016)[2] who build on a learning model in which changing economic environment is described by a state that is fixed, but unknown by the players. Instead, I build on directly on the standard real options framework, see e.g. Dixit and Pindyck (1994). My contribution is to develop the methods further and show that the problem has a significantly simpler solution than previously thought.

[1] The review is by no means complete, Sannikov (2013) provides an excellent survey.

[2] See He et al. (2016); Prat and Jovanovic (2014); Williams (2011, 2015); Strulovici (2011) for related recent contributions.

2 Setting

I examine a game with two players, a principal (she) and an agent (he). Both discount the future by the same rate $r > 0$. The principal has access to unlimited funding but the agent is protected by limited liability. In my setting this implies that the agent cannot make negative payments. Also, I abstract away from private savings by the agent.

At time t the firm cash-flow X_t evolves according to

$$dX_t = \mu X_t dt + \sigma X_t dZ_t \tag{1}$$

with μ denoting the drift and $\sigma > 0$ the volatility of the percentage growth rate. I assume that $\mu < r$ to guarantee that the efficient solution is well defined.

The agent delivers the cash-flow to the principal and has the opportunity to divert part of it to earn a private benefit. Specifically, I assume that the cash-flow process that agent delivers to the principal evolves according to

$$d\hat{X}_t = dX_t - l_t dt. \tag{2}$$

(2) implies that the agent can only make absolutely continuous changes in the cash-flow process. That is, the cash-flow that the principal receives from the agent, always follows a geometric Brownian motion. The assumption is without loss of generality since I focus on full commitment contracts.[3] Moreover, I assume that $X_0 = \hat{X}_0$ such that there is no asymmetric information at time 0 and the problem is one of pure moral hazard.

Cash-flow diversion is inefficient since the assets are not in their most efficient use, but there is a social cost of $1 - \lambda \in (0, 1)$ of diversion. The private benefit that the agent earns from the diverted cash-flow is

$$\lambda(X_t - \hat{X}_t). \tag{3}$$

It follows from limited liability, together with the assumption that the agent cannot save privately, that $\hat{X} \leq X_t$. Thus, the agent can deliver at most the realized cash-flow to the principal.

The contract determines an intertemporal flow of nonnegative payments from the principal to the agent $\{c_t \geq 0 : t \geq 0\}$ and a liquidation policy τ as functionals of the publicly observable path of cash-flows that the principal receives from

[3] To see why this is the case, notice that if the agent would deliver the principal a cash-flow that did not satisfy (2), she would immediately detect the deviation. Therefore, the principal can design the contract such that the firm is liquidated instantaneously as soon as such a history is observed. Such a contract would make it suboptimal for the agent to choose strategies that violate (2).

the agent. Liquidation is irreversible, and we denote the principal's payoff upon liquidation by $L \geq 0$ and the agent's by $R \geq 0$. We assume that the players can fully commit to the contract.

Formally, the optimal contract maximizes the principal's expected profit at time 0

$$E\left[\int_0^\tau e^{-rt}(X_t - c_t)dt + e^{-r\tau}L\right],$$

delivers the agent the expected value

$$W_0 = E\left[\int_0^\tau e^{-rt}c_t dt + e^{-r\tau}R\right]$$

at time 0 and satisfies the agent's incentive compatibility constraint

$$W_0 \geq E\left[\int_0^\tau e^{-rt}(c_t + \lambda(X_t - \hat{X}_t))dt + e^{-r\tau}R\right]$$

for all strategies $\{l_t : t \leq \tau\}$ that satisfy the limited liability constraint $\hat{X}_t \leq X_t$. The incentive constraint guarantees that the agent always delivers the whole realized cash-flow to the principal.

3 First-Best Solution

I first shortly review the efficient solution to the problem. The efficient solution is attainable if $\lambda = 0$, i.e. there is no incentive problem. The first-best optimal liquidation policy solves a standard real options problem, which chooses an optimal stopping time τ to maximize the sum of the players' utilities

$$s(x) = \sup_\tau E\left[\int_0^\tau e^{-rt}X_t dt + e^{-r\tau}(L+R)\right]$$
$$= \frac{x}{r-\mu} - E[e^{-r\tau^*}]\frac{x^*}{r-\mu} + E[e^{-r\tau^*}](L+R).$$

For the standard real options problem, the expectation $E[e^{-r\tau^*}]$ can be solved in closed form.

The optimal liquidation policy is a threshold policy: the firm is operated so long as the cash-flow stays above a threshold x^* and is liquidated as soon as x^* is reached. At any point $x \in (0, \infty)$, the optimal liquidation threshold can be solved from the players' Hamilton-Jacobi-Bellman equation

$$rs(x) = x + \mu x s_x(x) + \frac{\sigma^2}{2}x^2 s_{xx}(x)$$

with the boundary conditions $s(x^*) = L + R$ and $s_x(x^*) = 0$.

The value function can be solved in closed form and admits the solution

$$s(x) = \frac{x}{r-\mu} - \frac{x^*}{r-\mu}\left(\frac{x}{x^*}\right)^{\gamma} + (L+R)\left(\frac{x}{x^*}\right)^{\gamma} \tag{4}$$

with

$$x^* = (r-\mu)\frac{\gamma}{\gamma-1}(L+R) \tag{5}$$

and

$$\gamma = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\frac{1}{4} + \frac{\mu^2}{\sigma^4} + \frac{r-\mu}{\sigma^2}}. \tag{6}$$

The firm value (4) consists of the expected discounted stream of future cash-flows x minus the expected loss from interrupting the cash-flows at x^* plus the gain from the realized outside options $L + R$. The termination payoffs are attained with probability $E[e^{-r\tau^*}] = (x/x^*)^{\gamma}$.

4 Incentive Compatibility

In this section, I consider the agent's incentives to deliver the cash-flow to the principal. I first derive a set of necessary conditions that make a particular kind of deviation suboptimal for the agent. Then I derive a smaller set of sufficient conditions that guarantee that deviation is suboptimal for an arbitrary strategy of the agent. Later I will solve for the optimal contract from the principal's problem subject to the necessary incentive condition and verify ex post that the solution satisfies the sufficient conditions.

If the agent diverts cash-flows, he gains both by (*i*) earning an instantaneous private benefit; and (*ii*) having superior information about the future cash-flows. For the agent to be willing to deliver the cash-flow to the principal, he needs to be compensated for both losing the instantaneous private benefit and the future information rents.

As is standard in the literature, I start by deriving the representation of the agent's continuation value as a functional of the changes of the cash-flow observed by the principal. In the absence of deviations, the agent's continuation value evolves according to[4]

$$dW_t = (rW_t - c_t)dt + \beta_t(dX_t - \mu X_t dt). \tag{7}$$

Of course, if the agent deviates, the principal observes the process $\hat{X}$ instead of X as defined in (2) and (7) describes the evolution of the agent's promised value at the contract.

[4] See e.g. Sannikov (2008).

To derive necessary conditions for incentive compatibility, I follow the first-order approach by Williams (2011, 2015). To be more concrete, I first rewrite the agent's problem on a new domain using as control variables the relative likelihood ratio Γ_t between the probability measure induced by the actual cash-flow and the one induced by the agent's alternative strategy and the gap $g_t \equiv X_t - \hat{X}_t$ between the actual cash-flow and the one that the principal delivers to the agent. Then I use a stochastic maximum principle to solve for a necessary condition for the agent not to divert cash-flows on the equilibrium path.

In the following, I restrict the attention on a strict subset of strategies with $\{l_t \geq 0\}$ to derive a necessary condition for incentive compatibility.[5] The restriction seems appropriate since I look for condition to make delivering the true cash-flow profitable on the equilibrium path. The proof of sufficiency allows for arbitrarily complex strategies of the agent.

Recall that the agent's different strategies induce different probability distributions over the future outcomes. In particular, using the relative likelihood ratio process Γ, I can calculate the change of probability measure to express the agent's expected utility under strategy $\{l_t\}$ as

$$E^l\left[\int_0^\tau e^{-rt}(c_t + \lambda(X_t - \hat{X}_t))dt + e^{-r\tau}R\right]$$
$$= E^0\left[\int_0^\tau \Gamma_t e^{-rt}(c_t + \lambda(X_t - \hat{X}_t))dt + \Gamma_\tau e^{-r\tau}R\right].$$

By Girsanov theorem, the relative likelihood ratio Γ_t admits the law of motion

$$d\Gamma_t = -\frac{l_t}{\sigma X_t}\frac{dX_t - \mu X_t dt}{\sigma X_t}.$$

with the initial condition $\Gamma_0 = 1$. Moreover, by (2), the gap g_t between the actual cash-flow and the one that the agent delivers to the principal evolves according to

$$dg_t = l_t dt.$$

In particular, under strategy $\{l_t = 0\}$, $\Gamma_t = 1$ and $g_t = 0$ for all $t \leq \tau$. Both the agent's utility on and off the equilibrium path and the players' expectations coincide.

The agent's optimal strategy among the particular class of strategies that satisfy the constraint $\{l_t \geq 0\}$ can be derived by using a stochastic maximum principle. Using the costate variables p^i and q^i with $i \in \{\Gamma, g\}$ for the drift and volatility

[5] Limited liability requires that $\hat{X}_t \leq X_t$; i.e. the constraint $l_t \geq 0$ only applies on the equilibrium path. A suitable mathematical concept to handle the more general problem remains yet unexplored. I thank Eduardo Faingold for the observation.

terms of the controls, the Hamiltonian of the agent's problem can be written as

$$\mathcal{H}(t, \Gamma, g, l, p^\Gamma, p^g, q^\Gamma, q^g) = \Gamma(c + \lambda g) + p^g l - q^\Gamma \Gamma \frac{l}{\sigma x}.$$

Taking the first-order with respect to l, I find that if the condition

$$p_t^g - q_t^\Gamma \frac{\Gamma_t}{\sigma X_t} \leq 0 \tag{8}$$

holds, the agent has no incentive to depart from the equilibrium path. I show in the Appendix that $q_t^\Gamma = \sigma \beta_t X_t$ and $p_t^g = \lambda(1 - E[e^{-r\tau}])/r$. Notice that here q_t^Γ is the shadow cost of inducing false expectation to the principal and p_t^g is the shadow marginal benefit that the agent obtains by diverting a dollar.

Now since $\Gamma_t = 1$ at the equilibrium path, the necessary condition (8) for incentive compatibility becomes

$$\beta_t \geq \frac{\lambda}{r}(1 - E[e^{-r(\tau - t)}]). \tag{9}$$

The right hand side describes the change of the agent's continuation value if he delivers an additional dollar to the principal. The left hand side describes the expected gain of concealing the additional dollar. Since output is persistent, the gain of diversion is persistent, too. In particular, the agent earns a flow of private benefit of λ for the dollar until the termination time τ. For the agent to be willing to deliver the cash-flow to the principal, the expected gain of diversion must not exceed the increase in continuation value.

The result is summarized in the following proposition

Proposition 1 (Necessary Conditions for Incentive Compatibility)**.** *Consider a contract in which the agent always delivers all the realized cash-flow to the principal; i.e* $\hat{X}_t = X_t$ *for all* $t \leq \tau$*. Then (9) is a necessary condition for incentive compatibility.*

Proof. See Appendix. □

I next derive a sufficient condition for incentive compatibility. To obtain a sufficient condition, I look at a set of conditions that satisfy the necessary condition (9) with equality and verify that it is optimal for the agent to let $l_t = 0$ for all $t \leq \tau$.

In particular, I verify in the Appendix that if

$$\beta_t = \frac{\lambda}{r}(1 - E[e^{-r(\tau - t)}]), \tag{10}$$

the agent has no incentive to divert cash-flows. The result is summarized in the following proposition

Proposition 2 (Sufficient Conditions for Incentive Compatibility)**.** *Consider a contract in which the agent always delivers all the realized cash-flow to the principal; i.e* $\hat{X}_t = X_t$ *for all* $t \leq \tau$*. Then (10) is a sufficient condition for incentive compatibility.*

The challenge with persistent private information is that the stopping policy τ in (9) generally depends on the entire path of past outcomes. The expectation $E[e^{-r\tau}]$ admits a closed form solution only in very specific cases that I examine in more detail below. To solve for the more general case, it is useful to derive an expression for the agent's information rent as a stochastic process. I then use the information rent as a state variable to solve the optimal contract from the principal's relaxed problem.[6]

The agent's information rent at time t is the shadow interest rate p^g_t that he earns by keeping the gap between the actual cash-flow X_t and the cash-flow $\hat{X}_t$ that he delivers to the principal. The proof of Lemma 1 in the Appendix shows that the agent earns an expected information rent of

$$\xi_t = \frac{\lambda}{r}(1 - E[e^{-r(\tau-t)}]), \tag{11}$$

for each dollar that he keeps hidden from the principal. Notice that we can now write the incentive constraint (9) as

$$\beta_t \geq \xi_t \tag{12}$$

By comparing (11) and we can immediately see that if the necessary incentive constraint binds, the agent's off equilibrium path information rent ξ and the compensation β that the receives for the additional cash-flow that he delivers to the principal coincide. The agent has no incentive to divert cash-flows, on or off the equilibrium path, as confirmed by Proposition 2.

The next lemma summarizes the representation of the agent's information rent

Lemma 1 (Information Rent)**.** *The agent's information rent at time t admits the following representation*

$$d\xi_t = r\xi_t dt - \lambda dt + \chi_t X_t \sigma dZ_t \tag{13}$$

for some process χ_t *that is chosen by the principal.*

Proof. (11) follows from (34) in the Appendix since the agent's information rent is by definition $\xi_t = p^g_t$. (13) follows then from (30). □

[6]The approach borrows from Williams (2011, 2015); DeMarzo and Sannikov (2016).

Connection to DeMarzo and Sannikov (2016). In my model, the agent's diversion leads to a persistent flow of private benefits λ for each dollar until time τ at which the project is terminated, cf. (11). In DeMarzo and Sannikov private benefits are temporary, $\lambda a_t dt$, but diverting leads to a persistent information rent that decays exponentially, see equation (5) in their paper. The volatility of learning process causes the belief to gradually converge and the agent is rewarded by an increase of β_t in his continuation value for each unit of belief. As a consequence, the information rent depends on β_t, see equation (17).

In my framework, the agent has the possibility to return to the equilibrium path, by closing the gap between the actual cash-flow and the one that he delivers to the principal, losing an information rent of λ for each dollar that goes from him to the principal. Consequently, the agent's information rent only depends on λ and not on β. Then as soon as the incentive constraints bind, the contract is globally incentive compatible.[7]

5 Contracting Problem

I next derive the optimal contract from the principal's problem. I particular, I first verify the dynamics of the optimal contract from the principal's relaxed problem subject to the necessary incentive constraint. Then I show that the solution satisfies the sufficient conditions for incentive compatibility. I present a heuristic derivation in the body of the paper, some of the proofs are delegated in the Appendix.

Formally, I write the principal's relaxed problem as solving

$$v(w,x,\xi) = \sup\ E\left[\int_0^\tau e^{-rt}(X_t - c_t)dt + e^{-r\tau}L\right]$$

subject to the controlled processes (1), (7) and (13) and the controls ($c_t \geq 0, \beta_t \geq \xi_t, \chi_t$) satisfying the agent's limited liability conditions, Proposition 1 and Lemma 1.

The optimal controls can be derived from the principal's Hamilton-Jacobi-Bellman equation

$$\begin{aligned} rv(w,x,\xi) = F(w,x,\xi) + \sup_{c\geq 0,\beta\geq\xi,\chi}\Bigg\{ -c(1+v_w(w,x,\xi)) + \sigma^2 x^2\left(\frac{\beta^2}{2}v_{ww}(w,x,\xi)\right.\\ \left. + \frac{\chi^2}{2}v_{\xi\xi}(w,x,\xi) + \beta\chi v_{w\xi}(w,x,\xi) + \beta v_{wx}(w,x,\xi) + \chi v_{\xi x}(w,x,\xi)\right)\Bigg\} \quad (14)\end{aligned}$$

[7] Similar observations have earlier been made by Tchistyi (2005).

with the terms that are independent of the controls collected in

$$F(w,x,\xi) = x + rwv_w(w,x,\xi) + (r\xi - \lambda)v_\xi(w,x,\xi) + \mu x v_x(w,x,\xi) + \frac{\sigma^2}{2}x^2 v_{xx}(w,x,\xi).$$

5.1 Contract Properties

I start the analysis by proving some key properties of the principal's value function. The properties are then used to derive the optimal controls.

Firstly, I show that the marginal cost of providing the agent incentives is at most 1. The principal can always reward the agent by delivering him an immediate payment. Since both players are risk-neutral, the cost of delivering an immediate payment is 1. The next lemma summarizes the result

Lemma 2. *The marginal cost of compensating the risk-neutral agent is at most 1; i.e.* $v_w(w,x,\xi) \geq -1$.

Proof. Define the firm value in the second best optimal contract, $s(w,x,\xi) \equiv v(w,x,\xi) + w$. Since $dC_t \geq 0$ is feasible, $s(w,x,\xi)$ is nondecreasing in w. Letting $\varepsilon > 0$

$$s(w+\varepsilon,x,\xi) \geq s(w,x,\xi)$$

or,

$$\frac{v(w+\varepsilon,x,\xi) - v(w,x,\xi)}{\varepsilon} \geq -1$$

The result follows by letting $\varepsilon \to 0$. □

Next, I prove that the value function is concave in the state variables w and ξ. The contract needs to provide the agent both a nonnegative expected future payoff and a nonnegative information rent. Increasing the volatility of either of the variables increases the risk that either of them falls too low, an event that triggers inefficient termination.

I prove joint concavity in w and ξ; concavity in w and ξ follows. The result is summarized in the following lemma

Lemma 3. *The principal's value function* $v(w,x,\xi)$ *is jointly concave in* w *and* ξ.

Proof. See Appendix. □

Finally, I prove that the marginal cost of of providing the agent incentives is increasing in the volatility direction. Tightening the inefficient liquidation threat is more costly if the cash-flow is higher. The principal's loss of inefficient liquidation is higher while the agent always earns a constant share of each dollar that he conceals. Formally,

Lemma 4. *The marginal gain of tightening the inefficient liquidation threat is nonincreasing along the trajectory of* (w, x, ξ)*; i.e.* $\xi v_{ww}(w, x, \xi) + \chi v_{w\xi}(w, x, \xi) + v_{wx}(w, x, \xi) \leq 0$.

Proof. See Appendix. □

5.2 Payments and Incentive Constraints

I next verify the optimal payment schedule, show that the incentive constraints bind and determine the optimal volatility of the agent's information rent in the relaxed problem. The results can be derived by examining the principal's Hamilton-Jacobi-Bellman equation (14).

I first determine the optimal intertemporal allocation of the payments from the principal to the agent. First, recall from Lemma 2 that the marginal cost of providing the agent an immediate payment is at most 1. Indeed, taking the first condition of (14) with respect to c I find that if the condition

$$v_w(w, x, \xi) \geq -1 \tag{15}$$

holds with strict inequality, it is optimal to set $c = 0$ to delay the payments to the agent. If the condition (15) binds, initiating payments to the agent is feasible.

Delaying payments increases the agent's expected payoff in the future. Such back loading of payments is standard when the agent is risk-neutral and allows for the implementation of a more efficient intertemporal liquidation policy. I will verify below that payments are delayed until the state variables (w, x, ξ) hit a boundary that I call fixed liquidation boundary. Along this boundary $v_w(w, x, \xi) = -1$ and $c > 0$ is (weakly) optimal to the principal.

In next derive the optimal volatility χ of the agent's information rent. Taking the first-order condition of (14) I find that implies that the principal sets χ such that it satisfies

$$\chi v_{\xi\xi}(w, x, \xi) + \beta v_{\xi w}(w, x, \xi) + v_{\xi x}(w, x, \xi) = 0. \tag{16}$$

The relaxed optimal contract sets χ such that the marginal cost of increasing the agent's information rent, $v_\xi(w, x, \xi)$, is constant along the volatility trajectory. The result reflects the fact that the agent's information rent is always λ for each diverted dollar, regardless of the firm value.

I next verify that the incentive constraints bind and that the principal minimizes the volatility of the agent's continuation value. Taking the first-order condition of (14) with respect to β, I find that it is optimal to let the incentive constraint bind if

$$\begin{aligned}
&\beta v_{ww}(w, x, \xi) + \chi v_{w\xi}(w, x, \xi) + v_{wx}(w, x, \xi) \\
&\qquad = (\beta - \xi) v_{ww} + \xi v_{ww}(w, x, \xi) + \chi v_{w\xi}(w, x, \xi) + v_{wx}(w, x, \xi) \leq 0.
\end{aligned}$$

Indeed, the first part is minimized at $\beta = \lambda$ since the agent's value function is concave by Lemma 3 above. Moreover, Lemma 4 implies that the second part is always nonpositive.

6 Fixed Liquidation Regime

With persistent private information, the agent needs to be compensated, not only for his instantaneous private benefit, but also for all expected future private benefits that he loses when he delivers the cash-flow to the principal.

Formally, when the payments to the agent are delayed; i.e. $c_t = 0$, his continuation value drifts up according to (7). Moreover, since $r\xi_t = \lambda(1 - E[e^{-r(\tau-1)}]) \leq \lambda$, the agent's information rent drifts down according to (13). It turns our that there is a boundary at which processes "meet" such that $v_w(w, x, \xi) = 1$.

Once the state variables have reached the payout boundary, relaxing the inefficient termination threat would decrease the agent's information rent too much relative to his continuation value. Instead, it is optimal to implement termination at a fixed threshold x_L. The problem resembles the standard real options problem, with the difference that termination is inefficient. I say that the contract enters a fixed liquidation regime. Notice, in particular, that if $x_L = x^*$, the first-best solution is implemented.

6.1 Optimal Long-Term Distortions

I next characterize the fixed liquidation regime. In particular, I solve the principal's problem of choosing the optimal liquidation threshold x_L subject to the agent earning a given information rent $\xi^L(x, x_L)$. Then I compute the agent's continuation value that is needed to implement liquidation at x_L. The optimal contract enters the liquidation regime as soon as W_t reaches $w(X_t, x_L)$.

In the fixed liquidation regime, the agent's information rent can be written as

$$\xi^L(x, x_L) \equiv \frac{\lambda}{r}\left(1 - \left(\frac{x}{x_L}\right)^{\gamma}\right) \tag{17}$$

with γ as defined in (6). Then using Proposition 2,

$$\beta^L(x, x_L) = \frac{\lambda}{r}\left(1 - \left(\frac{x}{x_L}\right)^{\gamma}\right). \tag{18}$$

I can then solve for a lower bound of the agent's continuation value $w^L(x, x_L)$ that the contract has to promise him such that liquidation at the threshold x_L is feasible. Using (17), I find that

$$w^L(x, x_L) = \int_{x_L}^{x} \xi^L(y, x_L)dy + R$$

$$= \frac{\lambda}{r}(x - x_L) + \frac{\lambda x_L}{r(\gamma+1)}\left(1 - \left(\frac{x}{x_L}\right)^{\gamma+1}\right) + R. \quad (19)$$

I verify in the Appendix that (19) solves (7) with (18) when the payout policy is determined appropriately as discussed below.

The principal's value function $v^L(x, x_L)$ can now be solved similarly to the real options problem and admits the solution

$$v^L(x, x_L) = \frac{x}{r-\mu} - \frac{x_L}{r-\mu}\left(\frac{x}{x_L}\right)^{\gamma} + (L+R)\left(\frac{x}{x_L}\right)^{\gamma} - w^L(x, x_L). \quad (20)$$

The results are summarized in the following lemma[8]

Lemma 5 (Fixed Liquidation Regime)**.** *Suppose that the condition*

$$rR \geq -\frac{\lambda\gamma}{r}\frac{\sigma^2}{2} \quad \text{and} \quad \gamma \geq -1 - \frac{r}{\mu} \quad (21)$$

holds, $\beta_t = \xi^L(x, x_L)$ with $\xi^L(x, x_L)$ as defined in (17) and the payment to the agent is

$$c^L(x, x_L) = rR + \lambda(x - x_L) + \frac{\lambda x_L}{\gamma+1}\left(1 - \left(\frac{x}{x_L}\right)^{\gamma+1}\right)$$
$$- \frac{\lambda}{r}\mu x\left(1 - \left(\frac{x}{x_L}\right)^{\gamma}\right) + \frac{\lambda\gamma}{r}\frac{\sigma^2}{2}\left(\frac{x}{x_L}\right)^{\gamma}. \quad (22)$$

Then the termination policy achieves the fixed liquidation threshold x_L. That is, the contract is terminated as soon as x_t reaches the fixed cutoff value x_L.

Proof. See Appendix. □

I next verify that (15) binds in the fixed liquidation regime and is slack to the left of the payment boundary. The result is trivial and is given for completeness.

Lemma 6. *The optimal contract delays payments until it reaches the fixed liquidation regime. In the fixed liquidation regime, $v_w(w, x, \xi) = -1$.*

[8] The condition (21) is sufficient to guarantee that the payment process described in the lemma is nonnegative - recall from (6) that $\gamma < 0$. If (21) does not hold, the payment process that is implemented may not be absolutely continuous.

Proof. Now the liquidation policy $\tau^L = \inf\{t : X_t \leq x_L\}$ is independent of W_t. Let $\varepsilon > 0$, and notice that

$$\frac{\partial}{\partial\varepsilon}(v(w+\varepsilon, x, \xi) - v(w, x, \xi))$$
$$= \frac{\partial}{\partial\varepsilon}\left(E\left[\int_0^\tau X_t dt + e^{-r\tau}L\right] - (W_t^L + \varepsilon) - E\left[\int_0^\tau X_t dt + e^{-r\tau}L\right] + W_t^L\right)$$
$$= -1.$$

Next, since $v(w, x, \xi)$ is concave by Lemma 3 and $v_w(w, x, \xi) = -1$ in the fixed liquidation regime, $v(w, x, \xi) > -1$ before the contract enters the liquidation regime. Then $c_t = 0$ is optimal by (15). □

6.2 Evolution of Information Rents

The last step is to determine the volatility of the agent's information rent χ. To the right of the payment boundary, the optimal control can be solved in closed form. By substituting (17) into (13), I find that the agent's information rent solves the stochastic differential equation

$$d\xi_t = -\lambda\left(\frac{X_t}{x_L}\right)^\gamma + \chi_t \sigma X_t dZ_t. \tag{23}$$

The process χ_t is determined such that ξ_t hits 0 the same time that X_t hits x_L. Lemma 7 below shows how to use this fact to solve $\chi(x, x_L)$ in closed form. The solution is

$$\chi(x, x_L) = \frac{\sqrt{2}}{x\sigma}\sqrt{\frac{\xi(x, x_L)}{\gamma(\log(x) - \log(x_L))}A(x, x_L)} \tag{24}$$

with

$$A(x, x_L) = \frac{\xi(x, x_L)}{\gamma(\log(x) - \log(x_L))} + \lambda\left(\frac{x}{x_L}\right)^\gamma \tag{25}$$

It is straightforward to check that $\chi(x, x_L) \geq 0$ for all $x \geq x_L$, $\chi \to \infty$ as $x \to x_L$ and $\chi \to 0$ as $x \to \infty$.

The result is summarized in the following lemma

Lemma 7. *To the right of the payment boundary, the volatility of the agent's information rent, $\chi(x, x_L)$ admits the solution (24).*

Proof. Let $f(\xi(x, x_L))$ denote the hitting time distribution for $\xi(x, x_L)$. For a given x, $f(\xi)$ has to satisfy the following Kolmogorov equation

$$rf(\xi) = -\lambda\left(\frac{x}{x_L}\right)^\gamma f'(\xi) + \frac{\sigma^2 x^2}{2}\chi^2 f''(\xi)$$

with the boundary conditions $f(0) = 1$ and $\lim_{\xi\to\infty} f(\xi) = 0$.

A straightforward calculation shows that the solution is

$$f(\xi) = \exp\left\{\left(\lambda\left(\frac{x}{x_L}\right)^{\gamma} - \sqrt{\lambda^2\left(\frac{x}{x_L}\right)^{2\gamma} + 2\chi^2x^2\sigma^2}\right)\xi/(\chi^2x^2\sigma^2)\right\}$$

Moreover, we know that $f(\xi(x, x_L)) = (x/x_L)^{\gamma}$. (24) follows by combining and reorganizing. □

7 Dynamics of the Optimal Contract

In this chapter, I summarize the results from the previous sections and discuss how the principal's value function can be written in the form that is easier to interpret and that can be computed numerically.

7.1 Characterization

The optimal contract delays payments to the agent in the beginning of the relationship to increase his continuation value for the future. At the same time, the agent's information rent decays. Payments are delayed until the state variables hit the payment boundary and the contract enters the fixed liquidation regime. Payments to the agent are initiated and the contract terminates if the cash-flow hits the cutoff x_L.

In the relaxed optimal contract, the necessary incentive constraint (9) binds, i.e. $\beta_t = \xi_t$ for all $t \leq \tau$. The sufficient condition of Proposition 2 is satisfied, and therefore, the relaxed optimal contract is the solution of the principal's full optimization problem.

The results are summarized in the following theorem[9]

Theorem 1 (Optimal Contract)**.** *The optimal contract admits the following dynamics*

1. *At any point $w \in (R, w^L(x, x_L))$, the principal's value function is the unique solution of the follow system of partial differential equations*

$$rv = x + rwv_w + (r\xi - \lambda)v_\xi + \mu x v_x + \sigma^2x^2\left(\frac{\xi}{2}^2 v_{ww} - \frac{\chi}{2}^2 v_{\xi\xi} + \frac{1}{2}v_{xx} + \xi v_{wx}\right). \quad (26)$$

[9] To simplify the expressions, I drop the state variables (w, x, ξ) in the following.

The payment to the agent is $c_t = 0$. The state of the world (w, x, ξ) evolves according to (1), (13) and

$$dW_t = rW_t dt + \xi_t X_t \sigma dZ_t. \tag{27}$$

2. *As soon as W_t reaches $w^L(x, x_L)$ with $w^L(x, x_L)$ as defined in (19), the contract enters the fixed liquidation regime. If the condition (21) holds, the payment to the agent, $c(x, x_L)$ is (22). The volatility of the agent's information rent $\chi(x, x_L)$ is (24).*

3. *As soon as W_t reaches R, the contract is terminated. The principal's continuation payoff is L.*

Proof. A standard verification argument verifies optimality in the relaxed program. Since $\beta_t = \xi_t$ for all $t \leq \tau$, the contract is globally incentive compatible by Proposition 2. The solution of the relaxed program is the optimal contract that solves the principal's full program. □

7.2 Reformulation of the Problem

I next discuss an alternative representation of state variables that allow for a more tractable representation of the principal's value function. In particular, the partial differential equation (26) becomes parabolic. Standard methods have been developed to deal with such an equation.

In particular, consider $t = \beta x - (w - R)$ and $z = x - \xi/\chi$ and $x = x$. With abuse of notation, let $v(t, z, x)$ denote the principal's value function over the new domain. A straightforward calculation shows that the value function becomes

$$\begin{aligned} rv(t,z,x) &= x - ((1-\mu)\chi(x-z)x + r(t-R))v_t(t,z,x) \\ &\quad - \left(r(x-z) - \frac{\lambda}{\chi} - \mu x\right) v_z(t,z,x) + \mu x v_x(t,z,x) + \frac{\sigma^2}{2} x^2 v_{xx}(t,z,x) \end{aligned} \tag{28}$$

over the domain $(t, z, x) \in (0, \infty) \times (x_L, \infty) \times (x_L, \infty)$. The boundary conditions are $v(0, x, x) = L$ and (17), (19), (20) and (24). The formulation (28) is useful since some of the proofs simplify tremendously when using the new representation of state variables. Also, the equation can be solved numerically.

8 Discussion

This paper examines a dynamic principal-agent problem in which the agent has persistent private information. It shows that the problem admits a tractable solution that can be characterized using standard methods. The next step is to derive a numerical solution to the principal's value function. This allows me to examine the dynamics of the optimal contract in more detail, in particular, the dynamic distortions that arise as a result of the agent's persistent private information.

9 Appendix

I first derive necessary conditions for incentive compatibility

Proof of Proposition 1. The costate variables admit the dynamics

$$dp_t^\Gamma = rp_t^\Gamma - \mathcal{H}_\Gamma dt + q_t^\Gamma dZ_t = rp_t^\Gamma dt - (c_t + \lambda g_t)dt + q_t^\Gamma dZ_t, \tag{29}$$
$$dp_t^g = rp_t^g dt - \mathcal{H}_g dt + q_t^g dZ_t = rp_t^g dt - \lambda \Gamma_t dt + q_t^g dZ_t, \tag{30}$$

with the terminal conditions $p_\tau^\Gamma = \partial/\partial\Gamma\ \Gamma R = R$ and $p_\tau^g = \partial/\partial g\ \Gamma R = 0$.

Evaluating at the equilibrium path, along which $\Gamma_t = 1$ and $g_t = 0$, (29) and (30) become

$$dp_t^\Gamma = rp_t^\Gamma dt - c_t dt + q_t^\Gamma dZ_t, \tag{31}$$
$$dp_t^g = rp_t^g dt - \lambda dt + q_t^g dZ_t. \tag{32}$$

Then, solving the backward stochastic differential equation (31) I find that

$$p_t^\Gamma = E_t^0 \left[\int_t^\tau e^{-r(s-t)} c_s ds + e^{-r(\tau-t)} R \right] = W_t.$$

Now by comparing (31) with (7), we find that

$$q_t^\Gamma = \sigma X_t \beta_t. \tag{33}$$

Next, solving the backward stochastic differential equation (32), I find that

$$p_t^g = E_t^0 \left[\int_t^\tau e^{-r(s-t)} \lambda ds \right] = \frac{\lambda}{r}(1 - E[e^{-r(\tau-t)}]). \tag{34}$$

(9) follows by substituting (33) and (34) into (8). □

The next step is to verify sufficient conditions for incentive compatibility. In contrast to Williams (2011); DeMarzo and Sannikov (2016), my necessary conditions use a particular kind of deviation since I impose the condition $l_t \geq 0$ on the agent's strategy which only holds on equilibrium path. Therefore, I cannot derive sufficient conditions by examining the properties of Hamiltonian or the agent's incentives on path. Instead, I provide sufficient conditions for incentive compatibility by verifying incentive compatibility for an arbitrary strategy and verify later that the optimal contract satisfies the conditions

Proof of Proposition 2. Consider an arbitrary full strategy l of the agent. The agent's expected payoff from the strategy is

$$E^0\left[\int_0^\tau e^{-rt}(c_t+\lambda(X_t-\hat{X}_t))dt+e^{-r\tau}R\right]$$
$$=E^0\left[\int_0^\tau e^{-rt}c_t dt+e^{-r\tau}R\right]+E^0\left[\int_0^\tau e^{-rt}\lambda\int_0^t l_s ds dt\right].$$

Using standard arguments, we can write the agent's global incentive constraint as

$$U_0^l=U_0-E^0\left[\int_0^\tau e^{-rt}\left(\beta_t l_t-\lambda\int_0^t l_s ds\right)dt\right]\le 0.$$

I verify that the incentive constraint is satisfied. Substituting for (10) and reorganizing, I find that

$$U_0^l-U_0=\lambda E^0\left[\int_0^\tau e^{-rt}\int_0^t l_s ds-\frac{e^{-rt}}{r}(1-E[e^{-r(\tau-t)}])l_t dt\right]$$
$$=\lambda E\left[\int_0^\tau\int_t^\tau e^{-rs}l_t ds-e^{-rt}\int_0^t l_s ds dt\right]$$
$$=\lambda E\left[\int_0^\tau\int_0^t e^{-rt}l_s ds-e^{-rt}\int_0^t l_s ds dt\right]=0$$

almost surely. The third equality uses Fubini's theorem to exchange the order of integration. This verifies that cash-flow diversion is (weakly) suboptimal for the agent. □

I next discuss the relaxed optimal contract at the fixed liquidation regime

Proof of Lemma 5. In the fixed liquidation regime, the contract only depends on the pair (x, x_L). The agent's continuation value has to satisfy the Hamilton-Jacobi-Bellman equation

$$rw^L(x,x_L)-c^L(x,x_L)=\mu x w_x(x,x_L)+\frac{\sigma^2}{2}x^2 w_{xx}(x,x_L)$$

in which

$$\frac{\partial}{\partial x}w^L(x,x_L)=\frac{\lambda}{r}\left(1-\left(\frac{x}{x_L}\right)^\gamma\right)$$

and

$$\frac{\partial^2}{\partial x^2}w^L(x,x_L)=-\frac{\lambda}{r}\frac{\gamma}{x}\left(\frac{x}{x_L}\right)^\gamma.$$

Then using (7) with (18), I find that the agent's continuation value $w^L(x, x_L)$ solves the stochastic backward differential equation

$$dW(X_t, x_L) = \frac{\lambda}{r} X_t \left(\mu \left(1 - \left(\frac{X_t}{x_L} \right)^{\gamma} \right) - \frac{\sigma^2}{2} \gamma \left(\frac{X_t}{x_L} \right)^{\gamma} \right) + \sigma X_t \frac{\lambda}{r} \left(1 - \left(\frac{X_t}{x_L} \right)^{\gamma} \right) dZ_t$$

with the boundary condition $W(x_L, x_L) = R$. The solution is (19), as confirmed by Ito's lemma.

Next, I show that choosing the fixed liquidation threshold x_L is optimal for the principal, conditional on the agent earning the information rent $\xi^L(x, x_L)$. Specifically, let $\phi \geq 0$ denote the multiplier for the constraint (17) and consider the problem of choosing an optimal stopping time τ to maximize the social surplus subject to the constraint that the agent has to receive a fixed information rent

$$\max_{\tau} \left\{ E\left[\int_0^{\tau} e^{-rt} X_t + e^{-r\tau}(L + R) \right] - \phi \underbrace{\frac{\lambda}{r} \left(1 - E[e^{-r\tau}] \right)}_{=\xi_0} \right\}.$$

The difference to the standard real options problem is that the liquidation payoff is now $L + R + \phi\lambda/r$, the additional payoff consisting of savings on the agent's information rents. The optimal termination threshold x_L maximizes

$$x^{-\gamma} \left(L + R + \phi \frac{\lambda}{r} - \frac{x}{r - \mu} \right).$$

The first-order condition for maximum implies that

$$\frac{\lambda}{r} \phi = \frac{\gamma + 1}{\gamma} \frac{x_L}{r - \mu} - (L + R) \tag{35}$$

It follows from (5) that $\phi \geq 0$ for all $x_L \geq x^*$, with equality only if $x_L = x^*$. □

I next show that the value function $s(w, x, \xi) \equiv v(w, x, \xi) + w$ is jointly concave in w and ξ The state of the world (w, x, ξ) evolves according to the controlled processes (1), (7) and (13) with the controls $dC_t \geq 0$ $\beta_t \geq \xi$ and χ_t. Concavity of $v(w, x, \xi)$ follows since $v(w, x, \xi) = s(w, x, \xi) - w$.

Proof of Lemma 3. Consider two controlled processes (W_t^1, X_t^1, ξ_t^1) and (W_t^2, X_t^2, ξ_t^2) with the optimal controls $(dC_t^1, \beta_t^1, \chi_t^1)$ and $(dC_t^2, \beta_t^2, \chi_t^2)$ and $X_0^1 = X_0^2$. Notice that $X_t^1 = X_t^2 = X_t$ by (1). Let $\underline{\tau} = \tau^1 \wedge \tau^2$ and $\overline{\tau} = \tau^1 \vee \tau^2$ denote the stopping time under which the earlier and the later process terminate.

Let $\theta \in [0,1]$ and consider a process $(W_t^\theta, X_t^\theta, \xi_t^\theta)$ starting at the state $W_0^\theta = \theta W_0^1 + (1-\theta)W_0^2$, $X_0^\theta = X_0$ and $\xi_0^\theta = \theta\xi_0^1 + (1-\theta)\xi_0^2$. Let $(dC_t^\theta, \beta_t^\theta, \chi_t^\theta) \equiv \theta(dC_t^1, \beta_t^1, \chi_t^1) + (1-\theta)(dC_t^2, \beta_t^2, \chi_t^2)$.

(11) implies that

$$E[e^{-r\tau^i}] = 1 - \frac{r}{\lambda}\xi_t^i \tag{36}$$

for each $i \in \{1, 2, \theta\}$. Then before time $\underline{\tau}$, I have

$$\xi^\theta = \theta\underline{\xi} + (1-\theta)\overline{\xi}.$$

From time $\underline{\tau}$ onwards, $\underline{\xi} = 0$ and

$$\xi^\theta = (1-\theta)\overline{\xi}.$$

By combining with (36), I find that $\tau^\theta = \overline{\tau}$, with τ^θ denoting the stopping time at which the process θ is terminated.

Since the set of controls $(dC_t^\theta, \beta_t^\theta, \chi_t^\theta)$ is feasible in the control problem starting from the initial state $(W_0^\theta, X_0^\theta, \xi_0^\theta)$, I find that

$$\begin{aligned}
&s(w^\theta, x, \xi^\theta) \geq E\left[\int_0^{\tau^\theta} e^{-rt}X_t dt + e^{-r\tau^\theta}(L+R)\right] \\
&= (1-\theta)E\left[\int_0^{\overline{\tau}} e^{-rt}X_t dt + E[e^{-r\overline{\tau}}](L+R)\right] + \theta E\left[\int_0^{\overline{\tau}} e^{-rt}X_t dt + E[e^{-r\overline{\tau}}](L+R)\right] \\
&\geq (1-\theta)E\left[\int_0^{\overline{\tau}} e^{-rt}X_t dt + E[e^{-r\overline{\tau}}](L+R)\right] + \theta E\left[\int_0^{\underline{\tau}} e^{-rt}X_t dt + E[e^{-r\underline{\tau}}](L+R)\right] \\
&\qquad = \theta s(w^1, x, \xi^1) + (1-\theta)s(w^2, x, \xi^2).
\end{aligned}$$

The inequality in the third line follows since

$$E\left[\int_{\underline{\tau}}^{\overline{\tau}} e^{-rt}X_t dt + e^{-r\overline{\tau}}(L+R)\right] \geq E[e^{-r\underline{\tau}}(R+L)]$$

since $\overline{\tau} \geq \underline{\tau}$ and the contract never continues beyond the first-best solution. □

I am now ready to verify that the marginal cost of compensation is nondecreasing along the trajectory of the state variables. I proof the result using the new representation of state variables and the social value function $s(t, z, x)$.

Proof of Lemma 4. I prove the result using the new representation of state variables. Now $v_{ww}(w, x, \xi) = y_{tt}(t, z, x)$, $v_{w\xi}(w, x, \xi) = v_{tz}(t, z, x)/\chi$ and $v_{wx} =$

$-\beta v_{tt}(t,z,x) - v_{tz}(t,z,x) - v_{tx}(t,z,x)$. By substituting into the condition of Lemma 4, I find that

$$\beta v_{ww}(w,x,\xi) + \chi v_{w\xi}(w,x,\xi) + v_{wx}(w,x,\xi) = -v_{tx}(t,z,x).$$

To verify the result, I need to verify that $v_{tx}(t,z,x) \geq 0$.

Using the new representation of state variables, I find that $s(t,z,x) = v(t,z,x) + R + \beta x - t$. It follows that $s_{tx}(t,z,x) = v_{tx}(t,z,x)$. I need to verify that this is nonnegative.

Now recall from the proof of Lemma 2 that $s(w,x,\xi)$ is nondecreasing in w. Hence, $s(t,z,x)$ is nonincreasing in t. That is, $s(t+\varepsilon,z,x) \leq s(t,z,x)$ for $\varepsilon > 0$.

Next, a trivial argument implies that $s(t+\varepsilon,z,x+\delta) \geq s(t+\varepsilon,z,x)$. By combining the results, I find that

$$s(t+\varepsilon,z,x+\delta) - s(t+\varepsilon,z,x) - (s(t+\varepsilon,z,x) - s(t,z,x)) \geq 0$$

from which the result follows. □

References

Bruno Biais, Thomas Mariotti, Guillaume Plantin, and Jean-Charles Rochet. Dynamic security design: Convergence to continuous time and asset pricing implications. *The Review of Economic Studies*, 74(2):345–390, 2007.

Peter M DeMarzo and Michael J Fishman. Optimal long-term financial contracting. *Review of Financial Studies*, 20(6):2079–2128, 2007.

Peter M DeMarzo and Yuliy Sannikov. Optimal security design and dynamic capital structure in a continuous-time agency model. *The Journal of Finance*, 61(6):2681–2724, 2006.

Peter M. DeMarzo and Yuliy Sannikov. Learning, termination, and payout policy in dynamic incentive contracts. *Forthcoming in The Review of Economic Studies*, 2016.

Avinash K Dixit and Robert S Pindyck. *Investment under uncertainty*. Princeton university press, 1994.

Zhiguo He, Bin Wei, Jianfeng Yu, and Feng Gao. Optimal long-term contracting with learning. 2016.

Julien Prat and Boyan Jovanovic. Dynamic contracts when the agent's quality is unknown. *Theoretical Economics*, 9(3):865–914, 2014.

Yuliy Sannikov. A continuous-time version of the principal-agent problem. *The Review of Economic Studies*, 75(3):957–984, 2008.

Yuliy Sannikov. Dynamic security design and corporate financing. *Handbook of the Economics of Finance*, 2:71–122, 2013.

Bruno Strulovici. Contracts, information persistence, and renegotiation. *Working Paper, Northwestern University*, 2011.

Alexei Tchistyi. Security design with correlated hidden cash flows: The optimality of performance pricing. *NYU Working Paper*, 2005.

Noah Williams. Persistent private information. *Econometrica*, 79(4):1233–1275, 2011.

Noah Williams. A solvable continuous time dynamic principal–agent model. *Journal of Economic Theory*, 159:989–1015, 2015.

Bank of Finland Research Discussion Papers 2017

ISSN 1456-6184, online

1/2017 Gonçalo Faria – Fabio Verona
Forecasting the equity risk premium with frequency-decomposed predictors
ISBN 978-952-323-144-3, online

2/2017 Sari Pekkala Kerr – William Kerr – Çağlar Özden – Christopher Parsons
Global talent flows
ISBN 978-952-323-145-0, online

3/2017 Sari Pekkala Kerr – William R. Kerr
Global collaborative patents
ISBN 978-952-323-146-7, online

4/2017 Aino Silvo
House prices, lending standards, and the macroeconomy
ISBN 978-952-323-148-1, online

5/2017 Paavo Miettinen
Information acquisition during a descending price auction with asymmetrically informed players
ISBN 978-952-323-149-8, online

6/2017 Giovanni Caggiano – Efrem Castelnuovo – Giovanni Pellegrino
Estimating the real effects of uncertainty shocks at the zero lower bound
ISBN 978-952-323-150-4, online

7/2017 Sari Pekkala Kerr – William Kerr – Çağlar Özden – Christopher Parsons
High-skilled migration and agglomeration
ISBN 978-952-323-152-8, online

8/2017 Giovanni Caggiano – Efrem Castelnuovo – Gabriela Nodari
Uncertainty and monetary policy in good and bad times
ISBN 978-952-323-158-0, online

9/2017 Guido Ascari – Anna Florio – Alessandro Gobbi
Controlling inflation with switching monetary and fiscal policies: expectations, fiscal guidance and timid regime changes
ISBN 978-952-323-159-7, online

10/2017 Gene Ambrocio – Esa Jokivuolle
Should bank capital requirements be less risk-sensitive because of credit constraints?
ISBN 978-952-323-162-7, online

11/2017 Ville Voutilainen
Wavelet decomposition of the financial cycle: An early warning system for financial tsunamis
ISBN 978-952-323-165-8, online

12/2017 Mathias Drehmann – Mikael Juselius – Anton Korinek
Accounting for debt service: The painful legacy of credit booms
ISBN 978-952-323-169-6, online

13/2017 Tomasz Łyziak – Maritta Paloviita
Formation of inflation expectations in turbulent times. Can ECB manage inflation expectations of professional forecasters?
ISBN 978-952-323-170-2, online

14/2017 Suvi Vasama
Contracting with long-term consequences
ISBN 978-952-323-171-9, online

15/2017 Suvi Vasama
On moral hazard and persistent private information
ISBN 978-952-323-172-6, online

http://www.suomenpankki.fi/en • email: Research@bof.fi
ISBN 978-952-323-172-6, ISSN 1456-6184, online