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Controlling inflation with switching monetary and fiscal policies: expectations, fiscal guidance and timid regime changes

Controlling Inflation with switching monetary and fiscal policies: expectations, fiscal guidance and timid regime changes[*]

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Abstract

Inflation depends on both monetary and fiscal policies and on how agents believe that these policies will evolve in the future. Can monetary policy control inflation, when both monetary and fiscal policies are allowed to change over time? To analyse this problem, we study a model in which both monetary and fiscal policies may switch according to a Markov process. Controlling inflation entails a unique and Ricardian solution. We propose a natural generalisation of the original Leeper (1991) taxonomy, introducing the concepts of *globally active (or passive)* and *globally switching* policies to define the conditions that allow monetary policy to control inflation under Markov switching. First, monetary and fiscal policies need to be globally balanced to guarantee a unique equilibrium: globally active monetary policies need to be coupled with globally passive fiscal policies, and switching monetary policies with switching fiscal policies. Second, this distinction characterises the nature of the solutions: a *globally AM/PF regime* is Ricardian, while a *globally switching* regime features expectation and wealth effects. Third, the strength of policy deviations across regimes is key, insofar a *globally active (or passive)* policy allows only timid deviations. Finally, our framework can rationalise the impulse responses from a Bayesian VAR on U.S. data for the recent zero lower bound period as being due to "timidity" in fiscal actions that have been unable to spur inflation.

Keywords: Monetary-fiscal policy interactions, Markov-switching, determinacy, expectation effects.
JEL classification: E58, E63.

1 Introduction

The Great Recession deteriorated the fiscal positions of advanced economies. Figure 1 shows the expansion in both the fiscal deficit and the debt/GDP ratio for the United States starting in 2008. With the beginning of the crisis, monetary authorities set their interest rate at a low level and, in contrast to the Great Moderation era, in a entirely disconnected way from inflation (see panel (c) of Figure 1 for the United States). Despite these low interest rates and expansionary fiscal stance, inflation has remained alarmingly low.
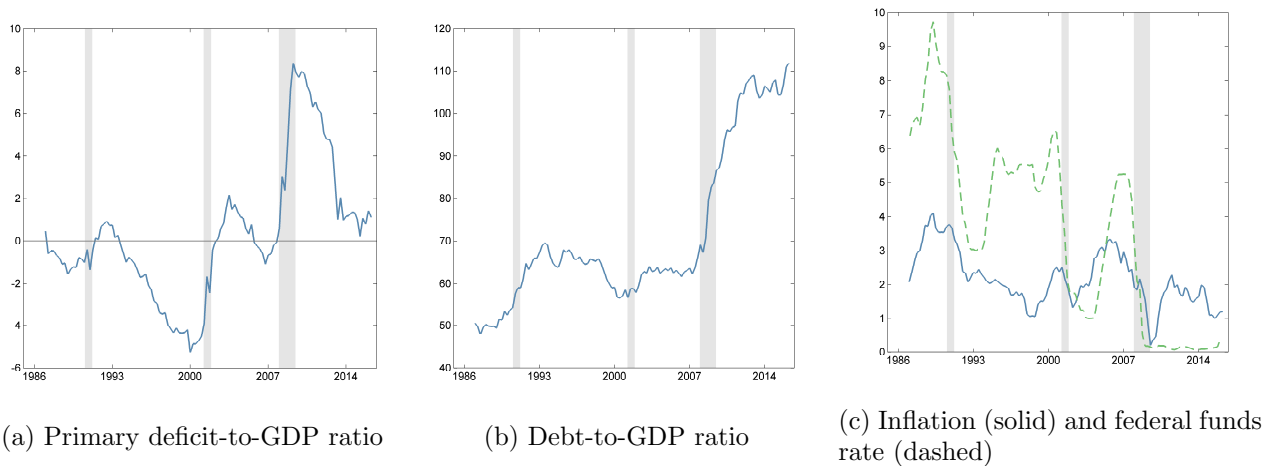

| (a) Primary deficit-to-GDP ratio | (b) Debt-to-GDP ratio | (c) Inflation (solid) and federal funds rate (dashed) |

Figure 1: The paths of the primary deficit-to-GDP ratio, debt-to-GDP ratio, inflation and policy rate in the U.S.
Notes: All variables are in percentage points. The shaded areas indicate the recession periods as computed by the NBER.

Under conventional views of price level determination, two conditions should be satisfied for monetary policy to be able to control inflation: a determinate solution and a Ricardian regime. The former is a desirable feature of monetary policy implementation because the presence of multiple stable equilibria would expose inflation (and output) to endogenous fluctuations; the latter assures the absence of wealth effects from public debt dynamics that would foster spending and create inflation.

Both conditions are satisfied in the New Keynesian framework in which an active monetary and passive fiscal regime is assumed to operate.[1] In that regime, monetary policy controls inflation while fiscal policy, satisfying the intertemporal government budget constraint, exhibits Ricardian equivalence and controls debt. In this case, the well-known Friedman (1963) proposition that inflation is always

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[1]We apply the terminology in Leeper (1991). Active monetary (AM) policy arises when the response of the nominal interest rate to inflation is more than one-to-one. Otherwise, we have passive monetary (PM) policy. Analogously, passive fiscal (PF) policy occurs when taxes respond sufficiently to debt to prevent its explosion; otherwise we have active fiscal (AF) policy.

and everywhere a monetary phenomenon holds.

The influential contribution by Davig and Leeper (2007) shows that the central bank can deviate to a passive monetary policy and still obtain determinacy once one allows for regime changes in the parameter that controls the response of the interest rate to inflation in the Taylor rule. A passive monetary policy—indeterminate in a static context—could then return determinacy if monetary policy is expected to be sufficiently aggressive in the future regime. The authors conclude that this long-run Taylor principle dramatically expands the determinacy region relative to the constant-parameter setup.[2]

Consistent with much of the literature, Davig and Leeper (2007) place fiscal policy in the background by assuming a passive fiscal policy. However, the seminal paper by Leeper (1991) analyses the determinacy properties of four different regimes in a fixed-coefficient setting, where both fiscal and monetary policy can be active or passive. He finds that coordination is essential because a unique bounded equilibrium requires one active and one passive policy. As a consequence, in addition to the standard active monetary/passive fiscal regime (AM/PF), a passive monetary and active fiscal regime (PM/AF) also yields a unique solution where fiscal policy determines inflation because the price level adjusts to keep the real value of debt consistent with the intertemporal budget constraint of the government. The so-called fiscal theory of the price level (FTPL) holds, where the absence of Ricardian equivalence produces wealth effects that, in turn, affect inflation. Therefore, as Sims (2016) claims, inflation becomes always and everywhere even a fiscal phenomenon. Moreover, expectations about future policies are crucial. Peoples' perceptions of whether government debt will bring about a higher tax burden in the future contribute to determine the inflation outcome. The ability to control inflation now depends on private sector beliefs about whether and how fiscal stress will be resolved. More generally, the expectation of future regimes should be taken into account since *"interest rate policy, tax policy and expenditure policy, both now and as they are expected to evolve in the future, jointly determine the price level"* (Sims, 2016).

Inflation dynamics are therefore generally a joint monetary-fiscal phenomenon and are crucially determined by agents' expectations about whether and how monetary and fiscal policies will change in the future. To study the role played by expected policy changes, we extend to fiscal policy the Markov switching (MS) framework that Davig and Leeper (2007) apply only to monetary policy. Our intent is to investigate, in this context, the conditions under which the central bank can control inflation. This is the case when two conditions hold: first, there should be a unique solution (i.e., determinacy),

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[2]See Barthelemy and Marx (2015) for a recent generalisation of the results in Davig and Leeper (2007).

and second, the model should imply Ricardian dynamics.[3]

Regarding the first condition, when agents' expectations incorporate the possibility of policy regime switches, the analysis of uniqueness or multiplicity of rational expectations equilibria differs from the standard literature on determinacy/indeterminacy (e.g., Lubik and Schorfheide, 2004) in which the change in regime comes as a complete surprise and is perceived to last forever. It is possible that policy combinations that lead to an explosive or indeterminate equilibrium in a fixed-regime model do not do so in a Markov-switching model because agents anticipate the probability of reverting to a different policy mix in the future. The second condition requires us to characterise the dynamics implied by the different regime switching combinations, not an easy task.

We apply the perturbation method developed by Foerster et al. (2015, henceforth FRWZ) to a simple New Keynesian model with MS in monetary and fiscal policies. Despite the complexity of the solution algorithm under MS, we are able to provide some analytical insights into the nature of the solutions regarding both determinacy and implied inflation dynamics.

The focus of our analysis will be primarily on the case in which one regime is AM/PF, which is the benchmark parametrisation in the New Keynesian literature.[4] Many works in this literature study the consequences for inflation of the transition from the Great Inflation to the Great Moderation era, the latter commonly considered an AM/PF regime.[5] Furthermore, we believe that this approach is particularly suited to studying the inflationary consequences of returning to the benchmark AM/PF regime after a period –such as the aftermath of the recent crisis- in which monetary policy has been constrained by the zero lower bound (ZLB) on nominal rates. In this case, could fiscal policy be set to reach the desired effect on inflation? Is an active fiscal policy sufficient, or does the outcome depend on the degree of activeness in fiscal policy, meaning that the "timidity trap" feared by Krugman (2014) becomes relevant?

Our approach extends Davig and Leeper's (2007) intuition and proposes a natural generalisation of Leeper (1991): we introduce the concepts of *globally active* (or *passive*) and *globally switching* policies to explain the determinacy properties of the model under MS. A timid deviation into passive (or active) monetary (or fiscal) policy in one of the regimes returns a globally active (or passive) monetary (or fiscal) policy. A substantial deviation instead results in a globally switching monetary

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[3]A growing body of literature uses models with recurring regime changes to estimate and study monetary and fiscal policy interactions. We mention several contributions without the aim of being exhaustive: Davig and Leeper (2006, 2007); Chung et al. (2007); Davig and Leeper (2008, 2011); Bianchi (2013); Bianchi and Melosi (2013); Foerster (2013); Bianchi and Ilut (2014); Leeper et al. (2015); Leeper and Leith (2016).

[4]The results, however, can be easily extended to any other regime combination.

[5]Note that some authors consider the Great Inflation an indeterminate regime, e.g., Bhattarai et al. (2012), while others consider it a PM/AF regime, e.g., Bianchi (2012).

(or fiscal) policy.

Our first result is that monetary and fiscal authorities, to yield determinacy, should coordinate not only on whether to be active or passive but even on the extent of activeness or passiveness. Monetary and fiscal policies need to be globally balanced to guarantee the existence of a unique equilibrium: a globally active monetary policy needs to be coupled with a globally passive fiscal policy and globally switching monetary policies with globally switching fiscal policies.

The second main result regards the nature of the solutions. Our taxonomy delivers a direct link between the concept of balanced policies and the presence of wealth effects or Ricardian dynamics. Usually, one can refer to the AM/PF regime as Ricardian and to the PM/AF regime as non-Ricardian only when agents are assumed to be unaware of regime changes. In a model with recurrent regime changes, as Bianchi and Melosi (2013) note, the policy mix is insufficient to establish whether a regime is Ricardian. However, in our setup, a globally AM/PF regime, which admits only timid deviations into PM/AF in one of the two regimes, implies Ricardian dynamics with neither expectation nor wealth effects. A *globally AM/PF regime* is therefore definitively Ricardian. Thus, our analysis establishes whether a regime is Ricardian in a model in which agents are aware of recurrent regime changes. Conversely, non-Ricardian dynamics prevail in a *globally switching regime*, that is, where the regime changes are sufficiently large.[6]

Third, our global AM/PF regime is consistent with the case advanced by Krugman (2014) of a "timidity trap". Consider an unbacked fiscal expansion under an PM/AF regime, engineered to escape a liquidity trap. If the policy action is too timid, that is, if that PM/AF policy deviates only timidly from the previous AM/PF regime, it would not bring about the wealth effects needed to reflate the economy. To have the desired effects, there should be a clear departure from the previous regime, hence a globally switching regime. Our theoretical framework suggests that at the ZLB and in the presence of a passive fiscal policy, now and in the future, or of a timid or short-lasting active fiscal policy, there would be multiple equilibria, one of which implies Ricardian dynamics. A BVAR on United States data for the recent ZLB period exhibits impulse response functions where a deficit shock is unable to spur inflation. Our theoretical framework can rationalise the impulse response functions as due to "timidity" in the fiscal action and agents coordinating on the Ricardian solution. A more aggressive fiscal policy would eliminate such equilibrium, leading to a unique *globally switching* regime

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[6]Previous work showed that any switching that entailed periodic excursion to a non-Ricardian regime generates non-Ricardian dynamics in every regime (Davig and Leeper (2006) and Chung et al. (2007)). In our setup and given our methodology, this is not always true: the fiscal theory of the price level may not be at work even if agents assign a positive probability to moving towards a regime with active fiscal policy, if deviations in policies are sufficiently timid.

in which agents perceive an expansionary fiscal policy as unbacked by higher taxes, and is thus able to reflate the economy and spur output. Moreover, if we maintain this multiple equilibria scenario as the relevant one to explain the recent U.S. experience, then the subdued inflation dynamics could abruptly revert to an inflation upswing due to the other non-Ricardian admissible rational expectations solutions, if expectations were to suddenly switch towards future unbacked fiscal policy expansions due, for example, to a newly elected government.

In a MS context, moreover, the expected duration of policies matters, in addition to their aggressiveness. Hence, given the ZLB, a determinate *globally switching* policy regime is more likely to arise the more active fiscal policy is and the longer it lasts. If agents expect the ZLB to last for a long period of time and to be accompanied by an active fiscal policy, the resulting *globally switching* regime yields expectation effects: the fiscal theory of the price level would apply, and an unbacked fiscal expansion would spur output, increase inflation and lower real debt.

Our analysis has implications for monetary policy: for both the timing of any exit strategy and for forward guidance. Two important points stem from these results. The first is the shortcoming of not considering inflation as a joint monetary-fiscal phenomenon, as is the case in most analysis of the conventional New Keynesian model, which disregard active fiscal policies. The other is the importance of "forward guidance", on both the monetary and fiscal side. Even if agents expect to return in the future to the virtuous AM/PF regime, the relevant consideration is not the policy prevailing at present but, rather, the one expected in the future. As Sims (2016) notes: *"big current deficits will not work without a change of perceptions of future fiscal policy from passivity"*. To obtain a unique equilibrium in which fiscal expansion fosters inflation and output, agents must be convinced of a long-lasting deviation from the virtuous regime both through the promise, on the monetary side, of a long period of zero interest rates and, on the fiscal side, of a long period without tax increases or spending cuts.

The paper is structured as follows. Section 2 presents the model and methodology. Section 3 analyses determinacy areas when one of the two regimes is AM/PF, explains what we mean by *globally balanced* and *globally switching* regimes, and describes the implications for policy coordination. Section 4 focuses on the dynamics of the model and on the expectation effects of regime shifts. Section 5 shows how our results can be applied to current economic issues such as ZLB policy. Section 6 concludes.

2 Model and methodology

2.1 The model

We consider a basic New Keynesian model with monetary and fiscal policy rules, as in Bhattarai et al. (2014). The model is well-known; thus a more complete description is offered in the Appendix. In non-linear form, the equations of the model are the following:

$$1 = \beta \mathbb{E}_t \left(\frac{Y_t - G}{Y_{t+1} - G} \frac{R_t}{\Pi_{t+1}} \right), \tag{1}$$

$$\phi_t \left(1 - \alpha \Pi_t^{\theta-1} \right)^{\frac{1}{1-\theta}} = \frac{\mu \theta \left(1 - \alpha \right)^{\frac{1}{1-\theta}}}{\theta - 1} Y_t + \alpha \beta \mathbb{E}_t \left[\phi_{t+1} \Pi_{t+1}^{\theta} \left(1 - \alpha \Pi_{t+1}^{\theta-1} \right)^{\frac{1}{1-\theta}} \right], \tag{2}$$

$$\phi_t = \frac{Y_t}{Y_t - G} + \alpha \beta \mathbb{E}_t \left[\Pi_{t+1}^{\theta-1} \phi_{t+1} \right], \tag{3}$$

$$\frac{b_t}{R_t} = \frac{b_{t-1}}{\Pi_t} + G - \tau_t, \tag{4}$$

$$\tau_t = \tau \left(\frac{b_{t-1}}{b} \right)^{\gamma_{\tau,t}} e^{u_{\tau,t}}, \tag{5}$$

$$R_t = R \left(\Pi_t \right)^{\gamma_{\pi,t}} e^{u_{m,t}}. \tag{6}$$

Equation (1) is a standard Euler equation for consumption, where Y_t is output, R_t the nominal interest rate, Π_t the gross inflation rate and G government spending, which is assumed to be exogenous and constant. Equations (2) and (3) describe the evolution of inflation in the non-linear model. ϕ_t is an auxiliary variable (equal to the present discounted value of expected future marginal revenues) that allows us to write the model recursively. Equation (4) is the government's flow budget constraint, where $b_t = B_t/P_t$ is real government debt. We follow Leeper (1991) in using lump-sum taxes (τ_t), which are set according to the fiscal rule (5): taxes react to the deviation of lagged real debt from its steady-state level (b) according to the parameter $\gamma_{\tau,t}$. Equation (6) describes monetary policy. It is a simple Taylor rule whereby the central bank reacts to current inflation according to the parameter $\gamma_{\pi,t}$. A variable without the time index (i.e., τ, b and R) indicates the value at the steady state. β is the intertemporal discount factor; θ is the Dixit-Stiglitz elasticity of substitution between goods; and α is the Calvo probability that a firm is unable to optimise its price.

The key parameters of our analysis are $\gamma_{\pi,t}$ and $\gamma_{\tau,t}$, which describe the time-varying stance of monetary and fiscal policy, respectively. We assume that these parameters follow an underlying two-state Markov process and are equal to $(\gamma_{\pi,i}, \gamma_{\tau,i})$ when the economy is in regime i, for $i = 1, 2$. The transition probabilities of going from regime i to regime j are denoted by p_{ij}. Thus, p_{ii} is the

probability of remaining in regime i, and $p_{ij} = 1 - p_{ii}$.

2.2 Solution method

As our model includes fiscal policy, we need to account for the dynamics of public debt, which is a state variable. We thus employ the perturbation method developed by FRWZ, whose logic is analogous to an undetermined coefficient method applied to a MS context. It allows us to solve for all the minimal state variable (MSV) solutions of a Markov-switching model in the presence of predetermined variables.[7] Following FRWZ, our model can be written as

$$\mathbb{E}_t \boldsymbol{f}\left(\boldsymbol{y}_{t+1}, \boldsymbol{y}_t, b_t, b_{t-1}, \boldsymbol{\varepsilon}_{t+1}, \boldsymbol{\varepsilon}_t, \boldsymbol{\theta}_{t+1}, \boldsymbol{\theta}_t\right) = \boldsymbol{0}, \tag{7}$$

where b_t is the only predetermined variable, while the remaining non-predetermined variables are stacked in vector $\boldsymbol{y}_t' \equiv [Y_t, \Pi_t, \phi_t]$. The exogenous shocks appear in vector $\boldsymbol{\varepsilon}_t' \equiv [u_{m,t}, u_{\tau,t}]$, and $\boldsymbol{\theta}_t' \equiv [\gamma_{\pi,t}, \gamma_{\tau,t}]$ is the vector of Markov-switching parameters. The first-order Taylor expansions of the recursive solutions are

$$b_t \approx b + h_i(b_{t-1} - b) + \boldsymbol{h}_{i,\varepsilon} \boldsymbol{\varepsilon}_t + h_{i,\chi} \chi, \tag{8}$$

$$\boldsymbol{y}_t \approx \boldsymbol{y} + \boldsymbol{g}_{i,b}(b_{t-1} - b) + \boldsymbol{g}_{i,\varepsilon} \boldsymbol{\varepsilon}_t + \boldsymbol{g}_{i,\chi} \chi, \tag{9}$$

for $i = 1, 2$, where χ is the perturbation parameter. Note that the slope coefficients of the solutions are regime-dependent, while the steady state is not. The coefficients h_1 and h_2 govern the stability properties of the solution and are therefore the main focus in the analysis of determinacy. FRWZ show that h_i and $\boldsymbol{g}_{i,b}$ can be jointly found after solving a system of quadratic equations. As this system cannot be solved using traditional approaches such as the generalised Schur decomposition, we follow FRWZ and adopt the Groebner basis algorithm to find all existing solutions.

Once all solutions belonging to the class of equilibria defined above have been found, a stability criterion needs to be imposed to select the stable ones. We use the concept of *mean square stability* (MSS) proposed by Costa et al. (2005) and Farmer et al. (2009).[8] The MSS condition constrains

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[7] Hence, by using this method, we consider only MSV solutions. While some other non-MSV solutions may still exist, the class of MSV solutions is usually that employed in the estimation of DSGE models. At the time of this writing, the analysis of rational expectation solutions in a Markov-switching context is a very active research area. In addition to FRWZ, see, among others, Farmer et al. (2009, 2011), Blake and Zampolli (2011), Cho (2015), Maih (2015), Barthelemy and Marx (2015).

[8] Davig and Leeper (2007) employs a different concept of stability, boundedness, which requires bounded paths and thus rules out temporarily explosive paths in one of the two regimes. See Farmer et al. (2009) and Barthelemy and Marx (2015) for a discussion in the context of Markov-switching DSGE models.

the values of the autoregressive roots in the state variable policy function in the two regimes. In the Appendix, we show that the exact conditions for MSS are

$$(p_{11} + p_{22} - 1) h_1^2 h_2^2 < 1, \tag{10}$$

$$p_{11} h_1^2 \left(1 - h_2^2\right) + p_{22} h_2^2 \left(1 - h_1^2\right) + h_1^2 h_2^2 < 1, \tag{11}$$

for $p_{11} + p_{22} > 1$. Therefore, any given parameter configuration could lead to: (i) determinacy, when a unique stable MSV solution exists; (ii) indeterminacy, when multiple stable MSV solutions exist; or (iii) explosiveness, when no stable MSV solutions exist. In what follows, we seek to explore the parameter space to identify the regions corresponding to these three cases.[9]

2.3 Determinacy under fixed coefficients: Leeper (1991)

Assume for the moment that both $\gamma_{\pi,t}$ and $\gamma_{\tau,t}$ are constant over time and not subject to regime changes, as in Leeper (1991). The log-linearised model is a trivariate dynamic system in the two jump variables $\hat{Y}_t$ and $\hat{\Pi}_t$ and the predetermined variable $\hat{b}_t$

$$\frac{1}{\bar{c}}\hat{Y}_t = \frac{1}{\bar{c}}\mathbb{E}_t\hat{Y}_{t+1} - \left(\hat{R}_t - \mathbb{E}_t\hat{\Pi}_{t+1}\right), \tag{12}$$

$$\hat{\Pi}_t = \frac{\lambda}{\bar{c}}\hat{Y}_t + \beta\mathbb{E}_t\hat{\Pi}_{t+1}, \tag{13}$$

$$\hat{R}_t = \gamma_\pi \hat{\Pi}_t + u_{m,t}, \tag{14}$$

$$\hat{b}_t = \frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma_\tau\right)\hat{b}_{t-1} - \frac{1}{\beta}\hat{\Pi}_t + \hat{R}_t - \frac{1}{\beta}\frac{\tau}{b}u_{\tau,t}, \tag{15}$$

where $\bar{c}$ is the steady-state consumption-to-GDP ratio, $\lambda \equiv (1-\alpha)(1-\alpha\beta)/\alpha$ determines the slope of the Phillips curve, and hatted variables indicate log-deviations from steady-state values. It is useful here to recall the necessary and sufficient conditions for determinacy of the rational expectations equilibrium (REE) in a fixed-coefficient model. Using Leeper's (1991) well-known taxonomy, fiscal policy is said to be *passive* if the fiscal rule guarantees debt stabilisation in (15), that is, if

$$\left|\frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma_\tau\right)\right| < 1. \tag{16}$$

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[9]Note that the term indeterminacy is used here in a different way from that used in the sunspot literature. Since we only consider MSV solutions, we do not consider sunspots in our model. Indeterminacy here means that there is more than one (generally a discrete number of) stable, and thus admissible, MSV solutions.

In the case of passive fiscal policy, it is easy to show that the following conditions have to hold to yield determinacy:

$$\gamma_\pi > 1 \tag{17}$$

and

$$\gamma_\pi > \frac{\beta - 1}{\lambda}. \tag{18}$$

The first condition is the Taylor principle, and it implies the second, which then becomes redundant. According to Leeper's taxonomy, monetary policy is labelled *active* if it satisfies the Taylor principle; otherwise, it is labelled *passive*. Hence, the famous result in Leeper (1991) follows: when fiscal policy is passive, monetary policy needs to be active (i.e., $\gamma_\pi > 1$) to yield determinacy.

Conversely, in the case of active fiscal policy (i.e., when (16) holds with the opposite direction), monetary policy should be passive to guarantee determinacy: $\gamma_\pi < 1$. In this case, the REE is non-Ricardian, and thus, a change in lump-sum taxation has real effects, and the so-called fiscal theory of the price level holds.[10] In summary, in a fixed-coefficient model, as in Leeper (1991), the determinacy region is defined by the following conditions:

- Active monetary/passive fiscal (AM/PF):

$$\gamma_\pi > 1 \quad \text{and} \quad (1 - \beta)\frac{b}{\tau} < \gamma_\tau < (1 + \beta)\frac{b}{\tau};$$

- Passive monetary/active fiscal (PM/AF):

$$\gamma_\pi < 1 \quad \text{and either} \quad \gamma_\tau < (1 - \beta)\frac{b}{\tau} \quad \text{or} \quad \gamma_\tau > (1 + \beta)\frac{b}{\tau}.$$

The REE is indeterminate under PM/PF configurations and explosive under AM/AF configurations.

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[10]See Bhattarai et al. (2014) for a thorough analysis of the dynamics implied by such a parameter configuration.

2.4 Determinacy under regime switching

2.4.1 The general case

Applying the FRWZ method, the Appendix shows that solutions need to satisfy the following system of equations for the general case with $p_{11}, p_{22} \in (0,1)$:

$$0 = g_{\pi,1} \left[1 + \lambda \gamma_{\pi,1} - p_{11} h_1 \left(1 + \beta + \lambda \right) + p_{11}^2 \beta h_1^2 \right] + \left(1 - p_{11} \right) \left(1 - p_{22} \right) \beta h_1 h_2 g_{\pi,1}$$
$$+ \left(1 - p_{11} \right) h_1 g_{\pi,2} \left[p_{11} \beta h_1 + p_{22} \beta h_2 - \left(1 + \beta + \lambda \right) \right], \tag{19}$$

$$0 = g_{\pi,2} \left[1 + \lambda \gamma_{\pi,2} - p_{22} h_2 \left(1 + \beta + \lambda \right) + p_{22}^2 \beta h_2^2 \right] + \left(1 - p_{11} \right) \left(1 - p_{22} \right) \beta h_1 h_2 g_{\pi,2}$$
$$+ \left(1 - p_{22} \right) h_2 g_{\pi,1} \left[p_{11} \beta h_1 + p_{22} \beta h_2 - \left(1 + \beta + \lambda \right) \right], \tag{20}$$

$$g_{\pi,1} = \frac{\frac{1}{\beta} \left(1 - \frac{\tau}{b} \gamma_{\tau,1} \right) - h_1}{b \left(\frac{1}{\beta} - \gamma_{\pi,1} \right)}, \tag{21}$$

$$g_{\pi,2} = \frac{\frac{1}{\beta} \left(1 - \frac{\tau}{b} \gamma_{\tau,2} \right) - h_2}{b \left(\frac{1}{\beta} - \gamma_{\pi,2} \right)}, \tag{22}$$

where the 4 unknowns are h_1, h_2, $g_{\pi,1}$ and $g_{\pi,2}$. Debt, b_t, is the state variable of the system, h_i is the response of debt to its lag in regime i, and $g_{\pi,i}$ is the response of inflation to the lagged debt in regime i (i.e., the element of $g_{i,b}$ that corresponds to inflation). Determinacy obtains when a single ordered pair (h_1, h_2) satisfies the MSS conditions (10)-(11).

2.4.2 The absorbing case

In the subsequent analysis, we will at times refer to the case in which regime 1 is absorbing, and hence, $p_{11} = 1$. This simplification allows us to derive analytical results on determinacy and, in turn, develop intuition concerning the numerical results in the general case. We refer the interested reader to the Appendix for the derivations of the analytical results for the absorbing case. If $p_{11} = 1$, equations (19) and (21) reduce to

$$0 = \frac{\frac{1}{\beta} \left(1 - \frac{\tau}{b} \gamma_{\tau,1} \right) - h_1}{b \left(\frac{1}{\beta} - \gamma_{\pi,1} \right)} \left[1 + \lambda \gamma_{\pi,1} - h_1 \left(1 + \beta + \lambda \right) + \beta h_1^2 \right] \tag{23}$$

and the conditions for MSS, i.e., (10) and (11), simplify to $|h_1| < 1$ and $|h_2| < \frac{1}{\sqrt{p_{22}}}$. Note that the Markov-switching nature of the economy only affects the condition in the non-absorbing regime. In particular, with respect to the fixed-coefficient case, the stability condition is less binding the lower the probability of remaining in the second state is.

3 Determinacy: Ricardian and FTPL solutions

This section contains the main results of the determinacy analysis in our model. First, starting from an AM/PF regime, we seek to understand how determinacy varies as the policy mix in the other regime changes. We will first provide analytical results for the case in which regime 1 is absorbing (Section 3.1) and then numerical results for the more general case in which the probability of remaining in each regime is lower than one (Section 3.2). Section 3.3 extends the analysis of Davig and Leeper (2007) to fiscal policy, studying the conditions that this policy must satisfy under the two regimes to yield a unique REE. We will here introduce our new taxonomy that links the determinacy analysis with the dynamics of the different solutions, analysed in the subsequent Sections. An important insight is that determinacy depends on the existence of globally balanced policies, thereby demanding coordination between fiscal and monetary authorities (Section 3.4).

3.1 The absorbing AM/PF case

Assume that regime 1 is absorbing and hence that $p_{11} = 1$. If the economy is already in this regime, the conditions for determinacy are clearly the same as under fixed coefficients. Hence, if fiscal policy is passive, condition (16) must hold ($\frac{b}{\tau}(1-\beta) < \gamma_{\tau,1} < \frac{b}{\tau}(1+\beta)$),[11] and monetary policy must be active ($\gamma_{\pi,1} > 1$). Figure 2a depicts the combinations of the monetary ($\gamma_{\pi,2}$) and the fiscal ($\gamma_{\tau,2}$) coefficients for the second regime (setting $p_{22} = 0.95$) that return determinacy of the global equilibrium given an absorbing AM/PF regime 1 ($\gamma_{\pi,1} = 1.5$, $\gamma_{\tau,1} = 0.2$). Notably, there are two regions in the ($\gamma_{\pi,2}, \gamma_{\tau,2}$) space that return determinacy: an upper-right zone and a lower-left zone.

First, let us analyse the upper-right zone. In this case, there is MSS if the following conditions concerning regime 2 hold:

$$\gamma_{\tau,2} \in \left(\bar{\gamma}_{\tau,2}, \frac{b}{\tau} \left(1 + \frac{\beta}{\sqrt{p_{22}}} \right) \right), \tag{24}$$

$$\gamma_{\pi,2} > \bar{\gamma}_{\pi,2}, \tag{25}$$

where

$$\bar{\gamma}_{\tau,2} \equiv \frac{b}{\tau} \left(1 - \frac{\beta}{\sqrt{p_{22}}} \right), \quad \text{and} \quad \bar{\gamma}_{\pi,2} \equiv \sqrt{p_{22}} - \frac{\left(1 - \beta\sqrt{p_{22}} \right) \left(1 - \sqrt{p_{22}} \right)}{\lambda}$$

Determinacy clearly emerges when the second regime is AM/PF, too. However, the threshold values

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$\bar{\gamma}_{\tau,2}$ and $\bar{\gamma}_{\pi,2}$ imply that both intervals for $\gamma_{\tau,2}$ and $\gamma_{\pi,2}$ widen, relative to the fixed-coefficients result: there is determinacy even if the second regime deviates from Leeper's (1991) definition of the AM/PF mix. Careful consideration of the above conditions reveals that $\bar{\gamma}_{\tau,2}$ is negative, if $\sqrt{p_{22}} < \beta$, while $\bar{\gamma}_{\pi,2}$ is lower than one because $\sqrt{p_{22}} < 1$. In other words, to have determinacy, fiscal and monetary policy in the second regime are not constrained to always be passive and active, respectively. Rather, they can now vary "timidly" and be (to a certain extent) active and passive, respectively. This effect is more pronounced the lower p_{22} is. The timid changes for fiscal and for monetary policy are given by the following intervals (visualised by the dotted and solid arrows in Figure 2a):

$$\bar{\gamma}_{\tau,2} < \gamma_{\tau,2} < \frac{b}{\tau}(1-\beta), \tag{26}$$

$$\bar{\gamma}_{\pi,2} < \gamma_{\pi,2} < 1. \tag{27}$$

Consider now what happens in the lower-left zone. There is global MSS if for regime 2 the following holds:

$$\gamma_{\tau,2} < \bar{\gamma}_{\tau,2} \text{ and } \gamma_{\tau,2} > \frac{b}{\tau}\left(1 + \frac{\beta}{\sqrt{p_{22}}}\right), \tag{28}$$

$$\gamma_{\pi,2} < \bar{\gamma}_{\pi,2}. \tag{29}$$

In this case, to have determinacy, fiscal and monetary policy in the second regime are constrained to always be "more than" active and "more than" passive, respectively, relative to Leeper's (1991) conditions. Hence, *both* monetary policy *and* fiscal policy must deviate "substantially" from the other AM/PF regime.

The absorbing case usefully provides easy intuition about these results, linking the determinacy analysis with the nature of the solutions that characterise the different regions in Figure 2a. First, the MSS condition $|h_2| < \frac{1}{\sqrt{p_{22}}}$ permits the relaxation of Leeper's (1991) original conditions and determines the extent of admissible "timid" and "substantial" deviations.

Second, as the absorbing regime is AM/PF, given (23), we know from Leeper (1991) that the only stable solution for this regime implies $g_{\pi,1} = 0$.[12] As from the fixed-coefficient New Keynesian model, this solution for the AM/PF regime is Ricardian, because inflation dynamics do not depend on the debt level, while they would do in the FTPL case (see Bhattarai et al., 2014). Then, given (20) and (22) it is easy to show that the upper-right zone in Figure 2a admits only one stable Ricardian

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[12] As the value of $\gamma_{\pi,1}$ is greater than 1, then it does not exist a value of $h_1 < 1$, such that the square bracket in (23) is equal to zero.

solution for the second regime (i.e. $g_{\pi,2} = 0$), while the lower-left zone admits only one stable non-Ricardian/FTPL solution for the second regime (i.e., $g_{\pi,i} \neq 0$).[13] It follows that in the upper-right zone the only stable solution (h_1, h_2) for the whole MS model implies Ricardian dynamics in both regime, while in the lower-left zone it implies FTPL dynamics in the second regime.

In what follows, when *both regimes admit a stable Ricardian solution*, we name this solution for the whole MS model the *"fiscal backing solution"*. In this case, the fiscal rule guarantees debt stabilisation, and monetary policy controls inflation in both regimes. Inflation is monetary determined and the traditional monetarist (AM/PF) solution applies. On the contrary, when *both regimes admit a stable non-Ricardian solution* with $g_{\pi,i} \neq 0$, we name this solution for the whole MS model the *"fiscal unbacking solution"*.[14] This solution would imply spillover effects across regimes and FTPL dynamics, because there is no fiscal backing in the economy. Monetary policy would not be able to control inflation because the price level jumps to stabilise the debt: inflation is fiscally determined. Depending on parameter configurations, in case of multiple stable solutions, these two type of solutions could co-exist.

3.2 The general case

Figure 2b shows that the general case in which both regimes are non-absorbing ($p_{11}, p_{22} < 1$) exhibits the same qualitative results.[15] In particular, it remains true that the unique stable solution in the upper-right zone is the fiscal backing solution, and hence, the dynamics will be Ricardian in both regimes. In contrast, the unique stable solution in the lower-left zone is a fiscal unbacking solution, and hence, the dynamics of the system will be non-Ricardian in both regimes.

Note that determinacy could arise from very different policy mixes. Nothing ensures that switching between two regimes that would yield determinacy in the fixed-coefficient case consistently yields determinacy in the MS model: it depends on the choice of policy coefficients. Point B in the upper-right zone and point A on its left return determinacy and indeterminacy, respectively. This is true even if both points exhibit the same fiscal policy in both regimes and correspond to an economy that switches between an AM/PF and a PM/AF mix: two regimes that, taken in isolation, are determinate.[16] The same result obtains if one compares point B_1 in the lower-left zone and point

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[13]From (21) and (22), the Ricardian solution in each regime depends only on the fiscal coefficient $\gamma_{\tau,i}$.

[14]For the stability of such a solution, the value of $\gamma_{\pi,i}$ should be less than a certain threshold that depends on model parameters. In the general case (see below) it also depends on the parameters defining the fiscal rule, i.e., $\gamma_{\tau,i}$.

[15]As Appendix A5 shows, for the general non-absorbing case with our calibration, the threshold values for the fiscal policy coefficient are $-0.02 < \gamma_{\pi,2} < 3.93$.

[16]The coordinates of the points in Figure 2b are A: ($\gamma_{\pi,2} = 0.9$; $\gamma_{\tau,2} = 0$); B: ($\gamma_{\pi,2} = 0.97$; $\gamma_{\tau,2} = 0$); B_1: ($\gamma_{\pi,2} = 0.9$; $\gamma_{\tau,2} = -0.05$).
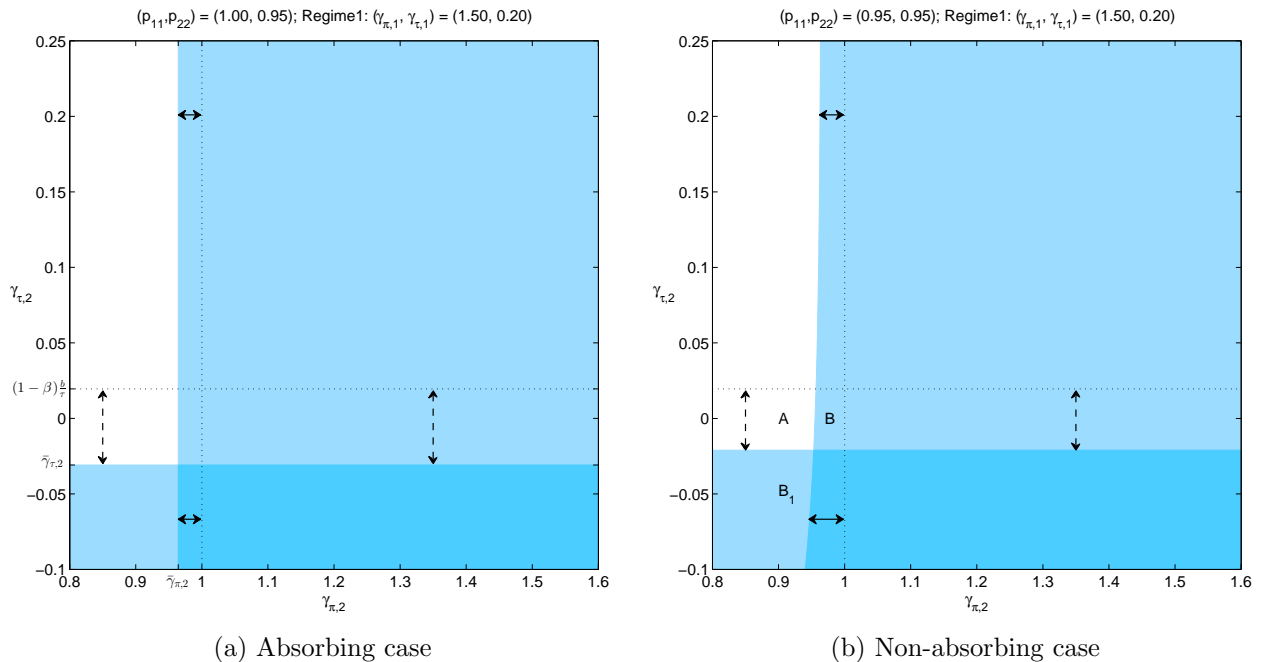
Figure 2: Determinacy regions given an AM/PF regime 1.

Notes: Light blue: unique solution; white: indeterminacy; dark blue: explosiveness. The solid lines with arrows indicate a timid deviation from active monetary policy; the dashed lines with arrows indicate a timid deviation from passive fiscal policy.

A, characterised by the same monetary policy in both regimes. To explain these apparently puzzling findings, we need to introduce some new theoretical concepts (i.e,. the fiscal frontier) that lead us to define a new taxonomy for the MS model. We will then go back to points A, B, B_1.

3.3 Globally balanced policies

Davig and Leeper (2007) indicate the conditions that a switching monetary policy needs to satisfy in the two regimes to yield a unique REE in the Markov-switching framework, assuming a passive fiscal policy. As we wish to extend the analysis to a switching fiscal policy, we search for the conditions that fiscal policy needs to satisfy to yield a unique REE, first assuming time-invariant active monetary policy (Section 3.3.1) and then switching monetary policy (Section 3.3.2).

3.3.1 The fiscal and monetary frontiers

The fiscal frontier. Assume now that monetary policy is always active ($\gamma_{\pi,1} = \gamma_{\pi,2} = 1.5$). This will be the symmetric case with respect to Davig and Leeper (2007), who implicitly consider passive fiscal policy in both regimes. Figure 3a displays what we label *the fiscal frontier* (henceforth FF) because it shows the combinations of fiscal policy rule coefficients ($\gamma_{\tau,1}$ and $\gamma_{\tau,2}$) under the two regimes that

deliver determinate equilibria for the given monetary rule coefficients.

Proposition 1. *The Fiscal Frontier and the fiscal backing solution. For any policy parameter combination, there always exists a particular solution such that in each regime: $h_i = \frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma_{\tau,i}\right) \equiv \bar{h}_i(\gamma_{\tau,i})$ and $g_{\pi,i} = 0$, for $i = 1, 2$. This solution thus depends only on $\gamma_{\tau,i}$ for $i = 1, 2$. Then:*

(i) *For this solution to be MSS, it must be true that h_1 and h_2 satisfy*

$$p_{11}\left[\bar{h}_1(\gamma_{\tau,1})\right]^2\left\{1 - \left[\bar{h}_2(\gamma_{\tau,2})\right]^2\right\} + p_{22}\left[\bar{h}_2(\gamma_{\tau,2})\right]^2\left\{1 - \left[\bar{h}_1(\gamma_{\tau,1})\right]^2\right\} + \left[\bar{h}_1(\gamma_{\tau,1})\bar{h}_2(\gamma_{\tau,2})\right]^2 < 1, \tag{30}$$

which defines the fiscal frontier (FF) in the space $(\gamma_{\tau,1}, \gamma_{\tau,2})$.

(ii) *The fiscal frontier is independent of the monetary policy coefficients;*

(iii) *If this solution satisfies (30), it yields Ricardian dynamics in both regimes because $g_{\pi,i} = 0$, so it is a fiscal backing solution.*

Proposition 1 defines the FF and establishes two important results for the general case: (i) the solution that depends only on $\gamma_{\tau,i}$ is stable only above the FF, and (ii) this solution yields Ricardian dynamics in both regimes.

As the figure shows, we have determinacy above the FF: the fiscal policy combinations then admit only timid deviations into active fiscal behaviour in one of the two regimes. Thus, we say that there is a *globally PF* policy. In contrast, the fiscal policy combinations below the FF in Figure 3a do not admit a mean square stable solution. In these cases, fiscal policy *substantially* deviates from PF (in the sense of being below the threshold values given by the MSS conditions) under at least one of the two regimes.[17]

The monetary frontier. We can map the insights of the FF onto a specular graph, call it the monetary frontier (henceforth MF), that provides the combinations of monetary policy rule coefficients under the two regimes that deliver determinate equilibria, for given fiscal rule coefficients. Figure 3b displays such a frontier for passive fiscal policy in both regimes. Note that the monetary policy combinations above the MF admit only timid deviations into passive monetary policy, and hence, we name them *globally AM regimes*. These combinations correspond to those above the FF and similarly yield Ricardian dynamics under both regimes. In contrast, the other monetary policy combinations in

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[17]Fiscal policy is no longer globally PF, but we label it as either globally AF (lower-left zone where fiscal policy is active in both regimes) or globally switching between AF and PF (when fiscal policy switches between AF and PF and it is below the FF).
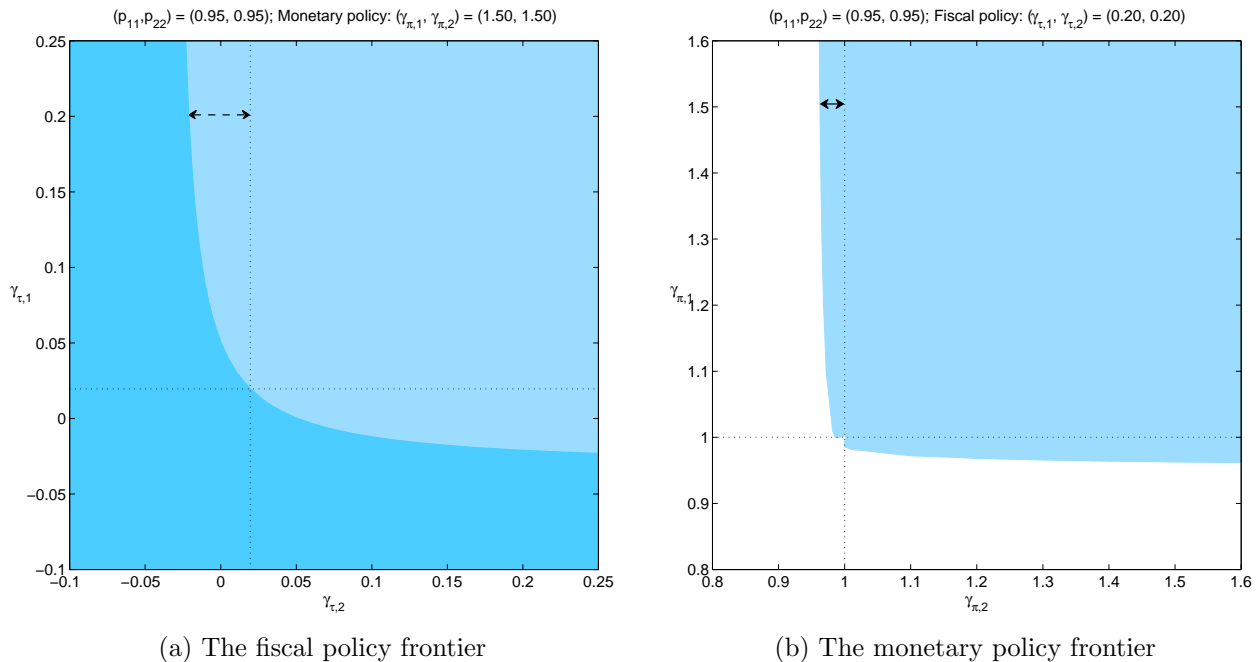
(a) The fiscal policy frontier   (b) The monetary policy frontier

Figure 3: The fiscal and monetary policy frontiers.

Notes: Light blue: unique solution; white: indeterminacy; dark blue: explosiveness.

the figure admit a solution that satisfies the MSS conditions and that depends on $\gamma_{\pi,i}$ under at least one of the regimes, yielding more than one stable solution.[18] The MF reproduces, in our framework, the main result in Davig and Leeper (2007): the long-run Taylor principle.[19]

Two important new results stem from our analysis. First, consider the fiscal stance underlying the long-run Taylor principle. It entails an always-passive fiscal policy: the central bank can stabilise the economy by following the Taylor principle or deviating from it, substantially for brief periods or timidly for longer periods, provided that it is backed by a government that implements the fiscal adjustments necessary to stabilise debt. Symmetrically, Figure 3a shows what we can analogously name the *long-run fiscal principle*, given by equation (30): also fiscal policy can deviate substantially from passive behaviour for brief periods or timidly for longer periods and still return determinacy, provided that monetary policy is always active.[20]

Second, our analysis suggests that the long-run Taylor principle holds, as long as the long-run fiscal principle does. Consider a policy mix that lies above the FF with passive fiscal policy under

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[18] In these cases, monetary policy substantially deviates from AM under at least one of the two regimes. Thus, we name monetary policy either globally PM or globally switching between AM and PM, according to the monetary policy coefficients being passive under both regimes or under only one.

[19] As in Davig and Leeper (2007), asymmetric mean duration would expand the determinacy region in favour of the more transient regime both for the MF and the FF. Unfortunately, given the complexity of the system, a meaningful analytical expression for MF is not possible.

[20] See Section 5 for an example of determinacy after a substantial variation for a brief period.

regime 1 and a timidly active fiscal policy under regime 2. The corresponding MF for this globally PF regime is very similar to that in Figure 3b.[21] In other words, the MF is largely unaffected as long as the fiscal stance is globally passive, i.e., as long as the long-run fiscal principle is satisfied. It follows that the long-run Taylor principle assures determinacy not only when fiscal policy is always passive, as Davig and Leeper (2007) maintain, but also when it deviates timidly into active fiscal territory for some time - provided that the long-run fiscal principle is satisfied.

3.3.2 The fiscal frontier when monetary policy deviates from the active regime

Assuming AM/PF under the first regime, we now analyse the case in which there is a deviation to passive monetary policy under the second regime. Again, the aim is to study how fiscal policy should behave to yield determinacy. To define the FF for this general case, we need to distinguish two cases, according to whether $\gamma_{\pi,2}$ deviates from the Taylor principle to a lesser (case 1) or to a greater (case 2) extent.

Case 1: A timid $\gamma_{2,\pi}$ deviation. Assume that monetary policy is active under the first regime and deviates only timidly under the second regime ($\gamma_{\pi,1} = 1.5$, $\gamma_{\pi,2} = 0.97$). If fiscal policy remains passive under both regimes, we are above the MF in Figure 3b and determinacy obtains as the long-run Taylor principle is satisfied. The deviation is so timid that it does not admit a stable fiscal unbacking solution, and thus, only the fiscal backing solution is allowed.[22] Consider now Figure 4a. Imagine that fiscal policy under regime 2 becomes *active*. Then, determinacy is preserved if $\gamma_{\tau,2}$ has only a timid deviation into the AF territory and thus if the fiscal policy mix is above the FF. In this case, when deviations from the current AM/PF regime are timid, we have a *globally AM regime* combined with a *globally PF regime*, which returns a *globally AM/PF regime*. Both the long-run Taylor principle and the long-run fiscal principle are satisfied.

Case 2: A substantial $\gamma_{2,\pi}$ deviation. Now assume a substantial deviation of $\gamma_{\pi,2}$ from the AM case (e.g., $\gamma_{\pi,2} = 0.9$). Determinacy generally requires fiscal policy to deviate substantially from passive behaviour, too. If fiscal policy remains largely passive under both regimes, according to the long-run Taylor principle, there would be indeterminacy (see Figure 3b). Figure 4b shows that a timid deviation from passive fiscal policy returns indeterminacy. Fiscal policy combinations need to be below the FF to yield determinacy, that is, the long-run fiscal principle should not be satisfied.

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[21]For the sake of brevity, we omit that figure, which is available from the authors upon request.

[22]The Appendix provides the analytics for the absorbing case. See Section 3.1 for an analytical condition for timid and substantial deviations in the absorbing case.
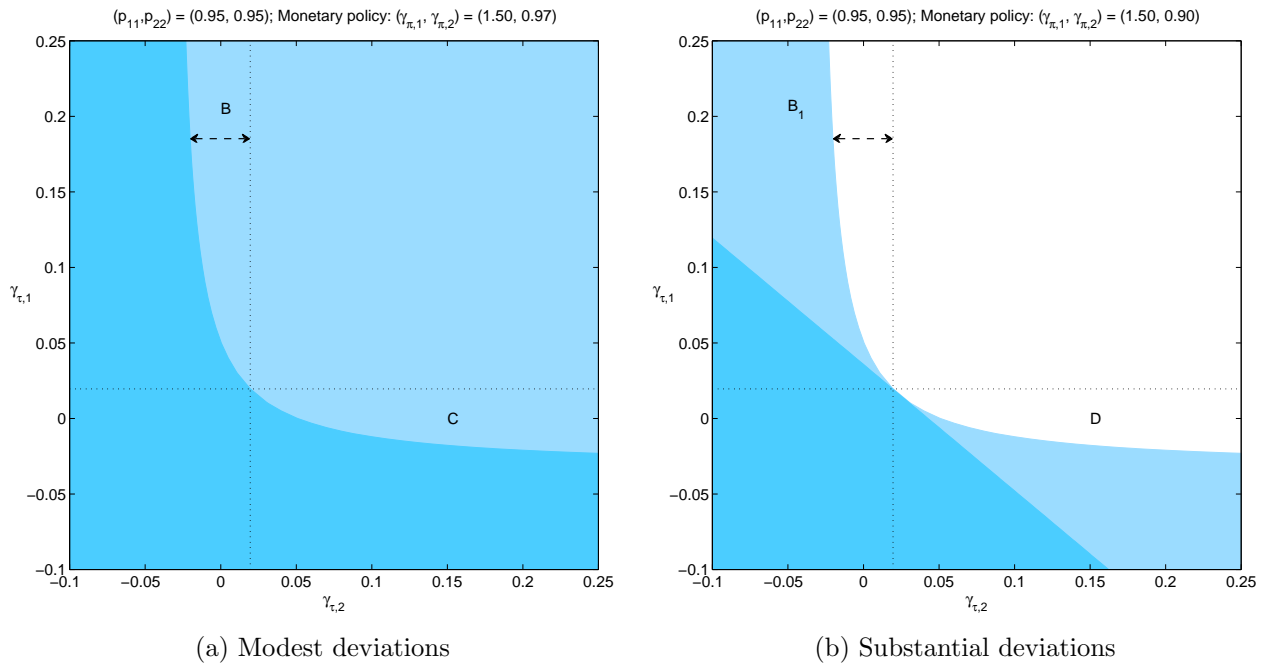
(a) Modest deviations (b) Substantial deviations

Figure 4: The fiscal policy frontier for different monetary regimes.
Notes: Light blue: unique solution; white: indeterminacy; dark blue: explosiveness.

Note that this is merely a sufficient but not a necessary condition because switching fiscal policies could also return instability. In Figure 4b, monetary policy is switching and there is only a limited set of fiscal policy combinations that yield determinacy, and they imply a switching fiscal policy. When deviations between the two regimes are substantial, we refer to the global regime as "switching". Therefore, in the presence of substantial deviations between the two regimes in both polices, we have both a *globally switching monetary regime* and a *globally switching fiscal regime* that return a *globally switching regime*.

To gain further insight into this result, note that in Figure 4b a new condition appears as a straight line in the space $(\gamma_{\tau,1}, \gamma_{\tau,2})$. This line indicates the threshold for the existence of a fiscal unbacking solution: above the line, the parameter combinations (i.e., the monetary policy coefficients $\gamma_{\pi,i}$ and the probabilities of switching) are such that at least one fiscal unbacking solution exists, while below the line no stable solution exists. We know from Proposition 1 that a fiscal backing solution always exists above the FF, while below the line it would be unstable. In Figure 4b, the threshold line is below the FF.[23] Thus, there are at least two stable solutions above the FF (one fiscal backing solution and at least one fiscal unbacking solution), meaning that there is indeterminacy. Below the line, no stable solution exists. Between the FF and the line, however, determinacy obtains, as there is only

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[23]To be clear, the line lies below the FF and is not tangent to the FF, as it might appear from the figure.

one fiscal unbacking solution and no fiscal backing solution. Thus, Ricardian equivalence does not hold and the dynamics in this globally switching regime would imply wealth effects.[24]

The general message from this analysis is that when monetary policy varies timidly, determinacy of the global equilibrium requires that fiscal policy also varies timidly. By contrast, when monetary policy varies substantially, determinacy generally requires fiscal policy to also vary substantially. In summary, monetary and fiscal policy need to be *globally balanced* to guarantee the existence of a unique stable equilibrium: globally active monetary policies need to be coupled with globally passive fiscal policies; globally switching monetary policies must be paired with globally switching fiscal policies. Moreover, globally switching policies imply wealth effects, while globally AM/PF regimes do not.

3.4 The importance of coordination

The previous analysis explains why nothing ensures that switching between two determinate regimes under fixed-coefficients yields determinacy. We are then ready to go back to the points A, B, B_1 in Figure 2b. Consider points A and B. As these two points entail the same timid deviation in regime 2 from the passive fiscal policy under regime 1, to have determinacy of the global equilibrium, monetary policy should also vary timidly. This does not happen at point A, as monetary policy is insufficiently active, while it does at point B (which indeed lies in the determinate area in Figure 4a). Point B is above both the MF and the FF, that is, it satisfies both the long-run Taylor principle and the long-run fiscal principle. Point A satisfies only the latter. Compare now points A and B_1. As these two points share the same substantial deviation in regime 2 from the active monetary policy under regime 1, they do not satisfy the long-run Taylor principle, yielding a globally switching monetary regime. To have determinacy of the MS system, fiscal policy is also required to vary substantially. This is not the case for point A, that lies above the FF, as fiscal policy is only timidly active. In contrast, this is the case for point B_1 which is above the FF (and thus lies in the determinate area in Figure 4b).

Furthermore, switching from a double active regime (AM/AF, explosive in fixed coefficients) to a double passive one (PM/PF, indeterminate in fixed coefficients) can return determinacy. Consider, for example, point C in Figure 4a and point D in Figure 4b. They share the same fiscal policy coefficients,

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[24] The Appendix contains the full analytical characterisation and an analytical expression for the threshold condition that defines the line in the absorbing case. Such an expression is not available in any meaningful way for the general case. In general, the slope and position of the line depend on the monetary policy coefficients $\gamma_{\pi,i}$ and the switching probabilities p_{ii}. For the parameter combinations in Figure 4b, the line lies below the FF. For larger switches into PM (i.e., lower $\gamma_{\pi,2}$) or different regime persistences, the line could also intersect the FF. However, our general message remains valid because there will always be a *globally switching fiscal* regime that yields a unique determinate solution. However, in this case, particular combinations of AF under the two regimes (with both regimes deviating timidly from PF) could also return determinacy. This again simply reflects that the fact that the monetary frontier generally depends on the fiscal policy mix in the two regimes.

satisfying the long-run fiscal principle above the FF: a passive fiscal policy under regime 2 and a timid deviation from it under regime 1. In both cases, monetary policy is active in the first regime and passive in the second. This would thus be a shift from a double active to a double passive regime that returns determinacy in Figure 4a but not in Figure 4b. According to our interpretation, this is because at point C the global regime is balanced (as in Figure 4a, there is also a timid change in monetary policy that satisfies the long-run Taylor principle), while at point D it is not (as in Figure 4b, there is a substantial change in monetary policy).

If policies should be balanced to obtain a determinate equilibrium, then coordination is not merely a question of being active or passive but the extent to which it is active or passive is essential, and the expectation of a stable regime in the future is not *per se* sufficient to achieve determinacy.

4 The expectation effects of regime shifts

This section considers the dynamics implied by the different solutions in greater detail to clarify their link with our definitions of global regimes derived from the determinacy analysis. Recall that we identified two sets of possible solutions: a fiscal backing one that yields Ricardian dynamics and a fiscal unbacking one that implies wealth effects. These two solutions could be considered the counterparts, in a Markov-switching context, of the two original determinate combinations in Leeper (1991). However, cross-regime spillovers are at work (see Davig and Leeper, 2007) once one allows policy regimes to change, because the economy's equilibrium properties are contaminated by both the characteristics of the other regimes and the probability of shifting towards those alternative regimes. Davig and Leeper (2008) define "expectation effects" as the difference between the equilibrium outcomes of a model with fixed coefficients and those of a model that accounts for expected changes in regimes.

Consider again points B and B_1 in Figure 2b. Both of them are characterised by a shift from the same AM/PF regime to a PM/AF regime with transition probabilities $p_{11} = p_{22} = 0.95$, and they both return determinacy. While point B entails a timid deviation of both monetary and fiscal policy from the AM/PF regime, for point $B1$, the deviation is substantial. As a consequence, point B is a globally AM/PF regime (see Figure 4a), and point B_1 is a globally switching regime (see Figure 4b).

Figure 5 shows the impulse responses to a positive fiscal shock (i.e., an unexpected reduction in lump-sum taxes) for the policy combinations implied by points B and B_1 (panels a and b, respectively).[25] The impulse response functions depend on the particular policy regime in place, and thus,

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[25] Recall that the policy combinations are: for point B, regime 1: ($\gamma_{\pi,1} = 1.5; \gamma_{\tau,1} = 0.2$) and regime 2: ($\gamma_{\pi,2} = 0.97; \gamma_{\tau,2} = 0$); for point B_1, regime 1: ($\gamma_{\pi,1} = 1.5; \gamma_{\tau,1} = 0.2$) and regime 2: ($\gamma_{\pi,2} = 0.9; \gamma_{\tau,2} = -0.05$).

each panel displays two columns of graphs corresponding to each of the two regimes. Moreover, the dashed lines in Figure 5 are the responses of the variables under a fixed-coefficients model, while the solid lines are the responses under a Markov-switching model. The difference between the solid and the dashed lines in each graph represents the expectation effects.

For the *globally AM/PF* regime (point B) we have the following:

1. The solid lines across the two regimes in Figure 5a are coincident except for the path of debt. In the PM/AF regime, the possibility of moving towards the Ricardian regime (with $p_{21} = 0.05$) makes the impulse responses behave as in the Ricardian regime (i.e., inflation does not increase).

2. Now consider the differences between the solid and dashed lines. The expectation effects are asymmetric in the two regimes. In the AM/PF regime, the expectations effects actually are absent, because there is no difference between these two lines.

Regarding the *globally switching* regime (point B_1) we have the following:

1. The solid lines no longer coincide. Now, beginning from a PM/AF regime, the possibility of switching to an AM/PF regime does not make the impulse responses behave as in this last regime (inflation now increases under both regimes)

2. The expectation effects are again asymmetric, and there are now wealth effects under the AM/PF regime.

Why do impulse responses for these two points, which entail a switch from an AM/PF to a PM/AF regime, return such strikingly different results?[26] Our determinacy analysis explains the underlying mechanisms that drive these results. We labelled a globally AM/PF regime one in which only timid deviations from AM/PF are allowed. In this case, there is only one type of admissible stable solutions: those above the FF in Figure 4a. However, as implied by Proposition 1, we know that these solutions yield Ricardian dynamics because they are fiscal backing solutions. The two regimes behave the same (except for the path of debt, as noted): there are no wealth effects under the AM/PF regime, and there are strong inflation-anchoring expectation effects under the PM/AF regime. The possibility of switching to the AM/PF regime, once in a PM/AF one, is here sufficient to stabilise inflation under both regimes. Conversely, for switching policies, Figure 4b shows that determinacy requires both policies to substantially switch across regimes, and the unique stable solutions in this case are fiscal

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[26] Both the results of asymmetric expectation effects and the coincidence of the solid lines under point B but not under point B_1 hold even when considering a monetary policy shock. These results are available from the authors upon request.
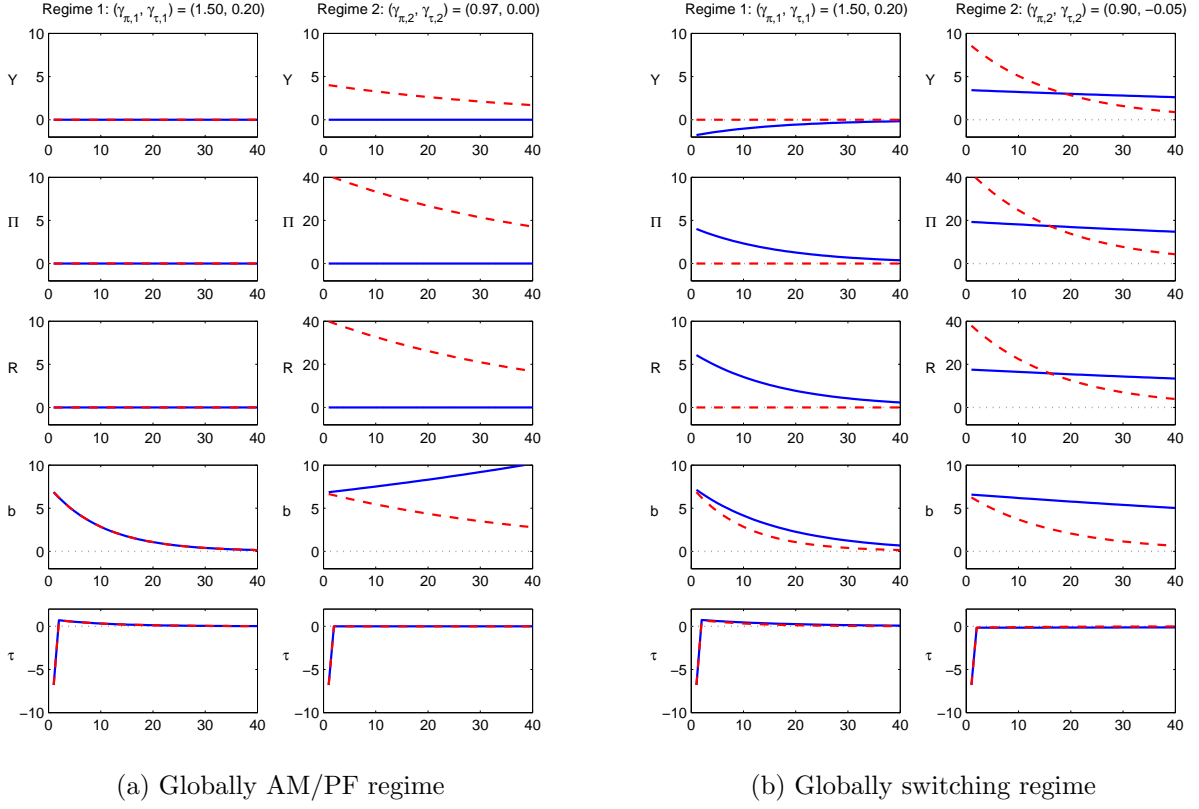
| Regime 1: $(\gamma_{\pi,1}, \gamma_{\tau,1}) = (1.50, 0.20)$ | Regime 2: $(\gamma_{\pi,2}, \gamma_{\tau,2}) = (0.97, 0.00)$ | Regime 1: $(\gamma_{\pi,1}, \gamma_{\tau,1}) = (1.50, 0.20)$ | Regime 2: $(\gamma_{\pi,2}, \gamma_{\tau,2}) = (0.90, -0.05)$ |

(a) Globally AM/PF regime (b) Globally switching regime

Figure 5: Impulse response function to a positive fiscal shock. $(p_{11} = p_{22} = 0.95)$

Notes: Blue solid lines: MS model; red dashed lines: fixed coefficients model.

unbacking solutions that admit non-Ricardian dynamics under both regimes. There are wealth effects also under the AM/PF regime: inflation increases under both regimes, although to a larger extent under the PM/AF one. The possibility of switching to the AM/PF regime is in this case not sufficient to stabilise inflation under both regimes.

Finally, do wealth effects disappear if agents are confident in a once-and-for-all switch to an AM/PF regime? Under our new taxonomy, the expectation of an absorbing AM/PF regime for the future is neither a necessary nor a sufficient condition to avoid wealth effects in the PM/AF regime. It is not necessary because we do not find wealth effects in the global AM/PF case even when $p_{11} = p_{22} = 0.95$. It is not sufficient because we detect wealth effects in the PM/AF regime in the globally switching case even when the AM/PF regime is absorbing.

5 Some theoretical and policy implications

Our framework and methodology has several implications. First, our new taxonomy provides an answer to the problem of establishing whether a regime is Ricardian in a model in which agents are aware of

recurrent regime changes. Usually, one can refer to the AM/PF regime as Ricardian and to the PM/AF regime as non-Ricardian only when agents are assumed to be unaware of regime changes.[27] In a model with recurrent regime changes, as Bianchi and Melosi (2013) note, the policy mix is insufficient to establish whether a regime is Ricardian. However, we find that neither expectation effects nor wealth effects are present under an AM/PF regime when agents expect a regime shift and the policy mix is *globally AM/PF*. Even more so, a *globally AM/PF* mix is definitively Ricardian in both regimes AM/PF and PM/AF.

Second, the global AM/PF regime is consistent with the case advanced by Krugman (2014) of a "timidity trap". Take an unbacked fiscal expansion engineered to escape a liquidity trap. If that (PM/AF) policy deviates only timidly from the previous (AM/PF) regime, that is, if the policy action is too timid, it would not bring about the wealth effects needed to reflate the economy. To have the desired effects, there should be a clear departure from the previous regime, hence a globally switching regime. This insight is particularly relevant for the recent zero lower bound episode, as we show in the next Section 5.1.

Third, our results are consistent with Liu et al. (2009) who find that the expectation effects are asymmetric, analysing regime shifts in monetary policy in a context of an always-passive fiscal policy.[28] The shift from a dovish (or a less hawkish) monetary regime to a hawkish one reduces inflation volatility to a greater extent than an inverse shift raises it: *inflation anchoring expectations* prevail.

Fourth, our methodology do not replicate the finding in Chung et al. (2007) that the fiscal theory is always at work when agents assign a positive probability of moving towards active fiscal policy (e.g., point B). More generally, the bulk of the literature[29] that estimates Markov-switching monetary-fiscal regimes and employs impulse responses to study the impact of policy shocks reports results consistent with those that we obtain under the *globally switching regime* case (e.g., point B_1): inflation increases under both regimes after an expansionary fiscal shock. This is why this literature concludes that whenever agents believe that it is possible for fiscal policy to become active, monetary and tax shocks always produce wealth effects. However, at a point to the right of B_1 in the white area in Figure 4b, monetary and fiscal policies are unbalanced, and thus there is indeterminacy. In this context, indeterminacy means the simultaneous existence of at least two stable solutions: a fiscal backing and a fiscal unbacking solution. In that region of the parameter space, it might be possible that the

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[27]See note 4 in Bianchi and Ilut (2014).

[28] This result holds even if the two regimes have the same transition probabilities.

[29]See Davig and Leeper (2008, 2011), Chung et al. (2007), Bianchi (2012) and Bianchi and Ilut (2014).

estimation selects the fiscal unbacking solution. In this case, however, it is not possible to conclude that the existence of a PM/AF regime is sufficient for wealth effects, without checking for the possible existence of another admissible solution (the fiscal backing one) for which this is not true.
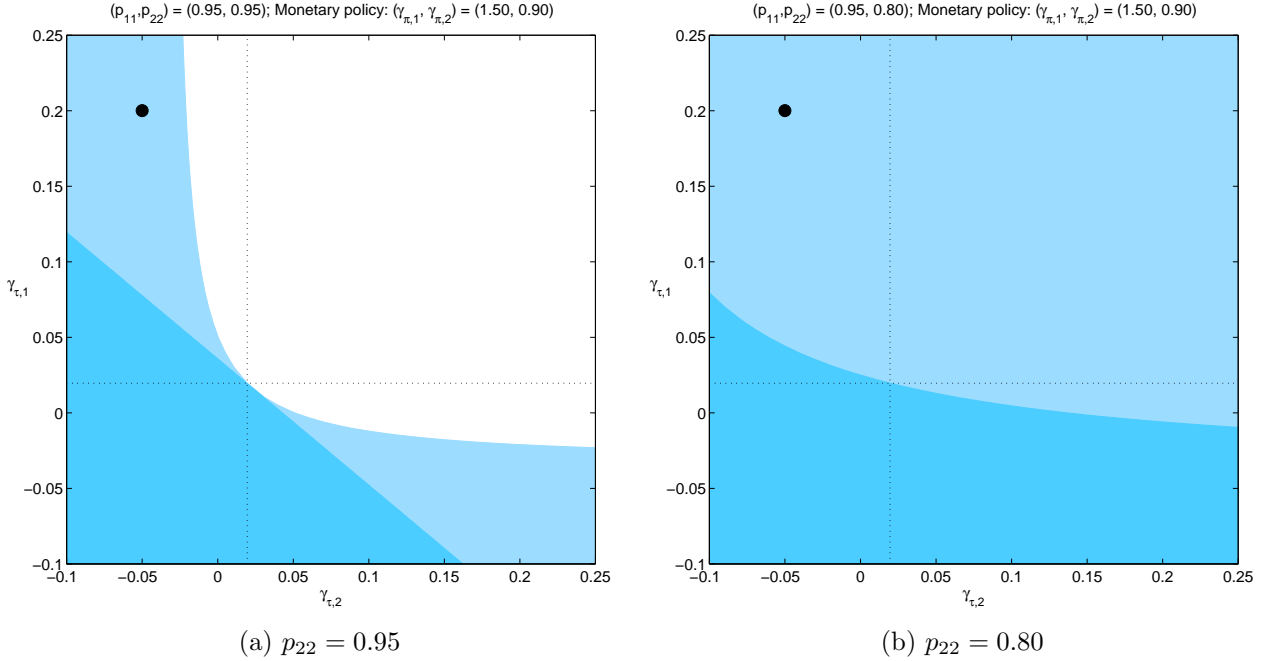

Figure 6: The monetary policy frontier for different levels of persistence of regime 2.
Notes: Light blue: unique solution; white: indeterminacy; dark blue: explosiveness.

Fifth, our paper is consistent with the results in Bianchi and Melosi (2013). Contrary to the results just discussed above and similar to us, they find that after a deficit shock under an AM/PF regime or under a short-lasting deviation towards a PM/AF regime, there are no effects on inflation (or output). Inflation and output, however, increase under a long-lasting deviation towards the same PM/AF. As both the short- and the long-lasting deviations are towards the same PM/AF, the authors consider the regime's persistence to be the key determinant to establish whether a regime is Ricardian.[30] Our taxonomy is consistent with this finding because, although we have not focused on the role of transition probabilities thus far, our definition of timid deviation depends on them (see Section 3.1). Consider a numerical example in the case of globally switching policies described by point B_1, reported as a black dot in Figure 6. Under this policy combination, if the second regime is long-lasting, say $p_{22} = 0.95$, a timid deviation is defined by $\gamma_{\tau,2} \in [-0.021; 0.02]$ and $\gamma_{\pi,2} \in [0.955; 1]$. Instead, if regime 2 is less persistent, say $p_{22} = 0.8$, a timid deviation is defined by $\gamma_{\tau,2} \in [-0.16; 0.02]$ and $\gamma_{\pi,2} \in [0.67; 1]$.[31]

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[30]See also Bianchi and Ilut (2014) on this point. They do not find any effect of a tax shock on inflation when the AM/PF regime is perceived to be fully credible (if agents expect to remain there forever) or if, being in a PM/AF regime, agents are confident in a return to the AM/PF regime.

[31] These intervals can be obtained following the procedure in Section 3.1 and Appendix A5.

Therefore, with a long-lasting deviation, B_1 would correspond to a globally switching regime (see Figure 6a), while with a less permanent deviation, we would have a globally AM/PF regime (see Figure 6b). In the first case, the impulse responses to a fiscal shock would display a hike in inflation (see Figure 7a) because the unique stable solution is the fiscal unbacking solution. In the second, there would not be inflationary effects (see Figure 7b) because the unique stable solution is the fiscal backing solution.
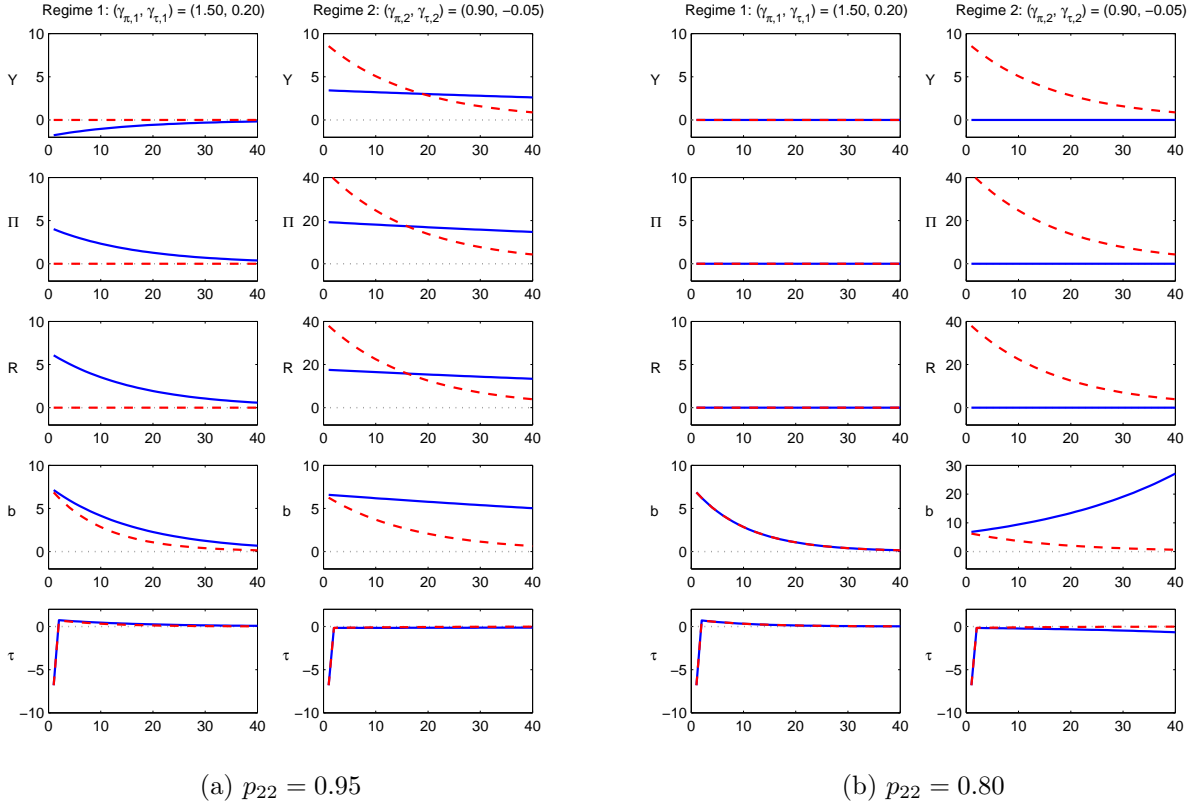

(a) $p_{22} = 0.95$ (b) $p_{22} = 0.80$

Figure 7: Impulse response function to a positive fiscal shock for different levels of persistence of regime 2.
Notes: Blue solid lines: MS model; red dashed lines: fixed coefficients model.

This analysis could have notable consequences for monetary policy: both for the timing of any exit strategy and for forward guidance. During the recent crisis, the accumulated credibility of the Federal Reserve permitted well-anchored inflation expectations, despite that the U.S. was potentially in a PM/AF regime. If we are prepared to believe that during the crisis monetary policy deviated substantially from an AM regime, then the only way to avoid a future spike in inflation is to make this deviation short-lasting. A long-lasting deviation, conversely, could either de-anchor inflation expectations and make inflation unavoidable or generate multiple solutions, depending on the behaviour of fiscal policy. Indeed, if fiscal policy remains only timidly active, it may be difficult for policy makers

to predict an inflationary surge because anything is possible.[32] If we believe this scenario to be the relevant one, then it might be that the observed subdued path of inflation is due to agents coordinating on a fiscal backing solution. However, these dynamics could abruptly revert into an inflation upswing if expectations about the behaviour of fiscal policy were to suddenly switch.

5.1   An application to the ZLB

During the Great Recession, the Federal Reserve, the European Central Bank, the Bank of England and the Bank of Japan all brought their policy rates towards the ZLB with the aim of stimulating economic activity. This is a very special case of passive monetary policy, which can be described in our model by assuming $\gamma_\pi = 0$: the central bank avoids moving interest rates as inflation changes. In this section we want to conduct two exercises relative to the ZLB regime. The first is the following. Imagine being in a crisis regime with interest rates stuck at the ZLB and agents expecting a return to business as usual — the traditional AM/PF regime. Then, how should fiscal policy be fixed in the current regime to guarantee a unique equilibrium solution? Which kind of solution would that be? How long should the ZLB regime be in place? Figure 8 depicts determinacy results for different fiscal coefficients ($\gamma_{\tau,2}$) and durations (p_{22}) of crisis regime 2, when $\gamma_{\pi,2} = 0$ and when an AM/PF regime 1 is expected with probability $p_{11} = 0.95$.
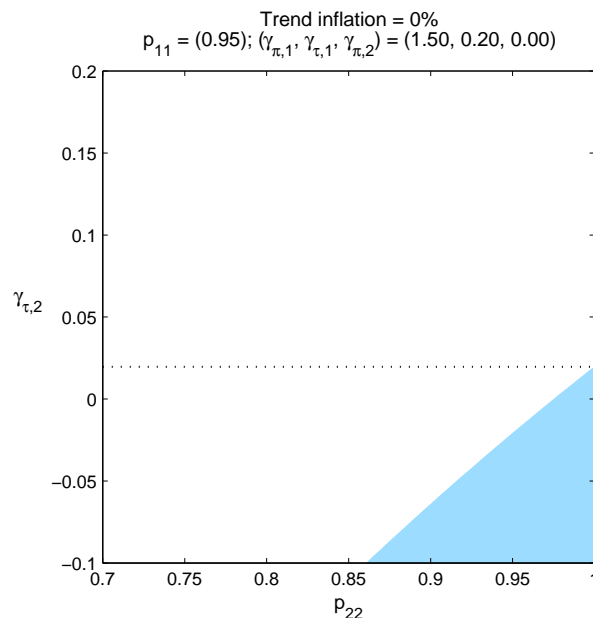

Figure 8: Determinacy and ZLB.
Notes: Light blue: unique solution; white: indeterminacy.

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[32]Both the fiscal backing and the fiscal unbacking solutions are admissible: an inflationary surge would be possible in the latter case.

The first clear-cut result is that if the ZLB regime 2 is short-lasting ($p_{22} \ll 1$), then there is indeterminacy whichever fiscal policy might be adopted. Second, determinacy is unattainable when fiscal policy is passive, because $\gamma_{\tau,2}$ should be below the dotted line that represents the cutoff value for passive fiscal policy. Third, the upper bound of the determinacy area is an upward-sloping line: the more the ZLB is short-lived, the more active fiscal policy must be. To have determinacy, agents must expect the ZLB to last for a long period of time and to be accompanied by an active fiscal policy. In the event that there were such a unique stable solution, it would be the result of a globally switching policy regime (a switching monetary policy combined with a switching fiscal policy), and hence, expectation effects would kick in. As in Figure 5b, the fiscal theory of the price level would apply, and an unbacked fiscal expansion would spur output, increase inflation and lower real debt: all desirable outcomes in the current period of mild economic growth, below-target inflation and high indebtedness.

Two important points stem from these results. The first is the inadequacy of the conventional New Keynesian model that does not consider active fiscal policies. The other is the importance of "forward guidance", both on the monetary and on the fiscal side. Even if agents expect to return in the future to the virtuous AM/PF regime, it is not the policy prevailing at present that is important but, rather, the policy expected in the future. According to Sims (2016), *"big current deficits will not work without a change of perceptions of future fiscal policy from passivity"*. To obtain determinacy, agents must be convinced of a long-lasting deviation from the virtuous regime both through the promise, on the monetary side, of a long period of zero interest rates and, on the fiscal side, of a long period of no tax increases or spending cuts.

On the contrary, in the presence of a passive fiscal policy, now and in the future, or of a short-lasting deviation from it, there would be multiple equilibria under a ZLB (the white area in the figure). These consist of two stable solutions: one characterised by a fiscal backing solution with Ricardian dynamics and the other by a fiscal unbacking solution where non-Ricardian dynamics prevail. In this case, inflation would become indeterminate. This could be the outcome for those countries, currently at their ZLB, where austerity imposes constraints on fiscal policy (the Eurozone) or where fiscal policy is mainly passive (Japan).

We now perform a second exercise: to empirically investigate the policy mix in place in the U.S. during the ZLB period, through the lenses of our model. We examine the empirical impulse response functions computed using a Bayesian VAR fitted on quarterly data for the sample period 2008q4–2015q4. To assess the effect of a fiscal shock, we consider data for the primary deficit-to-GDP
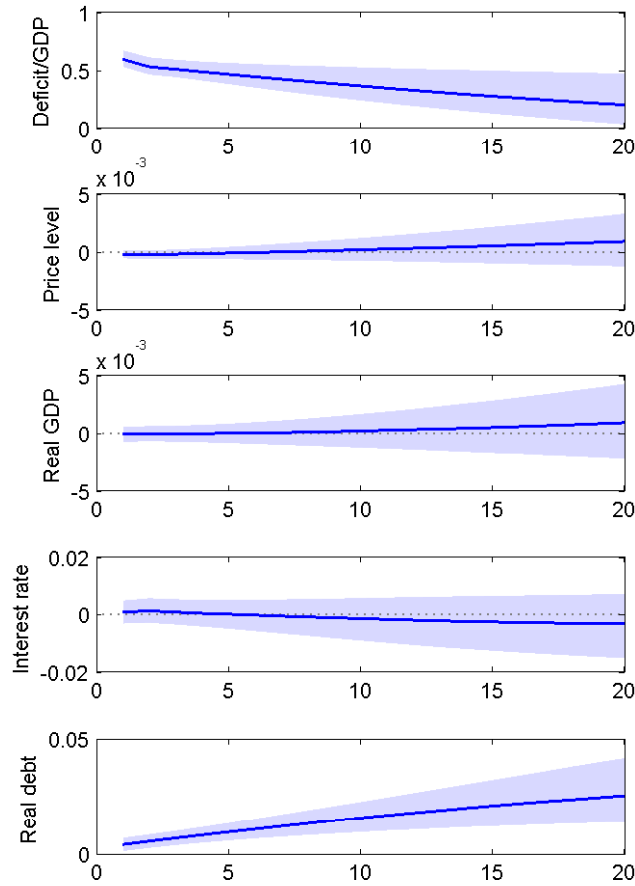
Figure 9: Impulse response functions from a BVAR

ratio, the federal funds rate, real GDP, GDP deflator and real debt.[33] During this period, monetary policy can be safely assumed to be passive, while there is more room for debate about the stance of fiscal policy. According to our model, we can gain some insights about the policy mix by comparing the pattern of VAR-based impulse responses following a positive fiscal shock to the corresponding theoretical responses shown in Figure 5.

As Figure 9 shows, the impulse response functions are consistent with the PM/AF case of a globally AM/PF regime: while output and inflation do not move, there is a run-up in real debt. Assuming

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[33]The primary deficit-to-GDP ratio was constructed from NIPA data using the same procedure illustrated in Bianchi and Melosi (2014), while real debt was computed by dividing the Market Value of U.S. Government Debt (FRED code: MVGFD027MNFRBDAL) by the GDP deflator. The remaining three variables were also taken from the FRED database. Real GDP, real debt and the GDP deflator are considered in log-levels. Our VAR includes two lags and a constant term, and we identify fiscal shocks by means of a recursive scheme in which the deficit-to-GDP ratio is ordered before all other variables. We use a Normal-Wishart prior augmented with two sets of dummy observations, i.e., both sums-of-coefficients and dummy initial observations, to accommodate the presence of unit roots and cointegration in the data (see Sims and Zha, 1998).

that agents expect to return to the same AM/PF equilibrium as in Figure 8, we find that the run-up in real debt is consistent with timid AF policies in the white area in Figure 8.[34] Thus, the U.S. could well be in a "timidity trap" and an indeterminate equilibrium. Among the stable solutions of such an indeterminate equilibrium, we find that U.S. data for the ZLB period favour agents coordinating on the fiscal backing one. This justifies the observed subdued path of inflation. Note that a more aggressive fiscal policy would eliminate one of the two equilibria and guarantee determinacy. If the goal is to reflate the economy and spur output, an unbacked fiscal stimulus would likely be effective.

6   Conclusions

This paper studies the determinacy properties of the equilibrium in a New Keynesian model when both monetary and fiscal policies may switch according to a Markov process. Nothing ensures that the switching between two regimes, which would be determinate under a fixed-coefficient framework, returns determinacy. Davig and Leeper (2007) define the *long-run Taylor principle* as the condition that the coefficients in the Markov-switching Taylor rule need to satisfy to guarantee a unique equilibrium, given a passive fiscal policy. This can be graphically visualised as a monetary frontier. Equivalently, we define a fiscal frontier that visualises the *long-run fiscal principle* as the condition that the coefficients in the Markov-switching government tax rule need to satisfy to guarantee a unique equilibrium, given an active monetary policy.

We propose a new taxonomy that generalises the seminal paper of Leeper (1991) to a Markov-switching context. We name a timid deviation from an active monetary policy into passive monetary territory that respects determinacy - i.e., that satisfies the long-run Taylor principle - a "globally active monetary policy". Symmetrically, a timid deviation from a passive fiscal policy into active fiscal territory - that satisfies the long-run fiscal principle - is named "globally passive fiscal policy". Substantial shifts in monetary and fiscal policies are termed "switching policies". Monetary and fiscal policies need to be globally balanced to guarantee a unique equilibrium. Globally active monetary policies need to be coupled with globally passive fiscal policies (i.e., a globally AM/PF regime), and switching monetary policies with switching fiscal policies (i.e., a globally switching regime).

Our new taxonomy also establishes an explicit link between the determinacy analysis and the dynamic behaviour of a Markov-switching DSGE model. If the policy mix is globally switching, then the fiscal theory of the price level is always at work. This is not true if the policy mix is globally

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[34]Impulse response functions undertaken employing aggressive fiscal policies (either active or passive) return a decreasing path for the real debt, as in Figure 5. These results are available from the authors upon request.

AM/PF. Under this latter case, there are no wealth effects because fiscal policy is globally passive. The taxonomy thus settles the problem of establishing whether a regime is Ricardian in a model in which agents are aware of recurrent regime changes. A globally AM/PF mix is definitively Ricardian because there are no wealth effects under either regime. Moreover, the expectation of a fully credible (even absorbing) AM/PF regime for the future is neither a necessary nor a sufficient condition to avoid wealth effects under a PF/AM regime.

Our framework has a number of policy implications that we discussed in Section 5. In particular, our model suggests that a "timidity trap" and an indeterminate equilibrium explain the empirical evidence for U.S. data during the crisis. An important implication for the ability of the central bank to control inflation, is that there could be an inflation upswing if the expectations about the behaviour of fiscal policy were to suddenly switch.

The analysis suggests some directions for future research. Our results are based on a very simple New Keynesian model. The advantage of such a framework is to allow us to obtain a number of analytical results, to gain insightful intuitions into what drives determinacy and the linkage among determinacy, dynamics, expectation effects and wealth effects. The natural next step in this line of research would be to determine the extent to which our new taxonomy and results help to interpret the numerical results in a more realistic, and possibly estimated, DSGE model. Finally, our definition of timid deviation has the same flavour as Leeper and Zha's (2003) definition of "modest policy interventions." However, our definition is based on the determinacy region of the parameter space and not on their modesty statistic. Empirically evaluating whether these definitions are consistent could be a fruitful avenue for future research.

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A Appendix

A.1 Parametrization

Table 1. Calibration

Parameter	Value	Description
β	0.99	Intertemporal discount factor
θ	11	Dixit-Stiglitz elasticity of substitution
α	0.75	Calvo probability not to optimise prices
N	0.333	Hours worked
$\bar{b}$	0.4	Debt-to-GDP ratio
$\bar{c}$	0.8	Consumption-to-GDP ratio

A.2 The model

In the paper we use a simple New Keynesian model with fiscal policy.

Households. The representative household maximises lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \mu N_t \right) \tag{31}$$

under a sequence of budget constraints given by

$$P_t C_t + (1 + i_t)^{-1} B_t \leq P_t w_t N_t + F_t - \tau_t + B_{t-1}. \tag{32}$$

E_0 is the expectations operator conditional on time $t = 0$ information, C_t is real consumption, N_t is labour, w_t is the level of real wages, F_t are profits, τ_t are taxes, $R_t = 1 + i_t$ is the gross return on bonds purchased at date t (i.e., B_t). Maximization yields the first order conditions

$$1 = \beta \mathbb{E}_t \left[\frac{P_t}{P_{t+1}} (1 + i_t) \frac{C_t}{C_{t+1}} \right], \tag{33}$$

$$w_t = \mu C_t. \tag{34}$$

Final good producers. In each period, a final good Y_t is produced by perfectly competitive firms, using a continuum of intermediate inputs $Y_{i,t}$ indexed by $i \in [0,1]$ and a standard CES production function $Y_t = \left(\int_0^1 Y_{i,t}^{(\theta-1)/\theta} di \right)^{\theta/(\theta-1)}$, with $\theta > 1$. Final good producers' demand schedules for intermediate good quantities are $Y_{i,t} = (P_{i,t}/P_t)^{-\theta} Y_t$, where the aggregate price index is defined as

$$P_t = \left(\int_0^1 P_{i,t}^{1-\theta} di \right)^{1/(1-\theta)}.$$

Intermediate goods producers. There exists a continuum of intermediate goods produced by firms with constant returns to scale production function: $Y_{i,t} = N_{i,t}$. Intermediate goods producers compete monopolistically and set prices according to the usual Calvo mechanism. In each period each firm has a fixed probability $1 - \alpha$ to re-optimise its nominal price $P_{i,t}^*$ in order to maximise profits. With probability α, instead, the firm keeps its nominal price unchanged. Using the stochastic discount factor $Q_{t,t+j} = \beta^j \frac{P_t C_t}{P_{t+j} C_{t+j}}$, the first order condition of the firm's problem gives the optimal relative price

$$p_{i,t}^* \equiv \frac{P_{i,t}^*}{P_t} = \frac{\theta}{\theta - 1} \frac{\mathbb{E}_t \sum_{j=0}^{\infty} (\alpha\beta)^j \frac{Y_{t+j}}{C_{t+j}} \left(\frac{P_{t+j}}{P_t} \right)^{\theta} w_{t+j}}{\mathbb{E}_t \sum_{j=0}^{\infty} (\alpha\beta)^j \frac{Y_{t+j}}{C_{t+j}} \left(\frac{P_{t+j}}{P_t} \right)^{\theta-1}}. \tag{35}$$

As in Ascari and Ropele (2009), we introduce two auxiliary variables that allow to rewrite the last expression recursively

$$\psi_t \equiv \mathbb{E}_t \sum_{j=0}^{\infty} (\alpha\beta)^j \frac{Y_{t+j}}{C_{t+j}} \left(\frac{P_{t+j}}{P_t} \right)^{\theta} w_{t+j} = \frac{Y_t}{C_t} w_t + \alpha\beta \mathbb{E}_t \left[\Pi_{t+1}^{\theta} \psi_{t+1} \right], \tag{36}$$

$$\phi_t \equiv \mathbb{E}_t \sum_{j=0}^{\infty} (\alpha\beta)^j \frac{Y_{t+j}}{C_{t+j}} \left(\frac{P_{t+j}}{P_t} \right)^{\theta-1} = \frac{Y_t}{C_t} + \alpha\beta \mathbb{E}_t \left[\Pi_{t+1}^{\theta-1} \phi_{t+1} \right], \tag{37}$$

where $\Pi_t \equiv P_t/P_{t-1}$ is the aggregate gross rate of inflation. As all re-optimizing firms face the same problem and pick the same relative price, aggregate inflation evolves according to

$$1 = \alpha \Pi_t^{\theta-1} + (1 - \alpha) \left(p_{i,t}^* \right)^{1-\theta}. \tag{38}$$

Individual firms demand labour according to the relation $N_{i,t} = (P_{i,t}/P_t)^{-\theta} Y_t$. Aggregating this expression yields $N_t = Y_t s_t$, where $N_t \equiv \int_0^1 N_{i,t} di$ and $s_t \equiv \int_0^1 (P_{i,t}/P_t)^{-\theta} di$. The variable s_t measures the dispersion of relative prices across intermediate firms. s_t is bounded below at one and it represents the resource costs (or inefficiency loss) due to relative price dispersion under the Calvo mechanism. s_t can be written recursively as

$$s_t = (1 - \alpha) \left(p_{i,t}^* \right)^{-\theta} + \alpha \left(\Pi_t \right)^{\theta} s_{t-1}. \tag{39}$$

Fiscal and monetary policies. The government budget constraint is given by

$$(1 + i_t)^{-1} b_t = \frac{b_{t-1}}{\Pi_t} + G - \tau_t, \tag{40}$$

where $b_t \equiv B_t/P_t$, G, and τ_t are the levels of government debt, expenditure, and taxes, all in real terms. Note that we assumed for simplicity that the government chooses a constant level of expenditure G. Taxes are set according to the fiscal policy rule

$$\tau_t = \tau \left(\frac{b_{t-1}}{b} \right)^{\gamma_{\tau,t}} e^{u_{\tau,t}}, \tag{41}$$

while the central bank sets the interest rate following the simple Taylor rule

$$R_t = R \Pi_t^{\gamma_{\pi,t}} e^{u_{m,t}}. \tag{42}$$

Note that the parameters $\gamma_{\tau,t}$ and $\gamma_{\pi,t}$ are indexed with time as they can take different values according to the underlying Markov-switching process.

Complete nonlinear model. After imposing market clearing (with $Y_t = C_t + G$), the dynamics of aggregate variables is described by the following set of equations (here reproduced for convenience)

$$1 = \beta \mathbb{E}_t \left[\frac{R_t}{\Pi_{t+1}} \frac{Y_t - G}{Y_{t+1} - G} \right]$$

$$w_t = \mu \left(Y_t - G \right)$$

$$1 = \alpha \Pi_t^{\theta-1} + (1 - \alpha) \left(p_{i,t}^* \right)^{1-\theta}$$

$$p_{i,t}^* = \frac{\theta}{\theta - 1} \frac{\psi_t}{\phi_t}$$

$$\psi_t = \frac{Y_t}{Y_t - G} w_t + \alpha \beta \mathbb{E}_t \left[\Pi_{t+1}^{\theta} \psi_{t+1} \right]$$

$$\phi_t = \frac{Y_t}{Y_t - G} + \alpha \beta \mathbb{E}_t \left[\Pi_{t+1}^{\theta-1} \phi_{t+1} \right]$$

$$s_t = (1 - \alpha) \left(p_{i,t}^* \right)^{-\theta} + \alpha \Pi_t^{\theta} s_{t-1}$$

$$N_t = s_t Y_t$$

$$\frac{b_t}{R_t} = \frac{b_{t-1}}{\Pi_t} + G - \tau_t$$

$$\tau_t = \tau \left(\frac{b_{t-1}}{b} \right)^{\gamma_{\tau,t}} e^{\sigma_\tau u_{\tau,t}}$$

$$R_t = R \Pi_t^{\gamma_{\pi,t}} e^{\sigma_m u_{m,t}}$$

Zero-inflation steady state. If we switch off the exogenous processes $u_{\tau,t}$ and $u_{m,t}$, we can solve for the zero-inflation steady state

$$R = \beta^{-1}$$

$$w = \mu \bar{c} Y$$

$$p_i^* = 1$$

$$\psi = \frac{1}{\bar{c}\left(1 - \alpha\beta\right)} w$$

$$\phi = \frac{1}{\bar{c}\left(1 - \alpha\beta\right)}$$

$$s = 1$$

$$N = sY = Y$$

$$\tau = \left[(1 - \bar{c}) + \bar{b}\left(1 - \beta\right)\right] Y$$

where we used the ratios $\bar{c} \equiv C/Y$ and $\bar{b} \equiv b/Y$. Further, note that the equation

$$p_{i,t}^* = \frac{\theta}{\theta - 1} \frac{\psi_t}{\phi_t}$$

implies the following parameter restriction

$$1 = \frac{\theta}{\theta - 1} \frac{\psi}{\phi} = \frac{\theta}{\theta - 1} \mu Y \bar{c}. \tag{43}$$

Log-linearised model. The nonlinear model can be log-linearised around the non-stochastic zero-inflation steady state. Standard computations lead to

$$\frac{1}{\bar{c}} \hat{Y}_t = \frac{1}{\bar{c}} \mathbb{E}_t \hat{Y}_{t+1} - \left(\hat{R}_t - \mathbb{E}_t \hat{\Pi}_{t+1} \right),$$

$$\hat{\Pi}_t = \frac{\lambda}{\bar{c}} \hat{Y}_t + \beta \mathbb{E}_t \hat{\Pi}_{t+1},$$

$$\hat{R}_t = \gamma_{\pi,t} \hat{\Pi}_t + \sigma_m u_{m,t}$$

$$\hat{b}_t = \frac{1}{\beta} \left(1 - \frac{\tau}{b} \gamma_{\tau,t} \right) \hat{b}_{t-1} - \frac{1}{\beta} \hat{\Pi}_t + \hat{R}_t - \frac{1}{\beta} \frac{\tau}{b} \sigma_\tau u_{\tau,t},$$

with $\lambda \equiv (1 - \alpha)(1 - \alpha\beta)/\alpha$. These equations correspond to equations (12)-(15) in the main text.

A.3 The FRWZ solution method

The analysis of determinacy under Markov-switching coefficients can be performed by checking the existence of a unique stable MSV solution. In order to find all the MSV solutions, we adopt the perturbation method proposed by FRWZ. The method can be applied directly on the nonlinear version of the model. However, instead of considering the complete nonlinear model outlined above, it is convenient to manipulate the equations and reduce the dimensionality of the system. The smaller system turns out to be:

$$1 = E_t \left[\frac{\Pi_t^{\gamma_{\pi,t}} e^{u_{m,t}}}{\Pi_{t+1}} \frac{Y_t - G}{Y_{t+1} - G} \right],$$

$$\left(\frac{1 - \alpha \Pi_t^{\theta-1}}{1 - \alpha} \right)^{\frac{1}{1-\theta}} \phi_t = \frac{\theta}{\theta - 1} \mu Y_t + \alpha \beta E_t \left[\Pi_{t+1}^{\theta} \left(\frac{1 - \alpha \Pi_{t+1}^{\theta-1}}{1 - \alpha} \right)^{\frac{1}{1-\theta}} \phi_{t+1} \right],$$

$$\phi_t = \frac{Y_t}{Y_t - G} + \alpha \beta E_t \left[\Pi_{t+1}^{\theta-1} \phi_{t+1} \right],$$

$$\frac{b_t}{R \Pi_t^{\gamma_{\pi,t}} e^{u_{m,t}}} = \frac{b_{t-1}}{\Pi_t} + G - \tau \left(\frac{b_{t-1}}{b} \right)^{\gamma_{\tau,t}} e^{u_{\tau,t}}.$$

Using the notation of FRWZ, this system can be rewritten as

$$\mathbb{E}_t \boldsymbol{f} \left(\boldsymbol{y}_{t+1}, \boldsymbol{y}_t, b_t, b_{t-1}, \boldsymbol{\varepsilon}_{t+1}, \boldsymbol{\varepsilon}_t, \boldsymbol{\theta}_{t+1}, \boldsymbol{\theta}_t \right) = \boldsymbol{0},$$

where b_t is the only predetermined variable and the remaining non-predetermined variables are stacked in vector $\boldsymbol{y}_t' \equiv [Y_t, \Pi_t, \phi_t]$. The exogenous shocks appear in vector $\boldsymbol{\varepsilon}_t' \equiv [u_{m,t}, u_{\tau,t}]$, and $\boldsymbol{\theta}_t' \equiv [\gamma_{\pi,t}, \gamma_{\tau,t}]$ is the vector of Markov-switching parameters. We look for recursive solutions such as

$$b_t = h_t(b_{t-1}, \boldsymbol{\varepsilon}_t, \chi)$$

$$\boldsymbol{y}_t = \boldsymbol{g}_t(b_{t-1}, \boldsymbol{\varepsilon}_t, \chi)$$

perturbed around the non-stochastic zero-inflation steady state $[b, \boldsymbol{y}']'$, where χ represents the perturbation parameter. Note that in our model the solutions are regime-dependent (h_t and $\boldsymbol{g}_t$ follow the latent Markov process too), while the steady state is not. The stability properties of each solution is governed by parameters of the first order expansion of the solutions, which reads as follows under regime i

$$b_t \approx b + h_{i,b}(b_{t-1} - b) + \boldsymbol{h}_{i,\varepsilon} \boldsymbol{\varepsilon}_t + h_{i,\chi} \chi, \tag{44}$$

$$\boldsymbol{y}_t \approx \boldsymbol{y} + \boldsymbol{g}_{i,b}(b_{t-1} - b) + \boldsymbol{g}_{i,\varepsilon}\boldsymbol{\varepsilon}_t + \boldsymbol{g}_{i,\chi}\chi, \tag{45}$$

for $i = 1, 2$. In these expressions we used a matrix notation for the partial derivatives: for example, $\boldsymbol{g}_{i,\varepsilon}$ is a (3×2) matrix whose first column is given by the partial derivative of $\boldsymbol{g}_i$ with respect to $u_{m,t}$, and so forth.

The elements in $h_{i,b}$, $\boldsymbol{h}_{i,\varepsilon}$, $h_{i,\chi}$, $\boldsymbol{g}_{i,b}$, $\boldsymbol{g}_{i,\varepsilon}$, $\boldsymbol{g}_{i,\chi}$ are unknown and can be found by exploiting the fact that the derivatives of $\mathbb{E}_t \boldsymbol{f}$ are equal to zero. Proposition 1 in FRWZ uses the chain rule to state that the coefficients in $h_{i,b}$ and $\boldsymbol{g}_{i,b}$ can be obtained by solving a system of quadratic polynomial equations that corresponds to equation (A4) in FRWZ. To derive such system, we need to compute the partial derivatives of $\boldsymbol{f}$

$$
\begin{bmatrix} \boldsymbol{f}_{ij,\boldsymbol{y}_{t+1}} & \boldsymbol{f}_{ij,\boldsymbol{y}_t} \end{bmatrix} =
\begin{bmatrix}
\frac{1}{\bar{c}Y} & 1 & 0 & -\frac{1}{\bar{c}Y} & -\gamma_{\pi,i} & 0 \\
0 & \alpha\beta\frac{\alpha\theta-\alpha-\theta}{1-\alpha}\phi & -\alpha\beta & -\frac{\theta}{\theta-1}\mu & \frac{\alpha}{1-\alpha}\phi & 1 \\
0 & \alpha\beta(1-\theta)\phi & -\alpha\beta & \frac{1-\bar{c}}{\bar{c}^2Y} & 0 & 1 \\
0 & 0 & 0 & 0 & (1-\beta\gamma_{\pi,i})b & 0
\end{bmatrix},
$$

$$
\begin{bmatrix} \boldsymbol{f}_{ij,b_t} & \boldsymbol{f}_{ij,b_{t-1}} \end{bmatrix} =
\begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
\beta & \frac{\tau}{b}\gamma_{\tau,i} - 1
\end{bmatrix}.
$$

Note that the derivatives are indexed with ij to indicate that they must be evaluated at the steady state with $\boldsymbol{\theta}_t = i$ and $\boldsymbol{\theta}_{t+1} = j$ (refer to FRWZ for further details). With these derivatives in hand, we can apply formula (A4) of FRWZ and obtain a set of 8 equations in 8 unknowns

$$
0 =
\begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{\tau}{b}\gamma_{\tau,1} - 1 \end{bmatrix}
+
\begin{bmatrix}
-\frac{1}{\bar{c}Y} & -\gamma_{\pi,1} & 0 \\
-\frac{\theta}{\theta-1}\mu & \frac{\alpha}{1-\alpha}\phi & 1 \\
\frac{1-\bar{c}}{\bar{c}^2Y} & 0 & 1 \\
0 & (1-\beta\gamma_{\pi,1})b & 0
\end{bmatrix}
\begin{bmatrix} g_{y,1} \\ g_{\pi,1} \\ g_{\phi,1} \end{bmatrix}
+
\begin{bmatrix} 0 \\ 0 \\ 0 \\ \beta \end{bmatrix} h_1
$$

$$
+ p_{11}
\begin{bmatrix}
\frac{1}{\bar{c}Y} & 1 & 0 \\
0 & \alpha\beta\frac{\alpha\theta-\alpha-\theta}{1-\alpha}\phi & -\alpha\beta \\
0 & \alpha\beta(1-\theta)\phi & -\alpha\beta \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} g_{y,1} \\ g_{\pi,1} \\ g_{\phi,1} \end{bmatrix} h_1
+ p_{12}
\begin{bmatrix}
\frac{1}{\bar{c}Y} & 1 & 0 \\
0 & \alpha\beta\frac{\alpha\theta-\alpha-\theta}{1-\alpha}\phi & -\alpha\beta \\
0 & \alpha\beta(1-\theta)\phi & -\alpha\beta \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} g_{y,2} \\ g_{\pi,2} \\ g_{\phi,2} \end{bmatrix} h_1,
$$

$$0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{\tau}{b}\gamma_{\tau,2} - 1 \end{bmatrix} + \begin{bmatrix} -\frac{1}{\bar{c}Y} & -\gamma_{\pi,2} & 0 \\ -\frac{\theta}{\theta-1}\mu & \frac{\alpha}{1-\alpha}\phi & 1 \\ \frac{1-\bar{c}}{\bar{c}^2Y} & 0 & 1 \\ 0 & (1-\beta\gamma_{\pi,2})\,b & 0 \end{bmatrix} \begin{bmatrix} g_{y,2} \\ g_{\pi,2} \\ g_{\phi,2} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \beta \end{bmatrix} h_2$$

$$+ p_{21} \begin{bmatrix} \frac{1}{\bar{c}Y} & 1 & 0 \\ 0 & \alpha\beta\frac{\alpha\theta-\alpha-\theta}{1-\alpha}\phi & -\alpha\beta \\ 0 & \alpha\beta(1-\theta)\phi & -\alpha\beta \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} g_{y,1} \\ g_{\pi,1} \\ g_{\phi,1} \end{bmatrix} h_2 + p_{22} \begin{bmatrix} \frac{1}{\bar{c}Y} & 1 & 0 \\ 0 & \alpha\beta\frac{\alpha\theta-\alpha-\theta}{1-\alpha}\phi & -\alpha\beta \\ 0 & \alpha\beta(1-\theta)\phi & -\alpha\beta \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} g_{y,2} \\ g_{\pi,2} \\ g_{\phi,2} \end{bmatrix} h_2,$$

where, with a slight abuse of notation, the h_i and $g_{x,i}$ coefficients are the elements of $h_{i,b}$ and $\boldsymbol{g}_{i,b}$ defined above. This system can be simplified by subtracting the third equation from the second, and the seventh from the sixth, to eliminate $g_{\phi,1}$ and $g_{\phi,2}$. We then arrive at

$$0 = \frac{1}{\bar{c}Y}g_{y,1} + \gamma_{\pi,1}g_{\pi,1} - h_1\left[p_{11}\left(g_{\pi,1} + \frac{1}{\bar{c}Y}g_{y,1}\right) + p_{12}\left(g_{\pi,2} + \frac{1}{\bar{c}Y}g_{y,2}\right)\right], \tag{46}$$

$$0 = g_{\pi,1} - \frac{\lambda}{\bar{c}Y}g_{y,1} - \beta h_1\left(p_{11}g_{\pi,1} + p_{12}g_{\pi,2}\right), \tag{47}$$

$$0 = \beta h_1 + b\left(1 - \beta\gamma_{\pi,1}\right)g_{\pi,1} + \frac{\tau}{b}\gamma_{\tau,1} - 1, \tag{48}$$

$$0 = \frac{1}{\bar{c}Y}g_{y,2} + \gamma_{\pi,2}g_{\pi,2} - h_2\left[p_{21}\left(g_{\pi,1} + \frac{1}{\bar{c}Y}g_{y,1}\right) + p_{22}\left(g_{\pi,2} + \frac{1}{\bar{c}Y}g_{y,2}\right)\right], \tag{49}$$

$$0 = g_{\pi,2} - \frac{\lambda}{\bar{c}Y}g_{y,2} - \beta h_2\left(p_{21}g_{\pi,1} + p_{22}g_{\pi,2}\right), \tag{50}$$

$$0 = \beta h_2 + b\left(1 - \beta\gamma_{\pi,2}\right)g_{\pi,2} + \frac{\tau}{b}\gamma_{\tau,2} - 1. \tag{51}$$

Note that the term λ appears after exploiting the restriction (43). Finally, these equations can be further combined to obtain

$$0 = g_{\pi,1}\left[1 + \lambda\gamma_{\tau,1} - p_{11}h_1(1 + \beta + \lambda) + p_{11}^2\beta h_1^2\right] + (1 - p_{11})(1 - p_{22})\beta h_1 h_2 g_{\pi,1}$$
$$+ (1 - p_{11})h_1 g_{\pi,2}\left[p_{11}\beta h_1 + p_{22}\beta h_2 - (1 + \beta + \lambda)\right],$$

$$0 = g_{\pi,2}\left[1 + \lambda\gamma_{\tau,2} - p_{22}h_2(1 + \beta + \lambda) + p_{22}^2\beta h_2^2\right] + (1 - p_{11})(1 - p_{22})\beta h_1 h_2 g_{\pi,2}$$
$$+ (1 - p_{22})h_2 g_{\pi,1}\left[p_{11}\beta h_1 + p_{22}\beta h_2 - (1 + \beta + \lambda)\right],$$

$$g_{\pi,1} = \frac{\frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma_{\tau,1}\right) - h_1}{b\left(\frac{1}{\beta} - \gamma_{\pi,1}\right)},$$

$$g_{\pi,2} = \frac{\frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma_{\tau,2}\right) - h_2}{b\left(\frac{1}{\beta} - \gamma_{\pi,2}\right)},$$

which correspond to equations (19)-(22) in the main text.

As this system cannot be solved using traditional approaches such as the generalised Schur decomposition, we follow FRWZ and adopt the Groebner basis algorithm to find all existing solutions, i.e., all the possible 8-tuples made by coefficients h_1, $g_{y,1}$, $g_{\pi,1}$, $g_{\phi,1}$, h_2, $g_{y,2}$, $g_{\pi,2}$, $g_{\phi,2}$ that satisfy the system of equations.

Note that to characterise the first order expansion of the MSV solution we still have to determine the other coefficients $\boldsymbol{h}_{i,\varepsilon}$, $h_{i,\chi}$, $\boldsymbol{g}_{i,\varepsilon}$, $\boldsymbol{g}_{i,\chi}$ that appear in equations (44) and (45). Fortunately, doing so is an easy task. Proposition 1 in FRWZ shows that one has to solve two separate systems of linear equations corresponding to their equations (A5) and (A6).

A.4   Mean square stability under regime switching

To asses the stability of the MSV solutions when some parameters are allowed to switch, FRWZ use the notion of mean square stability (MSS), which is discussed by Costa et al. (2005) and Farmer et al. (2009).

MSS requires the existence of

$$lim_{t\to\infty}\mathbb{E}_0\left(\begin{bmatrix} b_t \\ \boldsymbol{y}_t \end{bmatrix}\right), \quad \text{and} \quad lim_{t\to\infty}\mathbb{E}_0\left(\begin{bmatrix} b_t \\ \boldsymbol{y}_t \end{bmatrix}\begin{bmatrix} b_t \\ \boldsymbol{y}_t \end{bmatrix}'\right)$$

The MSS condition constrains the values of the autoregressive roots in the state variable policy function weighted by the probability of switching regimes. In our context with one state variable and two regimes, MSS formally states that one solution is stable if an only if all the eigenvalues of the matrix

$$\begin{bmatrix} p_{11} & 1-p_{22} \\ 1-p_{11} & p_{22} \end{bmatrix}\begin{bmatrix} h_1^2 & 0 \\ 0 & h_2^2 \end{bmatrix} = \begin{bmatrix} p_{11}h_1^2 & (1-p_{22})h_2^2 \\ (1-p_{11})h_1^2 & p_{22}h_2^2 \end{bmatrix}$$

are inside the unit circle. The characteristic polynomial of this matrix is

$$z^2 + a_1 z + a_0 = z^2 - \left(p_{11}h_1^2 + p_{22}h_2^2\right)z + \left(p_{11} + p_{22} - 1\right)h_1^2 h_2^2 = 0.$$

As discussed in LaSalle (1986, p. 28), both eigenvalues are inside the unit circle if and only if both the following conditions hold

$$|a_0| < 1$$

$$|a1| < 1 + a_0,$$

which in our case give

$$\left|(p_{11} + p_{22} - 1) h_1^2 h_2^2\right| < 1,$$

$$p_{11}h_1^2 + p_{22}h_2^2 < 1 + (p_{11} + p_{22} - 1) h_1^2 h_2^2.$$

If we assume $p_{11} + p_{22} \geq 1$, the two conditions can be be rewritten as

$$(p_{11} + p_{22} - 1) h_1^2 h_2^2 < 1,$$

$$p_{11}h_1^2 \left(1 - h_2^2\right) + p_{22}h_2^2 \left(1 - h_1^2\right) + h_1^2 h_2^2 < 1,$$

which correspond to equations (10) and (11) in the main text.

A.5   Figure 2b: A numerical example for the timid fiscal deviations

Consider the case with $p_{11} = p_{22} = p < 1$ in Figure 2b. If regime 1 is AM/PF so that $g_{\pi,1} = g_{y,1} = 0$, from system (46)-(51) we can derive the equations

$$0 = \underbrace{\frac{\frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma_{\tau,1}\right) - h_1}{b\left(\frac{1}{\beta} - \gamma_{\pi,1}\right)}}_{g_{\pi,1}} \left[1 + \lambda\gamma_{\pi,1} - p_{11}h_1 \left(1 + \beta + \lambda\right) + p_{11}^2\beta h_1^2\right], \tag{52}$$

$$0 = \underbrace{\frac{\frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma_{\tau,2}\right) - h_2}{b\left(\frac{1}{\beta} - \gamma_{\pi,2}\right)}}_{g_{\pi,2}} \left[1 + \lambda\gamma_{\pi,2} - p_{22}h_2 \left(1 + \beta + \lambda\right) + p_{22}^2\beta h_2^2\right]. \tag{53}$$

Call a solution stemming from $g_{\pi,i} = 0$, that therefore depends only on the fiscal coefficient $\gamma_{\tau,1}$, $\bar{h}_i(\gamma_{\tau,i})$. Take a passive fiscal policy in regime 1 with $\gamma_{\tau,1} = 0.2$ and $p = 0.95$. In this case the stable solution is $\bar{h}_1(\gamma_{\tau,1})$ that, under our calibration becomes $h_1 = \frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma_{\tau,1}\right) = 0.9068$. In order to have MSS, conditions (10) and (11) must hold. Under this case the most stringent one of the two turns out to be equation (11) that becomes $0.822p\left(1 - h_2^2\right) + 0.178ph_2^2 - 1 + 0.822h_2^2 < 0$. Then, in order to have MSS the stable solution in regime 2 must satisfy the following: $-1.0209 < h_2 < 1.0209$.[35] If the stable solution in regime 2 is again $\bar{h}_2(\gamma_{\tau,2})$ then: $h_2 = \frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma_{\tau,2}\right) \in (-1.0209, 1.0209)$. In this

---

[35]Condition (10) instead gives $-1.162 < h_2 < 1.162$.

case we get a "timid" fiscal deviation for:

$$-0.02 < \gamma_{\tau,2} < 3.93$$

Substituting $h_2 = 1.0209$ in the square bracket in equation (53) we get $\gamma_{\pi,2} = 0.955$ which is the lower bound of the correspondent "timid" monetary deviation:

$$0.955 < \gamma_{\pi,2} < 1$$

Note that these results hold for $\gamma_{\tau,1} = 0.2$; obviously, for every given $\gamma_{\tau,1}$ we could obtain different $\gamma_{\tau,2}$ and $\gamma_{\pi,2}$ coefficients.

A.6   The absorbing case

When regime 1 is absorbing ($p_{11} = 1$), MSS requires the eigenvalues of the following matrix to lie inside the unit circle:

$$\begin{bmatrix} h_1^2 & (1-p_{22})h_2^2 \\ 0 & p_{22}h_2^2 \end{bmatrix}$$

The two eigenvalues are equal to h_1^2 and $p_{22}h_2^2$, so that the conditions for MSS are

$$|h_1| < 1, \tag{54}$$

$$|h_2| < \frac{1}{\sqrt{p_{22}}}. \tag{55}$$

Moreover, by plugging $p_{11} = 1$ in equations (19) and (21) we obtain equation (23), that is

$$0 = \frac{\frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma_{\tau,1}\right) - h_1}{b\left(\frac{1}{\beta} - \gamma_{\pi,1}\right)} \left[1 + \lambda\gamma_{\pi,1} - h_1(1+\beta+\lambda) + \beta h_1^2\right].$$

This equation has three solutions for h_1: one $\bar{h}_1(\gamma_{\tau,1})$ that depends on the fiscal coefficient $\gamma_{\tau,1}$ (first term), the other two $\bar{h}_1(\gamma_{\pi,1})$ that depend on the monetary coefficient $\gamma_{\pi,1}$, (square brackets).

When regime 1 is AM/PF then $g_{\pi,1} = g_{y,1} = 0$, since debt has no impact on inflation and output. Using these restrictions, equations (20) and (22) yield equation (53) for the non-absorbing regime.

A.6.1 Figure 2a

Consider an AM/PF regime 1. The stability condition for the absorbing state (54) is the same as under fixed coefficients. As fiscal policy is passive, then

$$\left| \frac{1}{\beta} \left(1 - \frac{\tau}{b} \gamma_{\tau,1} \right) \right| < 1,$$

that is $(1 - \beta)\frac{b}{\tau} < \gamma_{\tau,1} < (1 + \beta)\frac{b}{\tau}$. Employing our calibration, we have $\gamma_{\tau,1} \in (0.019, 3.892)$. The condition for not having another stable solution is that the two solutions in the square bracket of (23) should be outside the unit circle, that is, $\gamma_{\pi,1} > 1$: monetary policy needs to be active. We now analyse the MSS condition (55) for regime 2, given that regime 1 is AM/PF. To do so, we have to solve the third order equation (53) for h_2, and obtain one solution $\bar{h}_2(\gamma_{\tau,2})$ and two solutions $\bar{h}_2(\gamma_{\pi,2})$. Let us distinguish two cases according to the stability of the $\bar{h}_2(\gamma_{\tau,2})$ solution in regime 2.

A stable $\bar{h}_2(\gamma_{\tau,2})$ solution. In this case the solution must satisfy: $\left| \frac{1}{\beta} \left(1 - \frac{\tau}{b} \gamma_{\tau,2} \right) \right| < \frac{1}{\sqrt{p_{22}}}$, that gives equation (24)

$$\frac{b}{\tau} \left(1 - \frac{\beta}{\sqrt{p_{22}}} \right) < \gamma_{\tau,2} < \frac{b}{\tau} \left(1 + \frac{\beta}{\sqrt{p_{22}}} \right)$$

which, employing our calibration, returns: $\gamma_{\tau,2} \in (-0.032, 3.952)$.

To have a unique stable $\bar{h}_2(\gamma_{\tau,2})$ solution the other (two $\bar{h}_2(\gamma_{\pi,2})$) solutions must be both unstable, which translates into equation (25)

$$\gamma_{\pi,2} > \sqrt{p_{22}} - \frac{\left(1 - \beta\sqrt{p_{22}} \right) \left(1 - \sqrt{p_{22}} \right)}{\lambda},$$

that is, $\gamma_{\pi,2} > 0.964$. This first case describes the upper-right zone in Figure 2a.

An unstable $\bar{h}_2(\gamma_{\tau,2})$ solution. Under this case $\left| \frac{1}{\beta} \left(1 - \frac{\tau}{b} \gamma_{\tau,2} \right) \right| > \frac{1}{\sqrt{p_{22}}}$ which corresponds to equation (28). Under our calibration, we have $\gamma_{\tau,2} < -0.032, \gamma_{\tau,2} > 3.952$.

In order to have only one stable solution, the two $\bar{h}_2(\gamma_{\pi,2})$ solutions of (53) must be one inside and the other outside the unit circle, which yields equation (29)

$$\gamma_{\pi,2} < \sqrt{p_{22}} - \frac{\left(1 - \beta\sqrt{p_{22}} \right) \left(1 - \sqrt{p_{22}} \right)}{\lambda}.$$

Employing our calibration, we get $\gamma_{\pi,2} < 0.964$. Again, monetary policy can be passive, and the more so, the lower p_{22}. This second case describes the lower-left zone in Figure 2a.

As the absorbing regime is AM/PF, we know that the only stable solution for this regime corresponds the fiscal backing one while, as the value of $\gamma_{\pi,1}$ is greater than 1, a stable $\bar{h}_1(\gamma_{\pi,1})$ solution does not exist. To have determinacy, there should only be one corresponding stable solution, h_2. The threshold values $\bar{\gamma}_{\pi,2}$ and $\bar{\gamma}_{\tau,2}$ for the timid changes in monetary and fiscal policies define the conditions for the existence of a stable solution in the second regime. In particular, starting from an AM/PF absorbing regime, determinacy in regime 2 admits either only timid deviations from AM/PF (upper-right zone, $\gamma_{\pi,2} > \bar{\gamma}_{\pi,2}$; $\gamma_{\tau,2} > \bar{\gamma}_{\tau,2}$) or large deviations in both monetary and fiscal policy, such that we are definitely in a PM/AF regime (lower-left zone, $\gamma_{\pi,2} < \bar{\gamma}_{\pi,2}$; $\gamma_{\tau,2} < \bar{\gamma}_{\tau,2}$). More precisely: (i) if fiscal policy is not too active (if $\gamma_{\tau,2} > \bar{\gamma}_{\tau,2}$), there exists a fiscal backing solution, where dynamics are Ricardian in both regimes; (ii) if monetary policy is sufficiently passive (if $\gamma_{\pi,2} < \bar{\gamma}_{\pi,2}$), there exists a fiscal unbacking solution, with wealth effect in the second regime.[36] Hence, to have only one solution, either (i) or (ii) should be satisfied. When both are satisfied, we have two solutions as in the white region; when neither is satisfied, we have no stable solutions as in the dark blue region.

A.6.2 Globally balanced policies: analytical results for an absorbing regime 1

The fiscal frontier Consider the absorbing case and assume that monetary policy is always active ($\gamma_{\pi,1} = \gamma_{\pi,2} = 1.5$). The equation to be solved for h_1 is (23). If $\gamma_{\pi,1} > 1$, the roots of the second order equation in the square brackets are out of the unit circle. If in the first regime there is a PF policy, the equation for the second regime reduces to equation (53). Given that we assumed an active monetary policy even in regime 2, we have global determinacy whenever fiscal policy is passive (or timidly active): $\frac{b}{\tau}\left(1 - \frac{\beta}{\sqrt{p_{22}}}\right) < \gamma_{\tau,2} < \frac{b}{\tau}\left(1 + \frac{\beta}{\sqrt{p_{22}}}\right)$, which corresponds to equation (25) in the text. So we have determinacy for the absorbing regime 1 when $\gamma_{\tau,1} > \frac{b}{\tau}(1 - \beta)$ and for the non-absorbing regime 2 when $\gamma_{\tau,2} > \bar{\gamma}_{\tau,2}$. Figure A1 displays what we label *the fiscal frontier*: the fiscal policy combinations above this frontier admit only timid deviations into active fiscal behaviour in the second regime. The other fiscal policy combinations in Figure A1, by contrast, do not admit a mean square stable solution $\bar{h}_i(\gamma_{\tau,i})$. In these cases, if the $\bar{h}_1(\gamma_{\tau,1})$ solution in the first regime is outside the unit circle ($h_1 = \frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma_{\tau,1}\right) > 1$), that is if there is an AF policy, then all solutions are explosive, independently from what happens in the second regime.

The monetary frontier Figure A2 displays the monetary frontier in the absorbing case. Take an always passive fiscal policy, we know that a fiscal backing solution always exists under the two regimes,

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[36]Obviously, there are no wealth effects in the first because it is absorbing, so that it does not admit spillovers.
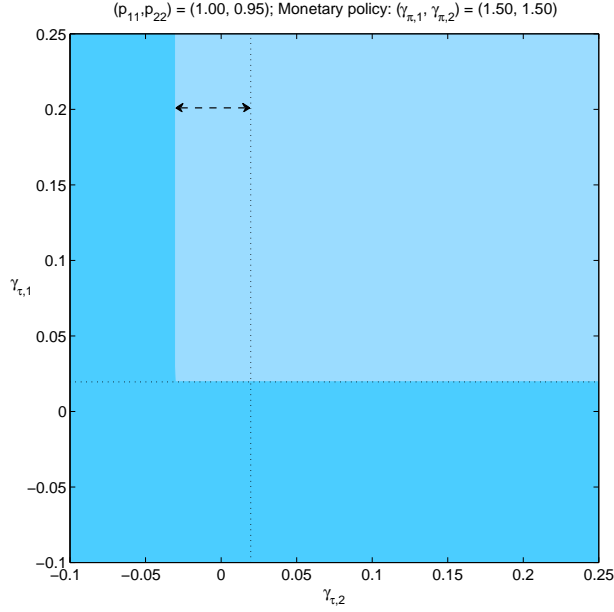
Figure A1: The fiscal policy frontier in the absorbing case.

and thus, determinacy requires all solutions $\bar{h}_i(\gamma_{\pi,i})$ to be unstable. The condition for the absorbing state is (23). If in the absorbing regime 1 fiscal policy is passive, then the solution $\bar{h}_1(\gamma_{\tau,1})$ is inside the unit circle and $(1-\beta)\frac{b}{\tau} < \gamma_{1,\tau} < (1+\beta)\frac{b}{\tau}$. The condition for not having another stable solution is that the two $\bar{h}_1(\gamma_{\pi,1})$ solutions should be outside the unit circle, that boils down to the usual $\gamma_{\pi,1} > 1$: monetary policy needs to be active. As for the non-absorbing regime 2, consider again equation (53). If fiscal policy is passive then to have a unique stable solution the other (two) (53) solutions must be both outside the unit circle, which gives (25): $\gamma_{\pi,2} > \sqrt{p_{22}} - \frac{\left(1-\beta\sqrt{p_{22}}\right)\left(1-\sqrt{p_{22}}\right)}{\lambda}$ that is, we have determinacy when $\gamma_{\pi,2} > \bar{\gamma}_{\pi,2}$.

A.6.3   Figure 4b under an absorbing PM regime 1

Suppose now monetary policy is PM (with $\gamma_{\pi,1} = 0.9$) in the first (absorbing) regime and AM (with $\gamma_{\pi,2} = 1.5$) in the second regime. The condition for the absorbing state is, as usual, (23). That for the non-absorbing state is derived from (20) evaluated at $p_{11} = 1$ where, to simplify notation, we define $z = h_2\sqrt{p_{22}}$

$$
\begin{aligned}
0 = {} & g_{\pi,2}\left\{1 + \lambda\gamma_{\pi,2} - z\sqrt{p_{22}}\left(1 + \beta + \lambda\right) + \beta z^2 p_{22}\right\} \\
& + (1-p_{22})g_{\pi,1}z\left[\beta z - \frac{1}{\sqrt{p_{22}}}\left(1 + \beta + \lambda - \beta h_1\right)\right]
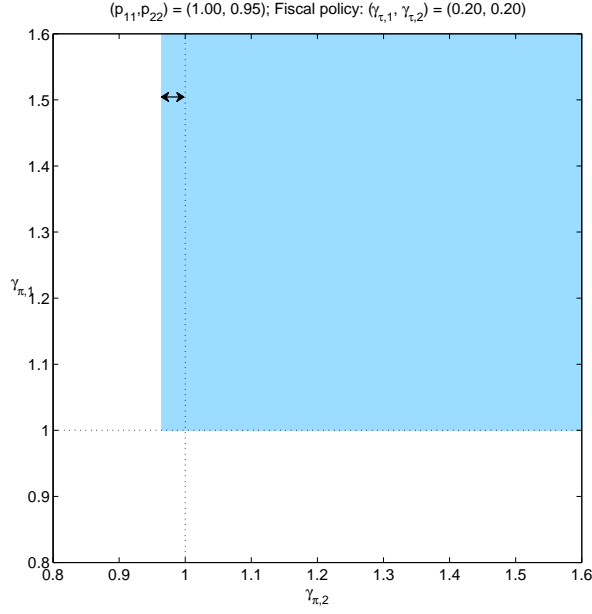\end{aligned}
\tag{56}
$$

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Figure A2: The monetary policy frontier in the absorbing case.

where $g_{\pi,1}$ and $g_{\pi,2}$ are given, as usual, by equations (21) and (22) in the text. We can re-write the condition for the non-absorbing state as

$$z^3 + b_2 z^2 + b_1 z + b_0 = 0$$

where

$$b_2 = -\frac{\left(1 - \frac{\tau}{b}\gamma_{\tau,2}\right) p_{22} + (1 + \beta + \lambda) + g_{\pi,1}(1 - p_{22})b\left(\frac{1}{\beta} - \gamma_{\pi,2}\right)\beta}{\beta\sqrt{p_{22}}},$$

$$b_1 = \frac{\sqrt{p_{22}}\left(1 + \beta + \lambda\right)\frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma_{\tau,2}\right) + \frac{(1 + \lambda\gamma_{\pi,2})}{\sqrt{p_{22}}} + g_{\pi,1}\frac{(1 - p_{22})}{\sqrt{p_{22}}}\left(1 + \beta + \lambda - \beta\bar{h}_1\right)b\left(\frac{1}{\beta} - \gamma_{\pi,2}\right)}{\beta\sqrt{p_{22}}},$$

$$b_0 = -\frac{\left(1 - \frac{\tau}{b}\gamma_{\tau,2}\right)\left(1 + \lambda\gamma_{\pi,2}\right)}{\beta^2\sqrt{p_{22}}}.$$

The necessary and sufficient condition for determinacy is that this cubic equation has exactly one solution inside the unit circle and the other two outside. By proposition C.2 in Woodford (2003), this is the case if and only if either of the following two cases is satisfied:

- Case I: $1 + b_2 + b_1 + b_0 < 0$ and $-1 + b_2 - b_1 + b_0 > 0$;

- Case II: $1 + b_2 + b_1 + b_0 > 0$, $-1 + b_2 - b_1 + b_0 < 0$, and $b_0^2 - b_0 b_2 + b_1 - 1 > 0$ or $|b_2| > 3$.

Let us study now how determinacy varies according to the fiscal policy undertaken in regime 1.

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AF in the absorbing regime 1. Consider the case of an AF policy in the absorbing regime: regime 1 will have a PM/AF mix hence a unique stable solution. In this case the $\bar{h}_1(\gamma_{\pi,1})$ solutions in (23) should have a root inside and one outside the unit circle while the $\bar{h}_1(\gamma_{\tau,1})$ solution, being AF, is outside. The monetary coefficient $\gamma_{\pi,1} = 0.9$ generates a stable solution $h_1 = 0.94343$ (and an unstable one, that we discard, $h_1 = 1.1534$).

Studying the necessary and sufficient conditions for determinacy, we find that, under these parameters, Case I is never satisfied (since the second inequality never holds) while Case II is. In particular, the condition to have exactly one solution inside the unit circle for regime 2 (from the first condition in Case II) reads

$$
\gamma_{\tau,1} > \frac{b}{\tau}(1 - \beta h_1)
$$
$$
- \frac{\left[1 + \lambda\gamma_{\pi,2} - \sqrt{p_{22}}\left(1 + \beta + \lambda\right) + \beta p_{22}\right]}{(1 - p_{22})\left[\beta - \frac{1}{\sqrt{p_{22}}}\left(1 + \beta + \lambda - \beta h_1\right)\right]\left(\frac{1}{\beta} - \gamma_{\pi,2}\right)}\left(\frac{1}{\beta} - \gamma_{\pi,1}\right)\frac{b}{\tau}\left[\beta\frac{1}{\sqrt{p_{22}}} - 1 + \frac{\tau}{b}\gamma_{\tau,2}\right],
$$

(57)

that is represented by a negative sloped line in the space $(\gamma_{\tau,1}, \gamma_{\tau,2})$ and that depends, among other things, on h_1. Note that (57) corresponds to (56) for $z = 1$.

Hence there is a unique stable solution when $\gamma_{\tau,1}$ is above this line for $h_1 = 0.94343$. As a result, when the first regime is PM/AF we have a global determinate equilibrium in the hatched area of Figure A3.
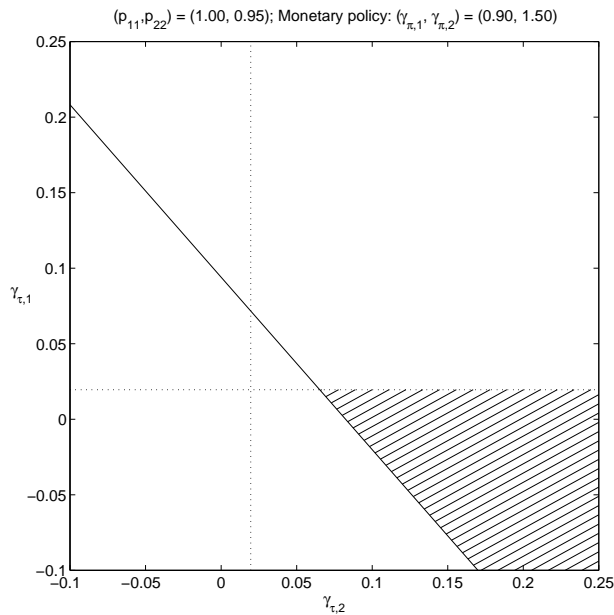

$(p_{11}, p_{22}) = (1.00, 0.95)$; Monetary policy: $(\gamma_{\pi,1}, \gamma_{\pi,2}) = (0.90, 1.50)$

Figure A3: Stability for an absorbing PM/AF regime 1.

PF in the absorbing regime 1. When the first regime is PM/PF, the two $\bar{h}_1(\gamma_{\pi,1})$ solutions are one inside ($h_1 = 0.94343$) and the other outside while the $\bar{h}_1(\gamma_{\tau,1})$ solution is inside with $g_{\pi,1} = 0$.

Now we have to consider two areas, one for each $h_1 < 1$ in regime 1. The solution $h_1 = 0.94343$, returns the same negative sloped straight line as before. So, again, there is a unique stable solution for regime 2 when $\gamma_{\tau,1}$ is above this line. Hence, when the first regime is PM/PF we have a global determinate equilibrium in the hatched area of Figure A4.
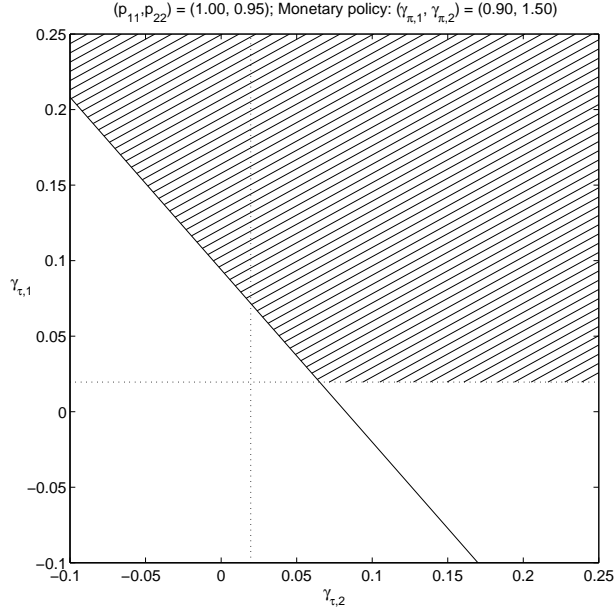
Figure A4: Stability for absorbing PM/PF regime 1: stable monetary solution case.

When we consider the other stable solution, with $g_{\pi,1} = 0$, equation (56) reduces to

$$0 = g_{2,\pi,b}\left[1 + \lambda\gamma_{\pi,2} - z\sqrt{p_{22}}\left(1 + \beta + \lambda\right) + \beta z^2 p_{22}\right].$$

Hence, we have just one stable solution if and only if the regime 2 is AM/PF (in this case there are 2 $\bar{h}_2(\gamma_{\pi,2})$ solutions outside and 1 $\bar{h}_2(\gamma_{\tau,2})$ solution inside) and the area characterised by just one stable solution is the hatched area in Figure A5.

Overlapping these two figures (A4 and A5) we get the areas with just one stable solution when fiscal policy in regime 1 is passive: the two triangular areas in Figure A6.

Putting everything together. Putting together Figures A3 and A6, one obtains A7, which is the counterpart of Figure 4b for the case of absorbing PM regime 1.
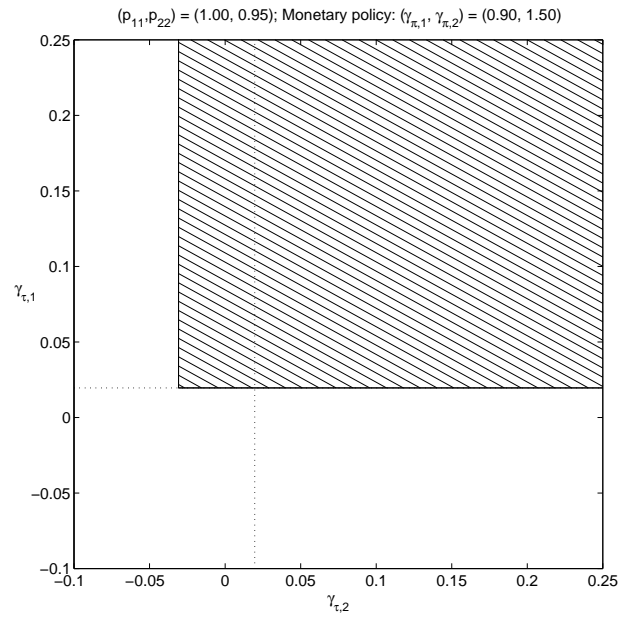
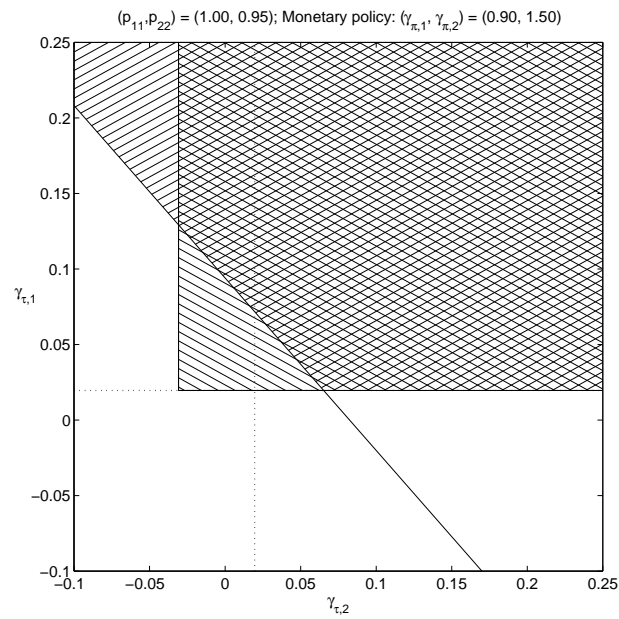Figure A5: Stability for absorbing PM/PF regime 1: stable fiscal solution case.

Figure A6: Stability for absorbing PM/PF regime 1: complete case.
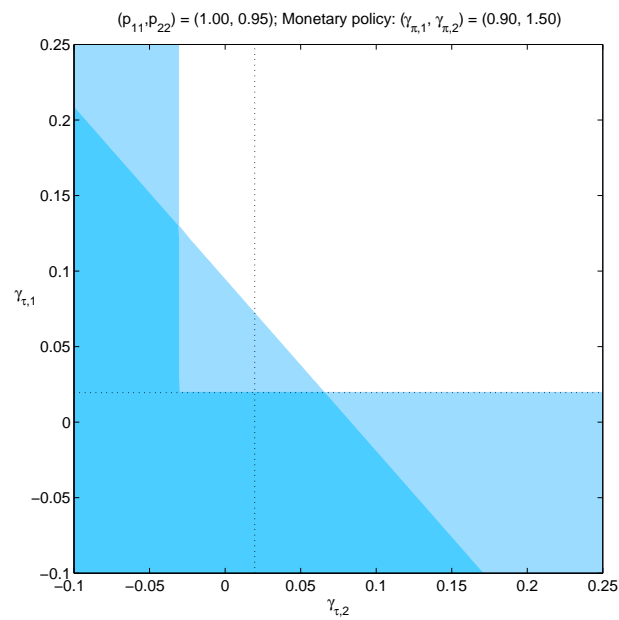
Figure A7: The fiscal policy frontier with substantial deviations in monetary policy and absorbing regime 1 with PM.

Notes: Light blue: unique solution; white: indeterminacy; dark blue: explosiveness.

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