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ANALYSIS OF THE BALANCE BETWEEN U.S. MONETARY AND FISCAL POLICY USING SIMULATED WAVELET-BASED OPTIMAL TRACKING CONTROL

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Abstract. This paper uses wavelet-based optimal control to simulate fiscal and monetary strategies under different levels of policy restrictions. The model applies the Maximal Overlap Discrete Wavelet Transform (MODWT) to United States quarterly GDP data, and then uses the decomposed variables to build a large 80 dimensional state-space linear-quadratic tracking model. Using a political targeting design for the frequency range weights, we simulate jointly optimal fiscal and monetary policy where: (1) both fiscal and monetary policy are dually emphasized, (2) fiscal policy is unrestricted while monetary policy is restricted to achieving a steady increase in the market interest rate, and (3) only monetary policy is relatively active, while fiscal spending is restricted to achieving a target growth rate. The results show that fiscal policy must be more aggressive when the monetary authorities are not accommodating the fiscal expansion, and that the dual-emphasis policy leads a series of interest rate increases that are balanced between a steadily increasing target and a low, fixed rate. This research is the first to construct integrated fiscal and monetary policies in an applied wavelet-based optimal control setting using U.S. data.

Keywords: Fiscal Policy, Linear-Quadratic, Monetary Policy, Optimal Tracking Control, Discrete Wavelet Analysis

JEL classifications C49 . C61 . C63 . C88 . E58 . E61

1 Introduction

Different versions of the accelerator-based optimal control model have recently been employed to analyze economic policy. Kendrick and Amman (2010), Kendrick and Shoukry (2013), and Hudgins and Na (2016) all present strong cases for the improvements gained by implementing a quarterly fiscal policy rather than an annual policy. Kendrick and Shoukry (2013) simulate the tracking performance and debt structure of the quarterly and annual models within a closed-economy macroeconomic model that contains monetary and interest rate components. Hudgins and Na (2016) examine optimal robust policies using a similar model for the U.S. economy, but without money and interest rates in the model. Crowley and Hudgins (2015) first used a wavelet-based optimal control model in order to analyze fiscal policy in a closed-economy model using U.S. data.

Rather than estimating various macroeconomic time series models with alternating lags, the interplay of the short-term lags with the long term cyclical components can be captured by using the quarterly lag structure gathered through wavelet filters in the time-frequency domain (Crowley and Hudgins, 2015). The use of longer cycles obtained from wavelet analysis is consistent with the findings of recent neoclassical research, including Leeper et al. (2010), who find that the fiscal adjustment speed impacts the effectiveness of the policy.

Crowley and Hughes Hallett (2014) gain insight into the lag structure through the time-frequency domain by employing a Maximal Overlap Discrete Wavelet Transform (MODWT) to obtain the frequency domain cyclical decomposition of U.S. GDP component data for the period 1947 – 2012. The authors found that fiscal policy has not been destabilizing or procyclical over the various business cycle frequencies, but also found that it has not been an effective countercyclical stabilizer either. Using the MODWT, Crowley and Hudgins (2015) built the first optimal control model that utilized the time-frequency domain decomposition to formulate fiscal policy, but the model did not include any channel for interest rates or monetary policy. This analysis expands the wavelet estimation and optimal control model used in Crowley and Hudgins (2015) to include macroeconomic responses to interest rates that are influenced by monetary authorities.

1.1 Purpose and Scope

The purpose of this paper is to construct optimally integrated fiscal and monetary policies by utilizing an optimal LQ (Linear-Quadratic) tracking control model that is formulated within the time-frequency domain based on a MODWT wavelet decomposition. This is the first research to design macroeconomic policy by coupling monetary and fiscal policy into an optimal control framework based on discrete wavelet analysis, and using U.S. data. Section 2 examines the MODWT wavelet decomposition of data in the time-frequency domain, using U.S. data over the period 1947 – 2015. Section 3 uses the method employed by Crowley and Hudgins (2015) and Hudgins and Na (2016) to build a macroeconomic time-frequency accelerator model that is estimated and used within an optimal control system to determine optimal control feedback rules for monetary and fiscal policy. We develop a MATLAB software program that computes the optimal joint fiscal and monetary policy based on the large-scale state-space framework.

Section 4 simulates the time-frequency model developed in section 3. The simulations explore a politically motivated targeting strategy, as in Crowley and Hudgins (2015), which allows the policymakers to place larger weights on the tracking errors for government spending, consumption, investment and the interest rate, at frequency ranges between 1 and 8 years. This strategy is not possible under an aggregate model without time-frequency decomposition, and thus constitutes an entirely new operational procedure for constructing both optimal fiscal and monetary policy, and for determining the likely effects over each frequency range. This paper only renders simulation results for the deterministic *LQ* tracking control design, although the design is developed to allow the policymaker to also employ both stochastic LQG (Linear-Quadratic Gaussian) and robust controller designs, which will be explored in a future paper.

The simulations analyze three different scenarios: (1) dual emphasis on the active use of both fiscal and monetary policy, (2) emphasis on active use of fiscal policy with relatively uncooperative passive monetary policy that closely tracks a fixed interest rate target, and (3) active use of monetary policy with relatively uncooperative passive fiscal policy that closely tracks a government spending target. The findings suggest that dual emphasis with distributed flexibility in both monetary and fiscal policy induces a moderated tracking response in market interest rate movements along with reduced volatility in optimal government spending levels, while generating improved performance in the aggregate consumption and investment trajectories.

The ameliorated performance under the dual emphasis strategy prompts monetary authorities to avoid being overly restrictive in tracking their intermediate market interest rate target too closely. Instead, the preferred strategy should embrace enough flexibility to allow the market interest rate to partially adjust based on the current levels of consumption and investment at each frequency range. This approach refrains from overly committing to a slow sequence of *ad hoc* small discrete jumps in the operating target interest rates (the discount rate and federal funds rate) with arbitrary timing and/or magnitude, or to a predetermined steady growth path in the market interest rate. Similarly, when the fiscal authorities restrict government spending so that it closely follows a prescribed target growth path, the optimal path of the interest rate consistently strays farther from its desired target, and the resulting performance of consumption and investment substantially deteriorates.

2 MODWT Wavelet Analysis

Time-domain methods generally omit any information contained in cyclical activity embedded in time series that can only be revealed by extracting the different frequency modes at work within a given time series. When modal frequencies are changing, time-domain analysis cannot provide a proper basis for analysis, so we need to appeal to a different toolbox of techniques, namely time-frequency analysis.

One particular form of time-frequency analysis, namely wavelet analysis, stems from multiresolutional analysis which is an approximation operation through a dense vector space (Hilbert space) with empty intersects from coarsest to less detailed information, developed by Meyer (1986), Mallat (1989a,b), Strang (1989), and Daubechies (1992). Using Mallat's pyramid algorithm and multiresolutional analysis, the value of a variable x at time instant k, x_k, can be expressed as:

$$x_k \approx S_{J,k} + d_{J,k} + d_{J-1,k} + \ldots + d_{1,k} \tag{1}$$

As expounded in Crowley and Hudgins (2015), the $d_{j,k}$ are wavelet "crystals", $j = 1,\ldots, J$; $S_{J,k}$ is a trend component, called the wavelet "smooth", and J represents the number of scales (frequency bands). There are many different wavelet filter functions that are used in discrete wavelet analysis to direct a filtering process that utilizes pairs of low pass and high pass filters, including the Biorthogonal, Coiflet, Daubechies, Discrete Meyer, Haar, and Symlet. For this paper, we utilize the asymmetric Daubechies 4-tap (D4) wavelet function, and employ the MODWT as our method of time-frequency decomposition. Using the MODWT avoids the dyadic data requirements and non-shift invariant shortcomings of the DWT (Crowley, 2007). We repeated the wavelet decompositions for national output (Y) using the Symlet 8-tap wavelet, and find virtually identical results, thus confirming the robustness of the results.

2.1 MODWT Wavelet Decomposition Analysis

Following Crowley and Hudgins (2015), this paper applies the MODWT to the data using a two-step procedure that extracts the crystals and the smooth trend at frequencies j = 1, …, 5. First, the wavelet decomposition was undertaken in terms of differences of the absolute values of each series, to obtain crystals, and then these crystals were then summed to create level equivalents. To ensure consistency in the terms of the decomposition for each data point, a residual was calculated to create a *modified smooth* (S), so that the sum of the level equivalent crystals and the *modified smooth* equals the actual observation.

Table 1

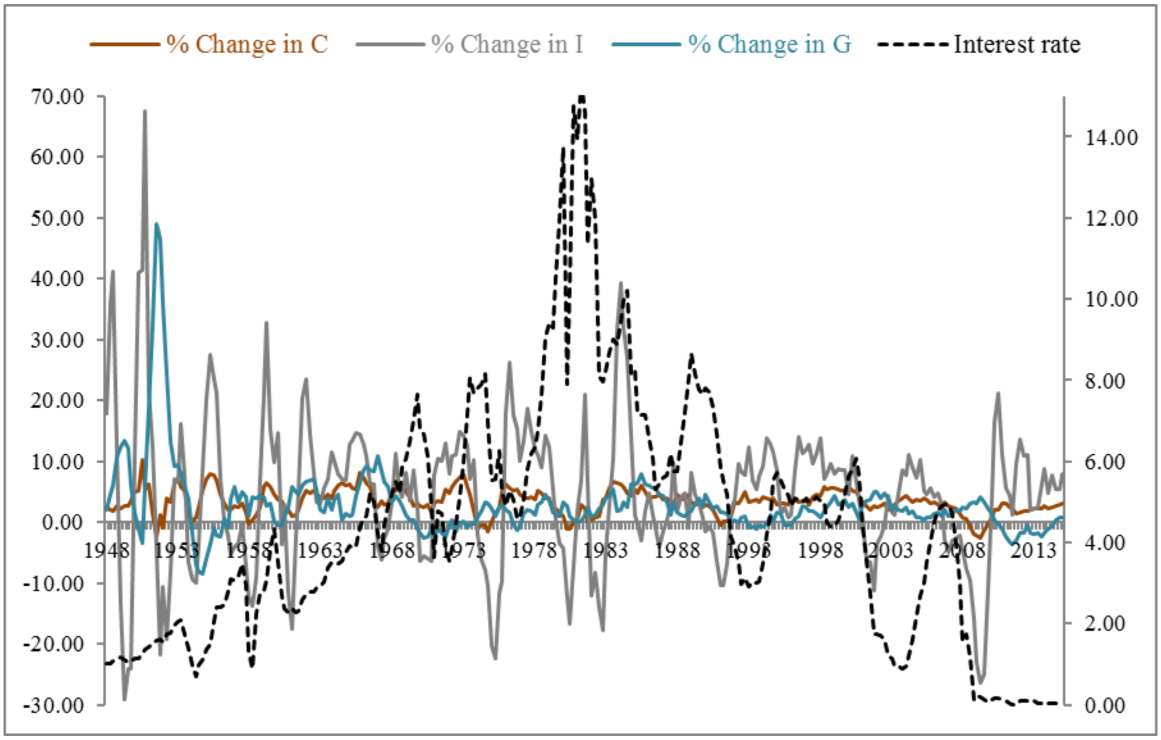
The time intervals associated with each of the frequencies

J	*Time interval*
1	6 months to 1 year
2	1 – 2 years
3	2 – 4 years
4	4 – 8 years
5	8 – 16 years

Table 1 defines the time-frequency ranges for all of the wavelet decompositions. Figure 1 provides an overview of the aggregate data for consumption, investment, government spending, and the short-term interest rate, plotting this data in percent change terms for the main GDP components, and in percent terms for the interest rate. The data is sourced from the U.S. Bureau of Economic Analysis (BEA) and consists of Real GDP component data in billions of dollars for the period 1947 quarter 1 to 2015 quarter 1, together with 3 month T-bill interest rate data which is obtained from the Federal Reserve's FRED database.

Figure 1

Overview of U.S. Data

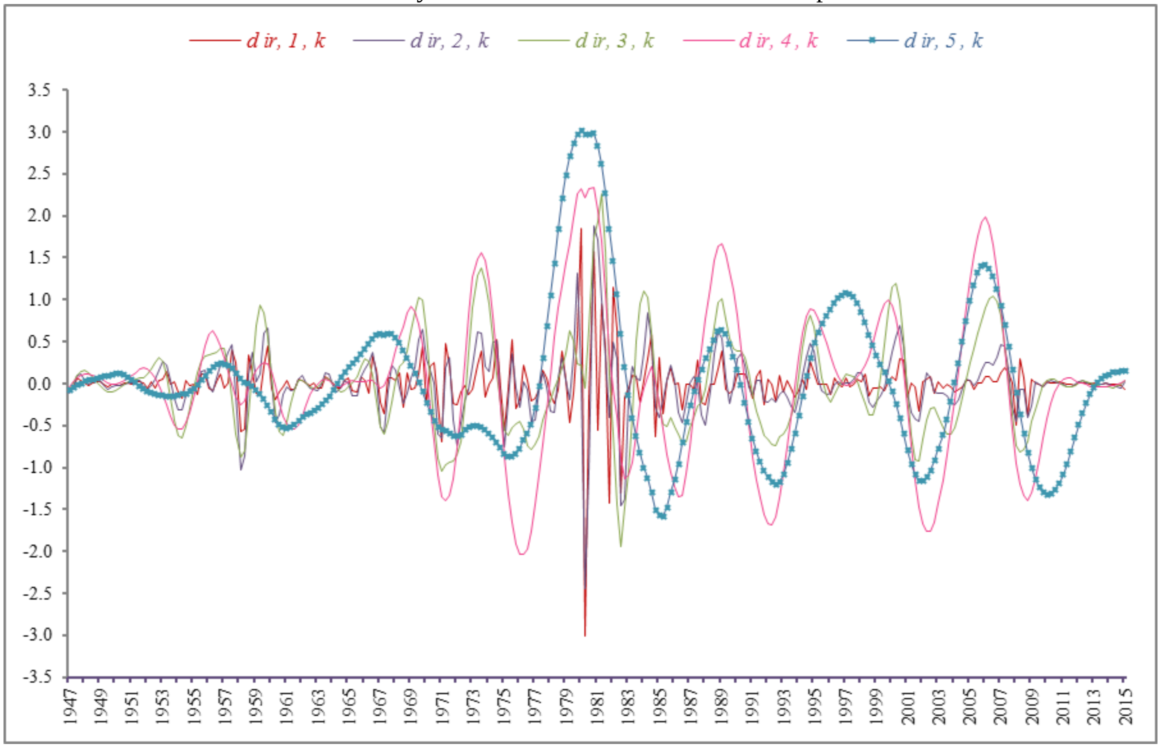


It is interesting to note the symmetry of the move upwards in interest rates to 1980 and then the gradual trend lower in interest rates thereafter. The interest rate is clearly procyclical, increasing as the GDP component growth slows, and then falling as recessions take hold, usually prompting a trough in the growth of the components. The pattern of government spending growth though appears to be largely independent of the interest rate. As expected, the investment component of GDP is much more volatile than other components of GDP.

Figure 2 shows the time-frequency decomposition of the interest rate, but without the modified smooth shown in this figure. It is clear that there are cycles in interest rate movements, with crystals $d_{5,k}$ and $d_{4,k}$ containing most cyclical activity. This conforms with what one might expect in terms of the procyclical movement of interest rates, but it also suggests that the procyclical activity may be contained in 2 frequency bands. Given that the business cycle is usually thought to have periodicity of 4 to 10 years, the frequencies contained in $d_{4,k}$ (4 to 8 years) and $d_{5,k}$ (8 to 16 years) encompass the business cycle frequency.

Figure 2

U.S. Treasury Bill Interest Rate: MODWT-decomposed



3 Macroeconomic Model Derivation and Estimation

This section employs the MODWT decomposed series within a macroeconomic accelerator model, which extends the closed-economy partial accelerator model of Crowley and Hudgins (2015) to include the interest rate. The GDP components of domestic output (Y) are indexed in blocks, so that personal consumption is nested as (1), private domestic investment as (2), and government purchases as (3). At each frequency range, the wavelet-based GDP components remove the effects at all other four frequency ranges, so that the each component only includes the crystal (d) at that frequency range and the modified smooth base-level trend (S). The wavelet-based components are defined in equation (2) as follows:

$$Y_{j,k}^{(i)} = d_{j,k}^{(i)} + S_k^{(i)} \qquad i = 1, 2, 3; \qquad j = 1, \ldots, 5 \tag{2}$$

Expanding the linear accelerator block framework of Crowley and Hudgins (2015) results in the reduced-form block component matrix system for each frequency range defined in table 1, where the β_{0i} coefficients are constants, the $ir^{(1)}$ block represents the wavelet decomposition of the short-term interest rate, and the ω terms represent blocks of random disturbance errors:

$$
\begin{bmatrix} Y_k^{(1)} \\ Y_k^{(2)} \\ Y_k^{(3)} \end{bmatrix} = \begin{bmatrix} \beta_{01} \\ \beta_{02} \\ 0 \end{bmatrix} + \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} & \beta_{14} \\ \beta_{21} & \beta_{22} & \beta_{23} & \beta_{24} \\ 0 & 0 & \beta_{33} & 0 \end{bmatrix} \begin{bmatrix} Y_{k-1}^{(1)} \\ Y_{k-1}^{(2)} \\ Y_{k-1}^{(3)} \\ ir_{k-1}^{(1)} \end{bmatrix} + \begin{bmatrix} \omega_{k-1}^{(1)} \\ \omega_{k-1}^{(2)} \\ \omega_{k-1}^{(3)} \end{bmatrix} \tag{3}
$$

The coefficients (β_{33}) of the government spending block ($Y^{(3)}$) are modelled to extract the current trend, since future government spending will be determined by an optimal control system in section 4, which simulates the optimal policy forecasts during the period starting in 2015 quarter 2. The estimated government spending coefficients (β_{33}) are all slightly larger than 1.004, which means that the average quarterly growth rate is slightly above .004 per quarter (about 1.7% per year) at all frequency ranges.

Table 2

Empirical Estimation of the MODWT-decomposed Accelerator Model

j	β_{01}	β_{11}	β_{12}	β_{13}	β_{14}	β_{02}	β_{12}	β_{22}	β_{23}	β_{24}
1	4.1327	0.9890	0.0336	0.0292	-2.7213	0.2733	-0.0048	0.9911	0.0286	-2.0558
	(0.483)	*(134.586)*	*(5.367)*	*(3.950)*	*(3.424)*	*(0.024)*	*0.497*	*(46.640)*	*(1.238)*	*(1.930)*
2	5.7179	0.9659	0.0848	0.0609	-2.3078	-1.3823	-0.0139	1.0152	0.0400	-2.4774
	(0.804)	*(160.197)*	*(6.305)*	*(4.214)*	*(3.440)*	*(0.157)*	*(1.862)*	*(60.772)*	*(2.228)*	*(2.974)*
3	6.9645	0.9674	0.0867	0.0537	-1.7930	-0.2659	-0.0100	1.0093	0.0311	-1.8777
	(1.070)	*(176.071)*	*(4.365)*	*(4.019)*	*(2.848)*	*(0.032)*	*(1.421)*	*(62.642)*	*(1.811)*	*(2.319)*
4	7.6667	0.9750	0.0715	0.0401	-0.8723	4.3327	0.0035	0.9838	0.0047	-0.3824
	(1.141)	*(180.918)*	*(5.564)*	*(2.910)*	*(1.318)*	*(0.453)*	*(0.451)*	*(53.860)*	*(0.242)*	*(0.406)*
5	1.6010	0.9738	0.0701	0.0487	-1.2583	3.5019	0.0027	0.9865	0.0062	-0.6458
	(0.223)	*(186.806)*	*(5.603)*	*(3.576)*	*(1.886)*	*(0.348)*	*(0.365)*	*(56.333)*	*(0.327)*	*(0.691)*

Table 2 shows the *OLS* regression coefficient estimates with *t-statistics* (absolute values in parentheses) for the MODWT decomposition accelerator system in equation (3) using the data for the period 1947 quarter 1 – 2015 quarter 1. For each of the equations, $R^2 > 0.99$. Table 2 shows that the estimated equations for consumption and investment equations over each frequency range have a good fit. Each coefficient has the expected sign, and almost all of the coefficients are statistically significant. The positive sign on each the estimated consumption equation coefficients β_{12} and β_{13} produces a crowding-in effect on consumption due to both investment and government spending, respectively. The investment equation coefficients β_{21} are negative for frequency ranges 1 – 3 and positive for frequency ranges 4 and 5, suggesting a consumption-based crowding-out effect at the highest three frequencies, and a crowding-in effect at the lowest two frequencies. The positive sign of β_{23} shows that government spending has a crowing-in effect on investment at all frequencies, with the most substantial influence occurring at frequency range 2. Each of the interest rate coefficients, β_{14} and β_{24}, have negative signs, which means that higher interest rates have a crowding-out effect on both consumption and investment at all frequency ranges.

Equation (4) models the modified smooth trend processes for consumption, investment, and government purchases as first-order difference equations, where the ω_S terms represent random disturbances term in each equation satisfying the standard assumptions.

$$S_k^{(i)} = s_1^{(i)} S_{k-1}^{(i)} + s_2^{(i)} Y_{k-1}^{(i)} + \omega_{S,k-1}^{(i)} \tag{4}$$

In equation (4), the coefficients on the lagged modified smooth trend variable, and the coefficients on the lagged component variables (aggregate consumption, investment, and government spending), produce a weighted average growth contribution toward the current trend values of each output component series. Equation (5) specifies the same process for the modified smooth trend of the market interest rate:

$$S_k^{(4)} = s_1^{(4)} S_{k-1}^{(4)} + s_2^{(4)} ir_{k-1} + \omega_{S,k-1}^{(4)} \tag{5}$$

Table 3

Empirical Estimation of the *Modified Smooth* Trend Series

	$s_1^{(1)}$	$s_2^{(1)}$	$s_1^{(2)}$	$s_2^{(2)}$	$s_1^{(3)}$	$s_2^{(3)}$	$s_1^{(4)}$	$s_2^{(4)}$
Coefficient	0.9347	0.0714	0.8999	0.1066	0.9186	0.0858	0.8651	0.1399
(*t-statistic*)	*(104.80)*	*(8.03)*	*(106.58)*	*(12.65)*	*(139.17)*	*(13.01)*	*(78.22)*	*(13.06)*

Table 3 gives the estimates for the coefficients of the modified smooth trends (with *t-statistics* in parentheses) for the 4 series: (1) consumption, (2) investment, (3) government purchases, and (4) interest rate, as specified in equations (4) and (5). Summing the two coefficients in each of the equations produces a weighted average trend growth rate. In the consumption trend series equation, the coefficient on the lagged value of the series is $s_1^{(1)} = 0.9347$, which is much larger than coefficient on the lagged value of aggregate consumption, given by $s_2^{(1)} = 0.0714$. This pattern holds for the investment, government spending, and interest rate modified smooth trend series, where the coefficients on the lagged value of each trend series exceeds 0.8, while the coefficients on the lagged aggregate variable of the series are less than 0.2. All four equations achieve a good fit, with statistically significant coefficients and $R^2 > 0.99$ in each equation.

The model includes a rational expectations component by allowing changes in the current national debt level to influence consumption and investment through changes in expected national output. The adjustment is implemented by introducing the following variables:

$DEBT_k$ = the total stock of government debt in quarter k

$\hat{G}_{j,k}^{d}$ = the trend government obligations at frequency range j in quarter k

$G_{j,k}^{e}$ = expected contribution of government purchases to national output

The trend process for government purchases at each frequency range is defined in equation (6) by letting the current trend value depend on the lagged value of the actual level of government purchases, where ρ is the growth coefficient, estimated by β_{33} in equation (3).

$$\hat{G}^{\,d}_{j,k} = \rho Y^{(3)}_{j,k-1} + \omega^{(3)}_{k-1} \qquad j = 1, \ldots, 5 \qquad (6)$$

Fiscal policy is specified to cause adjustments in consumption and investment in period k based on the expected contribution of government spending at frequency range j in period k, as modeled by equation (7).

$$G^{e}_{j,k} = \phi_{j,k}\left[Y^{(3)}_{j,k-1} - \pi_k\,(DEBT_{k-1} - DEBT_0)\right] + (1-\phi_{j,k})\hat{G}^{\,d}_{j,k-1}\,;\ 0 < \phi < 1;\ j = 1,\ldots,5 \quad (7)$$

Equation (7) causes the expected value of government spending in any period k to be determined based on a weighted average of the actual spending in the previous period, and the trend value along the wavelet frequency range in the previous period. Since government spending only affects the economy through expected national output, all government spending changes have a limited impact, where larger values of ϕ result in greater effectiveness of fiscal policy at any given frequency range. Equation (7) renders rational expectations behavior, where the current contribution of government spending toward national output production is both limited and crowded out by any stock of national debt that exceeds its initial value. The implementation of any new fiscal initiative pulses the current cycle at each frequency range. Our analysis explores the case where government spending is determined through optimal tracking control.

Substitute the variable G^e into equation (3) so that it replaces $Y^{(3)}$, and augment the system with the government debt and government purchases trend spending to obtain the reduced form system for determining consumption and investment at each frequency:

$$\begin{bmatrix} Y^{(1)}_k \\ Y^{(2)}_k \\ \hat{G}^{\,d}_k \end{bmatrix} = \begin{bmatrix} \delta_0 \\ \lambda_0 \\ 0 \end{bmatrix} + \begin{bmatrix} \delta_1 & \delta_2 & \delta_3 & \delta_4 & \delta_5 & \delta_6 \\ \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 & \lambda_5 & \lambda_6 \\ 0 & 0 & \rho & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} Y^{(1)}_{k-1} \\ Y^{(2)}_{k-1} \\ Y^{(3)}_{k-1} \\ ir^{(1)}_{k-1} \\ \hat{G}^{\,d}_{k-1} \\ DEBT_{k-1} \end{bmatrix} + \begin{bmatrix} \omega^{(1)}_{k-1} \\ \omega^{(2)}_{k-1} \\ \omega^{(3)}_{k-1} \end{bmatrix} \qquad (8)$$

where

$\delta_0 = \beta_{01} + \beta_{13}\phi\pi\, DEBT_0$; $\delta_1 = \beta_{11}$; $\delta_2 = \beta_{12}$; $\delta_3 = \beta_{13}\phi$; $\delta_4 = \beta_{14}$;
$\delta_5 = \beta_{13}(1-\phi)$; $\delta_6 = -\beta_{13}\phi\pi$;
$\lambda_0 = \beta_{02} + \beta_{23}\phi\pi\, DEBT_0$; $\lambda_1 = \beta_{21}$; $\lambda_2 = \beta_{22}$; $\lambda_3 = \beta_{23}\phi$; $\lambda_4 = \beta_{24}$;
$\lambda_5 = \beta_{23}(1-\phi)$; $\lambda_6 = -\beta_{23}\phi\pi$

The model is closed by equations (9) through (13). Equation (9) specifies the national income identity, and equation (10) focuses on the domestic economy by assuming that the value of net exports is constant.

$$Y_k = C_k + I_k + G_k + NX_k \tag{9}$$

$$NX_k = n_0 \qquad \text{for all } k = 1, \ldots, K \tag{10}$$

Net taxes (T_k), defined as the total government tax and income minus total government transfer payments in quarter k, are assumed to be generated as a constant percentage (τ) of national output, as specified by equation (11).

$$T_k = \tau Y_k \tag{11}$$

Following Kendrick and Shoukry (2013) and Crowley and Hudgins (2015), this analysis computes government tax income and transfer payments as passively determined variables, rather than modeling them as separate fiscal policy variables. This treatment is consistent with the results of Kliem and Kriwoluzky (2014), which find little evidence in the U.S. for the typical simple fiscal policy rules derived in Dynamic Stochastic General Equilibrium (DSGE) models where tax rates respond to output. Thus, the only actively determined fiscal policy variables are government spending levels over each frequency range.

Equation (12) records the resulting government budget deficit (or surplus, when the value is negative) in quarter k, which is given by DEF_k :

$$DEF_k = G_k - T_k \tag{12}$$

$$DEBT_k = 0.25 DEF_k + (1 + i_k) DEBT_{k-1} \tag{13}$$

In equation (13), the national debt ($DEBT_k$) is the sum of the current budget deficit (converted from annualized rates to quarterly levels) and the previous period debt stock, which grows at the quarterly interest rate of i_k.

This model in equations (8) through (13) has several advantages. It can be specified with either constant coefficients, as in this paper, or with time varying coefficients. It is derived from the macroeconomic accelerator framework which has been widely used, such as in Kendrick (1981), Kendrick and Shoukry (2013), Crowley and Hudgins (2015), and Hudgins and Na (2016). It includes an interest rate outlet for the transmission of monetary policy, and also includes a rational expectations component where the level of government debt affects fiscal policy effectiveness.

The matrix equation (8) also does not explicitly require a full specification of the structural equations underlying the final reduced form, since many theoretical underpinnings would result in the same reduced form block equation. This model is therefore not meant to represent a complete econometric forecasting model. Its purpose is to simulate the optimal monetary and fiscal and policy through tracking control in the time-frequency domain, and thereby illustrate how to employ this technique to explore some specific scenarios. Our methods could be implemented within a larger model, such as an augmented form of the 135-equation model used in Taylor (1993). Since deterministic, stochastic, and robust optimal feedback control designs can all be simulated within this MODWT wavelet-based accelerator framework, our model offers considerable insight.

3.1. Optimal Control

The LQ tracking problem is stated as follows, where the target variables are defined with a superscript (*). The objective is for the monetary authorities to choose the short-term interest rate at each of the five frequency ranges, while the fiscal policymakers choose the level of government spending at each of the five frequencies, so that the joint policy choices will minimize the quadratic performance index given in (14) subject to the linear state equations given by (8) – (13).

$$
\begin{aligned}
\min_{Y^{(3)}_{j,k};\, ir^{(1)}_{j,k}} J = {} & \sum_{i=1}^{2}\left[q^{(i)}_{1,f}\,(Y^{(i)}_{K+1} - Y*^{(i)}_{K+1})^2\right] + \sum_{i=1}^{2}\left[q^{(i)}_{2,f}\,(S^{(i)}_{K+1} - S*^{(i)}_{K+1})^2\right] \\
& + \sum_{i=1}^{2}\sum_{j=1}^{5}\left[q^{(i)}_{3,j,f}\,(Y^{(i)}_{j,K+1} - Y*^{(i)}_{j,K+1})^2\right] + \sum_{k=1}^{K}\sum_{i=1}^{3}\left[q^{(i)}_{1,k}\,(Y^{(i)}_{k} - Y*^{(i)}_{k})^2\right] \\
& + \sum_{k=1}^{K}\sum_{i=1}^{4}\left[q^{(i)}_{2,k}\,(S^{(i)}_{k} - S*^{(i)}_{k})^2\right] + \sum_{k=1}^{K}\sum_{i=1}^{2}\sum_{j=1}^{5}\left[q^{(i)}_{3,j,k}\,(Y^{(i)}_{j,k} - Y*^{(i)}_{j,k})^2\right] \\
& + \sum_{k=1}^{K} q_{4,k}(DEF_k - DEF_k^*)^2 + \sum_{k=1}^{K} q_{5,k}(DEBT_k - DEBT_k^*)^2 \\
& + \sum_{k=1}^{K}\sum_{j=1}^{5} q_{6,j,k}\left[\left(Y^{(3)}_{j,k} - Y^{(3)}_{j,k-1}\right) - \left(Y^{(3)}_{j,k} - Y^{(3)}_{j,k-1}\right)^*\right]^2 \\
& + \sum_{k=1}^{K} q_{7,k}(ir_k - ir_k^*)^2 + \sum_{k=1}^{K} q_{8,k}\left[\left(ir_k - ir_{k-1}\right) - \left(ir_k - ir_{k-1}\right)^*\right]^2 \\
& + \sum_{k=1}^{K}\sum_{j=1}^{5} q_{9,j,k}\left[\left(ir^{(1)}_{j,k} - ir^{(1)}_{j,k-1}\right) - \left(ir^{(1)}_{j,k} - ir^{(1)}_{j,k-1}\right)^*\right]^2 \\
& + \sum_{k=1}^{K}\sum_{j=1}^{5} r^{(1)}_{j,k}\left(Y^{(3)}_{j,k} - Y*^{(3)}_{j,k}\right)^2 + \sum_{k=1}^{K}\sum_{j=1}^{5} r^{(2)}_{j,k}\left(ir^{(1)}_{j,k} - ir*^{(1)}_{j,k}\right)^2
\end{aligned} \tag{14}
$$

The LQ tracking model in this section utilizes 10 control variables, including government spending (${Y^{(3)}}_j$) at each frequency range (j = 1,…, 5), and the interest rate (${ir^{(1)}}_j$) at each frequency range, and eighty state variables. Kendrick (1981) discusses the well-known benefits and drawbacks of symmetric quadratic performance indices for economic and engineering applications, thus eliminating the need for discussion here. When the disturbance terms in equation (8) are zero, then the model is deterministic. When the disturbance terms in equation (8) fluctuate, then the model can be simulated as a stochastic LQG design, as in Chow (1975), Kendrick (1981), and Kendrick and Shoukry (2013), or as a robust design as in Basar and Bernhard (1991) and Hudgins and Na (2016), or as a mixed H^∞ /stochastic LQG design (Hudgins and Na, 2016).

In equation (14), the first term penalizes the tracking errors for aggregate consumption and investment in the final period at the end of the planning horizon. The second term specifies penalties for the final period tracking errors of the modified smooth trends for consumption and investment. The third term in penalizes the final period tracking errors over each frequency range for consumption and investment. Policymakers will assign higher weights to frequency ranges where that time cycle interval is emphasized. For example, the weighting parameters on the final respective tracking errors for consumption and investment at frequency 3 will be assigned large values if policymakers are primarily concerned with 2 to 4 year cycles. Under a political targeting objective, the tracking errors in frequencies 3 and 4 receive the highest weights, since these frequencies enclose the standard 2 to 8 year political cycle.

The fourth term assigns penalties tracking errors in each period for aggregate consumption, investment, and government spending, while the fifth term assigns a penalty for the tracking errors for the modified smooth trends for consumption, investment, government spending, and the interest rate in each period. The sixth term introduces penalties for the tracking errors associated with consumption and investment over each individual frequency range for each period. Terms seven and eight assign penalties for the tracking errors associated with the current period budget deficit, and the current period national debt, respectively.

The ninth term penalizes the policymaker for large changes in government spending at each frequency between periods. Hudgins and Na (2016) employ this term as a pragmatic consideration, because government policymakers prefer stable spending patterns in the ongoing budget appropriation process, and do not desire, nor generally allow, large fluctuations from the previous budget. This term thus incorporates the fact that most new budgets are primarily designed by adjusting the prevailing budget on a line-by-line basis. This specification supports a constant frequency range structure where the resistance to change across each frequency range can be decoupled and penalized separately. Hudgins and Na (2016) find that a substantial additional cost savings in control effort results when this term is employed within robust modeling, as compared to the case where the government is not penalized for changing its spending above or below its stated quarterly target growth between periods, for each of the 5 frequency ranges given in table 1.

The tenth term specifies a penalty for the tracking error on the aggregate interest rate, which is the intermediate target for the central bank. The eleventh term assigns a penalty for large changes in the aggregate interest rate between periods, thus effectively penalizing the monetary authorities for an unstable market interest rate. Similarly, the twelfth term assigns a penalty for large changes in the interest rate between periods at each frequency range. The thirteenth term represents the control variables that are directly selected by the fiscal policymakers, and it provides a penalty for the government spending tracking error at each of the five frequencies. The last term specifies the control variables that are determined by the monetary authorities, and it assigns a penalty for the interest rate tracking errors across each of the five frequencies.

We transform the LQ-tracking problem into a LQ-regulator problem by restructuring the state-space equations using the conversion method used in Crowley and Hudgins (2015) and Hudgins and Na (2016). Although this is a large scale system, the state-space construction procedures and the accompanying MATLAB program that we have developed have proven to be efficient and feasible to employ. The model allows the optimal aggregate consumption, investment, government spending, and modified smooth values for each series to grow at distinct quarterly target rates of $g(.)$, that are specified by the policymakers, which results in annual growth rates of $\{[1 + g(.)]^4 - 1\}$ per year. Equation (15) defines target variable equations for quarterly consumption, investment, government spending, and interest rate for each frequency range, followed by the aggregate consumption, investment, government spending, and interest rate target equations. It then defines the modified smooth target equations for each of these respective series.

$$Y*_{j,k+1}^{(i)} = (1+g_{j,k}^{(i)})Y*_{j,k}^{(i)}\,; \quad ir*_{j,k+1}^{(1)} = (1+g_{j,k}^{(1)})ir*_{j,k}^{(1)}\,; \tag{15}$$
$$Y*_{k+1}^{(i)} = (1+g_k^{(i)})Y*_k^{(i)}; \quad ir*_{k+1} = (1+g_k^{(ir)})ir*_k\,; \quad S*_{k+1}^{(i)} = (1+g_{S,k}^{(i)})S*_k^{(i)}$$

The 80-dimensional state vector is defined by equation (16):

$$x_k = \begin{bmatrix} x_{1,k}\,;\ x_{2,k}\,;\ \ldots\,;\ x_{80,k} \end{bmatrix}^T \tag{16}$$

where

$$\begin{aligned} x_k = [\ & Y_{j,k}^{(1)};\ S_k^{(1)};\ Y_{j,k}^{(2)};\ S_k^{(2)};\ c_k;\ Y*_{j,k}^{(i)};\ \hat{G}_{j,k}^{\,d};\ S_k^{(3)};\ Y_k^{(i)};\ Y*_k^{(i)};\ NX_k;\ Y_k; \\ & T_k;\ DEF_k;\ DEBT_k;\ Y_{j,k-1}^{(3)};Y_{j,k-1}^{(3)} - Y_{j,k-2}^{(3)};\ \left(Y_{j,k-1}^{(3)} - Y_{j,k-2}^{(3)}\right)^*; \\ & DEF_k^*;\ DEBT_k^*\ ;\ S*_k^{(i)};\ S_k^{(4)};\ ir_k^*;\ ir_{k-1};\ ir_{j,k-1}^{(1)};\ ir_{j,k-1}^{(1)} - ir_{j,k-2}^{(1)};\ ir_{k-2}\ ]^T \end{aligned}$$

The control vector elements are the tracking errors between the actual and targeted level of the fiscal and monetary variables at each frequency:

$$u_k = \begin{bmatrix} u_{1,k}\,;\ u_{2,k}\,;\ \ldots\,;\ u_{m,k} \end{bmatrix}^T \tag{17}$$
$$u_{j,k} = Y_{j,k}^{(3)} - Y*_{j,k}^{(3)} \text{ for } m = 1, \ldots, 5; \quad u_{j,k} = ir_{j,k}^{(1)} - ir*_{j,k}^{(1)}\ ; m = 6, \ldots, 10$$

The disturbance vector for stochastic and robust design cases is defined by

$$\omega_k = \begin{bmatrix} \omega_{j,k-1}^{(i)}\,(i=1,\ldots,3);\ \omega_{S,k-1}^{(i)}\,(i=1,\ldots,4) \end{bmatrix}^T \qquad j = 1, \ldots, 5 \tag{18}$$

The disturbance vector is $[0;0]^T$ for the deterministic case.

The ten control variables ($u_{j,k}$) contain the negative of the targeted levels of government spending and the interest rate at each frequency; thus, these target variables are added to state equations for consumption and investment in equation (8), which converts the specification into standard *LQ*-regulator format. The target level of government purchases and the interest rate can be added to $u_{j,k}$ over each frequency range in order to recover the component values for the simulations. These values for government spending and the interest rate at each frequency are also automatically recorded after a one period lag in the 80-equation matrix state-space equation system given by (19).

$$x_{k+1} = A_k x_k + B_k u_k + D_k \omega_k \tag{19}$$

$dim\, x = (80, 1)$ $dim\, u = (10, 1)$ $dim\, \omega = (19, 1)$
$dim\, A = (80, 80)$ $dim\, B = (80, 10)$ $dim\, D = (80, 19)$

3.2. Transformed Deterministic Regulator Design

The deterministic *LQ*-regulator problem either sets the disturbance vector to be zero ($\omega_k = 0$), or alternatively, it defines the disturbance coefficient matrix to be zero ($D_k = 0$). Rewrite expression (14) based on the state space specifications in (15) – (19) so that the objective is to minimize the performance index

$$\min_u J(u) = x_{K+1}^T Q_f x_{K+1} + \sum_{k=1}^{K} \left[x_k^T Q_k x_k + u_k^T R_k u_k \right] \tag{20}$$

subject to

$$x_{k+1} = A_k x_k + B_k u_k \quad ; \quad x(1) = x_1 \tag{21}$$

where the penalty weighting matrices have sizes as follows:

$dim\, Q_f = (80, 80)$ $dim\, Q_k = (80, 80)$ $dim\, R_k = (10, 10)$

The solution is computed by solving the recursive equations (22) and (23) offline in retrograde time, as in Crowley and Hudgins (2015).

$$F_k = \left[B_k^T P_{k+1} B_k + R_k \right]^{-1} B_k^T P_{k+1} A_k \tag{22}$$

$$P_k = Q_k + A_k^T P_{k+1} \left[A_k - B_k F_k \right]; \quad P_{k+1} = Q_f \tag{23}$$

Equation (24) generates the unique optimal feedback control policy in forward time, by using the matrices from equations (22) and (23).

$$u_k^{Optimal} = -F_k x_k \tag{24}$$

The optimal closed-loop state trajectory is given by

$$x_{k+1} = \left[A_k - B_k F_k\right] x_k \quad ; \qquad x(1) = x_1 \tag{26}$$

The solution to the stochastic *LQG* design with perfect state information utilizes the same control equations from (22) and (23), but the state variable trajectory employs equation (19), rather than (26). In equation (24), the control vector must be computed by using equation (22) in conjunction with (19).

4 Simulation Analysis

After experiencing the Great Recession from 2008 – 2009, the U.S. government and the Federal Reserve (Fed) closely monitored the weak recovery throughout the period 2011 to 2015. From 2008 to 2009, the Fed targeted the interest rate, and held it close to 0. After forecasting economic growth of 2.4% for 2016, and an unemployment rate of just below 5%, the Fed raised the federal funds rate from the range of 0 to 0.25% to the range of 0.25% to 0.50% in December, 2015, which was the first interest rate hike since 2006.[1] Within minutes of the official announcement, the U.S. prime rate increased from 3.25% to 3.5%. In the two months after the announcement, the U.S. 3-month Treasury Bill (T-Bill) interest rates increased from 0.23% to over 0.3%.[2] As it enacted the widely anticipated interest rate increase, the Federal Reserve announced a "liftoff" strategy, whereby it aimed to implement a series of gradual interest rate increases across the ensuing quarters. The rate and timing of the interest rate adjustments was to be determined by the Federal Open Market Committee (FOMC) as it monitored macroeconomic performance over the following quarters.

After the U.S. government incurred large budget deficits during the Great Recession, the deficits decreased considerably in 2014 and 2015, where they achieved a level of 2.7% of GDP, and were projected by the Congressional Budget Office (CBO) to fall below this percentage over the next four years.[3] After remaining steady at about 19% of GDP in the first part of the 2000's, the U.S. federal spending increased to 25% of GDP during the Great Recession, but had fallen back to about 20% of GDP by 2015. Thus, the CBO did not forecast the need for strongly expansionary fiscal policy moving forward.

The initial values for the simulations set the state variables in period 1 to correspond to the U.S. data in 2015, quarter 1, measured in billions of constant dollars. The initial values for net taxes, the government deficit, and stock of government debt are respectively given by T_0 = 2,606.03, DEF_0 = 287.17, and $DEBT_0$ = 16,287.7.[4] Following Kendrick and Shoukry (2013) and Hudgins and Na (2016), the tax rate as a percentage of national income is fixed at τ_0 = 0.16. Net exports are held at a constant value of n_0 = –572.2. The quarterly interest rate on the government debt is set at i_0 = .0025, which is 1%

[1] See http://www.federalreserve.gov/newsevents/testimony/yellen20160210a.htm
[2] See https://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/TextView.aspx?data=billrates
[3] See https://www.cbo.gov/publication/49973
[4] The initial value of the national debt is assigned to be 100% of GDP, but the actual value does not affect the simulations, since only the discrepancy between the current value and initial value has an impact on the state equations and the tracking errors. Thus, the initial debt value can be chosen arbitrarily.

per year. Following Crowley and Hudgins (2015), the expectation formation equation (7) sets the weight for the current level of government spending at ϕ = 0.90, and the parameter weight for the adjustment for the national debt differential in the expectation at π = 0.0005 for all frequency ranges in all periods.

The simulations consider a 4-year (16-quarter) planning horizon. The state variables are assigned their initial values at period k = 1. The fiscal authorities choose the optimal level of government spending, and the monetary authorities determine the optimal market interest rate at each frequency range, j = 1, …, 5, starting in period k = 1. So, the government spending and interest rate and policy variables in period k = 1 have their first effects on the state variables in period k = 2. At the end of the planning horizon, the optimal government spending and interest rate in quarter K = 16 determines the levels of consumption, investment, and the other state variables in period K + 1 = 17.

Based on the CBO's initially stronger projected fiscal position, the quarterly target growth rate for government purchases and its modified smooth are set at g = 0.005, which is about 2% per year. The consumption and investment values at each frequency and the aggregate levels are initially set at their 2015 levels at the end of the data set. The initial targets for consumption and investment are set to be 1% above the aggregate consumption and investment levels, respectively. These same initial targets are also respectively assigned to the decomposed series for consumption and investment at each frequency. The quarterly target rates for aggregate and decomposed consumption and investment at all frequency ranges are all fixed at g = 0.006, which is consistent with the Fed's forecasted growth rate of 2.4% per year.

Both fiscal and monetary policymakers face tradeoffs. Due to the economic recovery, the initial crystals for consumption and investment are positive at each frequency range, and thus the decomposed consumption and investment variables are initially above their trends at all frequencies. The fiscal policymakers are attempting to achieve the consumption and investment targets that are predicated upon solid economic growth, while sustaining budget deficit reductions through limited increases government spending at each frequency range. The performance index in (14) also penalizes the government policymakers for large changes in spending between periods. Meanwhile, the central bank is attempting to implement a contractionary stance that nudges the interest rate upward without stifling the tenuous economic recovery. An overly expansionary continuation of expanded credit and depressed interest rates contributes to long-term inflation concerns, while interest rate hikes contribute to a drag on consumption and investment at all frequency ranges. The simulated trajectories from the wavelet-based model produce forecasts for the optimal fiscal and monetary policies over each period of the planning horizon.

The fiscal and monetary policymakers could use the wavelet decompositions to place relatively more importance to consumption and investment performance by placing the most weight on achieving targets at the desired frequency ranges, as explored by Crowley and Hudgins (2015). The simulations in this analysis all assume political cycle targeting, where frequency ranges 3 and 4 get the most weight, followed by frequency 2, then lastly frequency ranges 1 and 5. The political cycle target reflects the cross between the political and economic motivations of the policymakers, with primary emphasis being on the cycles between 2 and 8 years. The simulations consider three cases, where the interactions span the most generally relevant policy stances: (1) dual emphasis on fiscal and monetary policy; (2) fiscal policy emphasis with a heavily weighted interest growth target; and (3) monetary policy emphasis with a heavily weighted fiscal growth target. The values of the performance index coefficients are provide in appendix table A1.

4.1 Dual Emphasis on Fiscal and Monetary Policy: Fiscal and Monetary Compromise

Case (1) assumes that both fiscal and monetary policymakers take an active stance to track consumption and investment, especially at the frequency ranges 3 and 4, but each policymaker is also substantially concerned with tracking its respective policy growth target. In this scenario, both fiscal and monetary authorities are willing to deviate from their own growth path in order to pursue goals in consumption and investment; hence, the fiscal authorities are not forced to attempt stabilization in output without some concessions from the central bank, and the central bank is not propelled to oversee output stabilization without fiscal intervention. Figures 3 panels (a), (b), (c), and (d), show the simulated forecast trajectories for government spending, the short-term interest rate, consumption, and investment, respectively.

Figure 3 panel (a) shows that the aggregate and trend government spending is consistently above the target in all periods. Aggregate government spending is the largest in quarters 1 and 2. It then falls and levels off, before increasing again in the last two quarters of the policy period. The modified smooth trend closely follows frequency 1 spending, with strong growth after period 2. With the exception of frequency range 5 (8 to 16 years) during the early and middle quarters, government spending is also consistently above the target for all frequency ranges. Government spending is the largest at frequency ranges 2, 3, and 4, where the tracking errors in consumption and investment are the most heavily weighted. Spending at frequency range 1 increases towards the end of the horizon, where it surpasses spending at all other frequencies. The simulations call for higher levels of government spending at frequencies 2 and 3 early in the horizon, which then tapers off from the middle to the end of the horizon. The spending at frequency range 4 continually increases, and ends the planning period above the spending at frequencies 2, 3, and 5.

The government spending trajectories are somewhat different than the political cycle targeting scenario in Crowley and Hudgins (2015), which do not include the interest rate, nor any other outlet for monetary policy. In Crowley and Hudgins (2015), the aggregate spending trajectory has a relatively smaller initial increase in the first two periods, and then after a decrease in periods 3 through 5, it maintains a steady increase that closely tracks the target trend thereafter, except for a slight drop in periods 13 and 14. In Crowley and Hudgins (2015), the aggregate spending trajectory exceeds the initial bump in spending by period 7, and ends the horizon at its largest amount, after a substantial increase in the final period. This contrasts with the simulations in figure 3(a), since aggregate spending has a large decline in the seven periods after its large initial increase in periods 1 and 2, and it then levels off throughout the remaining horizon until its increase in the last two periods, which is much smaller than the final increase in Crowley and Hudgins (2015).

Government spending at the individual frequency ranges also follows a different pattern than that in Crowley and Hudgins (2015). In Crowley and Hudgins (2015), the spending a frequency 4 is continuously well-above spending at all other frequencies, followed by spending at frequencies 3, 5, 2, and 1, respectively. In figure 3(a), spending at frequency 4 does not surpass spending at frequencies 2 and 3 until the middle of the horizon, and spending at frequency 5 is consistently the lowest.

Figure 3
(a) *Government Spending* Optimal Forecast Trajectories
Dual Emphasis on Fiscal and Monetary Policy

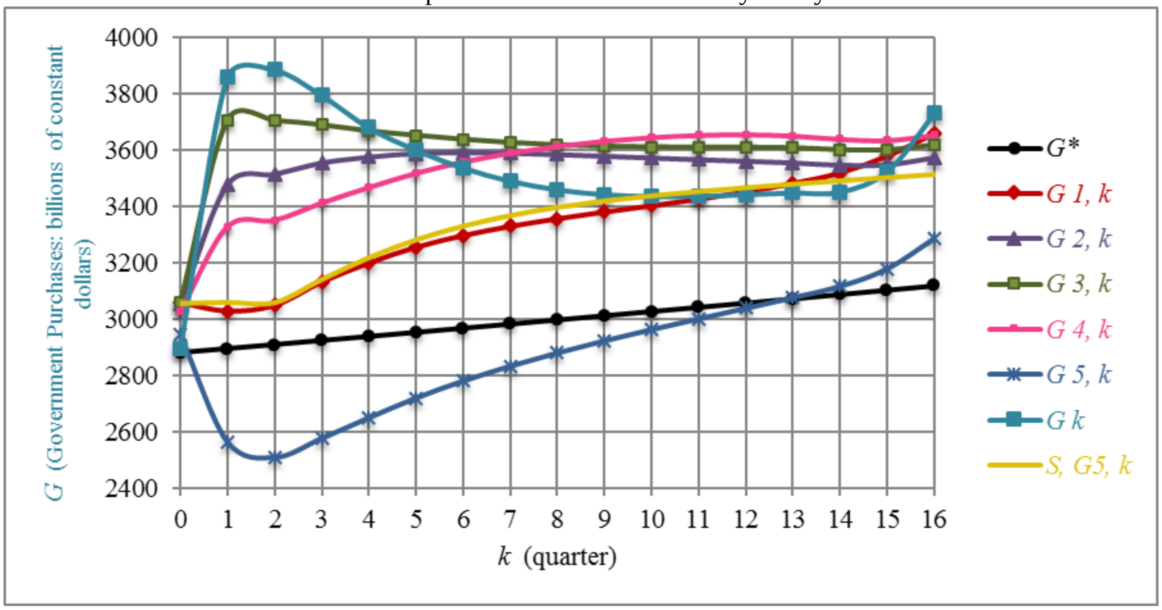


Figure 3
(b) *Short-term Market Interest Rate* Optimal Forecast Trajectories
Dual Emphasis on Fiscal and Monetary Policy

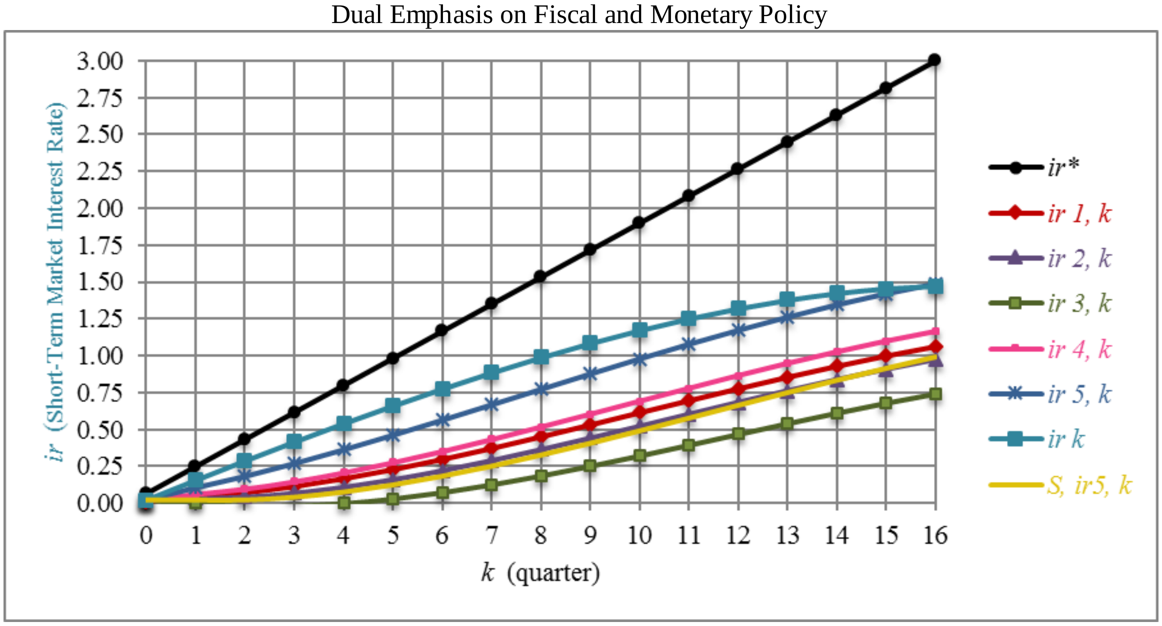


Figure 3
(c) *Consumption* Optimal Forecast Trajectories
Dual Emphasis on Fiscal and Monetary Policy

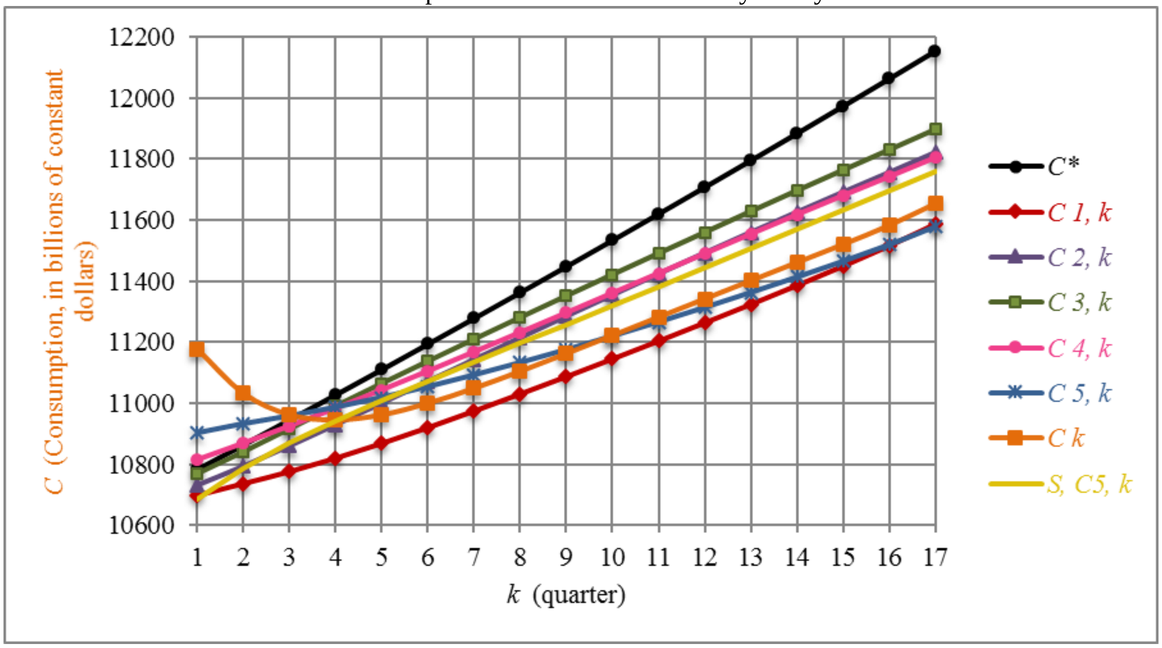


Figure 3
(d) *Investment* Optimal Forecast Trajectories
Dual Emphasis on Fiscal and Monetary Policy

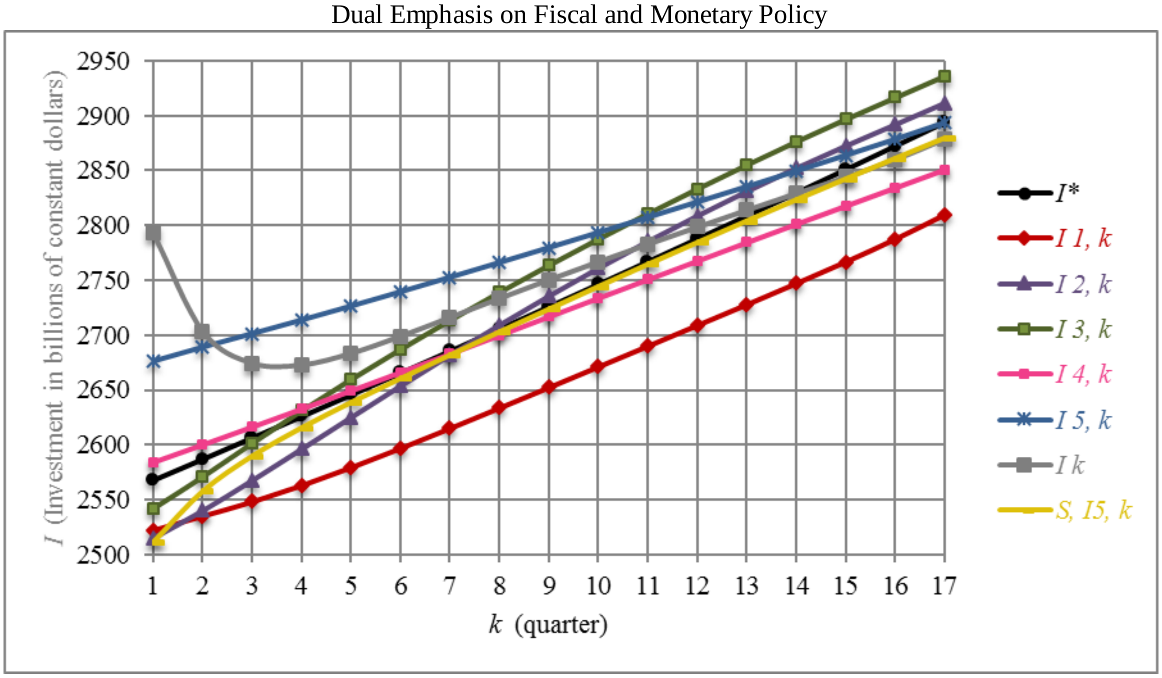


Figure 3(b) shows the trajectories for the short-term market interest rate. In the U.S., the Federal Reserve (Fed) sets the target ranges for the federal funds rate and the discount rate, and in turn, these interest rates indirectly impact market interest rates. The simulations model the Fed's announced liftoff strategy, where the interest rate begins with a target value in period 1 of 0.25%. The central bank seeks to evenly grow the market interest rate to 3% (which is the long-run economic growth target) by the end of the target horizon, which is in quarter 16 (end of year 4). Thus, the target interest rate grows linearly at 0.1833 per quarter. The short-term interest rate is an intermediate target, and not a policy instrument. Thus, the central bank would still conduct policy through discrete changes in the federal funds rate and the discount rate, which would still be slightly below the market interest rate. This can be seen by slight increases in the Treasury bill yields and the 0.25% increase in the prime rate that occurred after the Fed announced its new liftoff strategy at the end of year 2015. As a result, the market interest rate will move along relatively smooth trajectories as it responds to the underlying discrete changes in the federal funds rate.

Although the estimation results used the short-term Treasury bill rate as a proxy, there are many different short-term rates. Thus, the market interest rate in the simulations is meant to serve as a general proxy for an average of the low-risk short-term interest rates, rather than a specific market rate. It is also important to note that, unlike government spending, the Fed in practice has very little control over the short-term interest rate movements at individual frequencies, even though the data allows for wavelet decomposition. The control systems model yields the optimal value of the interest rate at each frequency, as well as the aggregate interest rate; however, the central bank can only set the aggregate market interest rate, rather focusing on cycles at specific frequencies. As a practical matter, the simulations inform the Fed about the timing of the aggregate market interest rate increases, where the decomposed interest rates are not individually implemented. Nonetheless, the computation of the optimal path of the aggregate interest rate is not possible without the decomposition, since these frequency cycles impact consumption and investment at each frequency range. The central bank's optimal strategy is to attempt to align the aggregate market rate with the trajectories in the simulations, so that it is consistent with the underlying frequency decomposition, even though that aggregate rate could have resulted from other decompositions.

In figure 3(b), the aggregate interest rate is initially set at 0.23%, which was the Treasury bill rate before the Fed's announced liftoff strategy. The interest rate increases to 0.29% in period 2, and to 0.54% in period 4. The interest rate achieves a value of about 1% by quarter 8 (the end of year 2), and continues to grow at a decreasing rate, where it levels of at about 1.5% at the end of the planning horizon. Thus, the dual emphasis simulations suggest that the central bank should implement a truncated trajectory that, at the end of the planning horizon, leaves the interest rate at a level that is only about half as large as its targeted interest rate of 3%.

The interest rate is consistently the largest at frequency range 5, followed in order by frequencies 4, 1, 2, and 3. The smooth trend closely aligns with the interest rate at frequency range 2. If the central bank were able target specific frequency ranges, then it would most aggressively track its target with the longer cycles, rather than shorter cycles, thus suggesting a U-shaped curve in the time-frequency domain.

In figure 3 panels (c) and (d), consumption and investment begin above their trends, since the consumption and investment at each frequency have positive crystals, and are experiencing a weak recovery in the aftermath of the Great Recession. The consumption target and the investment target start at the average of the consumption levels and investment levels, respectively, at each of the 5 frequency ranges, and each target grows at a quarterly rate of 0.6% thereafter. Since fiscal policy is targeting a slower growth rate in government spending, and the central bank is tightening monetary policy, both consumption and investment decrease in the early periods. Consumption is consistently the largest at frequency range 3, followed by consumption at frequencies 2 and 4, which trace at almost identical paths. The consumption trajectory for frequency 1 is consistently the lowest. Consumption is initially the largest at frequency range 5, but grows the most slowly along that trajectory. The smooth trend consumption follows a trajectory slightly below frequency range 2. After its initial fall, aggregate consumption eventually recovers to surpass its initial value after period 9, and ends the planning horizon with an increasing upward trend.

The consumption trajectories show some contrast with the political cycle targeting case in Crowley and Hudgins (2015). In Crowley and Hudgins (2015), consumption is consistently the largest at frequency 4, followed by the smooth trend, then consumption at frequencies 5, 3, 1, and 2, respectively. In figure 3(c), consumption is the largest at frequency range 3, followed by 4, 2, 5, and 1, respectively. Thus, unlike Crowley and Hudgins (2015), consumption is the largest at the political cycle emphasis frequencies.

The aggregate investment trajectory initially falls, but eventually experiences fast growth, and ends the horizon just short of the target. Investment is consistently the largest at frequency 3. Investment at frequency 2 has a high rate of growth, and surpasses investment at frequency 3 in the middle of the horizon. At frequency 1, investment is consistently the lowest, while investment at frequency 5 begins high due to the large initial value of its crystal, but slows to converge with the target at the end of the horizon. Thus, the dual emphasis policy employed by the fiscal and monetary authorities leaves consumption somewhat below its final target, but allows investment to achieve a final value very close to its target.

The investment trajectories in figure 3(d) vary greatly from those in Crowley and Hudgins (2015). In Crowley and Hudgins (2015), aggregate investment initially increases to a level that is well-above the trend. It then grows consistently at a slightly smaller rate than the target, and thus has a final value that is still considerable above the target value. This contrasts with figure 3(d), where rather than an initial increase, investment initially decreases for the first 3 periods, before beginning its steady increase. In Crowley and Hudgins (2015), investment at each of the individual frequency ranges follow trajectories that begin well-below the target, and then increase at a large rates before levelling off in the later periods, so that investment at all frequencies ends the horizon well above the target. In contrast, only investment at frequencies 2 and 3 end the horizon above the target in figure 3(d).

4.2 Fiscal Policy Emphasis with Heavily Weighted Interest Growth Target

Case (2) simulates the model when the Fed commits to achieving its liftoff strategy, whereby it regularly undertakes discrete interest rate step increases so that the market interest rate steadily increases, and closely tracks its target. This results in a passive monetary policy that defers the vast majority of the active optimal policy adjustments to the fiscal policymakers. This case is a viable option in countries where government spending and tax policy are highly flexible, such as the United States. Case (2) is currently not a viable case for the Euro area, since the “Fiscal Compact” restricts the ability to use large fiscal stabilization measures. The simulations for case (2) are shown in figure 4.

Fiscal policy is therefore the only variable being aggressively utilized to achieve the growth targets in consumption and investment. Thus, government spending is higher than in case (1) in each period, although the general behavior of the trajectories is similar in both cases, whereby government spending reaches its highest values in quarters 2 and 3, and ends the planning horizon with an increase in quarter 16. The target locked contractionary monetary stance in case (2) results in additional aggregate government spending increase of $350.8 billion over the entire 16-period planning horizon when compared to case (1). This demonstrates that the model is able to compute the forecasted dollar cost of a given non-cooperative monetary action, relative to a dual policy strategy where both policymakers have a large degree of flexibility.

The results from Crowley and Hudgins (2015) show the same contrast with figure 3(a) and figure 4(a). The spending in figure 4(a) further exceeds the target, and thus deviates even more from the trajectories in Crowley and Hudgins (2015). Comparing the results of Crowley and Hudgins (2015), which do not consider an increasing interest rate, with figure 4(a), illustrates the difference in the optimal government spending forecasts that results when the monetary authorities stringently pursue interest rate normalization.

The aggregate interest increase is roughly consistent with a 0.25% increase in quarter 1, quarter 2, and quarter 3. This would be consistent with the Fed following a steady pattern of 0.25% interest rate hikes as part of its normalization policy, regardless of economic conditions. The forecasted market interest rate levels off slightly in the latter part of the horizon, and achieves its final value of 2.7% in quarter 16, which is 0.30% less than the final target value of 3.0%.

Figure 4

(a) *Government Spending* Optimal Forecast Trajectories

Fiscal Policy Emphasis with Heavily Weighted Interest Growth Target

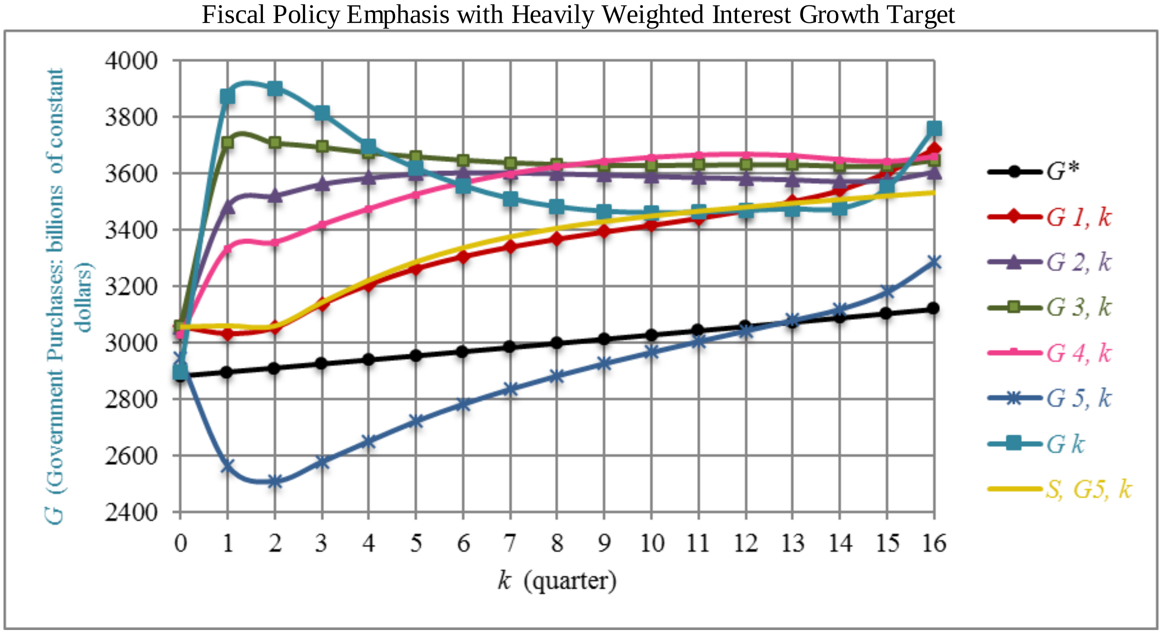


Figure 4
(b) *Short-term Market Interest Rate* Optimal Forecast Trajectories
Fiscal Policy Emphasis with Heavily Weighted Interest Growth Target

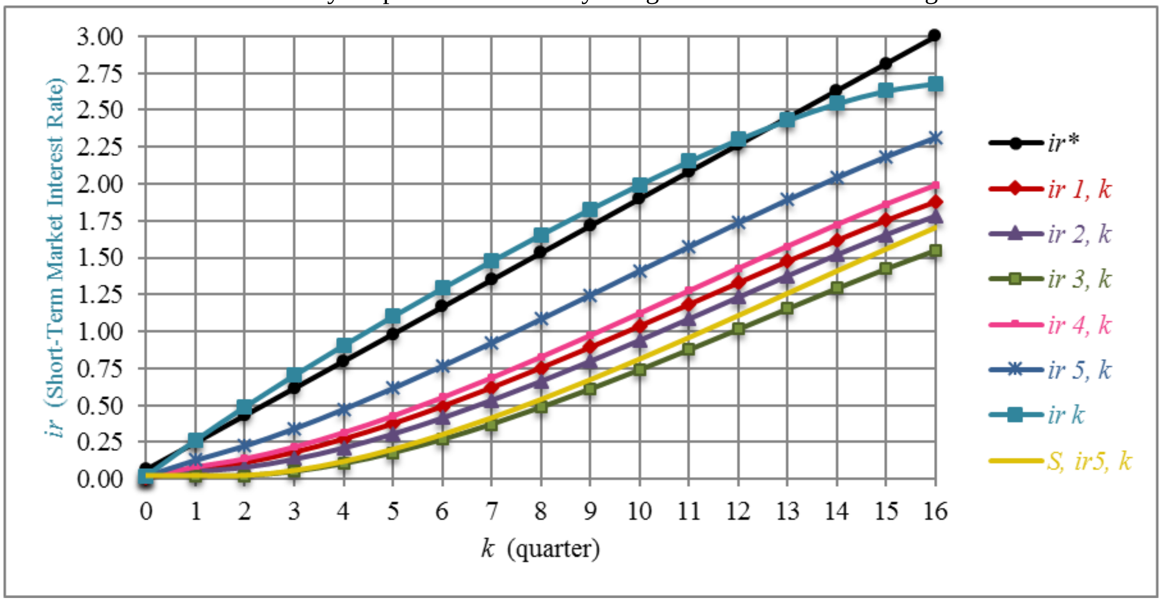


Figure 4
(c) *Consumption* Optimal Forecast Trajectories
Fiscal Policy Emphasis with Heavily Weighted Interest Growth Target

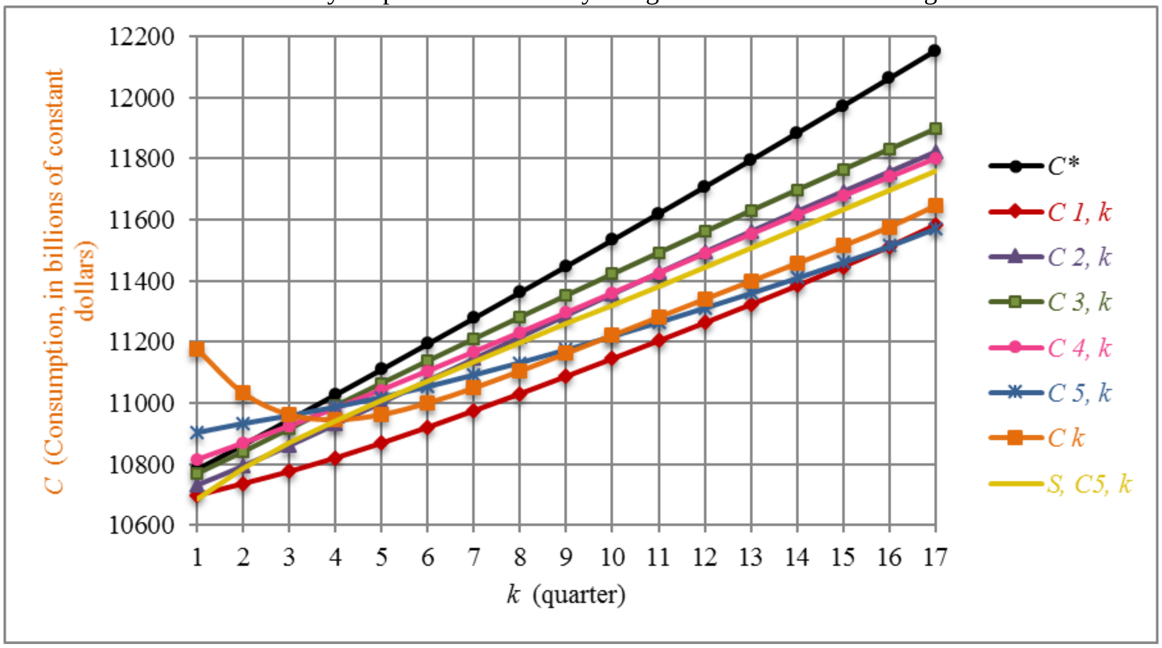
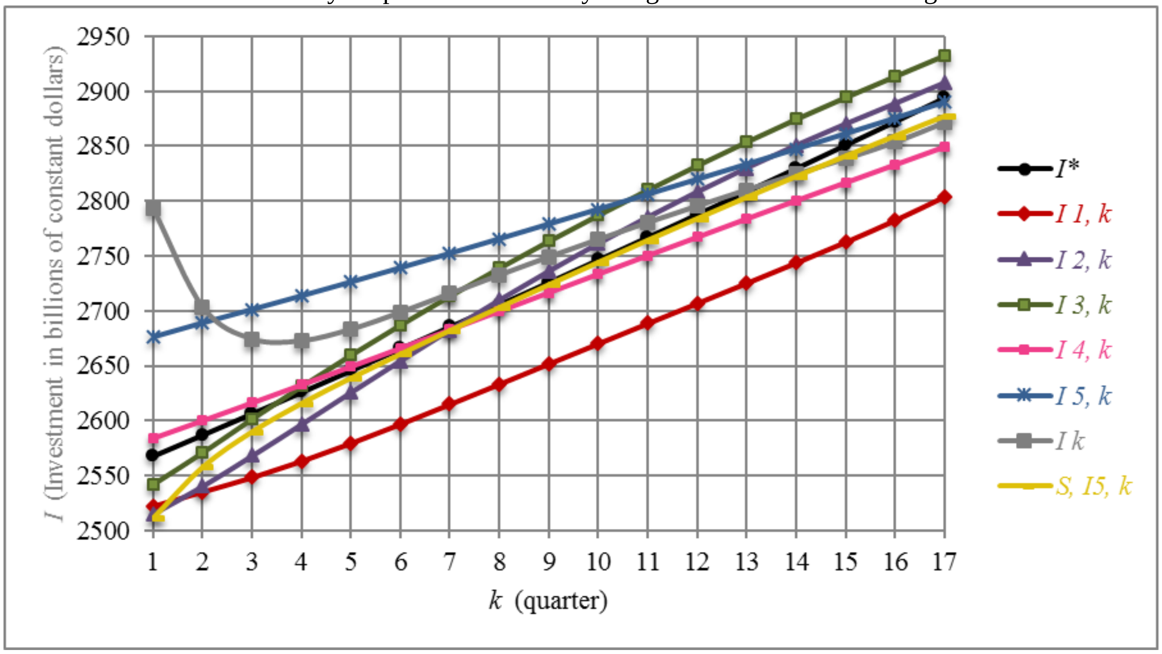


Figure 4

(d) *Investment* Optimal Forecast Trajectories

Fiscal Policy Emphasis with Heavily Weighted Interest Growth Target



This restrictive interest rate policy leads to crowding-out in both consumption and investment, but it is partially counteracted by the more active fiscal policy. The consumption and investment trajectories are lower in case (2) than in case (1). The lowest aggregate consumption levels in period 4 are about the same for both cases, but the final level of consumption in case (2) is about \$6.7 billion less than in case (1). The same pattern holds for aggregate investment, where the lowest level of investment is about the same in the two cases, but the quarter 17 level of aggregate investment in case (2) is about \$7.5 billion dollars less than final period investment in case (1). In summary, this illustrates that fiscal policymakers must be more aggressive when the monetary authorities are not accommodating the fiscal expansion, but after the measurable increases required in government spending, consumption and investment still follow measurably lower forecasted trajectories.

4.3 Monetary Policy Emphasis with Heavily Weighted Fiscal Growth Target

Figure 5 considers case (3), where fiscal policy is restricted so that aggregate government expenditure closely tracks its annual 2% (monthly 0.5%) growth target. Case (3) is generally more consistent with the current situation in the Euro area, due to the constraints imposed by the “Fiscal Compact” and ECB (European Central Bank) initiatives to institute a QE (Quantitative Easing) program. The case does still have relevance for the U.S. though, as if the U.S. were to introduce deficit constraints at the federal level, this scenario would be more appropriate. In case (3), the monetary stabilization policy is more aggressive, and the central bank actively balances between its own target (that steadily increases by 0.1833 per quarter toward 3% at the end of the period), and achieving the targeted values for consumption and investment.

Figure 5

(a) *Government Spending* Optimal Forecast Trajectories

Monetary Policy Emphasis with Heavily Weighted Fiscal Growth Target

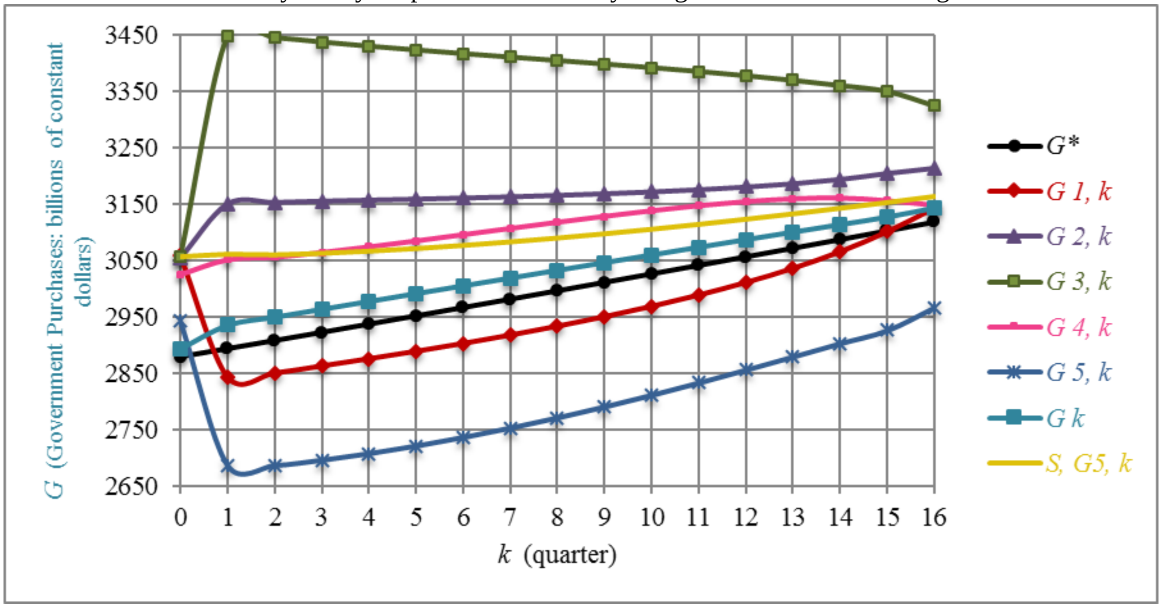


Figure 5

(b) *Short-term Market Interest Rate* Optimal Forecast Trajectories

Monetary Policy Emphasis with Heavily Weighted Fiscal Growth Target

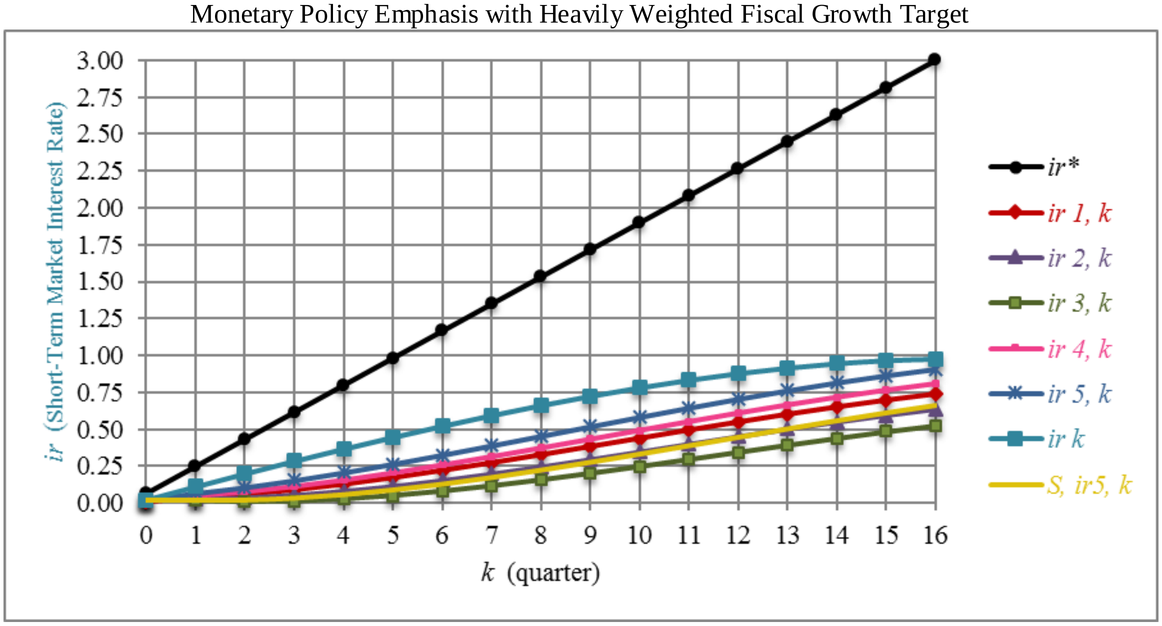


Figure 5
(c) *Consumption* Optimal Forecast Trajectories
Monetary Policy Emphasis with Heavily Weighted Fiscal Growth Target

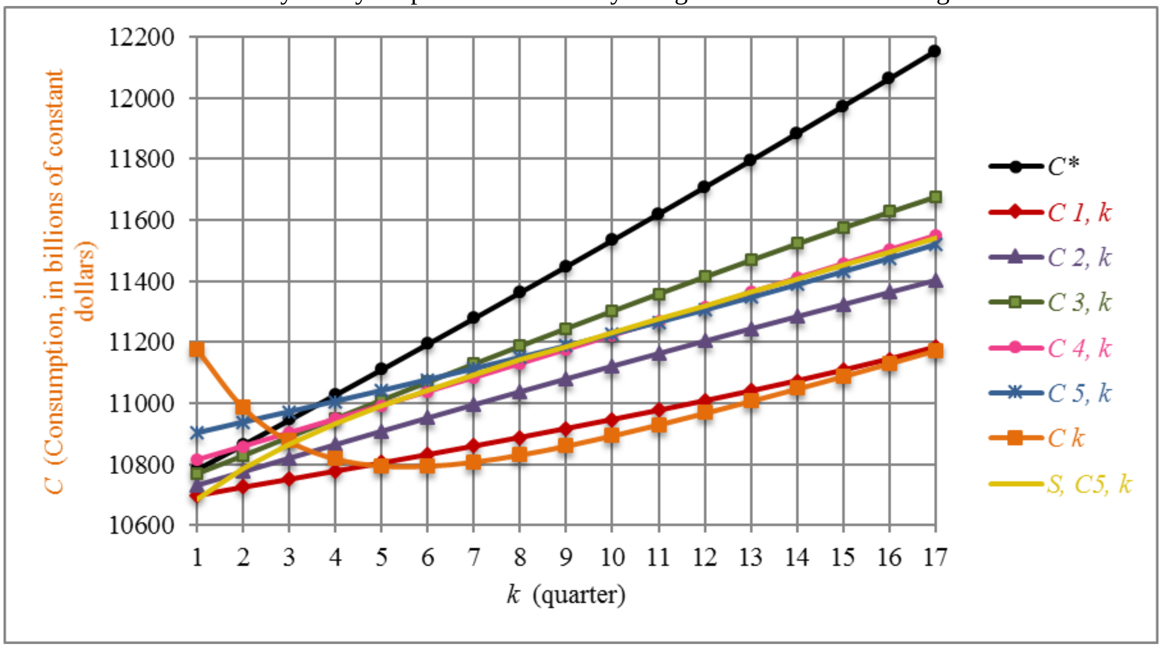
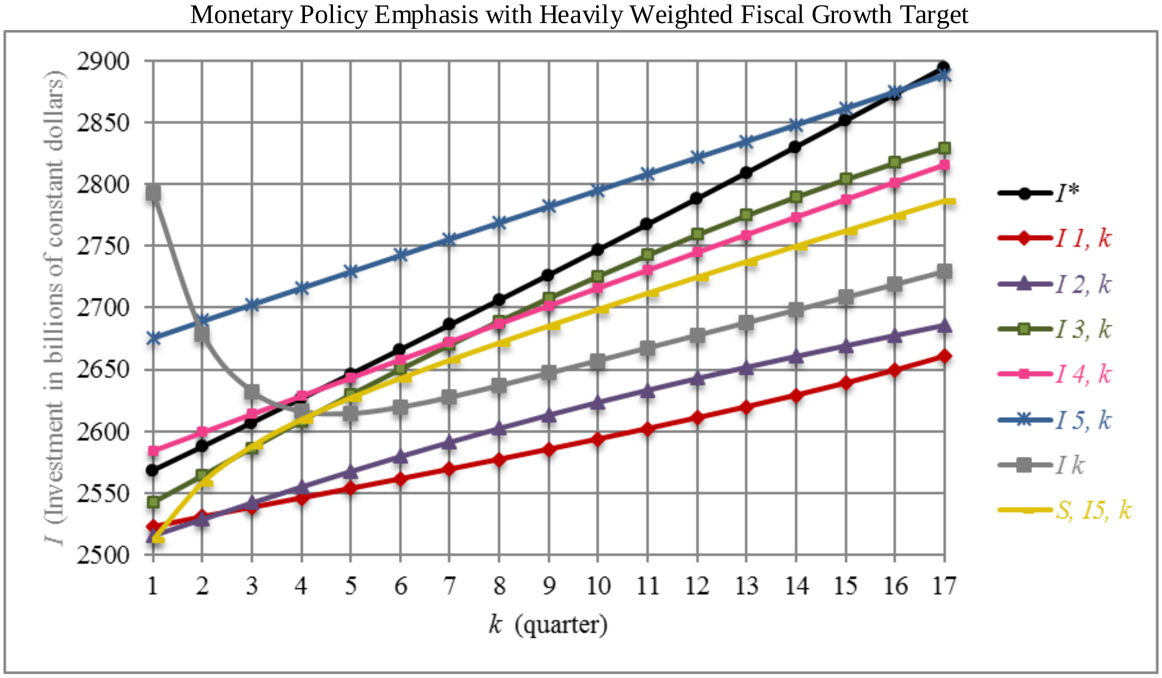


Figure 5
(d) *Investment* Optimal Forecast Trajectories
Monetary Policy Emphasis with Heavily Weighted Fiscal Growth Target



In figure 5(a), the government spending trajectories are considerably different than in the previous two cases. Aggregate government spending closely tracks its target; however, spending consistently slightly above the targeted values due to the fact that the government still has some (diminished) concern for achieving consumption and investment targets. The allocation of spending is skewed toward frequency ranges 3, 2 and 4, which are the more heavily weighted frequencies due to the political cycle targeting. The spending trajectories for frequencies 1 and 5, the shortest and longest cycles, respectively, both encounter substantial declines, and then increase throughout the remainder of the horizon. The total government spending over the horizon is about $8,582 billion less in case (3) than in case (1), and about $8,933 billion less in case (3) than in case (2), which is a substantial measurable savings under fiscal restrictions.

This leads to an aggregate interest rate in figure 5 panel (b) that does not increase above 0.25% until after period 3, and it reaches 0.52% and 0.78% in quarters 6 and 10, respectively. This means that the central bank is increasing the market interest rate by approximately 0.25% increments about every third quarter until period 10. The interest rate growth then slows, and the final interest rate in quarter 16 is about 1%. The final interest rate in case (1) was 1.5%, which falls in the mid-range between the case (3) final interest rate of 1% and the case (2) final interest rate of 2.7%.

The consumption and investment trajectories in case (3) are considerably lower than in the other two cases, since government spending closely tracks its target, and does not adjust to pursue economic growth. After a larger initial drop, aggregate consumption begins to increase, but its value at the end of the horizon is just below its initial value. Aggregate investment also suffers a larger decline in case (3), and has a final value that is about $64 billion below its initial value. Only investment at the heavily targeted frequency 3 closely tracks the target value. Even if the initial values for consumption and investment are at a peak in the business cycle, case (3) demonstrates the downside of relegating the vast majority of active policy to central banks, when deficit reduction is being pursued through limited levels of government spending that track low-growth targets.

5 Conclusion

The wavelet-based optimal control model illustrates the benefits of analyzing the forecasts of different mixes of fiscal and monetary strategies. The analysis demonstrates that simulations can forecast losses in consumption and investment spending that result from restricting either monetary or fiscal policy. Although the monetary authorities can take an expansionary stance and dampen its interest rate hikes when fiscal policy is passively tight, the decline in consumption and investment could be substantial. The findings also demonstrate that when the central bank rigidly follows a restrictive monetary policy of scheduled interest rate increases, this causes consumption and investment to diverge from their targets, and more aggressive increases in fiscal spending at each frequency, and in the aggregate, can be forecasted. The dual approach where fiscal and monetary policy are flexibly balanced can yield substantial gains in consumption and investment, while still exerting an emphasis on the policymaker's objectives.

The model can be used to consider scenarios where the policymakers place more weight on short-term stabilization, or conversely on long-term cycles. The wavelet-based framework is also capable of comparing the effects of different policies under stochastic error structures, a worst-case disturbance structure, and this is the subject of a future paper. The paper could also be extended to include an exploration of monetary policy through the exchange rate and the foreign sector, thereby necessitating a much larger state-space system. Finally, the model could be expanded to explicitly include the price level and inflation considerations. While the current model is not meant to be a complete forecasting tool, it demonstrates some considerable insights that can be gained through wavelet-based optimal control, and opens the paths for a variety of further extensions.

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Appendix

Table A1

Performance Index Coefficients

	Emphasis		
	Dual	Fiscal	Monetary
$q_{1,f}^{(1)}$	100	100	100
$q_{1,f}^{(2)}$	100	100	100
$q_{1,k}^{(1)}$	10	10	10
$q_{1,k}^{(2)}$	10	10	10
$q_{1,k}^{(3)}$	1.0	1.0	500.0
$q_{3,1,f}^{(1)}$	10	10	10
$q_{3,2,f}^{(1)}$	40	40	40
$q_{3,3,f}^{(1)}$	200	200	200
$q_{3,4,f}^{(1)}$	200	200	200
$q_{3,5,f}^{(1)}$	10	10	10
$q_{3,1,k}^{(1)}$	1	1	1
$q_{3,2,k}^{(1)}$	4	4	4
$q_{3,3,k}^{(1)}$	20	20	20
$q_{3,4,k}^{(1)}$	20	20	20
$q_{3,5,k}^{(1)}$	1	1	1
$q_{3,1,f}^{(2)}$	10	10	10
$q_{3,2,f}^{(2)}$	40	40	40
$q_{3,3,f}^{(2)}$	200	200	200
$q_{3,4,f}^{(2)}$	200	200	200
$q_{3,5,f}^{(2)}$	10	10	10
$q_{3,1,k}^{(2)}$	1	1	1
$q_{3,2,k}^{(2)}$	4	4	4
$q_{3,3,k}^{(2)}$	20	20	20
$q_{3,4,k}^{(2)}$	20	20	20
$q_{3,5,k}^{(2)}$	1	1	1
$q_{4,k}$	0.2	0.2	0.2
$q_{5,k}$	0.2	0.2	0.2

	Emphasis		
	Dual	Fiscal	Monetary
$q_{6,1,k}$	0.2	0.2	0.2
$q_{6,2,k}$	0.2	0.2	0.2
$q_{6,3,k}$	0.2	0.2	0.2
$q_{6,4,k}$	0.2	0.2	0.2
$q_{6,5,k}$	0.2	0.2	0.2
$q_{7,k}$	20,000,000	200,000,000	11,000,000
$q_{8,1,k}$	10,000,000	10,000,000	100,000,000
$q_{8,2,k}$	10,000,000	10,000,000	100,000,000
$q_{8,3,k}$	10,000,000	10,000,000	100,000,000
$q_{8,4,k}$	10,000,000	10,000,000	100,000,000
$q_{8,5,k}$	10,000,000	10,000,000	100,000,000
$q_{8,k}$	2,000,000,000	2,000,000,000	2,000,000,000
$q_{2,f}^{(1)}$	10	10	10
$q_{2,f}^{(2)}$	10	10	10
$q_{2,k}^{(1)}$	1.0	1.0	1.0
$q_{2,k}^{(2)}$	1.0	1.0	1.0
$q_{2,k}^{(3)}$	0.2	0.2	0.2
$q_{2,k}^{(4)}$	10,000	10,000	10,000
$r_{1,k}^{(1)}$	1.0	1.0	1.0
$r_{2,k}^{(1)}$	1.0	1.0	1.0
$r_{3,k}^{(1)}$	1.0	1.0	1.0
$r_{4,k}^{(1)}$	1.0	1.0	1.0
$r_{5,k}^{(1)}$	1.0	1.0	1.0
$r_{1,k}^{(2)}$	10,000	10,000	10,000
$r_{2,k}^{(2)}$	10,000	10,000	10,000
$r_{3,k}^{(2)}$	10,000	10,000	10,000
$r_{4,k}^{(2)}$	10,000	10,000	10,000
$r_{5,k}^{(2)}$	10,000	10,000	10,000

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