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## Abstract

Concern that government may not guarantee bank deposits in a future crisis can cause a bank run. The government may break its guarantee during a severe crisis because of time-inconsistent preferences regarding the use of public resources. However, as deposits are withdrawn during the bank run, the size of the government's liability to guarantee the remaining deposits is gradually reduced, which increases the government's incentive to provide the promised guarantee. This in turn reduces depositors' incentive to withdraw, which may explain why bank runs sometimes remain partial. Our model yields an endogenously determined probability and size of a partial bank run. These depend on a common signal as to the future state of the economy, the cost of liquidity provision to banks, and the government's reputational cost of breaking the deposit guarantee. We apply the model to a multi-country deposit insurance scheme, an idea that has been aired in the context of the European Banking Union.

Keywords: Bank crises, Information induced bank runs, Deposit guarantee, Bank regulation

*JEL-codes:* G21, G28

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# 1 Introduction

Empirical evidence suggests that even if bank deposits are protected by a deposit guarantee of a government, a distress that the bank or the government faces might induce depositors to bank run-like large-scale withdrawals of deposits. An example of such behavior was seen in Greece during the period from 2009 to June 2012 as the aggregate amount of Greek bank deposits decreased from €245bn to less than €174bn (Siegel, 2014). It is estimated that only one third of the funds had been withdrawn because of decreasing living standards, and that two thirds either left the country or were stored within Greece outside the Greek banking system (*ibid.*)<sup>1</sup>

The Greek "bank jog", i.e., the withdrawing of deposits only gradually, and only a part of them, would not have made much sense if depositors had during the crisis years 2009-2012 had either no trust at all, or a perfect trust in the deposit guarantee. This is because in the former case it would have been rational to withdraw all deposits immediately, whereas in the latter case there would have been no reason for withdrawing any deposits. These two polar cases are described by the classical bank run model of Diamond and Dybvig (1983), which leads to the conclusion that bank runs are complete whenever they occur. However, subsequent literature has identified a variety of explanations for partial bank runs. For example, Azrieli and Peck (2012) show that a bank run might remain partial when there is more variety in consumer preferences than Diamond and Dybvig (1983) postulated. Ennis and Keister (2010) consider a setup in which depositors withdraw their deposits sequentially and the government can respond to an emerging bank run by changing its policies in order to stop the run.

In this paper, we consider a new explanation for partial bank runs, and present a framework which can be used for studying the relationship between bank run sizes and government (rather than bank) characteristics. Our second contribution is to show that because of the new mechanism causing partial bank runs, our model has a unique equilibrium although we do not make use of the global games framework (cf. Goldstein and Pauzner, 2005). The crux of this mechanism is the following. A bank run may start if depositors fear, based on a common signal of the future state of the economy, that the government may break its guarantee in a severe future crisis. However, as deposits are withdrawn during the run, the government's future liability of guaranteeing the remaining deposits is reduced. This increases the likelihood that the government will actually honor its guarantee, given a fixed reputational cost of breaking the guarantee. As rational depositors anticipate this, no further depositors will run after a certain point. This implies a unique likelihood and size of the bank run in equilibrium.

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<sup>1</sup>Cf also Brown et al. (2013), who have studied bank run-like withdrawals of deposits in Switzerland during the crisis years 2008-2009. They compare the distress which various Swiss banks were facing with the tendency of the depositors of each bank to withdraw their deposits. According to *ibid.* (pp. 2-3), households were 16 percentage points more prone to withdraw some deposits and 11 percentage points more prone to withdraw at least half of their deposits from a distressed than from a non-distressed bank. Cf. discussion below.

More specifically, we consider an economy in which bank deposits have been guaranteed by the government and in which the depositors do not suspect that a bank run could by itself make the government break its promises. Bank runs may nevertheless emerge because depositors suspect that the promised guarantee might fail in some very adverse economic circumstances (a crisis) which might arise in the future. We show that when the switching costs of the depositors are low, large and costly bank runs may be triggered also by the fear of future events which the depositors view as quite unlikely.

Our model has three periods,  $T=0$ ,  $T=1$ , and  $T=2$ . Following Diamond-Dybvig (1983) and most of the subsequent bank run models, we assume that banks make at  $T=0$  investments which mature only at  $T=2$ , although the depositors are allowed to withdraw their deposits already at  $T=1$ . Similarly with most earlier literature, we present a bank run as a situation in which some of the depositors who wish to consume only at  $T=2$  withdraw their deposits already at  $T=1$  out of fear that the deposit might lose its value. However, we depart from earlier models in several other ways. Most importantly, we assume that the deposit guarantee never fails at  $T=1$  but that it may fail at  $T=2$ . In particular, we assume that the government always provides the bank with sufficient liquidity in case of a bank run at  $T=1$ .<sup>2</sup> This implies that, unlike in typical bank run models, banks do not have to liquidate part of their investment project at  $T=1$  for unfavorable terms.

Most earlier models have followed Diamond and Dybvig (1983) in assuming that the deposit contract does not specify the amount of funds that may be withdrawn at  $T=2$ , but simply allows the remaining depositors to receive all the funds that remain in the bank. This assumption implies that there cannot be a bank failure or a deposit guarantee failure at  $T=2$  because there is no specific amount to which the remaining depositors are entitled at  $T=2$ . In contrast, we specify a demand deposit contract which offers a fixed promised payment also in period  $T=2$ . It follows that for sufficiently low returns on investments, banks may fail at  $T=2$ .

We consider a perfectly competitive banking sector whose banks are run by owner-bankers. The owner-banker is at  $T=2$  the residual claimant if the revenue from the bank's investment exceeds its liabilities. A bank fails at  $T=2$  if its liabilities (which include the deposits of the remaining depositors) exceed the revenue from the investment. In this case the government deposit guarantee is activated. However, there is a dynamic inconsistency in the preferences of the government: while at  $T=1$  it is in the interest of the government to promise full deposit guarantee in order to stop bank runs, in a crisis the welfare-maximizing way to spend government wealth might not consist of transferring it to the depositors of failed banks. In order to state this idea more rigorously, we introduce - similarly with Hasman et al. (2011) and Keister (2012) - two uses for government wealth, transfers to consumers and the production of public goods. We assume that if there was no deposit guarantee, and if the depositors were left penniless after bank failures, the welfare-maximizing way to divide government wealth between public goods and transfers to depositors would not consist of paying out the total value of the deposits. Rather, in this case

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<sup>2</sup>See footnote 8 below for a discussion of the justification of this assumption.

the depositors would receive transfers from the government, which would be essentially smaller than the value of their deposits.<sup>3</sup>

However, we further assume that if the government decides to not provide the promised deposit guarantee, it incurs a fixed welfare cost  $F$  which we interpret as a reputational cost from reduced trust in government institutions. The depositors receive at  $T=1$  a signal concerning the state of the economy at  $T=2$ , and their fear of deposit guarantee breakdown is based on the suspicion that paying the cost  $F$  and not providing the deposit guarantee might in a sufficiently bad future crisis be the welfare-maximizing choice by the government. In this way, the fiscal preferences of a country may ultimately affect the credibility of its government's deposit guarantee and hence be a source of partial bank runs if the country's economic prospects turn bleak enough.

In our setting bank runs are always partial. As already sketched above, there is a simple intuition behind this result: as during a bank run at  $T=1$  more and more depositors withdraw their deposits from the bank, the cost from providing the deposit guarantee to the remaining depositors decreases. But as the reputational cost  $F$  from deposit guarantee failure does not decrease in a similar manner, the likelihood of the government's desire to honor the deposit guarantee gradually increases so that the bank run eventually stops. In other words, in our model the attractiveness of withdrawing deposits decreases with the amount of deposits that have already been withdrawn. This monotonous decrease implies that for each signal there can be only one size of a bank run for which the options of withdrawing and not withdrawing yield the same utility. Hence, the probability of a partial bank run and the probability distribution of its size are well-defined in our model, and they may be subject to rigorous comparative static analysis.

The way in which we arrive at a unique equilibrium is, to the best of our knowledge, different from the uniqueness proofs in the earlier literature. In its original form the Diamond - Dybvig (1983) model has two equilibria, the one with a bank run and the one without it, and by itself the Diamond - Dybvig model does not predict which equilibrium will be realized. This will not be changed if one supplements the Diamond-Dybvig model by postulating stochastic returns at  $T=2$  and a signal, which is common to all depositors, and which gives (already at  $T=1$ ) information on the returns at  $T=2$  (cf. Allen - Gale, 1998, pp. 1268-9). However, a unique equilibrium exists in the global games framework of Goldstein-Pauzner (2005) in which each depositor receives at  $T=1$  an inaccurate signal and uses it for deducing a probability distribution for the correct signal and further, for the revenue from the bank's investment at  $T=2$ .<sup>4</sup> A unique equilibrium has been proved to emerge also when the depositors coordinate their behaviour in an exogenously given manner,<sup>5</sup> and

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<sup>3</sup>Cf. Engineer et al. (2013) and Allen et al. (2014) who also consider deposit guarantee schemes which are partial (more specifically, perfectly reliable but such that they cover only a part of the value of the deposits).

<sup>4</sup>Cf also e.g. Takeda (2001), who applies a global games model to international capital flows, Moreno and Takalo (2012) who interpret the dispersion in the signals of the global games framework as a measure of bank transparency, and Silva (2008), who analyzes the effects of the design of partial deposit guarantee schemes on bank run probabilities utilizing a global games framework.

<sup>5</sup>The equilibrium becomes unique when one postulates that the depositors coordinate

when the demand deposit contracts are suitably modified.<sup>6</sup> However, although we restrict attention to real world-style, simple demand deposit contracts and do not postulate any exogenous coordination mechanisms, we are able to prove the existence of a unique equilibrium without resorting to the mathematically more elaborate global games framework.

The paper is organized as follows. Section 2 introduces the main features of the model. Sections 3 and 4 show that the model has a unique equilibrium, allowing us to present a comparative static analysis of the government-related parameters of the model in Section 5. As a simple application of our model, we discuss in Section 6 a pooled deposit insurance scheme, which some have envisaged for the European Banking Union. Section 7 concludes.

## 2 The model

We consider a sequential game whose participants are a government and consumers who deposit their funds in banks. The banks are not agents of the game. This is because the optimal strategy of each bank in the model (i.e., the strategy which maximizes a bank's expected profit) is independent of the choices of both the depositors and the government, and also of those model parameters, whose comparative statics we focus on. This model structure implies that the decisions of banks are exogenously given, and we do not have to include them among the moves of the game.

The timeline of our setup is illustrated by Figure 1. There are three periods ( $T=0$ ,  $T=1$ , and  $T=2$ ) and two forms of wealth, liquid funds which represent all forms of wealth which are easily convertible to each other and to consumption goods, and public resources which will be discussed in more detail in Section 2.3. By assumption, the consumers form a continuum, the size of which we normalize to 1. Each consumer is allocated the amount  $\zeta$  of liquid funds at the beginning of  $T=0$ . The consumers consume only at  $T=2$ , and at  $T=0$  they deposit their funds in the banks, which offer them demand deposit contracts. The demand deposit contract allows the consumers to withdraw their funds at either  $T=1$  or  $T=2$ .

Note that we differ from bank run models such as Diamond and Dybvig (1983) in that there is no division of consumers into patient and impatient, the latter of which would end up withdrawing their deposits for consumption in period  $T=1$ . Hence, in our idealized setting there is no obvious reason

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their behaviour (in accordance with some exogenously given rule) on the basis of a sunspot signal (see e.g. in Peck - Shell, 2003). Cf. also Engineer et al. (2013, p. 534) and the recent paper Dermine (forthcoming). Dermine (forthcoming) considers a Diamond-Dybvig style setting and postulates that the bank has also capital and not just deposits, and that a bank run emerges only when the bank's loan losses are (according to the information which becomes known in the interim period) excessively large, given the bank's amount of capital.

<sup>6</sup>E.g., as Allen - Gale (1998) point out, a unique equilibrium can be found in a Diamond-Dybvig style model with a shared signal if the bank is allowed to make the contract conditional on the information, received at  $T=1$ , concerning the returns that the bank obtains at  $T=2$ .

why the banks should allow the depositors withdraw their deposits already at  $T=1$ . Indeed, the possibility of a bank run would no longer exist in our model if the banks offered the consumers time deposits which allowed withdrawals at  $T=2$  only. We present our justification for not considering time deposits in Appendix 2. Appendix 2 discusses a generalized version of our model with impatient (or, as we call them, early) consumers, and demonstrates that all our results concerning the expected profit-maximizing choice of the banks, and concerning the equilibrium behaviour of consumers and the government, remain unchanged in the generalized setting.

The signal which gives information concerning the future state of the economy (i.e., concerning its state at  $T=2$ ) is denoted by  $\eta$ . The value of  $\eta$  is unknown at  $T=0$ , and by assumption,  $\eta$  has a uniform distribution in  $[0, 1]$  relative to the information which is available at  $T=0$ . The signal becomes known at the beginning of  $T=1$ , after which each consumer makes his choice. This is a choice between two options, switching (withdrawing the deposit already at  $T=1$  and storing it until  $T=2$  by means of an outside option) and staying (withdrawing the deposit only at  $T=2$ ). We denote the share of switching consumers among all consumers by  $\lambda$ . The situation in which  $\lambda > 0$  represents the partial bank run in our model, and  $\lambda$  can be taken as the measure of the size of bank run. The move of the government takes place at  $T=2$ , and it includes the decision whether to honor the promise of a deposit guarantee during a crisis. We postpone the more detailed discussion of the government's move until Section 2.3.

## 2.1 The banks

There is a continuum of identical banks, which operate in a perfectly competitive market and have increasing marginal costs (cf., e.g., Van Hoose, 2010, pp. 32-40). The banks are run by owner-bankers and for simplicity we assume that they have no equity. However, we assume that setting a bank causes the owner-banker a sunk cost  $B$ . The owner-banker is the residual claimant when the revenue from a bank's investments exceed its liabilities at  $T=2$ , and his profit equals the difference of the bank's net worth at  $T=2$  and  $B$ . Each owner-banker aim at maximizing the expected profit of his bank.

The banks offer consumers at  $T=0$  demand deposit contracts, which allow deposits to be withdrawn at either  $T=1$  or  $T=2$ . For simplicity, we assume that a demand deposit contract specifies a single interest rate  $r$ , which applies to all deposits independently of the time at which they are withdrawn. This assumption is motivated by the idea that  $T=1$  does not represent a prespecified point in time but rather a random arrival time of the signal  $\eta$ , which concerns the future state of the economy.

Having received the deposits (each of size  $\zeta$ ) from  $d$  consumers, each bank makes an investment of size  $i$  which matures at  $T=2$ . The cost of the investment  $i$  is given by a function  $\kappa(i)$ . Recalling that the cost  $B$  is a sunk cost which has already been paid when the bank has been set up, we may write the budget

constraint of each owner-banker as<sup>7</sup>

$$\zeta d = \kappa(i) \tag{2.1}$$

By assumption  $\kappa(0) = 0$ ,  $\kappa'(i) > 0$  and  $\kappa''(i) > 0$ . The government gives depositors a deposit guarantee, i.e., a promise that it will pay out the withdrawn deposits, should banks be unable to do so.

As we saw, the signal  $\eta$  becomes known at the beginning of period  $T=1$ , and the case in which some consumers choose to switch because of the signal  $\eta$  is the counterpart of a (partial) bank run in our model. Since investments of banks mature only at  $T=2$ , the banks cannot use the revenue from them for paying out the deposits (if any) which are withdrawn already at  $T=1$ . A traditional full-scale bank run might emerge at  $T=1$ , if the depositors suspected that the government might break its promise of a deposit guarantee already at  $T=1$  if the number of withdrawals at  $T=1$  is sufficiently large. Our model is not concerned with this situation. Rather, we analyze bank runs which are caused by the fear that deposits could not be withdrawn in the future (in our model, at  $T=2$ ). Accordingly, we assume that deposits withdrawn at  $T=1$  will always be fully paid out.<sup>8</sup>

In the real world the central bank would normally provide banks with the needed extra liquidity when the amount of withdrawn deposits turns out to be surprisingly large. However, we do not introduce a central bank into the model. Rather, we assume for simplicity that because of the promised guarantee the government directly provides at  $T=1$  the extra bank liquidity if need be. The closest real world counterpart for the role of the government at  $T=1$  might be a government support program for banks, which is introduced because of a banking crisis, and which has explicit payback clauses.

Hence, we assume that the government lends to banks the funds which they need for avoiding failure at  $T=1$ . More specifically, we assume that when a share  $\ell$  of the depositors of some bank choose to switch, the government always gives the bank at  $T=1$  a liquidity loan of the size

$$g = (\ell d)(1+r)\zeta \tag{2.2}$$

which just suffices for paying out the deposits which are withdrawn at  $T=1$ .

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<sup>7</sup>We do not explicitly consider the possibility that a bank would make an investment  $i$  which is smaller than the largest possible investment, i.e. an investment for which  $\kappa(i) < \zeta d$ . Footnote 16 in the proof of Remark 1 explains why a combination of  $d$  and  $i$  for which  $\kappa(i) < \zeta d$  cannot be the optimal (i.e. expected profit-maximizing) choice of the bank.

<sup>8</sup>This assumption might seem *ad hoc*, since we assume (see Section 2.2) that at  $T=2$  the government may choose not to fulfill its obligations. If one wished to avoid the assumption that deposits may always be withdrawn at  $T=1$ , one could generalize the model by assuming that, similarly with breaking the promise of deposit guarantee at  $T=2$ , also the practice of not providing the necessary liquidity at  $T=1$  was associated with a welfare cost, say  $F_{liq}$ , which was large enough to prevent a deposit guarantee failure at  $T=1$  in equilibrium. The assumption that  $F_{liq} > F$  (where, as we shall shortly see,  $F$  is the cost of breaking the promise of deposit guarantee at  $T=2$ ) is natural, since below the question whether the cost  $F$  should be paid emerges only under exceptionally adverse economic circumstances, and it is natural to assume that the social costs from e.g. decreased trust in government institutions which the disregard for obligations causes are smaller under such circumstances.



As Figure 1 illustrates, the period  $T = 2$  may be thought of as consisting of three separate events. First, the investment of each bank produces a revenue. By assumption, revenue from investment  $i$  is  $\rho i$  where the coefficient  $\rho$  is a random variable whose value is identical for all banks and independent of the values  $i$  that banks have chosen. The probability distribution of  $\rho$  depends on the signal  $\eta$ . We assume that the value of  $\rho$  always belongs to an interval  $[\rho_{\min}, \rho_{\max}]$  and we denote the distribution function of  $\rho$  for each  $\eta$  in  $[0, 1]$  by  $H_\eta(\rho)$ . We postulate that  $H_\eta(\rho)$  is a continuous function in both  $\eta$  and  $\rho$  and that

$$\text{when } \rho_{\min} < \rho < \rho_{\max}, \quad H_\eta'(\rho) > 0 \quad (2.3)$$

In our model larger values of the signal  $\eta$  correspond to better economic situations. Accordingly, we assume that if  $\eta$  and  $\vartheta$  are signals for which  $\eta < \vartheta$ ,  $H_\vartheta$  is second-order stochastically dominant over  $H_\eta$ . As a matter of fact, we introduce the slightly stronger assumption that

$$\text{when } \rho_{\min} < \rho < \rho_{\max}, \quad \frac{dH_\eta(\rho)}{d\eta} < 0 \quad (2.4)$$

The second event in period  $T=2$  is the dissolution of banks. There are two possible cases. First, by definition, a bank fails at  $T=2$  if its assets do not suffice for covering its liabilities. The assets of a bank consist of the revenue  $\rho i$  from its investment. Its liabilities, assuming that the share  $\ell$  of the depositors of the bank have switched, consist of the government liquidity loan  $g$  (2.2) and the deposits of the  $(1 - \ell) d$  staying consumers. Taken together the liabilities amount up to

$$g + d(1 - \ell)(1 + r)\zeta = d(1 + r)\zeta.$$

If a bank fails, its net worth is by definition zero, and its assets are taken over by the government. Second, if a bank does not fail, it pays out both the loan  $g$  and the deposits, and its net worth equals the difference between its assets  $\rho i$  and its liabilities.

The third event in period  $T=2$  in the time line of Figure 1, the move by the government, will be explained in Section 2.3. It takes place after the banks have dissolved and does not affect their profits. The following remark explains why we have not included banks among the agents of our model.

**Remark 1.** The expected profit-maximizing values of the interest rate  $r$ , the investment of each bank  $i$ , and the aggregate investment  $I$  are independent of the choices of the consumers and of the choice by the government.

Given Remark 1, the interest rate  $r$  and aggregate investment  $I$  can be taken to be exogenous constants. Accordingly, we do not include their choice among the moves of our game. We give to  $\zeta$  by normalization (i.e. by a choice of the unit of liquid funds) the value

$$\zeta = \frac{1}{1 + r} \quad (2.5)$$

This normalization implies that the withdrawals of the consumers are of the size  $\zeta(1+r) = 1$ . Now Remark 1 allows us to characterize the aggregate profits of the banks as follows.

**Remark 2.** The banks fail if and only if  $\rho I < 1$ . The aggregate profit of the banks is

$$\Pi = \max\{\rho I - 1, 0\} - \frac{B\zeta}{\kappa(i)}$$

We wish to consider the non-trivial case in which banks sometimes, but not always, fail. Given Remark 2, this will be the case if

$$\rho_{\max} I > 1 > \rho_{\min} I \tag{2.6}$$

## 2.2 The consumers

Each consumer aims at maximizing a utility function  $u(c)$ , which specifies the utility as a function of the consumption  $c$  in period  $T=2$ . By assumption, the utility function  $u$  satisfies the conditions  $u(0) = 0$ ,  $u'(c) > 0$ , and  $u''(c) < 0$  and, by normalization, also the condition  $u'(0) > 1 > u'(1)$ . Together these assumptions imply that there must be a unique  $c_{\min}$  with  $0 < c_{\min} < 1$  for which

$$u'(c_{\min}) = 1 \tag{2.7}$$

We assume that  $c_{\min}$  satisfies

$$\rho_{\min} I > c_{\min} \tag{2.8}$$

The interpretation of  $c_{\min}$  and the significance of the assumption (2.8) will be discussed in Section 3 after Remark 3.

As already explained, in our model a bank run is a situation in which some consumers *switch*, i.e. withdraw their deposits at  $T=1$  and store it until  $T=2$  using an outside option. By assumption, the government cannot influence the level of consumption of the switching consumers. However, the outside option is associated with a switching cost  $\delta$ . Since we made the normalization (2.5), this means that the utility of the consumers who switch is  $u(1 - \delta)$ .

The staying consumers withdraw their deposits only at  $T=2$ , and the government may influence their level of consumption. We denote the consumption level of the staying consumers by  $\tau$  and their utility by  $u(\tau)$ . As will be explained in more detail in Section 2.3, the deposit guarantee can be viewed as a promise by the government that  $\tau$  is at least 1.

We may think of the switching cost  $\delta$  as a cost that a depositor may be ready to incur in order to protect himself from the risk that the deposit could not be withdrawn despite of the government's guarantee. We are mainly interested in the case in which  $\delta$  is relatively small. In this case the consumers might

withdraw their deposits even if they view the probability of deposit guarantee failure to be small. We assume that<sup>9</sup>

$$1 - \delta > c_{\min} \tag{2.9}$$

### 2.3 The Government

The wealth of the government consists originally of a large amount  $Z_0 > 1$  of *public resources*, which may represent all kinds of resources that are specific to the various tasks of the public sector. By assumption, liquid funds of which consumer wealth initially consists may be used for creating public resources, and public resources may be converted to liquid funds which are transferred to consumers. The transfers that we consider are, of course, transfers which are motivated by the deposit guarantee rather than e.g. social transfers which have been planned in advance. The transfers might involve unwanted and unexpected changes in the allocation of resources, such as selling public property at unfavorable "fire sales" prices.

We capture the unplanned and unwanted nature of the transfers by assuming that public resources and liquid funds are not convertible to each other at the same rate in both ways. Rather, while the amount  $\Delta c$  of liquid funds from  $m$  consumers could be transformed into  $m(\Delta c)$  units of public resources, the amount of public resources which is needed for creating the amount  $\Delta c$  of liquid funds for  $m$  consumers is larger. This amount is, by assumption,  $\Gamma m(\Delta c)$ , where the constant  $\Gamma$  satisfies<sup>10</sup>

$$Z_0 > \Gamma > 1 \tag{2.10}$$

At  $T=1$  the only possible source for the extra liquidity (if any) that the government provides the banks with are the government's public resources. According to (2.2) and (2.5) the liquidity loan which the government gives to a bank with  $ld$  switching depositors equals

$$g = (ld)(1+r)\zeta = ld$$

and hence, the aggregate liquidity which is needed at  $T=1$  is simply the aggregate number of switching depositors, i.e.  $\lambda$ . Hence, at the end of period  $T=1$  the public resources of the government will amount up to

$$Z_1 = Z_0 - \Gamma\lambda \tag{2.11}$$

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<sup>9</sup>The significance of the assumption (2.9) will be made clear in section 4 below.

<sup>10</sup>Since we have normalized the continuum of the consumers to size 1, and since according to (2.5) the liquid funds to which each depositor is entitled,  $\zeta(1+r)$ , equal  $\zeta(1+r) = 1$ , the liquid funds which the government might have to pay as deposit guarantee payments cannot exceed 1. The assumption  $Z_0 > \Gamma$ , which is motivated by realism, states that the government is never *strictly speaking* unable to provide the deposit guarantee it has promised to provide, i.e. that the deposit guarantee could not be provided even if the government cancelled all other public expenses and sold all public property.

We assume that the government cannot influence the utility of the switching consumers, which equals  $u(1 - \delta)$ , or the profits of the banks which are given by Remark 2. However, the government may choose an arbitrary value  $\tau$  for the amount of liquid funds that the staying depositors are allowed to consume at  $T = 2$ . All the government's wealth that remains after each remaining depositor has received  $\tau$  in liquid funds will be spent on public resources. Recalling that each depositor is entitled to withdraw  $(1 + r)\zeta = 1$ , we present the deposit guarantee as the government's promise that  $\tau \geq 1$ .

If the government breaks its promise and chooses  $\tau < 1$ , it must by assumption pay a fixed cost  $F$ . This represents the indirect, reputational costs from distrust in government institutions. Although we assume that the government does not tax the profits of the banks when the banks do not fail, we assume, for simplicity, that at  $T=2$  the government might choose a value  $\tau < 1$  even in the absence of bank failures. However, we only consider values of  $F$  which are sufficiently large to prevent this from happening in equilibrium.<sup>11</sup> Formally, we define the cost from deposit guarantee failure to be

$$\widehat{F}(\tau) = \begin{cases} F, & \tau < 1 \\ 0, & \text{otherwise} \end{cases} \quad (2.12)$$

Next we determine the amount of public resources at the end of the game. We distinguish between the case in which  $\rho I < 1$  (in which case, according to Remark 2, banks fail), and the case  $\rho I \geq 1$  (in which case banks do not fail).

If banks fail, their liquid funds amount to  $\rho I$  and are taken by the government. If banks do not fail, they repay  $\lambda$  to the government for the liquidity loans. In addition, the government is able to decide about the use of the funds of the remaining depositors, which are equal to  $1 - \lambda$ . Hence, in this case the liquid funds at the government's disposal are  $(1 - \lambda) + \lambda = 1$ . Summing up, the liquid funds which the government allocates in period  $T=2$  are

$$L = \min\{\rho I, 1\} \quad (2.13)$$

The difference between the available funds  $L$  and the amount of funds that the staying consumers receive is

$$\Delta L = L - (1 - \lambda)\tau = \min\{\rho I, 1\} - (1 - \lambda)\tau \quad (2.14)$$

Liquid funds will be converted to public resources whenever  $\Delta L$  is positive, and public resources are converted to liquid funds if  $\Delta L$  is negative. The amount of public goods that remain for each given  $\tau$  equals

$$Q(Z_1, \Delta L) = \begin{cases} Z_1 + \Delta L, & \Delta L \geq 0 \\ Z_1 - \Gamma(\Delta L), & \Delta L < 0 \end{cases} \quad (2.15)$$

By assumption, the welfare function that the government maximizes depends on the amount of public goods, the utility of the consumers, and the

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<sup>11</sup> The hypothetical case in which  $\tau < 1$  although banks do not fail, can be interpreted as an unexpected introduction of a property tax of size  $1 - \tau$  on bank deposits, or any other government action that reduces the real value of deposits. Remark 4(a) shows that the equilibrium of our model would not change if we did not allow for such government actions (i.e. if the choice of  $\tau$  was constrained to  $\tau \geq 1$  when banks do not fail and to  $\tau \geq 0$  only when banks do fail).

possible welfare costs  $\widehat{F}$  of deposit guarantee breakdown. Because banks' profits are not influenced by government decisions (see Remark 2), for simplicity they are not included in the welfare function. Hence, we define our welfare function as

$$W(\tau) = \widetilde{U}(\tau) + Q(Z_1, L - (1 - \lambda)\tau) - \widehat{F}(\tau) \quad (2.16)$$

where  $\widehat{F}$  is given by (2.12),  $Q$  is given by (2.15), and  $\widetilde{U}$ , the aggregate utility, is given by

$$\widetilde{U}(\tau) = \lambda u(1 - \delta) + (1 - \lambda)u(\tau) \quad (2.17)$$

### 3 The equilibrium transfers by the government

We solve our model by backward induction, starting with the optimal choice of  $\tau$  by the government, when  $\lambda$  (i.e. the share of consumers who have switched at T=1) is known. The following result narrows the set of  $\tau$  values that we must consider.

**Remark 3 (a)** Among the strategies for which  $\tau < 1$ , the welfare maximizing choice is  $\tau = c_{\min}$ .

**(b)** Among the strategies for which  $\tau \geq 1$ , the welfare maximizing choice is  $\tau = 1$ .

To clarify Remark 3, recall that we are considering a problem of dynamic inconsistency in the preferences of a welfare-maximizing government. The government has at T=1 the incentive to stop bank runs with a deposit guarantee, i.e., by promising that each staying depositor will receive at T=2 at least the sum  $\tau = 1$  under all circumstances. However, in (2.16) we defined the welfare function  $W$  so that under sufficiently adverse circumstances the government might prefer to break its promise at T=2.

More specifically, the utility function  $u$  which measures the welfare effects of private consumption in (2.17) is by assumption concave. This implies that the marginal welfare from private consumption decreases with the level of consumption. However, according to (2.16), the marginal welfare from public resources is a constant (more specifically, 1), and according to (2.7) the borderline value of consumption at which both uses of liquid funds yield the same welfare is  $c_{\min}$ . Since  $c_{\min} < 1$ , this implies, as Remark 3(b) states, that the government never allows a higher level of private consumption than the "promised level of consumption", i.e. the value of the deposit  $\tau = 1$ .

Turning to part (a) of Remark 3, we note that according to assumption (2.6) the liquid funds of the government suffice for making the payment  $c_{\min}$  to each staying depositor even in the worst case scenario (i.e. when  $\rho = \rho_{\min}$ ).

This simplifying assumption implies that when the deposit guarantee fails, the welfare-maximizing choice of  $\tau$  is the choice for which the marginal welfare from liquid funds is identical in their two possible uses, i.e. the choice  $\tau = c_{\min}$ .<sup>12</sup>

The problem of the government reduces now to the choice between two alternatives,  $\tau = 1$  and  $\tau = c_{\min}$ . The choice  $\tau = 1$  corresponds to providing the promised minimum deposit guarantee if banks fail, and with not interfering with bank deposits when banks do not fail, whereas the choice  $\tau = c_{\min}$  corresponds to a deposit guarantee breakdown. Using (2.12) and (2.14)-(2.17), we conclude that in the former case welfare is given by

$$W_{DG} = \lambda u(1 - \delta) + (1 - \lambda) u(1) + Q(Z_1, \min\{\rho I, 1\} - (1 - \lambda)) \quad (3.1)$$

and that in the latter case welfare is given by

$$W_{NDG} = \lambda u(1 - \delta) + (1 - \lambda) u(c_{\min}) + Q(Z_1, \min\{\rho I, 1\} - (1 - \lambda)c_{\min}) - F \quad (3.2)$$

In these formulas  $\min\{\rho I, 1\} = \rho I$  when the banks fail and  $\min\{\rho I, 1\} = 1$  when banks do not fail. As already stated, the government can choose  $\tau = c_{\min}$  even if the banks do not fail, but from now on we assume that the fixed cost  $F$  is sufficiently large to prevent this from happening in equilibrium. We postulate that

$$F > 1 - c_{\min} \quad (3.3)$$

**Remark 4.** Assume that  $F$  satisfies (3.3).

(a)  $W_{DG} > W_{NDG}$  whenever  $\rho I \geq 1 - \lambda$ .

(b) In particular, it is welfare-maximizing for the government to let the depositors withdraw their full deposits if banks do not fail.

When banks fail, the condition  $\rho I \geq 1 - \lambda$  of Remark 4(a) means that the liquid funds of banks suffice for the remaining deposits but not all the liabilities of banks (which include also the liquidity loans from period  $T=1$ ).<sup>13</sup> Recalling (2.8), we observe that when  $\rho I < 1 - \lambda$ , the welfare difference  $W_{DG} - W_{NDG}$  between provision and non-provision of deposit guarantee is given by the function

$$\Xi(\lambda, \rho) = (1 - \lambda) [u(1) - u(c_{\min})] - \Gamma(1 - \lambda - \rho I) - (\rho I - (1 - \lambda)c_{\min}) + F \quad (3.4)$$

and that  $\Xi(\lambda, \rho)$  is increasing in  $\rho$  for each  $\lambda$ . Hence, keeping  $\lambda$  fixed and remembering that the possible values of  $\rho$  lie in  $[\rho_{\min}, \rho_{\max}]$ , we are left with two possible cases. If the deposit guarantee can fail for the given  $\lambda$ , there is

<sup>12</sup>Observe that if (2.6) is not valid and the realized value of  $\rho$  satisfies  $\rho I < c_{\min}$ , the government might have to convert public resources to liquid funds in order to produce the part  $\tau = c_{\min}$  of the promised deposit guarantee payments. In this case a choice  $\tau < c_{\min}$  might be optimal, and Remark 3(a) might not be valid.

<sup>13</sup>In other words, when  $F$  satisfies (3.3), the deposit guarantee can according to Remark 4 fail only when its provision involves a costly transformation of public resources to liquid funds.

a borderline value  $\bar{\rho}(\lambda) > \rho_{\min}$  which is such that the deposit guarantee fails when  $\rho < \bar{\rho}(\lambda)$ , but not otherwise. This borderline value is the unique solution of

$$\Xi(\lambda, \bar{\rho}(\lambda)) = 0$$

Secondly, it may also be the case that  $\Xi(\lambda, \rho)$  is positive for all possible values of  $\rho$ . Now the deposit guarantee never fails for the given  $\lambda$ . In this case we define  $\bar{\rho}(\lambda) = \rho_{\min}$ .

We summarize the result that we have just proved as the following theorem.

**Theorem 1.** Assume that the share of switching consumers is  $\lambda$ , and let  $\rho$  (i.e. the return from investment) obtain an arbitrary value from  $[\rho_{\min}, \rho_{\max}]$ . Then the deposit guarantee will fail if and only if  $\rho < \bar{\rho}(\lambda)$ . The level of consumption  $\tau$  of the staying consumers is  $\tau = c_{\min}$  if the deposit guarantee fails and  $\tau = 1$  otherwise. In particular, if  $\bar{\rho}(\lambda) = \rho_{\min}$ , the deposit guarantee never fails and  $\tau = 1$  for all possible values of  $\rho$ .

## 4 The equilibrium size of the bank run

Having found the equilibrium choice  $\tau$  of the government for each  $\lambda$  (i.e. each size of the bank run) and  $\rho$  (i.e. the return on banks' investments), we now solve for the equilibrium value of  $\lambda$ . When the consumers make their choice between staying and switching at  $T=1$ , they know the signal  $\eta$  but not  $\rho$ . At  $T=1$  the probability distribution of  $\rho$  is given by  $H_{\eta}(\rho)$ . We conclude from Theorem 1 that when the signal is  $\eta$  and the share of switching depositors is  $\lambda$ , the expected utility of the staying depositors is

$$(E_{\eta}u_B)(\lambda) = H_{\eta}(\bar{\rho}(\lambda))u(c_{\min}) + (1 - H_{\eta}(\bar{\rho}(\lambda)))u(1) \quad (4.1)$$

**Remark 5.** The expected utility from staying is an increasing function of the size of the bank run  $\lambda$ . More rigorously, the function  $\bar{\rho}$  satisfies

$$\frac{d\bar{\rho}(\lambda)}{d\lambda} \leq 0,$$

the expected utility from staying satisfies

$$\frac{d(E_{\eta}u_B)(\lambda)}{d\lambda} \geq 0,$$

and both inequalities are strict whenever  $\bar{\rho}(\lambda) > \rho_{\min}$ , i.e. whenever  $\lambda$  is such that there is a positive probability of a deposit guarantee failure.

Remark 5 states that the expected utility from staying increases with the number of switching consumers. This result has a simple intuition. The costs from deposit guarantee payments decrease if the number of the staying consumers decreases, and this increases the incentive of the government to make the promised payments. This shows up as a decrease in the range of values of  $\rho$  for which deposit guarantee breaks down. Consequently, the probability of deposit guarantee failure decreases and the expected utility from keeping deposits in the bank increases.

While a larger value of  $\lambda$  corresponds to smaller costs from deposit guarantee payments at  $T = 2$ , it also corresponds to larger costs for the government during the period  $T = 1$  when the government provides the banks with the extra liquidity that they need in case there is a bank run. However, at  $T = 2$  when the decision regarding the promised payments is made, the costs of a bank run at  $T = 1$  are sunk costs from the perspective of the government, and hence they do not affect the governments decision at  $T = 2$ .

If the share of the switching consumers  $\lambda$  was close to one, the costs from making the deposit guarantee payments of period  $T = 2$  would be small, and for sufficiently large value of  $\lambda$  they would necessarily be smaller than the fixed cost  $F$ . Hence, if  $\lambda$  were sufficiently large, consumers would know for sure that the deposit guarantee will not fail and that the utility from staying is  $u(1)$ . On the other hand, as we saw in Section 2.2, the utility from switching is always  $u(1 - \delta)$ . Hence, in case of a very large bank run staying is preferable to switching, implying that such bank runs cannot occur in equilibrium.

We formulate this argument more precisely in the following remark.

**Remark 6.** If all consumers switch (i.e. if  $\lambda = 1$ ), staying yields a larger expected utility than switching. Hence, in equilibrium  $\lambda < 1$  and bank runs are always partial.

Remark 6 implies that there can be at most two types of equilibria, the "partial bank run" equilibrium and the "no bank run" equilibrium. The following theorem describes these two equilibria.

**Theorem 2.** For each value of the signal  $\eta$ , there is unique share  $\lambda^* = \lambda^*(\eta)$  of switching consumers which corresponds to an equilibrium of the model. The equilibrium value of  $\lambda^*$  satisfies one of the following conditions:

(a)  $0 < \lambda^* < 1$  so that there is a partial bank run, and  $\lambda^*$  is determined by  $(E_\eta u_B)(\lambda^*) = u(1 - \delta)$ ,

i.e., by the condition that staying and switching yield the same expected utility.

(b)  $\lambda^* = 0$  so that there is no bank run and  $\lambda^*$  satisfies

$(E_\eta u_B)(\lambda^*) \geq u(1 - \delta)$ ,

i.e., the condition that staying yields at least the same expected utility as switching.

Combining Theorems 1 and 2, it follows that the conditional probability of deposit guarantee failure (given signal  $\eta$ ) is

$$P_{DGF}(\eta) = H_\eta(\bar{\rho}(\lambda^*(\eta))) \quad (4.2)$$

We conclude from Theorem 2 and (4.1) that the probability  $P_{DGF}(\eta)$  has a maximum value.

**Theorem 3.** When there is bank run after the signal  $\eta$ , the probability  $P_{DGF}(\eta)$  of a deposit guarantee failure given  $\eta$  has the value  $P_{DGF,\max}$ , which equals



$$P_{DGF,\max} = \frac{u(1) - u(1 - \delta)}{u(1) - u(c_{\min})}$$

The value  $P_{DGF,\max}$  is the maximum value that deposit guarantee failure probability can have in equilibrium.

The failure probability of the deposit guarantee,  $P_{DGF}(\eta)$ , has the maximum,  $P_{DGF,\max}$ , because if  $P_{DGF}(\eta)$  exceeded  $P_{DGF,\max}$ , the utility from staying would be less than  $u(1 - \delta)$  and the staying consumers would have an incentive to switch. This would reduce the number of staying consumers until the deposit guarantee failure probability had sunk to  $P_{DGF,\max}$ . When there is a partial bank run after a signal  $\eta$ , the condition that the probability of deposit guarantee failure must obtain its maximum value, i.e., condition

$$H_\eta(\bar{\rho}(\lambda^*(\eta))) = P_{DGF,\max} \quad (4.3)$$

suffices to determine the size  $\lambda_B^*(\eta)$  of the bank run, which corresponds to  $\eta$ .

The two types of equilibria, the partial bank run equilibrium and the no bank run equilibrium, are illustrated by Figures 2 and 3. Figures 2(a) and 3(a) show the part of the graph of the distribution function  $H_\eta(\rho)$  which corresponds to the low values of  $\rho$ . The probability  $P_{DGF,\max}$ , which is shown on the vertical axis, is the probability of deposit guarantee failure in all partial bank run equilibria.

The figures 2(a) and 3(a) depict also the value  $\rho = 1/I$ , which is the borderline between the values of  $\rho$  for which banks fail and do not fail, as well as the value  $\hat{\rho}$  which we define by

$$\hat{\rho} = H_\eta^{-1}(P_{DGF,\max})$$

We may conclude from (4.3) that if signal  $\eta$  corresponds to a partial bank run equilibrium, the value  $\hat{\rho}$  must be the "borderline" value

$$\hat{\rho} = \bar{\rho}(\lambda^*(\eta))$$

between the values of  $\rho$  for which the deposit guarantee fails and does not fail.

Figure 2(b) shows the curve of the "borderline value"  $\bar{\rho}(\lambda)$  of the return on investment as a function of size of the bank run  $\lambda$ . In accordance with Remark 5,  $\bar{\rho}(\lambda)$  is strictly decreasing in  $\lambda$  except for the region in which  $\bar{\rho}(\lambda) = \rho_{\min}$  and a deposit guarantee failure is impossible. The values of  $\lambda$  for which  $\bar{\rho}(\lambda) > \hat{\rho}$  are cases in which switching is more attractive than staying, and the values of  $\lambda$  for which  $\bar{\rho}(\lambda) < \hat{\rho}$  are cases in which the opposite is true. There is just one value of  $\lambda$  for which  $\bar{\rho}(\lambda) = \hat{\rho}$ , so we may conclude this value must be  $\lambda^*(\eta)$ ; i.e., the size of the bank run in equilibrium.

Figure 3(a) illustrates the case in which the signal  $\eta$  is "better" so that the distribution function  $H_\eta(\rho)$  implies smaller probabilities of low return on investment,  $\rho$ . Accordingly,  $\hat{\rho}$  is larger than in Figure 2(a). As Figure 3(b) shows, the value  $\hat{\rho}$  is now so large that  $\bar{\rho}(\lambda) < \hat{\rho}$  for all values of the size of the bank run  $\lambda$ . This implies that staying is more attractive than switching for all values of  $\lambda$ , and that  $\lambda^*(\eta) = 0$ , i.e., that in equilibrium all consumers stay.

Note that the size of a bank run depends on the relative but not on the absolute magnitudes of  $P_{DGF,\max}$  and  $H_\eta(\rho)$ . Hence, if both  $P_{DGF,\max}$  and the

function  $H_\eta$  were multiplied by the same positive constant (however small), the equilibrium size of the bank run would not change. It should also be observed that the value  $\rho = 1/I$ , which indicates "borderline revenue" between the regions in which banks fail and do not fail, does not directly affect the bank run size  $\lambda^*(\eta)$ , and that  $\rho = 1/I$  could be much larger than  $\hat{\rho}$  in a bank run equilibrium. Hence, there could be a bank run even when  $H_\eta(\hat{\rho})$  (the equilibrium probability of deposit guarantee failure) is very small, not just in an absolute sense, but also in comparison with  $H_\eta(1/I)$  (the probability of bank failure). In other words, there can be a bank run also when in equilibrium the conditional probability of deposit guarantee failure on the condition that there will be a bank crisis is very low.

These results have a clear intuition because according to Theorem 3 small values of  $P_{DGF,\max}$  correspond to small switching costs  $\delta$ . Clearly, even if it were extremely unlikely that the deposit guarantee fails, the depositors might have an incentive to withdraw their deposits from banks if also the switching costs are extremely small.

We formulate this observation as a separate remark.

**Remark 7.** An arbitrarily small danger of a deposit guarantee failure can cause a large partial bank run, if the switching costs of depositors are sufficiently small.

The result (4.3) has the following implication.

**Remark 8.** The size of the bank run is a decreasing function of the signal  $\eta$ . More rigorously, the number of switching consumers  $\lambda^*(\eta)$  satisfies  $\frac{d\lambda^*(\eta)}{d\eta} \leq 0$  and this inequality is strict for the signals  $\eta$  which correspond to a bank run (i.e. for which  $\lambda^*(\eta) > 0$ ).

In the comparative static analysis of the next section we restrict attention to the non-trivial case in which some signals, but not all signals, cause a bank run. Remark 8 implies that in this case the values of  $\eta$  for which there is a bank run form an interval  $[0, \bar{\eta})$  which consists of all signals which are worse than some borderline signal  $\bar{\eta}$ . We assume that

$$0 < \bar{\eta} < 1 \tag{4.4}$$

## 5 The Welfare and Stability Effects of Deposit Guarantee

We now turn to the comparative statics of our model. The model contains two parameters,  $\Gamma$  and  $F$ , that characterize the deposit guarantee scheme, and below we will analyze their effects on financial stability and welfare.

We use the expected value of the welfare function (2.16) as our measure of welfare, and we measure financial stability by the probability of a bank run and by the size of the bank run (if any) which corresponds to each signal  $\eta$ . In addition, we also consider comparative statics of the deposit guarantee failure probability. We shall not discuss the comparative statics of bank failure probability because according to Remark 2 the probability of bank failures is simply the probability that  $\rho < 1/I$  and hence, independent of the government-related parameters of the model.

## 5.1 Deposit Guarantee and Financial Stability

As we saw at the end of the previous section, a bank run occurs whenever  $\eta < \bar{\eta}$ . We have assumed that  $\eta$  has a uniform probability distribution over  $[0, 1]$  and hence, the probability of a bank run is simply

$$P_{BR} = \bar{\eta} \tag{5.1}$$

The size of the bank run is measured by the number of switching consumers,  $\lambda^*(\eta)$ . The function  $\lambda^*$  depends on the reliability of the government's promise of the deposit guarantee because, as we saw when discussing Figures 2 and 3, in case of a bank run  $\lambda^*$  is implicitly defined by the condition

$$H_\eta(\bar{\rho}(\lambda^*(\eta))) = P_{DGF, \max}$$

This condition states that staying and switching yield the same utility. Note that the function  $\bar{\rho}$  depends on  $\Gamma$  and  $F$  because  $\bar{\rho}$  is implicitly defined by

$$\Xi(\lambda, \bar{\rho}(\lambda)) = 0$$

and according to (3.4),  $\Xi$  depends on  $\Gamma$  and  $F$ . Definition (3.4) leads easily to the following result.

**Theorem 4.** (a) The unconditional probability of bank a run is strictly increasing in  $\Gamma$ , i.e., the funding costs of the deposit guarantee, and strictly decreasing in  $F$ , i.e., the indirect (reputational) social cost of deposit guarantee failure. In other words,

$$\frac{d\bar{\eta}}{d\Gamma} > 0 \text{ and } \frac{d\bar{\eta}}{dF} < 0$$

(b) When there is a bank run after the signal  $\eta$ , its size is strictly increasing in  $\Gamma$  and strictly decreasing in  $F$ . I.e., if  $\eta < \bar{\eta}$ , then

$$\frac{d\lambda^*(\eta)}{d\Gamma} > 0 \text{ and } \frac{d\lambda^*(\eta)}{dF} < 0.$$

Turning to the comparative statics of the probability  $P_{DGF}(\eta)$  of deposit guarantee failure, we observe that  $P_{DGF}(\eta)$  is not affected by changes in  $F$  and  $\Gamma$  when there is a bank run after the signal  $\eta$ , since in this case  $P_{DGF}(\eta)$  is determined by the condition (4.3). When there is no bank run,  $P_{DGF}(\eta)$  is according to (4.2) given by

$$P_{DGF}(\eta) = H_\eta(\bar{\rho}(0))$$

where  $\bar{\rho}(0)$  is implicitly defined by

$$\Xi(0, \bar{\rho}(0)) = 0$$

It follows easily from (3.4) that in this case the probability  $P_{DGF}(\eta)$  increases with  $\Gamma$  and decreases with  $F$ .

## 5.2 Expected welfare and the small switching costs approximation

Consider now the comparative statics of expected welfare. As we saw above, welfare depends on two random variables, the signal  $\eta$  which determines the size of the bank run, if any, at  $T = 1$  and  $\rho$  which determines the return on the investments of banks and the subsequent choices by the government. In this section we are mostly interested in the expected welfare  $E_\rho W$ , calculated by taking the expectation with respect to the distribution of  $\rho$  while keeping the signal  $\eta$  fixed.

From (2.16) we obtain

$$E_\rho W = (E_\eta u_B)(\lambda^*(\eta)) + E_\rho Q - P_{DGF}(\eta) F \quad (5.2)$$

Recalling (4.1), we observe that the first term of (5.2),  $(E_\eta u_B)(\lambda^*(\eta))$  expresses the expected value of staying in equilibrium. This value must be identical with the expected value of the utility of all consumers (because in a "no bank run" equilibrium all consumers stay, and because in a bank run equilibrium the utility of staying and switching is identical). Since the size of the continuum of consumers is normalized to one,  $(E_\eta u_B)(\lambda^*(\eta))$  is also equal to the expected value of the aggregate utility  $\tilde{U}(\tau)$ . From Theorem 2,

$$(E_\eta u_B)(\lambda^*(\eta)) = \begin{cases} u(1 - \delta), & \eta < \bar{\eta} \\ H_\eta(\bar{\rho}(0)) u(c_{\min}) + (1 - H_\eta(\bar{\rho}(0))) u(1), & \eta \geq \bar{\eta} \end{cases} \quad (5.3)$$

The second term of (5.2),  $E_\rho Q$ , is the expected value of  $Q(Z_1, \Delta L)$ , i.e., the amount of public goods. According to (2.11)  $E_\rho Q$  is given by

$$E_\rho Q = \int_{\rho_{\min}}^{\rho_{\max}} Q(Z_0 - \Gamma \lambda^*(\eta), \Delta L) dH_\eta(\rho) \quad (5.4)$$

where  $\Delta L$  is the (positive or negative) amount of liquid funds that the government converts to public resources at  $T=2$ . According to (2.14)

$$\Delta L = \min\{\rho I, 1\} - (1 - \lambda) \tau$$

where  $\tau$  is the consumption level of the staying depositors. By Theorem 1  $\tau$  is equal to  $c_{\min}$  when  $\rho < \bar{\rho}(\lambda^*(\eta))$  and to 1 when  $\rho \geq \bar{\rho}(\lambda^*(\eta))$ . Using (2.15), we may rewrite (5.4) as

$$E_\rho Q = Z_0 - \Gamma \lambda^*(\eta) + \int_{\rho_{\min}}^{\rho_{\max}} \Delta Q(\lambda^*(\eta), \rho) dH_\eta(\rho) \quad (5.5)$$

Here the function  $\Delta Q$  expresses the contribution of the events of period  $T=2$  to the amount of public goods, and it is given by

$$\Delta Q(\lambda, \rho) = \begin{cases} \rho I - (1 - \lambda) c_{\min} & \rho < \bar{\rho}(\lambda(\eta)) \\ -\Gamma(1 - \lambda - \rho I), & \bar{\rho}(\lambda) \leq \rho < (1 - \lambda)/I \\ \rho I - (1 - \lambda), & (1 - \lambda)/I \leq \rho < 1/I \\ \lambda, & \rho \geq 1/I \end{cases} \quad (5.6)$$

In the first case of this piecewise definition,  $\rho < \bar{\rho}(\lambda(\eta))$ , both the bank and the deposit guarantee fail. The government takes over the assets  $\rho I$  of the failed bank, makes the transfer  $c_{\min}$  to each of the  $1 - \lambda$  staying consumers, and uses the rest of the liquid funds for producing public goods. The second and the third case of (5.6) correspond to situations in which banks fail but the deposit guarantee does not fail. When  $\bar{\rho}(\lambda) \leq \rho < (1 - \lambda)/I$ , the liquid funds  $\rho I$  that the government receives from the failed banks do not suffice for paying out the remaining  $1 - \lambda$  deposits. In this case the government pays out the rest of the deposits which total  $1 - \lambda - \rho I$  by converting public resources into liquid funds. In the third case,  $(1 - \lambda)/I \leq \rho < 1/I$ , the assets of the banks suffice for the remaining deposits but not for all liabilities which include also liquidity loans from  $T=1$ . In this case the government takes over the funds of the banks, pays out the remaining  $1 - \lambda$  deposits, and uses the rest of the banks' assets for producing public goods. Finally the last case of (5.6),  $\rho \geq 1/I$ , represents the case in which banks do not fail and pay out both the remaining deposits to the depositors and the liquidity loans, which amount up to  $\lambda$ , to the government.

When (5.5) and (5.6) are combined with Theorem 4(b) which states that the size  $\lambda^*(\eta)$  of a bank run increases with the funding costs  $\Gamma$  of the deposit guarantee payments, we see that an increase of  $\Gamma$  decreases welfare in two ways: directly, because an increase in  $\Gamma$  decreases the amount of available public goods, and indirectly because also an increase in the size of the bank run decreases welfare.

We now consider the parameter  $F$  which represents the social cost of a deposit guarantee failure. Theorem 4(b) implies that an increase in  $F$  decreases the size of bank runs. According to (5.5) and (5.6) this has a positive effect on  $E_\rho Q$ , the expected utility from public goods when  $\eta$  corresponds to a bank run equilibrium. On the other hand, when the signal  $\eta$  corresponds to the no bank run equilibrium (i.e. when  $\eta \geq \bar{\eta}$ ), (3.4) implies that an increase in  $F$  reduces the danger of a bank run and (5.3) implies that the decrease in the danger of a bank run increases expected consumer utility.<sup>14</sup> According to (5.2) each of these effects increases expected welfare indirectly if  $F$  increases, but a larger cost  $F$  also decreases welfare directly whenever the cost  $F$  is paid.

In what follows we do not assess the relative magnitudes of these opposing direct and indirect effects, since we focus on the case in which both the probability  $P_{DGF}(\eta)$  of deposit guarantee failure and the switching cost  $\delta$  of the consumers are quite low. As Remark 7 states, large and costly partial bank runs can occur also in this case. It is clear that in this case the indirect positive welfare effect from an increase of  $F$ , caused by the reduction in the probability and size of bank runs, must be much larger than the direct negative effect from paying the cost  $F$ .

To prove this point more rigorously, we now introduce a small switching costs approximation for welfare calculations. This approximation applies to the situation in which both the deposit guarantee failure probability  $P_{DGF}(\eta)$

<sup>14</sup>More rigorously, since (3.4) implies that  $\Xi(0, \rho)$  is increasing in both  $\rho$  and  $F$ , we may conclude that the borderline value  $\bar{\rho}(0)$  - which is defined by  $\Xi(0, \bar{\rho}(0)) = 0$  - is decreasing in  $F$ . However, now (5.3) implies that in a "no bank run" equilibrium, i.e. when  $\eta \geq \bar{\eta}$ , the aggregate utility from consumption, i.e.  $(E_\eta u_B)(\lambda^*(\eta)) = (E_\eta u_B)(0)$ , is increasing in  $F$ .

and the switching costs are negligibly low. We conclude from Theorem 2 that

$$u(1 - \delta) \leq (E_\eta u_B)(\lambda^*(\eta)) \leq 1$$

so that when  $\delta \approx 0$ , the expected utility of the consumers is approximately

$$(E_\eta u_B)(\lambda^*(\eta)) \approx 1$$

Further, the assumption  $P_{DGF}(\eta) \approx 0$  immediately implies that also the last term of (5.2), the expected welfare loss from deposit guarantee failure, is negligibly small. Hence, when  $\delta$  and  $P_{DGF}(\eta)$  are both very small, the expected welfare (5.2) is approximately given by

$$E_\rho W \approx 1 + E_\rho Q$$

Now the welfare loss from a bank run (which affects the term  $E_\rho Q$  in (5.2)) is not necessarily small.

Using also (5.5) and (5.6), we define the small switching costs approximation for expected welfare as

$$(E_\rho W)_{SSC} = 1 + Z_0 - \Gamma \lambda^*(\eta) + \int_{\rho_{\min}}^{\rho_{\max}} (\Delta Q)_{SSC}(\lambda^*(\eta), \rho) dH_\eta(\rho) \quad (5.7)$$

Here  $(\Delta Q)_{SSC}$  is a slightly simplified version of the function  $(\Delta Q)$  which is defined by (5.6). Since now the probability with which  $\rho < \bar{\rho}(\lambda^*(\eta))$  - i.e. the probability with which the deposit guarantee fails - is negligibly small, we define  $(\Delta Q)_{SSC}$  by

$$(\Delta Q)_{SSC}(\lambda, \rho) = \begin{cases} -\Gamma(1 - \lambda - \rho I), & \rho < (1 - \lambda)/I \\ \rho I - (1 - \lambda), & (1 - \lambda)/I \leq \rho < 1/I \\ \lambda, & \rho \geq 1/I \end{cases} \quad (5.8)$$

It is clear that  $(E_\rho W)_{SSC}$  is independent of  $F$  for the signals for which no bank run occurs (i.e. when  $\lambda^*(\eta) = 0$ ) and an increasing function of  $F$  in case of a bank run (i.e., for the signals for which  $\lambda^*(\eta) > 0$ ), since according to Theorem 4(b) the size of the bank run  $\lambda^*(\eta)$  decreases as  $F$  increases.

We may now summarize the above results as the following theorem.

**Theorem 5. (a)** An increase in  $\Gamma$  (the funding costs of the deposit guarantee) always decreases expected welfare.

**(b)** An increase of  $F$  (the indirect social cost of deposit guarantee failure) increases  $(E_\rho W)_{SSC}$  when  $\eta < \bar{\eta}$  but does not affect  $(E_\rho W)_{SSC}$  when  $\eta > \bar{\eta}$ . Hence, when both the switching costs of consumers and the danger of deposit guarantee failure are very small, the welfare effects of an increase of  $F$  are positive in case of a bank run and negligibly small otherwise.

## 6 Application: Pooled Deposit Insurance

The European banking union consists of the Single Supervisory Mechanism (SSM), Single Resolution Mechanism (SRM), and more harmonized deposit

insurance schemes. The idea has also been aired that national deposit insurance schemes might perhaps, at a later stage, develop into a shared deposit insurance scheme. The shared scheme might be accomplished by either simply pooling the existing European national deposit guarantee schemes, or by introducing a two-tier system, in which a European Reinsurance fund would cover the losses from only those bank crises that are too large to be covered by the national deposit guarantee scheme (see e.g. Gros, 2013). Motivated by these plans and developments, we now apply our model to the pooling of the separate deposit insurance schemes. We ask how pooling in the light of our model affects bank stability and welfare in countries (henceforth, "the union") which participate in it.

A pooled deposit insurance scheme might be expected to improve financial stability in countries in which citizens have more trust in a guarantee provided by the union than in a guarantee provided by domestic authorities. The opposite might well be the case in the countries in which domestic authorities are trusted more. To analyze the effects of pooling deposit guarantee schemes, we now think of the union as consisting of two parts: C (for "core") in which the trust for domestic institutions is high, and part P (for "periphery") in which trust is lower. We view C and P as countries in the sense of our model, i.e. we assume that they correspond to different specifications for the two government-related parameters of the model. Accordingly, we define  $\Gamma = \Gamma_C$  and  $F = F_C$  in case of country C, and  $\Gamma = \Gamma_P$  and  $F = F_P$  in case of country P.

We model distinct national deposit insurance schemes as a situation in which the two countries participate simultaneously in the game of our model, and we assume that the signal  $\eta$ , the realized value of  $\rho$ , and the switching cost  $\delta$  are identical in the two countries. We assume that

$$\Gamma_C \leq \Gamma_P, \quad F_C > F_P \tag{6.1}$$

The assumption that  $F_C > F_P$  is motivated by the idea that the loss of confidence in a highly trusted institution decreases welfare more than the loss of confidence in a less trusted one. Similarly, the assumption that  $\Gamma_C \leq \Gamma_P$  is motivated by the idea that a more trusted government might find it easier (or at least not more difficult) to find funding for unexpected expenses than a less trusted government.

When we apply our model separately to either of the two countries, we normalize the size of the country to one. The model yields for each signal  $\eta$  the size of a bank run for country C and for country P. These are denoted by  $\lambda_C^*(\eta)$  and  $\lambda_P^*(\eta)$ , respectively. As we saw in Section 4, the values of  $\eta$  for which a bank run occurs; i.e., the values of  $\eta$  for which the function  $\lambda^*$  is positive, form an interval  $[0, \bar{\eta})$  in each country. We denote the "borderline signal" which separates the "bank run" and "no bank run" regions of the countries C and P by  $\bar{\eta}_C$  and  $\bar{\eta}_P$ , and note that according to Theorem 4

$$\bar{\eta}_C < \bar{\eta}_P \tag{6.2}$$

In other words, the values  $\bar{\eta}_C$  and  $\bar{\eta}_P$  divide the set  $[0, 1]$  of possible signals into three parts, the region  $[0, \bar{\eta}_C)$  in which there is a bank run in both countries, the region  $[\bar{\eta}_C, \bar{\eta}_P)$  in which there is a bank run just in the periphery, and

the region  $[\bar{\eta}_P, 1]$  in which there is no bank run in either country. Theorem 4 implies that

$$\text{when } \eta < \bar{\eta}_P, \lambda_C^*(\eta) < \lambda_P^*(\eta) \quad (6.3)$$

i.e., that when there is a bank run in the periphery, the bank run (if any) in the core is smaller.

We denote the relative sizes of the economies C and P by  $n_C$  and  $n_P$ . By normalization,  $n_C + n_P = 1$ . If C and P have separate deposit guarantee schemes, the share of the switching consumers within the whole union is

$$\bar{\lambda}^*(\eta) = n_C \lambda_C^*(\eta) + n_P \lambda_P^*(\eta) \quad (6.4)$$

Similarly, we denote  $(E_\rho W)_{SSC}$ , the value of the expected welfare, calculated using the small switching costs approximation, in the two countries by  $(E_\rho W)_{SSC,C}$  and  $(E_\rho W)_{SSC,P}$ , respectively. When the deposit guarantee schemes are kept separate, the value of  $(E_\rho W)_{SSC}$  which applies to the whole union is given by

$$\overline{(E_\rho W)}_{SSC} = n_P (E_\rho W)_{SSC,P} + n_C (E_\rho W)_{SSC,C} \quad (6.5)$$

We model the pooled deposit guarantee scheme as if it were a single country. In other words, we define single parameters  $\Gamma = \Gamma_{EU}$  and  $F = F_{EU}$  for the whole union. This specification yields for each signal  $\eta$  a size of the bank run which we denote by  $\lambda_{EU}^*(\eta)$ , and an approximate expected welfare  $(E_\rho W)_{SSC,EU}$ .

We contrast the values  $\bar{\lambda}^*(\eta)$  which measure average financial stability in the union when deposit guarantee schemes are kept separate, with the values  $\lambda_{EU}^*(\eta)$  which correspond to a pooled deposit guarantee scheme. Similarly, we contrast the welfare measures  $\overline{(E_\rho W)}_{SSC}$  and  $(E_\rho W)_{SSC,EU}$ . We divide our discussion of the pooling into two parts. First, we ask how welfare and bank stability would be affected in each country if the value of  $\Gamma$  was set to  $\Gamma = \Gamma_{EU}$  in them. Second, we ask how stability and welfare change when  $F$  is given in both countries the value  $F_{EU}$ , assuming that the  $\Gamma$  values are identical.

To proceed, we must specify how the parameter values  $\Gamma = \Gamma_{EU}$  and  $F = F_{EU}$  are determined by the corresponding values  $\Gamma_C$  and  $\Gamma_P$ , and  $F_C$  and  $F_P$ , of the two countries. As we saw above, in the simplified world of our model which contains no deposit insurance funds the parameter  $\Gamma$  expresses the size of a cost which must be paid whenever banks fail or run out of liquidity. In each case, the unified scheme has two sources of funding for the promised guarantee as we assume that the union can use the funding sources of each country. Assuming that the cheaper funding source is chosen in equilibrium, we can conclude that

$$\Gamma_{EU} = \min \{\Gamma_P, \Gamma_C\} = \Gamma_C$$

Using Theorems 4(b) and 5(a) we trivially arrive at the result that if the cost  $\Gamma$  is different in the core and the periphery, then setting it to the pooled



scheme value of  $\Gamma = \Gamma_{EU}$  would increase both bank stability and welfare in the periphery without decreasing them in the core.<sup>15</sup>

A more interesting analysis concerns the case of the social cost,  $F$ , of deposit guarantee failure. The following result describes the effects of pooling when the values of  $\Gamma$  are identical in the two countries before the pooling and the social cost  $F_{EU}$  in the pooled system is

$$F_{EU} = n_P F_P + n_C F_C \quad (6.6)$$

i.e., when  $F_{EU}$  is the average of the corresponding values of the separate systems.

**Theorem 6.** Assume that the funding cost of deposit guarantee payments is identical in the two countries, i.e. that  $\Gamma_P = \Gamma_C = \Gamma_{EU}$ .

(a) The probability of a bank run in the pooled system,  $\bar{\eta}_{EU}$ , is between its probabilities in the core and the periphery when the deposit guarantees are kept separate. In other words,  $\bar{\eta}_C < \bar{\eta}_{EU} < \bar{\eta}_P$ .

(b) The pooling reduces the size of bank runs. More precisely, for each signal  $\eta$

$$\lambda_{EU}^*(\eta) \leq \bar{\lambda}^*(\eta)$$

and the above inequality is strict when the signal  $\eta$  is between  $\bar{\eta}_C$  and  $\bar{\eta}_P$ , but not otherwise.

(c) The pooling increases aggregate expected welfare when switching costs and deposit guarantee failure probability are low. More precisely,

$$(E_\rho W)_{SSC,EU} \geq (E_\rho W)_{SSC}$$

and the above inequality is strict for each signal  $\eta < \bar{\eta}_P$  (i.e. whenever there is a bank run in the periphery).

Part (a) of the Theorem 6 is a straightforward implication of Theorem 4(a). We also observe that the inequality of (b) is trivial when  $\eta \geq \bar{\eta}_P$ , since in this case there are no bank runs in either region and both sides of the inequality are zero. When  $\eta \leq \bar{\eta}_C$ , the part (b) follows in a straightforward manner from the linearity assumption (6.6) and the fact that also the function  $\Xi$ , which we defined by (3.4), is linear in  $F$ . The more interesting part of Theorem 6(b) concerns the case in which  $\bar{\eta}_C < \eta < \bar{\eta}_P$ , i.e., in which there is a bank run in the periphery but not in the core when the guarantee schemes are kept separate. Theorem 6 states that regardless of our linearity assumptions pooling reduces in this case the average size of the bank run.

To understand the intuition of this result, we interpret the cost  $F$  from deposit guarantee failure as a measure of the trust that depositors have in the guarantee. When the signal  $\eta$  satisfies  $\bar{\eta}_C < \eta < \bar{\eta}_P$ , the amount of trust measured by  $F_P$  in the periphery does not suffice to prevent the bank run,

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<sup>15</sup>Extending the interpretation of our model, we might think of  $\Gamma$  as representing also the costs of transfers funded by sovereign debt. Under this interpretation, the assumption  $\Gamma_{EU} = \min\{\Gamma_P, \Gamma_C\}$  would mean that the funding costs of euro bonds would be on the level of the funding costs of the most creditworthy euro countries. A more realistic setting would presumably yield the result that  $\Gamma_{EU}$  is only close to but not identical with  $\Gamma_C$ . In this case the change in the  $\Gamma$  values due to pooled scheme would not constitute a Pareto improvement, as welfare in the core would be somewhat decreased.

but in the core there is "excess trust" in the guarantee. That is, the value  $F_C$  prevents the bank run, but even a somewhat smaller value of  $F'_C < F_C$  would suffice to prevent it. In this case a more equal division of trust can decrease the average size of bank runs, even if the pooled deposit guarantee is not more trustworthy than the separate guarantees are on the average.

The positive welfare effect stated in Theorem 6(c) is caused by both the lowered average size of partial bank runs and the more efficient use of funding sources in a pooled system. As we saw, in our model the deposit guarantee has two sources of funding, public resources and liquid funds from the banks' investments. Even if the  $\Gamma$  values of the two countries should be identical, there is an inefficiency in the use of these funding sources when one of the governments converts liquid funds into public resources while the other government is involved in the opposite conversion, since by assumption, the resources do not convert to each other at the same ratio. Such inefficiencies will be eliminated by pooling, since pooling makes the size of the bank runs (if any) identical in the pooled area.

Since pooling reduces the expected welfare in the core, it does not constitute a Pareto improvement. However, part (c) of the theorem shows that, at least in theory, the reduced expected welfare could be compensated with transfers from the periphery to the core, because the aggregate expected welfare is increased by pooling. Although we do not explicitly consider deposit guarantee fees paid by the beneficiaries in our model, they might in principle be used to achieve the Pareto improvement in the pooled deposit guarantee scheme.

## 7 Concluding Remarks

We have considered bank runs which are caused by the suspicion that, in spite of its promises, the government might not protect deposits during a severe future crisis. In this setting bank runs are quite different from those in traditional models, in which they occur in the absence of a deposit guarantee and are caused by the fear that a shortage of liquidity might lead to an immediate bank failure.

In the absence of a deposit guarantee traditional models of bank runs (e.g. Diamond and Dybvig, 1983) have two equilibria: the one in which no one has an incentive to withdraw his deposits (except for immediate consumption needs) because other depositors do not withdraw theirs, and the other in which all depositors withdraw simultaneously.

In contrast, we have assumed that the government always bails out banks by providing a liquidity loan if banks have a liquidity shortage in the absence of a crisis. Nonetheless, as the government may break its deposit guarantee in a severe crisis because it prefers to spend scarce public resources on more vital public goods, bank runs may still occur.

Our model provides a simple explanation for why in the presence of a government deposit guarantee bank runs are gradual and partial as has been recently often observed; e.g., in the euro area. As deposits are withdrawn during

a bank run, the government's future liability of guaranteeing the remaining deposits is gradually reduced. This increases the government's incentive to honor its promised deposit guarantee because the government's reputational cost of breaking its guarantee is fixed (by assumption). This in turn decreases the remaining depositors' incentive to withdraw, given that they face a switching cost from withdrawing. Eventually, there is a unique point when the bank run stops. This depends on the common signal that the depositors receive concerning the future state of the economy, the cost of the government's liquidity provision to banks, and the government's reputational cost of breaking the deposit guarantee. It is worth emphasizing that with this mechanism our model has a unique equilibrium in the presence of bank runs without making use of the global games framework (cf. Goldsteing and Pauzner, 2005).

A complementary interpretation of the model is the following. By providing liquidity to banks so that banks can weather a partial run on deposits, the government reduces the future cost of its own deposit guarantee liability. The cost of the government's liquidity provision before a crisis has been assumed to be a sunk cost. In effect, this implies that the larger the liquidity provision is, the lower is the likelihood that the government would break its guarantee in a future crisis because the reputational cost of doing that is fixed. This immediately suggests an extension to our analysis; to consider the case in which the liquidity provision is not a sunk cost. Rather, it might increase sovereign debt which might contribute to the government's financial distress at the same time as the deposit guarantee breaks down in a crisis. We conjecture that this could lead to a full bank run in the setting of our model.

Motivated by the ideas aired in the context of the European banking union, we also applied the model to study the potential costs and benefits of a common deposit guarantee system between two countries; a core and a periphery. The results hinge upon how the model parameters which reflect the cost of the government's liquidity support to banks and the government's reputational cost from breaking the deposit guarantee are thought to change in a common system. We found that under certain plausible parameter choices a common deposit guarantee system could be welfare increasing. However, to obtain a Pareto improvement in the joint system, an ex ante transfer would likely be needed from the periphery to the core.

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# Appendix 1. Proofs of Theorems.

**Remark 1.** The expected profit-maximizing values of the interest rate  $r$ , the investment of each bank  $i$ , and the aggregate investment  $I$  are independent of the choices of the consumers, of the choice by the government, and of the government-related parameters  $\Gamma$  and  $F$ .

**Proof of Remark 1.** As we saw, at  $T=2$  the assets of a bank consist of the return on its investment,  $\rho i$ .<sup>16</sup> The liabilities of the bank consist of the government loan  $g = (\ell d)(1+r)\zeta$  and the value of the remaining deposits  $(1-\ell)d(1+r)\zeta$ . The net worth of the bank is the difference of the assets and liabilities whenever the difference is positive, and zero otherwise, and hence, the net worth equals

$$\begin{aligned} & \max \{0, \rho i - (\ell d)(1+r)\zeta - (1+r)(1-\lambda)d\} \\ & = \max \{0, \rho i - d(1+r)\zeta\} \end{aligned}$$

The profit of the bank is the difference of its net worth and the sunk cost  $B$ . Remembering the budget constraint (2.1), the profit of the bank can be expressed as a function of  $i$  as

$$\pi = \max \{0, \rho i - (1+r)\kappa(i)\} - B$$

We let  $H$  denote the distribution function

$$H(\rho) = \int_0^1 H_\eta(\rho) d\eta$$

and we define  $\rho_0(r, i)$  by

$$\rho_0(r, i) = \frac{1}{i} (1+r)\kappa(i)$$

and  $\bar{\rho}(r, i)$  by

$$\bar{\rho}(r, i) = \int_{\rho_0(r, i)}^{\rho_{\max}} \rho dH(\rho).$$

We may write the expected profit of the bank as

$$\begin{aligned} E\pi(r, i) &= \int_0^1 \left[ \int_{\rho_0(r, i)}^{\rho_{\max}} (\rho i - (1+r)\kappa(i)) dH_\eta(\rho) \right] d\eta - B \\ &= \bar{\rho}(r, i) i - (1 - H(\rho_0(r, i))) (1+r)\kappa(i) - B \end{aligned} \quad (A1)$$

In a competitive banking sector, each bank chooses  $i$  in order to maximize  $E\pi(r, i)$ , taking  $r$  as given. We now define  $i^*(r)$  to be the value of  $i$  which maximizes  $E\pi(r, i)$  for each given  $r$ . Differentiating (A1) with respect to  $i$ , and observing the terms which express the dependence of  $\bar{\rho}(r, i)$  and  $\rho_0(r, i)$  on  $i$  cancel out, it follows that  $i^*(r)$  is implicitly given by

$$\kappa'(i^*(r)) = \frac{\bar{\rho}(r, i^*(r))}{(1+r)(1 - H(\rho_0(r, i^*(r))))} \quad (A2)$$

In a competitive equilibrium, the value of  $r$  is determined by the condition  $E\pi(r, i^*(r)) = 0$ . We denote the value of  $r$  which satisfies this condition by

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<sup>16</sup>Below we assume that the budget constraint (2.1) is valid with equality without considering the possibility that the bank would have more liquid funds that are needed for its investment, i.e. that  $\zeta d > \kappa(i)$ . This is because in our model the costs which are caused by the lack of liquidity in case of a bank run are paid completely by the government. When  $\zeta d > \kappa(i)$ , the bank will have to pay interest for the extra deposits  $\zeta d - \kappa(i)$  without earning any extra revenue from them. It is easy to see that in this case the profit  $\pi$ , which we calculate below, would be given by

$$\pi = \max \{0, \rho i - (1+r)\kappa(i) - r[\zeta d - \kappa(i)]\}$$

Hence, a choice  $\zeta d > \kappa(i)$  is never the expected profit-maximizing choice of the bank.

$r^*$ . The equilibrium amount of depositors  $d^*$  of each bank is now determined by the condition

$$\zeta d^* = \kappa(i^*(r^*)) \quad (\text{A3})$$

Since the banks are of an identical size, the total number of banks is fixed by the number of depositors (which we have normalized to 1). Hence in equilibrium there are  $1/d^* = \zeta/\kappa(i^*(r^*))$  banks and the aggregate investment equals

$$I = \frac{\zeta i^*(r^*)}{\kappa(i^*(r^*))} \quad (\text{A4})$$

Clearly, the value of  $r^*$  and the corresponding value  $I$  are independent of both the choices of the consumers (which are expressed by the value of  $\lambda$ ), of the choice of the government, and trivially, also of the parameters  $\Gamma$  and  $F$  which we have not yet discussed.

**Remark 2.** The banks fail if and only if  $\rho I < 1$ . The aggregate profit of the banks is

$$\Pi = \max\{\rho I - 1, 0\} - \frac{B\zeta}{\kappa(i)}$$

**Proof of Remark 2.** In the proof of Remark 1 we concluded that the banks are of an identical size (i.e. at  $T=0$  they accept the same amount  $\zeta$  of deposits from the same number  $d^* = \kappa(i^*(r^*))/\zeta$  depositors, and that make the same investment  $i = i^*(r^*)$ ). Since we set  $\zeta(1+r) = 1$  in (2.5), at  $T=2$  the liabilities of each bank (i.e. the value of the remaining deposits  $d^*(1-\ell)(1+r)\zeta$  and a possible liquidity loan  $\ell d^*(1+r)\zeta$ , where  $\ell$  is the share of the depositors of the bank who withdraw their deposits at  $T=1$ ) amount up to  $d^*(1+r)\zeta = d^*$ . Hence, the bank fails if and only if  $d^* > \rho i$ . We now observe that in our model either all banks fail or none of the banks fail, and that the banks fail if at  $T=2$  if their aggregate liabilities exceed the aggregate revenue  $\rho I$  from investments. However, in the proof of Remark 1 we saw that there are altogether  $1/d^*$  banks and hence, their aggregate liabilities equal  $(1/d^*)d^* = 1$ .

When none of the banks fail, the sum of their net worths must be the difference of the aggregate assets  $\rho I$  and aggregate liabilities 1. When all banks fail, there aggregate net worth must be zero. Hence, the aggregate net worth of the banks is

$$\max\{\rho I - 1, 0\}$$

The sunk costs are  $B$  for each of the  $1/d^*$  banks, and recalling (A3), we observe that the aggregate sunk costs equal

$$\frac{B}{d^*} = \frac{B\zeta}{\kappa(i)}$$

The result concerning the aggregate profit  $\Pi$  follows now by observing that  $\Pi$  must equal the difference of the aggregate net worth and aggregate sunk cost of the banks.

**Remark 3** (a) Among the strategies for which  $\tau < 1$ , the welfare maximizing choice is  $\tau = c_{\min}$ .

(b) Among the strategies for which  $\tau \geq 1$ , the welfare maximizing choice is  $\tau = 1$ .

**Proof (a).** Assume  $\tau < 1$ . Together with (2.12), (2.13), and (2.17), the definition (2.16) of the welfare function  $W$  implies that

$$\frac{dW}{d\tau} = (1 - \lambda) u'(\tau) + \frac{d}{d\tau} Q(Z_1, \min\{\rho I, 1\} - (1 - \lambda)\tau) \quad (\text{A5})$$

We may now conclude from (2.15) that the latter term in (A5) satisfies

$$\begin{aligned} & \frac{d}{d\tau} Q(Z_1, \min\{\rho I, 1\} - (1 - \lambda)\tau) \\ &= \begin{cases} -(1 - \lambda), & \min\{\rho I, 1\} > (1 - \lambda)\tau \\ -\Gamma(1 - \lambda), & \min\{\rho I, 1\} < (1 - \lambda)\tau \end{cases} \end{aligned} \quad (\text{A6})$$

We first consider the maximization of welfare function  $W$  with respect to  $\tau$  when  $\tau$  satisfies  $0 \leq \tau < c_{\min}$ . In this case (2.6) implies that

$$\min\{\rho I, 1\} - (1 - \lambda)\tau > \min\{\rho I, 1\} - c_{\min} \geq 0$$

and we may conclude from (A5) and (A6) that

$$\frac{dW}{d\tau} = (1 - \lambda)(u'(\tau) - 1)$$

However, according to (2.7)  $u'(c_{\min}) = 1$ . Hence, the concavity of  $u$  implies that

$$\frac{dW}{d\tau} > 0 \text{ whenever } \tau < c_{\min}.$$

Hence,  $\tau = c_{\min}$  is the welfare-maximizing value of  $\tau$  in the range  $0 \leq \tau \leq c_{\min}$ .

Secondly, assume that  $c_{\min} < \tau < 1$ . Now (A5) and (A6) imply that

$$\frac{dW}{d\tau} \leq (1 - \lambda)(u'(\tau) - 1)$$

while (2.7) and the concavity of  $u$  imply  $u'(\tau) < 1$ . Hence,

$$\frac{dW}{d\tau} < 0$$

and  $\tau = c_{\min}$  is the welfare-maximizing value of  $\tau$  also when  $c_{\min} \leq \tau < 1$ .

**(b)** Assume  $\tau > 1$ . The definitions (2.12), (2.13), (2.15), (2.16), and (2.17) imply that the results (A5) and (A6) remain valid also in this case. Hence, we may once more conclude from (A6), (A5), (2.7), and the concavity of  $u$  that

$$\frac{dW}{d\tau} \leq (1 - \lambda)(u'(\tau) - 1) < 0$$

Hence, the welfare-maximizing value of  $\tau$  in the range  $\tau \geq 1$  is  $\tau = 1$ .

**Remark 4.** Assume that F satisfies (3.3).

(a)  $W_{DG} > W_{NDG}$  whenever  $\rho I \geq 1 - \lambda$ .

(b) In particular, it is welfare-maximizing for the government to let the depositors withdraw their full deposits if banks do not fail.

**Proof of Remark 4. (a)** Assume  $\rho I \geq 1 - \lambda$ . We conclude from definition (2.15) that

$$Q(Z_1, \min\{\rho I, 1\} - (1 - \lambda)\tau) = Z_1 + \min\{\rho I, 1\} - (1 - \lambda)\tau$$

both when  $\tau = 1$  and when  $\tau = c_{\min}$ . Hence, definitions (3.1) and (3.2) imply that

$$\begin{aligned} W_{DG} - W_{NDG} &= (1 - \lambda)(u(1) - u(c_{\min})) - (1 - \lambda)(1 - c_{\min}) + F \\ &> F - (1 - \lambda) \end{aligned}$$

and now (3.3) implies that  $W_{DG} - W_{NDG} > 0$ .

**(b)** Assume that the banks do not fail. According to Remark 1, this implies that  $\rho I \geq 1 \geq 1 - \lambda$ , and we may conclude from part (a) and Remark 3 that the welfare maximizing choice of the government is  $\tau = 1$ . Hence, the government lets the depositors withdraw their deposits in equilibrium.

(**The proof of Theorem 1** is presented in the passage which immediately precedes it.)

**Remark 5.** The expected utility from staying is an increasing function of the size of the bank run  $\lambda$ . More rigorously, the function  $\bar{\rho}$  satisfies

$$\frac{d\bar{\rho}(\lambda)}{d\lambda} \leq 0,$$

the expected utility from staying satisfies

$$\frac{d(E_{\eta}u_B)(\lambda)}{d\lambda} \geq 0,$$

and both inequalities are strict whenever  $\bar{\rho}(\lambda) > \rho_{\min}$ , i.e. whenever  $\lambda$  is such that there is a positive probability of a deposit guarantee failure.

**Proof of Remark 5.** We defined to  $\bar{\rho}(\lambda)$  to be the value of  $\rho$  which satisfies

$$\Xi(\lambda, \bar{\rho}(\lambda)) = 0, \tag{A7}$$

if the solution of this equation satisfies  $\rho \geq \rho_{\min}$ , and by  $\bar{\rho}(\lambda) = \rho_{\min}$ , otherwise. In the latter case sufficiently small changes in  $\lambda$  leave  $\bar{\rho}(\lambda)$  unchanged, implying that

$$\frac{d\bar{\rho}(\lambda)}{d\lambda} = 0$$

In this case (4.1) implies that

$$\frac{d(E_{\eta}u_B)(\lambda)}{d\lambda} = 0$$

so that the remark is trivially valid.

Consider now the non-trivial case in which  $\bar{\rho}(\lambda)$  satisfies (A7). The concavity of  $u$  and (2.7) imply that  $u'(c) < 1$  whenever  $c > c_{\min}$ . Hence,

$$u(1) - u(c_{\min}) < 1 - c_{\min}$$

However, now (3.4) and (2.10) imply that

$$\frac{\partial \Xi(\lambda, \rho)}{\partial \lambda} = \Gamma - c_{\min} - (u(1) - u(c_{\min})) > 0 \tag{A8}$$

and also that

$$\frac{\partial \Xi(\lambda, \rho)}{\partial \rho} = (\Gamma - 1)I > 0 \tag{A9}$$

Differentiating (A7) with respect to  $\lambda$ , it follows that

$$\left( \frac{\partial \Xi(\lambda, \rho)}{\partial \lambda} \right)_{\rho=\bar{\rho}(\lambda)} + \frac{d\bar{\rho}(\lambda)}{d\lambda} \left( \frac{\partial \Xi(\lambda, \rho)}{\partial \rho} \right)_{\rho=\bar{\rho}(\lambda)} = 0$$

and now (A8) and (A9) imply that

$$\frac{d\bar{\rho}(\lambda)}{d\lambda} < 0$$

This shows that part the first result in this remark is valid with strict inequality.

To prove the second inequality, we conclude from (4.1) and (2.3) that

$$\frac{d(E_{\eta}u_B)(\lambda)}{d\lambda} = - (u(1) - u(c_{\min})) \left( \frac{dH_{\eta}(\rho)}{d\rho} \right)_{\rho=\bar{\rho}(\lambda)} \frac{d\bar{\rho}(\lambda)}{d\lambda} > 0.$$

**Remark 6.** If all consumers switch (i.e. if  $\lambda = 1$ ), staying yields a larger expected utility than switching. Hence, in equilibrium  $\lambda < 1$ , and bank runs are always partial.

**Proof of Remark 6.** If  $\lambda = 1$ , i.e. if all consumers switch, we may conclude from Remark 4(a) that the deposit guarantee does not fail at  $T=2$ .



(In this case there are no deposits left at  $T=2$ .) In this case staying yields the expected utility  $u(1)$  but switching yields only the expected utility  $u(1 - \delta)$ . Hence, the considered situation cannot be an equilibrium.

**Theorem 2.** For each value of the signal  $\eta$ , there is unique share  $\lambda^* = \lambda^*(\eta)$  of switching consumers which corresponds to an equilibrium of the model. The equilibrium value of  $\lambda^*$  satisfies one of the following conditions:

(a)  $0 < \lambda^* < 1$  so that there is a partial bank run, and  $\lambda^*$  is determined by  $(E_\eta u_B)(\lambda^*) = u(1 - \delta)$ ,

i.e., by the condition that staying and switching yield the same expected utility.

(b)  $\lambda^* = 0$  so that there is no bank run and  $\lambda^*$  satisfies

$(E_\eta u_B)(\lambda^*) \geq u(1 - \delta)$ ,

i.e., the condition that staying yields at least the same expected utility as switching.

**Proof of Theorem 2.** First, we let  $\eta$  have an arbitrary but fixed value in  $[0, 1]$ . We conclude from Remark 4(a) that if  $\lambda = 1$ , the deposit guarantee fails for no values of  $\rho$ . Hence,  $\bar{\rho}(1) = \rho_{\min}$ . The function  $\bar{\rho}(\lambda)$  is continuous, satisfies  $\bar{\rho}(\lambda) \geq \rho_{\min}$  for all  $\lambda$ , and is according to Remark 5 non-increasing in  $\lambda$ . Hence, the values of  $\lambda$  which satisfy the condition  $\bar{\rho}(\lambda) = \rho_{\min}$  must form a closed interval, say  $[\lambda', 1]$ . Now (4.1) implies that when  $\lambda$  belongs to  $[\lambda', 1]$ ,

$$E_\eta u_B(\lambda) = u(1) > u(1 - \delta)$$

Hence, if  $\lambda' = 0$ , staying yields a greater expected utility than switching for all possible values of  $\lambda$ . In this case all the consumers stay in equilibrium,  $\lambda^* = 0$ , and the condition (b) is valid.

Consider now the non-trivial case in which  $\lambda' > 0$ . In this case none of the  $\lambda$  values in  $[\lambda', 1]$  corresponds to an equilibrium, since these values correspond to cases in which some consumers switch although staying would yield a greater expected utility. Further, Remark 5 implies that  $(E_\eta u_B)(\lambda)$  is a strictly increasing function in the interval  $[0, \lambda')$ . Hence, the equation

$$(E_\eta u_B)(\lambda^*) = u(1 - \delta) \tag{A10}$$

has either a unique solution or no solutions. If a solution  $\lambda^*$  exists, it corresponds to the unique equilibrium and satisfies either (a) (if  $\lambda^* > 0$ ) or (b) (if  $\lambda^* = 0$ ). If, on the other hand, (A10) is not valid for any of the possible values of  $\lambda^*$ , it must be the case that

$$(E_\eta u_B)(0) > u(1 - \delta),$$

i.e. that staying yields a greater expected utility than switching when all consumers stay. In this case the unique equilibrium is the equilibrium in which (b) is valid. This completes the proof.

**Theorem 3.** When there is bank run after the signal  $\eta$ , the probability  $P_{DGF}(\eta)$  of a deposit guarantee failure given  $\eta$  has the value  $P_{DGF, \max}$ , which equals

$$P_{DGF, \max} = \frac{u(1) - u(1 - \delta)}{u(1) - u(c_{\min})}$$

The value  $P_{DGF,\max}$  is the maximum value that deposit guarantee failure probability can have in equilibrium.

**Proof of Theorem 3.** According to (4.2) the conditional deposit guarantee failure probability  $P_{DGF}(\eta)$  (given  $\eta$ ) equals

$$P_{DGF}(\eta) = H_\eta(\bar{\rho}(\lambda^*(\eta)))$$

and according to (4.1) the the expected utility from staying (given  $\eta$ ) equals in equilibrium

$$(E_\eta u_B)(\lambda^*(\eta)) = P_{DGF}(\eta) u(c_{\min}) + [1 - P_{DGF}(\eta)] u(1) \quad (11)$$

If there is a bank run, the expected utility from staying must according to Theorem 2(a) equal also  $u(1 - \delta)$ . I.e.,

$$(E_\eta u_B)(\lambda^*(\eta)) = u(1 - \delta) \quad (A12)$$

Eliminating  $(E_\eta u_B)(\lambda^*(\eta))$  from (11) and (A12) and solving for  $P_{DGF}(\eta)$ , it follows that

$$P_{DGF}(\eta) = \frac{u(1) - u(1 - \delta)}{u(1) - u(c_{\min})} = P_{DGF,\max}$$

To prove the maximality claim, we assume that for some  $\eta$

$$P_{DGF}(\eta) > P_{DGF,\max}.$$

Using (11) once more, it follows that

$$(E_\eta u_B)(\lambda^*(\eta)) = [P_{DGF,\max} u(c_{\min}) + (1 - P_{DGF,\max}) u(1)] - (P_{DGF}(\eta) - P_{DGF,\max}) (u(1) - u(c_{\min}))$$

The expression in square brackets equals  $u(1 - \delta)$  and hence,

$$(E_\eta u_B)(\lambda^*(\eta)) < u(1 - \delta)$$

In other words, staying yields a smaller expected utility than switching. This could only happen in equilibrium if all consumers switched at  $T=1$ , but according to Remark 6, there are no equilibria in which all consumers switch. This contradiction shows that the assumption is false and that

$$P_{DGF}(\eta) \leq P_{DGF,\max}.$$

for all  $\eta$ .

(The proof of **Remark 7** is presented in the passage preceding it and illustrated by Figure 2.)

**Remark 8.** The size of the bank run is a decreasing function of the signal  $\eta$ . More rigorously, the number of switching consumers  $\lambda^*(\eta)$  satisfies

$$\frac{d\lambda^*(\eta)}{d\eta} \leq 0$$

and this inequality is strict for the signals  $\eta$  which correspond to a bank run (i.e. for which  $\lambda^*(\eta) > 0$ ).

**Proof of Remark 8.** Let the signal  $\eta$  be arbitrary. We distinguish between two cases. Consider first the case in which  $\eta$  satisfies

$$(E_\eta u_B)(0) \geq u(1 - \delta) \quad (A13)$$

We conclude from Theorem 2 that in this case  $\lambda^*(\eta) = 0$  and there is no bank run in equilibrium. Let  $\vartheta$  be such that  $\eta < \vartheta \leq 1$  but otherwise arbitrary. We now conclude from (2.4) that

$$H_\vartheta(\bar{\rho}(0)) \leq H_\eta(\bar{\rho}(0))$$

and from (4.1) and (A13) that

$$\begin{aligned} (E_{\vartheta}u_B)(0) &= u(1) - H_{\vartheta}(\bar{\rho}(0))(u(1) - u(c_{\min})) \\ &\geq u(1) - H_{\eta}(\bar{\rho}(0))(u(1) - u(c_{\min})) = (E_{\eta}u_B)(0) \geq u(1 - \delta) \end{aligned}$$

In other words, if all consumer choose to stay when the signal is  $\eta$ , all consumers choose to stay also when the signal has a value  $\vartheta > \eta$ . Letting  $\bar{\eta}$  be the infimum of the values of  $\eta$  for which (A13) is valid, we observe that (A13) is valid for all signals  $\eta > \bar{\eta}$ , and by continuity, also for  $\eta = \bar{\eta}$ . Hence, the  $\eta$  values (if any) which satisfy (A13) form an interval  $[\bar{\eta}, 1]$ , in which the remark is trivially valid.

Secondly, assume that (A13) is not valid for the considered signal  $\eta$ . We may now conclude from Theorem 2 that  $\lambda^*(\eta) > 0$  and from Theorem 3 that

$$H_{\eta}(\lambda^*(\eta)) = \frac{u(1) - u(1 - \delta)}{u(1) - u(c_{\min})}$$

Differentiating, we get

$$\begin{aligned} &\frac{d}{d\eta} H_{\eta}(\bar{\rho}(\lambda^*(\eta))) \\ &= \left( \frac{d}{d\eta} H_{\eta}(\rho) \right)_{\rho = \bar{\rho}(\lambda^*(\eta))} + H_{\eta}'(\bar{\rho}(\lambda^*(\eta))) \bar{\rho}'(\lambda^*(\eta)) \frac{d}{d\eta}(\lambda^*(\eta)) = 0 \quad (\text{A14}) \end{aligned}$$

Now (2.4) and (2.3) imply that  $(d/d\eta)(H_{\eta}(\rho))$  is negative and that  $H_{\eta}'(\rho) > 0$  is positive when  $\rho = \bar{\rho}(\lambda^*(\eta))$ . Further, according to Remark 5 the derivative of  $\bar{\rho}(\lambda_B)$  is negative. Together with these three facts, (A14) implies that

$$\frac{d}{d\eta}(\lambda^*(\eta)) < 0$$

This completes the proof of Remark 8.

To proceed, we present three lemmas which are concerned with the function  $\bar{\rho}(\lambda)$ . We defined  $\bar{\rho}(\lambda)$  in Section 3 by the equation

$$\Xi(\lambda, \bar{\rho}(\lambda)) = 0$$

whenever the value  $\bar{\rho}(\lambda)$  thus defined satisfies  $\bar{\rho}(\lambda) \geq \rho_{\min}$ , and by  $\bar{\rho}(\lambda) = \rho_{\min}$  otherwise.

**Lemma 1.**  $\bar{\rho}(\lambda^*(\eta)) > \rho_{\min}$  for each signal  $\eta$ .

**Proof of Lemma 1.** Assume, on the contrary, that  $\bar{\rho}(\lambda^*(\eta)) = \rho_{\min}$  for some signal  $\eta$ . It this case (4.1) implies that

$$(E_{\eta}u_B)(\lambda^*(\eta)) = u(1) > u(1 - \delta)$$

and hence, the considered signal  $\eta$  must correspond to a "no bank run" equilibrium so that  $\lambda^*(\eta) = 1$ . Now our assumption becomes

$$\bar{\rho}(1) = \rho_{\min}$$

However, now (4.1) implies that for *any* signal  $\eta$

$$(E_{\eta}u_B)(1) = u(1) > u(1 - \delta)$$

and that all signals correspond to a no bank run equilibrium. We have, however, introduced the "non-triviality assumption" (4.4), according to which there are signals (namely, the signals in the non-empty interval  $[0, \bar{\eta})$ ) which are followed by a bank run. The contradiction shows that it cannot be the case that  $\bar{\rho}(\lambda^*(\eta)) = \rho_{\min}$  for any signal  $\eta$ .

**Lemma 2** The "borderline signal"  $\bar{\eta}$  (i.e. the signal which lies at the borderline of the bank run equilibria and the no bank run equilibria) satisfies

$$\bar{\rho}(0) = H_{\bar{\eta}}^{-1}(P_{DGF, \max})$$

and the signals  $\eta > \bar{\eta}$  satisfy

$$\bar{\rho}(0) < H_{\bar{\eta}}^{-1}(P_{DGF,\max})$$

**Proof of Lemma 2.** By definition  $\bar{\eta}$  is the borderline between the signals  $\eta$  which satisfy the condition (a) of Theorem 2 and the ones which satisfy the condition (b) of Theorem 2. Hence, none of the signals  $\eta < \bar{\eta}$  satisfy the condition

$$(E_{\eta}u_B)(0) \geq u(1 - \delta)$$

but this condition is valid when  $\eta = \bar{\eta}$ . By continuity of  $(E_{\eta}u_B)(\lambda)$  with respect to  $\eta$ , we conclude that

$$(E_{\bar{\eta}}u_B)(0) = u(1 - \delta) \tag{A12}$$

Using (4.1), we see that (A12) is equivalent with

$$H_{\bar{\eta}}(\bar{\rho}(0)) = \frac{u(1) - u(1 - \delta)}{u(1) - u(c_{\min})}$$

Combined with the definition  $P_{DGF,\max}$  (in Theorem 3) this proves the result concerning signal  $\eta = \bar{\eta}$ . According to Lemma 1  $\bar{\rho}(0)$  does not depend on  $\eta$ , and the validity of the other result follows now from (2.4), and Lemma 1.

**Lemma 3.** If  $\bar{\rho}(\lambda) > \rho_{\min}$ , it must be the case that  $\frac{d\bar{\rho}(\lambda)}{d\Gamma} > 0$  and  $\frac{d\bar{\rho}(\lambda)}{dF} < 0$

**Proof of Lemma 3.** The function  $\bar{\rho}$  depends implicitly on the government-related parameters  $\Gamma$  and  $F$ , because according to definition (3.4) of the function  $\Xi$  depends on  $\Gamma$  and  $F$ . If  $\bar{\rho}(\lambda) > \rho_{\min}$  for some  $\lambda$ , the definition of the function  $\bar{\rho}$  implies that

$$\Xi(\lambda, \bar{\rho}(\lambda)) = 0$$

and the definition of  $\Xi$ , (3.4), further implies that

$$(1 - \lambda)[u(1) - u(c_{\min})] - \Gamma(1 - \lambda - \bar{\rho}(\lambda)I) - (\bar{\rho}(\lambda)I - (1 - \lambda)c_{\min}) + F = 0$$

Solving for  $\bar{\rho}(\lambda)$ , one gets

$$\bar{\rho}(\lambda) = \frac{1 - \lambda}{I} - \frac{1 - \lambda}{(\Gamma - 1)I} [u(1) - u(c_{\min}) - 1 + c_{\min} + \frac{F}{1 - \lambda}]$$

Remembering (3.3) we observe that the expression in square brackets is positive, and that hence,

$$\frac{d\bar{\rho}(\lambda)}{d\Gamma} > 0 \text{ and } \frac{d\bar{\rho}(\lambda)}{dF} < 0$$

This proves Lemma 3.

**Theorem 4. (a)** The unconditional probability of bank a run is strictly increasing in  $\Gamma$ , i.e., the funding costs of the deposit guarantee, and strictly decreasing in  $F$ , i.e., the indirect (reputational) social cost of deposit guarantee failure. In other words,

$$\frac{d\bar{\eta}}{d\Gamma} > 0 \text{ and } \frac{d\bar{\eta}}{dF} < 0$$

**(b)** When there is a bank run after the signal  $\eta$ , its size is strictly increasing in  $\Gamma$  and strictly decreasing in  $F$ . I.e., if  $\eta < \bar{\eta}$ , then

$$\frac{d\lambda^*(\eta)}{d\Gamma} > 0 \text{ and } \frac{d\lambda^*(\eta)}{dF} < 0.$$

**Proof of Theorem 4. (a)** We conclude from Lemma 2 that

$$H_{\bar{\eta}}(\bar{\rho}(0)) = P_{DGF,\max} \tag{A13}$$

Differentiating with respect to  $\Gamma$ , we get

$$H_{\bar{\eta}}'(\bar{\rho}(0)) \frac{d\bar{\rho}(0)}{d\Gamma} + \frac{d\bar{\eta}}{d\Gamma} \frac{d}{d\eta} (H_{\eta}(\bar{\rho}(0)))_{\eta=\bar{\eta}} = 0 \quad (A14)$$

According to Lemma 1,  $\bar{\rho}(0) = \bar{\rho}(\lambda^*(\bar{\eta})) > \rho_{\min}$ , and we may conclude from (2.3) that

$$H_{\bar{\eta}}'(\bar{\rho}(0)) > 0 \quad (A15)$$

from Lemma 3 that

$$\frac{d}{d\Gamma} \bar{\rho}(0) > 0$$

and from (2.4) that

$$\frac{d}{d\eta} (H_{\eta}(\bar{\rho}(0)))_{\eta=\bar{\eta}} < 0 \quad (A16)$$

When the three last inequalities are combined with (A14), it follows that

$$\frac{d\bar{\eta}}{d\Gamma} > 0.$$

Similarly, differentiating (A13) with respect to  $F$  one gets

$$H_{\bar{\eta}}'(\bar{\rho}(0)) \frac{d\bar{\rho}(0)}{dF} + \frac{d\bar{\eta}}{dF} \frac{d}{d\eta} (H_{\eta}(\bar{\rho}(0)))_{\eta=\bar{\eta}} \quad (A17)$$

Lemmas 1 and 3 imply also that

$$\frac{d}{dF} \bar{\rho}(0) < 0$$

Combining this result with (A15), (A16), and (A17), it follows that

$$\frac{d\bar{\eta}}{dF} < 0.$$

**(b)** Assume that there is a bank run after the signal  $\eta$ . According to (4.3) the size of the bank run  $\lambda^*(\eta)$  is determined by

$$\bar{\rho}(\lambda^*(\eta)) = H_{\eta}^{-1}(P_{DGF, \max}) \quad (A18)$$

Differentiating (A18) with respect to  $\Gamma$ , we obtain.

$$\left( \frac{d\bar{\rho}(\lambda)}{d\Gamma} \right)_{\lambda=\lambda^*(\eta)} + \bar{\rho}'(\lambda^*(\eta)) \frac{d\lambda^*(\eta)}{d\Gamma} = 0 \quad (A19)$$

According to Lemmas 1 and 3, the first term of (A19) is positive. Lemma 1 and Remark 5 imply that

$$\bar{\rho}'(\lambda^*(\eta)) < 0 \quad (A20)$$

Hence, we may conclude from (A19) that

$$\frac{d\lambda^*(\eta)}{d\Gamma} > 0$$

Similarly, differentiating (A18) with respect to  $F$  we get

$$\left( \frac{d\bar{\rho}(\lambda)}{dF} \right)_{\lambda=\lambda^*(\eta)} + \bar{\rho}'(\lambda^*(\eta)) \frac{d\lambda^*(\eta)}{dF} = 0 \quad (A21)$$

The first term of (A21) according to Lemmas 1 and 3 negative, and using also (A20), we may conclude that

$$\frac{d\lambda^*(\eta)}{dF} < 0.$$

This completes the proof.

(The proof of **Theorem 5** is contained in the passage that precedes it.)

**Theorem 6.** Assume that the funding cost of deposit guarantee payments is identical in the two countries, i.e. that  $\Gamma_P = \Gamma_C = \Gamma_{EU}$ .

(a) The probability of a bank run in the pooled system,  $\bar{\eta}_{EU}$ , is between its probabilities in the core and the periphery when the deposit guarantees are kept separate. In other words,  $\bar{\eta}_C < \bar{\eta}_{EU} < \bar{\eta}_P$ .

(b) The pooling reduces the size of bank runs. More precisely, for each signal  $\eta$

$$\lambda_{EU}^*(\eta) \leq \bar{\lambda}^*(\eta)$$

and the above inequality is strict when the signal  $\eta$  is between  $\bar{\eta}_C$  and  $\bar{\eta}_P$ , but not otherwise.

(c) The pooling increases aggregate expected welfare when switching costs and deposit guarantee failure probability are low. More precisely,

$$(E_\rho W)_{SSC,EU} \geq \overline{(E_\rho W)}_{SSC}$$

and the above inequality is strict for each signal  $\eta < \bar{\eta}_P$  (i.e. whenever there is a bank run in the periphery).

**Proof of Theorem 6.** For short, we let  $\Gamma$  denote the value  $\Gamma_P = \Gamma_C = \Gamma_{EU}$ . The functions  $\Xi$  and  $\bar{\rho}$  depend implicitly on  $F$ , which has three different values in the cases that we consider (i.e. the values  $F_C$  and  $F_P$  which correspond to the separate deposit guarantees, and the value  $F_{EU}$  which corresponds to the pooled scheme.) For the sake of clarity, we introduce the notation  $\Xi_\psi$  ( $\psi = C, P, EU$ ) for the function  $\Xi$ , defined by (3.4), which correspond to these three cases. In other words, we put

$$\begin{aligned} \Xi_\psi(\lambda, \rho) &= (1 - \lambda) [u(1) - u(c_{\min})] \\ &- \Gamma(1 - \lambda - \rho I) - (\rho I - (1 - \lambda)c_{\min}) + F_\psi \end{aligned} \quad (\text{A22})$$

Analogously, we use  $\bar{\rho}_\psi$  ( $\psi = C, P, EU$ ) to denote the version of the function which corresponds to  $\Xi_\psi$ . I.e., we define  $\bar{\rho}_\psi(\lambda)$  implicitly by the equation

$$\Xi_\psi(\lambda, \bar{\rho}_\psi(\lambda)) = 0 \quad (\text{A23})$$

whenever the value  $\bar{\rho}_\psi(\lambda)$  thus defined satisfies  $\bar{\rho}_\psi(\lambda) > \rho_{\min}$  and by  $\bar{\rho}_\psi(\lambda) = \rho_{\min}$  otherwise.

(a) The assumptions (6.1) and (6.6) imply that

$$F_C > F_{EU} > F_P$$

and, hence, the result follows immediately from Theorem 4(a).

(b) According to (6.4) the result which is to be proved equivalent with

$$\lambda_{EU}^*(\eta) \leq n_C \lambda_C^*(\eta) + n_P \lambda_P^*(\eta) \quad (\text{A24})$$

We consider separately the cases in which  $\eta$  belongs to each of the intervals  $[0, \bar{\eta}_{EU})$ ,  $[\bar{\eta}_{EU}, \bar{\eta}_P)$ , and  $[\bar{\eta}_P, 1]$ . Assume first that  $\eta \geq \bar{\eta}_P$ . In this case there is according to part (a) no bank run in either country when the deposit guarantee schemes are kept separate, nor is there a bank run under the pooled deposit guarantee scheme. Hence,

$\lambda_{EU}^*(\eta) = \lambda_C^*(\eta) = \lambda_P^*(\eta) = 0$   
and (A24) is trivially valid.

Secondly, assume that  $\bar{\eta}_{EU} \leq \eta < \bar{\eta}_P$ . In this case

$$\lambda_{EU}^*(\eta) = 0$$

but since there is a bank run in the periphery,

$$\lambda_P^*(\eta) > 0$$

and (A24) is valid with strict inequality

Consider now the non-trivial case in which  $\eta < \bar{\eta}_{EU}$ . We now define  $\hat{\rho}_\eta$  by

$$\hat{\rho}_\eta = H_\eta^{-1}(P_{DGF, \max}) \quad (\text{A25})$$

Since (4.3) is valid in all bank run equilibria, it must be the case that

$$\bar{\rho}_{EU}(\lambda_{EU}^*(\eta)) = \bar{\rho}_P(\lambda_P^*(\eta)) = \hat{\rho}_\eta \quad (\text{A26})$$

The analogous result is valid also for  $\lambda_C^*(\eta)$  when  $\eta < \bar{\eta}_C$ , i.e. when there is a bank run also in the core. When  $\eta \geq \bar{\eta}_C$ , there is no bank run at the core, so that  $\lambda_C^*(\eta)$  is zero. Using Lemma 2, we conclude that

$$\begin{cases} \bar{\rho}_C(\lambda_C^*(\eta)) = \hat{\rho}_\eta, & \eta \leq \bar{\eta}_C \\ \bar{\rho}_C(\lambda_C^*(\eta)) < \hat{\rho}_\eta, & \eta > \bar{\eta}_C \end{cases} \quad (\text{A27})$$

Remembering Lemma 1, we observe that in each case ( $\psi = C, P, EU$ )

$$\Xi_\psi(\lambda_\psi^*(\eta), \bar{\rho}_\psi(\lambda_\psi^*(\eta))) = 0$$

Hence,

$$\begin{aligned} \Xi_{EU}(\lambda_{EU}^*(\eta), \bar{\rho}_{EU}(\lambda_{EU}^*(\eta))) \\ = n_C \Xi_C(\lambda_C^*(\eta), \bar{\rho}_C(\lambda_C^*(\eta))) + n_P \Xi_P(\lambda_P^*(\eta), \bar{\rho}_P(\lambda_P^*(\eta))) \end{aligned}$$

When we apply the definition (A22) to each term, simplify with (6.4), (6.6), and (A26), and remember that  $n_C + n_P = 1$ , it turns out that

$$\begin{aligned} (\lambda_{EU}^*(\eta) - \bar{\lambda}^*(\eta)) [\Gamma - c_{\min} - (u(1) - u(c_{\min}))] \\ = n_C (\bar{\rho}_C(\lambda_C^*(\eta)) - \hat{\rho}_\eta) (\Gamma - 1) I \end{aligned} \quad (\text{A28})$$

We conclude from (A27) that if  $\bar{\eta}_C < \eta < \bar{\eta}_{EU}$ , the right-hand side of (A28) is negative, implying that

$$\lambda_{EU}^*(\eta) < \bar{\lambda}^*(\eta)$$

i.e. that (A24) is valid with strict equality. Turning to the remaining case, i.e.  $\eta \leq \bar{\eta}_C$ , we conclude from (A27) that the right-hand side of (A28) is zero implying that

$$\lambda_{EU}^*(\eta) = \bar{\lambda}^*(\eta)$$

and that (A24) is valid with equality.

(c) We wish to evaluate

$$(\Delta W)_{SSC} = (E_\rho W)_{SSC, EU} - \overline{(E_\rho W)}_{SSC} \quad (\text{A29})$$

We saw in part (b) that if either  $\eta \leq \bar{\eta}_C$  or  $\eta \geq \bar{\eta}_P$ , the size of the bank run  $\lambda_{EU}^*(\eta)$  is the weighted average of the corresponding sizes in the two separate

schemes. In this case (A24) is valid with equality, and (5.7), (5.8), and (6.5) imply that

$$(\Delta W)_{SSC} = \int_{\rho_{\min}}^{\rho_{\max}} D(\eta, \rho) dH_{\eta}(\rho) \quad (\eta \leq \bar{\eta}_C \text{ or } \eta \geq \bar{\eta}_P) \quad (\text{A30})$$

where

$$D(\eta, \rho) = \Delta_{SSC}(\bar{\lambda}^*, \rho) - n_P \Delta_{SSC}(\lambda_P^*, \rho) - n_C \Delta_{SSC}(\lambda_C^*, \rho) \quad (\text{A31})$$

On the other hand, when  $\bar{\eta}_C < \eta < \bar{\eta}_P$ , the size of the bank run (if any) in the pooled system is smaller than  $\bar{\lambda}^*(\eta)$ . According to (5.7) and (5.8) a decrease in the size of the bank run  $\lambda_{EU}^*(\eta)$  increases the estimate of welfare  $(EW)_{SSC,EU}$ . Hence,

$$(\Delta W)_{SSC} > \int_{\rho_{\min}}^{\rho_{\max}} D(\eta, \rho) dH_{\eta}(\rho) \quad (\bar{\eta}_C < \eta < \bar{\eta}_P) \quad (\text{A32})$$

In a next step, we conclude from (5.8), (6.4) and (A31) that

$$D(\eta, \rho) = \begin{cases} 0, & \rho \leq (1 - \lambda_P^*(\eta)) / I \\ n_P(\Gamma - 1)(\rho I - (1 - \lambda_P^*(\eta))), & (1 - \lambda_P^*(\eta)) / I < \rho \leq (1 - \bar{\lambda}^*(\eta)) / I \\ n_C(\Gamma - 1)(1 - \lambda_C^*(\eta) - \rho I), & (1 - \bar{\lambda}^*(\eta)) / I \leq \rho < (1 - \lambda_C^*(\eta)) / I \\ 0, & \rho \geq (1 - \lambda_C^*(\eta)) / I \end{cases} \quad (\text{A33})$$

Hence,  $D(\eta, \rho)$  is strictly positive when  $(1 - \lambda_P^*(\eta)) / I < \rho < (1 - \lambda_C^*(\eta)) / I$  and zero otherwise.

When  $\eta \geq \bar{\eta}_P$ , there is no bank run under any of the three regimes,

$$\lambda_P^*(\eta) = \lambda_C^*(\eta) = \lambda_{EU}^*(\eta) = 0$$

and we may conclude from (A33) that  $D(\eta, \rho) = 0$  for all  $\rho$ , from (A30) that

$$(\Delta W)_{SSC} = 0$$

and from (A29) that

$$(EW)_{SSC,EU} = \overline{(EW)}_{SSC}$$

On the other hand, when  $\eta < \bar{\eta}_P$ , it must be the case that  $\lambda_P^*(\eta) > \lambda_C^*(\eta)$ . In this case there are values of  $\rho$  for which  $D(\eta, \rho)$  is positive. In this case (A30), (A32), and (A33) imply that

$$(\Delta W)_{SSC} \geq \int_{\rho_{\min}}^{\rho_{\max}} D(\eta, \rho) dH_{\eta}(\rho) > 0,$$

and (A29) implies that

$$(EW)_{SSC,EU} > \overline{(EW)}_{SSC}.$$



## Appendix 2. Early consumers and the liquidity reserve requirement

In the model which was developed above all the consumers wish to consume their wealth at  $T=2$ . Nevertheless, the banks offered them demand deposit contracts which allowed withdrawals also at  $T=1$ . As we already observed (in the beginning of Section 2), there is no obvious reason why bank runs could not be prevented by means of time deposit contracts, which allowed the depositors to withdraw deposits at  $T=2$  only. (More precisely, since the costs of bank runs are paid by the government rather than by the banks in our model, the banks could not increase their profits by introducing time deposits. However, in our setting a welfare-maximizing government has an incentive to introduce regulatory measures which force banks to offer time deposit contracts instead of demand deposit contracts.)

Above we did not consider time deposits, because our model can be easily generalized into a setup in which the consumers do not know their preferred moment of consumption at the time  $T=0$ , at which they deposit their liquid funds in banks. In the generalized version one knows at  $T=0$  only that a fixed share of consumers (to whom we refer as *early* consumers) wish to consume their wealth at  $T=1$  while the rest of the consumers (the *late* consumers) wish to consume it at  $T=2$ . The generalized version of the model shares otherwise the structure which was explained in Section 2. In it time deposit contracts are not welfare-maximizing, and they could not exist in the market equilibrium of a competitive banking sector. Nevertheless, the generalized version yields precisely the same results concerning bank runs and their welfare effects as our earlier model. Hence, the introduction of early consumers would only complicate our notation without bringing any interesting novelty to our results. In what follows, we give a short presentation of the generalized version with early consumers.

To avoid unnecessary changes of notation, we normalize the continuum of late consumers to 1 and the continuum of early consumers to  $\mu_E$ . Just like before, all consumers have originally the liquid funds  $\zeta$ . The banks offer them demand deposit contracts which specify the same interest rate (say  $r$ ) for deposits withdrawn at  $T=1$  and  $T=2$ . The consumers learn whether they are early or late at  $T=1$ , at the time at which they learn the signal  $\eta$ , but now they have three options to choose from: *consume* (i.e. consume already at  $T=1$ ), *switch*, and *stay*. The strategies "switch" and "stay" are defined just like we defined them in Sections 2.1 and 2.2 above, and the utility of all consumers is still given by the function  $u$  that was considered in Section 2.2. However, now the utility  $u(c_1)$  of the early consumers depends only on the consumption  $c_1$  at  $T = 1$ , while the utility  $u(c_2)$  of the late consumers depends only on the consumption  $c_2$  at  $T = 2$ . We denote the measure of the consumers who choose "consume" by  $\mu_C$  and the measure of the consumers who choose to switch by  $\lambda$ . This implies that the measure of the consumers who switch is  $1 + \mu_E - \mu_C - \lambda$ .

Just like before, we assume that the government provides the banks with extra liquidity at  $T=1$  when needed. Hence, both the switching and the consuming depositors can always withdraw their deposits at  $T=1$ . For an early con-

sumer the utility from the choice "consume" is  $u((1+r)\zeta) > 0$ , but the utility from switching and from staying is  $u(0) = 0$ , implying that all early consumers choose to consume. Similarly, for a late consumer the utility from the choice "consume" is  $u(0) = 0$  but the utility from switching is<sup>17</sup>  $u((1+r)\zeta - \delta) > 0$ . Hence, the strategy "switch" dominates the strategy "consume" and late consumers never choose "consume". In other words, precisely the early consumers consume,  $\mu_C = \mu_E$ , and the measure of the staying consumers is

$$(1 + \mu_E) - \lambda - \mu_C = 1 - \lambda$$

just like in our earlier model.

We now turn to a discussion of banks. Just like before, we postulate that there is a continuum of perfectly competitive and identical banks. We still assume that the cost of the investment  $i$  is given by the function  $\kappa(i)$ . Further, we assume that the share of early consumers among the depositors of each bank is identical with their share among all depositors, i.e.  $\mu_E / (1 + \mu_E)$ . As a new element to our model, we assume that the banks are subject to a *liquidity reserve requirement*, which restricts the investments that banks are allowed to make.

The liquidity reserve requirement has been meant to be the counterpart of real-world reserve requirements, which aim at guaranteeing sufficient liquidity in the absence of bank runs but are not unnecessarily large, given this aim.<sup>18</sup> To define it, we consider a bank which accepts deposits from  $d$  depositors and makes an investment of size  $i$ . In the absence of a bank run (i.e. when none of the bank's late depositors switch), the liquid funds that the bank needs at  $T=1$  amount up to

$$\frac{\mu_E}{1 + \mu_E} d (1 + r) \zeta$$

Accordingly, our liquidity reserve requirement states that the liquidity reserve of a bank must contain at least the share  $L_{req}$  of its deposits (i.e. be at least  $L_{req}\zeta d$ ) where

$$L_{req} = \frac{\mu_E}{1 + \mu_E} (1 + r) \tag{A34}$$

After the investment at  $T=0$ , the actual amount of liquid funds of the bank is  $d\zeta - \kappa(i) = L_0\zeta d$ , where

$$L_0 = 1 - \frac{\kappa(i)}{\zeta d} \tag{A35}$$

By definition, the liquidity requirement states that

$$L_0 \geq L_{req}$$

and it can be equivalently be formulated as the budget constraint

$$\kappa(i) \leq [1 - L_{req}] \zeta d \tag{A36}$$

---

<sup>17</sup>Just like we did above in(2.5) that (2.9), we shall assume that the switching cost  $\delta$  is sufficiently small to make  $(1+r)\zeta - \delta$  positive.

<sup>18</sup>In our model a bank run can only occur at  $T=1$ . Since in its earlier version banks did not need any liquidity at  $T=1$  in the absence of bank runs, in the earlier version the requirement which meets the above characterization is the trivial zero requirement.

Again, we assume that (if necessary) at  $T=1$  the government gives the bank a liquidity loan which just suffices to prevent the bank from failing. Denoting the share of switching consumers of the bank by  $\ell$  and remembering (A34), we observe that the liquidity that the bank needs is

$$\left(\frac{\mu_E}{1+\mu_E} + \ell\right) (1+r) d\zeta = (L_{req} + \ell(1+r)) \zeta d$$

while its actual liquidity is  $L_0 d\zeta$ . We conclude that the (possibly zero) liquidity loan amounts up to

$$g = \max \{0, (L_{req} + \ell(1+r)) \zeta d - L_0 \zeta d\} \quad (A37)$$

Above we have not ruled out the possibility that some of the liquid funds  $L_0 \zeta d$  might be left over after the withdrawals at  $T=1$ . Clearly, the (possibly) zero amount of liquid funds that remain after  $T=1$  is  $L_1 \zeta d$  where

$$L_1 = \max \{0, L_0 - (L_{req} + \ell(1+r))\} \quad (A38)$$

At  $T=2$ , the assets of the bank consist of liquid funds  $L_1 \zeta d$  and the revenue  $\rho i$  from the investment. The liabilities consist of the government loan  $g$  and the remaining deposits

$$\left(1 - \frac{\mu_E}{1+\mu_E} - \ell\right) (1+r) \zeta d = ((1-\ell)(1+r) - L_{req}) \zeta d$$

Still assuming that market entry causes the sunk cost  $B$  for each owner-banker, we observe that the profit of each bank equals

$$\pi = \max \{0, L_1 \zeta d + \rho i - g - ((1-\ell)(1+r) - L_{req}) \zeta d\} - B$$

Applying (A34), (A35), (A37), and (A38), and considering separately the cases in which  $g = 0$  and  $g > 0$ , this implies that

$$\pi = \max \{0, \rho i - \kappa(i) - r d \zeta\} - B \quad (A39)$$

Just like in the proof of Remark 1,  $\pi$  is maximized for each fixed  $i$  when  $d$  has the smallest value which is compatible with  $i$ , i.e the value which satisfies (A36) with equality. In this case profit turns out to be

$$\pi = \max \left\{ 0, \rho i - \left(1 + \frac{r}{1-L_{req}}\right) \kappa(i) \right\} - B$$

Analogously with the proof of Remark 1, this suffices to determine the expected profit  $E\pi(r, i)$  for each combination of  $r$  and  $i$ , the value  $i^*(r)$  of the investment which maximizes profits for each  $r$ , and the interest rate  $r = r^*$  of a perfectly competitive banking sector, i.e. the interest rate for which  $E\pi(r, i^*(r)) = 0$ .

The interest rate  $r = r^*$  is independent of the choices by the consumers and of government decisions, and we may view it as an exogenous constant. In particular, just like we did in (2.5), we may choose our unit of liquid funds so that

$$\zeta(1+r) = 1.$$

Now the switching consumers receive the funds  $1-\delta$ , the consuming consumers the funds 1, and the staying consumers are entitled to the funds 1.

Since (A36) is now valid with equality, we conclude from (A37) and (A35) that the liquidity loan  $g$  equals

$$g = \ell(1+r) \zeta d = \ell d$$

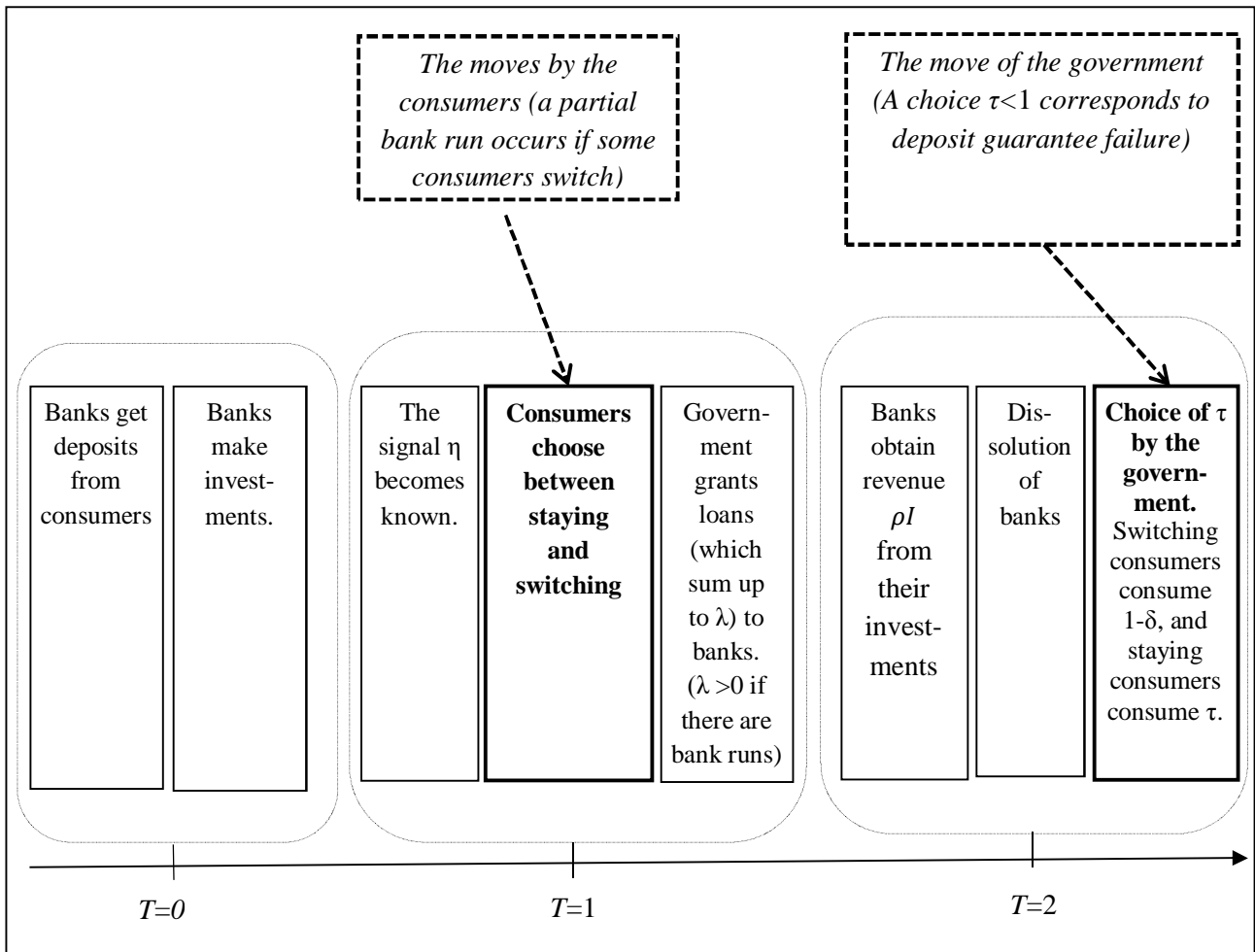
This implies that, just like in our earlier model, the aggregate liquidity loans in period  $T=1$  amount up to  $\lambda$ . Hence, the amount up of public resources

at the end of period  $T=1$  is still given by (2.11), and also the results (2.13), (2.14), and (2.15), which together define the amount of public resources after the government has made its choice at  $T=2$ , remain valid.

We still define welfare by (2.16), and the possible cost from a deposit guarantee breakdown by (2.12). However, since our model now contains also early consumers, the aggregate utility of the consumers is not given by (2.17), but by

$$\tilde{U}(\tau) = \mu_E u(1) + \lambda u(1 - \delta) + (1 - \lambda) u(\tau) \quad (\text{A40})$$

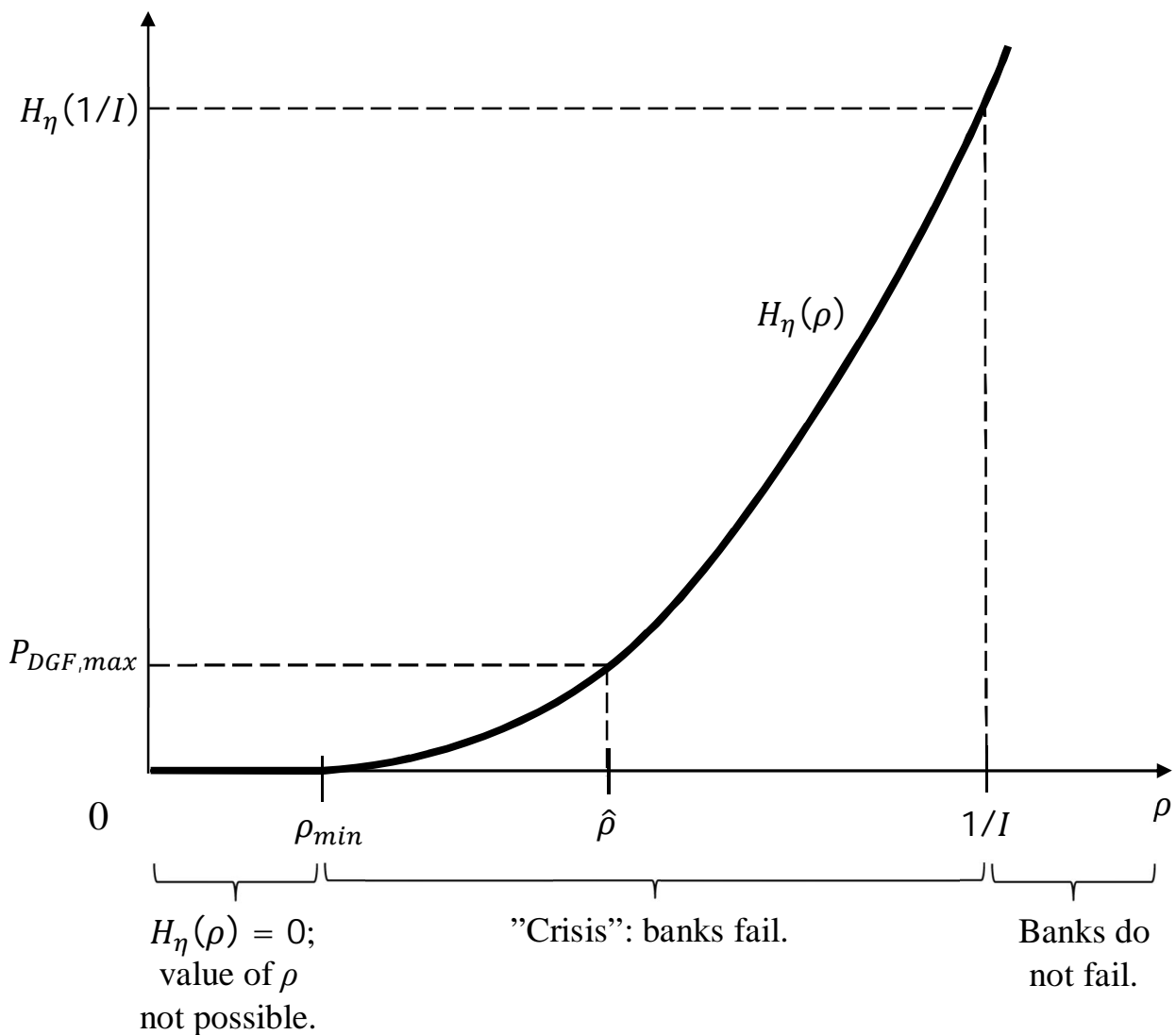
The additional first term of this definition is a constant, and does not affect the solution of the welfare-maximization problem of the government. Hence, Theorem 1 (which describes the optimal choice of  $\tau$  by the government) remains valid. This further implies that for the late consumers the expected utility  $(E_\eta u_B)(\lambda)$  (which is given by (4.1)) remains unchanged for each  $\lambda$ . Hence, the equilibrium value of  $\lambda$  is still determined by Theorem 2, and that also the rest of our analysis (which builds on Theorems 1 and 2) remains valid.



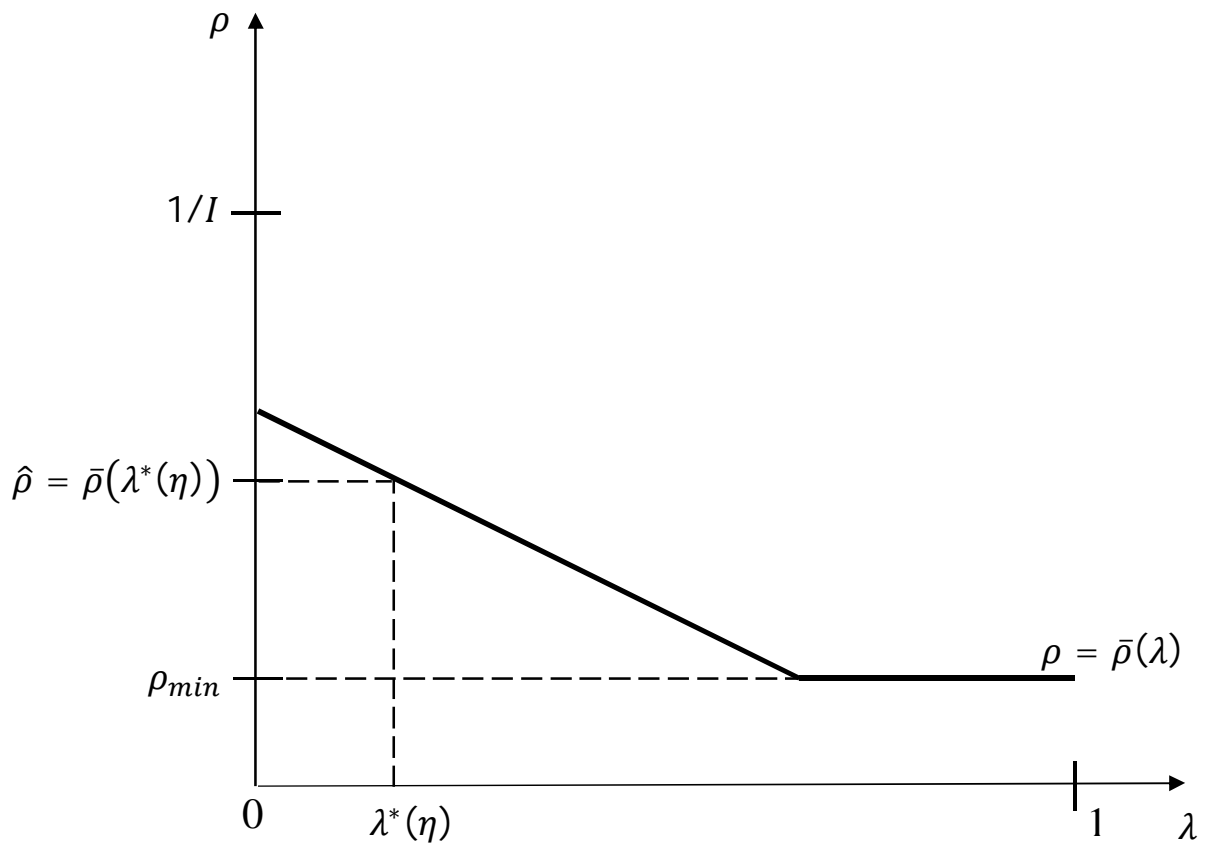
**Figure 1.** Time line

**Figure 2.** A partial bank run equilibrium.

**Figure 2, panel (a)** The distribution function  $H_\eta(\rho)$  for the small values of the per unit revenue  $\rho$  from the banks' investments. Here  $P_{DGF,max} = H_\eta(\hat{\rho})$  is the value of the deposit guarantee failure probability for which switching and staying would produce the same expected utility. The value  $H_\eta(1/I)$  is the probability with which the banks fail.

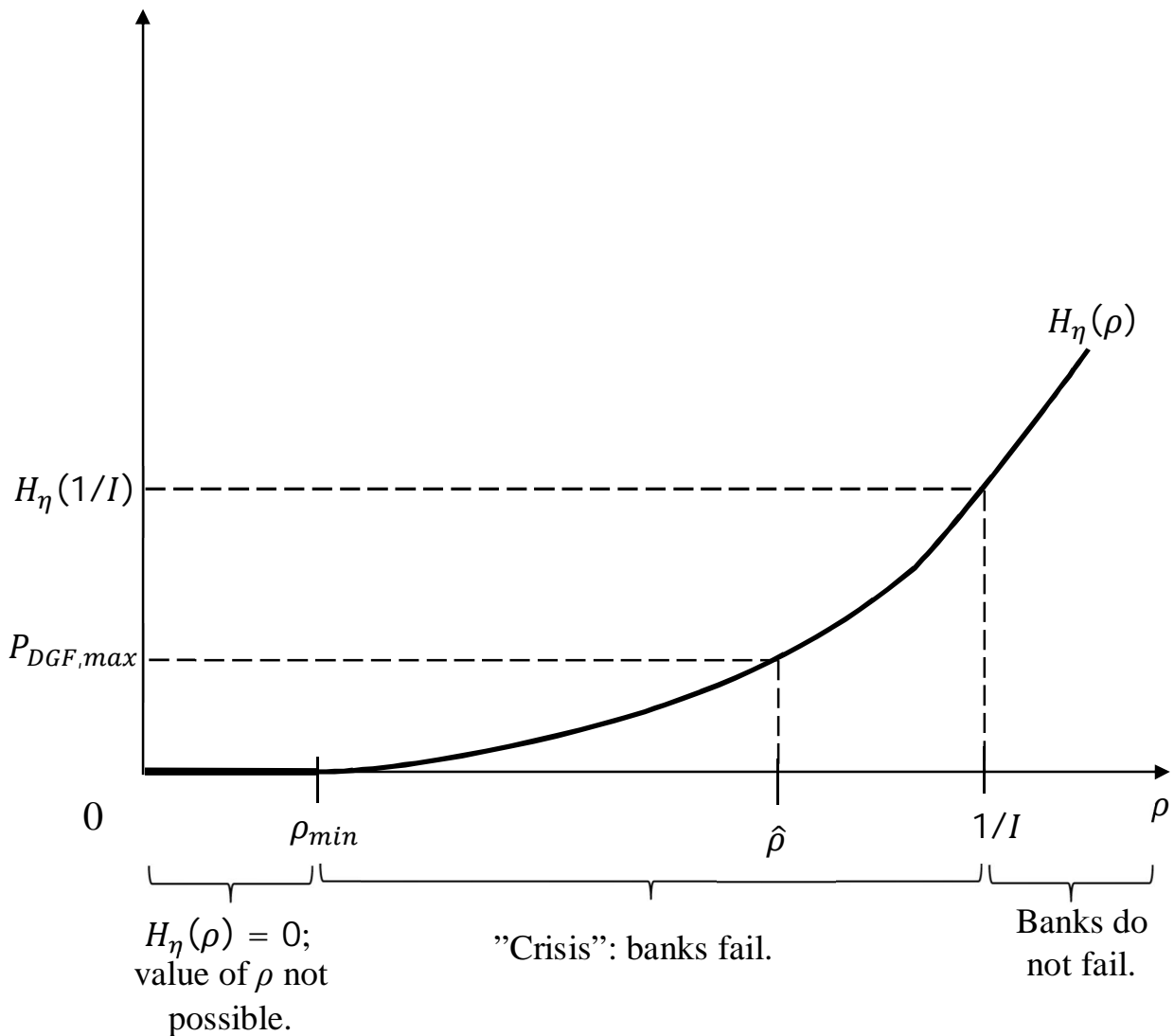


**Figure 2, panel (b).** The function  $\bar{\rho}(\lambda)$ , which shows the “borderline value” of the per unit revenue  $\rho$  (i.e. the value below which the deposit guarantee fails) as a function of the size  $\lambda$  of the bank run. As indicated in panel (a), switching and staying are (for the considered signal  $\eta$ ) equally attractive when the borderline value of deposit guarantee breakdown is  $\hat{\rho}$ . Since  $\bar{\rho}(0) > \hat{\rho}$ , switching is preferable to staying if there is no bank run. Hence, consumers switch until  $\lambda$  has obtained the equilibrium value by  $\lambda = \lambda^*(\eta)$  which is characterized by  $\bar{\rho}(\lambda^*(\eta)) = \hat{\rho}$ .



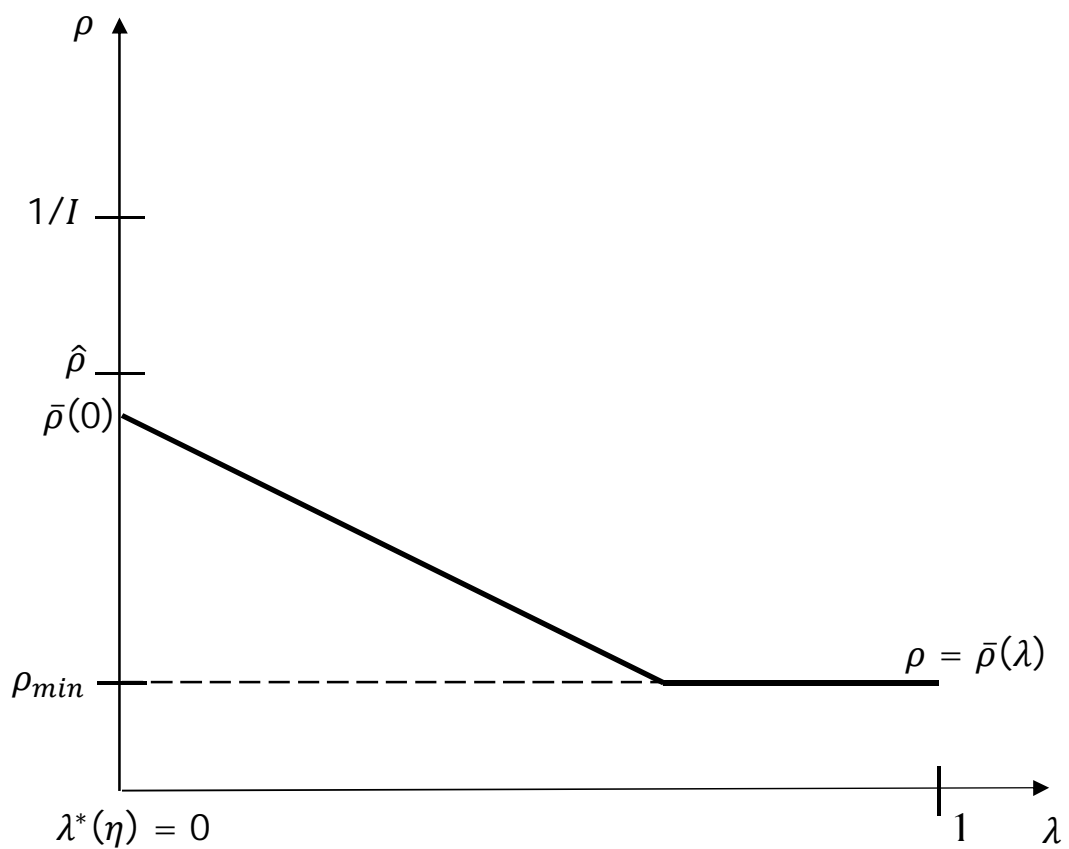
**Figure 3.** An equilibrium without a bank run.

**Figure 3, panel (a)** The distribution function  $H_\eta(\rho)$  for the small values of the per unit revenue  $\rho$  from the banks' investments. Here  $P_{DGF,max} = H_\eta(\hat{\rho})$  is the value of the deposit guarantee failure probability for which switching and staying would produce the same expected utility. The value  $H_\eta(1/I)$  is the probability with which the banks fail. In this figure the signal  $\eta$  is "better" than in Figure 2, since the value of the distribution function  $H_\eta(\rho)$  is for each  $\rho$  smaller than in Figure 2.





**Figure 3, panel (b).** The function  $\bar{\rho}(\lambda)$ , which shows the “borderline value” of the per unit revenue  $\rho$  (i.e. the value below which the deposit guarantee fails) as a function of the size  $\lambda$  of the bank run. As indicated in panel (a), switching and staying would be (for the considered signal  $\eta$ ) equally attractive if the borderline value of deposit guarantee breakdown were  $\hat{\rho}$ . However, since  $\bar{\rho}(0) < \hat{\rho}$ , the actual borderline value of deposit guarantee breakdown is always below  $\hat{\rho}$ . Hence, staying is preferable to switching even if there is no bank run, and in equilibrium no one switches and the equilibrium value of  $\lambda$  is  $\lambda^*(\eta) = 0$ .



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