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Comparing Inflation and Price Level Targeting: the Role of Forward Guidance and Transparency

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Abstract

We examine global dynamics under learning in New Keynesian models with price level targeting that is subject to the zero lower bound. The role of forward guidance is analyzed under transparency about the policy rule. Properties of transparent and non-transparent regimes are compared to each other and to the corresponding cases of inflation targeting. Robustness properties for different regimes are examined in terms of the domain of attraction of the targeted steady state and volatility of inflation, output and interest rate. We analyze the effect of higher inflation targets and large expectational shocks for the performance of these policy regimes.

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1 Introduction

The appearance of the zero lower bound (ZLB) as a constraint for policy interest rates led to the introduction of some new tools of monetary policy,

*Any views expressed are those of the authors and do not necessarily reflect the views of the Bank of Finland.

first in Japan early 2000s and during the financial crises in the United States and Europe since 2008. In this paper attention is focused on forward guidance which is one of these new tools.[1] Forward guidance typically consists of announcements about future plans for instruments of monetary policy - most commonly the policy interest rate. However, forward guidance can also be formulated in terms of a threshold of a target variable, such as the price level. In the latter case the interest rate is kept at the ZLB until the actual value of the target variable reaches its threshold level.[2]

It has been recently suggested that price-level targeting can be a more appropriate framework for monetary policy than inflation targeting. Carney (2012) and Evans (2012) discuss the need for additional guidance for the price level. Also other variables, such as a target path for nominal GDP, has been mentioned as candidate for a threshold in forward guidance.[3] Evans (2012) argues that price-level targeting could be used to combat the liquidity trap. Price-level targeting makes monetary policy history-dependent. This guidance is arguably good policy in a liquidity trap, see Eggertsson and Woodford (2003) for a modified form of price-level targeting under rational expectations (RE).

It is important to allow for significant uncertainties that private agents face in planning their economic activities. One case of these uncertainties arises if a move from inflation targeting to price-level targeting is announced as agents need to learn the new economic environment due to the switch in the policy regime. We consider the properties of price-level targeting under imperfect knowledge and with specification of learning by means of the adaptive learning approach which is increasingly used in the literature.[4] This view differs from the existing literature that is predominantly based on the RE hypothesis. RE is a very strong assumption about the agents' knowledge as under RE agents are able to perfectly predict the future path of the economy, except for the effects of unforecastable random shocks. There

[1] See for example Woodford (2012), Campbell, Evans, Fisher, and Justiniano (2012), Filardo and Hoffmann (2014), Bayomi (2014), Gavin, Keen, Richter, and Throckmorton (2014) and Weale (2013) for discussions of forward guidance.

[2] See for example Woodford (2012), pp.223-4 and Mendes and Murchinson (2014).

[3] Price-level targeting has received a fair amount of attention, see for example Svensson (1999) and Vestin (2006). A recent overview of nominal income targeting is given in Bean (2009).

[4] See Sargent (1993), Evans and Honkapohja (2001), Sargent (2008), and Evans and Honkapohja (2009) for discussions and techniques to study learning dynamics.

has recently been interest in relaxing the RE hypothesis in the context of macroeconomic policy analysis, see e.g. Taylor and Williams (2010) and Woodford (2013).[5]

Our objective is to compare several aspects of dynamics of price-level targeting to inflation targeting in a nonlinear New Keynesian (NK) model when private agents learn adaptively. The NK model is otherwise standard, so that there are no financial market imperfections and the policy frameworks are compared in the simplest setting. This paper emphasizes the transparent case, where private agents know the interest rate rule. We argue that the good performance of price-level targeting requires credible forward guidance when imperfect knowledge prevails and agents behave in accordance with adaptive learning. The companion paper Honkapohja and Mitra (2014) focuses on the case of opacity where the policy instrument rule is not known.

The nonlinear framework is used to assess the global properties of the policy regimes. Two important criteria for good dynamic performance are introduced: (i) the large size of the domain of attraction under learning dynamics and (ii) low volatility of aggregate variables during the learning adjustment. The need for the nonlinear model stems from the observation in Honkapohja and Mitra (2014) that, like inflation targeting, price-level targeting is subject to global indeterminacy problems caused by the ZLB. In a standard NK model a key equation for a nonstochastic steady state is the Fisher equation $R = \beta^{-1}\pi$, where R is the gross interest rate, β is the subjective discount factor and π is the gross inflation rate. Usually the interest rate rule has a specified target inflation rate $\pi^* \geq 1$ (and an associated output level) as a steady state. If policy sets $R = 1$, then $\pi = \beta < 1$ becomes a second steady state as the Fisher equation holds at that state.[6] The targeted steady state is locally stable under learning and the deflationary steady state is locally unstable for the price-level targeting regime without forward guidance but with transparency.

[5] Gaspar, Smets, and Vestin (2007) introduce aspects of imperfect knowledge in their discussion of price-level targeting. Williams (2010) suggests that price-level targeting under imperfect knowledge and learning may not work well. His work relies on simulations of a linearized model.

[6] Analysis of the ZLB and multiple equilibria for an inflation targeting framework and a Taylor-type interest rate rule has been carried out by Reifschneider and Williams (2000), Benhabib, Schmitt-Grohe, and Uribe (2001) and Benhabib, Schmitt-Grohe, and Uribe (2002). These issues have been considered under learning, e.g., in Evans and Honkapohja (2010), and Benhabib, Evans, and Honkapohja (2014).

A key finding in Honkapohja and Mitra (2014) is that the dynamic performance properties of learning depend heavily on whether private agents include the forward guidance provided by the price-level targeting regime in their forecasting and learning. Inclusion of forward guidance means that agents have full credibility of the adopted policy regime; lack of credibility of the monetary policy regime can be a reason for not using forward guidance from price-level targeting. We re-examine this result when the policy rule is transparent.

As noted above, volatility properties of learning adjustment are important in the comparisons of the two policy regimes. We compare the volatility properties of the two policy regimes under alternative assumptions about forward guidance in price level targeting. Our focus is on the transparent case, but some comparisons are made to the non-transparent case for both price-level and inflation targeting.

There have been some arguments made for raising the inflation target in an inflation targeting regime so that the possibility of deflationary risks and incidence of hitting the ZLB are reduced particularly when the economy is hit by large adverse shocks; see e.g. Reifschneider and Williams (2000), Williams (2009) and Blanchard, Dell Ariccia, and Mauro (2010). We examine this argument in our non-linear framework for the different policy regimes both when the economy is subject to small and large expectational shocks. The results are different in the two cases: when expectational shocks are small, IT may be the preferred policy regime. However, in the presence of large expectational shocks, PLT with guidance is a more robust policy regime in terms of the criteria of the domain of attraction, volatility measures and deflationary risks for the economy making it the preferred regime.

2 A New Keynesian Model

The analytical framework we use is a standard New Keynesian model.[7] A continuum of household-firms produce each a differentiated consumption good under monopolistic competition and price-adjustment costs. The government uses monetary policy, buys a fixed amount of output, finances spending by taxes, and issues of public debt.

[7] The same framework is developed in Evans, Guse, and Honkapohja (2008). It is also employed in Evans and Honkapohja (2010) and Benhabib, Evans, and Honkapohja (2014).

The objective for agent s is to maximize expected, discounted utility subject to a standard flow budget constraint (in real terms):

$$Max\ E_0 \sum_{t=0}^{\infty} \beta^t U_{t,s} \left(c_{t,s}, \frac{M_{t-1,s}}{P_t}, h_{t,s}, \frac{P_{t,s}}{P_{t-1,s}} - 1 \right) \tag{1}$$

$$st.\ c_{t,s} + m_{t,s} + b_{t,s} + \Upsilon_{t,s} = m_{t-1,s}\pi_t^{-1} + R_{t-1}\pi_t^{-1} b_{t-1,s} + \frac{P_{t,s}}{P_t} y_{t,s}, \tag{2}$$

where $c_{t,s}$ is the consumption aggregator, $M_{t,s}$ and $m_{t,s}$ denote nominal and real money balances, $h_{t,s}$ is the labor input into production, and $b_{t,s}$ denotes the real quantity of risk-free one-period nominal bonds held by the agent at the end of period t. $\Upsilon_{t,s}$ is the lump-sum tax collected by the government, R_{t-1} is the nominal interest rate factor between periods $t-1$ and t, $P_{t,s}$ is the price of consumption good s, $y_{t,s}$ is output of good s, P_t is the aggregate price level, and the inflation rate is $\pi_t = P_t/P_{t-1}$. The subjective discount factor is denoted by β.

The utility function has the parametric form

$$U_{t,s} = \frac{c_{t,s}^{1-\sigma_1}}{1-\sigma_1} + \frac{\chi}{1-\sigma_2} \left(\frac{M_{t-1,s}}{P_t} \right)^{1-\sigma_2} - \frac{h_{t,s}^{1+\varepsilon}}{1+\varepsilon} - \frac{\gamma}{2} \left(\frac{P_{t,s}}{P_{t-1,s}} - 1 \right)^2,$$

where $\sigma_1, \sigma_2, \varepsilon, \gamma > 0$. The final term displays the cost of adjusting prices in the spirit of Rotemberg (1982). This formulation rather than the Calvo model of price stickiness is used as it enables us to study global dynamics in the nonlinear system. The household decision problem is also subject to the usual "no Ponzi game" (NPG) condition.

Production function for good s is given by

$$y_{t,s} = h_{t,s}^{\alpha},$$

where $0 < \alpha < 1$. Output is differentiated and firms operate under monopolistic competition and face a downward-sloping demand curve

$$P_{t,s} = \left(\frac{y_{t,s}}{y_t} \right)^{-1/\nu} P_t. \tag{3}$$

Here $P_{t,s}$ is the profit maximizing price set by firm s consistent with its production $y_{t,s}$. The parameter ν is the elasticity of substitution between two goods and is assumed to be greater than one. y_t is aggregate output, which is exogenous to the firm.

The government's flow budget constraint in real terms is

$$b_t + m_t + \Upsilon_t = g_t + m_{t-1}\pi_t^{-1} + R_{t-1}\pi_t^{-1}b_{t-1}, \tag{4}$$

where g_t denotes government consumption of the aggregate good, b_t is the real quantity of government debt, and Υ_t is the real lump-sum tax collected. We assume that fiscal policy follows a linear tax rule for lump-sum taxes as in Leeper (1991)

$$\Upsilon_t = \kappa_0 + \kappa b_{t-1}, \tag{5}$$

where we will assume that $\beta^{-1} - 1 < \kappa < 1$. Thus fiscal policy is "passive" in the terminology of Leeper (1991).

We assume that g_t is constant and given by $g_t = \bar{g}$. From market clearing we have

$$c_t + g_t = y_t. \tag{6}$$

Next, we summarize the key behavioral equations. The Appendix provides a quick derivation of the equations (the basic framework is directly adopted from earlier papers including Evans and Honkapohja (2010) and Benhabib, Evans, and Honkapohja (2014)). It is assumed that (i) utilities are logarithmic, (ii) expectations are identical and (iii) price-setters have seen that in the past their price has been equal to the aggregate price.

With these assumptions the optimal price setting decisions imply an infinite horizon Phillips curve in which current decision depend on expectations over the infinite future.[8]

$$\begin{aligned} Q_t &= \frac{\nu}{\alpha\gamma} y_t^{(1+\varepsilon)/\alpha} - \frac{\nu-1}{\gamma} y_t (y_t - \bar{g})^{-1} + \\ &\quad \frac{\nu}{\gamma}\sum_{j=1}^{\infty} \alpha^{-1}\beta^j \left(y_{t+j}^e\right)^{(1+\varepsilon)/\alpha} - \frac{\nu-1}{\gamma}\sum_{j=1}^{\infty}\beta^j \left(\frac{y_{t+j}^e}{y_{t+j}^e - \bar{g}}\right) \\ &\equiv \tilde{K}(y_t, y_{t+1}^e, y_{t+2}^e ...), \text{ where} \\ Q_t &= \pi_t(\pi_t - 1). \end{aligned} \tag{7, 8}$$

It is also assumed that households act in a Ricardian way, i.e. they impose the intertemporal budget constraint (IBC) of the government $\lim_{T\to\infty} D_{t,t+T} b_{t+T} = 0$ in conjunction with (4). Here $D_{t,t+T}$ is the discount factor from t to $t+T$

[8] The formulation of infinite horizon learning in New Keynesian models is emphasized by Preston (2005) and Preston (2006), and is used in Evans and Honkapohja (2010) and Benhabib, Evans, and Honkapohja (2014) to study the properties of a liquidity trap.

and is formally defined in the Appendix. Using the consumer budget constraint and the iterated form of the Euler equation yields an intermediate consumption function (see the Appendix).[9] Then combining the household and government intertemporal budget constraints, yields the consumption function

$$c_t = (1-\beta)\left(y_t - \bar{g} + \sum_{j=1}^{\infty}(D^e_{t,t+j})^{-1}(y^e_{t+j} - \bar{g})\right). \quad (9)$$

3 Monetary Policy Frameworks

Our aim is to compare the performance of price-level targeting against inflation targeting (IT) under either transparent or non-transparent monetary policy. For concreteness and simplicity of the comparisons we model IT in terms of the standard Taylor rule

$$R_t = 1 + \max[\bar{R} - 1 + \psi_\pi(\pi_t - \pi^*) + \psi_y(y_t/y^* - 1), 0] \equiv F_{IT}(\pi_t, y_t), \quad (10)$$

where we have introduced the ZLB, so that the gross interest rate cannot fall below one. For analytical ease, we adopt a piecewise linear formulation of the interest rate rule. The inflation target π^* for the medium to long run is assumed to be known to private agents and agents know the interest rate rule (10) in the transparent case but do not know it under opacity.

For price-level targeting (PLT) we also employ a comparable simple formulation, where (i) the policy maker announces a target path for the price level as a medium to long run target and (ii) sets the policy instrument with the intention to move the actual price level gradually toward a targeted price level path which is specified exogenously. These kinds of instrument rules are called Wicksellian, see pp. 260-61 of Woodford (2003) and Giannoni (2012) for discussions of Wicksellian rules.[10]

We assume that the target price level path $\{\bar{P}_t\}$ involves constant inflation, so that

$$\bar{P}_t/\bar{P}_{t-1} = \pi^* \geq 1. \quad (11)$$

[9] Evans, Honkapohja, and Mitra (2012) state the assumptions under which Ricardian Equivalence holds along a path of temporary equilibria with learning if agents have an infinite decision horizon.

[10] Giannoni (2012) analyses a number of different versions of the Wicksellian rules. A different formulation of PLT is sometimes advocated as a way to achieve optimal policy with timeless perspective under RE locally near the targeted steady state; see Eggertsson and Woodford (2003).

The interest rate, which is the policy instrument, is set above (below, respectively) the targeted steady-state value of the instrument when the actual price level is above (below, respectively) the targeted price-level path $\bar{P}_t$, as measured in percentage deviations. The interest rate also responds to the percentage gap between targeted and actual levels of output. The target level of output y^* is the steady state value associated with π^*. This leads to a **Wicksellian interest rate rule**

$$R_t = 1 + \max[\bar{R} - 1 + \psi_p(X_t - 1) + \psi_y(y_t/y^* - 1), 0] = F_{PLT}(X_t, y_t), \quad (12)$$

where $X_t = P_t/\bar{P}_t$ and the max operation takes account of the ZLB on the interest rate. $\bar{R} = \beta^{-1}\pi^*$ is the gross interest rate at the targeted steady state. To have comparability to the IT rule (10), we adopt a piecewise linear formulation of the interest rate rule.

The target price level path becomes known to the private agents when the PLT policy regime is introduced. Given the ZLB for the interest rate, the price and output gap terms $X_t - 1 = (P_t - \bar{P}_t)/\bar{P}_t$ and $(y_t - y^*)/y^*$ act as triggers towards lifting of the interest rate from its lower bound if either actual price level or output meets its target value. Regarding interest rate setting it is assumed that the form of the interest rate (12) is or is not known to private agents. These are called transparent and opaque cases, respectively.

For expectations formation there are two possible assumptions. One possibility is that forecast future inflation in the same way as under IT, so that inflation expectations adjust in accordance with (20). In this case private agents do not use the additional guidance from the target price level path. We refer to this case as PLT without forward guidance (and under transparency or opacity). A second possibility is that private agents make use of the announced target price level path in the inflation forecasting. This case is called PLT with forward guidance (and under transparency or opacity). As will be seen below, this is a key issue for the properties of PLT regime.

4 Temporary Equilibrium and Learning

The formulation of learning needs to be discussed next. When learning, each agent is assumed to have a model for perceived dynamics of state variables, called the perceived law of motion (PLM), to make his forecasts of relevant variables. The PLM parameters are estimated using available data and the estimated PLM is used for forecasting. When new data becomes available in the next period the PLM parameters are re-estimated and the agent employs the re-estimated PLM in forecasting.

A common formulation is to postulate that the PLM is a linear regression model where endogenous variables depend on intercepts, observed exogenous variables and possibly lags of endogenous variables and a standard estimation method is employed. The regression formulation cannot be applied in the current non-stochastic setting.[11] We therefore assume that agents form expectations using so-called steady state learning. Steady-state learning with point expectations is formalized as

$$s^e_{t+j} = s^e_t \text{ for all } j \geq 1, \text{ and } s^e_t = s^e_{t-1} + \omega_t(s_{t-1} - s^e_{t-1}) \tag{13}$$

for variables $s = y, \pi, X, y/y^*$ (and R in the case of opacity). It should be noted that expectations s^e_t refer to future periods and not the current one. It is assumed that the newest available data point is s_{t-1}, i.e. expectations are formed in the beginning of the current period and current-period values of endogenous variables are not known at that moment.

ω_t is called the "gain sequence," and measures the extent to which the estimates adjust to the most recent forecast error. In stochastic systems one typically assumes "decreasing gain" learning $\omega_t = t^{-1}$ which corresponds to least-squares updating. The case $\omega_t = \omega$ for $0 < \omega \leq 1$, called "constant gain" learning, is also widely used. In this case it is assumed that ω is small and theoretical stability conditions are stated so that they hold for all sufficiently small gains.

We now list the temporary equilibrium equations with steady-state learning under policy transparency, i.e., when the monetary policy rule is known. Under the IT regime agents substitute current values and their own forecasts of inflation and output in the policy rule. Under the PLT regime agents use current values and their own forecasts of the price gap and output inside the rule. In the case of opacity the policy rule is not known, so that agents must formulate expectations of the interest rate R^e_t and have a learning rule for adapting R^e_t over time.

[11] There would be asymptotic perfect multicollinearity. See Evans and Honkapohja (1998) or Section 7.2 of Evans and Honkapohja (2001) for discussions of learning in deterministic and stochastic models.

1. The aggregate demand

$$
\begin{aligned}
y_t &= \bar{g} + (\beta^{-1} - 1)(y_t^e - \bar{g}) \left(\frac{\pi_t^e}{R_t} \right) \left(\frac{R_t^e}{R_t^e - \pi_t^e} \right) \\
&\equiv Y(y_t^e, \pi_t^e, R_t, R_t^e),
\end{aligned} \tag{14}
$$

where under IT $R_t = F_{IT}(\pi_t, y_t)$ and $R_t^e = F_{IT}(\pi_t^e, y_t^e)$, or under PLT $R_t = F_{PLT}(P_t/\bar{P}_t, y_t)$ and $R_t^e = F_{PLT}((P_t/\bar{P}_t)^e, y_t^e)$.

2. The nonlinear Phillips curve

$$
\pi_t = Q^{-1}[\tilde{K}(y_t, y_t^e, y_t^e ...)] \equiv Q^{-1}[K(y_t, y_t^e)] \equiv \Pi(y_t, y_t^e)], \tag{15}
$$

where $\tilde{K}(.)$ is defined in (7), $Q(\pi_t) \equiv (\pi_t - 1)\, \pi_t$ in (8) and

$$
\begin{aligned}
K(y_t, y_t^e) \;\equiv\; & \frac{\nu}{\gamma} \left(\alpha^{-1} y_t^{(1+\varepsilon)/\alpha} - \left(1 - \nu^{-1}\right) \frac{y_t}{(y_t - \bar{g})} \right) \\
& + \frac{\nu}{\gamma} \left(\beta (1-\beta)^{-1} \left(\alpha^{-1} (y_t^e)^{(1+\varepsilon)/\alpha} - \left(1 - \nu^{-1}\right) \frac{y_t^e}{y_t^e - \bar{g}} \right) \right).
\end{aligned} \tag{16}
$$

3. Bond dynamics and money demand

$$
b_t + m_t = g - \Upsilon_t + \frac{R_{t-1}}{\pi_t} b_{t-1} + \frac{m_{t-1}}{\pi_t}, \tag{17}
$$

$$
m_t = \chi \beta \frac{R_t}{R_t - 1} c_t. \tag{18}
$$

The state variables are b_{t-1}, m_{t-1}, and R_{t-1}. With Ricardian consumers the dynamics for bonds and money do not influence the dynamics of the endogenous variables, though the evolution of b_t and m_t is influenced by the dynamics of inflation and output.

Under IT and transparency the system has two expectational variables, output y_t^e and inflation π_t^e. The interest rate expectations R_t^e are also relevant in the case of IT and opacity as agents then do not known the policy rule. Analogously, under PLT and transparency the expectational variables are y_t^e, π_t^e, $X_t^e \equiv (P_{t+1}/\bar{P}_{t+1})^e$ and $relY_t^e \equiv (y_{t+1}/y^*)^e$. Here y^* is the output level associated with π^*. Under transparency agents know the interest rate rule, so they need to make the forecasts X_t^e and $relY_t^e$. In the case of PLT and opacity the expectational variables are output y_t^e, inflation π_t^e, and the interest rate R_t^e.

We now assume that evolution of expectations is given by

$$y_t^e = y_{t-1}^e + \omega(y_{t-1} - y_{t-1}^e), \tag{19}$$
$$\pi_t^e = \pi_{t-1}^e + \omega(\pi_{t-1} - \pi_{t-1}^e), \tag{20}$$
$$X_t^e = X_{t-1}^e + \omega(X_{t-1} - X_{t-1}^e), \tag{21}$$
$$relY_t^e = relY_{t-1}^e + \omega(relY_{t-1} - relY_{t-1}^e), \tag{22}$$
$$R_t^e = R_{t-1}^e + \omega(R_{t-1} - R_{t-1}^e). \tag{23}$$

Here the first four equations are applied in the different cases as explained in the paragraph just before these equations . The case of PLT with forward guidance is developed below in Section 6.2.

5 Multiple Steady States

A non-stochastic steady state (y, π, R) under PLT must satisfy the Fisher equation $R = \beta^{-1}\pi$, the interest rate rule (12), and steady-state form of the equations for output and inflation (14) and (15).

One steady state clearly obtains when the actual inflation rate equals the inflation rate of the price-level target path. Then $R = \bar{R}$, $\pi = \pi^*$ and $y = y^*$, where y^* is the unique solution to the equation

$$\pi^* = \Pi(Y(y^*, \pi^*, \bar{R}, \bar{R}), y^*)].$$

Moreover, for this steady state $P_t = \bar{P}_t$ for all t. The targeted steady state under PLT or IT is not unique. There is also a second steady state in which the ZLB condition is binding:[12]

Lemma 1 *Assume that $\beta^{-1}\pi^* - 1 < \psi_p$. Under the Wicksellian PLT rule (12), there exists a ZLB-constrained steady state in which $\hat{R} = 1$, $\hat{\pi} = \beta$, and $\hat{y}$ solves the equation*

$$\hat{\pi} = \Pi(Y(\hat{y}, \hat{\pi}, 1, 1), \hat{y}). \tag{24}$$

In the ZLB-constrained steady state the price level P_t converges toward zero.

[12] In what follows $\hat{R} = 1$ is taken as a steady state equilibrium. In principle, we then need to impose a finite satiation level in money demand or assume that the lower bound is slightly above one, say $\hat{R} = 1 + \varepsilon$. Neither of these assumptions is explicitly used below as our focus is on inflation and output dynamics.

The result states that, just like IT with a Taylor rule, a common formulation of price-level targeting suffer from global indeterminacy. The corresponding results for IT are well known, see e.g. Benhabib, Schmitt-Grohe, and Uribe (2001). The proof of the result is given in the companion paper Honkapohja and Mitra (2014). We remark that the sufficient condition $\beta^{-1}\pi^* - 1 < \psi_p$ is not restrictive e.g. when $\beta = 0.99$ and $\pi^* = 1.005$.[13]

6 Expectations Dynamics

6.1 Basic Considerations

We now begin to consider dynamics of the economy in these regimes when agents form expectations using adaptive learning. Expectations of output and inflation influence the behavior of actual output and inflation as is evident from equations (14) and (15). As an example consider the inflation targeting regime under transparency when the policy rule and its functional form are known. The agents also know the inflation target. The temporary equilibrium equations are (14), (15) and (10), while agents' expectations are given by equations (19) and (20) in accordance with steady-state learning.

If the policy regime is changed to PLT targeting, one possible assumption is that private agents' forecasting will continue as before. We remark that PLT targeting regime includes one further piece of dynamic information, namely the target path for the price level.[14] Thus, alternatively agents could use data about the gap between actual and target paths in their forecasting and below we show that this formulation has major implications. In this section the dynamics of PLT are analyzed under the assumption that agents forecast and learn according to the relevant equations from (19)-(23) and there is transparency of the policy rule. It is assumed in this section that agents do not use forward guidance in their learning, while forward guidance is formally introduced in Section 6.2.

Following Honkapohja and Mitra (2014) a general framework for analyzing the dynamics of the economy under steady state learning is now outlined. By introducing the variable $X_t = P_t/\bar{P}_t$ it is possible to analyze also the sit-

[13] For PLT a weaker sufficient condition is $\beta^{-1}\pi^* - 1 - \psi_p + \psi_y(\hat{y}/y^* - 1) < 0$, in which the term $\hat{y}/y^*$is complicated function of all model parameters.

[14] In IT regime, knowledge of π^* does not matter as π^* is a constant. Forecasting the future gap between actual π and π^* is equivalent to forecasting future π.

uation where the actual price level is explosive because of $\pi^* > 1$. We then have a further equation

$$X_t = \pi_t X_{t-1}/\pi^* \tag{25}$$

and for the PLT regime with transparency write the temporary equilibrium system (14), (15), (12), and (25) in the general form

$$F(x_t, x_t^e, x_{t-1}) = 0, \tag{26}$$

where the vector x_t contains the dynamic variables. The vector of state variables is $x_t = (y_t, \pi_t, X_t, relY_t)^T$.[15] The learning rules (19)-(22) can be written in vector form as

$$x_t^e = (1-\omega)x_{t-1}^e + \omega x_{t-1}. \tag{27}$$

The system formed by equations (26) and (27) is both high-dimensional and nonlinear, so that much of the analysis for its dynamics is necessarily numerical. Before embarking on this, local stability properties of steady states under PLT are studied. Linearizing around a steady state we obtain the system

$$x_t = (-DF_x)^{-1}(DF_{x^e}x_t^e + DF_{x_{-1}}x_{t-1}) \equiv Mx_t^e + Nx_{t-1}, \tag{28}$$

where for brevity we use the same notation for the deviations from the steady state. Recall that x_t^e refers to the expected future values of x_t and not the current one. Combining (28) and (27) we get the system

$$\begin{pmatrix} x_t \\ x_t^e \end{pmatrix} = \begin{pmatrix} N+\omega M & (1-\omega)M \\ \omega I & (1-\omega)I \end{pmatrix} \begin{pmatrix} x_{t-1} \\ x_{t-1}^e \end{pmatrix}. \tag{29}$$

This system can be analyzed in a standard way as a system of linear difference equations or alternatively using so-called expectational (E-stability) techniques.

We are interested in "small gain" results, i.e. stability obtains for all ω sufficiently close to zero. The steady state is then said to be **expectationally stable** or **(locally) stable under learning**. It turns out that theoretical derivation of local results for learning stability for the PLT regime is intractable, but theoretical results are available in the limiting case $\gamma \to 0$ of

[15] Under opacity the state variables would be $(y_t, \pi_t, X_t, R_t)^T$.

small price adjustment costs. The Appendix contain proofs for the theoretical results in the paper:[16]

Proposition 2 *Consider PLT with transparency but without forward guidance and the Wicksellian interest rate rule (12). The targeted steady state* π^* *is expectationally stable when* $0 < \psi_p$*. The low steady state defined by* $\hat{R} = 1$, $\hat{\pi} = \beta$ *is not E-stable.*

We remark that the corresponding results for the case of opacity are given in Honkapohja and Mitra (2014).

6.2 Forward Guidance in Price-Level Targeting

Above learning dynamics under PLT was formulated under the assumption that private agents continue to learn in the same way as is natural under IT, i.e., they do not incorporate the target price level path into their forecasting system. This might happen, for example, after a regime change from IT to PLT. Agents might continue use the natural method of inflation forecasting under IT, and then agents do not use the forward guidance that the PLT regime provides.

However, this is not the necessary outcome. If private agents fully trust the announcement about the target path under the PLT regime, then private agents' method of forming inflation expectations incorporates the forward guidance that is provided by the target path for the price level.[17] In the companion paper Honkapohja and Mitra (2014) it is shown that (when there is opacity) the inclusion of the forward guidance from PLT dramatically improves robustness of the learning process. We now consider robustness of the PLT regime under the alternate assumption of transparency about the interest rate rule.

The most straightforward formulation for including forward guidance is to assume that agents forecast the future values of gap between the actual and targeted price levels and infer the associated expected inflation from the forecasted gap. The gap is defined as the ratio $X_t \equiv P_t/\bar{P}_t$, so that

$$X_t \equiv X_{t-1} \times (\pi_t/\pi^*). \tag{30}$$

[16]The corresponding results for IT are known. See e.g. Honkapohja and Mitra (2014) for the case of IT with opacity with IH learning. See also the references therein for previous literature.

[17]Forward guidance in the form of announcements of the future path of the interest rate is studied from the learning viewpoint in Cole (2014) and Gauss (2014).

We remark that the variable X_t is also the relevant variable in the interest rate rule (12) in the PLT regime.

Moving (30) one period forward, agents can compute the inflation forecast from the equation

$$ {}_tX_t^e \pi_t^e = (X_t^e \times \pi^*), \tag{31} $$

where X_t^e denotes the forecasted value of the gap for the future periods and ${}_tX_t^e$ refers to the forecast of the current gap X_t in the beginning of period t.[18] This assumes as before that information on current values of endogenous variables is not available at the time of forecasting. The inflation forecasts π_t^e from (31) are then substituted into the aggregate demand function (14). The gap expectations X_t^e and ${}_tX_t^e$ are formed using constant-gain learning so that the learning rules are (21) and

$$ {}_tX_t^e = \omega_1 X_{t-1} + (1-\omega_1) X_{t-1}^e. \tag{32} $$

Thus ${}_tX_t^e$ is a weighted average of the most recent observation X_{t-1} and the most recent forecast X_{t-1}^e for period t. Here the weight ω_1 is positive.

Output expectations are assumed to be formed as before, see equation (19) and under transparency relative output expectations as in (22). The temporary equilibrium is then given by equations (31), (14), (15), (12) and the actual relative price is given by (30). The following Lemma (omitting a straight-forward proof) shows that the low steady state exists in this case:

Lemma 3 *Assume that agents include forward guidance from the target price path into their learning. Then the targeted steady state $R = R^*$, $\pi = \pi^*$, $y = y^*$ with $X^* = 1$ and the ZLB-constrained steady state $\hat{R} = 1$, $\hat{\pi} = \beta$, $y = \hat{y}$ with $\hat{X} = 0$ described in Lemma 1 exist.*

As regards local stability properties of the steady states, it is possible to obtain a theoretical result in the case of small adjustment costs $\gamma \to 0$:

Proposition 4 *Assume that the monetary regime is PLT with transparency and with forward guidance. The targeted steady state is stable under learning for all sufficiently small gain parameters ω and ω_1, provided $0 < \psi_p$. The low steady state defined by $\hat{R} = 1$, $\hat{\pi} = \beta$ is not stable under learning in this regime.*

[18] Note that ${}_tX_{t+1}^e = X_t^e$ in more detailed notation.

The corresponding result for PLT with forward guidance and opacity holds as shown in Honkapohja and Mitra (2014). Local instability of the low steady state is analyzed numerically below; it appears that the low steady state is totally unstable under forward guidance.[19]

7 Robustness of the Policy Regimes: Domain of Attraction

We now adopt a global viewpoint on convergence to the targeted steady state and consider robustness in terms of the domain of attraction for the targeted steady state. How far from the targeted steady state can the initial conditions be and still deliver convergence to the target?

This kind of analysis is necessarily numerical, so values for structural and policy parameters must be specified. We adopt the following calibration for a quarterly framework: $\pi^* = 1.005$, $\beta = 0.99$, $\alpha = 0.7$, $\gamma = 128.21$, $\nu = 21$, $\varepsilon = 1$, and $g = 0.2$. The calibrations of β, α, and g are standard. The chosen value of π^* corresponds to two percent annual inflation rate. We set the labor supply elasticity $\varepsilon = 1$. The value for γ is based on a 15% markup of prices over marginal cost suggested in Leeper, Traum, and Walker (2011) (see their Table 2) and the price adjustment costs estimated from the average frequency of price reoptimization at intervals of 15 months (see Table 1 in Keen and Wang (2007)). It is also assumed that expectations of the real interest rate revert to the steady state value β^{-1} for $j \geq T$.[20] We use $T = 28$. The gain parameter is set at $\omega = 0.002$, which is a low value.

The two steady states are $y^* = 0.943254$, $\pi^* = 1.005$ and the low steady state is $\hat{y} = 0.943026$, $\hat{\pi} = 0.99$. For most simulations we consider a grid of initial conditions and present mean paths. For policy parameters in the PLT regime we adopt the values $\psi_p = 0.25$ and $\psi_y = 1$, which are also used by Williams (2010). For the IT rule (10) the parameter values are assumed to be the usual values $\psi_\pi = 1.5$ and $\psi_y = 0.125$.

We now compare performance of the two rules by computing numerically partial domains of attraction for the targeted steady state under the different

[19] Note that in the limit ${}_tX_t^e, X_t^e \to 0$ equation (31) becomes $0 = 0$, so that inflation expectations are not defined by the equation. It is instead given by the steady state condition $\pi_t^e = \beta$.

[20] The truncation is done to avoid the possibility of infinite consumption levels for some values of the expectations. See Evans and Honkapohja (2010) for more details.

rules. (The computed domains are only partial as the dynamical system is high dimensional.) The computed domains are presented with respect to initial inflation and output expectations π_0^e and y_0^e. Other initial conditions for actual inflation and output are set at $y_0 = y_0^e + 0.0001$ and $\pi_0 = \pi_0^e + 0.0001$. Also $X_0^e = X_0 = 1.003$ and $relY_0 = relY_0^e = y_0^e/y^* - 0.003$ under PLT. The model is then simulated for ranges of values of initial inflation and output expectations. π_0^e ranges from 0.935 to 1.065 at steps of 0.002 while y_0^e varies from 0.923254 and 0.963254 at steps of 0.0005. We say convergence has been attained when both π_t and y_t are within 0.5% of the targeted steady state; otherwise we say the dynamics does not converge.[21]

7.1 The Case without Forward Guidance

The results are described in Figures 1 - 2 which present numerical computations of the domains of attraction for PLT and IT policy regimes under the assumption of transparency. Figure 1 for the PLT regime is computed under the assumption that private agents do not incorporate the forward guidance from the target price level path into their inflation forecasting. Inflation expectations then follow (20). The same way of forming expectations π_t^e is also assumed under the IT regime for which the results are given in Figure 2.[22]

FIGURES 1 AND 2 HERE

It is seen that neither of the two domains of attraction entirely contains the other. In a part of the state space, where π_0^e is fairly high and simultaneously y_0^e is fairly low (i.e. North-West of the target steady state), the domain for the PLT regime without forward guidance is larger than the domain for the IT regime. Otherwise, the IT regime does better than the PLT regime without forward guidance. This happens in particular if y_0^e is above the targeted value y^* and π_0^e is relatively low or high. In fact, even relatively significant deflationary expectations under IT lead to convergence to the targeted steady state as long as this is compensated by high enough y_0^e.

[21] Mathematica routines for the numerical analysis and for technical derivations in the theoretical proofs are available upon request from the authors.

[22] Recall that in the IT regime announcing a constant inflation target π^* does not necessitate a change in inflation forecasting as such announcement is just a constant shift in the forecasted value of inflation.

The results for the same comparison under the assumption of opacity are given in the companion paper Honkapohja and Mitra (2014). The latter results also suggest that IT performs somewhat better than PLT without forward guidance. In conclusion, in terms of robustness of convergence IT seems to perform somewhat better than PLT without forward guidance.

7.2 Domain of Attraction for PLT with Forward Guidance

We now examine robustness of the targeted steady state in terms of the domain of attraction when private agents employ the forward guidance from the PLT regime. The numerical computations are shown for displacements of initial output and relative price level expectations y_0^e and X_0^e from the targeted values in analogy with Figures 1 - 2 showing the case without forward guidance. For simplicity, the simulations assume through the rest of the paper that $\omega_1 = 1$.

The powerful result of Honkapohja and Mitra (2014) that (under the assumption of opacity) the domain of attraction is very large under the PLT regime also holds when there is transparency about the interest rate rule. In the computations the domain of initial conditions for X_0^e and y_0^e was made quite large and we set the initial values of the other variables at the deflationary steady state $\hat{R} = 1$, $\hat{\pi} = \beta$, $y = \hat{y}$ and $relY_0 = relY_0^e = y_0^e/y^* - 0.003$. The gaps X_0 and X_0^e were set at values slightly above 0. Also $X_0 = X_0^e$. Figure 3 presents numerical illustration of the domain of attraction for the PLT policy rule. The grid search for y_0^e was over the range 0.942 to 1 at intervals of 0.0005 and that for X_0^e over the range 0.1 to 2 at intervals of 0.02 with the baseline gain. For equation (32) it is assumed that $\omega_1 = 1$ for simplicity.

FIGURE 3 ABOUT HERE

It is seen that the domain covers the whole area above values $y_0^e = 0.942$, except the unstable low steady state where $X_0 = X_0^e = 0$.

The result in Figure 3 demonstrates the huge role for forward guidance in moving the economy out of the liquidity trap toward the targeted steady state. The mechanism works through deviations of the price level from the target path, i.e. the way the gap variable X_t influences inflation expectations. Identity (30) implies that $X_t/X_{t-1} = \pi_t/\pi^*$ and so the price gap variable X_t decreases whenever inflation is below the target value. In the region where ZLB is binding (and $R_t = R_t^e = 1$ imposed) the dynamics of gap expectations X_t^e translate into dynamics of inflation expectations taking the form

$$\pi_t^e = \pi_{t-1}^e(\pi^*/\Pi(y_{t-1}^e, \pi_{t-1}^e))(1-\omega) + \omega\pi^*, \tag{33}$$

where $\pi_{t-1} = \Pi(y_{t-1}^e, \pi_{t-1}^e) = \Pi(Y(y_{t-1}^e, \pi_{t-1}^e, 1, 1), y_{t-1}^e)$ by (15). Equation (33) results from combining equations (21) and (31) and assuming that $\omega_1 = 1$. (Note that the derivation of (33) assumes that X_t and X_t^e are not zero, so that the intersection of the isoclines not undefined.) It is seen from (33) that in case realized inflation falls far below the target π^* (so that $(\pi^*/\pi)(1-\omega) > 1$), then inflation expectation begins to increase from its previous value.

The key conclusion is that with forward guidance from PLT the target path for the price level continues to influence the economy through inflation expectations even when ZLB is binding. Monetary policy alone is able to pull the economy out of the liquidity trap if PLT is implemented to induce agents to use forward guidance in their expectations formation.

8 Further Aspects of Learning Dynamics

Our focus now shifts to the nature of adjustment paths under learning after a small shock has displaced the economy from the targeted steady state (which is locally stable for the different cases). The general approach is similar in spirit to the commonly used impulse response analysis in stochastic models, except that shocks are not normalized to have unit variance. The calibration specified in Section 7 is used in the simulations. We start by illustrating the dynamic paths for inflation output and the interest rate. Then we consider various measures of volatility for the adjustment paths. Finally, we look at the range of fluctuations in the adjustments for the different monetary policy regimes. In particular, we are interested in the frequency of paths that hit either the ZLB or the border of deflation $\pi < 1$.

8.1 The Nature of Adjustment

We first consider the basic features of the adjustment paths under adaptive learning. For the PLT regime the dynamics are studied both without and with agents including forward guidance in their expectations formation. Transparency about the interest rate rule is assumed (see Honkapohja and Mitra (2014) for corresponding analysis under opacity about the interest rate rule).

Figures 4 - 6 illustrate the dynamics of inflation, output and the interest rate for IT, PLT with forward guidance, and PLT without guidance. To generate the figures the model was simulated for various values of initial inflation, price gap and output expectations, π_0^e, X_0^e and y_0^e, in the neighborhood of the desired steady state. π_0^e ranges in an interval of 1% annually around π^* i.e. from 1.0025 to 1.0075 at steps of 0.0002 while y_0^e varies in an interval around y^*, specifically between 0.94303 and 0.94355 at steps of 0.00001. In the case with forward guidance X_0^e is between 0.9975 and 1.0025 (i.e. within 1% of its annual steady state value) at intervals of 0.0002. The gain parameter is at a baseline value of 0.002. For initial output and inflation we set $y_0 = y_0^e + 0.001$ and $\pi_0 = \pi_0^e + 0.001$ as before while in PLT the initial deviation for the target path is set at $X_0 = 1.003$ i.e. 0.3% off. As before $relY_0 = relY_0^e = y_0^e/y^* - 0.003$.[23] The runs for all the grid points are done for a time interval of 500 periods.

The figures show the mean paths of the endogenous variables for the first 200 periods.

FIGURES 4 - 6 ABOUT HERE

It is seen that convergence for IT is monotonic after the initial jump, whereas for PLT there is oscillatory convergence to the targeted steady state, but the oscillations die away much faster when agent use the forward guidance. By far the fastest convergence occurs in the case of PLT with forward guidance.

8.2 Measures of Adjustment Volatility

The differences in the adjustment dynamics just shown suggest that IT and PLT with/without forward guidance should be compared further in terms of the disequilibrium adjustment properties toward the targeted steady state. The further robustness property considered here is **volatility**: how big are the fluctuations during the adjustment path?

[23] This means that mean paths for inflation and output start from initial values that are above the corresponding steady state values. This delivers genuine adjustment dynamics.

Volatility in inflation, output and interest rate during the learning adjustment shown in Figures 4 - 6 is computed in terms of median unconditional variances of inflation, output and interest rate. We also calculate the value of a quadratic loss function in terms of the weights 0.5 for output, 0.1 for the interest rate and 1 for the inflation rate (the weights are taken from Williams (2010)) and the median ex post utility of the representative consumer. The utility function used to compute ex-post utility is

$$\sum_{t=0}^{Tend} \beta^t U_t, \text{ where}$$

$$U_t = \ln[y_t - g] + \chi \ln[\frac{\beta\chi R_{t-1}(y_{t-1} - g)}{(R_{t-1} - 1)\pi_t}] - \frac{y_t^{\frac{1+\varepsilon}{\alpha}}}{1+\varepsilon} - \frac{\gamma}{2}(\pi_t - 1)^2.$$

Here $Tend = 500$ and the money demand function has been used to substitute out the real balances in the utility function. $\chi = 1$ is assumed as in Chapter 2.5 of Gali (2008) and $R_{-1} = R_0$ and $y_{-1} = y_0$.

The grid searches are over $\pi_0^e \in [1.0025, 1.0075]$, $y_0^e \in [0.94303, 0.94355]$ and $X_0^e \in [0.9975, 1.0025]$. This means that for inflation and relative price expectations the range of allowable fluctuations is within 1% annually. In the next sub-section we allow for wider ranges of these fluctuations. The initial relative price is set at $X_0 = X_0^e + 0.0001$. The way the dynamics are generated is the same as for Figures 4 - 6 above. The reported results are the median magnitudes based on a run of 500 periods using our baseline gain of 0.002 for each monetary policy.

	$var(\pi)$	$var(y)$	$var(R)$	$LOSS$	$Utility$
IT	0.441806	0.345378	0.854288	0.699923	314.005
PLT nog	1.39809	1.13175	9.54669	2.91863	293.912
PLT wig	0.142463	1.17121	1.38875	0.866945	313.196

Table 1: Volatility of inflation, output and interest rate for IT and PLT with and without forward guidance under transparency.
Note: the numbers should be multiplied by 10^{-6} (except for utility).

In all tables we use the acronyms nog = without forward guidance and wig = with forward guidance. It is seen from Table 1 that in terms of output and interest rate fluctuations as well as in terms of loss and utility, IT clearly does best. For inflation fluctuations only, IT is trumped by PLT with guidance.

Central banks typically care about inflation and output volatility. In terms of the results of Table 1, there is a trade-off between these two volatilities for the policy regimes of IT and PLT with guidance: output volatility is higher but inflation volatility lower for PLT with guidance *vis-a-vis* IT. In sharp contrast, PLT without guidance is the worst regime (except for output volatility where it slightly outperforms PLT with guidance).

Next, we assess the significance of transparency for the dynamics by reporting the same volatility measures for IT and the two cases of PLT under opacity using the results in Honkapohja and Mitra (2014). The details for the grid searches are the same as those in Table 1. The reported results are the median volatilities based on a run of 500 periods using our baseline gain of 0.002 for PLT.

	$var(\pi)$	$var(y)$	$var(R)$	$LOSS$	$Utility$
IT	4.86463	0.34440	10.6563	6.10246	314.257
PLT nog	1.28178	1.00526	6.79211	2.46362	316.137
PLT wig	0.14772	1.18161	1.12792	0.85131	316.53

Table 2: Volatility of inflation, output and interest rate for IT and PLT with and without forward guidance under opacity.
Note: the numbers should be multiplied by 10^{-6} (except for utility).

By comparing Tables 1 and 2, we see that transparency has major benefits for IT. Inflation, interest rate volatility and loss are dramatically reduced for IT when opacity is replaced by transparency. Transparency has less obvious benefits for PLT both with and without guidance. In fact, PLT without guidance performs worse under transparency in all dimensions! We conclude that transparency is a major issue of importance for IT. Contrariwise, the central issue of relevance for PLT is whether agents incorporate forward guidance or not in their forecasting rather than transparency *per se*.

8.3 Extent of Fluctuations, ZLB and Deflation

Numerical simulations are now used to analyze the extent of fluctuations in inflation, output and interest rates under the three monetary policy regimes. We also look at how often do these fluctuations result in hitting either the ZLB for the interest rate or boundary for deflation, and how the level of inflation target affects these fluctuations and frequencies of hitting the ZLB or deflation boundary. In contrast to the previous sub-section we allow for a wider range of initial shocks to inflation and relative price expectations which may be a more realistic description of the situation experienced in recent times since the financial crisis.

The basic setting is as before except that initial inflation expectations are randomly picked from the range 0% to 4% under a uniform distribution. More precisely, when $\pi^* = 2\%$ annually, initial inflation expectations are assumed to be annually between $0 - 4\%$ and when $\pi^* = 4\%$ annually, initial inflation expectations are annually between $2 - 6\%$ for both IT and PLT without forward guidance; in other words, a 2% fluctuation around the target value is considered (as opposed to 1% in the previous sub-section). For PLT with forward guidance, relative price expectations fluctuates by 2% symmetrically annually around one i.e. from 2% below one to 2% above one (in contrast to 1% in the previous sub-section).[24] Note that output expectations always range in the interval $(0.94303, 0.94355)$ as in the previous sub-section.

Baseline Inflation target (π^* = 2% annually): Table 3 gives the same measures of volatility as in Tables 1 and 2 with an inflation target of 2% annually.

	$var(\pi)$	$var(y)$	$var(R)$	$LOSS$	$Utility$
IT	0.7675	0.3452	1.5789	1.09798	314.297
PLT nog	1.93947	1.58158	17.3474	4.465	293.612
PLT wig	0.1430	1.1712	1.6471	0.8933	313.836

Table 3: Volatility of inflation, output and interest rate for different policy rules.
Note: the numbers should be multiplied by 10^{-6} (except for utility).

Table 4 gives the fluctuation ranges and the frequencies of hitting ZLB (when interest rate is below 1.001) or deflation boundary (Def shows the percentage of times in all simulations when $\pi < 1$) corresponding to previous table. $\pi l = 1.0001$ below.

[24] We note that the distribution of inflation is often assumed to be independent of the value of the inflation target, see e.g. Williams (2009). This contrasts with the older literature that posits a positive relation between the average rate and variability of inflation, see e.g. Taylor (1981).

	$Range(\pi)$	$Range(y)$	$Range(R)$	ZLB	Def
IT	[0.9956,1.0174]	[0.9309,0.9531]	[1.0020,1.0324]	0	0.1731
PLT nog	[0.9877,1.0294]	[0.9291,0.9565]	[πl,1.0617]	0.4297	1.2469
PLT wig	[0.9926,1.0261]	[0.9053,0.9702]	[πl,1.0429]	0.1476	0.0394

Table 4: Fluctuation ranges for π, y and R and frequencies of hitting ZLB or deflation boundary under different policy rules.

We summarize some of the important results from the above simulations. From Tables 3 and 4 we see that, as before, PLT without guidance performs the worst compared to the other two regimes (IT and PLT with forward guidance): the volatilities of inflation, output and interest rate are all significantly higher as is the loss (with utility being correspondingly lower). Inflation falls into the deflationary zone in 1.25% of the time periods and the ZLB for interest rates is hit in 0.43% time periods. The volatility of output and interest rate is lower with IT compared to PLT with forward guidance but volatility of inflation is higher as before; thus, the trade-off between inflation and output volatility continues to hold true for these two regimes. Utility is slightly higher with IT though the loss is higher as well. In terms of the deflationary risks, PLT with forward guidance is better since inflation falls below zero in only 0.04% of time periods compared to 0.17% for IT. However, while IT does not hit the ZLB for interest rates, it is hit in 0.15% of the cases for PLT with guidance.

The risks associated with deflation are considered to be more serious than inflation because it can cause a vicious circle of rising real debt burdens leading in turn to more downward pressure on prices (i.e. the debt-deflation problem); on this count, therefore, PLT with forward guidance is to be preferred over IT. We also note that allowing for wider range of initial fluctuations in inflation or relative price expectations makes IT worse compared to PLT with guidance in terms of the loss function (compare Tables 1 and 3); this is due to a rise in inflation and interest rate volatility for IT even though output volatility is relatively unaffected.

Higher inflation target ($\pi^* = 4\%$ annually): Next we consider how the volatility measures depend on the level of the inflation target. It is assumed that the inflation target is raised from 2% to 4% in annual terms.[25]

[25]We remark that the domain of attraction results for IT, PLT with and without guid-

The range for initial shocks is as specified above. Table 5 reports volatility measures for the different policy regimes while Table 6 gives fluctuation ranges and the frequencies of hitting the ZLB or deflation boundary in the different policy regimes operating with the higher 4% inflation target.

	$var(\pi)$	$var(y)$	$var(R)$	$LOSS$	$Utility$
IT	0.77096	0.342517	1.58294	1.10051	287.641
PLT nog	1.91606	1.56086	17.5044	4.44693	270.251
PLT wig	0.1408	1.2004	1.6571	0.90667	283.336

Table 5: Volatility of inflation, output and interest rate for different policy rules with $\pi^* = 4\%$.
Note: the numbers should be multiplied by 10^{-6} (except for utility).

	$Range(\pi)$	$Range(y)$	$Range(R)$	ZLB	Def
IT	[0.9976,1.0193]	[0.9340,0.9561]	[1.003,1.033]	0	0.0312
PLT nog	[0.9881,1.0298]	[0.9319,0.9584]	[πl,1.0629]	0.1291	0.1717
PLT wig	[0.9930,1.0265]	[0.9205,0.9807]	[πl,1.0583]	0.0104	0.0371

Table 6: Fluctuation ranges for π, y and R and frequencies of hitting ZLB or deflation boundary under different policy rules with $\pi^* = 4\%$.

With a higher inflation target, the qualitative comparison across different policy regimes is generally unaffected. PLT without guidance continues to perform the worst (see Tables 7 and 8) while the comparison between IT and PLT with guidance is mixed; the volatility of output and interest rate is lower with IT but volatility of inflation is higher as is utility (but loss is higher). The trade-off between inflation and output volatility for IT and PLT with guidance continues to be observed. The ZLB is not hit with IT and it is hit in a small fraction (0.01%) of the cases for PLT with guidance. These results are similar to the ones obtained with a lower inflation target.

A higher inflation target, nevertheless, has two significant benefits for IT and PLT with guidance. In the case of IT, the deflationary risks are substantially reduced and in fact it is slightly smaller than PLT with guidance. In contrast, for PLT with guidance and the higher inflation target, the risks

ance outlined in Section 7 are unchanged when the inflation target is raised to 4%.

of hitting the ZLB are significantly reduced. The message for policymakers is that PLT with guidance may be a better policy regime than IT when the inflation target is low and when a larger range of expectational shocks hit the economy. On the other hand, IT is the better regime when the inflation target is low and when small expectational shocks hit the economy (as shown in the previous sub-section; see Table 1). In terms of utility, the lower inflation target is preferred for all regimes.

Finally, we consider the case where the inflation target is at the higher level of 4% and also the range of the initial inflation expectational shock is much higher, so that it is $\pi_0^e \in [0, 10]$ percent. The aim of this exercise is to see which regime is the best in terms of volatility measures etc. when the economy is subject to even bigger expectational shocks than before. As might be suspected from our previous discussion, PLT with guidance continues to be the preferred regime in this case. Tables 7 and 8 give the results.

	$var(\pi)$	$var(y)$	$var(R)$	$LOSS$	$Utility$
IT	2.2284	0.3431	4.7921	2.8792	292.533
PLT nog	5.9750	4.6930	106.262	18.9477	249.647
PLT wig	0.1408	1.1985	2.8934	1.0285	283.582

Table 7: Volatility of inflation, output and interest rate for different policy rules.
Note: the numbers should be multiplied by 10^{-6}(except for utility).

	$Range(\pi)$	$Range(y)$	$Range(R)$	ZLB	Def
IT	[0.9965,1.0251]	[0.9324,0.9574]	[1.0012,1.033]	0	0.0511
PLT nog	[0.9612,1.1308]	[0.8611,0.9846]	[πl,1.1995]	2.2692	1.9692
PLT wig	[0.9909,1.0285]	[0.9188,0.9814]	[πl,1.0604]	0.0131	0.0368

Table 8: Fluctuation ranges for π, y and R and frequencies of hitting ZLB or deflation boundary under different policy rules with $\pi^* = 4\%$, with big range for the initial shock.

To summarize, PLT without guidance continues to be the worst policy regime. Apart for output volatility and utility, PLT with guidance outperforms IT in terms of inflation, interest rate volatility and loss. An important observation is that with larger expectational shocks, interest rate volatility is much higher under IT which in turn results in a significantly higher loss compared to PLT with guidance (compare Tables 5 and 7). Another notable feature is that the incidence of deflationary risks goes up (slightly) with IT and in fact is higher than PLT with forward guidance. The ZLB is hit in 0.1% of the cases for PLT with guidance and is not hit under IT.

Somewhat surprisingly, despite the bigger range of 10% for the initial shock, the volatility measures for inflation and output are not much affected compared to the case when the range of the shock is smaller at 4%. In particular, output volatility does not change much under IT whereas both inflation and output volatility are not much affected for PLT with guidance with bigger expectational shocks. It is only in the case of PLT without guidance that all volatility measures for inflation, output and the interest rate are much higher.

The conclusion is that performance of PLT with forward guidance is relatively robust with respect to the level of inflation target and the magnitude of possible shocks in terms of volatility measures and the deflationary risks posed by the ZLB. This result is further strengthened when we consider the results of Section 7 on the domain of attraction where it was shown unambiguously to be the best policy regime. The other two policy regimes, IT and PLT without forward guidance are less robust in this sense. In the absence of credibility (guidance) IT is the best regime even when the ZLB is taken into account in our non-linear framework. However, if the central bank is able to build credibility which results in agents using (forward) guidance under PLT, then the latter is a better policy regime.

9 Conclusion

Our study focuses on the roles of forward guidance and transparency in assessing price-level vs. inflation targeting. The desirability of a particular regime depends on the magnitude of shocks hitting the economy. In the presence of small expectational shocks, a key result is that transparency has significant benefits for IT in sharp contrast to that for PLT (with or without guidance). This overturns the corresponding conclusion under opacity as then price-level targeting with forward guidance clearly dominates inflation targeting.

More specifically, one key result holds under both opacity and transparency: the domain of attraction for price-level targeting with forward guidance is very large with basically global convergence and is thus much larger than the domain of attraction for inflation targeting. However, under transparency, inflation targeting is better than price-level targeting in terms of volatility properties for adjustment paths around the target steady state (though the deflationary risks and inflation volatility under IT are higher than PLT with guidance). As mentioned, these results are valid when one allows for small expectational shocks.

If, on the contrary, the economy is hit by large expectational shocks (as may have happened during the recent Great Recession), PLT with forward guidance is a more robust regime and out-performs IT in terms of inflation, interest rate volatility and loss (and, of course, in terms of the domain of attraction criterion). Guidance is absolutely crucial for obtaining good outcomes for PLT. In the absence of guidance, PLT is clearly dominated by IT on all counts; it mostly has a bigger domain of attraction than PLT and also its volatility measures are better.

A Deriving the Phillips Curve and the Consumption Function

As in Evans, Guse, and Honkapohja (2008), the first-order conditions for an optimum yield

$$\begin{aligned} 0 \quad = \quad & -h_{t,s}^{\varepsilon} + \frac{\alpha\gamma}{\nu}(\pi_{t,s}-1)\pi_{t,s}\frac{1}{h_{t,s}} \\ & +\alpha\left(1-\frac{1}{\nu}\right)y_t^{1/\nu}\frac{y_{t,s}^{(1-1/\nu)}}{h_{t,s}}c_{t,s}^{-\sigma_1} - \frac{\alpha\gamma\beta}{\nu}\frac{1}{h_{t,s}}E_{t,s}(\pi_{t+1,s}-1)\pi_{t+1,s}. \end{aligned} \tag{34}$$

$$c_{t,s}^{-\sigma_1} = \beta R_t E_{t,s}\left(\pi_{t+1}^{-1}c_{t+1,s}^{-\sigma_1}\right) \tag{35}$$

and

$$m_{t,s} = (\chi\beta)^{1/\sigma_2}\left(\frac{\left(1-R_t^{-1}\right)c_{t,s}^{-\sigma_1}}{E_{t,s}\pi_{t+1}^{\sigma_2-1}}\right)^{-1/\sigma_2}, \tag{36}$$

where $\pi_{t+1,s} = P_{t+1,s}/P_{t,s}$.

A.1 The Nonlinear Phillips Curve

Equation (34) is the nonlinear New Keynesian Phillips curve describing the optimal price-setting by firms. The term $(\pi_{t,s}-1)\,\pi_{t,s}$ arises from the quadratic form of the adjustment costs, and this expression is increasing in $\pi_{t,s}$ over the allowable range $\pi_{t,s} \geq 1/2$. The first term on the right-hand side is the marginal disutility of labor while the third term can be viewed as the product of the marginal revenue from an extra unit of labor with the marginal utility of consumption. The terms involving current and future inflation arise from the price-adjustment costs.

Defining

$$Q_{t,s} = (\pi_{t,s} - 1)\,\pi_{t,s}, \tag{37}$$

using the appropriate root $\pi \geq \frac{1}{2}$ for given $Q \geq -\frac{1}{4}$, the production function and the demand curve we can iterate the Euler equation (34) as

$$Q_{t,s} = x_{t,s} + \sum_{j=1}^{\infty} \beta^j E_{t,s} x_{t+j,s}, \tag{38}$$

provided that the transversality condition $\beta^j E_{t,s} x_{t+j,s} \to 0$ as $j \to \infty$ holds.[26] Here

$$x_{t,s} \equiv \frac{\nu}{\alpha\gamma}(P_{t,s}/P_t)^{-(1+\varepsilon)\nu/\alpha} Y_t^{(1+\varepsilon)/\alpha} - \frac{\nu - 1}{\gamma} y_t (P_{t,s}/P_t)^{-(\nu-1)} c_{t,s}^{-\sigma_1}.$$

The variable $x_{t+j,s}$ is a mixture of aggregate variables and the agent's own future decisions.

We now make some further adaptive learning assumptions. First, agents are assumed to have point expectations. Second, agents are assumed to have learned from experience that $P_{t,s}/P_t = 1$ in temporary equilibrium, so that they set $(P_{t+j,s}/P_{t+j})^e = 1$. Third, similarly agents have learned from experience that $c_{t,s} = y_t - g_t$ in per capita terms and thus agents impose in their forecasts that $c^e_{t+j,s} = y^e_{t,t+j} - g^e_{t,t+j}$. With no fiscal policy changes this becomes $c^e_{t+j,s} = y^e_{t+j} - \bar{g}$.

Next, the representative agent assumption is invoked, so that all agents s have the same initial money and debt holdings and also make the same forecasts $c^e_{t+1,s}$ $\pi^e_{t+1,s}$, as well as forecasts of other relevant variables. Under these assumptions $h_{t,s} = h_t$, $y_{t,s} = y_t$, $c_{t,s} = c_t$ and $\pi_{t,s} = \pi_t$, and all agents make the same forecasts. For convenience, the utility of consumption and of money is also taken to be logarithmic ($\sigma_1 = \sigma_2 = 1$). Then (38) takes the

[26] The condition is an implication of the necessary transversality condition for optimal price setting, see Benhabib, Evans, and Honkapohja (2014).

form

$$Q_t = \frac{\nu}{\alpha\gamma} y_t^{(1+\varepsilon)/\alpha} - \frac{\nu-1}{\gamma} y_t (y_t - \bar{g})^{-1} + \frac{\nu}{\gamma} \sum_{j=1}^{\infty} \alpha^{-1} \beta^j \left(y_{t+j}^e\right)^{(1+\varepsilon)/\alpha} - \frac{\nu-1}{\gamma} \sum_{j=1}^{\infty} \beta^j \left(\frac{y_{t+j}^e}{y_{t+j}^e - \bar{g}} \right) \equiv \tilde{K}(y_t, y_{t+1}^e, y_{t+2}^e ...), \tag{39}$$

where $Q_t = (\pi_t - 1)\pi_t$. The expectations are formed at time t where at the time of forecasting variables at time t are not in the information set of the agents. Period t variables are known to agents at the moment of current decision making. (39) is the price-setting equation that determines π_t for given expectations $\{y_{t+j}^e\}_{j=1}^{\infty}$.[27]

A.2 The Consumption Function

Equation (35) is the standard Euler equation giving the intertemporal first-order condition for the consumption path. Equation (36) is the money demand function. To derive the consumption function one defines the asset wealth $a_t = b_t + m_t$ and the flow budget constraint

$$a_t + c_t = y_t - \Upsilon_t + r_t a_{t-1} + \pi_t^{-1}(1 - R_{t-1}) m_{t-1}, \tag{40}$$

where $r_t = R_{t-1}/\pi_t$. Note that we assume $(P_{jt}/P_t) y_{jt} = y_t$. Iterating (40) forward and imposing

$$\lim_{j\to\infty} (D_{t,t+j}^e)^{-1} a_{t+j}^e = 0, \text{ where} \tag{41}$$

$$D_{t,t+j}^e = \frac{R_t}{\pi_{t+1}^e} \prod_{i=2}^{j} \frac{R_{t+i-1}^e}{\pi_{t+i}^e}$$

[27] Note that inflation does not depend directly on the expected future aggregate inflation rate in (7). The representative agent assumption implies that firm's output equals average output in every period and we obtain (7). There is an indirect effect of expected inflation on current inflation via current output.

with $r^e_{t+i} = R^e_{t+i-1}/\pi^e_{t+i}$, we obtain the life-time budget constraint of the household[28]

$$\begin{aligned} 0 &= r_t a_{t-1} + \Phi_t + \sum_{j=1}^{\infty} (D^e_{t,t+j})^{-1} \Phi^e_{t+j} && (42) \\ &= r_t a_{t-1} + \phi_t - c_t + \sum_{j=1}^{\infty} (D^e_{t,t+j})^{-1} (\phi^e_{t+j} - c^e_{t+j}), && (43) \end{aligned}$$

where

$$\begin{aligned} \Phi^e_{t+j} &= y^e_{t+j} - \Upsilon^e_{t+j} - c^e_{t+j} + (\pi^e_{t+j})^{-1}(1 - R^e_{t+j-1}) m^e_{t+j-1}, && (44) \\ \phi^e_{t+j} &= \Phi^e_{t+j} + c^e_{t+j} = y^e_{t+j} - \Upsilon^e_{t+j} + (\pi^e_{t+j})^{-1}(1 - R^e_{t+j-1}) m^e_{t+j-1}. \end{aligned}$$

Invoking the relations

$$c^e_{t+j} = c_t \beta^j D^e_{t,t+j}, \tag{45}$$

which are an implication of the consumption Euler equation (35), and using (36) we obtain the consumption function

$$c_t \frac{1+\chi\beta}{1-\beta} = r_t b_{t-1} + \frac{m_{t-1}}{\pi_t} + y_t - \Upsilon_t + \sum_{j=1}^{\infty} (D^e_{t,t+j})^{-1} (y^e_{t+j} - \Upsilon^e_{t+j}).$$

Assuming that households act in a Ricardian way, i.e. they impose the intertemporal budget constraint (IBC) of the government, modifies the consumption function as in Evans and Honkapohja (2010). From (4) and assuming $\lim_{T\to\infty} D_{t,t+T} b_{t+T} = 0$, we get

$$0 = r_t b_{t-1} + \Delta_t + \sum_{j=1}^{\infty} D^{-1}_{t,t+j} \Delta_{t+j}, \tag{46}$$

where $\Delta_t = \bar{g} - \Upsilon_t - m_t + m_{t-1}\pi_t^{-1}$. Combining the two budget constraints, iterating forward and using (45) yields the consumption function

$$c_t = (1-\beta) \left(y_t - \bar{g} + \sum_{j=1}^{\infty} (D^e_{t,t+j})^{-1} (y^e_{t+j} - \bar{g}) \right). \tag{47}$$

[28] Here all expectations are formed in period t, which is indicated in the notation for $D^e_{t,t+j}$ but is omitted from the other expectational variables.

B Local Stability of Steady States: Theoretical Results

We first provide a proof of Proposition 2:

Proof. In the case of PLT policy regime with transparency but without forward guidance the learning system (28) and (29) has state variables $(y_t, \pi_t, X_t, relY_t)$, where $relY_t = y_t/y^*$. In the limit $\gamma \to 0$ for (28) it can be shown that the equations for y_t and $relY_t$ are separable from other equations and take the form

$$\begin{aligned} y_t &= \frac{\beta}{\beta - 1} y_t^e \\ relY_t &= \frac{\beta}{(\beta - 1)y^*} relY_t^e \end{aligned}$$

for which the steady state is E-stable. Eliminating the first and fourth rows and columns leads to a two-dimensional system for variables (π_t, X_t), which has coefficient matrices

$$\tilde{M} = \begin{pmatrix} \frac{\pi^*}{\psi_p \beta(1-\beta)} & \frac{\pi^* \beta}{\beta - 1} \\ \frac{1}{\psi_p \beta(1-\beta)} & \frac{\beta}{\beta-1} \end{pmatrix}, \tilde{N} = \begin{pmatrix} 0 & -\pi^* \\ 0 & 0 \end{pmatrix}.$$

The coefficient matrix of (29) is 4×4 and its characteristic polynomial has one root equal to 0. The remaining eigenvalues are roots of the cubic $\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 = 0$. The Schur-Cohn conditions are

$$SC1 : \quad |a_0 + a_2| < 1 + a_1 \tag{48}$$

$$SC2 : \quad |a_1 - a_0 a_2| < 1 - a_0^2. \tag{49}$$

It can be computed that at $\omega = 0$ we have $sc1 = (1 + a_1) - |a_0 + a_2| = 0$ and $sc2 = (1 - a_0^2) - |a_1 - a_0 a_2| = 0$ and so we need to compute the derivatives

$$\begin{aligned} \partial sc1/\partial\omega &= \frac{4\pi^* - 4\beta\psi_p + 2\beta^2\psi_p}{\beta\psi_p(1-\beta)}, \\ \partial sc2/\partial\omega &= \frac{\beta - 2}{\beta - 1} \end{aligned}$$

at $\omega = 0$. Requirements $\partial sc1/\partial\omega > 0$ and $\partial sc2/\partial\omega > 0$ at $\omega = 0$ are necessarily satisfied.

In the low steady state $R = 1$ is a binding constraint and the result then follows from the corresponding result in Honkapohja and Mitra (2014). ■

Next with prove Proposition 4:

Proof. For PLT regime with transparency and forward guidance the learning system (28) and (29) has state variables $(y_t, \pi_t, X_{t,t} X_t^e, relY_t)^T$, where ${}_tX_t^e$ is defined in Section 6.2. Note that we treat equation (32) as part of state dynamics. In the limit $\gamma \to 0$ the learning system can be simplified as the equation for y_t

$$y_t = \frac{\beta}{\beta - 1} y_t^e,$$

is separable from the rest of the system and in conducive to E-stability. Eliminating the first row and column from the matrices M and N leads to the 4×4 coefficient matrices

$$\check{M} = \begin{pmatrix} 0 & \frac{\pi^*(\psi_p \pi^* - 1)}{\psi_p(\pi^* - 1)} & \frac{-\psi_y \pi^*}{\psi_p(\pi^* - 1)} & 0 \\ 0 & \frac{\psi_p \pi^* - 1}{\psi_p(\pi^* - 1)} & \frac{-\psi_y}{\psi_p(\pi^* - 1)} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \check{N} = \begin{pmatrix} 0 & \frac{\pi^*(\omega_1 + \psi_p(1 - \pi^*))}{\psi_p(\pi^* - 1)} & 0 & \frac{\pi^*(1 - \omega_1)}{\psi_p(\pi^* - 1)} \\ 0 & \frac{\omega_1}{\psi_p(\pi^* - 1)} & 0 & \frac{1 - \omega_1}{\psi_p(\pi^* - 1)} \\ 0 & 0 & 0 & 0 \\ 0 & \omega_1 & 0 & 1 - \omega_1 \end{pmatrix}.$$

The matrix (29) is now 8×8 and it has two zero eigenvalues and three eigenvalues equal to $1 - \omega$. The remaining eigenvalues are roots of a cubic $\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 = 0$ and the Schur-Cohn conditions (48)-(49) can be applied. In the limit $\omega, \omega_1 \to 0$ we get that $1 + a_1 - |a_0 + a_2| = 1 + \psi_p^2$ and $1 - a_0^2 - |a_1 - a_0 a_2| = 1 + \psi_p^2$, so that SC1 and SC2 are both satisfied for $\psi_p > 0$. ■

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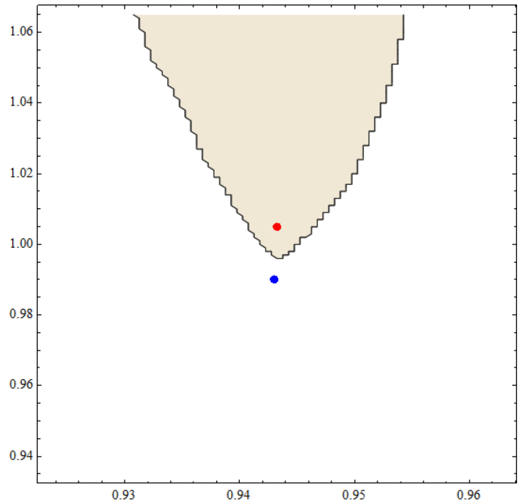

Figure 1: Domain of attraction for PLT without guidance. Horizontal axes gives y_0^e and vertical axis π_0^e. Shaded area indicates convergence. The circle in the shaded region denotes the intended steady state and the other circle is the unintended one in this and subsequent figures.

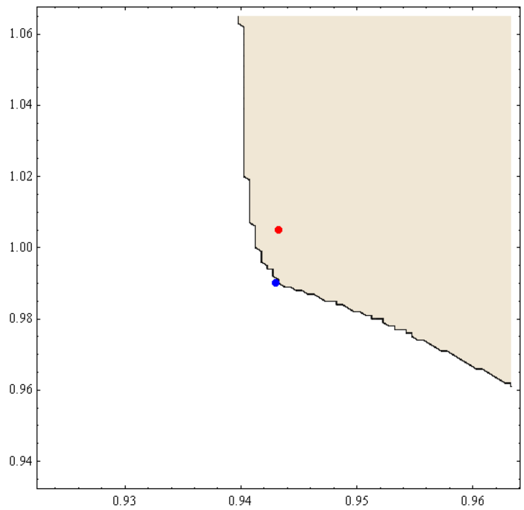

Figure 2: Domain of attraction for IT.
Horizontal axes gives y_0^e and vertical axis π_0^e. Shaded area indicates convergence.

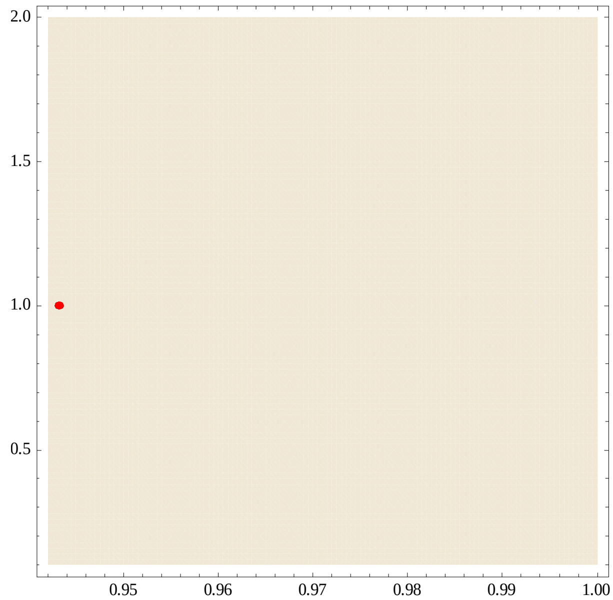

Figure 3: Domain of attraction for PLT with forecasting of gaps. Horizontal axis gives y_0^e and vertical axis X_0^e. The dot is the targeted steady state. Shaded area indicates convergence.

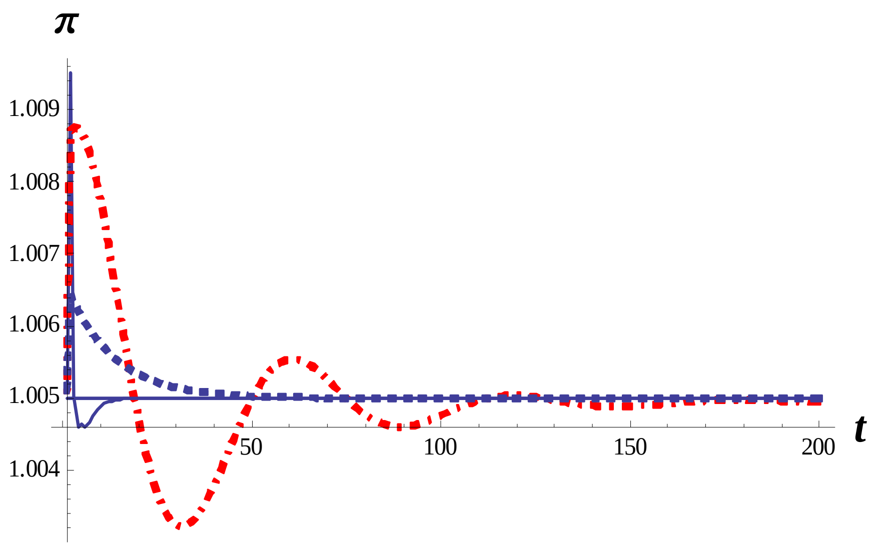

Figure 4: Inflation mean dynamics under IT (dashed line), PLT without forward guidance (mixed dashed line) and PLT with forward guidance (solid line).

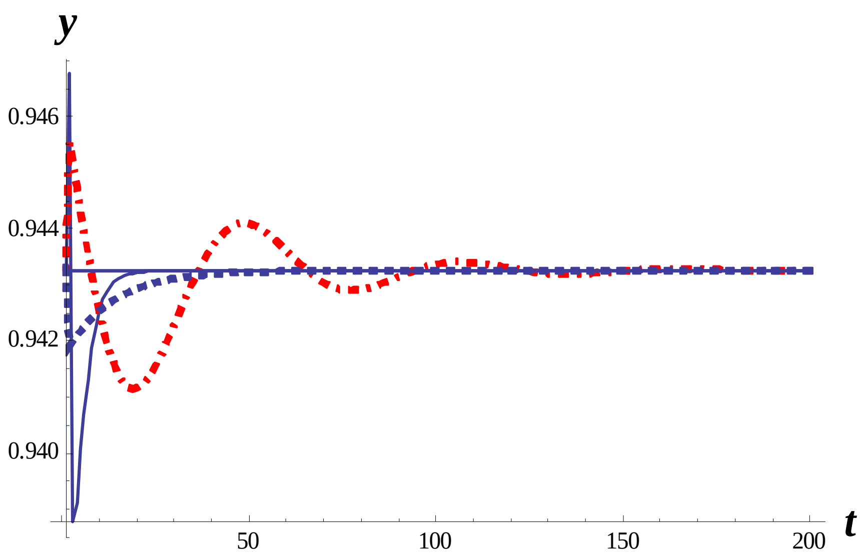

Figure 5: Output mean dynamics under IT (dashed line), PLT without forward guidance (mixed dashed line) and PLT with forward guidance (solid line).

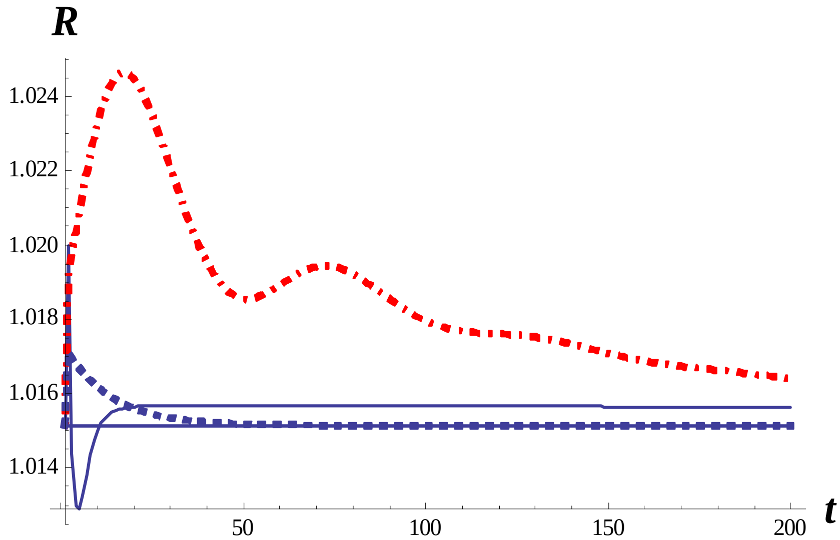

Figure 6: Interest rate mean dynamics under IT (dashed line), PLT without forward guidance (mixed dashed line) and PLT with forward guidance (solid line).

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