

Juha Kilponen – Kai Leitemo

**Robustness in monetary  
policymaking:  
a case for the Friedman rule**



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The views expressed are those of the author and do not necessarily reflect the views of the Bank of Finland.

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# Robustness in monetary policymaking: a case for the Friedman rule

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## Abstract

Inflation targeting involves using all available information in stabilizing inflation around some target rate (Svensson, 2003). Inflation is typically at the very end of the transmission mechanism and hence its determination is subject to much model uncertainty which the central bank will want to guard against using robust policies. Such robustness comes however with the cost of increased social loss under the most likely description of the economy. We show that with a sufficiently high degree of model uncertainty, adherence to the Friedman rule of increasing the money stock by  $k$  percent will be superior as the price paid for robustness is smaller.

Key words: policy robustness, money growth targeting, inflation targeting, Friedman rule

JEL classification numbers: E42, E52, E58, E61

# Friedmanin sääntö ja robusti rahapolitiikka

## Suomen Pankin tutkimus Keskustelualoitteita 4/2006

Juha Kilponen – Kai Leitemo  
Rahapolitiikka- ja tutkimusosasto

### Tiivistelmä

Inflaatiotavoitteeseen perustuva rahapolitiikka edellyttää, että keskuspankki perustaa rahapolitiikan kaikkeen käytettävissä olevaan informaatioon. Inflaatio on kuitenkin havaittavissa vasta rahapolitiikan välittymismekanismien lopussa, joten siihen liittyvä epävarmuus voi olla merkittävä. Keskuspankin rahapolitiikassa epävarmuus voidaan ottaa huomioon perustamalla rahapolitiikka robusteihin ohjaussääntöihin, jotka ovat vähemmän herkkiä rahapolitiikan välittymismekanismiin liittyvän epävarmuuden suhteen kuin perinteiset optimaaliset säännöt. Robustisuus ei kuitenkaan synny ilman hyvinvointikustannuksia. Tässä tutkimuksessa osoitetaan, että Friedmanin k-prosentin sääntö on talouden hyvinvoinnin kannalta parempi kuin robusti inflaatiotavoitesääntö, jos epävarmuus on riittävän suuri. Tämä johtuu siitä, että Friedmanin säännöstä koituvat robustisuuden kustannukset ovat pienemmät.

Avainsanat: robustit ohjaussäännöt, rahan määrän kasvu, inflaatiotavoite, Friedmanin sääntö

JEL-luokittelu: E42, E52, E58, E61

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# 1 Introduction

During the last 15 years in many countries, monetary policy has been formulated with an objective to stabilize inflation directly, a regime known as inflation targeting. Thus, many central banks, with perhaps the prominent exception of the European Central Bank, have abandoned regimes of targeting variables such as money growth or the exchange rate. Svensson (2003) argues that inflation-targeting central banks should use all relevant information and not restrict itself by looking at only a few indicator variables. Accordingly, central banks put much resources into understanding the link between the instruments of monetary policy and inflation in order to establish what information is relevant and how to use this information most efficiently. Regardless of the effort put into describing the monetary transmission mechanism, however, it is unlikely that central banks ever achieve a complete understanding of this mechanism and thereby eliminate all model uncertainty. Incomplete understanding creates a need for monetary policy design to work well under alternative descriptions of the economy and thus be robust to model uncertainty.

Robustifying monetary policy to model uncertainty has been a central topic in monetary policy for a long time, dating at least back to Friedman (1959) and his book *A Program for Monetary Stability*. In his famous book, Friedman discusses the choice of monetary policy target. He advocates money growth targeting and specifically warns against adopting price stability as an operational objective for monetary policy due to the high degree of model ('link') uncertainty between monetary policy and prices:

[...] the link between price changes and monetary changes over short periods is too loose and too imperfectly known to make price level stability an objective and reasonably unambiguous guide to policy. [...] [T]here is much evidence that monetary changes have their effect only after a considerable lag and over a long period and that the lag is rather variable. (p. 87)

Friedman doubts the usefulness of state-contingent rules in monetary policymaking due to the uncertainty related to magnitude and lags in which the economy reacts to the policy stimulus.<sup>1</sup>

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<sup>1</sup>Another insight from Friedman is that model uncertainty creates data uncertainty which is another reason to be cautious in adopting state-contingent rules. The quotation in Friedman (1959) is illustrative:

The proposal to increase the money stock at a fixed rate month-in and month-out is certainly simple. [...] Surely, it would be better to 'lean against the wind,' [...] rather than to stand straight upright whichever way the wind is blowing.[...] We seldom in fact know which way the economic wind is blowing until several months after the event, yet to be effective, we need to know which way the wind is going to be blowing when the measures we take now will be effective [...] Leaning today against next year's wind is hardly an easy task in the present state of meteorology. (p. 93)

Under these circumstances, the price level – or for that matter any other set of economic indicators – could be an effective guide only if it were possible to predict, first, the effects of non-monetary factors on the price level for a considerable period of time in the future, second, the length of time it will take in each particular instance for monetary policy actions to have their effect, and third, the amount of effect of alternative monetary actions. (p. 88)

For these reasons, Friedman suggests using policy rules that are invariant to detailed knowledge about the dynamics of the economy. Accordingly, he advocates the well-known constant  $k$ -% money growth rule:

[...] The stock of money [should be] increased at a fixed rate year-in and year-out without any variation in the rate of increase to meet cyclical needs. (p. 90)

It is widely agreed that if money demand is not hit by permanent velocity shocks, such a strategy will stabilize inflation around a constant rate and hence pin down inflation expectations. However, the rule will typically deviate from the optimal policy derived from minimizing the impact of distortions on economic welfare in a setting with complete understanding of the economy. Given that such complete understanding is absent in practical policymaking, the policymaker may engage in strategies to improve policy robustness – that is, trade off precision for greater assurance that the policy does not contribute to increasing the distortions in the true (but unknown) model of the economy. In this regard, we interpret Friedman's constant money growth rule as a strategy that contributes to improving this trade-off if model uncertainty is sufficiently high.

We compare social welfare under the Friedman rule and under the now-a-days common inflation targeting approach for different degrees of uncertainty about the monetary policy transmission mechanism. Both regimes can be interpreted as delegation of monetary policy to the central bank, who has a mandate to set its own policy instrument, the interest rate, in order to achieve a pre-specified target – either for money growth or inflation.

The results suggest that if the central bank doubts its model sufficiently and therefore has relatively strong preferences for policy robustness, the Friedman's approach of targeting the money growth could be superior in terms of economic welfare. Since inflation is at the very far end of the transmission mechanism and therefore subject to several, potentially misspecified transmission channels, there are simply more things that can go wrong under inflation targeting than under money growth targeting. If uncertainty with regards these channels is sufficiently high, the inflation-targeting policymaker has more reasons to guard against model misspecifications and therefore is ready to pay a higher price for robustness. The price is lower economic welfare in the most likely description of the economy. This price is not paid under (base) money growth targeting, since the central bank has complete control of the money stock.

## 2 Model, policy preferences and robust control

We assume that the best available description of the monetary transmission mechanism is given by a New-Keynesian model. The work-horse log-linearized New-Keynesian model is derived in Woodford (2003) and is given by

$$\pi_t = \beta\pi_{t+1|t-1} + \gamma x_{t|t-1} + \varepsilon_t \quad (2.1)$$

$$x_t = x_{t+1|t} - \sigma (i_t - \pi_{t+1|t} - r_t^n) \quad (2.2)$$

where  $\pi_t \equiv p_t - p_{t-1}$  is the inflation rate,  $x_t$  is the output gap,  $r_t^n$  is the normal real interest rate, and  $\varepsilon_t$  is a cost-push shock. Equation (2.1) is a forward-looking Phillips curve which allows for delayed effects of monetary policy on inflation using a one-period implementation lag. Equation (2.2) is an expectational IS curve. Both equations are structural and derived from microfoundations. The properties of the model are described extensively by Clarida et al. (1999) and Woodford (2003). The model is parameterized as follows:  $\beta = 0.99$ ,  $\gamma = 0.024$ ,  $\sigma = 0.16$ ,  $\kappa = 0.05$  and  $\rho_\varepsilon = \rho_\pi = \rho_v = 0.5$  and the standard error of the shocks are all set to one percent.<sup>2</sup>

In order to relate the interest rate to base money,  $m_t$ , we introduce a money market. A log-linearized money demand equation can be derived from microfoundations (see, e.g., Walsh, 2003). An inverted representation is given as

$$i_t = (x_t + v_t - m_t + p_t) / \kappa, \quad (2.3)$$

where  $v_t$  is a velocity shock.

The disturbances to the model follows AR(1) processes, i.e.,

$$\varepsilon_{t+1} = \rho_\varepsilon \varepsilon_t + \hat{\varepsilon}_{t+1}, \quad (2.4)$$

$$r_{t+1}^n = \rho_r r_t^n + \hat{r}_{t+1}^n, \quad (2.5)$$

$$v_{t+1} = \rho_v v_t + \hat{v}_{t+1}. \quad (2.6)$$

The model described in equations (2.1–2.6) is denoted the reference model (see Giordani and Söderlind, 2004), which structure and parameterizations represent the best available knowledge of the transmission of monetary policy to the monetary policymaker.

### 2.1 Robust monetary policy

The policymaker doubts the reference model but is not able to specify a probability distribution over the potential misspecification errors. Instead, the policymaker wants to guard against misspecifications that would lead to

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<sup>2</sup>The  $\beta, \gamma$  and  $\sigma$  parameter values are taken from Giannoni and Woodford (2005, Section 1). The remaining parameters are set at values that do not seem a priori unreasonable.

severe outcome.<sup>3</sup> In order to describe general uncertainty surrounding the reference model, we adopt the standard approach from robust control literature (see Hansen and Sargent, 2003a) and augment the reference model with a vector of additive misspecification terms  $\eta_{t+1}$ . The model including potential misspecifications terms can be set up in state-space form as<sup>4</sup>

$$\begin{bmatrix} x_{1,t+1} \\ E_t x_{2,t+1} \end{bmatrix} = A \begin{bmatrix} x_{1,t} \\ x_{2,t} \end{bmatrix} + B i_t + C (\eta_{t+1} + \xi_{t+1}), \quad (2.7)$$

where  $A$  and  $B$  are matrices of model parameters,  $C$  is a vector that scales the impact of the vector of error terms  $\xi_{t+1}$ ,  $x_{1,t}$  is the vector of predetermined variables with  $x_{1,0}$  given and  $x_{2,t}$  is a vector of forward-looking variables. Appendix A specifies the  $A$ ,  $B$  and  $C$  matrices in the case of the model given above.

The misspecifications are assumed to be bounded as

$$E_0 \sum_{t=0}^{\infty} \beta^t \eta'_{t+1} \eta_{t+1} \leq \eta_0, \quad (2.8)$$

where  $\eta_0$  reflects specifically how large the potential misspecifications can be.

Monetary policy is delegated to an independent central bank that minimizes under discretion a period loss function that reflects the delegated objectives in a robust manner. Under the Friedman rule, the central bank's period loss function is  $L_t = (\Delta m_t)^2$  while under inflation targeting the period loss function is  $L_t = \pi_t^2$ . The policymaker assumes that misspecifications are of the worst kind and maximizes policy loss, subject to the constraint (2.8). It is shown by Hansen and Sargent (2003a) and Giordani and Söderlind (2004) that this problem can be stated as

$$\min_{i_t} \max_{\eta_t} E_0 \sum_{t=0}^{\infty} \beta^t (L_t - \theta \eta'_{t+1} \eta_{t+1})$$

subject to (2.7).  $L_t$  denotes periodic loss function assigned to the central bank by the society, while  $\theta$  summarizes the central bank's attitude towards model misspecifications in setting its policy. In particular,  $\theta > 0$  relates to  $\eta_0$  in such way that in the case with no misspecifications allowed  $\lim_{\eta_0 \rightarrow 0} \theta = \infty$ , while a smaller  $\theta$  allows for greater misspecifications. A policymaker that is confident about the reference model will typically choose a high  $\theta$  as he does not want to pay a high price in terms of social costs by deviating too much from the optimal policy in a model he believes in. The problem solves for the

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<sup>3</sup>For similar approaches in dealing with model uncertainty, see Hansen and Sargent (2003a,b), Giordani and Söderlind (2004), Levin et al. (2003), Kilponen (2003, 2004), Levin and Williams (2003), Leitmo and Söderström (2004, 2005), Walsh (2004, 2005b,a), Onatski and Williams (2003) and Tetlow and Muehlen (2001).

<sup>4</sup>In setting up the state space form of the model we are assuming that policy is implemented using the short-term interest rate,  $i_t$ . An alternative representation of the state space is more appropriate using money growth as the instrument of monetary policy. For the results, however, either state space form can be used.

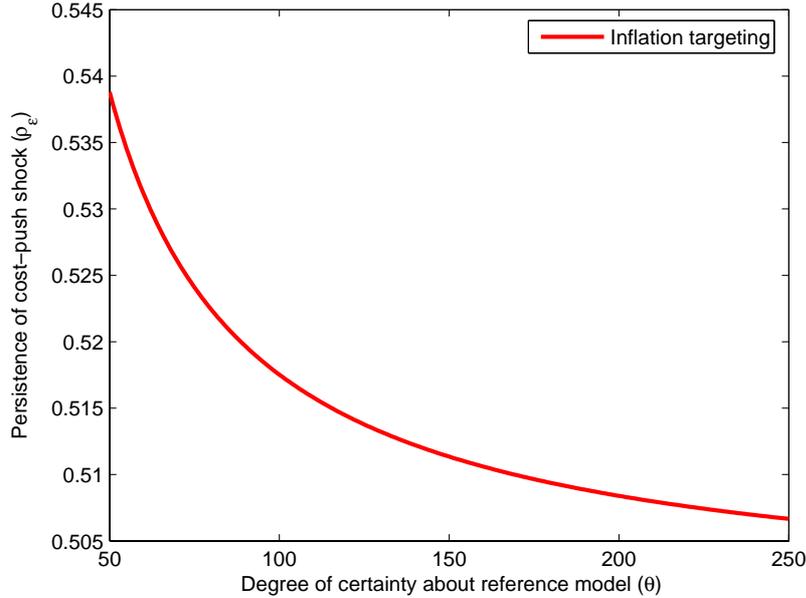


Figure 1: The figure shows the negative relationship between the persistence parameter ( $\rho_\varepsilon$ ) in the worst-case model and the confidence the inflation-targeting central bank has about the reference model ( $\theta$ ). The inflation-targeting central bank fears that cost-push shocks are more persistent than in the reference model where  $\rho_\varepsilon = 0.5$ .

optimal choice of  $i_t$  and  $\eta_{t+1}$ .<sup>5</sup> The equilibrium in the worst-case model can then be described by substituting these solutions into (2.7) and then solved in the usual manner. This system then describes the worst-case transition laws the central bank and the private sector wants to guard against.

Woodford (2003) shows that a quadratic approximation of economic welfare in the above reference model is measure by the social loss function

$$L^s = E_0 \sum_{t=0}^{\infty} \beta^t \left( \pi_t^2 + \frac{\gamma}{\psi} x_t^2 \right), \quad (2.9)$$

where  $\psi$  is the elasticity of substitution between alternative differentiated goods. Following Woodford (2003), it is parameterized as  $\psi^{-1} = 0.13$ . We use (2.9) in computing the social loss under the two strategies.

## 2.2 Misspecifications in the worst-case model

Under inflation targeting, the central bank fears that shocks to inflation is more persistent than in the reference model. Figure 1 shows how the cost-push shocks become more persistent as the confidence in the model declines. Since inflation is the only argument in the loss function and the central bank can

<sup>5</sup>The computer code is available from the authors upon request.

only influence inflation with a lag, the central bank sets policy so that the inflation forecast is equal to the inflation target in the next period (see, e.g., Svensson, 1997). Since the central bank fears that a cost-push shock that raises inflation is more persistent, the central bank depresses the output gap more under robust monetary policy. Interest rate also becomes more volatile. Under Friedman's approach of constant money growth targeting, the central bank sets its policy without any variation in its policy instrument to meet cyclical needs. Since the central bank controls the money base perfectly, its policy is completely isolated from concerns for misspecifications.

The resulting worst-case dynamic equilibrium is not the most likely outcome. The model without misspecification errors (reference model) is by construction the policymaker's best attempt at describing the monetary transmission mechanism. By assuming that there are no misspecification errors  $\eta_t = 0$  for all  $t$ , but retaining the robust policy and expectations formation under the worst-case scenario, we get the approximating model which describes equilibrium dynamics under robust decisions making by the private sector and the policymaker.

### 2.3 Comparing targeting regimes in the approximating equilibrium

In this section we compare inflation targeting with money growth targeting in the approximating equilibrium where the policymaker and the private sector has preference for robustness. In Figure 2 we plot the social loss (2.9) under different assumptions about the preference for robustness as represented by  $\theta$ .

If the policymaker has complete confidence in the model ( $\theta = \infty$ ), the rational expectations equilibrium with the non-robust optimal solutions to policy in the reference model applies. In this case, inflation targeting outperforms money growth targeting due to the usual arguments. The policymaker has preferences that are closer to those of society and given a correct description of monetary policy transmission mechanism, the policymaker will get closer to minimizing social loss. Money growth targeting is moreover subject to the inefficiency of being subjected to persistent velocity shocks.

Now consider the case of declining confidence in the reference model. The reduction in confidence induces the policymaker to pay more and more attention to model uncertainty and allows for the possibility of greater misspecifications in the model ( $\eta_0$  increases  $\rightarrow \theta$  decreases). Under inflation targeting, the policymaker worries that all the channels of monetary policy transmission from instrument to inflation are potentially misspecified. Since the policymaker, in principle, uses all channels leading up to inflation in order to stabilize inflation efficiently, the policymaker adopts a policy strategy and the private sector forms expectations that are designed to work well when these misspecifications are of the worst kind. The cost of this, however, is that the outcome in the case where the reference model is the true representation of the economy is worsened. This is the price paid for being robust under inflation

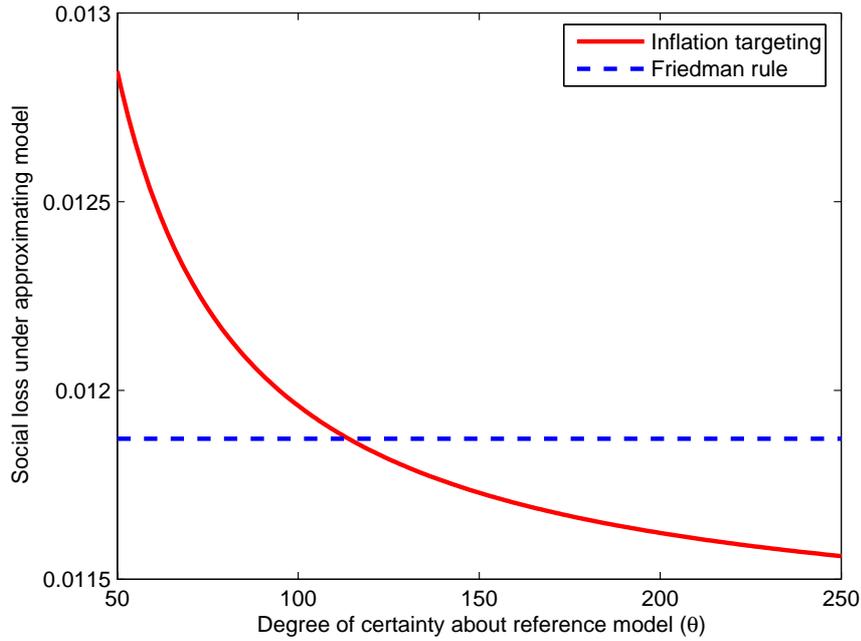


Figure 2: The figure shows the social loss (2.9) under different assumptions about preference for robustness ( $\theta$ ) under inflation targeting and money growth targeting (Friedman rule).

targeting.

For the money growth targeting, the policymaker does not worry about misspecifications in the transmission mechanism since he controls money growth perfectly. Hence, there is no need to worry about misspecifications in the transmission mechanism and neither to pay the price of protecting against such misspecifications.

Increased worries about misspecifications eventually lead the inflation targeting regime to produce an outcome that is worse from a social point of view than under money growth targeting. The conclusion is that if a policymaker have small confidence in his description of the transmission mechanism, it would do better by adopting the Friedman rule.

### 3 Conclusions

Inflation is at the very end of the transmission mechanism of monetary policy. Between inflation and the monetary instrument there are time lags and several transmission channels of monetary policy that the central bank might fear be misspecified. By targeting inflation, the central bank is exposed to misspecifications in all transmission channels as policy is tweaked so as to using these channels in an efficient way. Robustness in monetary policymaking implies that the policymaker wants to design policy so as to allow possible misspecifications to have the least impact on the objectives of monetary

policy. Since the inflation targeting policymaker uses all, possibly misspecified, transmission channels, his need to guard against misspecification is higher than a policymaker that only has a target for his instrument (money growth). Consequently, the inflation-targeting policymaker will be willing to pay a higher price for being robust against misspecification. This price is paid in the form of increased social loss. With sufficient model uncertainty, this leads to an inferior outcome than under money growth targeting in the most likely description of the economy.

In more general, our exercise suggests that the presence of model uncertainty should be taken into account in delegation of monetary policy to an independent central bank. Setting advanced objectives for an independent monetary authority can be welfare reducing if the central bank does not have sufficient confidence to its knowledge about the monetary policy transmission.

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## A The model in state-space form

In order to cast the model in its state-space form, lead equation (2.1) by one period, i.e.,

$$\pi_{t+1} = \beta\pi_{t+2|t} + \gamma x_{t+1|t} + \varepsilon_{t+1}, \quad (\text{A.1})$$

and take expectations at time  $t$ ,

$$\pi_{t+1|t} = \beta\pi_{t+2|t} + \gamma x_{t+1|t} + \varepsilon_{t+1|t}. \quad (\text{A.2})$$

By isolating  $\pi_{t+2|t}$  on the left side, we get the representation of the forward-looking equation treating  $\pi_{t+1|t}$  as a forward-looking variable,

$$\begin{aligned} \pi_{t+2|t} &= \frac{1}{\beta}\pi_{t+1|t} - \frac{1}{\beta}\gamma x_{t+1|t} - \frac{1}{\beta}\varepsilon_{t+1|t}, \\ &= \frac{1}{\beta}\pi_{t+1|t} - \frac{1}{\beta}\gamma x_{t+1|t} - \frac{1}{\beta}\rho_\varepsilon\varepsilon_t \end{aligned}$$

We need, however, also a state equation for  $\pi_t$ . By subtracting equation (A.2) from equation (A.1), and rearranging, we get

$$\begin{aligned} \pi_{t+1} &= \pi_{t+1|t} + \varepsilon_{t+1} - \varepsilon_{t+1|t}, \\ &= \pi_{t+1|t} + \hat{\varepsilon}_{t+1}, \end{aligned}$$

where in the last step we used equation (2.4).

The expectational IS-curve can be stated directly in state space by isolating  $x_{t+1|t}$  on the left hand side, treating  $x_t$  as a forward-looking variable, i.e.,

$$x_{t+1|t} = x_t + \sigma (i_t - \pi_{t+1|t} - r_t^n).$$

Expected inflation may then be expressed as

$$\begin{aligned} \pi_{t+2|t} &= \frac{1}{\beta}\pi_{t+1|t} - \frac{1}{\beta}\gamma (x_t + \sigma (i_t - \pi_{t+1|t} - r_t^n)) - \frac{1}{\beta}\rho_\varepsilon\varepsilon_t \\ &= \frac{1}{\beta}(1 + \gamma\sigma)\pi_{t+1|t} - \frac{1}{\beta}\gamma x_t - \frac{\sigma}{\beta}\gamma i_t + \frac{1}{\beta}\gamma\sigma r_t^n - \frac{1}{\beta}\rho_\varepsilon\varepsilon_t. \end{aligned}$$

It will be useful for the analysis to define real money balances as  $\hat{m}_t \equiv m_t - p_t$  and then use the money demand equation in (2.3) as a state equation, so that

$$\hat{m}_t = x_t - \kappa i_t + v_t.$$

Since we use money growth as a target variable, we need a measurement equation that relates money growth to the state variables. By subtracting with lagged real money balances on both sides in equation (2.3) and rearranging, we get the measurement equation

$$\Delta m_t = \pi_t + x_t - \kappa i_t - \hat{m}_{t-1} + v_t.$$

We may then set the model up in state space form using matrix notation as

$$\begin{bmatrix} z_{1,t+1} \\ z_{2,t+1|t} \end{bmatrix} = A \begin{bmatrix} z_{1,t} \\ z_{2,t} \end{bmatrix} + B i_t + C \begin{bmatrix} V_{t+1} \\ 0 \end{bmatrix},$$

where  $z_{1,t} \equiv [v_t \ r_t^n \ \varepsilon_t \ \hat{m}_{t-1} \ \pi_t]'$ ,  $z_{2,t} \equiv [\pi_{t+1|t} \ x_t]'$ ,  $V_{t+1} \equiv [\hat{v}_{t+1} \ \hat{r}_{t+1}^n \ \hat{\varepsilon}_{t+1} \ 0 \ \hat{\varepsilon}_{t+1}]'$ ,

$$A = \begin{bmatrix} \rho_v & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_r & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_\varepsilon & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & \frac{1}{\beta}\gamma\sigma r_t^n & -\frac{\rho_\varepsilon}{\beta} & 0 & 0 & \frac{1}{\beta}(1+\gamma\sigma) & -\frac{1}{\beta}\gamma \\ 0 & -\sigma & 0 & 0 & 0 & -\sigma & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\kappa \\ 0 \\ -\frac{\sigma}{\beta}\gamma \\ \sigma \end{bmatrix}.$$

The measurement equations are given by

$$\begin{bmatrix} \Delta m_t \\ \pi_t \\ x_t \end{bmatrix} = M_0 \begin{bmatrix} z_{1,t} \\ z_{2,t} \end{bmatrix} + M_1 i_t$$

where

$$M_0 = \begin{bmatrix} 1 & 0 & 0 & -1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

and

$$M_1 = \begin{bmatrix} -\kappa \\ 0 \\ 0 \end{bmatrix}.$$

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