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Consumption and Uncertain Access to the Asset Market

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Abstract

This paper presents a new approach to modelling credit restrictions by considering uncertain access to the asset market. The asset market and the stochastic process governing access are considered fully exogenous and independent of income. The model generates stable debt trajectories for a broader array of interest rate levels than the one corresponding to the agent's rate of time preference. The agent exhibits excess sensitivity of consumption to current period income, even for low probabilities of constraints. Because this sensitivity is inversely related to the maturity of debt contracts, the availability of long-term debt contracts reduces the income-sensitivity of consumption. A very tractable approximative Euler equation for the model is presented.

Key words: consumption, liquidity constraints, permanent income hypothesis

Kulutus ja epävarma pääsy rahoitusmarkkinoille

Suomen Pankin keskustelualoitteita 23/2000

Leena Rudanko
Tutkimusosasto

Tiivistelmä

Tutkimuksessa esitetään uusi, epävarmaan rahoitusmarkkinoille pääsyyn perustuva tapa mallintaa luottorajoitteiden vaikutusta kulutuskysyntään. Rahoitusmarkkinoita ja niille pääsyä määrittelevää stokastista prosessia käsitellään täysin eksogeenisina ja tuloista riippumattomina. Mallin mukaan velkaantumisurat ovat stabiileja silloinkin, kun korkotaso ei ole täsmälleen sama kuin kuluttajan aikapreferenssi. Jos on pienikin epävarmuus rahoitusmarkkinoille pääsystä, kuluttaja ylireagoi tulojen muutoksiin. Tämä ylireagointi riippuu negatiivisesti velkasopimusten maturiteetista: mitä pitemmät velkasopimukset, sitä vähemmän kulutuksella ylireagoidaan tulojen muutoksiin. Tutkimuksessa johdetulle mallille esitetään myös hyvin käyttökelpoinen likimääräinen Eulerin yhtälö.

Asiasanat: kulutus, likviditeettirajoitteet, pysyväistulohypoteesi

Contents

1 Introduction

The permanent income hypothesis of consumption, developed by Friedman (Friedman 1957) is based on the assumption that individuals can optimize with respect to their consumption expenditure subject only to the constraint that the present value of their expenditure cannot exceed their total wealth, defined as the present value of their expected future resources. Hall (Hall 1978) formalized the permanent income hypothesis in the rational expectations context and showed that, under certain assumptions,[1] this implies that unexpected changes in consumption must result from revisions of expectations of the present value of future income. Consumption expenditure thus follows a martingale. Furthermore, transitory changes in current income should have only a very small impact on current consumption.

The failure of this implication of the permanent income theory to hold in empirical data is referred to as the excess sensitivity of consumption. Changes in consumption seem to depend on current and previous income (see eg (Flavin 1981); (Campbell and Mankiw 1991)). As the assumptions in Hall's version of the permanent income model are quite restrictive, there has been room to generalize the model in several directions in order to reconcile the theory with the excess sensitivity and other anomalies. In this paper, we will consider one such generalization, namely credit restrictions which may hinder the individuals' ability to move their lifetime resources over time.

Credit restrictions have been considered before (see eg. (Deaton 1991)).The usual way to incorporate credit restrictions to the theory of consumption is to assume that lenders impose some given maximum on the amount of debt the individual may incur. Deaton considers a model where the debt maximum is set to zero. He finds that in such a models, the reaction of consumption to income "news" depends on the relative impatience of consumers and the time series properties of income in a complicated way. Saving may behave countercyclically in some cases, or procyclically in others, or there might even be no saving at all (the last case is obtained when income follows a random walk).

In empirical work attempting to take credit restrictions into account, the most common approach has been to assume that households are heterogeneous in such a way that a fraction of the population is permanently excluded form the credit markets, and the rest are free to operate as assumed in permanent income theory (see (Campbell and Mankiw 1989), (Campbell and Mankiw 1991); (Hall and Mishkin 1982)). The consumption of the credit constrained households is assumed to equal their income. If the fraction of the credit constrained households is π, aggregate consumption is given by $c = (1 - \pi)c_u + \pi c_c$. Consumption of the unconstrained households c_u would follow the permanent income hypothesis, and the consumption of the credit constrained

[1] The assumptions are (Muellbauer 1994): 1)no credit restrictions or other nonlinearities in the budget constraint, 2) separable lifetime utility with quadratic periodic utility, 3)no habits or adjustment costs, 4) non-durable goods, 5)the subjective discount rate is constant across consumers, 6)no measurement errors or transitory shocks to consumption, 7)coincidence of the frequency of consumers' decision making with the the frequency of the data, 8)constant real interest rate, 9)rational expectations.

would be simply equal to their income $c_c = y_c$. While this kind of an auxiliary hypothesis may serve to reconcile the permanent income theory to aggregate data, it is highly arbitrary and lacking in theoretical justification.

Our paper explores another way of including credit constraints to the intertemporal theory of consumption. We do not divide the households into permanently different categories regarding their access to the credit market (that would be theoretically unwarranted in any case, if the households are equal in other respects). Instead, we study the possibility that credit constraints are stochastic: In each period any individual has a fixed probability of being unable to borrow. For simplicity, we consider the symmetric case that the same stochastic restriction applies also to saving. Therefore, our model actually describes a situation in which there is exogenous uncertainty concerning access to the asset markets. The probability of restrictions is constant and independent over time. It is also independent of income changes.

The assumption of stochastic access to the assets market might be justified by arguing on the basis of some exogenous noise in the operation of the markets. Unless there is symmetric information on the creditworthiness of individuals and on the quality of assets, there is always a possibility that some individuals may be classified as not creditworthy even though they themselves would see scope for further borrowing. Similarly, some assets may be occasionally difficult to sell. Our model, then, looks at the saving and borrowing behavior of individuals who take these risks prudently into account.

It turns out that our approach yields simple and tractable first order conditions, at least as approximations (exact first order conditions are also simple for the basic case when the period length of the model is the same as the maturity of the debts and investments considered). Therefore, due to tractability, the model may be a promising starting point for theoretical macroeconomic analyses.

We find that for even small probabilities of constraints occurring, the model produces strong excess sensitivity of consumption to transitory income shocks. Another finding is that the stochastic credit constraints approach yields stable debt trajectories even when the interest rate and the rate of time preference of the consumers are constant but different. This is a property which is lacking from the basic permanent income model of Hall (Hall 1978). Finally, our model has the interesting property that the maturity of available debt matters for saving and the demand for debt. The longer is the maturity (in the sense that the ratio of amortizations to debt is low), the larger amounts of debt (or assets) the individuals are willing to have in equilibrium.

In what follows we begin by introducing the model and discussing the analytical properties of the resulting asset positions. This is mainly done by considering the respective Euler equation or an approximation of it. For the general case a simple exact Euler equation cannot be found but a tractable approximation is readily available. We then explore the implications of the model in the case of logarithmic utility with the help of numerically solved value functions for the (exact) corresponding Bellman equation. In the case of stochastic income we compute resulting correlations of current period consumption and income for the representative agent, obtaining values in the range of 0.17-0.76 for the parameter values considered. Finally we go on to inspect the goodness of the approximate Euler equation compared to the numerically obtained solution of the exact Bellman equation. We find that the goodness of the approximation depends on parameter values so that a small probability of restricted periods occurring appears to make the approximation better.

2 The life cycle model: an extension

In the basic infinite horizon life cycle model, the consumer chooses his consumption c_t each period based on the available knowledge about his present and future income y_t, $t = t_0, t_1, \dots$, ($y_t > 0$), interest rates R_t and his own preferences. Instead of having to spend what he earns each period, the consumer can use the asset market to redistribute his income between time periods. We denote by a_t the asset position (i.e. net wealth) of the consumer at the beginning of period t. If $a_t > 0$ the consumer has savings and if $a_t < 0$ the consumer is in debt.

Preferences over consumption streams are modeled with an intertemporal utility function $U : \{c_t\}_{t=t_0}^{\infty} \to \mathbb{R}$, where $c_t \in \mathbb{R}$, $c_t > 0 \ \forall t$. The consumption decisions are made to maximize the utility function $U(c)$, where $c = \{c_{t_0}, c_{t_1}, \dots\}$ and in the presence of uncertainty, to maximize expected utility $E_{t_0}U(c)$. By E_{t_0} we refer to the expected value conditional on the information available in the present period $t = t_0$. The consumption decision is restricted by the requirement that the discounted value of future consumption must equal the expected discounted wealth of the consumer. In the absence of borrowing restrictions the consumer is free to distribute his income over time as long as consumption remains positive for each period.

To this basic model we add the feature, that in each future period, there is a possibility of the consumer not having access to the asset market. This means that the amount consumed in those periods will be predetermined and not subject to optimization. We denote by s_t the stochastic variable governing market access. Let $s_t = 0$ if access is free, and $s_t = 1$ if access is restricted. Furthermore assume s_t to be independently and identically distributed, with a constant probability $p(> 0)$ of restrictions occurring each period.

Since there are periods when the consumer cannot enter into new contracts on the asset market, it becomes relevant how the previous contracts are implemented. We will assume that the implementation is such that a constant percentage $d \cdot 100\%$ of debts or savings is paid back each period ($0 < d \leq 1$). Whether or not the consumer has the possibility to take part in the asset market this period, he will either receive or have to pay $\frac{r_t+d}{R_t}a_t$, where $R_t = 1 + r_t$, depending on whether a_t is positive or negative. This means that for a restricted period t we have $a_{t+1} = (1-d)a_t$, but for unrestricted periods, as new asset contracts are available, a_{t+1} can be more freely chosen.

We will consider a time separable utility function with a constant rate of time preference $\rho \Rightarrow \beta_t = \frac{1}{1+\rho} = \beta$ and a constant real gross interest rate $R = 1 + r$. The consumer's optimization problem can then be written as follows:

$$
\begin{aligned}
&\sup_c E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} u(c_t), \\
&\text{where } a_{t+1} = R(y_t + a_t - c_t), \\
&a_{t+1} = (1-d)a_t, \text{ if } s_t = 1, \\
&c_t > 0, \\
&\text{and } \lim_{t->\infty} \frac{a_t}{R^t} = 0. \qquad (1)
\end{aligned}
$$

The function u is the utility function for one period, $u : c_t \to \mathbb{R}$, where $c_t \in \mathbb{R}$, $c_t > 0$. We assume that $u \in C^2(0, \infty)$, $u' > 0$ and $u'' < 0$. Typically the utility function is formulated so that $\lim_{c_t \to 0+} u'(c_t) = \infty$, implying that the positivity constraint $c_t > 0$ is not binding. We will assume this to hold. For the parameters β and R it is assumed that $R \geq 1$ and $0 < \beta < 1$.

To write this in the form of an optimal control problem we introduce as the control variable v, the asset position at the end of a period, after consumption has taken place. Between periods there holds the relationship $a_{t+1} = Rv_t$. The choice of v each period depends on the realization of s. If the asset market is not accessible, then v is predetermined. Otherwise, v can be chosen to maximize utility.

Let income follow a given process $y_{t+1} = F(y_t, \epsilon_{t+1})$, where ϵ_t is a stochastic disturbance, independently and identically distributed for each period. We particularly assume y and s to be independent of each other. We then have for the state variables a_t , y_t and s_t and the control variable v_t:

$$
\begin{aligned}
&\sup_{v_t} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} u(y_t + a_t - v_t), \\
&\quad \text{where } a_{t+1} = Rv_t, \\
&\quad\quad y_{t+1} = F(y_t, \epsilon_{t+1}), \\
&\quad\quad v_t \in \Gamma(a_t, y_t, s_t) \\
&\quad \text{and } \lim_{t->\infty} R^{-t} a_t = 0, \qquad (2)
\end{aligned}
$$

where $\Gamma(a, y, 0) = \{v \in \mathbb{R}\}$ and $\Gamma(a, y, 1) = \{v \in \mathbb{R} | v = \frac{1-d}{R} a\}$. The positivity of consumption also restricts v as negative consumption can never be optimal for our utility function.

The Bellman equation for the problem is:

$$
\hat{V}(a, y, s) = \sup_{v \in \Gamma(a,y,s)} \{u(y + a - v) + \beta E_\epsilon E_s \hat{V}(Rv, F(y, \epsilon), s)\}. \qquad (3)
$$

As market restrictions and income are assumed independent, we write the equation for the expected value of the value function with respect to the market restrictions $V(a, y) = E_s \hat{V}(a, y, s)$, in the form:

$$
\begin{aligned}
V(a, y) = &p[u(y + \frac{r + d}{R} a) + \beta E_\epsilon V((1 - d)a, F(y, \epsilon))] \\
&+ (1 - p) \sup_v \{u(y + a - v) + \beta E_\epsilon V(Rv, F(y, \epsilon))\}. \qquad (4)
\end{aligned}
$$

This gives the value function V for time periods $t > t_0$. The corresponding value function for the beginning of the planning period is then given by:

$$V_0(a, y) = \sup_v \{u(y + a - v) + \beta E_\epsilon V(Rv, F(y, \epsilon))\}, \tag{5}$$

since we can assume that for the planning period the asset market is open. Still it is clear from (4) and (5) that the maximization problems in both cases are equivalent. From this point on all references to the value function will refer to the solution of equation (4), assuming that it exists and is unique.

Restricting consumption, $c_t = y_t + a_t - v_t$, to be positive, the model implies a lower bound for possible asset positions a_t and therefore an upper bound for possible v_t. Since $y_t + \frac{r+d}{R} a_t$ must always be positive, we must have $a_t > -\frac{R}{r+d} y_t$. Considering that the consumer may not have access to the asset market in the following periods either, if $d < 1$ we must also have $a_t > -\frac{R}{r+d} \frac{y_{t+n}}{(1-d)^n}$ for $n = 1, 2, \ldots$. In the deterministic case the process $y_{t+1} = F(y_t)$ thus determines, along with parameter values, the lower bound on a_t.

In the case of stochastic income, the consumer cannot allow for any such a_t, where there is a possibility that the income level is so low that $y_t + \frac{r+d}{R} a_t \leq 0$. Assuming that every period there exists a lowest possible value of income $\hat{y}_t$, which may depend on the income attained the previous period, he must always have: $a_t > -\frac{R}{r+d} \hat{y}_t$. In addition, if $d < 1$ we must have: $a_t > -\frac{R}{r+d} \frac{\hat{y}_{t+n}}{(1-d)^n}$ for $n = 1, 2, \ldots$, for positive consumption in the future.

Thus the decision of v_{t-1} made in the previous period is restricted from below since $a_t = Rv_{t-1}$. It also follows that v_{t-1} is restricted from above, because we must have $v_{t-1} < y_{t-1} + a_{t-1}$. Therefore the optimization problem of the consumer on each period is such that there is a restricted feasible set, depending partly on the current state. If the utility function is such that zero consumption implies an infinite marginal utility of consumption, the optimum will be an interior point.

We consider the case where income is constant or has a constant lowest possible value $\hat{y}$, which may be reached from any previous state of income from one period to the next. If $d < 1$, for positive consumption in the future we must have: $a_t > -\frac{R}{r+d} \frac{\hat{y}}{(1-d)^n}$ for $n = 0, 1, 2, \ldots$. This lower bound decreases as n grows so that the $n = 0$ condition is sufficient. If $d = 1$ we obtain the same condition. This implies that the value function is in this case defined for $a > -\frac{R}{r+d} \hat{y}$.

It follows that for the control variable v_t we have the bounds: $-\frac{\hat{y}}{r+d} < v_t < y_t + a_t$, where a_t determined the previous period may be anything above $-\frac{R}{r+d} \hat{y}$. The value of y_t is either the constant level of income or given by the stochastic process and for both cases there holds: $y_t \geq \hat{y}$. Examining the bounds on v_t closer, we see that the set of possible values of v_t will not be empty for any allowed a_t as long as $d > 0$. In the case of the market being closed v_t is fixed at: $v_t = \frac{1-d}{R} a_t$, which can easily be seen to lie within the bounds.

Considering the previous discussion, to formulate the feasible set for optimization including the constraint $c_t > 0 \ \forall t$, we have

$$\Gamma(a, y, 0) = \{v \in \mathbb{R} | -\frac{1}{r+d} \min\{F(y, \epsilon)\} < v < y + a\}, \tag{6}$$

$$\Gamma(a, y, 1) = \{v \in \mathbb{R} | v = \frac{1-d}{R} a\}, \tag{7}$$

where we assume that $F(y, \epsilon)$ has a fixed lower bound and that each outcome may be attained from any previous state of income from one period to the next.

In this model the asset position cannot become one of ever increasing indebtedness, because the consumer must always make sure that he will survive through "bad" periods. Because of this we know that the value function that solves the Bellman equation is defined only for values of the state variable a larger than a lower bound that depends on the values of R, d and the process $\{y_t\}$.

In characterizing the solutions of the model we will consider both the Euler equation and the Bellman equation approaches because in general we only have an approximative Euler equation in use. It turns out that we are not able to derive a neat and exact Euler equation except for the case when $d = 1$. This is because each choice of v will have a positive probability of having direct effects on consumption indefinitely far in the future and these possible effects will have to be considered in the decision making. If optimization were free in every period this direct effect would not be present and as we will see in the following section, all relevant information about the future could be expressed in next period's optimal control value. In the case of $d = 1$, this period's choice will only directly affect next period's consumption and the problem is avoided. For the general case however, an approximate Euler equation can be derived. In the Bellman equation approach we do not have similar problems, but the nonlinear functional equation can only be solved numerically.

2.1 Deriving an approximate euler equation

From the Bellman equation we can for the $d = 1$ case derive a simple Euler equation. We will begin with a general d so that the analytical problems resulting from $d < 1$ become evident. We will assume in the following that the value function solving the Bellman equation is smooth enough to be at least twice continuously differentiable. This may restrict, for example, possible income processes. The value function is also assumed to be increasing and concave in a so that $V_a > 0$, $V_{aa} < 0$.

We first differentiate the right hand side of the Bellman equation (4) to find the maximizing v:

$$-u'(y + a - v) + \beta R E V_a(Rv, F(y, \epsilon)) = 0. \tag{8}$$

Since both the utility and value functions are twice continuously differentiable and strictly concave, (8) determines v as a differentiable function of (a, y), $v = h(a, y)$.

Differentiating $V(a, y)$ in the Bellman equation (4) with respect to a along the maximizing curve $v = h(a, y)$ and using (8) we have:

$$\begin{aligned} V_a(a, y) = p[\frac{r+d}{R}u'(y + \frac{r+d}{R}a) + (1-d)\beta E_\epsilon V_a((1-d)a, F(y, \epsilon))] \\ + (1-p)u'(y + a - v). \end{aligned} \tag{9}$$

Combining (8) and (9) we obtain:

$$\begin{aligned} u'(y + a - v) = \beta E_\epsilon[p[(r+d)u'(F(y, \epsilon) + (r+d)v) \\ + (1-d)\beta R E_{\hat{\epsilon}} V_a((1-d)Rv, F(F(y, \epsilon), \hat{\epsilon}))] \\ + (1-p)Ru'(F(y, \epsilon) + Rv - \tilde{v})]. \end{aligned} \tag{10}$$

If we let $d = 1$, thus assuming that debts taken in each period must be paid back in full the following period (and for savings respectively), we have the following Euler equation on the optimal path:

$$\begin{aligned} u'(y + a - v) = \beta R E_\epsilon[pu'(F(y, \epsilon) + Rv) \\ + (1-p)u'(F(y, \epsilon) + Rv - \tilde{v})], \end{aligned} \tag{11}$$

where $\tilde{v}$ is the optimal control value for next period.

When $d \neq 1$ (and $p \neq 0$) we are in (10) left with the term containing the value of the function $V_a(a, y)$ at a point $(1-d)Rv$ which is generally not on the optimal path $R\tilde{v}$ as specified by the first order condition (8). We don't know the value of $V_a((1-d)Rv)$ and can't use equation (8) to substitute for it in equation (10). (We could use equation (9) repeatedly, but this would not simplify matters.)

To find an approximate form of the Euler equation for the more general case we use a linear approximation of the value function in a around the optimal path:

$$V_a((1-d)a, y) \approx V_a(Rv, y),$$

where $(1-d)a$ is the state the consumer is in after a market restriction and Rv is the state the consumer is in after optimizing v.

Using this and the first order condition (8) we can write (10) in the form:

$$\begin{aligned} u'(y + a - v) = \beta E_\epsilon[p(r+d)u'(F(y, \epsilon) + (r+d)v) \\ + (R - p(r+d))u'(F(y, \epsilon) + Rv - \tilde{v})], \end{aligned} \tag{12}$$

obtaining an approximate Euler equation.

Writing this with time indices we have:

$$\begin{aligned} u'(y_t + a_t - v_t) = \beta E_{y_{t+1}}[p(r+d)u'(y_{t+1} + (r+d)v_t) \\ + (R - p(r+d))u'(y_{t+1} + Rv_t - v_{t+1})]. \end{aligned} \tag{13}$$

The qualitative effects of the linear approximation on the chosen control values for the constant income case can be deduced since we assumed for the value function: $V_a > 0$ and $V_{aa} < 0$. Since income is held constant we denote in the following $V(a, y) = V(a)$.

Given an initial asset position a, assume that v is the optimal control value this period, defined by the equation (10). Consider the effects of using the linear approximation of the value function, assuming the resulting value function is has a third derivative with respect to a in the relevant neighborhood. We may write:

$$V'((1-d)a) = V'(Rv) + V''(Rv)((1-d)a - Rv) + \mathrm{Err}((1-d)a - Rv),$$

where it is known that the error term Err is smaller in absolute value than the second term on the right hand side if $|(1-d)a - Rv|$ is small enough. Since $V'' < 0$, the sign of the term $((1-d)a - Rv)$ determines whether we overestimate or underestimate $V'((1-d)a)$.

If for the optimal control v there holds $Rv < (1-d)a$, we approximate $V'((1-d)a)$ to be larger (or equal) than it is for each a. From equation (10) we see that we then overestimate the size of the right hand side and since $u'(y+a-v)$ is increasing in v, we therefore choose the control parameter v_{approx} too large (v_{approx} is the solution of the approximate Euler equation whereas v is the true solution). It follows that from the approximate Euler equation we obtain a control value v_{approx} which is larger then the true v. Respectively, if $Rv > (1-d)a$, we obtain a value v_{approx} smaller than v. This means that if the optimal next period's state is further from zero than $(1-d)a$ we approximate it closer to zero than it is and if the optimal state is closer to zero than $(1-d)a$, we approximate it to be further from zero than it is. The approximation thus brings the asset position a little closer to $(1-d)a$, the asset position after a liquidity constraint.

In the constant income case it may be expected that there exists a fixed optimal asset level, towards which the consumer moves whenever he is free to optimize. The optimal level may be very large, but not arbitrarily small, because of the possibility of market restrictions. Since in some periods the consumer's asset position is forced towards zero, it can be assumed that in the long run the consumers asset position is typically increasing in absolute value whenever the consumer may optimize. If we then use the approximate Euler equation to estimate chosen asset position, we will estimate is to be closer to zero than it really is.

2.2 Undisturbed steady-state solutions

In this subsection we consider a steady-state solution to the consumer's maximization problem. By the undisturbed steady-state we mean a constant asset position solution of the approximate Euler equation resulting in the constant income case when restricted periods do not realize. We thus now set both stochastic variables, income and access to the asset market, to fixed values.

Considering the case of fixed income, the development of the asset position (in time) in the model is such that during some periods the consumer optimizes his asset position and in some periods he is forced to take a position which is a fixed proportion closer to zero than the previous position was. If the market is closed to the consumer for an infinite sequence of periods the asset position will converge to zero. We can therefore expect that whatever the consumer initially had, at some point he will have reached a situation where his asset position fluctuates between some ideal value and zero. The position may be generally increasing or decreasing, but depending on the market conditions, from time to time it will be reduced toward zero by a proportion. Since we know that the forced limit is zero, we would also want to find the upper/lower limit, if one exists, where the consumer would be happiest at being, if he avoided restricted periods. We therefore consider the case where the ever present possibility of not having access to the asset market never realizes, so that the Euler equation holds in all periods. Ideally we would then expect the asset position to converge to this steady-state, when restrictions are avoided.

Solving the approximate Euler equation for a constant v we have:

$$u'(y+rv) = \beta[p(r+d)u'(y+(r+d)v) + (R-p(r+d))u'(y+rv)], \tag{14}$$

$$\Leftrightarrow \frac{u'(y+rv)}{u'(y+(r+d)v)} = \frac{p\beta(r+d)}{1-\beta R+p\beta(r+d)}, \tag{15}$$

assuming $1-\beta R+p\beta(r+d) \neq 0$.

From the shape of the utility function ($u' > 0$, $u'' < 0$) and (15) it follows that for the existence of a steady-state solution we must have: $1-\beta R + p\beta(r+d) > 0$. It also follows that for a steady-state solution v there holds: $\beta R < 1 \Leftrightarrow v < 0$, $\beta R > 1 \Leftrightarrow v > 0$ and $\beta R = 1 \Leftrightarrow v = 0$.

Considering the logarithmic utility function $u(c) = \ln c$ we obtain from (15):

$$v = -[\frac{1-\beta R}{1-\beta R+pd\beta}]\frac{y}{(r+d)}. \tag{16}$$

We therefore find that a steady-state solution exists iff $R < \frac{1}{\beta}+pd$. If this holds the solution is of the form (16), which is for $\beta R < 1$ negative, for $\beta R = 1$ zero and for $1 < \beta R < 1+pd\beta$ positive, approaching infinity as R approaches the upper limit $\frac{1}{\beta}+pd$.

Note that we are assuming p to be positive. Considering the standard $p = 0$ case we can directly see from (14) that a steady-state solution exists only if $\beta R = 1$.

To consider the effects of income y, interest rate R, probability of not having access to the asset market p and the length of asset contracts d, on the resulting constant asset position v, we compute partial derivatives of v. For all CRRA utility functions we find all the derivatives have definitive signs.[2]

For the effects of the interest rate we have: $\frac{\partial v}{\partial r} > 0$, which implies that as we would expect a higher interest rate always makes the optimal asset position higher: debts become smaller and savings larger.

For income we have: $\frac{\partial v}{\partial y} = \frac{v}{y}$. Increasing income therefore increases both savings and debts moving the asset position further away from zero. Particularly we see that unit elasticity holds for v with respect to income. If $\beta R = 1$, the asset position of zero is not affected by changes in income.

For $x = p, d$ we have the result that if $\beta R < 1$, then $\frac{\partial v}{\partial x} > 0$, if $\beta R > 1$, then $\frac{\partial v}{\partial x} < 0$ and if $\beta R = 1$, then $\frac{\partial v}{\partial x} = 0$. This means that an increased risk of not having access to the asset market brings the undisturbed steady-state asset position closer to zero. Similarly an increase in the amount of debt and savings to be paid back each period makes debts and savings smaller.[3]

The effects of changes in parameters y, p and d do not include changing the asset position from debt to savings or from savings to debt, only the level adjusts. Changes in the interest rate however always cause a nonzero effect on the asset position, thus being able to change the quality of the position from savings to debt or debt to savings.

[2] For other utility functions of the CRRA type $u(c) = \frac{c^\gamma - 1}{\gamma}$, where $\gamma < 1$, $\gamma \neq 0$, the steady-state is:

$$v = -\frac{1 - B^{\frac{1}{1-\gamma}}}{d + r(1 - B^{\frac{1}{1-\gamma}})} y,$$

where $B = \frac{p\beta(r+d)}{1-\beta R + p\beta(r+d)} = 1 + \frac{\beta R - 1}{1 - \beta R + p\beta(r+d)}$.

When $\beta R \leq 1$ this gives a non-positive steady-state, but the case where $\beta R > 1$ we must set a stronger constraint than $1 - \beta R + p\beta(r + d) > 0$ to have $v > 0$. The requirements for the existence of the steady-state in this case can, after simple manipulation, be seen to be:

$$1 - \beta R + p\beta(r + d) > p\beta R(\frac{r}{r + d})^{1-\gamma}.$$

[3] For the CRRA functions the previous also holds for $x = \gamma$.

3 Numerical solution for the logarithmic utility function

3.1 The value function iteration

To explore the implications of our model on the consumer's behavior, we use the value function iteration (18) to find a numerical approximation of the value function solving the Bellman equation (4).

$$\begin{aligned} V(a,y,t) =& p[u(y+\frac{r+d}{R}a)+\beta E_\epsilon V((1-d)a,F(y,\epsilon),t+1)] \\ &+(1-p)\max_v\{u(y+a-v)+\beta E_\epsilon V(Rv,F(y,\epsilon),t+1)\}, \\ &\text{for } t<T, \\ V(a,y,T) =& V_T(a,y). \end{aligned} \tag{18}$$

We present no analytical convergence results of the algorithm for this stochastic setting, but find that in practice the iteration does typically converge in the parameter range described in the following, particularly if the interest rate is not too high. The implementation of the algorithm is described in Rudanko (Rudanko 2000).[4]

In the following we will always consider $p>0$, which gives the problem the specific structure that has been discussed previously. The essential difference to the $p=0$ case is the boundedness of the asset position from below, which makes it possible to have optimal control sequences that stay inside a bounded region. In the $p=0$ case it appears to be the case that the optimal paths tend to grow out of any region (except for $\beta R=1$), which means that we would need an infinite grid for the value function iteration.

Of the CRRA utility functions we choose to use the logarithmic one in order to be able to compare results more easily with the ones presented for the Euler equation. The basic case of parameter values which we will consider in the following is such that: $\beta=0.95$, $d=0.1$, $p=0.05$ and $R=1.05$. The consumer has a rate of time preference of about 5.26% and faces a slightly lower market interest rate. The probability of access to the asset market being denied is 5% per period and 10% percent of asset positions will resolve by contract in all periods. We will also consider ceteris paribus changes in the three latter parameters one by one.

In the calculations we consider the cases of constant income and stochastic income which has two possible values. In the following sections we show the resulting behavior of the optimal control. We will see that for some parameter values consumption will tend to converge to a constant level if market restrictions are avoided.

[4] The computations were done with Matlab. The level of fineness of the used state grid (distance between grid points) in assets was from 0.05 to 0.125. As initial value function $V_T(a,y)=\ln(a-\frac{R}{r+d}\min\{y\})$ was used and for each maximization problem a linear interpolant of V_t was used. In particular the algorithm would dismiss state values (a) which would lead to following period state values outside the state grid. As references for writing the algorithm Kirk (Kirk 1970), Judd (Judd 1998) and Bazaraa (Bazaraa et al. 1993) were used.

3.2 Constant income

3.2.1 Value functions

We use the income level $y = 20$ for the constant income computations. In running the iteration we see that the interest rate has a clear effect on the convergence: for high interest rates we do not obtain a value function estimate. Specific values of R are considered in section 3.2.3.

The value function iteration typically converges at a speed of q^k, where $q \approx 0.95$ and is run until $\frac{\|V_k - V_{k-1}\|_2}{\sqrt{\dim(V_k)}} < 10^{-6}$.[5] As they should, the resulting functions appear continuous, monotonically increasing and concave.

Considering the effects of the parameters p, d and R on the shape of the value function in the basic case $p = 0.05$, $d = 0.1$ and $R = 1.05$, it seems that the effects of p and d are similar. A decrease in these parameters makes the function less curved, raising the value of extreme asset positions compared to mid value positions. The effects of R seem to be that an increase in the interest rate will make the whole value function more steep. The value of debt becomes smaller as it becomes more costly and the value of savings increases. This implies that the optimal control values determining asset positions become larger.

In addition to these changes in shape, the parameters R and d also effect the asset range over which the function is defined. For large R or d the lowest allowable asset position is higher than for smaller values, as the cost of restricted periods will be higher.

3.2.2 Effects of the probability of credit restrictions

We now consider the effects of varying the probability of market restrictions occurring when the other parameters are held fixed. We let $R = 1.05$ and $d = 0.1$, which imply a negative net asset position. In figure 1 we see the effect of p on the development of the optimal asset position given zero initial assets.

In the figure the paths have been plotted for values of p ranging from 1% to 30% with even intervals. The upper limit has been set at 30% since it would not be expected that the probability of disturbances taking place be any larger, even 30% may be quite extreme. The lowest probability of restrictions occurring corresponds to the largest debt position and the larger the probability of not obtaining credit, the smaller the debt position on which the curve settles. The corresponding path for $p = 1$ would be the path where the asset position always remains at zero.

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[5] $\frac{\|V_k - V_{k+1}\|_2}{\|V_{k-1} - V_k\|_2} \approx q$.

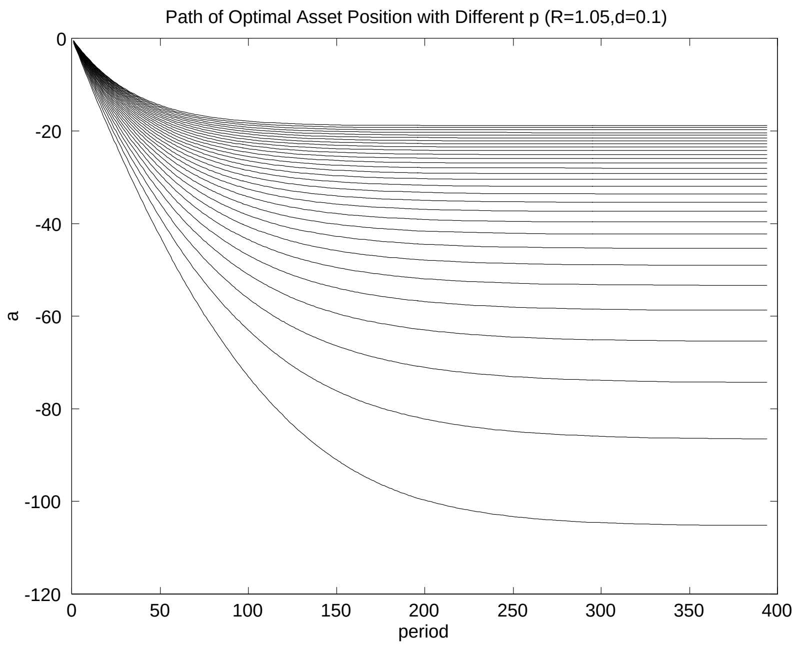

Figure 1. *Path of asset position in time for p=[0.01 : 0.01 :0.3] from zero initial value, when the consumer is not subject to borrowing or saving restrictions. The lowest path corresponds to p=0.01 and as p grows the paths settle higher. Parameter values are: d=0.1, R=1.05.*

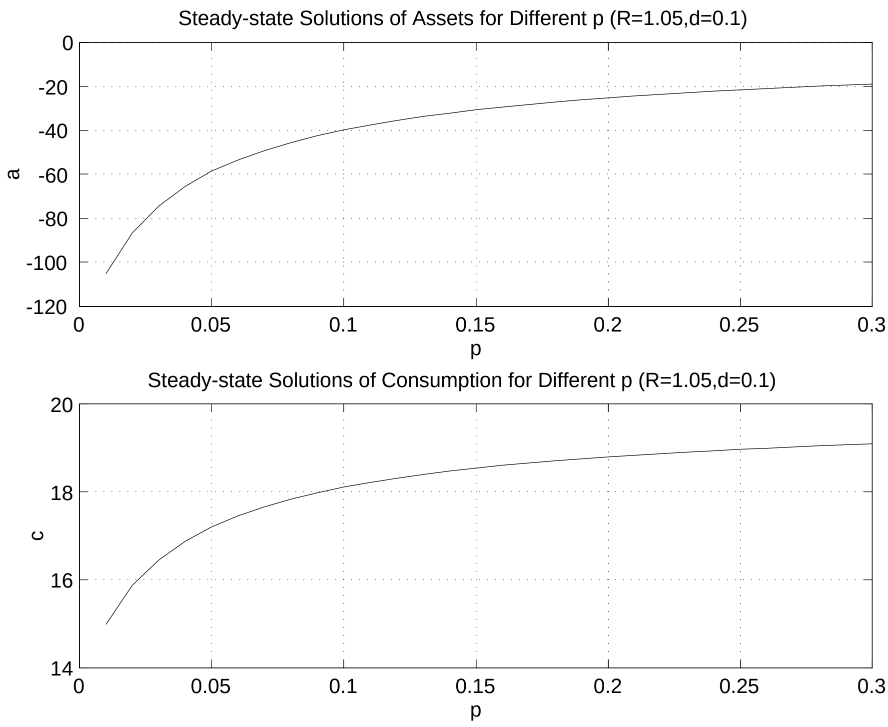

Figure 2: *Steady-states of asset position and consumption for different p, when the consumer is not subject to borrowing or saving restrictions. Parameter values are: d=0.1, R=1.05.*

Note in the figure the difference in the speed of adjustment of the asset position for different levels of debt. Clearly the speed of adjustment toward the steady-state is much faster when the debt position is small.

Plotting the reached steady-states from figure 1 we have figure 2. The optimal level of asset in p appears increasing and concave. Below the steady-state for assets in the figure we have plotted the corresponding steady state for consumption, which is a linear function of assets a. Consumption is only affected by p through the asset position, so the shape of the two curves in the figure is the same. Consumption is reduced from the income level ($y = 20$) since debt is held and interest must be paid.

3.2.3 Effects of interest rate

We now consider the effects of varying the interest rate when the other parameters are held fixed. We let $p = 0.05$ and $d = 0.1$. When R is given values $R = \{1, 1.0025, 1.0050, \ldots, 1.05, 1.0505, 1.051, \ldots, 1.06\}$, in the value function iteration, we find that solutions are obtained only for $R \leq 1.054$. For interest rates higher than this the optimal control values for high initial assets would exist above the upper limit of the grid. Since we must fix the upper limit on some level, here set at $a = 300$ units, it must be that the undisturbed optimal level of assets for larger interest rates, if one exists, lies above the upper limit. As a result of the optimal control leading outside of the state grid, we are not able to determine the value function value for the asset position in question.

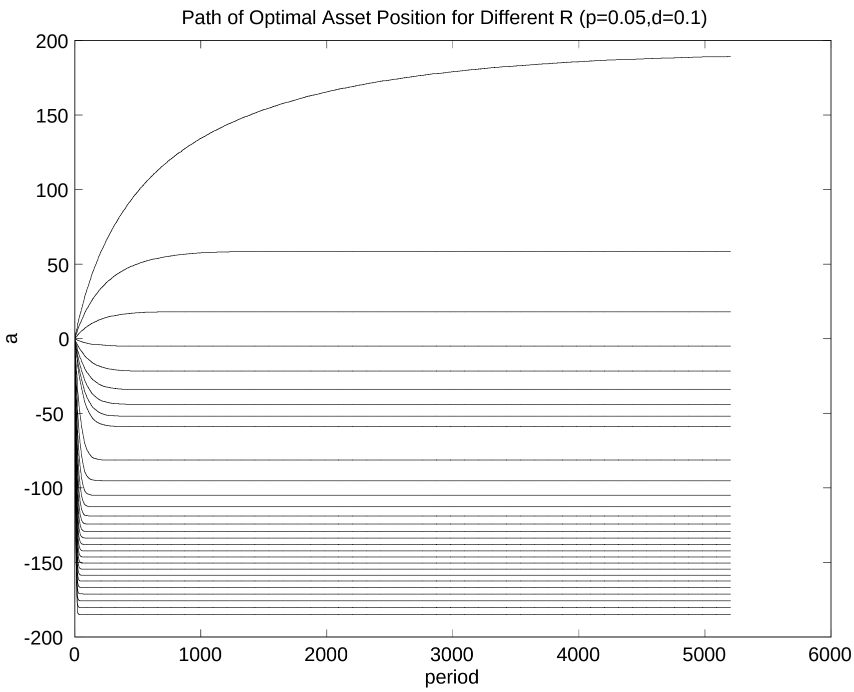

Figure 3: *Path of asset position in time for interest rates* $R = [1 : 0.0025 : 1.05]$ *and* $[1.05 : 0.0005 : 1.054]$ *from zero initial value, when the consumer is not subject to borrowing or saving restrictions. The lowest path corresponds to* $R = 1$ *and as* R *grows the paths settle higher. Parameter values are:* $d = 0.1$, $p = 0.05$.

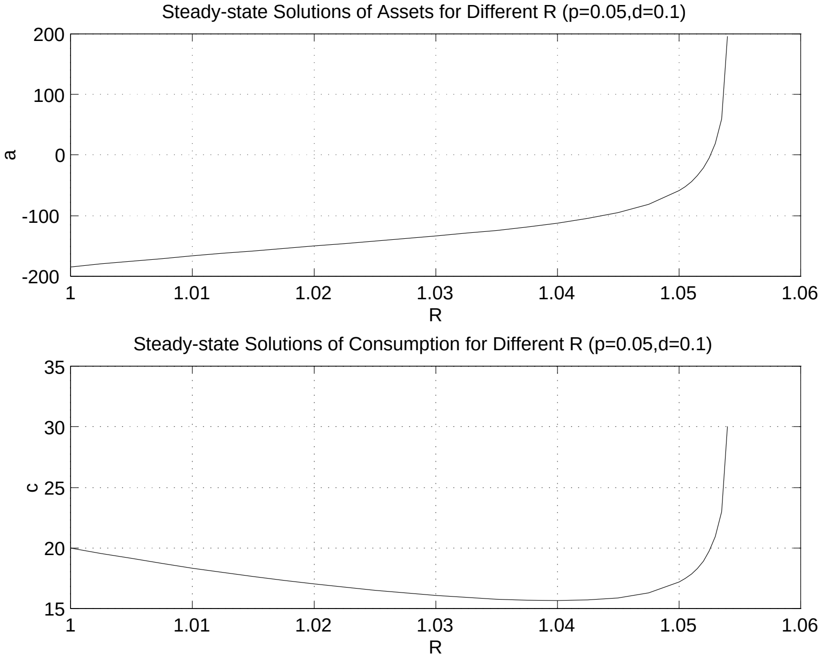

Figure 4: *Steady-states of asset position and consumption for different interest rates R, when the consumer is not subject to borrowing or saving restrictions. Parameter values are: $d = 0.1$, $p = 0.05$.*

In figure 3 we see the effect of the interest rate on the optimal asset path given zero initial assets. In the figure, the zero net interest rate $R = 1$ corresponds to the asset path that settles lowest, with the greatest speed of convergence. As the interest rate rises the debt levels that are optimal when market restrictions are avoided are smaller. Debt becomes more expensive. In choosing the interest rates plotted in the figure, the larger values were chosen closer together in order to obtain positive steady-states in the figure. This change in frequency is apparent in the figure. Considering that the upper limit of the grid was set at $a = 300$ it is acceptable in light of the figure that the next level of interest rates was not solvable.

Note again in the figure the difference in the speed of adjustment of the asset position for different levels of interest rate. The speed of adjustment toward the steady-state appears to be much faster when the interest rate is very low.

The steady-states that are reached in figure 3 are plotted in figure 4 as a function of the interest rate. These are the asset positions that would be optimal for the consumer and ones he will aim for whenever he is not restricted. We see that as we would expect, the level of asset position increases along with the interest rate. Note that the asset position a plotted is the position after interest has been added: $a = Rv$, where v is the amount borrowed or saved at the end of the previous period. Multiplication with R somewhat increases the curvedness of the relationship of the steady-state with R.

Below the steady-state for assets in the figure, is presented the corresponding steady-state of consumption: $c = y + (1 - \frac{1}{R})a$. Obviously debt leads to consumption below the income level as interest must be paid. In the figure we have the case where, for low interest rates, even though debt decreases as interest rate increases, the amount of interest paid still increases. This means that an increasing interest rate decreases consumption. As higher levels of R (> 1.04) are reached, the effect is in the opposite direction. An increase in the interest rate increases the steady-state level of consumption.

3.2.4 Effects of the length of debt contracts

Since the restriction on R from above generally implies that we obtain solutions mainly for negative asset positions and since the focus in the model is on them, we may call the parameter d as describing the length of debt contracts available in the model. Recall that d comes into effect only with the possibility of not having access to new debt contracts during a period in the future and it determines the relative size of the restriction. On those periods the consumer is forced to choose $v = \frac{(1-d)}{R} a_0$, which means paying back $\frac{d}{R} a_0$. This means that consumption for the period can not be affected during the period and the consumer has to have previously made sure that consumption will be adequate in this case as well. We will see that the effects of varying d are similar to those of varying p.

Figure 5 shows the time paths of asset positions from an initial position of zero with different debt parameter values. Here the curve that corresponds to ten percent of debts being paid back each period is the one that settles lowest and the curves above is correspond to evenly increasing pay back parameters $15\%, 20\%, \ldots, 100\%$. The resulting steady-states show the effect of debt length on the optimal long term asset position and corresponding consumption in figure 6. As debt decreases with an increasing d in the figure, consumption increases with the reduction in interest payments. Here consumption is only affected by d through the asset position, so the shape of the two curves in the figure is the same.

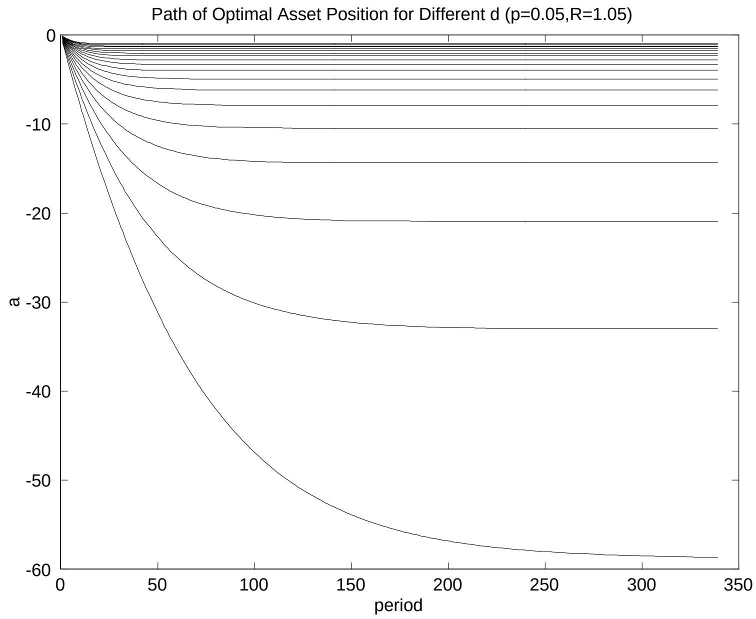

Figure 5: *Path of asset position in time for* $d = [0.1 : 0.05 : 1]$ *from zero initial value, when the consumer is not subject to borrowing or saving restrictions. The lowest path corresponds to* $d = 0.1$ *and as* d *grows the paths settle higher. Parameter values are:* $p = 0.05$, $R = 1.05$.

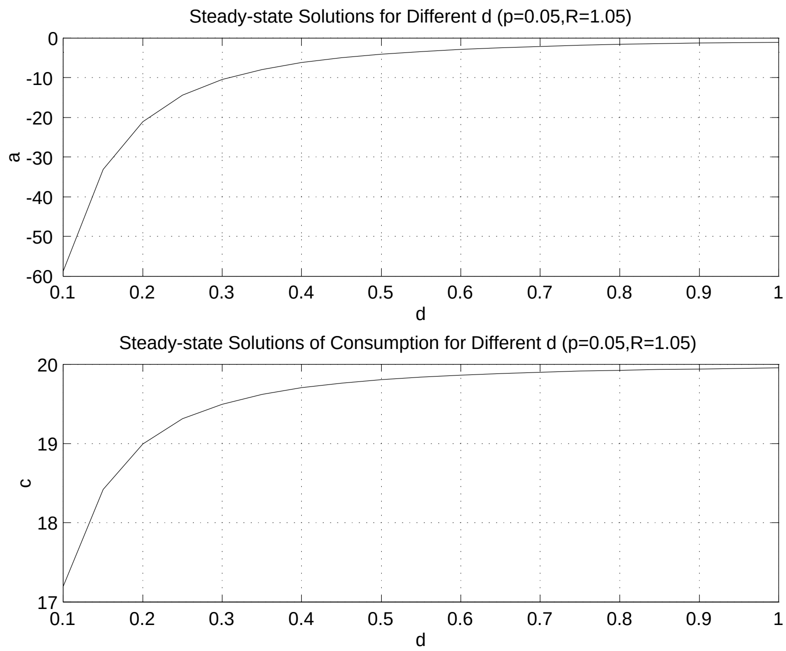

Figure 6: *Steady-states of asset position and consumption for different* d*, when the consumer is not subject to borrowing or saving restrictions. Parameter values are:* $p = 0.05$, $R = 1.05$.

This concludes the investigation of the effects of the parameter values on the constant income case. We have seen that in general the impatient consumer who has more freedom in his consumption choices (d and p are small), will have more debt for the pair of interest rate and rate of time preference considered. As a result of this he will also in the long run pay more interest leading to lower levels of consumption and suffer larger reductions in his consumption as result of occurring market restrictions.

3.3 Stochastic income

We now add a second source of stochastic shocks for the consumer to come to terms with, namely an uncertain income level. We consider the simplest possible case of two possible income levels, y_0 and y_1 and a constant probability of attaining either one in all periods, independent of other variables. The probability of enjoying either the higher income $y_1 = 22$ or the lower income $y_0 = 18$ during a period is set to 50%, so that the mean value of income is $y = 20$, as considered in the previous subsections, and the variance in income is relatively large.

The effects of stochastic income on consumption have been considered in eg. Caballero (Caballero 1991), Skinner (Skinner 1988), Zeldes (Zeldes 1989), where the implications entail precautionary savings. Here asset positions considered are negative, but similar buffering of consumption from income changes by changes in the asset position can be seen.

In figure 7 we plot the development of both assets and consumption over a longer time interval when restricted periods do not occur (but the consumer expects them with probability p). Clearly consumption fluctuates from period to period, but less than income. The asset position is used to smooth consumption over time, but not not fully.

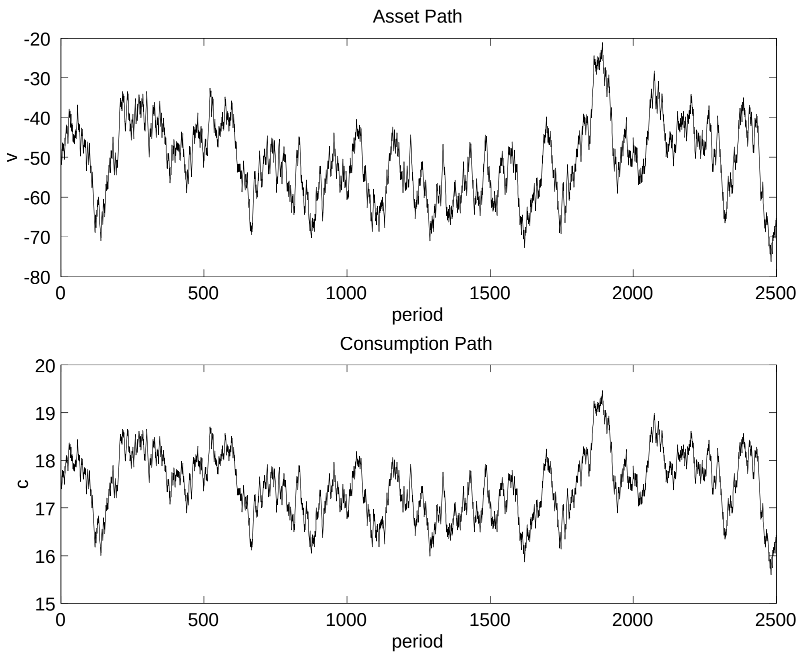

Figure 7: *Paths of asset position and consumption over* 2500 *periods after* 500 *iterations from an initial asset position of zero. Parameter values are:* $d = 0.1$, $p = 0.05$, $R = 1.05$.

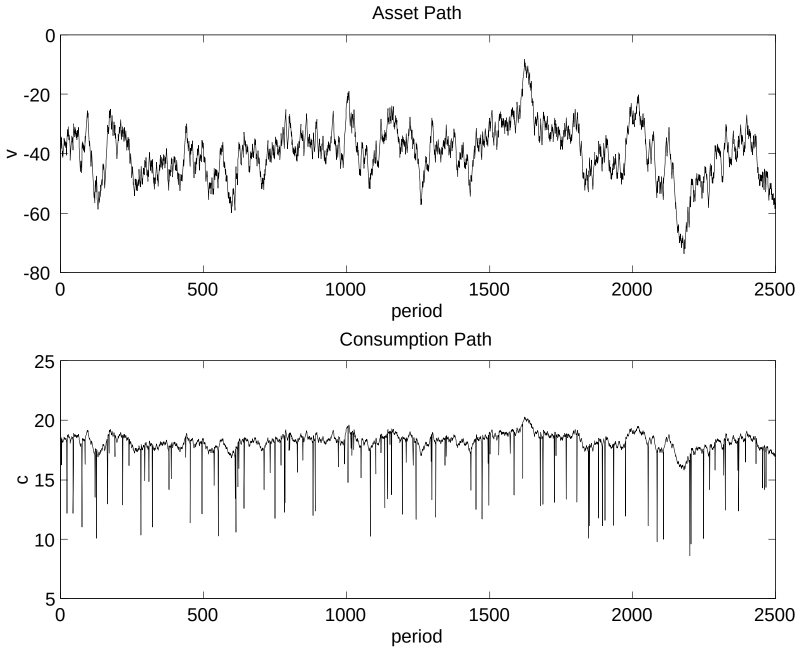

Figure 8: *Paths of asset position and consumption over* 2500 *periods after* 500 *iterations from an initial asset position of zero. Asset market access denied with probability* p *each period. Parameter values are:* $d = 0.1$, $p = 0.05$, $R = 1.05$.

To see the effects of the occurrence of restricted periods in figure 8 we have added stochastic market restrictions corresponding to $p = 0.05$. The downward spikes in the figure correspond to the restricted periods. The size of the spikes changes since it depends on the asset position at the beginning of the period and the current income. Even though the general level of consumption in unrestricted periods is not seemingly much affected by the restrictions, we will find that in some periods consumption will be much lower than if they didn't occur.

We then go on to explore the relationship between income and consumption further by computing correlations between current period income and resulting current period consumption. In table 1 we have a matrix of correlation coefficients between current period income and consumption resulting in the case where restricted periods never occur. The correlations have been computed over 2500 periods with the initial asset position first being allowed to adjust for 500 periods from zero. All correlation coefficients are positive, so that a good outcome of the income process generally implies increased consumption for the current period and a bad outcome decreased consumption.

We see that a larger debt parameter d implies a larger correlation regardless of the value of p. The shorter the available debt contracts are, the more closely consumptions follows income. Here market restrictions never realize, so that the effect of d is completely implicit in the consumer's optimization decisions. In the constant income case we had a larger d imply smaller debts. Here the correlation can be explained by the fact that a large d implies that when good income periods occur, increases in the asset position are relatively smaller, so that consumption suffers less from the change in asset position. A good income period implies a higher consumption. When debts are available for longer time periods, a good income period causes a larger change in assets, so that consumption increases less. For a bad income period respectively, if debt contracts are short the increase in debt is smaller than if longer debts were available so that consumption decreases more.

Correlation of income and consumption				
	d=0.1	d=0.2	d=0.5	d=1.0
p=0.3	0.2410	0.3458	0.5531	0.7529
p=0.2	0.2249	0.3252	0.5171	0.7115
p=0.1	0.2044	0.2905	0.4574	0.6344
p=0.05	0.1954	0.2568	0.4026	0.5622

Table 1: *Correlation between current income and consumption calculated over* 2500 *periods, after allowing asset position to develop for* 500 *periods from an initial position of zero.* $R = 1.05$.

Correlation of income and consumption				
	d=0.1	d=0.2	d=0.5	d=1.0
p=0.3	0.5416	0.5879	0.6533	0.7426
p=0.2	0.4252	0.4854	0.5905	0.6672
p=0.1	0.2586	0.3552	0.4771	0.5767
p=0.05	0.1676	0.2520	0.3572	0.4786

Table 2: *Correlation between current income and consumption calculated over* 2500 *periods, after allowing asset position to develop for* 500 *periods from an initial position of zero. Access to market is stochastic with corresponding probability* p *of access being restricted during a period.* $R = 1.05$.

A large probability of market restrictions occurring will also increase the correlation between current income and consumption regardless of the value of d. This may be understood by similar reasoning since a large p will make both large debts and saving less attractive for any $d(> 0)$. A large p makes changes in the asset position corresponding to the current income outcome smaller thus allowing consumption to fluctuate more.

Considering similar correlations when restricted periods occur with frequency corresponding to p, we obtain table 2. We would expect the difference to show mostly for large values of p since there will be more market restrictions taking place. Also a small d may mean that the restrictions would be more severely felt since levels of debt may be expected to be higher. What we see in the table is that correlation is clearly increased for small d and a large p and decreased for large d combined with a small p. The effects are clearly the strongest for large p and small d in the upper left corner.

The effect of a restriction taking place whatever the income outcome may be can be expected to be to reduce consumption considering the debt levels presented in the previous figures. The size of the reduction depends on the size of d and amount of assets, which in turn is affected by both p and d. The asset position is also directly affected by income levels of previous periods.

For a fixed small level of p it may be expected that longer debts (small d) imply consumption which is less dependent on income. The consumer is relatively free to borrow to satisfy his consumption needs whatever income outcomes realize since debts are only paid back in small portions and the risk of not being able to borrow during the next period is small. Here the effect of the market restrictions realizing is then to increase the dependence of consumption on income, as the asset position is not freely adjustable. The consumer is therefore made more dependent on the income process.

If however debts are short and the probability of not being able to borrow is relatively large, we would expect income to influence consumption quite much. Then the realization of disturbances is likely to disturb the positive relationship between income and consumption, since restricted periods may be expected to typically cause a withdrawal in consumption whatever income level realizes.

It is table 2 that presents the true implications of the consumer behavior resulting from the model discussed in this paper. In these parameter ranges and with $R = 1.05$, $\beta = 0.95$, $u(c) = \ln(c)$, correlation coefficients between current period income and consumption vary between 0.17 and 0.74. The prediction for correlation between current income and consumption based on our model and the parameter values of $d = 0.1$ and $p = 0.05$ thought realistic, is only 0.17.

As these results have been obtained for a single consumer they could therefore be considered incomparable with macro level consumption data. However, it happens in this case that as income is independently distributed for each period, if we consider a group of similar consumers (with all the considered parameters taking on equal values) the resulting correlations for aggregated consumption and income levels for the group will be the same as for one consumer. This can easily be verified analytically (and seen in numerical results). For groups of dissimilar consumers or ones with more general income processes such a result cannot be expected. Interesting further experiments would be to consider several groups of consumers with dissimilar properties and the resulting correlations of income and consumption on aggregate level or the case where income processes of individuals were correlated with each other.

4 Bellman and Euler equation solutions compared

4.1 Comparison of steady-states

In this final section we inspect the difference between asset paths resulting from the Bellman equation approach and the approximative Euler equation approach. We will consider only the constant income case. As previously mentioned, for $d = 1$ we have an exact Euler equation. The asset paths solving it should therefore completely correspond to ones obtained through the exact value function solving the Bellman equation. This turns out to be true for our computations on the scale of discretization. We will therefore consider the numerical solutions of the Bellman equation as representing the true solution to the consumer's optimization problem in order to use them to assess the goodness of the approximate Euler equation derived in section 2.1.

We begin by comparing undisturbed steady-state solutions and in the next subsection examine the adjustment of the asset paths resulting from both approaches.

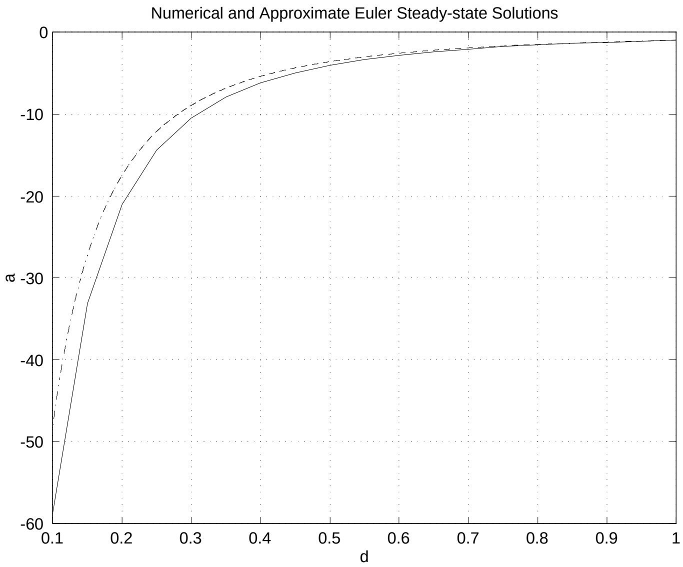

Figure 9: *Steady-states of asset position for different d. Values obtained from the Bellman equation are plotted with '-' and those from the approximate Euler equation with '-.'. Parameter values are: p = 0.05, R = 1.05.*

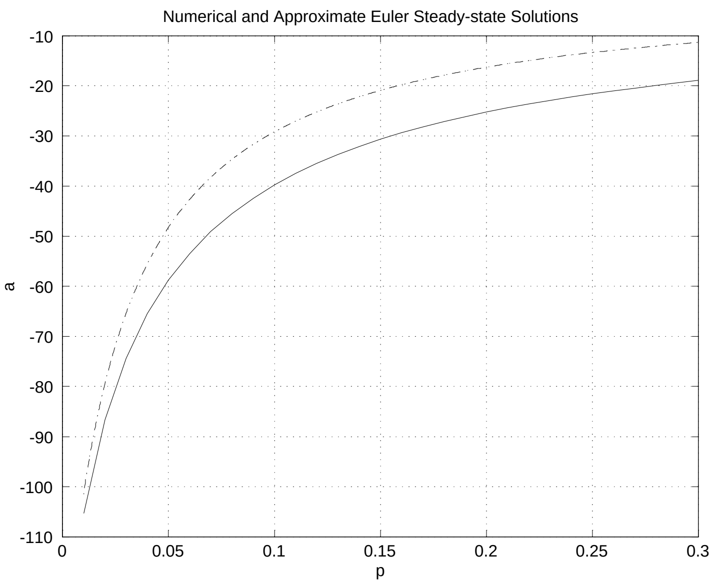

Figure 10: *Steady-states of asset position for different P. Values obtained from the Bellman equation are plotted with '-' and those from the approximate Euler equation with '-.'. Parameter values are: d = 0.1, R = 1.05.*

Letting $p = 0.05$ and $R = 1.05$, figure 9 shows the growing difference in the steady-states as d is reduced from one, where the curves coincide. On the level of accuracy of the figure, the difference is caused by the local linearization used in deriving the approximate Euler equation. The difference grows as d is reduced, which is acceptable in view of the linearization made. When d reaches 10%, the optimal level of debt is about 20% larger than what the approximate Euler equation forecasts for these parameter values.

Considering the effects of changes in p on the steady-states we have figure 10. As $d = 0.1$ and $R = 1.05$, the figure implies that the difference in steady-states would decline as p decreases. This is in line with the fact that the approximation made to obtain the used Euler equation concerns only the outcomes from market restrictions. If the probability of disturbances occurring is small, it is understandable that the error caused by the approximation becomes smaller. As mentioned previously, a small p has the effect of making the value function less curved and therefore closer to linearity, which also supports the finding. This decreasing of the error as p decreases is quite fortunate if the probability of market restrictions occurring is not expected to be large. The figure would imply an error of only about 5% for p in the range of $1-2\%$, when d is as small as 10%.

Concluding the comparison of steady-states we note that, in both figures presented comparing the results of approximate Euler equation to those of the numerical value function, the Euler equation levels have been closer to zero. This corresponds with our expectations based on the analytic properties of the approximation discussed in subsection 2.1.

In considering the steady-states we have obtained some information on the asset paths, but as the problem is of a stochastic nature, the adjustment of the asset position after market restrictions has a central role. The larger p is the more often restriction will be expected and the asset position will be shifted away from the steady-state. On the other hand for smaller p the level of debt tends to be larger so that the effects of the restriction will be larger and may take longer to even out. We shall next turn to viewing the adjustment differences between the two solutions.

4.2 Comparison of adjustment properties

To be able to view the adjustment properties of the asset paths solving approximate Euler equation, we first need to solve them. For the logarithmic utility functions considered here, this involves solving a second order, nonlinear difference equation, so that it has to be done numerically. We use the Newton iteration method to solve the problem.

Figure 11 shows the development of the asset position a in time from the initial value of zero and with the parameter values $p = 0.05$, $d = 0.1$, $R = 1.05$. The asset path obtained from the approximate Euler equation is seen to settle closer to zero than the true solution. Below in the figure we have plotted for both solutions the asset position each period relative to the corresponding steady-state value. We see that initially the asset path resulting from the approximative equation converges slightly faster then the true solution.

The goodness of the approximate Euler equation will depend on the parameter values used. This means that it will be more suitable for some circumstances than others. We have assumed in this paper that a probable range for the parameters were around $d = 0.1$, $p = 0.05$, $R = 1.05$ and $\beta = 0.95$ so that the annual interest rate is 5% compared to a rate of time preference of $1/0.95 - 1 \approx 0.0526$, debts are repaid 10% annually and the consumer faces each year a 5% chance of not having access to the asset market. The implications of the computations are that a small d will increase errors and a small p reduce them. If we have the case where the probability of facing restricted periods is small and debts are paid back relatively quickly, we can expect the approximate Euler equation to give relatively good results for the constant income case.

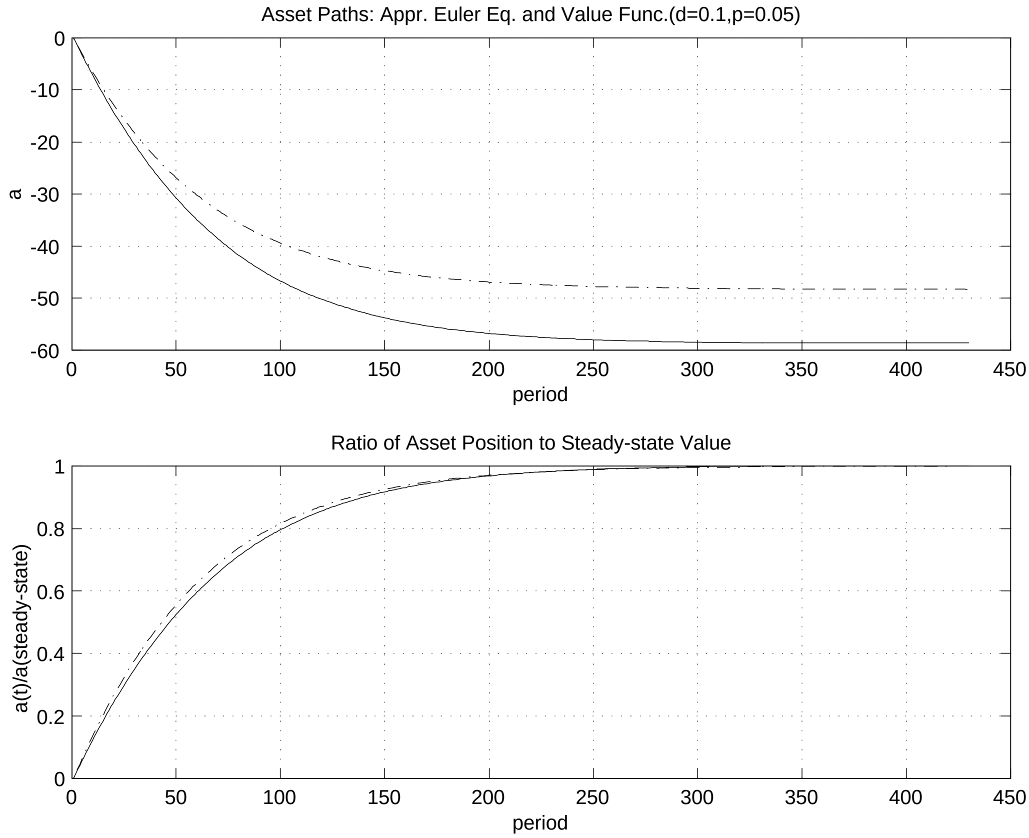

Figure 11: *Adjustement of assets for Euler equation and Bellman equation solutions from initial position of zero. Values obtained from the Bellman equation are plotted with '-' and those from Euler equation with '-.'. Parameter values are: d = 0.1, p = 0.05, R = 1.05.*

5 Conclusions

This paper has considered the idea of modeling credit restrictions through a stochastic process governing the representative agent's access to the asset market. With the simplified implementation of this idea considered here, we have obtained promising results. An important implication of stochastic access to the asset market is that the resulting debt trajectories remain bounded by a fixed level depending on the model parameters.

For the simple stochastic income process considered in section 3.3 it was seen that the model induces notable levels of correlation between current period consumption and income even for small probabilities of credit restrictions occurring. This sensitivity of consumption to current period income was also seen to be strengthened when the maturity of available debt contracts was reduced and conversely.

In addition to having a well-behaved form for numerical solving by the value function iteration, a tractable Euler equation could also be derived by a local linearization of the value function. In comparisons of the results of this approximate equation to those obtained in the value function iteration, it was seen that the approximate Euler equation seemed more suitable for use when the probability of constraints occurring was small and the debt contracts fairly short.

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