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Adaptive learning and multiple equilibria in a natural rate monetary model with unemployment persistence

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Adaptive learning and multiple equilibria in a natural rate monetary model with unemployment persistence

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Abstract

This paper demonstrates that the adaptive learning approach to modelling private sector expectations can be used as an equilibriumselection mechanism in a natural-rate monetary model with unemployment persistence. In particular, it is shown that only one of the two rational expectations equilibria is stable under least-squares learning, and that it is always the low-inflation equilibrium with intuitive comparative statics properties that is the learnable equilibrium. Hence, this paper provides a basic theoretical justification for focusing on the low-inflation equilibrium. Earlier contributions, in which the high-inflation equilibrium was ignored, mainly because of its unpleasant characteristics, are not theoretically satisfactory.

Key words: adaptive learning, monetary policy, multiple equilibria, persistence

JEL classification numbers: C62, D83, D84, E52

Adaptiivinen oppiminen ja monikäsitteiset tasapainot työttömyyden persistenssillä täydennetyssä luonnollisen työttömyysasteen rahataloudellisessa mallissa

Suomen Pankin keskustelualoitteita 30/2003

Anssi Rantala
Tutkimusosasto

Tiivistelmä

Tässä työssä osoitetaan, että adaptiivisen oppimisen lähestymistapaa yksityisen sektorin odotusten muodostamiseen voidaan käyttää tasapainon valintamekanismina työttömyyden persistenssillä täydennetyssä luonnollisen työttömyysasteen rahataloudellisessa mallissa. Osoittautuu, että vain toinen rationaalisten odotusten tasapainoista on stabiili pienimmän neliösumman oppimisen vallitessa. Opittavissa oleva tasapaino on aina vähäisen inflaation tasapaino, jonka komparatiivinen statiikka on intuitiivista. Näin ollen työssä esitetään teoreettinen perustelu sille, että mallin analysoinnissa voidaan keskittyä vähäisen inflaation tasapainon tarkasteluun. Aikaisemmat tutkimukset, joissa nopean inflaation tasapaino suljettiin tarkastelun ulkopuolelle lähinnä sen epämiellyttävien ominaisuuksien vuoksi, eivät ole teoreettisesti tyydyttäviä.

Avainsanat: adaptiivinen oppiminen, rahapolitiikka, monikäsitteiset tasapainot, persistenssi

JEL-luokittelu: C62, D83, D84, E52

Contents

1 Introduction

Recent years have witnessed a surge of interest in the application of adaptive learning approach on the analysis of monetary policy (see Evans and Honkapohja (2002b) for a recent survey of the literature). The starting point in the analysis of adaptive learning in macroeconomics is that some or all agents have imperfect knowledge about the structure of the economy. The agents rely on an econometric learning technology to form expectations and continuously update their estimates regarding the structure of the economy based on incoming data (see Evans and Honkapohja (2001) for a comprehensive treatment of learning in macroeconomics).

In the literature concerning learning and monetary policy there are, broadly speaking, two separate classes of papers. The first one looks at the case where the central bank has imperfect knowledge about the functioning of the economy, whereas the private sector has fully rational expectations (see eg Sargent (1999) and Cho et al (2002)). Another, more widely used approach models the private sector being boundedly rational and the central bank having full information about the structure of the economy (see eg Evans and Honkapohja (2002a)). However, these two modelling choices are not mutually exclusive, as is demonstrated in Honkapohja and Mitra (2002), where both the central bank and the private sector are learning. Another classification can be made in terms of the macroeconomic framework used in the analysis. Some contributions apply the natural rate model where the Phillips curve is of the Neo-Classical type, whereas the bulk of the latest literature uses a New-Keynesian framework with forward-looking agents.[1]

The purpose of this paper is to demonstrate that the adaptive learning approach of modelling private sector expectations can be used as a equilibrium selection mechanism in a dynamic monetary policy model of Lockwood and Philippopoulos (1994). Their model is an extension to the classic Kydland and Prescott (1977) and Barro and Gordon (1983) natural rate model.[2] Lockwood and Philippopoulos (1994) show that when unemployment rate is persistent, there are two rational expectations equilibria in the model.[3] One is associated with low inflation and intuitive comparative statics properties, and the other with high inflation and non-intuitive comparative statics properties. In particular, this paper shows that only one of the two rational expectations equilibria is stable under adaptive learning, and that it is always the low inflation equilibrium with intuitive comparative statics properties which is the learnable one. Earlier Sargent (1999) has shown that the unique rational expectations equilibrium of the standard Barro-Gordon model is stable under least-squares learning.

[1] Most notable examples of the former approach are Sargent (1999), Cho et al (2002) and Orphanides and Williams (2002). Evans and Honkapohja (2002b) contains a survey of the New-Keynesian strand of the literature.

[2] The standard natural rate model is usually called the Barro-Gordon model, a convention which is somewhat unfair to the original contribution by Kydland and Prescott (1977). For brevity, this usual practice is also followed in this paper.

[3] It is empirically well established that the unemployment rate is highly persistent in most industrialized countries (see eg Layard, Nickell and Jackman (1991)).

The learning approach to expectations formation is especially interesting in a situation where the economy has gone through a regime shift, like the formation of the Economic and Monetary Union (EMU) in Europe. Monetary policies have been quite different in the member countries before the EMU, and thus it is quite natural to assume that the agents don't exactly know all parameter values, eg the preferences of the common central bank, which affect the economy. After such a dramatic change in the economic environment as the EMU brought about, rational expectations is a particularly strong assumption and some sort of bounded rationality may provide a much more realistic starting point in thinking of the behavior of the agents.

In the earlier contributions which use the natural rate model with real persistence two solutions to the multiple equilibria problem have been adopted. The first solution is to ignore the 'bad' high inflation equilibrium by appealing to counter-intuitive comparative statics properties, and to the fact that the high inflation equilibrium appears only in the infinite horizon model. This strategy is followed eg in Svensson (1997, 1999), Beetsma and Jensen (1998, 1999), Lockwood et al (1998) and also in a companion paper Rantala (2003). The second solution treats the two equilibria more equally and considers both cases as being the possible outcomes of the model. This line of reasoning in followed eg in Lockwood and Philippopoulos (1994), Lockwood (1997) and Jensen (1999). This paper proposes a third solution to this problem, namely the use of adaptive learning as a selection tool. Hence, this paper provides a more elegant justification in a theoretical sense for focusing on the low inflation equilibrium than the first approach, where the high inflation equilibrium was ignored mainly because of its unpleasant characteristics.

2 The model

The model economy evolves over an infinite number of time periods $t = 1, 2, ...$ The economy consists of a private sector and a central bank. In the private sector there is a continuum of firms producing the same product, so that the goods market is perfectly competitive. In each firm the labor force is organized in a firm-specific labor union, which sets the nominal wage in each time period before the central bank sets inflation.

The assumption of an atomistic labor market is made in order to make inflation exogenous to each individual labor union. This assumption rules out any strategic interactions between the wage setters and the central bank. Assuming that labor union preferences are identical across firms and abstracting from firm-specific disturbances makes it possible to write the model in terms of a single firm-union pair, where the union takes inflation as given. Naturally, one could continue to work with the continuum of firm-union pairs and aggregate over firms later on. The approach adopted here is chosen purely for notational simplicity.[4]

[4]In what follows, the notions private sector and labor union are used interchangeably.

Labor demand determines employment in the model. For simplicity, the labor demand schedule is written directly in terms of unemployment instead of employment and the real wage elasticity is set to unity:

$$u_t = w_t - p_t, \tag{2.1}$$

where u_t is the unemployment rate, w_t is the log of the nominal wage and p_t is the log of the price level.

The labor union's loss function is of the form

$$U_t = (u_t - \overline{u}_t)^2, \tag{2.2}$$

where $\overline{u}_t$ is the time period t unemployment target. The unemployment target of the labor union is assumed to be time varying and affected by 'insider power'. In the spirit of Lindbeck and Snower (1986), and following the formulation by Blanchard and Summers (1986) with a minor modification (unemployment instead of employment), it is assumed that the unemployment target of the union is a weighted average of the currently unemployed workers, u_{t-1}, and the long-run unemployment target of the labor union, u^0, which is normalized to zero:[5]

$$\overline{u}_t = \rho u_{t-1} + (1-\rho)\, u^0, \tag{2.3}$$

where $u^0 = 0$ and $0 < \rho < 1$. This formulation captures the insider power in wage setting in a sense that when unemployment is above the long run target of zero, the currently employed insiders want to set the nominal wage at a level, which is consistent with the current unemployment rate u_{t-1}. This wage level is higher than the one consistent with the long-run target of zero unemployment. The parameter ρ is thus a measure of insider power in the model. If the insiders don't have more power than others, ρ equals zero and the unemployment target is constant. In this case the model collapses to the standard Barro-Gordon model. A high ρ implies that insiders dominate wage setting and the unemployment target of the union varies a lot with changes in the number of insiders.

The union's optimization problem is to set w_t so as to minimize the expected value of (2.2). Carrying out minimization yields a simple nominal wage rule of the form (after inserting (2.3)):

$$w_t = p_t^e + \rho u_{t-1}. \tag{2.4}$$

Combining the wage rule (2.4) with the labor demand schedule (2.1) and using the conventional definitions $\pi_t = p_t - p_{t-1}$ and $\pi_t^e = p_t^e - p_{t-1}$ for inflation and inflation expectations yields the Neo-Classical Phillips curve (a.k.a. the expectations augmented Phillips curve):

$$u_t = \rho u_{t-1} - (\pi_t - \pi_t^e). \tag{2.5}$$

[5] u^0 could equally well be non-zero. This normalization is made for analytical simplicity only.

In this simple framework the insider power parameter directly determines the persistence of the unemployment rate.[6] The long-run natural rate of unemployment is zero in the model, which results directly from the long-run unemployment target of the labor union being zero.

The intertemporal loss (or value) function of the central bank is of the standard quadratic type

$$V = \sum_{t=1}^{\infty} \delta^{t-1} \frac{1}{2} \left(\lambda u_t^2 + (1-\lambda) \pi_t^2 \right), \tag{2.6}$$

where δ, $(0 < \delta < 1)$ is the discount factor and λ and $(1-\lambda)$ are the relative weights of unemployment and inflation stabilization, respectively. It is noteworthy that the unemployment target of the central bank is zero, so it coincides with the long-run natural rate. This implies that the central bank doesn't have an over-ambitious unemployment target on average. Hence, there is no average inflation bias in the model. Inflation target of the central bank is, without loss of generality, normalized to zero for analytical convenience. Throughout the paper it is assumed that the central bank does not have access to a commitment technology and therefore sets inflation in a discretionary manner.[7]

3 Rational expectations equilibria

In this section the model is solved assuming that the private sector has rational expectations. The game between the private sector and the central bank is dynamic in nature. This results from unemployment persistence, because the central bank's decisions today will affect the future through unemployment rate. Obviously, another requirement for the game being dynamic is that the central bank is forward-looking in a sense that it cares about the future, that is, $\delta > 0$. The previous time period unemployment rate, u_{t-1}, is the state variable of the dynamic problem, since it summarizes the impact of the history of the game on the current state of the economy.

The solution concept used in the paper is the so-called Markov-perfect equilibrium, where the current actions of the players depend on the history of the game only through the state variable and trigger strategies are ruled out (see eg Fudenberg and Tirole (1991)). Although the labor union is setting the nominal wage, equally well it can be said that it chooses inflation expectations. Thereby, the choice variables of the central bank and the labor union are inflation and inflation expectations, respectively.

The linear-quadratic structure of the model implies that the value functions will be quadratic. The labor union has only one target and one instrument. Hence, it can choose inflation expectations in every period so that it will achieve its target. From (2.2)–(2.5) it is easy to see that by setting $\pi_t^e = \pi_t$, that is,

[6] See Lockwood et al (1998) for a formulation where both unemployment and real wage deviations enter the labor union's loss function.

[7] The commitment solution to the problem is presented eg in Svensson (1997).

having rational expectations, yields zero loss in each time period. The central bank's value function from time period t onwards will be of the form

$$V = \frac{\beta}{2} u_{t-1}^2, \tag{3.1}$$

where $\beta \geq 0$ is the parameter to be solved. Throughout the analysis it is assumed that β is not time dependent.

The dynamic game can now be written as:

$$\frac{\beta}{2} u_{t-1}^2 = \min_{\pi_t} (\lambda + \delta\beta) \frac{u_t^2}{2} + (1-\lambda) \frac{\pi_t^2}{2} \quad \text{given (2.5) and } \pi_t^e \text{ fixed}, \tag{3.2}$$

$$\pi_t^e = \pi_t, \tag{3.3}$$

where in (3.2) the left-hand side is the present value of losses from t onwards, and the right-hand side is the value of current period losses, plus the losses from $t+1$ onwards evaluated at the optimally chosen level of inflation. Monetary policy is discretionary and thereby the central bank takes inflation expectations as given. Equation (3.3) is the best response function of the union, which says that rational expectations are optimal.

The first-order condition of the central bank is given by

$$-(\lambda + \delta\beta) u_t + (1-\lambda) \pi_t = 0, \tag{3.4}$$

which after inserting (2.5) and solving for π_t yields

$$\pi_t = \frac{\lambda + \delta\beta}{1 + \delta\beta} \pi_t^e + \frac{\lambda + \delta\beta}{1 + \delta\beta} \rho u_{t-1}. \tag{3.5}$$

This is the best response function of the central bank, which is conditional on fixed β. The rational expectations equilibrium with fixed β is then obtained by setting $\pi_t^e = \pi_t$, which gives

$$\pi_t(\beta) = \pi_t^e(\beta) = \frac{\lambda + \delta\beta}{1-\lambda} \rho u_{t-1}. \tag{3.6}$$

Since the problem is linear-quadratic, the central bank's and the labor union's actions are linear functions of the state variable.

The next step is to solve β. Inserting (3.6) into (3.2) and reorganizing the terms results in the Riccati equation defining β:

$$\rho^2 \delta^2 \beta^2 + \left(\lambda - 1 + (1+\lambda) \delta\rho^2\right) \beta + \lambda\rho^2 = 0. \tag{3.7}$$

The Riccati equation is quadratic in β, which implies that there in general are two equilibrium values for β. From (3.6) it is then obvious that there are multiple rational expectations inflation equilibria in the model. The lower equilibrium value of β corresponds with the low inflation equilibrium, and the higher value corresponds with the high inflation equilibrium.

Solving (3.2) for β reveals that β is a non-linear function of the optimally chosen rate of inflation π_t and the given inflation expectations π_t^e:

$$\beta = \frac{\lambda \left(\rho u_{t-1} + \pi_t^e - \pi_t\right)^2 + (1-\lambda)\,\pi_t^2}{u_{t-1}^2 - \delta \left(\rho u_{t-1} + \pi_t^e - \pi_t\right)^2}, \tag{3.8}$$

where π_t is determined by (3.5).[8] Substitution of (3.8) into (3.5) and simplifying yields a quadratic

$$\begin{aligned}&(1-\lambda)\,\delta \left(\rho u_{t-1} + \pi_t^e\right) \pi_t^2 + \left(u_{t-1}^2 - (1-\lambda)\,\delta \left(\rho u_{t-1} + \pi_t^e\right)^2\right) \pi_t \\ &-\lambda \left(\rho u_{t-1} + \pi_t^e\right) u_{t-1}^2 = 0.\end{aligned} \tag{3.9}$$

This quadratic function of π_t has two roots, but one of them is always negative irrespective of π_t^e and u_{t-1}. This is inconsistent with (3.5) and $\beta \geq 0$, so the unconditional reaction function of the central bank is the larger root of (3.9)[9]:

$$\pi_t = \frac{-\left(u_{t-1}^2 - \varphi\left(\pi_t^e\right)\right) + \sqrt{\left(u_{t-1}^2 - \varphi\left(\pi_t^e\right)\right)^2 + 4\lambda\varphi\left(\pi_t^e\right) u_{t-1}^2}}{2\,(1-\lambda)\,\delta\left(\rho u_{t-1} + \pi_t^e\right)}. \tag{3.10}$$

where $\varphi\left(\pi_t^e\right) = (1-\lambda)\,\delta\left(\rho u_{t-1} + \pi_t^e\right)^2$. It is evident from (3.10) that the relationship between inflation and inflation expectations is highly non-linear.

The two rational expectations equilibria can now be solved by setting $\pi_t^e = \pi_t$ in (3.10) and simplifying, which yields real solutions:

$$\pi_t = \pi_t^e = \frac{1}{2\delta\rho}\left(\left(1-\delta\rho^2\right) \pm \sqrt{\left(1-\delta\rho^2\right)^2 - 4\lambda\,(1-\lambda)^{-1}\,\delta\rho^2}\right) u_{t-1}, \tag{3.11}$$

if, and only if, the following existence condition holds:

$$\lambda\,(1-\lambda)^{-1} \leq \frac{\left(1-\delta\rho^2\right)^2}{4\delta\rho^2}. \tag{3.12}$$

Obviously, this condition is assumed to hold, since complex solutions make no sense for rates of inflation and unemployment.

When the previous time period unemployment rate is above (below) the long-run natural rate of zero, there are two equilibrium levels of inflation, both of which are positive (negative). Naturally, when $u_{t-1} = 0$, inflation is at the target level of zero, since the is no average inflation bias in the model. It is noteworthy, that in addition to the multiple equilibria problem, there is also an indeterminacy problem in the model. With given parameter values either one of the two equilibria can emerge as an equilibrium of the dynamic game.

As mentioned in the introduction, the comparative statics properties for the two equilibria are quite different. At the low inflation equilibrium inflation

[8] (3.8) is derived from (3.2) assuming that β is not time varying. The $\min_{\pi_t}$ operator can be removed, since π_t is the optimal inflation chosen by the central bank.

[9] The value of losses at time perod t is non-negative, that is, $\beta \geq 0$. Giving π_t^e and u_{t-1} arbitrary positive values in (3.5) yields a positive π_t. Since the smaller root of (3.9) yields negative values for π_t with all possible combinations of π_t^e and u_{t-1}, it is ignored in the analysis.

increases when the central bank puts more weight on unemployment stabilization and less weight on inflation stabilization, that is, when λ becomes bigger. This is intuitive, since the central bank then cares more about the persistent unemployment fluctuations, and therefore reacts more strongly to unemployment by creating a higher inflation. Moreover, inflation increases when unemployment persistence ρ increases, or when the discount factor δ becomes bigger so that the central bank cares more about the future. An increase in ρ calls for a stronger inflation reaction, because the state dependent inflation bias in the future gets more serious when unemployment fluctuations become bigger and so the benefits of higher inflation this period are increased while the costs are unaffected. When the central bank cares more about the future, it pays more attention to persistence in unemployment and therefore reacts more strongly to unemployment deviations. All this is quite intuitive, but it holds only at the low inflation equilibrium. When the economy is at the high inflation equilibrium, changes in the values of these three parameters bring about inflation responses, which are exactly opposite in direction. That is, the comparative statics are counter-intuitive (see the Appendix for derivations).

There exists a unique rational expectations equilibrium in the model provided that the time horizon of the central bank is finite.[10] In addition, it can be shown that the equilibrium in question approaches the low inflation equilibrium of the infinite horizon model as the terminal period goes further away from the current period (see the Appendix for an illustration of this point).

The two equilibria in (3.11) can be illustrated graphically in (π_t, π_t^e)-space as intersections of the best response functions of the central bank and the labor union. Before doing that, it is useful to characterize the behavior of the best response function of the central bank (3.10). First, (3.10) can be shown to be positive at $\pi_t^e = 0$ when $u_{t-1} > 0$. Second, the slope of (3.10) can, after tedious manipulations, be shown to be is positive with all π_t^e (see the Appendix). These two properties together with the above mentioned fact that there are exactly two positive inflation equilibria when $u_{t-1} > 0$ guarantee that (3.10) cuts the 45 degree line twice, first from above and then from below. This is depicted in Figure 1, where the low and high inflation equilibria are denoted by π_- and π_+, respectively.

The multiplicity of equilibria in the model arises from unemployment persistence, which makes the best response function of the central bank non-linear in inflation expectations. In the standard Barro-Gordon model real persistence is absent and the reaction function of the central bank is linear in inflation expectations with a slope smaller than unity (see eg Sargent (1999)). Therefore, in that case the equilibrium is unique. Moreover, both inflation equilibria in the presence of real persistence yield a higher inflation than in the standard case.[11] This is quite intuitive, since with persistence the incentive to inflate the economy becomes stronger due to the fact that future inflation bias can be reduced by choosing a higher inflation today.

[10] Lockwood and Philippopoulos (1994) point out that a somewhat weaker condition than a finite horizon suffices to guarantee the uniqueness of equilibrium in the model. Namely, they show that if the inflation variability is eliminated with probability one at some known time period in the future, the high inflation equilibrium disappears.

[11] See Lockwood and Philippopoulos (1994) for an illustration of this point.

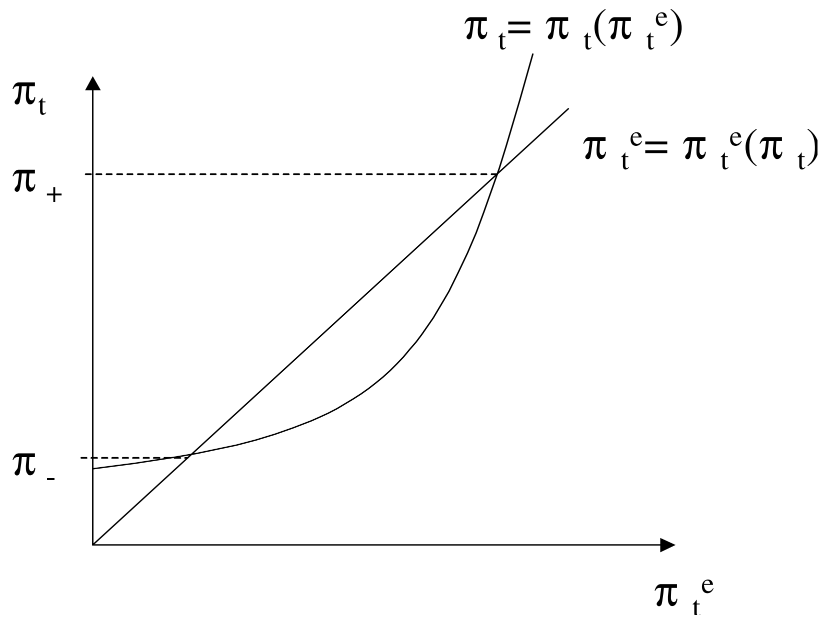

Figure 1: Determination of rational expectations equilibria

4 Expectational stability of the equilibria

Rational expectations from the private sectors point of view mean that the agents are endowed with perfect knowledge about the structure of the economy including eg the preferences of the central bank and the monetary transmission mechanism. Obviously, this is a very strict requirement which is relaxed in a specific manner in what follows. In particular, agents are assumed to be boundedly rational in a sense that they don't know the inflation process exactly, but they try to learn it by estimating the process with observed macroeconomic data at each time period. The setting is non-standard, because the system is self-referential in a sense that future data points are affected by the estimates produced today. Analyzing the dynamics of such a system calls for a special technique called stochastic approximation, which is discussed in the next section.

The agents apply the standard least-squares estimation procedure, in which the parameters to be estimated are assumed to be constants over time. However, since the agents are updating their estimates the true coefficient governing the inflation process is actually changing over time. Thereby, their model is dynamically misspecified, but as learning converges the misspecification problem disappears. Throughout the analysis it is assumed that the estimated model is of the same form as the rational expectations equilibria in (3.11), that is, inflation is assumed to be a constant times last period's unemployment rate (plus some noise in the next section).[12]

[12] In the next section a control error is added for technical reasons to the inflation process. Obviously, the stochastic component could have been introduced into the model already in section 2 without any changes in the main results.

This section uses the methodology of Evans and Honkapohja (2001, Ch. 2) to study the learnability of the rational expectations equilibria. The criterion used in the analysis is called expectational stability (a.k.a. E-stability). An equilibrium is said to be stable under learning if it fulfills the E-stability conditions. Next section demonstrates that the local convergence of real-time recursive least-squares learning in this model is governed by the very same E-stability condition.[13] The E-stability approach is a very useful one, since the E-stability of an equilibrium is in most cases technically much easier to establish than least-squares learnability.

The private sector has a perceived law of motion (PLM) of the same form as (3.11):

$$\pi_t = a u_{t-1}, \tag{4.1}$$

but it doesn't know the correct value of the coefficient in front of u_{t-1}. The parameter a is the private sector's perception of the coefficient. The forecast function used to form expectations is obtained from the PLM:

$$\pi_t^e = a u_{t-1}. \tag{4.2}$$

Combining (4.2) with the best response function of the central bank (3.10) and reorganizing gives the actual law of motion (ALM) the economy follows when the private sector has the PLM described above:

$$\pi_t = \frac{-\left(1-\omega\left(a\right)\right) + \sqrt{\left(1-\omega\left(a\right)\right)^2 + 4\lambda\omega\left(a\right)}}{2\left(1-\lambda\right)\delta\left(\rho+a\right)} u_{t-1}, \tag{4.3}$$

where $\omega\left(a\right) = \left(1-\lambda\right)\delta\left(\rho+a\right)^2$. Now, the mapping from the PLM to the ALM takes the form

$$T\left(a\right) = \frac{-\left(1-\omega\left(a\right)\right) + \sqrt{\left(1-\omega\left(a\right)\right)^2 + 4\lambda\omega\left(a\right)}}{2\left(1-\lambda\right)\delta\left(\rho+a\right)}. \tag{4.4}$$

Expectational stability is determined by the following differential equation:

$$\frac{da}{d\tau} = T\left(a\right) - a, \tag{4.5}$$

where τ denotes virtual time. The fixed points of (4.5) give the rational expectations equilibria presented in (3.11). An equilibrium $\overline{a}$ is said to be E-stable if the fixed point of the differential equation (4.5) is locally asymptotically stable at that point. Thus the E-stability condition is

$$T'\left(\overline{a}\right) < 1. \tag{4.6}$$

For the E-stability analysis it is possible to utilize previously derived results concerning the behavior of the best response function of the central bank (3.10). The expression (4.4) is just a special case of (3.10) where π_t^e is replaced by a

[13] This is a standard result in the literature.

and u_{t-1} is set to unity. Hence, it is immediately clear from Figure 1 that the slope coefficient $T'(a)$ is smaller than one at the low inflation equilibrium, $\overline{a}_-$, and bigger than one at the high inflation equilibrium, $\overline{a}_+$. The result of the analysis can be summarized as:

$$T'(\overline{a}_-) < 1 \Rightarrow \overline{a}_- \text{ is E-stable,} \tag{4.7}$$

$$T'(\overline{a}_+) > 1 \Rightarrow \overline{a}_+ \text{ is E-unstable.} \tag{4.8}$$

The low inflation equilibrium with intuitive comparative statics properties is always E-stable, whereas the high inflation equilibrium is never E-stable.

5 Real-time adaptive learning

The purpose of this section is to show that under real-time learning, where the private sector updates the coefficient estimate by running recursive least-squares on actual data, the local convergence to the rational expectations equilibrium is governed by the very same E-stability condition derived in the previous section. The mathematical method which is used in analyzing the convergence of least-squares learning is called the theory of stochastic recursive algorithms (SRA) or stochastic approximation. (see Evans and Honkapohja (2001, Ch. 6)).

The basic technique of applying the SRA approach in adaptive learning is presented in Evans and Honkapohja (2001, Ch. 2) and here it is adapted for this particular model. Following Sargent (1999), a disturbance term representing the central bank's imperfect control of inflation is added to the bank's best response function (3.10). The reason for doing that is mainly technical, since then it is guaranteed that the stochastic approximation tools can be applied. The bank's reaction function becomes:

$$\pi_t = \frac{-\left(u_{t-1}^2 - \varphi\left(\pi_t^e\right)\right) + \sqrt{\left(u_{t-1}^2 - \varphi\left(\pi_t^e\right)\right)^2 + 4\lambda\varphi\left(\pi_t^e\right)u_{t-1}^2}}{2\left(1-\lambda\right)\delta\left(\rho u_{t-1} + \pi_t^e\right)} + \eta_t, \tag{5.1}$$

where $\varphi\left(\pi_t^e\right) = (1-\lambda)\,\delta\left(\rho u_{t-1} + \pi_t^e\right)^2$ and η_t is an independent and identically distributed random term with zero mean.

Assuming rational expectations in this stochastic setup would mean that $\pi_t^e = E_{t-1}\pi_t$, where $E_{t-1}\pi_t$ is obtained from (5.1) remembering that $E_{t-1}\eta_t = 0$. Solving π_t^e from this expression gives exactly the same functional form as in (3.11). The inflation process under rational expectations can be written as

$$\pi_t = \pi_t^e + \eta_t, \tag{5.2}$$

and hence the rational expectations equilibria are of the same form as (3.11), but augmented with the control error η_t.

Now, the private sector knows the form of the rational expectations equilibria and updates the coefficient estimate each period after a new data point is observed. The PLM is thus time dependent:

$$\pi_t = a_{t-1}u_{t-1} + \eta_t, \tag{5.3}$$

where a_{t-1} is the coefficient estimate obtained at time period $t-1$,which is used to form inflation expectations for time period t:

$$\pi_t^e = a_{t-1}u_{t-1}. \tag{5.4}$$

The estimation problem can be written in recursive least-squares (RLS) form

$$a_t = a_{t-1} + t^{-1}R_t^{-1}u_{t-1}\left(\pi_t - a_{t-1}u_{t-1}\right), \tag{5.5}$$
$$R_t = R_{t-1} + t^{-1}\left(u_{t-1}^2 - R_{t-1}\right), \tag{5.6}$$

where R_t is the moment matrix for the regressors, which in this simple setup without an intercept and with only one regressor is actually a scalar $R_t = t^{-1}\sum_{i=1}^{t} u_{i-1}^2$. The reason for including the stochastic control error η_t to inflation can be seen from (5.5). Without η_t the model would be deterministic, and u_{t-1} would then approach the equilibrium level of zero asymptotically. But then R_t would be zero at the equilibrium and the estimation problem wouldn't be well defined, since R_t^{-1} enters (5.5). By introducing a stochastic element into the model, it is guaranteed that R_t is non-zero at the equilibrium.

The ALM determining π_t is now generated by the perceptions prevailing at time period $t-1$:

$$\begin{aligned}\pi_t &= \frac{-\left(1-\omega\left(a_{t-1}\right)\right) + \sqrt{\left(1-\omega\left(a_{t-1}\right)\right)^2 + 4\lambda\omega\left(a_{t-1}\right)}}{2\left(1-\lambda\right)\delta\left(\rho + a_{t-1}\right)} + \eta_t \\ &= T\left(a_{t-1}\right)u_{t-1} + \eta_t.\end{aligned} \tag{5.7}$$

Inserting (5.7) into (5.5) yields a stochastic recursive system

$$a_t = a_{t-1} + t^{-1}R_t^{-1}u_{t-1}\left(\left(T\left(a_{t-1}\right) - a_{t-1}\right)u_{t-1} + \eta_t\right), \tag{5.8}$$
$$R_t = R_{t-1} + t^{-1}\left(u_{t-1}^2 - R_{t-1}\right). \tag{5.9}$$

Before it is possible to study the convergence of this system by applying results from the stochastic approximation literature, a minor modification is needed. On the right-hand side of (5.8) there exists R_t, but the SRA approach allows only lagged values of a_t and R_t to appear. This problem is solved by defining a new variable $S_{t-1} = R_t$. The system (5.8)–(5.9) becomes

$$a_t = a_{t-1} + t^{-1}S_{t-1}^{-1}u_{t-1}\left(\left(T\left(a_{t-1}\right) - a_{t-1}\right)u_{t-1} + \eta_t\right), \tag{5.10}$$
$$S_t = S_{t-1} + t^{-1}\left(\frac{t}{t+1}\right)\left(u_t^2 - S_{t-1}\right), \tag{5.11}$$

which is now in the standard SRA form (see Evans and Honkapohja (2001, Ch. 2):

$$\theta_t = \theta_{t-1} + \gamma_t Q\left(t, \theta_{t-1}, X_t\right), \tag{5.12}$$

with

$$\theta_t = \left(\begin{array}{c} a_t \\ S_t \end{array} \right), \; X_t = \left(\begin{array}{c} u_t \\ u_{t-1} \\ \eta_t \end{array} \right), \gamma_t = t^{-1} \text{ and} \tag{5.13}$$

$$Q\left(t, \theta_{t-1}, X_t\right) = \left(\begin{array}{c} S_{t-1}^{-1} u_{t-1} \left(\left(T\left(a_{t-1}\right) - a_{t-1} \right) u_{t-1} + \eta_t \right) \\ \left(\frac{t}{t+1} \right) \left(u_t^2 - S_{t-1} \right) \end{array} \right).$$

The stochastic approximation approach associates an ordinary differential equation (ODE) with the SRA,

$$\frac{d\theta}{d\tau} = h\left(\theta\left(\tau\right)\right), \tag{5.14}$$

where $h(\theta)$ is obtained as

$$h\left(\theta\right) = \lim_{t\to\infty} EQ\left(t, \theta, X_t\right). \tag{5.15}$$

$EQ(t, \theta, X_t)$ denotes the expectation of $Q(\cdot)$ for fixed θ. The stochastic approximation results show that the ODE (5.14) approximates the behavior of the SRA (5.12) well with large t. Importantly, the limit points of the SRA correspond to locally stable equilibria of the ODE.[14]

Adapted for the problem at hand, (5.15) becomes

$$h_a\left(a, S\right) = \lim_{t\to\infty} ES^{-1} u_{t-1} \left(\left(T\left(a\right) - a \right) u_{t-1} + \eta_t \right), \tag{5.16}$$

$$h_S\left(a, S\right) = \lim_{t\to\infty} \left(\frac{t}{t+1} \right) E\left(u_t^2 - S\right), \tag{5.17}$$

where a and S are fixed. Writing $\lim_{t\to\infty} Eu_t^2 \equiv M$ and noting that $\lim_{t\to\infty} \left(\frac{t}{t+1}\right) = 1$ the second equation (5.17) becomes

$$h_S\left(a, S\right) = M - S. \tag{5.18}$$

The first equation (5.16) can be rewritten by noting that $E\left(u_{t-1}\eta_t\right) = 0$ and $\lim_{t\to\infty} Eu_{t-1}^2 = \lim_{t\to\infty} Eu_t^2 \equiv M$ which gives

$$h_a\left(a, S\right) = S^{-1} M \left(T\left(a\right) - a \right). \tag{5.19}$$

Finally, the associated ODE is then

$$\frac{da}{d\tau} = S^{-1} M \left(T\left(a\right) - a \right), \tag{5.20}$$

$$\frac{dS}{d\tau} = M - S. \tag{5.21}$$

The second equation of the recursive system is globally stable so that S approaches M from any starting point. The stability of the system is thereby

[14] The technical assumptions required for the convergence conditions to hold include regularity assumptions on Q, conditions on the rate at which $\gamma_t \to 0$ and assumptions on the properties of the stochastic process followed by X_t (see Honkapohja and Evans (2001, Ch. 6) for the precise assumptions).

governed by the first differential equation, which can be further simplified by noting that $S^{-1}M$ approaches unity:

$$\frac{da}{d\tau} = T(a) - a. \tag{5.22}$$

But, this is exactly the same stability condition as the E-stability condition derived in the previous section. Thus E-stability (E-instability) of an equilibrium implies also (in)stability under least-squares learning.

6 Concluding remarks

This paper has demonstrated that it is possible to use adaptive learning approach as an equilibrium selection device in a dynamic model of monetary policy by Lockwood and Philippopoulos (1994). When the private sector has imperfect information about the way economy works and try to learn the rational expectations equilibrium, it turns out that the economy always arrives at the low inflation equilibrium of the model. In the earlier literature the high inflation equilibrium was ignored by referring to counter-intuitive comparative statics properties or to the fact that it appeared only in an infinite horizon version of the model. These arguments are obviously quite weak in the theoretical sense. The contribution of this paper is that it provides a more elegant way based on adaptive learning to exclude the high inflation equilibrium from the analysis.

Obviously, the analysis could be extended to many different directions. It would be interesting to study learning with misspecification in this framework. What would happen if some or all agents try to learn a wrong model? For example, some agents could misinterpret the joint existence of inflation and unemployment as an (average) inflation bias, which would result from an over-ambitious long-run unemployment target of the central bank. Would the model still converge to some restricted perceptions equilibrium à la Evans and Honkapohja (2001), which would possibly be different from the rational expectations equilibria? Another extension would be to study the robustness of the learnability results when there exists a stabilization role for monetary policy. A potentially challenging extension is related to the solution technique of the dynamic problem adopted in the paper. The dependence of β on the optimal choice of inflation by the central bank is ignored (see equation (3.8)). If the central bank would take into account this interdependence, the dynamic problem would obviously become a lot harder to solve. The issues raised in this section call for a closer examination, but are left for future research.

A Appendix

A.1 Comparative statics properties of the rational expectations equilibria

The rational expectations equilibria are given by (3.11). Setting $u_{t-1} = 1$ and dropping time indices gives:

$$\pi = \frac{1}{2\delta\rho}\left(\left(1-\delta\rho^2\right) \pm \sqrt{\left(1-\delta\rho^2\right)^2 - 4\lambda\left(1-\lambda\right)^{-1}\delta\rho^2}\right).$$

In what follows the low and high inflation equilibria are denoted by π_- and π_+, respectively.

Now, it is easy to establish the following results concerning the weight of unemployment stabilization in the central bank loss function:

$$\frac{\partial\pi_-}{\partial\lambda} = -\frac{1}{4\delta\rho\sqrt{\left(\left(1-\delta\rho^2\right)^2 - 4\frac{\lambda}{1-\lambda}\delta\rho^2\right)}}\left(-\frac{4}{1-\lambda}\delta\rho^2 - 4\frac{\lambda}{\left(1-\lambda\right)^2}\delta\rho^2\right) > 0,$$

$$\frac{\partial\pi_+}{\partial\lambda} = \frac{1}{4\delta\rho\sqrt{\left(\left(1-\delta\rho^2\right)^2 - 4\frac{\lambda}{1-\lambda}\delta\rho^2\right)}}\left(-\frac{4}{1-\lambda}\delta\rho^2 - 4\frac{\lambda}{\left(1-\lambda\right)^2}\delta\rho^2\right) < 0,$$

where π_- behaves intuitively and π_+ counter-intuitively with respect to changes in λ.

Next, the comparative statics with respect to ρ are studied:

$$\frac{\partial\pi_-}{\partial\rho} = -\frac{1}{2}\left(1+\delta\rho^2\right)\frac{-1+\delta\rho^2+\sqrt{\left(\frac{\lambda-1+2\lambda\delta\rho^2+2\delta\rho^2+\delta^2\rho^4\lambda-\delta^2\rho^4}{\lambda-1}\right)}}{\sqrt{\left(\frac{\lambda-1+2\lambda\delta\rho^2+2\delta\rho^2+\delta^2\rho^4\lambda-\delta^2\rho^4}{\lambda-1}\right)}\delta\rho^2} > 0,$$

$$\frac{\partial\pi_+}{\partial\rho} = \frac{1}{2}\left(1+\delta\rho^2\right)\frac{-1+\delta\rho^2-\sqrt{\left(\frac{\lambda-1+2\lambda\delta\rho^2+2\delta\rho^2+\delta^2\rho^4\lambda-\delta^2\rho^4}{\lambda-1}\right)}}{\sqrt{\left(\frac{\lambda-1+2\lambda\delta\rho^2+2\delta\rho^2+\delta^2\rho^4\lambda-\delta^2\rho^4}{\lambda-1}\right)}\delta\rho^2} < 0.$$

It is clear that $\frac{\partial\pi_+}{\partial\rho}$ is negative, since $-1+\delta\rho^2 < 0$. In order to establish that $\frac{\partial\pi_-}{\partial\rho} > 0$ one needs to investigate the sign of the numerator of that expression:

$$-1+\delta\rho^2+\sqrt{\left(\frac{\lambda-1+2\lambda\delta\rho^2+2\delta\rho^2+\delta^2\rho^4\lambda-\delta^2\rho^4}{\lambda-1}\right)},$$

where the first part $-1+\delta\rho^2$ negative. Now, if the first term squared, that is, $\left(-1+\delta\rho^2\right)^2$ is bigger than the expression inside the square root, it follows that the numerator is negative and $\frac{\partial\pi_-}{\partial\rho} > 0$:

$$\left(-1+\delta\rho^2\right)^2 - \left(\frac{\lambda-1+2\lambda\delta\rho^2+2\delta\rho^2+\delta^2\rho^4\lambda-\delta^2\rho^4}{\lambda-1}\right) = 4\frac{\lambda}{1-\lambda}\delta\rho^2 > 0,$$

which verifies that the numerator is negative and $\frac{\partial \pi_-}{\partial \rho} > 0$. π_- behaves intuitively and π_+ counter-intuitively with respect to changes in ρ.

The comparative statics with respect to δ are as follows:

$$\frac{\partial \pi_-}{\partial \delta} = \frac{1}{2}\frac{(\lambda\delta\rho^2+\lambda-1+\delta\rho^2)+(1-\lambda)\sqrt{\left(\frac{\lambda-1+2\lambda\delta\rho^2+2\delta\rho^2+\delta^2\rho^4\lambda-\delta^2\rho^4}{\lambda-1}\right)}}{\delta^2\rho\sqrt{\left(\frac{\lambda-1+2\lambda\delta\rho^2+2\delta\rho^2+\delta^2\rho^4\lambda-\delta^2\rho^4}{\lambda-1}\right)}(\lambda-1)} > 0,$$

$$\frac{\partial \pi_+}{\partial \delta} = -\frac{1}{2}\frac{(\lambda\delta\rho^2+\lambda-1+\delta\rho^2)-(1-\lambda)\sqrt{\left(\frac{\lambda-1+2\lambda\delta\rho^2+2\delta\rho^2+\delta^2\rho^4\lambda-\delta^2\rho^4}{\lambda-1}\right)}}{\delta^2\rho\sqrt{\left(\frac{\lambda-1+2\lambda\delta\rho^2+2\delta\rho^2+\delta^2\rho^4\lambda-\delta^2\rho^4}{\lambda-1}\right)}(\lambda-1)} < 0.$$

First, it is useful to examine the expression inside the square root. For real solutions to exist, it must be non-negative. Thus the following inequality need to hold (note that the denominator $\lambda - 1 < 0$):

$$\lambda - 1 + 2\lambda\delta\rho^2 + 2\delta\rho^2 + \delta^2\rho^4\lambda - \delta^2\rho^4 < 0.$$

This restriction limits the term $\lambda\delta\rho^2+\lambda-1+\delta\rho^2$ in the numerators of the both derivatives $\left(\frac{\partial\pi_-}{\partial\delta} \text{ and } \frac{\partial\pi_+}{\partial\delta}\right)$ to negative values. This can be seen by subtracting the term $\lambda\delta\rho^2 + \lambda - 1 + \delta\rho^2$ from the restriction above, that is $\lambda - 1 + 2\lambda\delta\rho^2 + 2\delta\rho^2 + \delta^2\rho^4\lambda - \delta^2\rho^4$, which yields:

$$\begin{aligned} & \lambda - 1 + 2\lambda\delta\rho^2 + 2\delta\rho^2 + \delta^2\rho^4\lambda - \delta^2\rho^4 - \left(\lambda\delta\rho^2 + \lambda - 1 + \delta\rho^2\right) \\ = \;& \delta\rho^2\left(\lambda - \rho^2\delta + \rho^2\lambda\delta + 1\right) > 0. \end{aligned}$$

Subtracting a positive term from a negative term equals always something negative, but here the outcome is positive, which implies that the term $\lambda\delta\rho^2 + \lambda - 1 + \delta\rho^2$ is negative provided that the restriction derived above holds. Now, it is clear that $\frac{\partial\pi_+}{\partial\delta} < 0$, since the numerator is negative. To prove that $\frac{\partial\pi_-}{\partial\delta} > 0$ one must study the numerator of that expression more closely. The first term squared is bigger than the term inside the square root (with $1-\lambda$ moved inside):

$$\begin{aligned} & \left(\lambda\delta\rho^2 + \lambda - 1 + \delta\rho^2\right)^2 - (\lambda-1)^2\left(\frac{\lambda - 1 + 2\lambda\delta\rho^2 + 2\delta\rho^2 + \delta^2\rho^4\lambda - \delta^2\rho^4}{\lambda - 1}\right) \\ = \;& 4\delta^2\lambda\rho^4 > 0, \end{aligned}$$

which directly implicates that the numerator is negative, and hence $\frac{\partial\pi_-}{\partial\delta} > 0$. Again, π_- behaves intuitively and π_+ counter-intuitively.

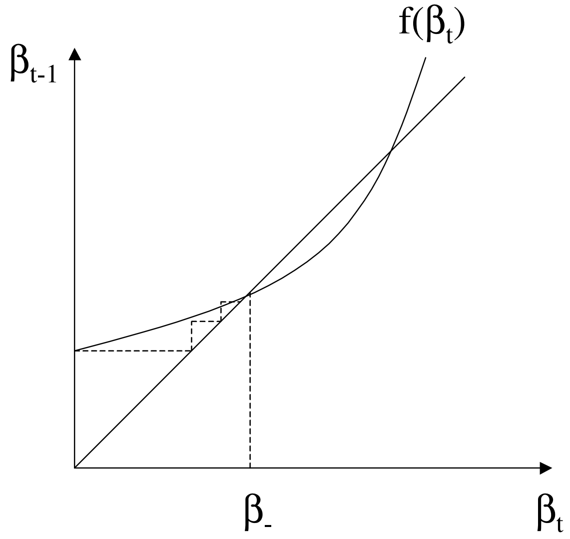

Figure 2: The determination of equilibrium in a finite horizon case

A.2 Finite horizon case

Assuming that there exists a terminal time period T, the central bank's optimizing problem needs to be modified so that future losses equal zero at T. This implies a terminal condition $\beta_T = 0$. The Riccati equation with time subscripts can be derived from (3.2)–(3.6) by letting β be time dependent. Then on the left-hand side of (3.2) β is replaced by β_{t-1}, whereas on the right-hand side β is replaced by β_t. This means that in (3.4)–(3.6) β becomes β_t. Now, inserting (3.6) with β_t into (3.2) yields a dynamic Riccati equation:

$$\beta_{t-1} = (\lambda + \delta\beta_t)\,\rho^2 + (\lambda + \delta\beta_t)^2\,\rho^2\,(1-\lambda)^{-1} = f(\beta_t)\,.$$

The function $f(\beta_t)$ is an increasing and convex function in the relevant range, that is $\beta_t \in [0,\infty)$:

$$\begin{aligned} f'(\beta_t) &= \delta\rho^2 + 2(\lambda + \delta\beta_t)\frac{\rho^2}{1-\lambda}\delta > 0, \text{ when } \beta_t \in [0,\infty)\,, \\ f''(\beta_t) &= 2\delta^2\frac{\rho^2}{1-\lambda} > 0. \end{aligned}$$

By drawing a phase diagram it is possible to investigate the backward iterations from the terminal condition $\beta_T = 0$.

The phase diagram makes evident that assuming a finite horizon guarantees a unique equilibrium, as iterating backwards from T makes β_t asymptotically converge to the lower equilibrium value of β_-, which is associated with the low inflation equilibrium. However, it is noteworthy, that the equilibrium is not exactly the low inflation equilibrium of the infinite horizon model, since $\beta_t < \beta_-$ with all $T < \infty$. The two equilibria coincide only when $T \to \infty$, but then also the high inflation equilibrium emerges. The more further away in the future T is, the closer the two equilibria are. In other words, since inflation is positively related to β, inflation approaches gradually zero as t approaches T.

A.3 Behavior of equation (3.10)

Setting $\pi_t^e = 0$ in (3.10) yields

$$\begin{aligned}\pi_t(0) &= \frac{-\left(u_{t-1}^2 - \varphi(0)\right) + \sqrt{\left(u_{t-1}^2 - \varphi(0)\right)^2 + 4\lambda\varphi(0)\,u_{t-1}^2}}{2(1-\lambda)\,\delta\rho u_{t-1}} \\ &= \frac{-\left(1-(1-\lambda)\,\delta\rho^2\right) + \sqrt{\left(1-(1-\lambda)\,\delta\rho^2\right)^2 + 4(1-\lambda)\,\lambda\delta\rho^2}}{2(1-\lambda)\,\delta\rho} u_{t-1} > 0.\end{aligned}$$

Taking the derivative of (3.10) with respect to π_t^e and collecting terms gives

$$\pi_t'(\pi_t^e) = \frac{1}{2}\left(u_{t-1}^2 + \varphi(\pi_t^e)\right)\frac{\left({u_{t-1}}^2 - \varphi(\pi_t^e)\right) + \sqrt{\left(u_{t-1}^2 - \varphi(\pi_t^e)\right)^2 + 4\lambda\varphi(\pi_t^e)}}{\varphi(\pi_t^e)\sqrt{\left(u_{t-1}^2 - \varphi(\pi_t^e)\right)^2 + 4\lambda\varphi(\pi_t^e)}} > 0.$$

References

[1] Barro, R.J. – Gordon, D.B. (1983) **A Positive Theory of Monetary Policy in a Natural Rate Model.** Journal of Political Economy 91, 589–610.

[2] Beetsma, R.M.W.J. – Jensen, H. (1999) **Optimal Inflation Targets, 'Conservative' Central Banks, and Linear Inflation Contracts: Comment.** American Economic Review 89, 342–347.

[3] Beetsma, R.M.W.J. – Jensen, H. (1998) **Optimal Inflation Targets, 'Conservative' Central Banks, and Linear Inflation Contracts: Comment.** Working Paper No. 1998-11, Economic Policy Research Unit, University of Copenhagen.

[4] Blanchard, O.J. – Summers, L.H. (1986) **Hysteresis and the European Unemployment Problem.** NBER Macroeconomics Annual 1986, The MIT Press, Cambridge, MA.

[5] Cho, I. – Williams, N. – Sargent, T.J. (2002) **Escaping Nash Inflation.** Review of Economic Studies 69, 1–40.

[6] Evans, G.W. – Honkapohja, S. (2002a) **Expectations and the Stability Problem for Optimal Monetary Policies.** forthcoming in Review of Economic Studies.

[7] Evans, G.W. – Honkapohja, S. (2002b) **Adaptive Learning and Monetary Policy Design.** Bank of Finland Discussion Papers, 29/2002.

[8] Evans, G.W. – Honkapohja, S. (2001) **Learning and Expectations in Macroeconomics.** Princeton University Press, Princeton, NJ.

[9] Fudenberg, D. – Tirole, J. (1991) **Game Theory.** The MIT Press, Cambridge, MA.

[10] Honkapohja, S. – Mitra, K. (2002) **Performance of Monetary Policy with Internal Central Bank Forecasting.** Bank of Finland Discussion Papers, 3/2002.

[11] Jensen, H. (1999) **Monetary Policy Cooperation and Multiple Equilibria.** Journal of Economic Dynamics & Control 23, 1133–1153.

[12] Kydland, F.E. – Prescott, E.C. (1977) **Rules Rather than Discretion: The Inconsistency of Optimal Plans.** Journal of Political Economy 85, 473–491.

[13] Layard, R. – Nickell, S. – Jackman, R. (1991) **Unemployment: Macroeconomic Performance and the Labour Market.** Oxford University Press, Oxford.

[14] Lindbeck, A. – Snower, D.J. (1986) **Wage Setting, Unemployment, and Insider-Outsider Relations.** American Economic Review 76, Papers and Proceedings, 235–239.

[15] Lockwood, B. (1997) **State-Contingent Inflation Contracts and Unemployment Persistence.** Journal of Money, Credit, and Banking 29, 286–299.

[16] Lockwood, B. – Miller, M. – Zhang, L. (1998) **Designing Monetary Policy when Unemployment Persists.** Economica 65, 327–345.

[17] Lockwood, B. – Philippopoulos, A. (1994) **Insider Power, Unemployment Dynamics and Multiple Inflation Equilibria.** Economica 61, 59–77.

[18] Orphanides, A. – Williams, J.C. (2002) **Imperfect Knowledge, Inflation Expectations, and Monetary Policy.** The Federal Reserve Board, Finance and Economics Discussion Series No. 27-2002.

[19] Rantala, A. (2003) **Labor Market Flexibility and Policy Coordination in a Monetary Union.** Bank of Finland Discussion Papers 11/2003.

[20] Sargent, T.J. (1999) **The Conquest of American Inflation.** Princeton University Press, Princeton, NJ.

[21] Svensson, L.E.O. (1999) **Price-Level Targeting versus Inflation Targeting: A Free Lunch?** Journal of Money, Credit, and Banking 31, 277–295.

[22] Svensson, L.E.O. (1997) **Optimal Inflation Targets, 'Conservative' Central Banks, and Linear Inflation Contracts.** American Economic Review 87, 98–114.

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