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Banks' option to lend, interest rate sensitivity, and credit availability

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Abstract

Interest rate risk is a major concern for banks because of the nominal nature of their assets and the asset-liability maturity mismatch. This paper proposes a new way to derive a bank's interest rate sensitivity, by examining separately the effects of interest rate changes on existing loans (loans-in-place) and potential loans (loans-in-process). A potential loan is shown to be equivalent to an American option to lend, and is valued using option theory. An increase in interest rates usually has a negative effect on existing loans. However, if both deposit and lending rates rise by the same amount, the value of a potential loan generally increases. Hence a bank's lending slack (ratio of loans-in-process to loans-in-place) will determine its overall interest rate risk. Empirical evidence indicates that low-slack banks indeed have significantly more interest rate risk than high-slack banks. The model also makes predictions regarding the effect of deposit and lending rate parameters on bank credit availability. Empirical tests with quarterly data are generally supportive of these predictions.

Key words: interest rate risk, option to lend, bank's lending capacity, maturity intermediation

JEL classification numbers: G21, G13

Pankkien lainaoptiot, korkoherkkyys ja luoton saatavuus

Suomen Pankin keskustelualoitteita 15/2002

Iftekhar Hasan – Sudipto Sarkar
Tutkimusosasto

Tiivistelmä

Pankeilla on merkittäviä korkoriskejä, jotka johtuvat niiden velkojen ja saamisten nimellisyydestä ja maturiteettierosta. Tässä tutkimuksessa esitetään uusi tapa arvioida pankkien korkoherkkyyttä tarkastelemalla erikseen korkomuutosten vaikutuksia vanhoihin lainoihin ja potentiaalisiin uusiin lainoihin. Potentiaalisen uuden lainan osoitetaan vastaavan amerikkalaista luotonanto-optiota, joten sen arvo voidaan johtaa optioteorian avulla. Korkojen nousulla on yleisesti negatiivinen vaikutus vanhojen lainojen arvoon. Jos sitä vastoin talletus- ja luottokorot vaihtelevat yhdessä, potentiaalisen uuden lainan arvo yleensä nousee koron myötä. Näin ollen pankin luotonantovara eli potentiaalisten uusien lainojen määrän suhde vanhaan lainakantaan määrää sen korkoriskin. Empiiristen tulosten mukaan korkoriski sellaisilla pankkeilla, joiden luotonantovara on pieni, on merkittävästi suurempi kuin pankeilla, joilla on paljon käyttämätöntä luotonantovaraa. Mallin avulla voidaan myös ennustaa luotto- ja talletuskorkojen vaikutuksia luoton saatavuuteen. Neljännesvuosiaineistolla saadut empiiriset tulokset yleisesti tukevat mallin ennusteita.

Asiasanat: korkoriski, luotonanto-optio, pankin luotonantokyky, maturiteettien muuttaminen

JEL-luokittelu: G21, G13

Contents

1 Introduction

Banks are different from other commercial firms in that they produce financial services, the reward to which is an interest rate, and most of their operations are financed by borrowings, the cost of which is also an interest rate. Banks are therefore more sensitive to interest rate fluctuations than most other businesses. The effect of interest rate changes on bank profits and values has been an important issue for the banking industry in recent years. It has been argued that banks' exposure to interest rate risk was perhaps the most important factor in precipitating the U.S. savings and loans crisis (Duan, et al (1995)). Whether higher market interest rates hurt or benefit banks will have crucial implications for monetary policy and government regulation of interest rates. Moreover, regulators in the banking industry are interested in the relationship between bank risk and the insurance liabilities of the FDIC, and interest rate risk exposure is one of the factors examined when determining a bank's capital adequacy situation.

To date, there is no consensus on whether higher interest rates hurt or help the banking industry. In fact, whether banks are at all sensitive to changes in interest rates is an unresolved issue. Samuelson (1945) argued that banks should benefit from rising interest rates, but empirical studies provide mixed results on this issue. While some have found no significant effect of interest rate changes on bank profitability or stock returns (Flannery (1981, 1983) and Chance & Lane (1980)), others have found that higher interest rates help banks (Hanweck & Kilcollin (1984) and Hancock (1985)), and still others have found that higher interest rates hurt banks (Booth & Officer (1985), Flannery & James (1984a) and Lynge & Zumwalt (1980)). Scott & Peterson (1986) found that equity values of unhedged S&L associations were more sensitive to unexpected interest rate changes than commercial banks and insurance companies, which more closely balance the maturities of their assets and liabilities. Importantly, bank stocks have been found to be significantly more sensitive to interest rate changes than industrial stocks (Lynge & Zumwalt (1980)).

The traditional theoretical approach has been to view the bank as a maturity intermediary which borrows short and lends long; thus its assets are long-maturity and liabilities are short-maturity. In case of an interest rate change, the effects on the assets and liabilities are computed, from which the net effect is derived. This has led to the concept of gap management, where the maturity/duration gap is the control variable in interest rate risk management (chapter 5 of Saunders (1994)).

While this approach may be adequate for measuring the effect of interest rate changes on the existing loan portfolio, it is not appropriate for the bank as a whole. This is because it ignores an important source of value for a bank: the ability to make additional loans at positive spreads between loan and deposit rates. Positive spreads between borrowing and lending rates survive in equilibrium to reward banks for interest rate risk, among others (Kane (1981)). This "ability to lend at a positive spread" in the future comes from the bank's special status as a financial institution, expertise in the area, specialized skills of its employees, etc. It is similar to the concept of "value as a going concern" (Samuelson (1945)) or "rent" (Hutchison & Pennacchi (1996)). Although this aspect has been largely ignored in the existing literature, it is necessary to examine the valuation effects of this "ability to lend" in order to properly evaluate the effect of interest rate changes. This is what our article attempts.

1.1 Valuing the "Ability to lend"

The bank has, by virtue of its position, the ability to lend money at a certain rate (say L%) and finance it by taking deposits at the deposit rate (say r%). With positive spreads (L – r), this ability contributes to bank value. If the spread is large enough, the bank will grant the loan immediately. If, on the other hand, the spread is narrow (or zero, or even negative), the bank will postpone granting the loan. We can recognize the correspondence between the ability to make a loan and an American option. Once the option to lend is exercised, the payoff is the spread over the life of the loan, ie, once the option is exercised, the option to lend is replaced by the loan itself. We can therefore use option theory to value the ability to lend, and study the overall effect of interest rate changes on bank values. This is the main contribution of our paper. Hutchison & Pennacchi (1996) also incorporate the value of future stream of profits from loans and deposits. However, in their model, the loan rate is set competitively and the deposit market is monopolistic where the bank chooses the deposit rate (and corresponding quantity) optimally; whereas in our model, the bank is a price-taker[1] in both loan and deposit markets (the rates being determined by the markets) and the bank decides whether or not to make the loan at the market rates, ie, the bank acts optimally in timing its loans/deposits.

1.2 Our approach

We examine the effect of interest rate risk on the net value of the bank's loan portfolio (existing and potential), since this is basically the means by which

[1] The "ability to lend at a positive spread" and banks being "price-takers" are consistent because (i) banks are "special" in lending because of the special position they occupy, from which they derive "rents" (Hutchison & Pennacchi (1996)); and (ii) banks take interest rate risk, for which the reward is the risk premium (higher loan rate) or the return to maturity intermediation. We thank the referee for pointing this out.

interest rate effects are transmitted to bank equity values. There are two points to note about our approach: (1) First, we make a distinction between the deposit rate or bank's cost of funds (which we call the short-term rate r_t) and the loan rate or bank's revenue (which we call the long-term rate L_t). This allows us to capture an essential feature of banks, ie, the mismatch between asset and liability maturities. The long rate is in general different from the short rate, although they tend to move together and are positively correlated. Hence the two rates are modelled as separate (possibly correlated) processes. (2) Second, we treat an existing loan differently from a potential loan. The former is termed a "loan-in-place" and the latter a "loan-in-process." For the former, the loan rate remains unchanged when interest rates change, whereas deposits (being short-term) are continuously renewed; therefore, interest rate changes will affect costs but not revenues. For a loan-in-process, interest rate changes will affect both revenues and cost, hence its valuation effects will differ from that of a loan-in-place.

It is obvious that, for a loan-in-place, the bank will be hurt by a rise in interest rates, since revenues will be unchanged but costs will rise. For a loan-in-process, a rise in interest rates will be reflected in the potential lending rate. Thus the effect of interest rate changes on a loan-in-process might be very different from the effect on a loan-in-place. As shown in this paper, a loan-in-process is equivalent to an option to lend,[2] and once the option is exercised, a loan-in-process becomes a loan-in-place. Therefore, option pricing techniques can be used to value a loan-in-process and gauge the effect of interest rate changes. This allows us to include the role of both existing loans and potential loans, when examining the net effect of interest rate risk on a bank's value.

[2] This is different from the conventional exchange-traded option. It is analogous to a "real option" (Dixit & Pindyck (1994)). In a partial-equilibrium setting, where both the loan and deposit rates are market-determined (and therefore exogenous for the bank), the bank's ability to grant loans when conditions are favorable and to withhold loans when conditions are unfavorable is similar to an American option. Since this option has real value, it has to be taken into account. The value comes from the bank's special position as a recognized financial institution that can accept deposits and lend the money at higher rates. It is well known in the real options literature that such options have positive values in equilibrium if there are barriers to entry, and there are significant barriers to establishing a bank. The set-up costs incurred in incorporating as a bank can be viewed as the price paid for the implicit option. Borrowers also have the option to borrow, but this option has zero value in equilibrium because there are no significant barriers to entry. The equilibrium or market-clearing loan rate is, however, affected by borrowers: if the loan rate is too high, they will not borrow, as a result of which L will fall to the appropriate level at which markets clear. At this level, the option to borrow will have zero value.

1.3 Maturity intermediation

A bank has many roles, eg, brokerage function, asset transformation, and maturity intermediation (Saunders (1994)). Of these, maturity intermediation is arguably the most important function of a bank, particularly in the context of interest rate risk. To quote McCulloch (1981), page 223, "… interest rate risk is particularly interesting in view of the traditional practice followed by financial intermediaries of 'transforming maturities' by borrowing short and lending long." For commercial banks and thrifts, the average maturity for assets is longer than that for liabilities, since they tend to hold large amounts of relatively long-term fixed-income assets such as loans and conventional mortgages, while issuing shorter-term liabilities such as certificates of deposit. A classic recent illustration of the effects of this maturity mismatch is the S&L disaster (Duan, et al (1995)). Recently, there have been attempts by banks to reduce their maturity intermediation role by matching the maturities of assets and liabilities, especially after the S&L crisis. In spite of these efforts, "… all financial institutions tend to mismatch their balance sheet maturities to some degree" (Saunders (1994), page 86).

In this paper, we focus on this particular role of banks: maturity intermediation. The bank borrows short and lends long,[3] and its primary function is to match the maturities of short-term lenders and long-term borrowers. The reward to this intermediation is the spread \$(L – r) per dollar of intermediation, where r = short-term (deposit) rate, and L = long-term (lending) rate.

Suppose that at time t = 0, a bank makes a \$1 T-period loan at a (fixed) lending rate of L_0. Then it will receive a constant cash flow stream of \$$L_0$ per unit time till the loan matures at t = T. In order to finance the loan, it borrows \$1 from short-term depositors whom it pays at the rate of \$$r_t$ per unit time. Since this is the short-term rate, it changes continuously. If the bank's role is reduced to this bare function, the cash flow can be viewed as the spread between the fixed return from the loan and the variable cost of funds, ie, \$(L_0 – r_t) per unit time per dollar intermediated, till the loan matures.

The rest of the paper is organized as follows. In Section 2, we present the model, value a loan-in-place, and use the option pricing methodology to value a

[3] This is equivalent to saying that its liabilities are short-term and assets long-term, a fairly accurate characterization for most banks. Although banks have (infinite-maturity) equity in the balance sheet, the equity/assets ratio for banks is generally much smaller than other firms. For instance, the ratio was 6.68% for the banking sector as a whole in 1991 (Saunders (1994)); thus, over 93% of total bank assets were funded by deposits or borrowed funds. Of these, deposits far outweighed the rest, accounting for \$2.48 trillion against \$0.49 and \$0.34 trillion respectively for borrowings and other liabilities. On the assets side, the most important component is loans (other assets include securities held as investments, such as treasury bonds, municipal bonds, and high-grade corporate bonds). Bank assets therefore have longer maturities than bank liabilities.

loan-in-process. Section 3 presents the numerical analysis and comparative statics: we show that the value of a loan-in-place decreases when interest rates rise, and the value of a loan-in-process rises when both lending rate and deposit rate rise by the same amount. Section 4 examines the overall impact of interest rate risk on a bank, and shows its relation to the lending slack maintained by the bank. It also examines the critical lending trigger and its role in credit availability. Finally, Section 4 includes some empirical tests of the model's implications; the results are found to be generally supportive of the implications. Section 5 summarizes and concludes.

2 The model

2.1 Assumptions

- The bank is a pure maturity intermediary; that is, it borrows short and lends long and in the process, is subject to interest rate risk
- There is no default risk. Although default risk is undoubtedly important for banks, our focus is on the pure effect of interest rate risk[4]
- The bank borrows money from depositors at the (variable) short rate r_t and lends to businesses at the (fixed) long rate L.[5] Both the rates are exogenously determined by the market, and follow random processes described below; the bank is thus a price-taker in both the loan and the deposit markets, as in Pyle (1971) and Jaffee (1986). While this implies perfect competition and lack of market power (which might not be completely true in real life), it is a reasonably good characterization of the way banks behave (Jaffee (1986)).
- It is by now well-established that interest rates generally follow a mean-reverting stochastic process (Campbell & Shiller (1984), Cox, Ingersoll & Ross or CIR (1985), etc). Therefore, we assume two separate (but possibly

[4] In fact, the pure effect of interest rate risk is a starting point in examining the combined effect of interest rate risk and default risk and the interactions between them. Duan, et al (1995), for instance, have derived an expression for interest rate elasticity of bank asset value, ϕ_v, which is a constant, with the objective of relating bank risk to insurer's risk. Our paper, on the other hand, focuses on this one aspect (ϕ_v), and we find that ϕ_v is not necessarily constant; also, it can be positive or negative, large or small, depending on the bank's lending position.

[5] The assumption that all loans are fixed-rate may be unrealistic since most banks do make *some* floating-rate loans today. What is necessary for our results, however, is that some loans be fixed-rate, not necessarily all. Floating-rate loans by their very nature are not subject to significant interest rate risk, hence they will not make a significant contribution to any effects of interest rate changes on bank values. Therefore, any interest rate risk should come via the fixed-rate loans, which is what this paper examines.

correlated) mean-reverting square-root stochastic processes (as in CIR) for the short-term and long-term interest rates,[6] as follows:

$$dr = \kappa(\theta - r)dt + \sigma\sqrt{r}\, dz_1 \tag{1}$$

$$dL = \gamma(\alpha - L)dt + \beta\sqrt{L}\, dz_2 \tag{2}$$

where r and L denote the short and long rates respectively, κ and γ the speeds of adjustment of the short and long rates, θ and α the long-run mean values of the two processes, σ and β the volatilities of the two processes, and z_1 & z_2 are two standard Brownian Motion Processes which are possibly correlated.[7] Let the correlation coefficient be given by ρ, ie

$$dz_1 dz_2 = \rho dt \tag{3}$$

- For ease of exposition, we look at only two maturities, long and short; although in real life there may be many possible maturities
- We ignore operating costs, and the only cash flows in the model are those related to the loan, ie, cash outflow to depositors and cash inflow from borrowers. While operating costs certainly affect the bank's bottom line, they are largely unrelated to interest rate changes and therefore not relevant here.
- When the bank makes a unit loan[8] (a $1 loan), ie, when the option to lend is exercised, it instantaneously raises $1 (by taking in an additional dollar in deposits) and lends the same amount to the borrower. The loan rate will be fixed at L_0 (since the loan is granted at t = 0) till the loan matures. However, the deposit rate r_t will vary continuously as in equation (1).

Our model is cast in a partial equilibrium setting with two exogenously specified state variables r and L. The bank takes these rates as given, and has no control over their determination. In a more complete general equilibrium model, banks would, through competition and optimizing behavior in their loan decisions,

[6] To be more precise, equations (1) and (2) describe the risk-adjusted stochastic processes, or the processes under the risk-neutral martingale measure (Duffie (1992), chapter 7). Therefore, for valuation purposes, all we need to do is to compute the expected present value of cash flows.

[7] This is similar to the two-factor term structure model of Brennan & Schwartz (1979 and 1982) that has two stochastic state variables, the short rate and the long rate (or the "consol" rate).

[8] We are considering the valuation effects on a unit loan. As for the total amount the bank can lend, the upper limit on lending (and borrowing, since loans are financed by borrowings) is set by regulatory constraints such as the capital adequacy requirement. The bank can borrow (at rate r) and lend (at rate L) till the limit is reached. This is similar to Crouhy and Galai's (1991) assumption: "In a competitive market, however, each bank is a rate-taker rather than a rate-setter. Hence, any quantity of deposits can be obtained at the 'right' price."

certainly influence the long-term rate. Our paper, however, models it from the perspective of an individual competitive bank, which by itself has no influence on the interest rate processes, and takes them as exogenously specified.

The deposit contract is essentially a unit deposit, ie, a \$1 deposit that is similar to a floating-rate coupon bond, with the depositor receiving a continuous stream of cash flow $\$r_t$ per unit time. It is like any coupon-bearing bond in continuous-time models, except that the coupon is stochastic. The deposit contract is a short-term one, the underlying assumption being that it matures instantaneously and is continuously rolled over. However, if the depositor wishes to withdraw at any time, the bank can get the \$1 from another depositor. The bank borrows \$1 to finance a unit (or \$1) loan [we ignore equity since most of the operations of banks are financed by deposits]. The dollar amount is just a normalization, and does not represent an optimal strategy by the bank. The optimal strategy comes in choosing when to extend the loans, and the deposit (borrowing) policy follows from that. As stated above, equity funding of short-term shortfalls is not allowed in the model since all future loans will be financed by deposits. Finally, there is no renegotiation of loan rates, which remain fixed till maturity.

2.2 Valuation of a loan-in-place

Let us look at a unit T-year loan. If the bank makes the loan today (at time $t = 0$), then it locks in a fixed cash inflow of $\$L_0$ per unit time from $t = 0$ to $t = T$. To finance this loan, the bank borrows \$1 from depositors, but since the depositors are short-term lenders the deposit has to be continuously rolled over. The resulting (stochastic) cash outflow is $\$r_t$ per unit time from $t = 0$ to $t = T$. The Value of the loan-in-place (EPV) is given by:[9]

$$EPV(L_0, r_0) = E\left[\int_0^T (L_0 - r_t) D_0(t) dt \mid L_0, r_0\right] \tag{4}$$

or

[9] Note that equation (4) is similar to Flannery's (1981, equation 1) representation of the value of a bank loan, with the difference being that we use the stochastic short-term rate as both the discount rate as well as the bank's cost of funds. Thus, we have two state variables (loan and deposit rates) while Flannery's model has three. In a stochastic interest rate environment with negligible default risk, with continuous discounting and compounding, we feel that the appropriate discount rate is the stochastic short-term rate. Our model could be extended to include a discount rate process different from the cost-of-funds process, but this would complicate the computations significantly.

$$EPV(L_0, r_0) = L_0 \int_0^T D_0(t)dt - E\int_0^T r_t D_0(t)dt \tag{5}$$

where $D_0(t)$ = value at time 0 of \$1 to be received at time t (or the current value of a unit discount bond maturing at time t). As shown in Cox, Ingersoll and Ross (1985), $D_0(t)$ is given by

$$D_0(t) = A(t)\, e^{-r_0 B(t)} \tag{6}$$

where

$$A(t) = \left[\frac{2\delta\, e^{(\kappa+\delta)t/2}}{(\kappa+\delta)(e^{\delta t}-1)+2\delta}\right]^{2\kappa\theta/\sigma^2} \tag{7}$$

$$B(t) = \frac{2(e^{\delta t}-1)}{(\kappa+\delta)(e^{\delta t}-1)+2\delta} \tag{8}$$

and

$$\delta = \sqrt{\kappa^2 + 2\sigma^2} \tag{9}$$

The first term on the right hand side of equation (5) denotes the value of a constant cash flow stream of L_0 per unit time from t = 0 to t = T; this has to be evaluated by numerical integration. The second term denotes the value of a continuous payment of r_t per unit time from t = 0 to t = T. We know that a bond paying a continuous coupon of \$$r_t$ per unit time from time t = 0 to t = T, as well as \$1 at t = T, has an expected value of \$1. Thus

$$E\int_0^T r_t D_0(t)\, dt = 1 - \text{Value of \$1 to be received at time } T = 1 - D_0(T) \tag{10}$$

Therefore, the value of making a T-period loan now (at t = 0) is:[10]

$$EPV(L_0, r_0) = L_0 \int_0^T A(t) e^{-r_0 B(t)} dt - 1 + A(T) e^{-r_0 B(T)} \tag{11}$$

More generally, the value of granting a T-period loan at time t is:

$$EPV(L_t, r_t) = L_t \int_t^{t+T} A(s) e^{-r_t B(s)} ds - 1 + A(T) e^{-r_t B(T)} \tag{12}$$

Now consider an existing loan (granted when the long rate was L_{ex}) with remaining time to maturity of T_{rem}. Then the value of the loan to the bank at time t = 0 is:

$$EPV(L_{ex}, r_0) = L_{ex} \int_0^{T_{rem}} A(t) e^{-r_0 B(t)} dt - 1 + A(T_{rem}) e^{-r_0 B(T_{rem})} \tag{13}$$

2.3 Valuation of loan-in-process

We have seen that a loan-in-process is equivalent to an option to grant the loan (ie, a call option on a bond), hence it can be valued as an American option to buy a bond. The option value will depend on the two state variables L_t and r_t, as well as the calendar time t; let us therefore denote it V(L,r,t). As shown in Appendix 1, V(L,r,t) must satisfy the following partial differential equation:

$$V_t + \frac{1}{2}\beta^2 L V_{LL} + \frac{1}{2}\sigma^2 r V_{rr} + \rho\sigma\beta\sqrt{Lr} V_{Lr} + \gamma(\alpha - L) V_L + \kappa(\theta - r) V_r - rV = L - r \tag{14}$$

[10] Note, however, that the bank's NPV from the deposit contract itself is zero. The depositor pays the bank \$1 at time t = 0, the deposit matures instantaneously and is continuously rolled over till time T, at which time the \$1 is returned to the depositor. During the period from t = 0 to t = T, the depositor receives the stochastic coupon stream $\$r_t$ per unit time. Thus the cash flows to the bank are: +\$1 at time t = 0, $-\$r_t$ per unit time from t = 0 to t = T, and –\$1 at time t = T. When the deposit is made at time t = 0, the bank's NPV for the deposit contract is given by: $NPV = +1 - E_0 \int_0^T r_t D_0(t) dt - D_0(T)$. From equation (10), we can see that $E_0 \int_0^T r_t D_0(t) dt = 1 - D_0(T)$. Thus the bank's NPV with respect to the deposit contract is given by $1-[1-D_0(T)]-D_0(T) = 0$. We thank an anonymous referee for clarifying this point.

where subscripts denote partial derivatives. In addition, a number of other conditions must be satisfied by V(L,r,t), as discussed below.

Let the option to grant the loan expire at time T^* (the time to expiration is discussed in more detail in Section 3). At expiry, the option should optimally be exercised if it is in the money, giving the terminal condition:

$$V(L,r,T^*) = \text{Max}\{EPV(L,r),0\}, \quad \forall\,(L,r) \tag{15}$$

When r is very low (approaches zero), the option will be exercised for sure, if in the money. This gives the lower boundary condition:

$$V(L,0,t) = \text{Max}\{EPV(L,0),0\}, \quad \forall\,(L,t) \tag{16}$$

On the other hand, when r approaches infinity, it is unlikely that the loan will be granted at all, hence the option value approaches zero:

$$\lim_{r\to\infty} V(L,r,t) = 0, \quad \forall\,(L,t) \tag{17}$$

Similarly, when L approaches zero, the option value will also approach zero:

$$V(0,r,t) = 0, \quad \forall\,(r,t) \tag{18}$$

When L approaches infinity, the option will be exercised for sure, if it is in the money:

$$\lim_{L\to\infty} V(L,r,t) = \text{Max}[EPV(L,r),0], \quad \forall\,(r,t) \tag{19}$$

Next consider the optimal exercise of the American option to grant a loan. From equation (12), it is clear that the option will be exercised when L is high enough and/or r is low enough. With two state variables, the optimal exercise policy is given by a free boundary (Brock & Rothschild (1986)) characterized by a function $L_t^*(r)$ such that, for a given r at time t, the option should be exercised as soon as the value of L equals or exceeds this critical value $L_t^*(r)$. Hence, the decision rule is: exercise when $L_t \geq L_t^*(r)$.[11] This optimal exercise condition is ensured by the smooth-pasting condition (Brock & Rothschild (1986)):

[11] The boundary could equivalently be expressed as $L_t^*(r)$, such that exercise should occur as soon as $L_t \leq L_t^*(r)$.

$$\frac{\partial V}{\partial r}(L^*(r), r, t) = \frac{d}{dr}[EPV(L^*(r), r)], \quad \forall\,(r, t) \tag{20}$$

or

$$\frac{\partial V}{\partial r}(L^*(r), r, t) = -L^*(r)\int_0^T B(t)A(t)e^{-rB(t)}dt - B(T)A(T)e^{-rB(T)}, \quad \forall\,(r, t) \tag{21}$$

where $L_t^*(r)$ = critical loan rate for option exercise at time t for a given deposit rate r.

We can compute the option value V(L,r,t) and the current critical lending rate $L_t^*(r)$ [henceforth written $L^*(r)$] by solving the system of equations (14)–(21). Since there is no analytical solution for this system of equations, it has to be solved numerically. Some numerical results are presented next.

3 Numerical analysis

3.1 Loans-in-place

Differentiating equation (13) with respect to the lending and deposit rates:

$$\frac{\partial EPV}{\partial L_{ex}} = \int_0^{T_{rem}} A(t)e^{-r_0 B(t)}dt > 0 \tag{22}$$

$$\frac{\partial EPV}{\partial r_0} = -L_{ex}\int_0^{T_{rem}} B(t)A(t)e^{-r_0 B(t)}dt - B(T_{rem})A(T_{rem})e^{-r_0 B(T_{rem})} < 0 \tag{23}$$

This gives

Result 1. *The value of a loan-in-place is an increasing function of the loan rate and a decreasing function of the deposit rate.*

This, of course, is not a surprising result. It is intuitively obvious that a higher loan rate (with deposit rate held constant) will make the loan more attractive, whereas a higher deposit rate (with loan rate held constant) will make the loan less attractive to the bank. Although this is not explicitly stated as a theoretical result in the existing literature, it is implicitly taken for granted, eg, Hancock (1985). In any event, we have included this result for completeness.

The exact magnitudes of these effects will of course depend on the parameter values (κ,θ,σ) used for the computations. The CIR process parameters have been estimated by various researchers, eg, Sun & Pearson (1989 and 1994), Gibbons & Ramaswamy (1993), etc. There is a wide variation in the different sets of estimates. Sun & Pearson (1994), for instance, get the following estimates for the period 1972–1986: $\kappa = 7.4525$, $\theta = 0.0264$, $\sigma = 0.0197$, while Gibbons & Ramaswamy (1993) get the following estimates: $\kappa = 12.43$, $\theta = 0.0154$, $\sigma = 0.49$. Moreover, there seems to be a problem with unit roots leading to an over-estimate of the speed of reversion parameter κ (Ball & Torous (1996)). Therefore, rather than examine the arguments for or against the various sets of parameter estimates, we decided to start with the following "plausible" parameter values as the base case (from Ball & Torous (1996)) for the deposit rate: $\kappa = 1$, $\theta = 0.05$, and $\sigma = 0.05$.[12]

The results with these parameter values are displayed in Table 1. With deposit and loan rates of r = 5% and L = 10% respectively, the value of a loan with 1 year left to maturity is $0.04877; this value increases with loan maturity, to $0.58201 for a loan with 20 years left.

Since five years is a reasonable term for a bank loan, we look at the results for a five-year loan to get some idea of the effect of interest rate changes. With r = 5% and L = 10%, the value of a unit 5-year loan is $0.22025. If the deposit rate were to fall by 1% to 4%, the loan value would increase to $0.23208 (a 5.37% increase). If, on the other hand, the loan rate L were to increase by 1% to 11%, the loan value would increase to $0.26438 (a 20.04% increase). We get similar results for other loan terms and other sets of parameter values. Our simple simulation results demonstrate.

Result 2. *A loan-in-place is generally more sensitive to changes in the loan rate than in the deposit rate.*

When the loan rate increases (deposit rate remaining unchanged) the only effect is to increase cash inflows, hence the loan value goes up. When the deposit rate r increases, the cash outflows increase but are discounted harder, which results in a smaller drop in loan value. Hence the loan value is less sensitive to r than to L. Again, this result has not been stated formally in the literature, but Hancock (1985) has presented empirical evidence consistent with this result. She has shown that bank profits rise more from a 1% increase in the loan rate than a 1% decrease in the deposit rate (33.66% and 26.46% respectively).

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[12] Although the parameter estimates would be slightly different under the risk-neutral measure, the results are robust to exact values of parameters used, since we get similar results with a wide range of parameter values.

Table 1. **Valuation of unit loan-in-place**

Part A of the table describes the impact of changes in deposit rate (r), loan rate (L), on the value of a loan-in-place on different loan terms (T). It shows the value (to the bank) of a T-year loan-in-place of $1. Both the deposit rate and the loan rate have flat term structures. Part B shows the valuation effect of the deposit rate volatility and Part C shows the effect of changes in speed reversion on loan values.

A. Effect of changes in short and long rates
(Base case parameter values: $\kappa = 1$, $\theta = 0.05$, $\sigma = 0.05$)

			Value of loan-in-place			
	r(%)	L(%)	T = 1 yr	T = 5 yrs	T = 10 yrs	T = 20 yrs
Base case	5	10	0.04877	0.22025	0.38582	0.58201
Change in	4	10	0.05519	0.23208	0.39968	0.59789
Short rate **r**	6	10	0.04240	0.20853	0.37210	0.56629
Change in	5	9	0.03902	0.17612	0.30793	0.46064
Long rate **L**	5	11	0.05852	0.26438	0.46372	0.70338

Note: Loan value (i) increases with loan maturity (ii) increases with loan rate L (iii) decreases with deposit rate r (iv) is more sensitive to changes in L than to changes in r.

B. Effect of changes in short rate volatility σ
(r = 5% and L = 10%; T = 5 years, and other parameter values same as above)

σ	Value of loan-in-place
2.5%	0.22010
5.0%	0.22025
7.5%	0.22050

Note: Loan value is an increasing function of volatility.

C. Effect of changes in speed of reversion κ
(r = 5% and L = 10%; T = 5 years, and other parameter values same as above)

κ	Value of loan-in-place
0.5	0.22056
1.0	0.22025
1.5	0.22015

Note: Loan value is a decreasing function of speed of reversion.

Part B of Table 1 shows the valuation effect of the deposit rate volatility σ. As σ increases (keeping both the rates unchanged), the loan value increases. This can be explained intuitively as follows: in good periods (ie, when r is low), bank profits (L – r) are higher since L is fixed and r is stochastic; these profits are discounted at a lower rate (since r is low) hence the present value is high. During bad periods (high r), the profit will be lower but will be discounted harder (since r is high). Therefore the bank will be better off (in present value terms, which determine "loan value") if r fluctuates more.

Unfortunately, an increase in interest rate volatility is almost always accompanied by an increase in the interest rate level, hence it is difficult to test the pure effect of volatility. When both the level and volatility increase, they have opposing effects on the bank profit, so the net effect might not be significant. In fact, Flannery (1981 & 1983) reports that interest rate volatility has very little effect on bank profits.

Part C of Table 1 shows that the loan value is a decreasing function of the speed of reversion κ. Increasing κ reduces the conditional volatility of the process (since r will revert back faster to the mean), hence it has the same effect as reducing the volatility σ, as a result of which the loan value falls.

3.2 Loans-in-process

A loan-in-process is equivalent to an American call option to grant the loan, and its value can be computed by solving the system of equations (14)–(21). As no analytical solution is available, we solved the system numerically using the explicit method of Schwartz (1977) to discretize the model. We also used the transformation of variables suggested by Hull & White (1990) to ensure convergence and stability of the solution, ie, r and L replaced by variables x and y respectively, where $x = 2\sqrt{r}$ and $x = 2\sqrt{L}$.

The parameters for the deposit rate process are unchanged from the previous section, ie, $\kappa = 1$, $\theta = 0.05$, and $\sigma = 0.05$. For the loan rate process, we use the same value for the speed of adjustment, $\gamma = 1$; since the loan rate is less volatile than the deposit rate, we set $\beta = 0.04$; also, we set $\alpha = 0.10$ since the loan rate, on average, is significantly higher than the deposit rate. Finally, we set the correlation $\rho = 0.95$, approximately equal to the measured correlation between the 3-month CD rate and the prime rate for the period 1980–1996 (Madura (1997)). The term of the loan (T) is 5 years. The time to expiration of the option to grant loan (T^*) was set equal to 1 month.[13] For the base case, we set both deposit and loan rates at their long-term mean levels (θ and α respectively), r = 5% and L = 10%. This means that both rates have a flat term structure. We later repeat all numerical exercises with upward-sloping ($r < \theta$, $L < \alpha$) and downward-sloping ($r > \theta$, $L > \alpha$) term structures.

With these parameter values, we get V(10%, 5%) = $0.23880; that is, the option to issue a $1 five-year loan when L = 10% and r = 5% is worth $0.23880, compared to $0.22025 if the loan is granted immediately. We can see that there is a small premium (8.42%) over the loan value; this is an outcome of the stochastic nature of deposit and loan rates. It also implies that, with these particular parameter values, the bank needs a spread somewhat larger than the current 5% in order to grant the loan.

Part A of Table 2 summarizes the effect on loans-in-process of changing the loan and deposit rates, L and r, one at a time. Not surprisingly, the value of the option to lend rises when L increases and falls when r increases. Further, it is more sensitive to changes in L than in r, consistent with Hancock's (1985) finding. The results are summarized below.

Result 3. *The value of a loan-in-process is an increasing function of L and a decreasing function of r, and is more sensitive to changes in L than in r.*

This is a new result since nobody has looked at a loan-in-process as an option yet. The intuition behind the result is as follows. The loan-in-process is an option to lend (or an option to buy a bond). When L increases, the option is more in-the-money or less out-of-the-money, hence the call option value rises. When r increases, the option becomes less in-the-money or more out-of-the-money, and option value falls. However, r is also the discount rate, and we know that a call option value is an increasing function of the discount rate. Hence the option value does not fall as far. As a result, option value is more sensitive to changes in L than in r.

Part B of Table 2 summarizes the effect on loan-in-process of changing both L and r by a similar margin, ie, the effect of a spread-preserving shift in interest rates. If both long and short rates rise by the same margin, the value of the option to lend rises. This is to be expected from Result 3 since the value is more sensitive to L than r, and is summarized below.

[13] Informal discussions with bank personnel suggest that the option to lend is considered a short-term option; one month seems a reasonable "time window" for making a loan decision. However, the results are not qualitatively sensitive to the value of T^*. The only problem with using a longer T^* (say 1 year) is that the value of waiting constitutes most of the value of the loan-in-process, which does not seem very realistic.

Table 2. **Valuation of unit loan-in-process (option to grant loan)**

Part A of the table portrays the effect on loan-in-process due to changes in deposit rate (r), loan rate (L), on the value of a loan-in-place on different loan terms (T). It shows the value (to the bank) of the option to grant a unit ($1) 5-year loan. Part B shows the effect on loan-in-process by changing L and r by a similar margin.

A. Effect of changes in deposit and loan rates

Base case parameter values: $\kappa = 1$, $\theta = 0.05$, $\sigma = 0.05$, $\gamma = 1$, $\alpha = 0.10$, $\beta = 0.04$ and $\rho = 0.95$; Loan term T = 5 years, Option matures in $T^* = 1$ month.

	r(%)	L(%)	Value of loan-in-process	Change
Change in **r**	4	10	0.24891	+4.23%
	5	10	0.23880	Base case
	6	10	0.22906	–4.08%
Change in **L**	5	9	0.21339	–10.64%
	5	10	0.23880	Base case
	5	11	0.26621	+11.48%

Note: The value of loan-in-process increases with L and decreases with r, and is more sensitive to changes in L than to changes in r.

B. Effect of spread-preserving changes in r and L

Parameter values same as above

r(%)	L(%)	Value of loan-in-process
3	8	0.20422
4	9	0.22284
5	10	0.23880
6	11	0.25373
7	12	0.28347

Note: If the spread between L and r is maintained, rising interest rates result in higher values of loans-in-place.

Result 4. *The value of a loan-in-process increases (decreases) when both r and L rise (fall) while maintaining the same spread between the two rates.*

Thus the effect of a spread-preserving change in loan and deposit rates on a loan-in-process is exactly the opposite of than on a loan-in-place. Also, although Result 4 seems to imply that loans-in-process are worth more in a higher interest rate environment, any such conclusion must be treated with caution. What it says is that banks are better off only if both deposit and loan rates rise by the same amount, leaving the spread unchanged. If the two rates do not rise by equal amounts, the net result will depend on the actual levels by which the loan and deposit rates rise.

3.3 Comparative static results

In order to establish the robustness of the model's results and also to derive comparative static results, we repeated the numerical simulations with a wide range of parameter values. The comparative static results are illustrated in Figures 1–5 for flat term structure (r = 5%, L = 10%), for upward-sloping term structure (r = %, L = 9%), and for downward-sloping term structure (r = 6%, L = 11%).

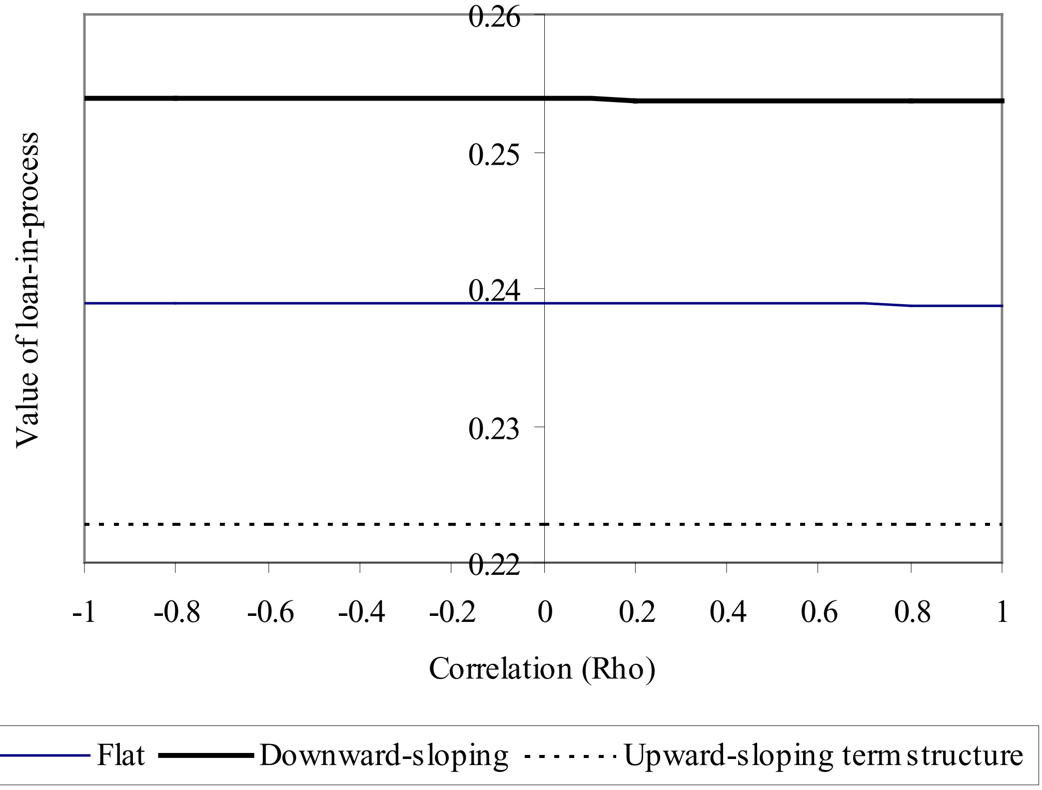
Effect of Correlation between deposit and loan rates (ρ). As Figure 1 shows, the value of the loan-in-process is a slightly decreasing function of the correlation ρ for all three term structure shapes. This is consistent with basic economic intuition: a higher correlation means that r and L will move more closely in the future, thus reducing the value of waiting and the option value.

Effect of Deposit and Loan Rate Volatilities (σ and β). As shown in Figures 2 and 3, the value of the loan-in-process is a decreasing function of the deposit rate volatility (β) and an increasing function of the loan rate volatility (σ) for all three term structures. The intuition is as follows. The value of the option to lend comes from the spread between lending and deposit rates, (L – r). Thus, from basic option theory, the option value should be an increasing function of the volatility of (L – r). Since L and r are so highly positively correlated, the standard deviation of (L – r) should be increasing in β (standard deviation of L) and decreasing in σ (standard deviation of r). Therefore, the option value is increasing in β and decreasing in σ.

Effect of Speeds of Reversion (κ and γ). The value of a loan-in-process is an increasing function of κ for flat and downward-sloping terms structures, but initially decreasing and then increasing in κ for upward-sloping term structures (Figure 4). Also, it is an increasing function of γ for flat and upward-sloping term structures and a decreasing function of γ for downward-sloping term structures (Figure 5).

Figures 1. **Comparative static results for different term structure.**

Figure 1 shows the value of a unit loan-in-process as a function of the correlation between deposit and lending rates, ρ (rho). The following base case parameters were used: $\kappa = 1$, $\theta = 0.05$, $\sigma = 0.05$, $\gamma = 1$, $\beta = 0.04$, $\alpha = 0.10$, and a loan term of 5 years. Three term structures are considered: flat (r = 5%, L = 10%), downward sloping (r = 6%, L = 11%), and upward sloping (r = 4%, L = 9%). In all three cases, the value of a loan-in-process is a slightly decreasing function of ρ.



r

Figure 2. **Comparative Static results for different term structure**

Figure 2 reports the value of a unit loan-in-process as a function of the volatility of the deposit rate, σ (sigma). The following base case parameters were used: κ = 1, θ = 0.05, γ = 1, β = 0.04, α = 0.10, ρ = 0.95, and a loan term of 5 years. Three term structures are considered: flat (r = 5%, L = 10%), downward sloping (r = 6%, L = 11%), and upward sloping (r = 4%, L = 9%). In all three cases, the value of a loan-in-process is a decreasing function of σ.

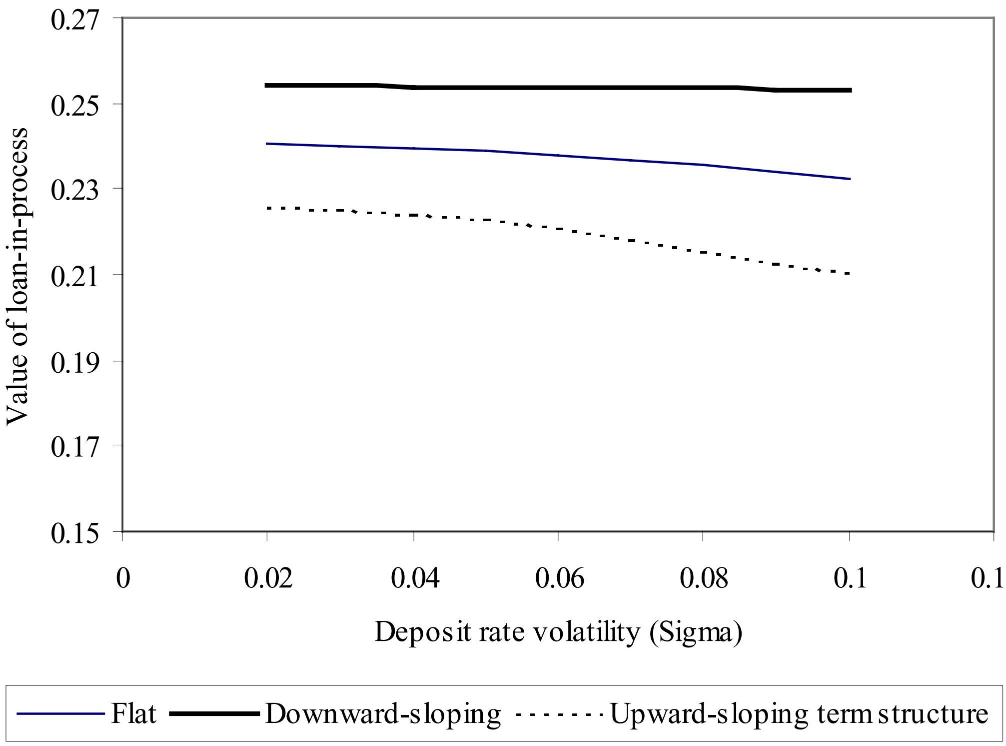


Figure 3. **Comparative Static results for different term structure**

Figure 3 reports the value of a unit loan-in-process as a function of the volatility of the lending rate, β (beta). The following base case parameters were used: $\kappa = 1$, $\theta = 0.05$, $\sigma = 0.05$, $\gamma = 1$, $\alpha = 0.10$, $\rho = 0.95$, and a loan term of 5 years. Three term structures are considered: flat (r = 5%, L = 10%), downward sloping (r = 6%, L = 11%), and upward sloping (r = 4%, L = 9%). In all three cases, the value of a loan-in-process is an increasing function of β.

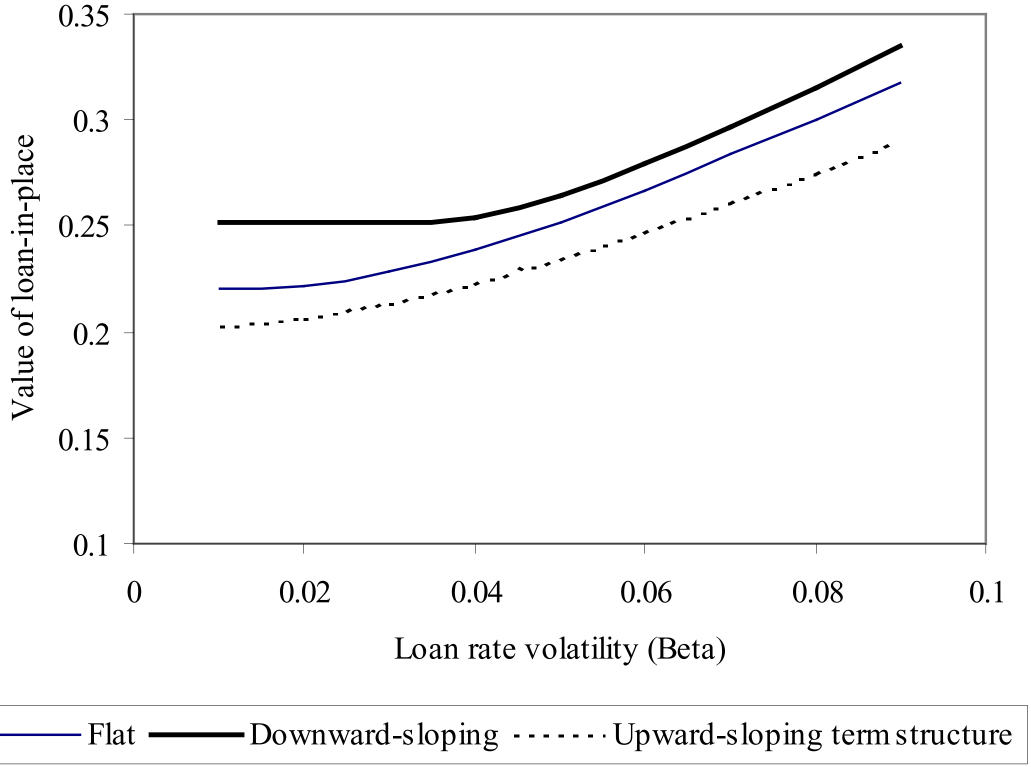


Figure 4. **Comparative Static results for different term structure**

Figure 4 reports the value of a unit loan-in-process as a function of the speed of reversion of the deposit rate, κ (kappa). The following base case parameters were used: $\theta = 0.05$, $\sigma = 0.05$, $\gamma = 1$, $\alpha = 0.10$, $\beta = 0.04$, $\rho = 0.95$, and a loan term of 5 years. Three term structures are considered: flat (r = 5%, L = 10%), downward sloping (r = 6%, L = 11%), and upward sloping (r = 4%, L = 9%). The effect of κ on the value of a loan-in-process is ambiguous, and depends on the shape of the term structure.

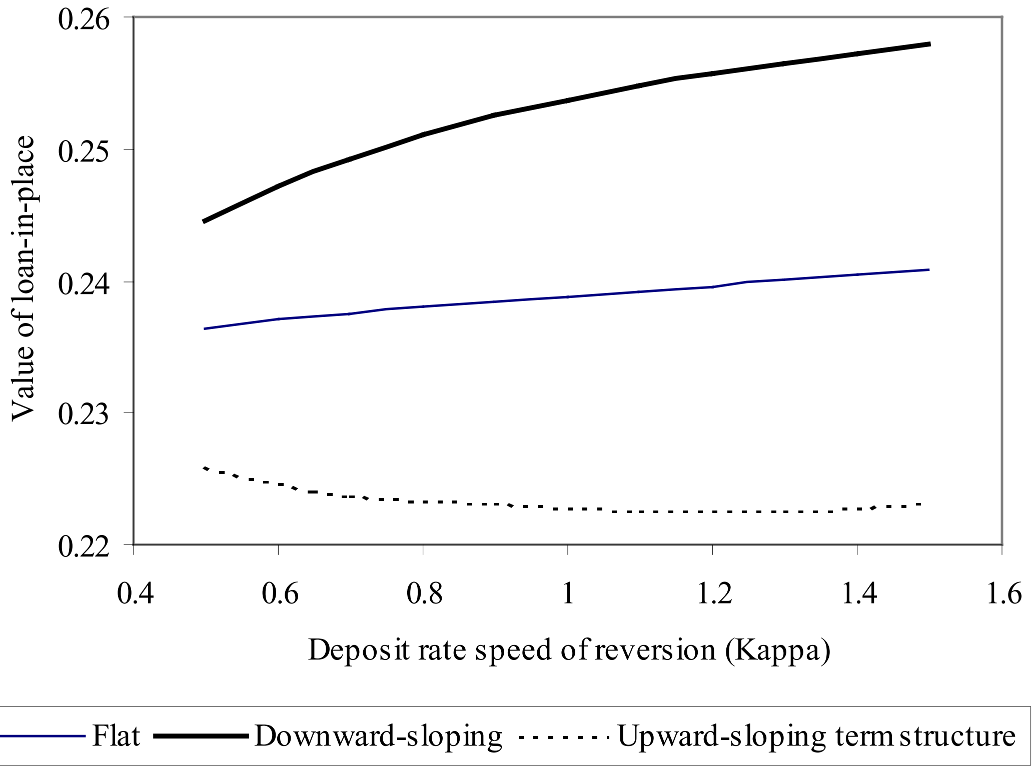
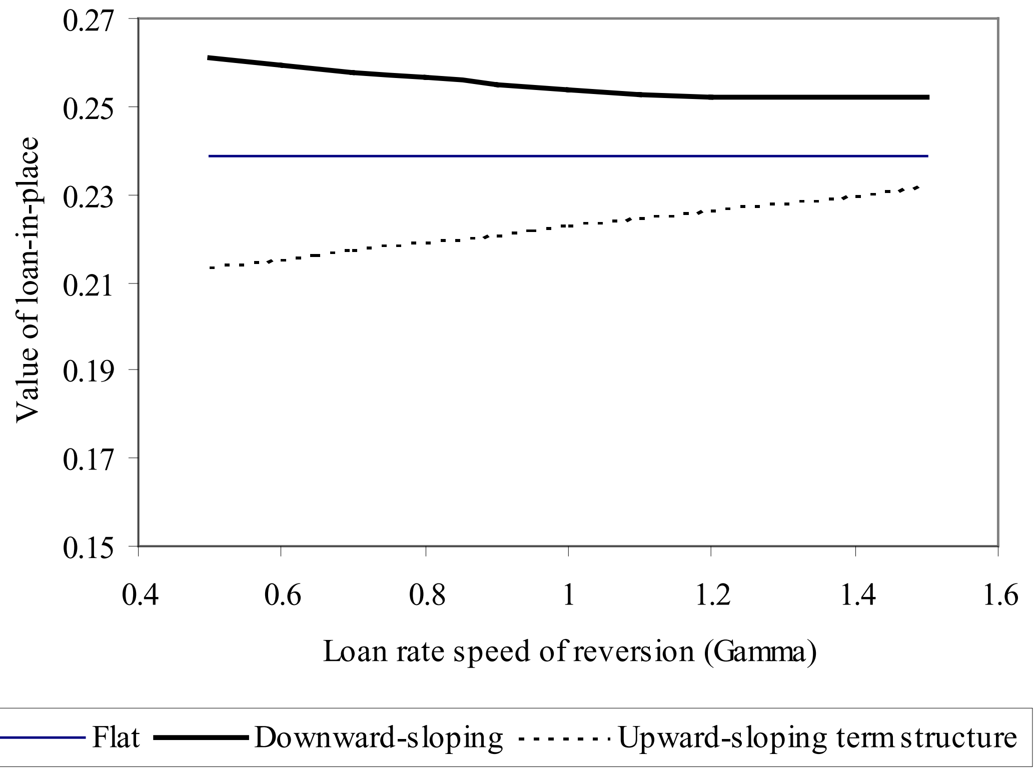


Figure 5. **Comparative Static results for different term structure**

Figure 5 reports the value of a unit loan-in-process as a function of the speed of reversion of the lending rate, γ (gamma). The following base case parameters were used: $\kappa = 1$, $\theta = 0.05$, $\sigma = 0.05$, $\alpha = 0.10$, $\beta = 0.04$, $\rho = 0.95$, and a loan term of 5 years. Three term structures are considered: flat ($r = 5\%$, $L = 10\%$), downward sloping ($r = 6\%$, $L = 11\%$), and upward sloping ($r = 4\%$, $L = 9\%$). The effect of γ depends on the shape of the term structure.



4 The main results

4.1 Overall effects of interest rate changes: theoretical results

We have seen how interest rate changes affect loans-in-place and loans-in-process separately. When interest (both loan and deposit) rates rise by the same amount, the value of a loan-in-place falls while that of a loan-in-process rises. Clearly, the overall effect of higher rates will depend on the ratio of loans-in-place and loans-in-process, since the two effects are in opposite directions. The smaller the bank's lending slack (or the closer the bank to its lending limit), the smaller the influence of loans-in-process. Therefore, such banks should be hurt by a spread-preserving increase in interest rates. As the bank moves away from its lending limit, loans-in-process become relatively more important, and the bank will be less negatively affected by a spread-preserving increase in interest rates. Therefore, banks with large lending slack should be less sensitive to interest rate changes.

The ratio of loans-in-place to loans-in-process is thus an important factor in determining the overall interest rate risk. This ratio is related to the bank's capital base by regulatory restrictions. Because of capital adequacy requirements, banks are able to increase the size of their loan portfolios only up to a certain limit, for a given level of equity.[14] This sets an implicit upper limit on the amount a bank can lend. If it lends more than this critical upper limit, it will be labeled "under-capitalized" and subject to disciplinary measures ("prompt corrective action", Saunders (1994), p. 324) by regulating bodies.

We present below some numerical illustrations to demonstrate quantitatively how interest rate changes might affect the overall value of a bank, and how this effect is related to the bank's lending slack. As a starting point, let us assume that the bank can make loans totaling \$100; this ceiling results from the capital adequacy requirement mentioned above. If it already has \$100 of loans-in-place, it can make no more loans (ie, has no lending slack), hence all the value comes from loans-in-place. If it has loans-in-place of less than \$100, it can potentially make more loans, hence a part of its value comes from loans-in-process. Table 3 shows the results for various scenarios, using the base case parameter values of Section 3.

[14] The capital adequacy requirement generally looks like the following: Capital/Assets ≥ some critical percentage, where the numerator and denominator can be defined more specifically depending on the context, eg, Capital/Assets ratio (Tier 1) or Total-Risk-based Capital Ratio. The equity goes into the numerator, and the value of the loan portfolio in the denominator. It is clear that, with a given level of equity, there is an upper limit on the amount of loans that can be made.

Table 3. **Net effect of interest rate changes**

This table shows the effects of spread-preserving changes in interest rates on the overall value of a bank, for different weights of loans-in-place and loans-in-process. Deposit rate is (r), loan rate is (L), loan term is (T). The loan-in-place capacity ranges from $100–$50. Parameter values: $\kappa = 1$, $\theta = 0.05$, $\sigma = 0.05$, $\gamma = 1$, $\alpha = 0.10$, $\beta = 0.04$ and $\rho = 0.95$; Loan term T = 5 years, Option matures in $T^* = 1$ month.

1. Loans-in-place $100, loans-in-process $0 (at lending limit)

Rate movement (%)	r(%)	L(%)	Total value ($)	Change in value (%)
Base case	5	10	22.025	
Up	6	11	20.853	–5.32
Down	4	9	23.208	+5.37

2. Loans-in-place $90, loans-in-process $10

Rate movement (%)	r(%)	L(%)	Total value ($)	Change in value (%)
Base case	5	10	22.211	
Up	6	11	21.305	–4.08
Down	4	9	23.116	+4.07

3. Loans-in-place $80, loans-in-process $20

Rate movement (%)	r(%)	L(%)	Total value ($)	Change in value (%)
Base case	5	10	22.396	
Up	6	11	21.757	–2.85
Down	4	9	23.023	+2.80

4. Loans-in-place $70, loans-in-process $30

Rate movement (%)	r(%)	L(%)	Total value ($)	Change in value (%)
Base case	5	10	22.582	
Up	6	11	22.209	–1.65
Down	4	9	22.931	+1.55

5. Loans-in-place $50, loans-in-process $50

Rate movement (%)	r(%)	L(%)	Total value ($)	Change in value (%)
Base case	5	10	22.953	
Up	6	11	23.113	+0.70
Down	4	9	22.746	–0.90

Note: Sensitivity of total bank value to interest rate changes is found to decrease as the bank moves away from its lending limit. Indeed, if the bank is very far from the lending limit (eg, 50% slack), higher interest rates will result in higher overall bank value.

Looking first at a bank that has no lending slack (in this case, \$100 in loans-in-place and \$0 in loans-in-process), we see that (i) it is very sensitive to interest rate changes, and (ii) it is hurt by an increase and helped by a decrease in interest rates. An increase of 1% in both the deposit and lending rates will reduce value by 5.32% whereas a 1% fall in rates will increase value by 5.37%.

If the bank had instead utilized 90% of its lending capacity (had a 10% lending slack; that is, \$90 of loans-in-place and \$10 of loans-in-process),[15] its interest rate sensitivity would decline substantially. In this case, a 1% increase (decrease) in both deposit and lending rates would reduce (increase) bank value by 4.08% (4.07%). As the lending slack is increased, the bank becomes less sensitive to interest rate changes; interest rate sensitivity falls to 1.65% (1.55%) when the slack is 30%. Indeed, with a lending slack of 50%, the interest rate sensitivity is reversed; for a 1% increase (decrease) in both rates, the bank value actually rises (falls) by 0.70% (0.90%). These numbers illustrate the main argument of this article, which is summarized in

Result 5. *The net effect of interest rate changes on a bank's value depends on its lending slack. If the slack is very small, the bank will be very sensitive (and negatively affected when interest rates rise). As the slack is increased, the effect becomes weaker; and for sufficiently large slack, the effect of higher interest rates can even be positive.*

This follows from Results 1 and 4, and is the most important result of the paper. It relates, for the first time, a bank's interest rate sensitivity to its lending position and how close it is to its lending limit. If a bank has sufficient lending slack, it could behave as if its asset and liability maturities have been matched. This suggests a possible explanation for Flannery & James's (1984b) finding that the "effective maturity" of demand, savings and small time deposits was longer than their very short contractual maturity.

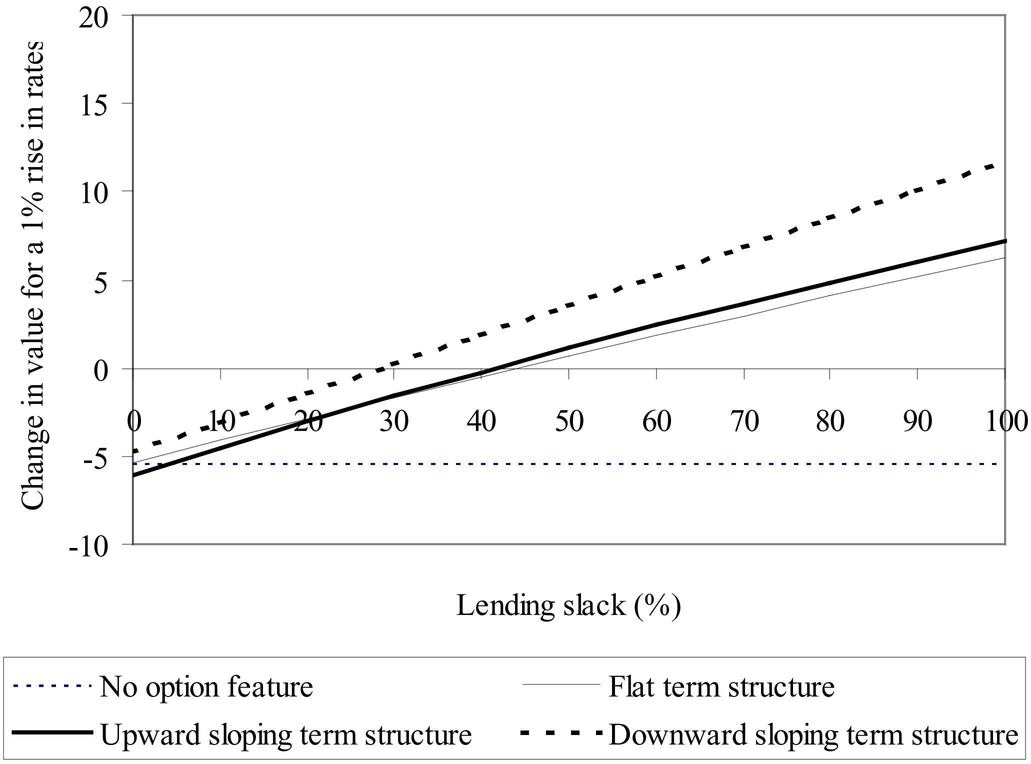
The effect of lending slack on a bank's interest rate risk is illustrated in Figure 6 for the base case parameter values, under four scenarios: (i) without the option feature (ie, there is no option to lend), (ii) for a flat term structure ($r = 5\%$, $L = 10\%$), (iii) for an upward-sloping term structure ($r = 4\%$, $L = 9\%$), and (iv) for a downward-sloping term structure ($r = 6\%$, $L = 11\%$). It is clear that, in the absence of an option-like feature in the bank's loan decisions, the lending slack will have no impact on interest rate risk. Since the existing banking literature has not looked at the option aspect, this link between interest rate risk and lending

[15] Most banks would probably not have too much excess capital, since it is expensive. However, as loans mature over time, it provides banks with significant slack in their lending capacity. If conditions are not favorable, a maturing loan might be replaced with an investment in short-term government securities. Thus, it is common for banks to have some unutilized lending capacity.

slack has been ignored to date. With the option feature, however, the bank's interest rate risk is decreasing in its lending slack for all three term structures. For a downward-sloping term structure, a lending slack slightly below 30% virtually eliminates all interest rate risk, and for the other cases a lending slack of 40–45% eliminates interest rate risk.

Figure 6.

Figure 6 describes the effect of lending slack on a bank's interest rate risk for the base parameters under different scenarios. It traces the change in bank value resulting from a 1% rise in interest rates, as a function of the bank's lending slack. Lending slack is defined as [1 – proportion of lending capacity in loans-in-place] · 100%. The following base case parameter values were used: $\kappa = 1$, $\theta = 0.05$, $\sigma = 0.05$, $\gamma = 1$, $\alpha = 0.10$, $\beta = 0.04$, $\rho = 0.95$, and a loan term of 5 years. Four scenarios are illustrated: (i) bank has no option to lend and term structure is flat, (ii) bank has option to lend and term structure is flat, (iii) bank has option to lend and term structure is upward-sloping, and (iv) bank has option to lend and term structure is downward-sloping. Note that the degree of lending slack makes no difference in the absence of an option to lend. In the other three scenarios, the change in bank value is an increasing function of lending slack. Higher interest rates hurt a bank with small lending slack but help a bank with large enough lending slack.



We note at this point that we made some simplifying assumptions when developing the model. It might be of interest to ask how the results might change if some of the assumptions are relaxed. One such assumption was that deposits are rolled over, and if there is a withdrawal it is replaced right away because there is a large depositor pool. But if there should be stochastic withdrawals that are not perfectly replaceable, the bank cannot ask for loan repayment without incurring significant penalty costs. What the bank can do is to pay off the depositor from its holdings of T-bills and other liquid security holdings (part of its lending slack). Thus the cost of the loan would no longer be the deposit rate but rather the opportunity cost of the liquid investments (which would be close, since both are short-term rates). Then the valuation of loans-in-place or loans-in-process would not change much. However, the bank will not be able to use its entire lending slack to exercise options to lend, because of the uncertainty regarding deposit withdrawal and replacement. Thus the effective lending slack will now be smaller, as a result of which the positive effect of loans-in-process will be diluted somewhat. This will weaken Result 5 of the paper. Also, the bank could, in principle, issue equity to cover the deposit withdrawals, but in practice it would be very difficult (because of the time requirements) and prohibitively expensive (high cost of equity and large issue costs).

Also, while we have discussed banks' lending capacity, we have not explicitly modeled the lending capacity or its determinants. For instance, if there is a loss of charter value upon default (arising from excess lending capacity), it will reduce the bank's lending capacity and slack. Although the valuation of loans-in-place and loans-in-process will not be different, Result 5 will be weakened because the importance of loans-in-process will be diluted, and the bank's overall interest rate risk will now be somewhat higher. Incorporating a formal capital requirement would introduce additional complexity to the model. To illustrate, if we specify a capital requirement of 5%, the bank's lending capacity will be 20E, where E is the equity value; and the lending slack will be (1 – loans/20E). When interest rates increase, equity value E generally falls, unless the slack is very large (as we have shown). If E falls, the slack also falls, from the above expression; and this will weaken Result 5 since loans-in-process will become less important. However, if the slack is large enough, E will rise and the slack will rise even more, thus strengthening Result 5. The effects will be exactly the opposite when interest rates fall. Therefore, including a capital requirement might weaken or strengthen our main result, depending on the situation.

4.2 Empirical implications and evidence

Result 5 and the accompanying discussion generates the following

Empirical Implication: *Banks with considerable lending slack (banks far from their lending limits imposed by capital requirements) will have lower levels of interest rate risk than banks with little lending slack.*

The importance of this empirical hypothesis comes from the fact that in the existing banking literature there has been no effort to separate low-slack and high-slack banks when testing for interest rate risk. This might have contributed to the absence of any conclusive findings (discussed in Section 1). As discussed below, empirical evidence indicates that low-slack banks are indeed significantly more (negatively) affected by higher interest rates than are high-slack banks.

Following the models used by U.S. saving and loans (S&L) and bank regulators in calculating interest rate risk (IRR),[16] we investigate the valuation impact of interest rate increase on different asset-liability and off-balance sheet items of approximately 1400 S&L institutions during the fourth quarters of 1993 and 1994 (1993Q4 and 1994Q4 respectively). Additionally, we analyze the sample data by groups based on their respective lending positions compared to their total potential or lending capacity (ie, based on their lending slack).

Descriptive statistics of key balance sheet variables (Appendix 3) reveal that the sample financial institutions held around 80% of their assets (on average) in loans-in-place, keeping about 20% of their capacity in loan-in-process.[17] Table 4 shows the impact of a 200-basis-point increase in interest rate on potential NPV changes in total assets, total liabilities, and total off-balance-sheet activities of sample institutions.

[16] For details on the estimation techniques and procedure followed here, see Appendix 2.

[17] In general, our separate estimates for 1993Q4 and 199Q4 provide evidence very similar to pooled estimates from the combined sample. The minor differences between the two are likely to be due to the changes in the thrift portfolio as well as changes in interest rates. Similar evidence is also obtained if we use the BA approach. Under the BA approach, assets were categorized as amortizing assets, non-amortizing assets, and deep-discounting assets. The BA estimation results are not reported here but are available upon request.

Table 4. **Change in NPV for a 200 basis point increase in interest rates ($ billions)**

The table describes the impact of interest rate change (a 200 basis point increase in interest rate in this case) on the change of Net Present Value of different asset, liability and off-balance sheet activities undertaken by the sample banking institutions. Asset categories include adjustable rate and fixed rate mortgages, Mortgage servicing assets and mortgage derivatives (IOs, POs, CMOs), other mortgages, non-mortgage loans such as consumer and commercial loans. Liability item categories include retail or core deposits consists of savings and demand deposits; time deposits; and borrowings and regulatory advances. All relevant and available off-balance sheet activities are combined into one category termed off-balance sheet activities.

Asset/liability category	1993:Q4 (n=1388)	1994:Q4 (n=1408)
Adjustable rate mortgages	–5.73	–4.98
Fixed-rate mortgages	–12.32	–11.21
IOs, POs, CMOs, REMIC tranches etc.	–1.40	–2.02
Other mortgages and second mortgages	–3.46	–4.05
Consumer and commercial loans	–1.53	–1.98
Investment securities, equity securities, zero-coupon securities, and debt securities	–1.04	–0.98
Rated below investment grade		
Liquid and other assets	0.29	0.43
TOTAL ASSETS OF THE SAMPLE	–25.19	–24.79
Core deposits	–9.23	–8.48
Fixed maturity deposits (interest bearing)	–6.85	–7.36
Advances and borrowing	–3.54	–2.65
TOTAL LIABILITIES	–19.62	–18.49
Impact of off-balance sheet activities	0.88	1.07
TOTAL CHANGE IN NET PRESENT VALUE	–4.69	–5.23

Note: Estimation followed OTS approach

In 1993Q4, the interest rate shock resulted in a decline in the value of balance sheet and off-balance sheet activities of sample institutions by $4.69 billion. A similar decline of $5.23 billion is observed in 1994Q4. In asset categories, the interest rate increase caused larger declines in asset items that can be categorized as “loans-in-place” (items listed in the first 5 rows). Indeed, these five items account for over 97% of the total asset value decline (in both samples), although they constitute only about 80% of the total assets. The impact on assets that can be categorized as “loan-in-process” (rows 6 and 7) is much smaller at 2–3% of the total decline, even though these items constitute about 20% of the assets. Also, the Other Asset category (item 7) produced a small but positive change in NPV. These findings are consistent with the major implication of our paper, that the impact of interest rate changes on loans-in-place will be very different from the impact on loans-in-process.

We also compared the overall effect of interest rate changes on "high-slack" and "other" institutions. As mentioned earlier, almost 20 percent of the assets on average could be categorized as loan-in-process. The standard deviation for these items was found to be $6.3 billion. In order to form two distinct groups of financial institutions differentiated by their lending slack, we identified all the institutions with loans-in-process more than two standard deviations above the mean. This group was defined as the "Slack Group". The rest of the institutions were put in the "Other Group". We then had two groups of financial institutions, one with substantial lending slack and the other with less slack. Table 5 illustrates the difference between the two categories.

Table 5. **Change in NPV For a 200 Basis Point Increase in Interest Rates ($ Billions)**

Comparison of the impacts of interest rate change between "Slack group" (institutions under this category have relatively higher loan-in-process, at least by two standard deviation above the average loan-in process of sample institutions) and "other group" (institutions that are not in the "slack" category).

	"slack group"	"other group"
Total change in NPV (1993Q4)	–1.54	–8.79
Total change in NPV (1994Q4)	–1.06	–9.03

Note: in both cases, the differences are statistically significant at all levels.

We find 55 thrifts in 1993Q4 and 62 thrifts in 1994Q4 that fall in the "slack" group. The table reports the total change in NPV resulting from a 200-basis-point increase in interest rate, for each category.[18] In 1993Q4, the change in NPV for the "substantial slack" group was a decline of $1.54 billion; however, this number was significantly lower than the $8.79 billion decline for the other group. A similar result is seen for 1994Q4, when the total decline for the "substantial slack" group was found to be $1.06 billion compared to a decline of $9.23 billion for the other group. In both cases, the difference is statistically significant at all levels. This evidence provides further support for the implications of Result 5.

Overall, the evidence indicates that an increase in interest rates generally has a significant negative effect on asset items categorized as "existing loans," and a substantially smaller effect on items categorized as "loans-in-process". Moreover, institutions with more slack in loan capacity (ie, farther from the lending limit) were found to be significantly less sensitive to interest rate changes than institutions closer to their lending limits.

[18] A detailed estimate by asset/liability categories and off-balance-sheet activities produced results very similar to Table 5.

4.3 The critical lending rate $L^*(r)$ and credit availability[19]

In Section 2 we had briefly discussed the critical lending rate $L^*(r)$, ie, the lending rate at which the bank will optimally grant the loan (exercise the option to lend), for a given deposit rate r. There is a large body of work on stopping times, optimal exercise policies and critical exercise triggers in the options literature (Barone-Adesi and Whaley (1987), Brock & Rothschild (1984), and others). There is also a large body of work on credit availability and rationing in the banking literature (Berger & Udell (1992), Shrieves & Dahl (1995), and others). However, to our knowledge, the role of the option aspect and critical exercise trigger in banking (credit availability) has not been investigated yet. Here we carry out a preliminary investigation into this issue.

Since $L^*(r)$ is the lending rate at which the bank is willing to grant the loan, a higher $L^*(r)$ means banks will be less willing to grant loans, and a lower $L^*(r)$ means banks will be more willing to grant loans. Since this reasoning applies to all banks, any factor that decreases $L^*(r)$ will result in an expansion of loans granted (ie, credit availability will rise) and any factor that increases $L^*(r)$ will reduce credit availability. In fact, if $L^*(r)$ rises high enough, it might lead to credit rationing.

This would seem to suggest that *all* banks make the "loan/no-loan" decision simultaneously, since our model has no inter-bank differences. It results from our assumption that all banks face the same deposit and loan rate processes. In real life, there would be some differences between banks, and all banks might not behave in an identical manner. However, the overall behavior of the banking sector is not entirely inconsistent with the notion of most banks behaving in a similar fashion. It has been noticed that most banks behave in a similar manner in extending or refusing loans, which is why credit availability from the banking sector as a whole goes through sharp expansions and contractions (Shrieves & Dahl (1995)).

We first look at the behavior of $L^*(r)$ with respect to r. When the deposit rate increases, we would expect the critical lending rate $L^*(r)$ to also go up, since the cost of a loan is now higher. Intuitively, therefore, $L^*(r)$ should be an increasing function of r. Unfortunately, $L^*(r)$ has to be determined numerically, and it is not possible to sign $dL^*(r)/dr$ analytically. We therefore determine L^* numerically for different values of r, and the results are displayed in Figure 7 for the base case parameter values used in Section 3.

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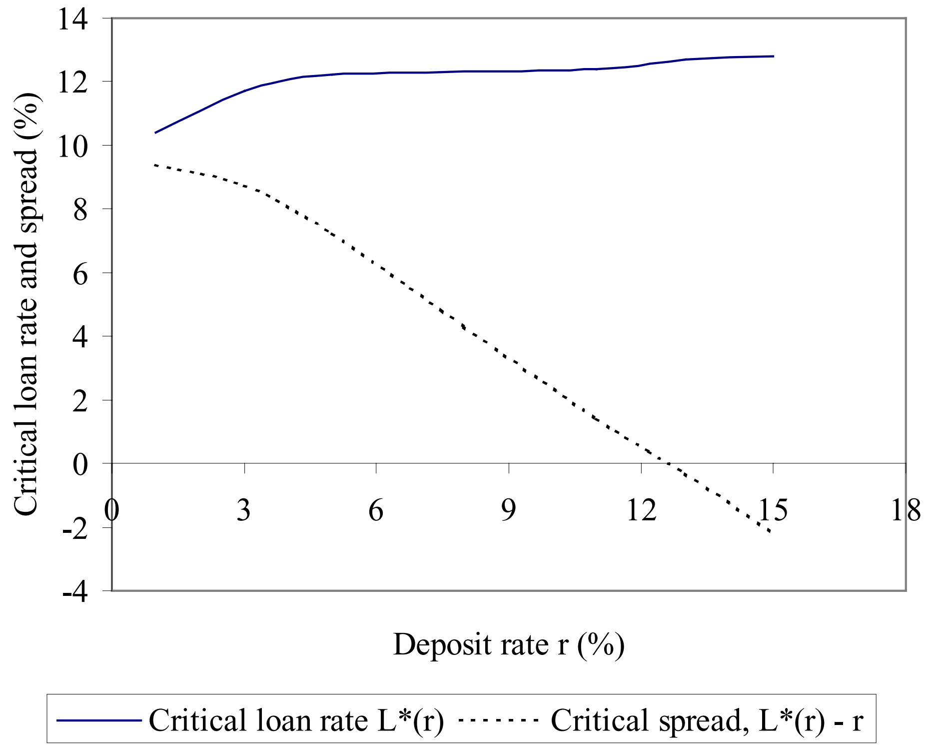
[19] We thank the referee for recommending the addition of this section to the paper.

Figure 7.

Figure 7 describes critical loan rate $L^*(r)$ and critical spread $[L^*(r)–r]$ as functions of deposit rate r. The following base case parameter values were used: $\kappa = 1$, $\theta = 0.05$, $\sigma = 0.05$, $\gamma = 1$, $\alpha = 0.10$, $\beta = 0.04$, $\rho = 0.95$, and a loan term of 5 years. Note that the critical lending rate is an increasing function of r, but the critical spread is a decreasing function of r (and can even be negative for large enough r).



$L^*(r)$ is indeed an increasing function of r, as expected from the above discussion. We also note that the spread required to grant a loan $[L^*(r)–r]$ is a decreasing function of r. This is an outcome of the mean-reverting nature of interest rates. If r is very low, it is expected to bounce back up because of mean reversion. But if a loan is granted, the loan rate does not change. Hence the bank is hesitant to grant the loan, unless the spread is large. This results in a larger critical spread for low r (upward-sloping term structure). The opposite is true when r is very high (downward-sloping term structure). Because of mean reversion, r is expected to fall, hence the bank is more willing to make the loan, ie, the critical spread $[L^*(r)–r]$ is lower. In fact, if the term structure is sufficiently inverted (here, for $r \geq 12.5\%$) the spread is negative; that is, the bank is willing to lend at a rate lower than the deposit rate, which we know from anecdotal evidence is observed when the term structure is sufficiently downward-sloping.[20]

Table 6. **Comparative static results for the critical loan rate L*(r)**

Describes comparative static results showing determinants of the critical loan rate $L^*(r)$ using different parameter values. Base case parameter values: $\kappa = 1$, $\theta = 0.05$, $\sigma = 0.05$, $\gamma = 1$, $\alpha = 0.10$, $\beta = 0.04$ and $\rho = 0.95$; r = 5%, loan term T = 5 years, and option matures in $T^* = 1$ month.

A. Effect of correlation between deposit and loan rates (ρ)

ρ	$L^*(r)$
–1.0	12.84%
0	12.54%
0.95	12.22%

B. Effect of deposit rate volatility (σ)

σ	$L^*(r)$
0.005	12.54%
0.05	12.22%
0.10	11.66%
0.15	11.08%

C. Effect of loan rate volatility (β)

β	$L^*(r)$
0.01	10%
0.02	10.82%
0.04	12.22%
0.08	17.31%
0.12	21.72%

D. Effect of deposit rate speed of reversion (κ)

κ	$L^*(r)$
0.0	11.08%
1.0	12.22%
2.0	12.54%

E. Effect of loan rate speed of reversion (γ)

γ	$L^*(r)$
0.0	21.07%
1.0	12.22%
2.0	11.08%

[20] Note that in Figure 7 the curves are not very smooth because of the discretization necessitated by numerical solution of the differential equation.

Determinants of $L^*(r)$. The critical loan rate $L^*(r)$ was numerically determined for a wide range of parameter values, and the results are summarized in Table 6. $L^*(r)$ is found to a decreasing function of ρ, σ and γ, and an increasing function of β and κ. The effect of ρ was captured intuitively by Santomero (1984), who wrote "… higher covariance reduces uncertainty around expected profits and encourages intermediation activity." Pyle (1971) had earlier made a similar prediction about the role of correlation in a somewhat different setting.

Since a higher (lower) $L^*(r)$ implies decreased (increased) credit availability, the comparative static results in Table 6 suggest the following

<u>Empirical Implication:</u> *Credit availability from the banking sector should be increasing in ρ, σ and γ, and decreasing in β and κ.*

Empirical test on credit availability. To test for the relationship between the above parameters and credit availability, we gathered quarterly data on aggregate loans granted by the banking sector, as a proxy for credit availability (source: *Historical Data, Federal Reserve Board*) and 3-month CD rates and prime rates (proxies for deposit and lending rates respectively, source: *Madura* (1997)). We next estimated the correlation series (ρ) by computing a running 20-period correlation between the two rates, and the deposit and lending rate volatilities (σ and β) by computing a running 20-period volatility for each series.[21] The estimates are shown in part A of Table 7.

We then ran a regression with the aggregate loans as the dependent variable and the previous quarter's ρ, σ and β as the explanatory variables (we ignored the speeds of reversion κ and γ which are difficult to estimate because of data limitations), ie,

$$\text{AGGLOAN}_t = \beta_0 + \beta_1 \text{RHO}_{t-1} + \beta_2 \text{SIGMA}_{t-1} + \beta_3 \text{BETA}_{t-1} + \tilde{\varepsilon}$$

where AGGLOAN is aggregate loans, and RHO, SIGMA and BETA are the parameters ρ, σ and β. From the above empirical hypothesis, the expected coefficient signs are positive for β_1 and β_2, and negative for β_3. The regression results are summarized in part B of Table 7. As predicted, the coefficients of RHO and SIGMA are positive and significant, while the coefficient of BETA is negative and significant. The adjusted R-squared statistic is 75.89%, and F-statistic is 46.15 (significance level of 1.213 E–13). Therefore, although this is a

[21] We used both 20 periods and 10 periods of quarterly data for computing the correlations and volatilities. The results are not sensitive to the exact number of data points used for the estimation.

simple test limited to only three explanatory variables, the regression results are quite consistent with the predictions of the model.[22]

Table 7. **Credit availability and the option approach**

A. Parameter estimates and aggregate loans

This table shows aggregate loans made by the banking sector, by quarter, from 1985 to 1996. It also shows, by quarter, the correlation between the 3-month CD rate and the prime rate, the volatility of the 3-month CD rate, and the volatility of the prime rate. The last three measures are estimated using a running 20-quarter window.

Year	Quarter	Aggregate loans ($B)	Correlation	Deposit rate volatility	loan rate volatility
1984	4	1326.9	0.97913812	0.030642083	0.034031952
1985	1	1361.6	0.98009992	0.03183423	0.035029844
	2	1394.7	0.98123599	0.031163472	0.033401785
	3	1425.6	0.98399612	0.031742952	0.034392332
	4	1466.3	0.98904969	0.032580935	0.035624365
1986	1	1509.5	0.98743005	0.030316959	0.032925922
	2	1521.7	0.98697202	0.029983892	0.032547373
	3	1551.4	0.98252	0.025799781	0.028130651
	4	1596.5	0.98036724	0.023848078	0.025187925
1987	1	1635.2	0.97875492	0.022932654	0.024326331
	2	1668.4	0.97073901	0.019576045	0.021478508
	3	1703	0.95817435	0.016275176	0.01719374
	4	1720.2	0.96262369	0.01599878	0.016336208
1988	1	1761	0.97400762	0.016225973	0.016259743
	2	1810.2	0.97462334	0.016374636	0.016418548
	3	1838.3	0.97781876	0.015990302	0.016329579
	4	1872.6	0.97709759	0.015754561	0.016040488
1989	1	1914.1	0.97504977	0.015648887	0.015846298
	2	1948	0.97412024	0.015278523	0.015534053
	3	1996.7	0.96449539	0.01277603	0.013715964
	4	2019.2	0.95540285	0.011357997	0.011873388
1990	1	2043.8	0.96432118	0.011362913	0.011641056
	2	2068.6	0.96531761	0.01133575	0.011423589
	3	2100.7	0.96549283	0.011387319	0.011527472
	4	2116.4	0.96466092	0.011414408	0.011624768
1991	1	2123.3	0.95646852	0.01145547	0.011626353
	2	2118	0.93428275	0.011766943	0.011593619
	3	2105.3	0.92512708	0.012009403	0.011434707
	4	2110.5	0.91512024	0.012328669	0.011054763

continued on the next page …

[22] This type of analysis does not go into the economic fundamentals, eg, why do the parameters change over time in a certain way? In addition, there are several possible reasons for a reduction in loans, eg, recession, decline in bank capital, tightened federal guidelines on certain types of loans, etc. These issues are discussed in papers such as Shrieves & Dahl (1995).

… continued

Year	Quarter	Aggregate loans ($B)	Correlation	Deposit rate volatility	loan rate volatility
1992	1	2119.8	0.93544721	0.014169406	0.012126847
	2	2110	0.95766229	0.016045937	0.013256747
	3	2109.1	0.97029047	0.018307307	0.015009818
	4	2113	0.98188847	0.020143381	0.016593971
1993	1	2115.2	0.98501131	0.021810844	0.017906041
	2	2146.6	0.98873814	0.023279641	0.019016129
	3	2166.4	0.99212732	0.024214569	0.019921331
	4	2196.9	0.99292277	0.024639295	0.020422807
1994	1	2220.3	0.99453303	0.024288842	0.02044367
	2	2249.5	0.99362092	0.022461279	0.019174712
	3	2313.5	0.9933417	0.021110169	0.017788236
	4	2378.2	0.99200299	0.019611935	0.016542637
1995	1	2448.2	0.99101459	0.018473121	0.015611712
	2	2530.2	0.98907572	0.016879651	0.014912879
	3	2585.5	0.98504253	0.015275794	0.014048648
	4	2617.1	0.98363217	0.013438612	0.013069627
1996	1	2656.2	0.97934684	0.012223673	0.012347491
	2	2689.7	0.97828102	0.011870143	0.011901874
	3	2730.8	0.97832103	0.011622184	0.01177475
	4	2779.3	0.97839515	0.011653082	0.011874204

B. Regression results

This table shows the results of the regression

$$AGGLOAN_t = \beta_0 + \beta_1 RHO_{t-1} + \beta_2 SIGMA_{t-1} + \beta_3 BETA_{t-1} + \tilde{\varepsilon}$$

where the dependent variable AGGLOAN denotes aggregate loans, RHO is the estimated correlation between the 3-month CD rate and the prime rate, and SIGMA and BETA are the estimated volatilities of the 3-month CD rate and the prime rate respectively. The explanatory variables RHO, SIGMA and BETA are proxies for the parameters ρ, σ and β of the model. All three explanatory variables have the predicted signs and are significant.

Independent variable	Expected sign	Coefficient	t-static	p-value
INTERCEPT		–5782.98	–3.10	0.0034
RHO	+	8816.09	4.45	0.0001
SIGMA	+	28804.84	1.76	0.0854
BETA	–	–70885.20	–5.23	0.0000

R-squared: 0.7589
Adjusted R-squared: 0.7424
F-statistic: 46.15
Significance (F-stat) = 1.213 E–13

5 Conclusion

Interest rate risk is very important for financial institutions, in light of their maturity intermediation function. This paper proposes a way of analyzing the interest rate sensitivity of a bank, explicitly taking into account the contribution of potential loans to the value of the bank. Based on the mismatch of asset and liability maturities, the revenue and cost streams depend on the stochastic long-term (lending) and short-term (deposit) rates in a partial equilibrium setting, where the two rates are possibly correlated. We also examine separately the effect on existing loans (loans-in-place) and potential loans (loans-in-process), and show that the latter can be valued using option pricing theory.

The major contribution of the paper is to show that a bank's interest rate sensitivity depends on how close it is to its lending limit imposed by capital requirement regulations. A bank that is close to its lending limit (has very little lending slack) will be very sensitive to interest rate changes, while a bank with substantial lending slack will be less sensitive. In fact, if the slack is large enough, it might even be helped by higher interest rates. This might be a reason for the inability of empirical studies to reach a unanimous conclusion regarding interest rate sensitivity of banks. The empirical evidence presented in this paper indicates that loans-in-place are more sensitive to interest arte changes than loans-in-process. Also, financial institutions that have more lending slack are less sensitive to interest rate changes.

An additional prediction from the option-like feature of the model is that credit availability from the banking sector should be affected by the parameters of the lending and deposit rate processes. This prediction is also supported by an empirical test carried out with quarterly data.

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Appendix 1.

Theoretical framework

We look at a unit loan (a \$1 loan), following the tradition of the "real options" literature (Dixit & Pindyck (1994), McDonald & Siegel (1986), etc). At time t, the bank can make a unit T-period loan at a rate of L_t. The loan is financed by a unit deposit, which matures instantaneously (very short-term) and is continuously rolled over, and has a rate r_t. Both the loan and deposit rates are market-determined, and the bank has no control over them. The bank's decision at any point in time is whether to make the loan, given the current values of L_t & r_t and their respective processes (equations (1) and (2) of the paper). The value of the loan to the bank (at the time of the loan, say at time t) is given by $EPV(L_t, r_t)$ (equation (12) of the paper):

$$EPV(L_t, r_t) = L_t \int_t^{t+T} A(s) e^{-r_t B(s)} ds - 1 + A(T) e^{-r_t B(T)} \qquad \text{(A1)}$$

If the loan has a positive (negative) value to the bank, it will result in a higher (lower) stock price when the loan is granted. The loan's value today (at time t = 0) is $E_0\{EPV(L_t, r_t)D_0(t)\}$, where $D_0(t)$ = value at time 0 of \$1 to be received at time t (ie, the current value of a unit discount bond maturing at time t), and E_0 is the expectations operator. The bank's objective therefore is to time the loan so as to maximize $E_0\{EPV(L_t, r_t)D_0(t)\}$. Let the maximum be given by $V(L_0, r_0, 0)$. Then:

$$V(L_0, r_0, 0) = \underset{t}{Max}[EPV(L_t, r_t) D_0(t)] \qquad \text{(A2)}$$

where $EPV(L_t, r_t)$ and $D_0(t)$ are given by equations (12) and (6) respectively of the paper. $V(L_t, r_t, t)$ is the value function.

Suppose that over the next instant dt, the loan is not granted. Then the change in the value function will be (using Ito's lemma):

$$dV = V_t dt + V_L dL + V_r dr + 0.5 V_{LL}(dL)^2 + 0.5 V_{rr}(dr)^2 + V_{Lr}(dL)(dr) \qquad \text{(A3)}$$

the expected value of which is (substituting for dL and dr from equations (1) and (2) of the paper):

$$E(dV) = V_t dt + V_L \gamma(\alpha - L) dt + V_r \kappa(\theta - r) dt + \frac{1}{2} V_{LL} \beta^2 L dt + \frac{1}{2} V_{rr} \sigma^2 r dt$$
$$+ V_{Lr} \rho\sigma\beta\sqrt{rL}\, dt \qquad (A4)$$
$$= dt\left[V_t + V_L \gamma(\alpha - L) + V_r \kappa(\theta - r) + \frac{1}{2} V_{LL} \beta^2 L + \frac{1}{2} V_{rr} \sigma^2 r + V_{Lr} \rho\sigma\beta\sqrt{rL}\right]$$

Therefore, at the next instant, the expected value of not granting the loan will be V + E(dV). The present value of this is

$$[V + E(dV)]e^{-rdt} = [V + E(dV)](1 - rdt) \qquad (A5)$$

using the approximation exp(–r dt) ≈ (1 – r dt), ignoring higher powers of dt since it is very small.

However, by not granting the loan, the bank is giving up the cash flow (L – r)dt ie, the spread it would have made from the loan. Therefore, the net expected present value of waiting is given by

$$[V + E(dV)](1 - rdt) - (L - r) dt = V - rVdt + E(dV) - (L - r) dt \qquad (A6)$$

(since dV dt = 0). This is the value if the bank does not grant the loan immediately. If it grants the loan immediately, the value will be EPV(L,r), given by equation (5) of the paper. Being a value-maximizer, the bank will choose that course of action which will result in the higher value, ie,

$$V(L, r, t) = \text{Max}[V - rVdt + E(dV) - (L - r) dt, \text{EPV}(L, r)] \qquad (A7)$$

If EPV(L,r) < V – rVdt + E(dV) – (L–r)dt, the bank should not exercise the option, ie, should not grant the loan. Rather, it should postpone its decision; from the above equation, we have V = V – rVdt + E(dV) – (L–r) dt, or E(dV) – rdt – (L–r)dt = 0. But if EPV(L,r) > V – rVdt + E(dV) – (L–r)dt, it should grant the loan immediately, so that V(L,r,t) = EPV(L,r).

This gives the following conditions:

1) For all (L,r) where it is optimal to postpone, we have the partial differential equation:

$$V_t + \frac{1}{2}\beta^2 L V_{LL} + \frac{1}{2}\sigma^2 r V_{rr} + \rho\sigma\beta\sqrt{Lr}\, V_{Lr}$$
$$+ \gamma(\alpha - L) V_L + \kappa(\theta - r) V_r - rV = L - r \qquad (A8)$$

and the inequality:

$$V(L, r, t) > EPV(L, r) \tag{A9}$$

2) When it is optimal to grant the loan, we have:

$$V(L,r,t) = EPV(L,r) \tag{A10}$$

and

$$\begin{aligned} &V_t + \frac{1}{2}\beta^2 L\, V_{LL} + \frac{1}{2}\sigma^2 r\, V_{rr} + \rho\sigma\beta\sqrt{Lr}\, V_{Lr} + \gamma(\alpha - L)\, V_L \\ &+ \kappa(\theta - r)\, V_r - rV - L - r < 0 \end{aligned} \tag{A11}$$

In this setting, choosing a value of t to maximize $E_0\{EPV(L_t,\ r_t)D_0(t)\}$ is isomorphic to choosing an optimal boundary $L_t^*(r)$ such that as soon as L reaches $L_t^*(r)$, the loan should be granted (see McDonald & Siegel (1986)).

As long as it is optimal to defer the decision, the above partial differential equation (14) must be satisfied by the function V(L,r,t). When is it optimal to grant the loan? Or, how do we compute the optimal $L_t^*(r)$ for each r and t? For this, we need the smooth-pasting condition (equation (20) of the paper).

The above argument establishes the theoretical framework for the option approach, and also the equivalence between value maximization and optimal timing of loans.

Appendix 2.

Interest rate risk measure

Over the years, the two approaches to measuring interest-rate risk (IRR), the OTS (Office of Thrift Supervision) approach and the BA (Federal Banking Agencies, ie, Federal Deposit Insurance Corporation, Office of the Comptroller of Currency, and the Federal Reserve Board) approach, have differed greatly. The OTS approach incorporates each institution's weighted average coupons (WACs), weighted average remaining maturities (WARMs), and other issues needed to calculate changes in net present values (NPVs) for different categories of assets, liabilities and off-balance sheet positions (see OTS (1990 & 1993)). This measure tells how interest-rate "shocks" are evaluated as to their effects on each institution's overall portfolio. The regulators decide on possible Capital charges (Risk-based capital requirement) on these NPV changes.

In contrast, the BAs follow the Bank of International Settlement's risk-based capital guidelines for assessing capital charges for IRR [for more details, see Notice of Proposed Rulemaking (NPR)]. The BAs' method applies four types of interest-rate "risk weights" to all banks, based on measures of industry average durations, and assumes that IRR can be measured by summing up these changes in values. Cordell, Gordon, and Anderson (1993) report the superiority of OTS model over the BA model. We have attempted both approaches in measuring the IRR for our sample institutions and the results overall were found to be quite similar, hence only the OTS-based results are reported. We used 1993Q4 and 1994Q4 samples of all savings and loan institutions reporting to the OTS. The sample period had experienced substantial interest rate changes thus helpful for our purposes. Our method closely follows the directives and procedures used by Cordell, Gordon, and Anderson (1993).

According to both of these models, Risk is defined as the calculated decline in value of an institution for a 200 basis point (bp) parallel upward shift in the yield curve. Decline in value is measured in the OTS model by the decline in net portfolio value (NPV), which equals the market value of assets minus market value of liabilities plus market value of off-balance sheet positions, in each interest rate scenario. In the BA approach, the decline in value is measured by adding the amount entered into each maturity band times the respective "risk weight".

We used information from the Consolidated Maturity and Rate Schedule (Schedule CMR) of the Thrift Financial Report for 1993Q4 (1,388) and 1994Q4 (1,408), for a total of 2,796 thrifts, to estimate IRR. Our reporting here is based on the OTS's Net Portfolio Value model that estimates discounted present values for assets, liabilities, and off-balance sheet items for each thrift's portfolio using interest rates prevailing at the end of each quarter. The IRR is the net decline in assets less liabilities plus off-balance sheet items associated with either a + or – 200 bp parallel yield-curve shift, whichever shift produces the greater decline.

$$IRR_{OTS} = \left(\frac{\Delta NPVA - \Delta NPVL + \Delta NPVOB}{NPVA} \right)_{\pm 200bp} - 0.02 \tag{A12}$$

where ΔNPVA = change in net portfolio value of assets valued at + or – 200 bp yield curve shifts, ΔNPVL = change in net portfolio value of liabilities valued at + or – 200 bp yield curve shifts, and ΔNPVOB = change in net portfolio value of off-balance sheet items valued at + or – 200 bp shifts.

We subtracted 2% as the Capital is assessed (in deciding risk based capital standards) against the decline in value exceeding two percent of assets. Our results are strikingly similar when we do not incorporate this 2 percent in the equation. The BAs use two alternative measures of risk in their NPR. In the first, they measure IRR as the decline in net risk weighted positions (NRWPs) of assets less liabilities plus off-balance sheet items associated with a + or – 100 bp parallel yield-curve shift, whichever shift produces the greater decline. The functional form of the BA model is

$$IRR_{BA1} = \left(\frac{\sum_{i=1}^{7}\sum_{j=1}^{3} D_{Aij} * BA_{ij} - \sum_{i=1}^{7} D_{Li} * BL_{i} + \sum_{i=1}^{7}\sum_{j=1}^{2} D_{Aij} * NOB_{ij}}{TBA} \right)_{\pm 100bp} - 0.01 \tag{A13}$$

where Dij = duration risk weight for asset j in time b and i, BAij = book value of asset group j in time band i, DLi = duration risk weight for liabilities in time band i, BLi = book value of liabilities in time band i, NOBij = notional amount of off-balance-sheet item j in time band i, and TBA = total book value of assets.

In the second alternative, a + or – 200bp yield-curve shift is considered, whichever produces the greater decline, with capital assigned against the decline in NRWP for the amount over one percent of assets.

$$IRR_{BA2} = \left(\frac{\sum_{i=1}^{7}\sum_{j=1}^{3} D_{Aij} * BA_{ij} - \sum_{i=1}^{7} D_{Li} * L_{i} + \sum_{i=1}^{7}\sum_{j=1}^{2} D_{Aij} * NOB_{ij}}{TBA} \right)_{\pm 200bp} - 0.01 \tag{A14}$$

[1 percent was subtracted as Capital is assessed against the decline in net risk weighted positions for the amount over 1 percent]

Static Discounted Cash Flow Method

$$PV = \sum_{m=1}^{n} \frac{CF_m}{(1 + i_m + s)^m}$$

where PV = present value of instrument, CF = estimated cash flows including constant assumptions on options, i_m = implied zero-coupon Treasury rate, and s = a residual representing the constant spread over the zero-coupon Treasury rate for the instrument.

[Used for: Consumer and Commercial Loans, other Non-mortgage Assets, Investment securities, Deposits and Borrowings, Second Mortgages, and Multi-Family and Non-Residential Mortgages.]

Option-Adjusted Spread Method

$$PV = \sum_{m=1}^{n} \frac{CF_m}{(1 + i_m + OAS)^m}$$

where OAS = a residual that equates the average of the prices from 200 interest rate paths to the current market price of a comparable security.

<u>Five Steps to Calculating Net Present Values under OAS approach:</u>

Generate 200 interest rate paths over 360-month period.
Generate 200 mortgage rate paths over 360-month period.
Generate prepayment rates along 200 paths over 360-month period.
Generate PV, discounting by implied forward rates plus OAS.
Calculate ΔNet Present Value at 200 basis points.

[Used for: 1–4 Family Fixed- and Adjustable-rate Mortgages, Mortgage Servicing Assets, and Mortgage-Derivative Products. Sources: The details on the interest rate risk measure described above and the organization of asset-liability breakdown categories, and estimation methodology used in the paper follows closely the categories and description of the methods introduced by Cordell, Gordon, and Anderson (1993), OTS (1990, 1993).]

Appendix 3.

Balance sheet items of sample thrift institutions

Asset/liability category	Amount ($ billions)	% of total assets
Adjustable rate mortgages***	568.8	37.6
Fixed-rate mortgages***	347.4	20.3
Mortgage servicing assets and derivatives***	73.5	4.7
Other mortgages and second mortgages**	229.2	13.5
Consumer and commercial loans**	70.4	5.0
Securities (equity securities, zero-coupon securities, and debt securities)**	151.7	11.3
Other assets**	102.6	7.6
TOTAL ASSETS OF THE SAMPLE	1,542.6	100
Core deposits (savings and demand deposits)**	414.3	31.3
Fixed-maturity deposits (time deposits)**	696.5	52.6
Advances and borrowing**	212.4	16.1

** indicates calculation with static discounted cash flow method.

*** indicates calculation with option-adjusted spread method.

Note: The breakdown of asset-liability items simply follows the breakdown categories used by the OTS (1990) and (1993) and Cordell, Gordon, and Anderson (1993).

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