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Tuomas Välimäki
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Variable rate liquidity tenders

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Variable rate liquidity tenders

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Abstract

This paper constructs an equilibrium model for the short-term money market, when the central bank provides liquidity via variable rate tenders. The relation between market rate of interest and liquidity is derived from a single bank's profit maximisation problem in the interbank market, and the CB determines its liquidity provision by minimising a quadratic loss function that contains both deviations of expected market rate from CB target rate and differences between liquidity supply and target liquidity. We model equilibrium bid behaviour in the tenders and explain the underbidding phenomenon resulting from the minimum bid rate. We also show that, when maturities of consecutive operations overlap, the expected market interest rate will rise above the CB's target whenever a target rate change (hike or cut) is expected to occur in the same reserve maintenance period. Finally, we review the data from the ECB variable rate tenders and find that the ECB has been fairly liquidity oriented in its allotment decisions.

Key words: money market tenders, liquidity policy, bidding, central bank operational framework

Vaihtuvakorkoiset likviditeettihuutokaupat

Suomen Pankin keskustelualoitteita 24/2002

Tuomas Välimäki
Tutkimusosasto

Tiivistelmä

Tutkimuksessa mallinnetaan lyhyen rahan markkinoiden tasapaino, kun keskuspankki ohjaa likviditeettiä vaihtuvakorkoisilla huutokaupoilla. Markkinakoron ja likviditeetin välinen suhde johdetaan yksittäisen pankin voitonmaksimointiongelmasta pankkienvälisillä rahamarkkinoilla. Keskuspankki mitoittaa tavoittelemansa likviditeetinjaon minimoimalla tappiofunktiota, joka sisältää sekä odotetun rahamarkkina- ja tavoitekoron välisen kvadraattisen poikkeaman että likviditeetin kvadraattisen poikkeaman keskuspankin tavoitemäärästä. Tutkimuksessa mallinnetaan lisäksi huutokauppojen tarjousmäärä tasapainossa sekä selitetään tavoiteltua vähäisemmän tarjousmäärän ongelma (nk. underbidding-ilmiö) minimitarjouskoron avulla. Lisäksi markkinakoron osoitetaan nousevan tavoitekoron yläpuolelle, jos peräkkäisten operaatioiden maturiteetit menevät limittäin ja keskuspankin odotetaan muuttavan tavoitekorkoaan jäljellä olevan vähimmäisvarantojen keskiarvoistuperiodin kuluessa – huolimatta siitä, odotetaanko koron nostoa vai laskua. Lopuksi tarkastellaan EKP:n vaihtuvakorkoisia huutokauppoja. Niiden pohjalta EKP näyttäisi likviditeetinjakopäätöksissään painottaneen nimenomaan likviditeetin poikkeamia tavoitetasolta.

Asiasanat: rahamarkkinahuutokaupat, likviditeettipolitiikka, huutokauppatarjoukset, keskuspankin toimintakehikko

Contents

1 Introduction

The operational framework a central bank applies has significant effects on the behaviour of the money market rates. Eg Prati, Bartolini and Bertola (2002) study the interbank overnight markets of the G-7 countries and Euroarea, and find that the day-to-day behaviour of short-term interest rates are more likely to reflect institutional arrangements than the market frictions. Hence, it's far from being clear that the models describing the federal funds market can be applied for the case of Euro money markets, as the operating procedures applied by the European Central Bank (ECB) differs in many respect from those applied by the Fed.[1] The purpose of this paper is to analyze the determination of the short-term money market equilibrium in an operational framework similar to the one applied by the ECB. Probably the most significant change in the operating procedures of the ECB during the first three years of operation, was the adoption of variable rate tenders in the main refinancing operations (the weekly liquidity auctions) instead of the fixed rate tenders that were applied until June 2000. In this paper we concentrate in the case where the MROs are executed as variable rate tenders. The analysis on fixed rate tenders can be found in Välimäki (2001 and 2002).

According to the ECB the shift to variable rate tenders was "*a response to the severe overbidding problem which has developed in the context of the fixed rate tender procedure*".[2] Välimäki (2001) shows that overbidding is an equilibrium feature in fixed rate liquidity tenders when the central bank applies proportional allotment procedure. Furthermore, Välimäki (2002) claims that the sharp increase of the bid amount vs the amount of reserves that actually was provided to the market in the ECB fixed rate tenders resulted from a combination of expectations of an interest rate hike and liquidity-oriented allotment policy of the ECB. Now, basically the problems the ECB experienced with the fixed rate tenders were a consequence from the goal of smooth reserve holding during a reserve maintenance period in the supply of liquidity together with having fixed price for it. If the monopoly supplier of a good is having troubles when it fixes both the price and the quantity, the natural way out is to let the market decide either one of them. A shift to variable rate tenders is one way of letting the price to adjust to the demand, while a change from proportional allotment into full allotment[3] would basically mean that the adjustment is done within the quantity side. The ECB adopted variable rate tenders in 28 June 2000. However, the auction format the ECB introduced is not a variable rate tender in its purest form, as the ECB also introduced a minimum bid rate, that is the lower limit for bids in the tender. *Therefore, when a rate cut is expected by the banks, the tender format the ECB chose functions very similarly to the fixed rate tenders.*

[1] The operational frame work of the ECB is described in full detail in "*The Single Monetary Policy in Stage Three; General Documentation on ESCB Monetary Policy Instruments and Procedures*" (ECB, 2000).

[2] ECB press release dated 8 June 2000.

[3] Full allotment refers to the case in which the central bank accepts all bids placed in a fixed rate tenders in full, ie the bids are never scaled back.

The closest reference to this paper is Ayuso and Repullo (2000). They analyze both the fixed and variable rate tenders of the ECB. They show that variable rate tenders also have multiple equilibria characterized by varying degrees of overbidding. However, by publishing the intended allotment volume an equilibrium without overbidding can be obtained. In their two period model there is one liquidity tender, in which the central bank minimizes a loss function that depends on the squared difference between the interbank rate and the central bank's target rate. The expected market rate of interest will differ from the central bank's target rate when the loss function penalizes more heavily market rates below the rate. In our model there is one operation for each day in a maintenance period that lasts for two days. Thus, *the expected central bank rates for the second period will affect the amount of liquidity demanded already at the first period*. Furthermore, our model central bank has a loss function that penalizes *both* differences between the market rate of interest and the central bank's target rate, *and* also deviations of the money market liquidity from the steady path of reserve holding. Therefore, *the expectations over the second period rates will affect the expected first period market rate of interest*. Also, the effect of a minimum bid rate is different in our model to that of the Ayuso and Repullo model.

Also Bindseil (2002) analyses the open market operations of the ECB. His approach differs, however, considerably from the one applied in this paper. Bindseil takes the martingale hypothesis as given, and he forms the equilibrium condition for aggregate bidding based on it, whereas the hypothesis does not necessarily hold with the micro foundations developed here. Furthermore, Bindseil analysis mainly the case where the allotment decision of the central bank is based on a rigid target for liquidity, while we allow for richer set of possible liquidity policies. Finally, a recent paper by Nyborg et al. (2002) analyses empirically the bidding in the ECB main refinancing operations by using microdata form the 53 first auctions after switching to the variable rate procedure.

The standard literature on multiple unit auction is not directly applicable in the central bank liquidity tenders, as in those auctions the seller is usually not maximizing its revenue from the auction. However, when deciding on the auction format to be applied, the central bank shouldn't be immune to the lessons that can be learned eg from Back and Zender (1993 and 2001), who show that a sealed bid uniform-price auction may lead to equilibria in which the price actually paid in the auction (here the tender rate) is considerably below the true value (here the corresponding market rate of interest).

The rest of the paper is organized as follows. Next, we'll briefly review the main features of the operational framework the ECB applies. In Chapter 2 we model the demand for liquidity in the interbank money market for both days of a two day reserves maintenance period. We can use these demand functions to derive the equations that determine the market rate of interest as a function of liquidity. In Chapter 3 we model the central bank behaviour in its allotment decisions. In the model the central bank will minimize a quadratic loss function in which deviations both in liquidity and in expected interest rate from their target levels can be taken into account. Chapter 4 describes the bidding of the banks in these tenders. Also the effects of minimum bid rate and overlapping maturities of consecutive tenders are analyzed. Chapter 5 briefly reviews the experience we have on the ECB variable rate main refinancing operations, and finally section 6 summarizes and draws the conclusions.

1.1 Brief outline over the operational framework of the ECB

To put it briefly, the ECB applies three different types of monetary policy instruments: i) active liquidity management is conducted via open market operations, ii) the banks are provided with standing facilities, and iii) all credit institutions are subject to reserve requirements.

First, the ECB conducts main refinancing operations (MRO) once a week. The role of the MROs is to provide liquidity to the banks, and to signal the monetary policy stance of the Eurosystem. These operations are liquidity providing tenders with the maturity of two weeks. They can be executed in the form of fixed rate (FRT) or variable rate tenders (VRT). In a FRT the interest rate is specified by the ECB in the tender announcement, while in the VRTs the counterparties of the ECB specify in their bids both the amount of reserves they want to transact with and the interest rate at which they wish to enter into transactions. The VRT may be conducted with multiple or single rate procedure[4]. The ECB arranges the bids in descending order (in terms of the bid rate) and accepts the highest bids until the amount of liquidity to be provided to the market is satisfied. The lowest rate at which bids are accepted is called the marginal rate. The ECB may restrict the supply of liquidity at the marginal rate of the allotment. If this is the case the, the ECB applies pro rata rationing for these bids. In the multiple rate procedure, the allotment interest rate for all the accepted bids is the interest rate offered at the given bid, while in the single rate procedure the marginal rate of the allotment is applied for all accepted bids.

In addition to the MROs, the ECB executes once a month a longer-term refinancing operation. However, as the aim of the ECB is not to signal monetary policy stance or to actively manage the liquidity conditions with these operations, we leave these operations out of the analysis in this paper.[5] Furthermore, the ECB may execute irregular operations to fine tune the liquidity conditions. Yet, these operations have been extremely rare during the first three years of Stage Three of the Monetary Union.[6]

Secondly, the two standing facilities the ECB offers to the banks are the *marginal lending facility* and the *deposit facility*. The banks may obtain overnight reserves from the ECB through the marginal lending facility, the rate for which is pre-specified by the ECB. This *marginal lending rate* (r^m) provides a ceiling for the interbank overnight rate of interest. Furthermore,

[4] The terms multiple and single rate procedures are applied by the ECB. In auctions literature these procedures are also known as discriminatory price or American auction and single price or Dutch auction respectively. Note that outside financial literature the term "Dutch auction" has a different meaning.

[5] "*In these operations, the Eurosystem does not, as a rule, intend to send signals to the market and therefore normally acts as a rate taker.*" (ECB, 2000 p. 15)

[6] Only 4 fine tuning operations were conducted between 1.1.1999 and 31.12.2001.

the banks can place overnight deposits to the deposit facility of the ECB at the pre-specified interest rate. This *deposit rate* (r^d) provides a floor for the market overnight rate.

Thirdly, in addition to the open market operations and the standing facilities, the ECB requires the banks to hold compulsory deposits with the Eurosystem. However, the ECB provides the banks with averaging provision in the reserve holding. That is, the compliance with the reserve requirement is judged by the average of the end-of-day balances of the reserve accounts held with the Eurosystem. The reserves maintenance period is one month (from 24th of a given calendar month to the 23rd of the following month).

2 Market rate as a function of liquidity

The evolution of liquidity during a day is assumed to be the following. The reserve balances of a bank at the beginning of the day are the end-of-day balances of the previous banking day (RB_{t-1}). The bank estimates the effect of changes in autonomous liquidity factors (a_t; which here is assumed to contain also the effect of maturing central bank operations). The forecast error of this estimate is the liquidity shock the bank encounters. We assume part of the shock (μ_i) to realize before the settlement of the overnight market (that is assumed to occur at a given moment of the day), while the rest of the shock (ε_i) realizes after the overnight market has been closed. The amount of reserves the bank is provided with at the tender is denoted by q_i, and is received by the bank before the interbank overnight trading. Thus, bank i's liquidity at the overnight market (l_i) equals $RB_{t-1,i}+ a_i + q_i + \mu_i$. The net borrowing of bank i from the interbank market is denoted by b_i. Therefore, the end-of-day balances are $RB_{i,t} = l_i + b_i + \varepsilon_i$, unless the bank has to use the standing facilities. If the cumulative reserve holdings of bank i is larger than the reserve requirement for the whole period, it will place the excess reserves into the deposit facility. Furthermore, the bank has to obtain reserves from the marginal lending facility, if it's end-of-day balances otherwise would be negative or if it otherwise would fail to comply with the reserve requirement (relevant only at the last day of the reserves maintenance period).

Reserve balances held on different evenings of the same reserves maintenance period are perfect substitutes for each other from the reserve requirement point of view. Also, all units of liquidity are identical irrespective of wether they are borrowed from the central bank or the interbank market. Now, as trading in the interbank market takes place later than the tender, we assume that the banks ask for liquidity at the interbank market to be able to comply with the reserve requirement at minimum cost, whereas the demand at the central bank tender depends solely on the expected profit opportunity between the price of liquidity at the tender and the expected value for it in the interbank market. The expected market rate depends on the amount of liquidity provided to the market at the tender, while the bidding of the banks depends on the expectation over the market rate. We will approach this problem by first modelling the demand for liquidity at the interbank market as a function of the total money market liquidity. Based on the demand functions we derive the market rate of interest as a function of money market liquidity. After that we model the central bank's intended liquidity supply so that it takes into account this relation as a constraint on the loss function it minimizes.

The cost of obtaining reserves from the market is the overnight rate of interest, while the return from the reserves depends on the second liquidity shock. The income from the reserves borrowed from the interbank market is the marginal lending rate (r^m) for the amount of reserves that is a substitute for acquiring liquidity from the marginal lending facility, ie if the reserve balances of the bank after the second liquidity shock are negative (or below the required reserves at the end of the last day of the maintenance period), while the income from balances in excess of the requirement for the whole maintenance period is the deposit rate. The expected value of positive balances (that are below the amount that would fulfill the reserve requirement for the whole period) is the expected value of the reserves on the following day(s), as the reserve balances of today and those of tomorrow are substitutes for each other.

The length of the reserves maintenance period must be at least two days in order for the interest rate expectations to affect the demand for euro. Hence, we will develop a model of the demand for reserves in a two day maintenance period in order to keep the model as tractable as possible while still maintaining the effect that arises from the interest rate expectations. The demand functions for longer maintenance periods can be found in Välimäki (2001). We begin the modelling from the second day of the period (when there is not the averaging provision available anymore), as the maximization problem of the first day (i.e. in the case with averaging possibility) must be solved recursively using the result of the following day.

2.1 Final day of the MP (no averaging)

The profit maximization problem of a risk neutral atomistic bank at the interbank market for the final day of the reserves maintenance period is the following:

$$\begin{aligned} \max_{b_{i,2}} \mathrm{E}(\Pi) &= r_2^m \left[\int_{-\infty}^{-l_{i,2}+rdb_{i,2}-b_{i,2}} (l_{i,2} - rdb_{i,2} + b_{i,2} + \varepsilon_2) f(\varepsilon_2) d\varepsilon_2 \right] \\ &\quad + r_2^d \left[\int_{-l_{i,2}+rdb_{i,2}-b_{i,2}}^{\infty} (l_{i,2} - rdb_{i,2} + b_{i,2} + \varepsilon_2) f(\varepsilon_2) d\varepsilon_2 \right] \\ &\quad - r_2 b_{i,2}, \end{aligned} \tag{1}$$

where $b_{i,2}$ is bank i's net borrowing from the interbank market, $f(\varepsilon_2)$ is *pdf* of the second shock of the day, and $rdb_{i,2}$ is the amount of reserves with which the bank would exactly meet its reserve requirement[7]. The first order condition with respect to interbank borrowing is:

$$(r_2^m - r_2^d)F\left(-l_{i,2} + rdb_{i,2} - b_{i,2}^*\right) + (r_2^d - r_2) = 0, \tag{2}$$

where $F(\cdot)$ is the *cdf* of ε_2. $F\left(-l_{i,2} + rdb_{i,2} - b_{i,2}^*\right)$ gives the probability of bank i being forced to use the marginal lending facility under the optimal borrowing. We can rewrite equation (2) as:

$$F\left(-l_{i,2} + rdb_{i,2} - b_{i,2}^*\right) = \frac{r_2 - r_2^d}{r_2^m - r_2^d}, \tag{3}$$

that relates the probability of using the marginal lending facility to the location of the market rate of interest within the interest rate corridor set by the standing facilities.

If the cumulative distribution function has an inverse function $(F^{-1}(\cdot))$, we can derive explicit form for bank i's borrowing function:

$$b_{i,2}^*(-l_{i,2}, r_2) = -l_{i,2} + rdb_{i,2} - F^{-1}\left(\frac{r_2 - r_2^d}{r_2^m - r_2^d}\right) \tag{4}$$

Bank i can act as a borrower or lender in the market. However, as long as the overnight market rate stays strictly inside the corridor, money market liquidity is constant, as there will not be any transactions with the central bank. Therefore, the aggregate borrowing must be zero $(\sum_{i=1}^{n} b_i = 0)$. We can get the market-clearing rate of interest from equation (3) simply by setting the aggregate borrowing to zero and aggregating over the unit measure of banks (i.e. $b_{i,2}^* = b_2^* = 0$ and $-l_{i,2} = -l_2$):

$$r_2 = r_2^m F(rdb_2 - l_2) + r_2^d(1 - F(rdb_2 - l_2)), \tag{5}$$

Equation (5) gives the market rate of interest as a probability weighted average of the two rates of the standing facilities. The higher the rates of the standing facilities the higher will the overnight rate be, whereas the more liquidity there exists (with respect to the required daily balances) at the market the lower the market rate of interest. Now, the required daily balances at the final day of the maintenance period is simply the requirement for the whole period less

[7] The required daily balances for the rest of the maintenance period $(rdb_{i,2})$ is calculated from the reserve requirement per day (R), and the amount of reserves already held as reserve balances within the current reserves maintenance period;

$$rdb_{i,t} = \frac{T * R_i - \sum_{k=1}^{t-1} RB_{i,k}}{T - (t-1)},$$

where T is the number of days in a maintenance period. Thus, at the first day of a two day maintenance period we have $rdb_{i,1} = R_i$, while at the second day $rdb_{i,2} = 2R_i - RB_{i,1}$.

the amount of reserves held during the first day of the maintenance period ($rdb_2 = 2R - z_1$, where z_1 denotes $l_1 - \mu_1 - sf_1$, with sf_1 denoting the possible use of the standing facilities at end of period 1). Thus, we can write equation (5) as:

$$r_2 = r_2^m F(2R - z_1 - l_2) + r_2^d(1 - F(2R - z_1 - l_2)), \tag{6}$$

Furthermore, the expected value of the market rate of interest on the second day is given by:

$$\mathrm{E}[r_2] = r_2^m G(rdb_2 - l_2) + r_2^d(1 - G(rdb_2 - l_2)), \tag{7}$$

where the expectations are taken over the distribution f_μ, and $G(\cdot)$ is the *cdf* of the sum of the two liquidity shocks μ_2 and ε_2. The proof for this can be found in Välimäki (2001). Equation (7) says that the expected market rate with a given allotment equals the probability weighted average of the standing facility rates, where the probability weights take into account both of the shocks and the amount allotted in the tender.

2.2 First day of the MP (averaging)

On the first day of the two day reserves maintenance period the amount of reserves (prior to the use of the standing facilities) bank i holds is divided between the reserve balances and use of the standing facilities as follows: for $l_{i,1} < 0$ the bank must acquire $-l_{i,1}$ units of liquidity from the marginal lending facility; if $l_{i,1} > 2R$ the bank will have $2R$ as reserve balances while $l_{i,1} - 2R$ units must be placed into the deposit facility; if $0 < l_{i,1} < 2R$ all the liquidity will be held as reserve deposits. The profit maximization problem of the bank takes the following form:

$$\begin{aligned}\max_{b_{i,1}} \mathrm{E}(\Pi) &= r_1^m \left[\int_{-\infty}^{-l_{i,1}-b_{i,1}} (l_{i,1} + b_{i,1} + \varepsilon_1) f(\varepsilon_1) d\varepsilon_1 \right] \\ &\quad + \mathrm{E}_1[r_2] \left[\int_{-l_{i,1}-b_{i,1}}^{\infty} (l_{i,1} + b_{i,1} + \varepsilon_1) f(\varepsilon_1) d\varepsilon_1 \right] \\ &\quad + \left(r_1^d - \mathrm{E}_1[r_2]\right) \left[\int_{2R_{i,1}-l_{i,1}-b_{i,1}}^{\infty} (l_{i,1} + b_{i,1} - 2R_{i,1} + \varepsilon_1) f(\varepsilon_1) d\varepsilon_1 \right] \\ &\quad - r_1 b_{i,1}, \end{aligned} \tag{8}$$

from which we get the FOC for the profit maximizing problem of interbank borrowing:

$$\begin{aligned} & r_1^m F(-l_{i,1} - b_{i,1}) + \mathrm{E}_1[r_2]\left[F(2R_{i,1} - l_{i,1} - b_{i,1}) - F(-l_{i,1} - b_{i,11})\right] \\ & + \left(r_1^d\right)\left[1 - F(2R_{i,1} - l_{i,1} - b_{i,1})\right] - r_1 b_{i,1} = 0, \end{aligned} \tag{9}$$

From equation (9) we can derive (after aggregation) the market rate of interest as a function of liquidity:

$$\begin{aligned} r_1 &= \mathrm{E}_1\left[r_2\right]\left\{F(2R-l_1)-F(-l_1)\right\} \\ &+r_1^m F(-l_1)+r_1^d\left[1-F(2R-l_1)\right]. \end{aligned} \tag{10}$$

The market rate of interest at the first day of the maintenance period will be a probability weighted average of the rates of the standing facilities and the market rate of interest expected to prevail at the following day of the maintenance period. Now, like in the case of the second day, the expected value for the market rate at the first day of the reserves maintenance period is given by:

$$\mathrm{E}\left[r_1\right]=\mathrm{E}_1\left[r_2\right]\left\{G(2R-l_1)-G(-l_1)\right\}+r_1^m G(-l_1)+r_1^d\left[1-G(2R-l_1)\right], \tag{11}$$

Next we will turn to the analysis of the central bank behaviour in the tenders.

3 Central bank behaviour: the supply of liquidity

In its liquidity allotment decision, the model central bank aims at maintaining the expected value of the market rate of interest as close as possible to a target that is derived from the ultimate goal of the central bank (eg maintaining price stability). However, the central bank might also like to stabilize the reserve holdings within a reserve maintenance period. For example, in ECB (2000) it is stated that in the final allotment decision the considerations of the ECB relate to *the smoothness of the reserve fulfillment path and the level of the interest rates*. Hence, here the model central bank minimizes a loss function that consists of two parts: the expected difference between the market rate and the target rate and the deviations of expected liquidity from the steady path of reserve holding. The precise functional form of the central bank's loss function is usually not explicitly announced. Here, the central bank is assumed to minimize a quadratic loss function, that consists of the weighted sum of squared percentage deviations of the expected market rate of interest and expected liquidity from their target values. The loss function takes the following form:

$$\begin{aligned} L_t &= \frac{1}{2}(1-\lambda_t)\left(\frac{\mathrm{E}\left[r_t\right]-\overline{r}_t}{\overline{r_t}}\right)^2+\frac{1}{2}\lambda_t\left(\frac{\mathrm{E}\left[m_t+\eta_t\right]-\overline{m_t}}{\overline{m}_t}\right)^2, \\ & s.t.\ r_t=r_t\left(m_t+\eta_t\right) \end{aligned} \tag{12}$$

where m_t is the central bank's estimate of the liquidity at the overnight market with a given supply of liquidity (ie $m_t = RB_{t-1}+ a_t^{CB} + q_t$, where a^{CB} is the central bank's estimate over the autonomous liquidity factors), η_t is a zero mean liquidity shock (ie the estimation error in the autonomous liquidity factors), λ_t measures the relative weight of the preferences over the two objectives, and finally the minimization is subjected to the inverse demand function $r_t = r_t(m_t + \eta_t)$ that gives the market rate of interest as a function of the money market liquidity. We can substitute the restriction directly into the loss function. Hence, the optimization problem can be written as:

$$\min_{m_t} L_t = \frac{1}{2}(1-\lambda_t)\left(\frac{\mathrm{E}\left[r(m_t+\eta_t)\right]-\overline{r}_t}{\overline{r}_t}\right)^2 + \frac{1}{2}\lambda_t\left(\frac{m_t-\overline{m}_t}{\overline{m}_t}\right)^2, \tag{13}$$

for which the first order condition is:

$$\frac{1-\lambda_t}{\overline{r}_t^2}\left\{\mathrm{E}\left[r_t(m_t^*+\eta_t)\right]-\overline{r}_t\right\}\frac{\partial \mathrm{E}\left[r_t(m_t^*+\eta_t)\right]}{\partial m_t} + \frac{\lambda_t}{\overline{m}_t^2}(m_t^*-\overline{m}_t) = 0. \tag{14}$$

The FOC implicitly determines the optimal liquidity supply, ie the amount of money market liquidity the central bank plans to supply in the tender operation $\left(q_t^* = m_1^* - RB_{t-1} - a_t^{CB}\right)$. From equation (14) we see, that the expected interest rate will be above (below) the target rate, if the optimal liquidity is above its target.[8] Consequently, if the expected market rate of interest with the target liquidity is above the target rate (ie $\mathrm{E}[r(\overline{m})] > \overline{r}$), both the optimal liquidity and the expected interest rate will be above their target values (ie $m^* > \overline{m}$, and $\mathrm{E}[r(m^*)] > \overline{r}$). Similarly, if $\mathrm{E}[r(\overline{m})] < \overline{r}$ ($\mathrm{E}[r(\overline{m})] = \overline{r}$) we'll have $m^* < \overline{m}$ and $\mathrm{E}[r(m^*)] < \overline{r}$ ($m^* = \overline{m}$ and $\mathrm{E}[r(m^*)] = \overline{r}$).

To illustrate the effect a change in the demand for liquidity has on the equilibrium liquidity and market rate of interest, figure 1 shows three different demand curves for money market liquidity (the thick curves), and an indifference curves based on the central bank's minimization problem (the thin curve). The (inverse) demand curves are derived by equation (10) with different values for the expected future interest rate. The higher is the expected interest rate for the second period, the larger is the demand with a given market rate for the first period. Now, only if the banks' demand for liquidity passes through $(\overline{m}, \overline{r})$, will the expected interest rate and the expected liquidity be at their target levels. Also, if the inverse demand curve at $\overline{m}$ is above (below) $\overline{r}$, both equilibrium liquidity and the expected market rate will be higher (lower) than the target values are.

Let's assume that the central bank chooses its liquidity target ($\overline{m}_t$) such that, first of all, it aims at supplying the market with the required reserves, and secondly, it wants to hold the market liquidity as stable as possible throughout the rest of the maintenance period. Hence, the liquidity target will always equal rdb_t, which at the first day of a two period maintenance period is simply the required reserves ($\overline{m}_1 = R$), and for the second day it's the sum of the daily reserve requirements minus the reserves held during the first day ($\overline{m}_2 = 2R - l_1 - \varepsilon_1 - sf_1$). Furthermore, we assume that the target rate $\overline{r}$ is chosen by the

[8] This results from the following:

$$\operatorname{sign}\left[\underbrace{\frac{1-\lambda}{\overline{r}^2}}_{>0}\left\{\mathrm{E}\left[r(l^*+\mu)\right]-\overline{r}\right\}\underbrace{\frac{\partial \mathrm{E}\left[r(l^*+\mu)\right]}{\partial l}}_{<0}\right] = \operatorname{sign}\left[\underbrace{-\frac{\lambda}{\overline{l}^2}}_{<0}(l^*-\overline{l})\right]$$

$$\Rightarrow \operatorname{sign}\left[\mathrm{E}\left[r(l^*+\mu)\right]-\overline{r}\right] = \operatorname{sign}\left[l^*-\overline{l}\right]$$

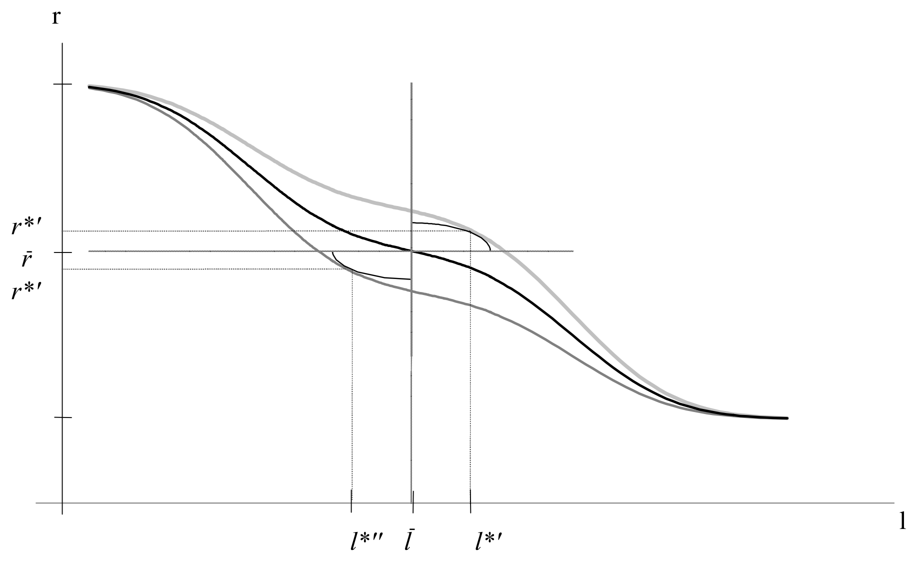

Figure 1: Money market equilibrium under different demand conditions.

ultimate goal of the central bank, and thus, is exogenous to the central bank's liquidity management. Next, we study what the optimal liquidity supply and the expected market rate of interest will be in a two period model.

3.1 Final day of the MP (no averaging)

Inserting equation (7) into the minimization problem of equation (13) gives the central bank's loss function for the final day of the reserves maintenance period[9]:

$$\begin{aligned} \min_{m_2} L_2 &= \frac{1}{2}(1-\lambda_2)\left(\frac{r_2^d + \left(r_2^m - r_2^d\right) G(2R - z_1 - m_2) - \overline{r}_t}{\overline{r}_t}\right)^2 \\ &+ \frac{1}{2}\lambda_2 \left(\frac{m_2 - \overline{m}_2}{\overline{m}_2}\right)^2, \end{aligned}$$

the FOC for which is:

$$\begin{aligned} & \frac{1-\lambda_2}{\overline{r_2}^2}\left\{r_2^d + \left(r_2^m - r_2^d\right) G(2R - z_1 - m_2^*) - \overline{r}_2\right\} \\ \times \quad & \left(r_2^m - r_2^d\right)\left(-g(2R - z_1 - m_2^*)\right) + \frac{\lambda_2}{\overline{m}_2^2}\left(m_2^* + \eta_2 - \overline{m}_2\right) = 0. \end{aligned} \tag{15}$$

[9]Note that we have substituted m_2 for l_2 of equation (7), as what matters here is the central bank's liquidite estimate instead of that of the banks.

Now, by inserting the liquidity target of the central bank ($\overline{m}_2 = 2R - z_1$), we can rewrite equation (15) as:

$$\frac{1-\lambda_2}{\overline{r}_2^{\,2}}\left\{r_2^d - \overline{r}_2 + \left(r_2^m - r_2^d\right)G(2R - z_1 - m_2^*)\right\} \quad (16)$$
$$\times \quad \left(r_2^m - r_2^d\right)\left(-g(2R - z_1 - m_2^*)\right) = \frac{\lambda_2}{\left(2R - z_1\right)^2}\left(2R - z_1 - m_2^*\right),$$

which implicitly defines the liquidity to be supplied as a function of the position of the interest rate target within the interest rate corridor, the unfulfilled amount of the reserve requirement for the whole period, and the distribution of shocks.

Now, the sign of the RHS of equation (16) depends on the equilibrium liquidity versus the required daily balances:

$$RHS\begin{cases} > 0, \text{ if } m_2^* < 2R - z_1 \\ = 0, \text{ if } m_2^* = 2R - z_1 \\ < 0, \text{ if } m_2^* > 2R - z_1 \end{cases},$$

while the sign of the LHS is given by sign $\left(\left(\overline{r}_2 - r_2^d\right)/\left(r_2^m - r_2^d\right) - G(2R - z_1 - m_2^*)\right)$. This means that the sign of the LHS depends on the location of the target rate within the interest rate corridor versus the probability of being forced to use the marginal lending facility to meet the reserve requirement with the equilibrium liquidity.

Let's assume for a while that the shock distribution is symmetric. Now, if the target rate of the central bank is the mid-point of the corridor (ie $\overline{r}_2 = 0.5\left(r_2^m + r_2^d\right)$), it's easy to see that the equilibrium supply of liquidity is just the target liquidity of the central bank regardless of the weight given to each of the two objectives (ie for all λ_2).[10] However, if the target rate lies in the upper half of the interest rate corridor (ie $\left(\overline{r}_2 - r_2^d\right)/\left(r_2^m - r_2^d\right) > 0.5$), the equilibrium supply of liquidity must be below the target (ie $m_t^* < 2R - z_1$) unless the weight given to interest rate in the objective function of the central bank is zero (ie as long as $\lambda_2 > 0$). In this case, the expected interest rate will be between the target rate and the mid-point ($\overline{r}_t \geq r_t^* \geq r_t^{mid}$, with strict inequalities when both objectives have positive weights, ie $0 < \lambda_2 < 1$). Similarly, when the target rate is in the lower half of the corridor, the equilibrium liquidity must be larger than the target amount, and the expected interest rate will again be between the target rate and the mid-point of the corridor ($\overline{r}_t \leq r_t^* \leq r_t^{mid}$, with strict inequalities when $0 < \lambda_2 < 1$).

If the shock distribution is left skewed (right skewed), we'll have $G(0) < 0.5$ ($G(0) > 0.5$). In this case, the central bank wouldn't be able to meet its targets for interest rate and liquidity with an interest rate corridor that is symmetric about the target rate. Hence, to have $r_t^* = \overline{r}_t$ and $m_2^* = \overline{m}_2$, the rates of the standing facilities should be set to locate the target rate at the lower (upper) half of the corridor. More specifically, the corridor should be set such that $\left(\overline{r}_2 - r_2^d\right)/\left(r_2^m - r_2^d\right) = G(0)$, which would produce $m_2^* = 2R - z_1$

[10] When the target rate is the mid-point of the interest rate corridor $\left(\overline{r}_2 - r_2^d\right)/\left(r_2^m - r_2^d\right) = 0.5$. Now, with symmetric shock distribution $G(2R - z_1 - m_2^*) = 0.5$ for $m_2^* = 2R - z_1$, in which case both RHS and LHS equal to zero.

and $\mathrm{E}[r_2] = r_2^d + \left(r_2^m - r_2^d\right) G(0) = \overline{r}_2$, ie the equilibrium liquidity and the expected market rate would equal their target values.

When the central bank wants to use the standing facilities as an independent signalling device, the equilibrium liquidity or the expected interest rate does not necessarily equal the corresponding target of the central bank. In such a case the relative deviations from the targets are determined by the preferences weighting parameter λ_2. The lower the value of λ_2 is, the more weight is put on the interest rate deviations. An extreme case is naturally $\lambda_2 = 0$, when the central bank is only interested in interest rate, and the liquidity supply will be determined simply by setting $r_2^d + \left(r_2^m - r_2^d\right) G(2R - z_1 - m_2^*) = \overline{r}_2$ (i.e. $G(2R - z_1 - m_2^*) = \left(\overline{r}_2 - r_2^d\right) / \left(r_2^m - r_2^d\right)$) and the expected market rate of interest will equal the target. At the other extreme ($\lambda_2 = 1$) the central bank cares only about providing the market with liquidity that will on average equal the reserve requirement. In this case, the central bank will provide the markets with $m_2^* = \overline{m}_2 = rdb_2$, and $\mathrm{E}[r_2] = r_2^d + \left(r_2^m - r_2^d\right) G(0)$. Hence, the relation with the expected market rate of interest and the target would depend on the asymmetry of both the shock distribution and the interest rate corridor.

For the rest of the paper we assume as a benchmark case that the central bank will set the rates of the standing facilities in order to meet the target for interest rate and the liquidity, or if the rates of the standing facilities are used as independent tools for signalling the monetary policy stance, the weight given to liquidity considerations during the *final day* of the reserves maintenance period is zero. The case in which the interest rate corridor is determined independently and there is a independent target also for liquidity is only discussed shortly.

3.2 First day of the MP (averaging)

We can rewrite equation (11) describing the expected market rate of interest for the first day of the maintenance period as:

$$\mathrm{E}[r_1] = \mathrm{E}_1[r_2] + (r_1^m - \mathrm{E}_1[r_2])\, G(-m_1) + \left(r_1^d - \mathrm{E}_1[r_2]\right)[1 - G(2R - m_1)]\,. \quad (17)$$

That is the expected value for the market rate at day 1 is the sum of the expected value for the following day and the probability weighted spreads between the current rates of the standing facilities and the overnight rate expected for the final day. Note that here the relevant liquidity estimate is m_1 instead of l_1 in equation (11).

Now, by inserting equation (17) and the central bank's target for liquidity ($\overline{m}_1 = R$) into the loss function, we get the following minimization problem for the central bank:

$$\begin{aligned} \min_{m_1} L_1 \;=\; & \frac{1}{2}\frac{1-\lambda_1}{(\overline{r}_1)^2}\{\mathrm{E}_1[r_2] + (r_1^m - \mathrm{E}_1[r_2])\, G(-m_1) \\ & + \left(r_1^d - \mathrm{E}_1[r_2]\right)[1 - G(2R - m_1)] - \overline{r}_1\}^2 \\ & + \frac{1}{2}\frac{\lambda_1}{R^2}\left(\mathrm{E}[m_1 + \eta_1] - R\right)^2, \end{aligned}$$

for which the FOC is:

$$\begin{aligned}&\frac{1-\lambda_1}{(\overline{r}_1)^2}\{\mathrm{E}_1[r_2]+(r_1^m-\mathrm{E}_1[r_2^*])\,G(-m_1^*)\\&+\left(r_1^d-\mathrm{E}_1[r_2]\right)[1-G(2R-m_1^*)]-\overline{r}_1\}\\&\times\left[\left(r_1^d-\mathrm{E}_1[r_2]\right)g(2R-m_1^*)-(r_1^m-\mathrm{E}_1[r_2])\,g(-m_1^*)\right]\\&+\frac{\lambda_1}{R^2}(m_1^*-R)=0.\end{aligned}\tag{18}$$

Equation (18) implicitly defines the optimal liquidity supply for the first day of the reserves maintenance period as a function of current and expected future central bank rates, the central bank preference weighting parameter, the liquidity shock distributions and the reserve requirement. Let's next study the optimal liquidity supply in two parts. First, we assume the interest rate expectations to be static, and after that we analyze the effect of a change in the interest rate expectations.

3.2.1 Static interest rate expectations

When the interest rate corridor is not used as an independent signalling device, the expected market rate for the final day of the maintenance period equals the target rate expected to prevail during that day (ie $\mathrm{E}_1[r_2]=\mathrm{E}_1[\overline{r}_2]$). With static expectations over the central bank target rate ($\mathrm{E}_1[\overline{r}_2]=\overline{r}_1\equiv\overline{r}$) we may write the FOC of equation (18) as:

$$\begin{aligned}&\frac{1-\lambda_1}{(\overline{r})^2}\left\{(r_1^m-\overline{r})\,G(-m_1^*)+\left(r_1^d-\overline{r}\right)[1-G(2R-m_1^*)]\right\}\\\times\quad&\left[\left(r_1^d-\overline{r}\right)g(2R-m_1^*)-(r_1^m-\overline{r})\,g(-m_1^*)\right]=\frac{\lambda_1}{R^2}(R-m_1^*).\end{aligned}\tag{19}$$

If the shock distribution is symmetric, the expected market rate of interest at the final day equals the mid-point of the interest rate corridor expected for that day, which under static expectations is the mid-point of today's corridor (ie $\mathrm{E}_1[r_2^*]=\mathrm{E}_1[\overline{r}_2]=\mathrm{E}\left[r_2^{mid}\right]=r_1^{mid}\equiv r^{mid}$). Hence, for symmetric shocks equation (19) can be further reduced into:

$$\begin{aligned}&\frac{1-\lambda_1}{\left(r^{mid}\right)^2}\left(r_1^d-r^{mid}\right)^2\{1-G(-m_1)-G(2R-m_1)\}\\\times\quad&[g(2R-m_1)+g(-m_1)]\\=\quad&\frac{\lambda_1}{R^2}(R-m_1^*)\end{aligned}\tag{20}$$

The RHS is decreasing in m_1^*, and is positive for $m_1^*<R$, zero for $m_1^*=R$ and negative for $m_1^*>R$. The sign for the LHS is given by:

$$\mathrm{sign}\,\{G(m_1-2R)-G(-m_1)\}=\begin{cases}-,\ \text{if } m_1^*<R\\0,\ \text{if } m_1^*=R\\+,\ \text{if } m_1^*>R\end{cases}$$

The FOC is fulfilled if and only if $m_1^* = R$. Thus, we may conclude that *under static interest rate expectations, the central bank will provide the markets with liquidity that equals the target liquidity, and the expected market rate of interest will be at the level targeted by the central bank regardless of the preferences weighting parameter if the shock distribution is symmetric.*[11]

For asymmetric shock distributions, the sign of the LHS of equation (19) is:

$$\text{sign } LHS\ (19) = \begin{cases} +, \text{ if } & \left(r_1^m - \overline{r}\right) G(-m_1^*) < \left(\overline{r} - r_1^d\right)\left[1 - G(2R - m_1^*)\right] \\ 0, \text{ if } & \left(r_1^m - \overline{r}\right) G(-m_1^*) = \left(\overline{r} - r_1^d\right)\left[1 - G(2R - m_1^*)\right] \\ -, \text{ if } & \left(r_1^m - \overline{r}\right) G(-m_1^*) > \left(\overline{r} - r_1^d\right)\left[1 - G(2R - m_1^*)\right] \end{cases}$$

That is the sign of the LHS depends on the size of the probability weighted expected cost of marginal lending versus the probability weighted expected cost of having to use the deposit facility. Now, the central bank will provide liquidity exactly according to the target (ie $m_1^* = R$) only if $[1 - G(R)] = G(-R)$. With asymmetric shock distribution this is not (necessarily) the case, hence, the liquidity provision is going to depend on the asymmetry of the distribution and the size of the reserve requirement. Eg if the probability of going overdrawn with $m_1 = R$ is larger (smaller) than the probability of having reserves in excess of the reserve requirement ($G(-R)/\left[1 - G(R)\right] > 1$ ($G(-R)/\left[1 - G(R)\right] < 1$)), the expected interest rate with such a liquidity policy would be above (below) the target rate, and consequently, the optimal liquidity provision would be $m_1^* > R$ ($m_1^* < R$). In this case, the liquidity supply would depend also on the preference weighting of the central bank. The higher the λ_1, the more the interest rate would differ from its target and the closer the equilibrium liquidity would be to the reserve requirement.

3.2.2 Change in central bank rates expected

The derivation of the optimal liquidity supply becomes a bit more complicated when the banks expect the central bank to change its target rate within the reserves maintenance period, as in such a case the expected change in the rate affects the demand for liquidity before the change actually occurs. Now, by envelope theorem we know that the change in optimal liquidity supply when the expected future interest changes is given by:

$$\frac{dm_1(\mathrm{E}_1\left[r_2\right])}{d\mathrm{E}_1\left[r_2\right]} = -\frac{\partial^2 L_1/\partial m_1 \partial \mathrm{E}_1\left[r_2\right]}{\partial^2 L_1/\partial m_1^2}.$$

As the central bank minimizes L_1, we know that the denominator on the right hand side is positive due to the second order condition for minimization. Thus,

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11

$$\begin{aligned} \mathrm{E}\left[r_1^{on}\right] &= r^{mid} + \left(r_1^m - r^{mid}\right) G(-m_1^*) + \left(r_1^d - r^{mid}\right)\left[1 - G(2R - m_1^*)\right] \\ &= r^{mid} + \left(r_1^m - r^{mid}\right) G(-R) + \left(r_1^d - r^{mid}\right) G(-R) = r^{mid} \end{aligned} \quad (21)$$

we have:

$$\text{sign}\frac{dm_1(\text{E}_1[r_2])}{d\text{E}_1[r_2]} = \text{sign} - \frac{\partial^2 L_1}{\partial m_1 \partial \text{E}_1[r_2]}.$$

That is, the sign of the derivative of the optimal liquidity w.r.t. the expected future interest rate is minus of the sign of the second cross-partial of the loss function w.r.t. m_1 and $\text{E}_1[r_2]$. For the first day's minimization problem we have:

$$\begin{aligned} \frac{\partial^2 L_1}{\partial m_1 \partial \text{E}_1[r_2]} &= \frac{1-\lambda_1}{(\overline{r}_1)^2}\{[G(2R-m_1)-G(-m_1)] \\ &\times \left[\left(r_1^d - \text{E}_1[r_2]\right)g(2R-m_1) - \left(r_1^m - \text{E}_1[r_2]\right)g(-m_1)\right] \\ &+ \left\{\text{E}_1[r_2] - \overline{r}_1 + \left(r_1^m - \text{E}_1[r_2]\right)G(-m_1) + \left(r_1^d - \text{E}_1[r_2]\right)\right. \\ &\times [1-G(2R-m_1)]\}[g(-m_1)-g(2R-m_1)]\}. \end{aligned} \tag{22}$$

Inserting the FOC into equation (22), we can derive the sign of the central bank's optimal liquidity response to an increase in the expectations over the future interest rate from:

$$\begin{aligned} &\text{sign} - \frac{1-\lambda_1}{(\overline{r}_1)^2}[G(2R-m_1)-G(-m_1)] \\ &\times \left[\left(r_1^d - \text{E}_1[r_2]\right)g(2R-m_1) - \left(r_1^m - \text{E}_1[r_2]\right)g(-m_1)\right] \\ &-\frac{\lambda_1}{R^2}(R-m_1^*)\frac{[g(-m_1)-g(2R-m_1)]}{\left[\left(r_1^d - \text{E}_1[r_2]\right)g(2R-m_1^*) - \left(r_1^m - \text{E}_1[r_2]\right)g(-m_1^*)\right]}. \end{aligned} \tag{23}$$

which is positive (at least when the shock distribution is symmetric and single peaked) as long as there is any weight given to the interest rate considerations (ie $\lambda < 1$).[12] Therefore, we may conclude that *if the central bank pays any attention to the money market interest rates, the central bank will provide more liquidity the higher the expected future interest rate is.*

Now, we know, that the expected value of the market rate for today will be above the target rate when an interest rate hike is expected. However, the simultaneous effect of both the expected interest rate hike and the increasing liquidity supply on the expected market rate for today is not necessarily monotonic. That is, when $m_1^* > \overline{m}_1$, we'll have $\text{E}_1[r_1] > \overline{r}_1$, however, if $m_1^* > \widehat{m}_1 > \overline{m}_1$ we can not conclude that $\text{E}_1[r_1]$ is above $\widehat{r}_1$, for $\widehat{r}_1 = \text{E}_1[r_1(\widehat{m}_1 + \eta_1)] > \overline{r}_1$. The effect of a change in the expected future rate on the expected rate for today is given by:

$$\begin{aligned} \frac{\partial \text{E}_1[r_1]}{\partial \text{E}_1[r_2]} &= \{G(2R-m_1)-G(-m_1)\} \\ &\quad - \left[\left(\text{E}_1[r_2] - r_1^d\right)g(2R-m_1) + \left(r_1^m - \text{E}_1[r_2]\right)g(-m_1)\right]\frac{\partial m_1}{\partial \text{E}_1[r_2]}, \end{aligned} \tag{24}$$

[12] This results from the following: $(1-\lambda_1)(\overline{r}_1)^2[G(2R-m_1)-G(-m_1)] > 0$; $\left[\left(r_1^d - \text{E}_1[r_2]\right)g(2R-m_1) - \left(r_1^m - \text{E}_1[r_2]\right)g(-m_1)\right] < 0$, as long as the expected future rate is within the interest rate corridor; if we initially have $m_1^* > R$, $(R-m_1^*) < 0$, and with symmetric single peaked shockdistribution $[g(-m_1)-g(2R-m_1)] < 0$, while when initially $m_1^* < R$ ($m_1^* = R$) we have $(R-m_1^*) > 0$ ($(R-m_1^*) = 0$) and $[g(-m_1)-g(2R-m_1)] > 0$.

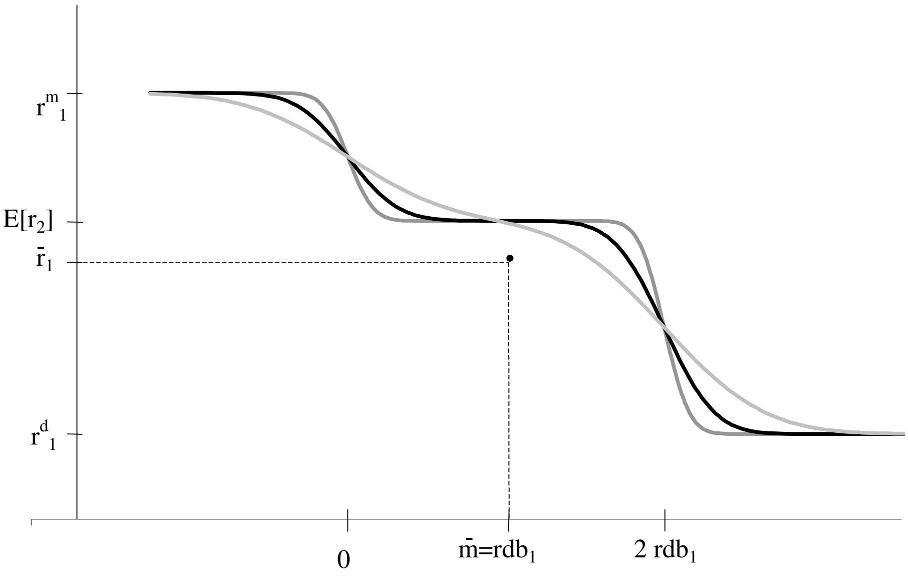

Figure 2: The effect of uncertainty on the demand for liquidity.

where the first term on the RHS is the probability of not having to rely on the standing facilities today with m_1 units of liquidity, while the second term gives the effect of the change in the probability of being forced to use the standing facilities on the expected cost of using them. From above we know that $\partial m_1 / \partial \mathrm{E}_1 [r_2] > 0$. Thus, the expected interest rate for today will increase due to an increase in the expected rate for tomorrow, if $\{G(2R - m_1) - G(-m_1)\} > \left[\left(\mathrm{E}_1 [r_2] - r_1^d \right) g(2R - m_1) + \left(r_1^m - \mathrm{E}_1 [r_2] \right) g(-m_1) \right] \partial m_1 / \partial \mathrm{E}_1 [r_2]$. The functional form of (24) is tedious, hence, we are satisfied with the fact that the higher the preferences weighting parameter is (ie the more weight is given to the liquidity considerations) the more likely it is that an increase in the expected future rate will lead to a positive change in the expected market rate for today. This comes from the fact that $\partial^2 m_1 / \partial \mathrm{E}_1 [r_2] \partial \lambda_1 < 0$.

Let us next examine graphically a few examples to get some intuition how the effect of expectations of future rates on the expected rate for today depends on the distribution of shocks (hence the interest rate elasticity of the demand for liquidity) and on the size of the expected change.

Figure 2 shows a set of inverse demand functions for liquidity at different levels of uncertainty about the development of liquidity. All the demand functions are calculated with normally distributed zero mean shocks, reserve requirement (ie the target liquidity) at 2000, standard deviations of (250; 500; 1000), and under interest rate hike expectations of 25 basis points[13]. The curvature of the inverse demand functions increase as the shock distributions get wider. Now, *the more there is liquidity uncertainty, the more likely the central bank's reaction to higher interest rate expectations is to let both the*

[13] We used the following rate assumptions while drawing the figure; target rate at 3%, deposit rate at 2%, marginal lending rate at 4% and the expected rate for the second period 3,25%.

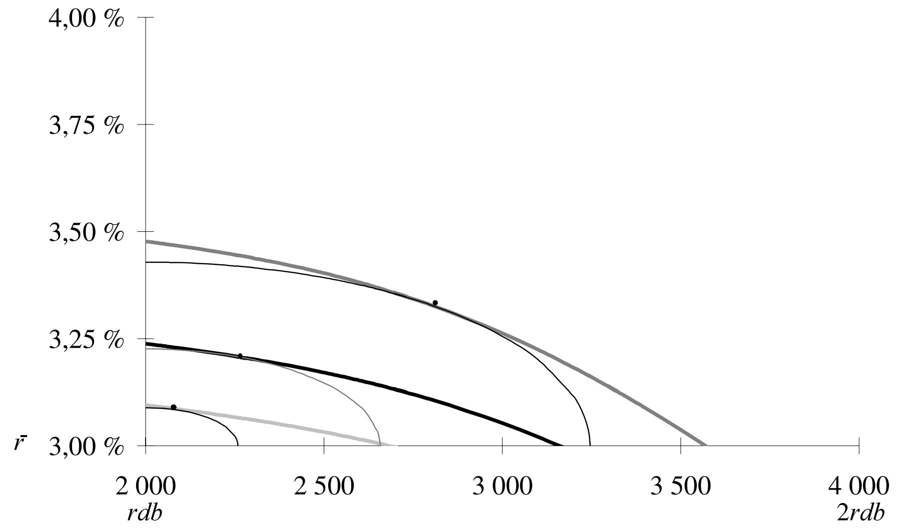

Figure 3: The effect of interest rate hike expectations on the equilibrium liquidity supply, with high liquidity uncertainty.

interest rate and the liquidity to differ from the target. Figure 3 illustrates this effect when the standard deviations of the liquidity shocks is half of the reserve requirement. The equilibrium levels for the interest rate and liquidity naturally depend crucially on the central bank's preferences weighting parameter. However, it's quite obvious that when the inverse demand function is nearly linear, they both increase with the expectations. The better the evolution of the money market liquidity can be estimated, the larger will the interest rate elasticity of the demand for liquidity be with liquidity itself close to the target level. Thus, with low liquidity uncertainty, it's likely that the equilibrium interest rate-liquidity point will lie close to either of the targets, ie the central bank will let the expectations to be shown mostly either in the liquidity or in the market rate. This is illustrated with figure 4, where the standard deviation of the liquidity shock is 10% of the reserve requirement. Furthermore, with reasonably low liquidity uncertainty it's possible that the optimal reaction of a central bank minimizing the quadratic differences is to jump from tight control of liquidity into tight control of interest rates as the interest rate hike expectations reach high enough level. This case is illustrated in figure 5. Next we turn to analyzing the bid behaviour of the banks in the tenders.

4 Bidding in tenders

There are n homogenous banks eligible for participating the tenders. Each of the banks can place up to 10 bids in every tender. A bid consists of a quantity-interest rate pair, in which the specified interest rate is the one at which the bidder wants to enter into transaction, and the quantity is the amount it wants to transact with at the given rate. Let's arrange the bids of bank i

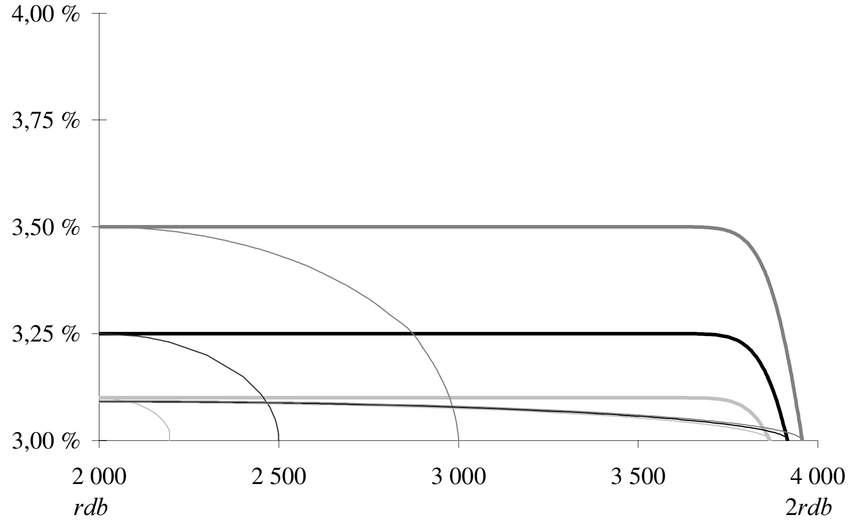

Figure 4: The effect of interest rate hike expectations on the equilibrium liquidity supply, with low liquidity uncertainty.

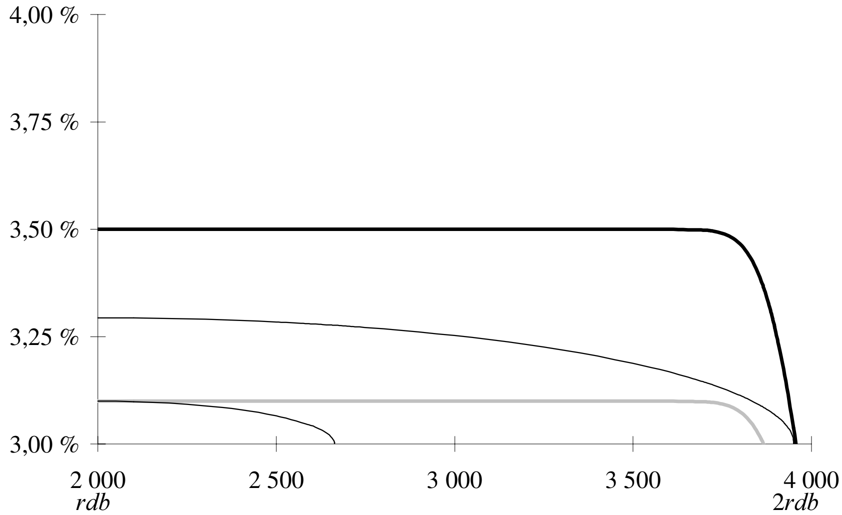

Figure 5:

in descending order, such that the bid amount with the highest bid rate $r_{i,1}^T$ is denoted by $b_{i,r_{i,1}^T}$, with the second highest rate $r_{i,2}^T$ by $b_{i,r_{i,2}^T}$, and so on (ie $r_{i,1}^T > r_{i,2}^T > ... > r_{i,10}^T$).

The monopoly supplier of the liquidity (the central bank) aims at supplying m^* (defined in previous chapter) units of liquidity to the market, regardless of the shape of the demand schedule given by the banks; ie unlike in most auctions here the seller is not trying to maximize its income from the tender. However, we assume that if collusion (or collusion like behaviour) between the bidders were detected by the central bank, it would reduce the supply below m^*, which is costly enough for the banks to deter any collusive behaviour.

The ex post value for a unit of liquidity is the secondary market price for it, and is common to all banks. As shown above, this price is a decreasing function of the liquidity, and it always lies within the interest rate corridor set by the rates of the standing facilities. Therefore, without a reserve price[14], there will always be enough bids for the central bank to be able to provide liquidity according to m^*.[15] The realizing market rate of interest is a random variable, due to the liquidity shock η. However, the expected market rate of interest is common to all banks, as *we assume that either there is an explicitly announced central bank target for the market rate (in which case $E[r] = r^{\text{target}}$) or the banks receive a common signal on the forthcoming market rate* (in case of the euroarea eg the quotations on the two week EONIA-swap rate could serve as such a signal). Hence, $\mathrm{E}_i\left[r\left(m^*+\eta\right)\right] = \mathrm{E}[r\left(m^*+\eta\right)]$ for all banks, as long as a reserve price is not used or if it is ineffective. We return later to the case where we have binding reserve price for the bids.

The analysis of the bid behaviour is divided into two parts. We begin with the multiple rate procedure (*American or discriminatory auction*), in which the allotment rate for accepted bids is the bid rate (ie the rate specified in the given bid). After that we move to single rate procedure (*Dutch or uniform price auction*), where the marginal rate[16] of the allotment is applied to all accepted bids. The ECB has used multiple rate auctions in all main refinancing operations conducted in the form of variable rate tenders and also in most of the longer term refinancing operations. Dutch auction has been applied so far only in the first two longer tern refinancing operations.

4.1 Multiple rate auctions

The profit for bank i from participating a variable rate tender, in which the multiple rate procedure is applied, is simply the allotment volume weighted sum of the differences between the expected market rate and the bid rate. It

[14]The reserve price is the lowest price at which the counterparties are allowed to submit bids in auctions.

[15]This follows from the fact that a bank can make positive profit by borrowing liquidity from the auction and lending it back to the central bank, if the price of liquidity at the tender is below the deposit rate.

[16]The *marginal rate of allotment* is a term used by the ECB for the lowest rate at which bids are accepted. In auction literature this kind of rate is usually refered to as the stop-out price.

is given by:

$$\Pi = \sum_{j=1}^{10} q_{i,r_{i,j}^T} \left(\mathrm{E}\left[r\left(m^* + \eta \right) \right] - r_{i,j}^T \right) \tag{25}$$

$$s.t.\ q_{i,r_{i,j}^T} = \begin{cases} b_{i,r_{i,j}^T},\ \text{if } B_{r_{i,j}^T} \leq m^* \\ \frac{m^* - B_{r_{j-1}^T}}{b_{r_{i,j}^T}} b_{i,r_{i,j}^T},\ \text{if } B_{r_{j-1}^T} < m^* < B_{r_{i,j}^T} \\ 0,\ \text{if } B_{r_{j-1}^T} \geq m^* \end{cases},$$

where $b_{r_{i,j}^T}$ is the banking sector wide aggregate bid at $r_{i,j}^T$, $B_{r_{i,j}^T}$ denotes the cumulative bid amount of all banks at interest rate equal to or above $r_{i,j}^T$, and $B_{r_{j-1}^T}$ denotes the cumulative bid amount at all rates higher than $r_{i,j}^T$. The highest rate at which $B_{r_{i,j}^T}$ exceeds the amount the central bank is willing to provide to the market is called the marginal rate of the allotment (ie $r^{\text{marginal}} \equiv r_j^T$ that satisfies $B_{r_{j-1}^T} < m^* < B_{r_j^T}$)[17]. The net profit from bid $b_{i,r_{i,j}^T}$ is the difference between the bid rate and the forthcoming market rate of interest times the amount of liquidity actually provided to the bank with the given rate. Now, the bank will not be allotted any liquidity for a bids at rates below the marginal rate, whereas the bids at rates above the marginal rate are accepted in full (ie in this case the allotted amount will equal the bid amount). Furthermore, for bids at the marginal rate, the central bank will use pro rata rationing in the allotment procedure. This means that the bids at the marginal rate are accepted only partially, such that the share of the total allotment for each bid at the marginal rate equals its fraction of the total bids at the rate.

It's easy to see that optimal bid is zero at all rates exceeding the expected market rate of interest (ie $b_{i,r_{i,j}^T}^* = 0$ when $r_{i,j}^T >$E$[r\,(m^*)]$), as otherwise the bid would yield negative expected profits. Also, bids at rates below the marginal rate yield zero profit, as there will not be any residual supply for liquidity left at these rates. *Hence, positive expected profit can be made only with bids at rates between the marginal rate and the expected market rate of interest.*

Let's suppose for a moment that the marginal rate of the allotment is lower than the expected market rate of interest (ie $r^{\text{marginal}} <$E$[r\,(m^* + \eta)]$). Now, consider bank i placing a bid for the entire quantity of the allotment at a rate infinitesimally above the marginal rate ($b_{i,r^{\text{marginal}}+\in} = m^*$, where $r^{\text{marginal}} + \in <E[r\,(m^* + \eta)]$). Now, the allotment for bank i with this bid must be at least as large as it would be with a bid at the marginal rate. Thus, the bank would be making positive profit with this kind of behaviour. However, this incentive applies for all banks, therefore, the optimal policy for another bank would be to place a bid large enough to satisfy the whole supply at a rate that is infinitesimally higher than that of bank i's, and so forth until the marginal rate reaches the expected market rate of interest. As a consequence, at the equilibrium all banks place large enough bids for the central bank to be able to provide liquidity according to its target (m^*) at the price equalling the expected market rate, ie $B_{\mathrm{E}[r(m^*+\eta)]} \geq m^*$ and $r_{i,j}^T =$E$[r\,(m^* + \eta)]$ for

[17]Note that we have omitted the bank index i from subscripts of the rates that apply for all banks.

all i and j. *The aggregate demand schedule the central bank receives is flat at* $E[r(m^* + \eta)]$ *at least up to* m^*, *and the bids at rates below the expected market rate can be ignored as they are ineffective.*

The analysis of this section applies to the pure case where the bidding of the banks was not restricted by a reserve price. The next section will deal with the case where the central bank explicitly states the minimum rate for bids to be accepted.

4.1.1 Minimum bid rate

The ECB has applied a reserve price for bids in its main refinancing operations conducted as variable rate tenders. This so called *minimum bid rate* (r^{minimum}) is the minimum interest rate at which the counterparties may place their bids in the tenders. The purpose of this section is to analyze the effect the minimum bid rate has on the bid behaviour.

From the section above, we know that the bids are never placed at higher rates than the expected market rate of interest. Let's first assume that the expected market rate of interest with the liquidity provided by the central bank was below the minimum bid rate (ie $\mathrm{E}\left[r\left(m^{\text{provided by the CB}} + \mu\right)\right] < r^{\text{minimum}}$). Now, it would be profitable for bank i not to participate the tender (even at the minimum bid rate) and to buy the missing liquidity from the interbank market. As this applies for all banks, the central bank would not be able to provide the market with liquidity in excess of the amount z implicitly given by $\mathrm{E}[r(z + \mu)] = r^{\text{mimimum}}$. Therefore, *the expected market rate of interest will always be at the minimum bid rate or above it.* As a consequence, the maximum value for the minimum bid rate should be the interest rate targeted by the central bank.[18]

Now, even if the minimum bid rate is set at or below the target rate of the central bank, the reserve price can either be restrictive or it might as well be inefficient. Let's first assume that the equilibrium liquidity estimated by the central bank (m^*) is such that with it the expected market rate of interest is at the level of or above the minimum bid rate, ie $\mathrm{E}[r(m^* + \mu)] \geq r^{\text{minimum}}$. Now, the reserve price set for the liquidity will not be restrictive, because, as shown in the section above, the banks will bid for at least m^* at the expected market rate. On the contrary, assume that the expected market rate with the estimated equilibrium liquidity was below the minimum bid rate (ie $\mathrm{E}[r(m^* + \mu)] < r^{\text{minimum}}$). In this case, if the central bank were able to provide the market with m^*, it would be profitable for all banks individually not to participate the tender, but to borrow the liquidity needed from the interbank market. As a consequence, the central bank would not be able to provide the estimated equilibrium liquidity to the market. This situation would be similar to the case of fixed rate tenders, with central bank fulfilling all the

[18] Assume the contrary: $r^{\text{minimum}} > \overline{r}$. Now, the expected market rate and the expected liquidity will be above those of the levels targetted by the central bank (ie $\mathrm{E}[r] > \overline{r}$ and $m > \overline{m}$). It is possible that this does not change the optimal reaction of the central bank when a rate hike is expected. However, this can never be optimal when the interest rate expectations are neutral or when a rate cut is expected.

bids (also referred to as full allotment).[19] Here the counterpart for the fixed tender rate would be the minimum bid rate that is fixed and made public in the tender announcement. Välimäki (2001 and 2002) show that in fixed rate tenders with full allotment procedure, the banks will bid for such liquidity that with it the expected market rate of interest will equal the tender rate (here the minimum bid rate). Therefore, we expect that the bidding in variable rate tenders where $\mathrm{E}[r\,(m^{*}+\mu)] < r^{\mathrm{minimum}}$, is such that the aggregate bid amount is implicitly given by $\mathrm{E}[r\,(b+\mu)] = r^{\mathrm{minimum}}$, and the central bank will accept all the bids placed into the tender. This means that *when the minimum bid rate is effective, the market liquidity will be below and the expected market rate will be above the level preferred by the central bank.*

When are we likely to see the minimum bid rate becoming effective? If the banks expect the central bank to lower its target rate between the operations still remaining during the ongoing reserves maintenance period, we know that estimated equilibrium rate will be below the current target rate (ie $\mathrm{E}[r\,(m^{*}+\eta)] < \overline{r}$) as long as the central bank gives positive weight to the market liquidity. Thus, in the case where the interest rate is expected to be cut, the smaller the difference between the target rate and the minimum bid rate is, the more likely it is that the equilibrium (estimated by the central bank) rate will below the minimum bid rate. However, with expectations of a higher interest rate, the minimum bid rate should never become effective, as in such a case we will have $\mathrm{E}[r\,(m^{*}+\eta)] > \overline{r} \geq r^{\mathrm{minimum}}$.

Now, after having established the consequences of the minimum bid rate, we turn to a phenomenon that is closely related to the case in which the minimum bid rate becomes an effective restriction for bidding.

4.1.2 Underbidding

When the ECB switched from fixed to variable rate tenders, it stated in the press release following the decision (dated 8 June 2000) that "*For the purpose of signalling the monetary policy stance, the minimum bid rate is designed to play the role performed, until now, by the rate in fixed rate tenders*". If this is interpreted as $\overline{r_t} = r_t^{\mathrm{minimum}}$, we expect (according to the analysis in the section above) the minimum bid rate to become effective whenever the banks expect the rate to be cut during the rest of the current reserves maintenance period.

In connection with the ECB variable rate tenders the term "underbidding" has been used several times by market players and financial press (Bindseil (2002)). According to Bindseil underbidding refers to the lack of bids in a fixed rate tender, such that the central bank can not allot the liquidity actually needed by the banks to fulfill smoothly their reserve requirements. Now, even though Bindseil applies the term to fixed rate tenders, this is also a feature of variable rate tenders with minimum bid rate, which is apparent as Bindseil

[19] For further information on the fixed rate tenders and the banks' behaviour in such tenders see Välimäki (2001 and 2002).

himself refers to four cases of underbidding within the period the ECB has been using the variable rate tenders.

If underbidding is understood as a phenomenon in which the central bank will not allot according to the smooth reserve holding, it does not necessarily refer only to cases in which the minimum bid rate is binding. If interpreted this way, there would be underbidding whenever $m^* < \bar{l}$, even though in this case the deviations from smooth reserves keeping would be intentional (as long as $b \geq m^*$). If the central bank is purely liquidity oriented (in the model of the previous chapter $\lambda_1 = 1$) underbidding is closely related to a binding minimum bid rate, as in such a case underbidding will occur when the optimal bid of the banks is lower than the amount desired by the central bank (ie $b < m^* = \bar{l}$). Furthermore, we will, here, restrict to use the term underbidding only to refer to cases in which the aggregate bid by the banks is lower than the equilibrium liquidity estimated by the central bank (ie $b < m^*$). This is also the situation when the reserve price for liquidity is efficient.

Is underbidding a problem for the central bank? To get an answer for this question, we must look at the motives of a central bank to introduce the reserve price for bidding into its operational framework. Now, if a liquidity-oriented central bank used the minimum bid rate as a signalling device of the monetary policy stance, underbidding would naturally be a problematic phenomenon, since underbidding would entail that the central bank is not in charge over the level of money market liquidity. In this case, underbidding and the subsequent loss of control over the liquidity is the price the central bank pays for having the minimum bid rate as a method for signalling the monetary policy stance.

On the contrary, let's assume for a moment that the loss function of the central bank depends on the interest rate expectations of the banks, such that the central bank is purely liquidity-oriented ($\lambda_1 = 1$) when the interest rates are expected to be raised in the near future, while it is purely interest rate-oriented ($\lambda_1 = 0$) when the rates are expected to be lowered. Now, if the minimum bid rate equals the central banks target rate, we know from above that when the banks expect a rate cut they will hold back their bids to the amount at which the expected market rate equals the minimum bid rate which also is the target of the central bank. Also, in this framework the banks will bid enough for the central bank to be in control of liquidity when the rate is expected to be raised. If the central bank had this kind of asymmetric reaction function to the interest rate expectations, the so called underbidding should not be a major problem for it.

Moreover, the underbidding might be a bit more problematic in the framework the ECB actually applies than in the model framework used in this paper. The difference between these two is that, the maturity of the ECB weekly main refinancing operations is two weeks whereas the maturity of the operations in the model developed in the previous chapter is one period. The overlapping nature of the ECB operations adds an extra incentive for the banks to lower their bids. We illustrate this with a simple example.

Underbidding and overlapping operations

Let's assume that in its liquidity provision the central bank uses variable rate tenders with minimum bid rate set at the level of its interest rate target ($r_t^{\text{minimum}} = \overline{r}_t$). Furthermore, assume that there are two operations left during the current reserves maintenance period, and the target rate (as well as the minimum bid rate) of the central bank is expected to be cut between the two operations. Moreover, the central bank is expected to allot liquidity according to its interest rate target in the last operation; consequently, the expected market rate for the last period (ie from the settlement of the last operation until the end of the maintenance period) equals the expected target rate for it.

Firstly, if the maturity of an operation is one period (ie to the settlement of the next operation), we know that the amount of liquidity the banks bid for (at the minimum bid rate) in the first operation is just the amount at which the expected market rate of interest equals the minimum bid rate. Thus, no positive profits can be made by substituting lending between the central bank operation and the interbank market.

On the contrary, let's assume that also the first operation matures at the end of the maintenance period, ie the maturity of the first operation is two periods. Now, in order for not to have an open profit opportunity between central bank and interbank lending, the average (effective) market rate for the two subperiods must equal the rate at which the liquidity is borrowed from the central bank for the two periods (ie the bid amount in the first operation is implicitly given by $0.5(\mathrm{E}[r(b_1+\mu_1)]+\mathrm{E}[\overline{r}_2]) = r_1^{\text{minimum}}$). If this was not the case, it would be profitable for all banks individually to increase the bid amount when $\mathrm{E}[r(b_1+\mu_1)] < 2r_1^{\text{minimum}} - \mathrm{E}[\overline{r}_2]$ or to decrease it when $\mathrm{E}[r(b_1+\mu_1)] < 2r_1^{\text{minimum}} - \mathrm{E}[\overline{r}_2]$. This would occur until the equality was established again. When the central bank is expected to lower its target rate, we have $\mathrm{E}[r(b_1+\mu_1)] = 2r_1^{\text{minimum}} - \mathrm{E}[\overline{r}_2] > r_1^{\text{minimum}}$; ie the expected market rate of interest is higher than the minimum bid rate. This means that the overlapping nature of the central bank operations increases the underbidding; the money market liquidity will be even further below the estimated equilibrium liquidity. The overlap of the maturities of consecutive operations together with reserve price for bidding lead to the perverse situation in which the expected market rate for the first period increases above the target rate when the target rate is expected to be lowered in the following operation. Furthermore, we have seen that if the central bank places any weight for liquidity considerations, the expected market rate for the first period will increase above the target when the target is expected to be raised in the following operation. *Thus, the expected market rate for the first period increases above the target rate whenever the central bank is expected to change its target, no matter in which directions the target is expected to be moved.*

Next we will turn into analyzing the other procedure available for the central bank while applying variable rate tenders – namely the single rate auctions.

4.2 Single rate auctions

The profit for bank i from participating in a single rate variable rate tender is given by:

$$\Pi = \sum_{j=1}^{10} q_{i,r_{i,j}^T} \left(\mathrm{E}\left[r\left(m^* + \eta \right) \right] - r^{\text{marginal}} \right) \tag{26}$$

$$s.t.\ q_{i,r_{i,j}^T} = \begin{cases} b_{i,r_{i,j}^T}, \text{ if } B_{r_{i,j}^T} \leq m^* \\ \frac{m^* - B_{j-1}}{b_{r_{i,j}^T}} b_{i,j}, \text{ if } B_{r_{j-1}^T} < m^* < B_{r_{i,j}^T} \\ 0, \text{ if } B_{r_{j-1}^T} \geq m^* \end{cases} .$$

The only difference between the profit maximization problem in multiple rate tenders and single rate tenders is that in the former the cost of liquidity acquired is the bid rate whereas in the latter the marginal rate for the whole allotted amount. It's easy to see that the equilibrium outcome of the case with multiple rate tenders constitutes an equilibrium also in this case. A single bank can not make positive profit by bidding at rates above or below the expected market rate, when the aggregate bids of the other banks at the expected market rate is at least m^*, as bids at rates below this rate will be disregarded, and bids at rates above this level will provide zero profit (or negative profit if the bid is large enough to raise the marginal rate of the allotment). However, with the single rate procedure, there are plenty of other potential bidding equilibria.

Back and Zender (1993) analyze auctions on divisible goods, and show that for any given price that falls between the reserve price applied in the auction and the value of the (divisible) good being auctioned, there is a symmetric pure-strategy equilibrium in which the money received by the seller is just that price. In this setting this would mean that, the banks should be able to maintain a bidding strategy in which the marginal rate of the tender would be below the expected market rate, by placing very steep demand curves in which the inframarginal bid rates are relatively high, as these bids are never marginal (ie they do not affect the marginal rate), thus, they are costless for the banks to submit. However, in a later paper Back and Zender (2001) show that if the seller has the option to cancel part of the supply after observing the bids, it would eliminate many of the "collusive seeming" equilibria of the auction. Furthermore, in equilibrium the seller will still always sell the full quantity.

To take into account the potentially adverse effect of the collusive equilibria under single rate auctions, we assume the central bank to cut back the intended supply form m^*, if it detects collusive behaviour by the banks. This punishes the banks immediately by raising the marginal rate above the expected market rate of interest. The banks know that the central bank will punish collusive bidding, which eliminates the equilibria with demand schedules that are steeper than the true inverse demand functions would suggest. *In this case, the outcome of the single rate auction procedure will resemble that of the multiple rate procedure, ie the central bank is bid for at least m^* units of liquidity at the expected market rate of interest.* However, the demand schedule up to m^* does not need to be flat in this case, as the inframarginal bidding is costless. Thus, with single rate tenders each bank placing a bid that reflects its true demand curve for liquidity on the interbank market would also be an equilibrium solution.

The effect of the minimum bid rate as well as the underbidding phenomenon are similar with single rate tenders as they are with multiple rate tenders.

After having developed a model on the central bank liquidity supply and the banks behaviour in the liquidity tenders, we will in the next section try to evaluate the experience we have on the ECB variable rate tenders in light of the model.

5 Experience with ECB variable rate tenders

The ECB applied multiple rate procedure in all 92 main refinancing operations conducted as variable rate tenders between 23.6.2000 and 30.3.2002. Also, a minimum bid rate was applied in all these operations. We assume here that the minimum bid rate was the short term operational target rate for the ECB, as it stated in a press release (8 June 2000) following the decision to change the tender procedure from fixed to variable rate that the *minimum bid rate will take the role that previously the tender rate had in signalling the monetary policy stance.* In this chapter we first study the liquidity provision of the ECB, and after that we analyze the bid behaviour of the banks in the ECB variable rate tenders.

5.1 Liquidity provision of the ECB

The liquidity management of the ECB is comprehensively described in the May 2002 issue of the ECB monthly Bulletin. It states that the baseline for the ECB liquidity provision is constituted by the so-called benchmark allotment, which basically consists of smooth fulfilling of the reserve requirements taking into account the banking sectors' liquidity needs arising from autonomous liquidity factors and the reserve requirement (ECB 2002). This means that when liquidity is provided according to the benchmark allotment rule, the reserve holdings of the banks (ie the money market liquidity) during a reserves maintenance period are expected to be stable. However, there is a natural exception to this rule. The analysis conducted in the previous chapter suggests that, with this kind of an operational framework, ECB should face underbidding when the banks expect it to cut its target (ie here the minimum bid rate) within the current reserves maintenance period. Accordingly, in four main refinancing operations (namely those that were settled on 14 February, 11 April, 10 October and 7 November 2001), the allotted amount was not de facto decided by the central bank; in these tenders the pro rata rationing was not used, and the bid amount was seemingly less than that of the benchmark allotment.

The equation according to which one can calculate the benchmark allotment for the main refinancing operations, is given in the annex to the ECB monthly bulletin article mentioned above. We estimated such benchmark allotments for the 92 MROs between 23 June 2000 and the end of March 2002, and regressed the excess supply of liquidity (ie the actual – benchmark allotment) in 88 of these tenders against a constant, the benchmark liquidity and the spread between one week Euribor rate and the minimum bid rate (from now on the Euribor-spread). From the regressions, we left out the four tenders in which the underbidding was obvious, as in these cases the decision over the allotment volume was not in the hands of the ECB. The regression equation took the following form:

$$\textit{excess supply} = a + b_1 \textit{benchmark allotment} + b_2 \textit{Euribor-spread} + \textit{error term}$$

The ex ante expectation for a liquidity oriented central bank (ie a central bank that is keen on stabilizing the liquidity holdings without paying much attention to the interest rate variability) is that, the parameter estimates for both explanatory variables should be statistically insignificant, as the amount of liquidity supplied in excess of the benchmark allotment should be determined only by the changes in the forecast of the evolution of the autonomous liquidity factors. On the contrary, if the central bank is not purely liquidity oriented in its liquidity decisions, we expect the banks expectations concerning the evolution of the central bank rate(s) to affect the liquidity provision, because these expectations will affect the banks' demand for liquidity in the interbank market. A significant positive parameter estimate for the Euribor-spread, is taken as an indication of the central bank paying attention also to the interest rate deviations. There might also be a natural spread between the one week Euribor rate and the target rate. This natural spread would make a negative contribution into the constant (when it's assumed to be constant).

Furthermore, the level of the benchmark liquidity is added to the set of explanatory variables, as it could enter the liquidity decision if the central bank is willing to stabilize the amount of liquidity to be provided in the two overlapping tenders. The central bank might not want the difference in the size of the two overlapping MROs to get too large; this kind of bias for equality in the amount provided in each operation could be reflected as a negative parameter estimate for the benchmark liquidity in the regression and as a positive contribution to the constant.

The parameter estimates, White Heteroskedasticity-consistent standard errors and the associated probabilities are given in table 1:

Dependent variable: Excess supply (in EUR bn)			
Variable	Coefficient	Std. Error	Probability
Constant	3.911	1.991	0.053
Benchmark	-0.063	0.027	0.024
Euribor-spread	0.094	0.033	0.006
adj R^2	0.163		
n	88		

Now, the parameter estimates for both the benchmark amount of liquidity and the Euribor-spread are significantly different from zero. Based on this we believe that besides the banks' benchmark need for liquidity, the ECB has both tried to smooth the difference in the allotted volumes of the consecutive operations and given positive weight to market expectations of the evolution of interest rates in its liquidity allotment decisions. However, the regression suggests that a 20 basis points increase in the Euribor-spread would be countered by the ECB only by allotting extra EUR 1.8 bn to the markets. This is a relatively small amount compared to the average (benchmark) liquidity of more than EUR 120 bn. Furthermore, when the liquidity is close to the benchmark level, we expect the interest rate elasticity of liquidity to be at lowest; thus, this extra allotment is not expected to have a large impact on the realizing interest rates, unless it affects the counterparties expectations over the future interest rate policy of the central bank. Moreover, the exogenous variables in the regression explain only some 20% of the total variation of the excess supply. Hence, the changes in the estimated effect in the autonomous liquidity factors between the publishing of the estimate and the time the liquidity allotment decision actually takes place seems to be responsible for most of the differences between the actual and the benchmark allotment. Consequently, despite the statistically significant values for both explanatory variables in the regression we are not willing to reject the idea of the ECB being a liquidity oriented central bank; the weight given to interest rate considerations might be positive, but it's effect on the liquidity provision is relatively small. Yet, how the ECB has reacted to the cases of underbidding needs to be further explored to get a full picture of the liquidity policy of the ECB.

5.1.1 Underbidding episodes

There were four tenders in which the banks bid for an amount that was clearly below the benchmark allotment for smooth reserve holding between the switch to variable rate tenders with minimum bid rate (in June 2000) and March 2002. These tenders were settled on 14 February, 11 April, 10 October and 7 November 2001. The reason for this so called underbidding was that, in each case the banks expected the ECB to cut its interest rates before the next main refinancing operation (still belonging to the same reserves maintenance period). The expectations were fulfilled only in the last underbidding episode as the ECB cut its main refinancing rate and the rates of the standing facilities in the operation that was settled on 14 November 2001.

The amount of liquidity actually allotted in these tenders versus the benchmark allotments were (in billion of euro as given in Bindseil (2002)) 65 vs 88, 25 vs 53, 60 vs 79 and 38 vs 66. According to the analysis of the previous chapter, we expect the bid amount to be such low that the shortest money market rate will be above the main refinancing rate (due to the overlapping nature of the weekly ECB main refinancing operations with two weeks maturity), if the ECB applies the benchmark allotment rule in the last tender of the reserves maintenance period. However, if the ECB were expected to punish the underbidding behaviour by supplying less liquidity than what the benchmark would suggest in the consecutive operation, the incentive to underbid would have diminished.

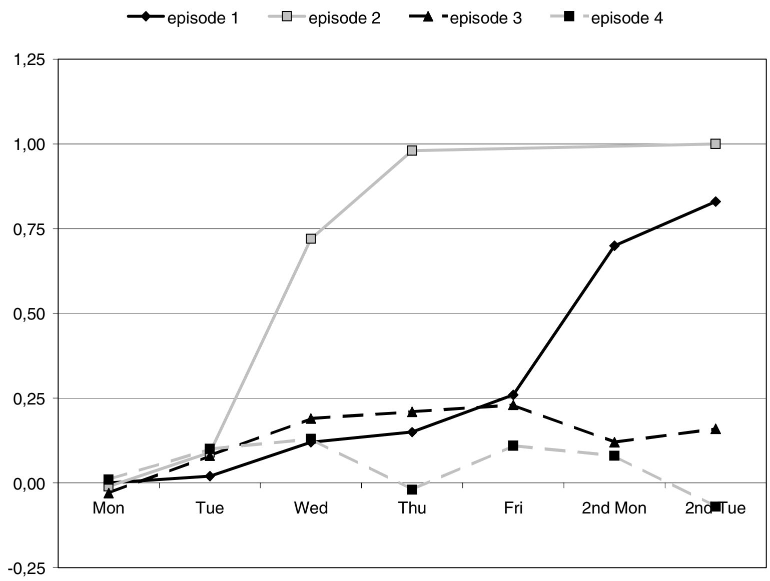

Figure 6: Evolution of the EONIA after the four underbidding episodes.

According to our calculation the actual liquidity provision versus the benchmark allotment in the each operation following those where underbidding occurred 155 vs 182.5, 172 vs 176, 82 vs 86 and 116 vs 119. That is, in the first underbidding episode, the ECB clearly provided too little liquidity (in the first operation after the underbidding) for the banks in order to fulfill the reserve requirement without using the marginal lending facility. Similar policy seems to have applied also for the second and third episodes, although to a lesser extent. During these incidences, the difference between the actual and benchmark allotment was only EUR 4 billion, which probably could have resulted from a change in the autonomous liquidity factor forecast. However, as the banks net recourse to the standing facilities before the end of the maintenance period was EUR 61 and 25 billion (as reported in Bindseil(2002)), we assume the ECB to have intentionally provided liquidity below the benchmark allotment. In the last episode, the actual liquidity provision in the first operation following the underbidding also was below the benchmark allotment, but in this case there was still one operation left within the same reserves maintenance period, and the net recourse to standing facilities was negative. Hence, according to our interpretation, the ECB applied benchmark liquidity provision in the last overbidding episode.

Figure 6 shows the evolution of EONIA during the week following the four cases of underbidding. At the penultimate main refinancing operation of the reserves maintenance period ending at 23 April 2001, the banks expected the ECB to cut its rates in the following operation (episode 1). Thus, they wanted to postpone the holding of reserves until the expected rate cut would have taken place, and the price of reserves would have come down. On aggregate level, the banks can do this kind of backloading only by bidding for less liquidity than smooth reserve holding path would suggest, ie by underbidding. Now, due to the underbidding, EONIA increased from the level of the main refinancing rate (*MRR*) up to 20-25 basis points above it. This is just what the analysis of the previous chapter would suggest to happen when the maturities of the two consecutive operations overlap. At the day of the announcement of the following operation, the spread between EONIA and *MRR* rose to 70 bps. It reached 83 bps at the allotment day (of the second operation), when it was clear that the ECB did not supply enough liquidity for the banks to fulfill their reserve requirements without recourse to the marginal lending facility (ie according to the benchmark rule).

With the banks having the experience of tight liquidity provision following the first underbidding episode, the EONIA-spread increased up to 72 bps already on the settlement day of the operation the underbidding took place for the second time (ie episode 2). The spread widened even further on the following days, so that the EONIA nearly equalled the marginal lending rate already on the day after the settlement. EONIA increased again considerably above the *MRR* during the week following the third underbidding episode in October 2001, but this time it never exceeded 23 bps. So, the evolution of the rate seems to have somehow followed the path we expect to see during a rate cut expectation together with overlapping operations. Hence, it seems likely that the banks did not expect the ECB to punish underbidding severely this time. Finally, during the last episode the EONIA-spread again behaved like the model would suggest; it remained below 13 bps until the next tender.

We find the bidding in the underbidding episodes to reflect rational behaviour of the banks that expect the central bank to cut its rates at the subsequent operation (still falling under the same reserves maintenance period). The fact that the very short market rate has risen above the tender rate after an operation with underbidding, can merely be a result of the overlapping nature of the main refinancing operations. However, *the large widening of the spread between the market rate and the MRR* (eg up to 70 bps or more like during the second episode) *reflects more the increasing probability of the central bank 'punishing' the underbidding by liquidity provision below the benchmark allotment, than the normal increase in market rate due to lower level of liquidity.* This interpretation is in contrast with the explanation for the positive spread proposed in Bindseil (2002). Bindseil suggests the spread to result from the banks' lack of ability to bring the aggregate bids in line with liquidity needs. Now, if the spread between the market rate and the tender rate is the result from a bank's inability to estimate precisely the bid volume of other banks, and hence, to make a correct bid reflecting its own liquidity demand, the aggregate liquidity should stochastically vary around the equilibrium level. This means that, at one underbidding episode the probability of the liquidity provided being too small should approximately be as large as the probability of it being too large.

Now, our claim is that, the *basic reason for underbidding is the combination of interest rate cut expectations and the minimum bid rate. The incentive to bid under the smooth path for reserve holding is further enhanced by the overlap of the maturities of consecutive tenders.* These features of the operational framework result in a positive spread between a market rate with shorter maturity than two weeks and the main refinancing rate (whose maturity is two weeks). The size of this spread reflects the size and the probability of the expected rate cut, as long as the central bank is expected to follow the benchmark strategy in the subsequent tenders. However, the if the central bank is expected to 'punish' underbidding by restricted liquidity provision in the coming tender(s), the incentive to underbid will be reduced. Although, it will be deterred totally only if the punishment expected to be large enough to compensate the effect of the expected rate cut. Whether the demand for liquidity is more stochastic due to the banks' inability to coordination the bids or not, will only affect the volatility of the short term market rates. Hence, the possible coordination problems is never the initial reason for the underbidding.

There were three rate cuts that were not preceded by underbidding during the period under examination. These took place in the tenders that were settled on 15 May, 5 September and 19 September 2001. We suggest that, the tight liquidity provision by the ECB after the two first underbidding episodes deterred the banks from underbidding before the May rate cut. The same may apply also for the first rate cut in September 2001. Yet, some banks seem to have underbid in the operation prior to the cut, as the bid amount totaled only up to EUR 72.9 billion, while the allotted amount was EUR 70 billion. Hence, in this operation the bid ratio (ie the aggregate amount of bids / allotted amount) was only 1.04 whereas on average the bid ratio was 2.0 in the tenders without underbidding. The third rate cut was implemented in the aftermath of the 11 September disaster. Hence, it really could not be foreseen by the banks (during the last operation before the cut), and consequently underbidding that is related to rate cut expectations did not occur. Furthermore, during that time the demand for liquidity most probably exceeded that of under normal time.

We will next focus on the bid behaviour in the tenders that did not suffer from the underbidding.

5.2 Bidding in ECB main refinancing operations

In this section we will examine the bid behaviour of the banks in ECB variable rate main refinancing operations.[20] According to the analysis in chapter 4.1, we expect the banks to place demand schedules that are flat at the expected market rate up to the amount the benchmark allotment rule would suggest, as the ECB seems to follow the benchmark rule quite closely. Consequently, the marginal rate of the allotment is expected to equal the comparable market rate of interest.

ECB (2002) reports the spreads between some market rates and the marginal rate in period from 27 June 2000 to 12 June 2001. The average spread between the marginal rate and the two-week general collateral repo rate was –0.6 basis points, while it was –3.4 bps against two-week EONIA swap rate. These figures indicate the marginal rate of the allotment to have been fairly close to the comparable market rate. The cumulative bid amount at rates above the marginal rate divided by the total allotment volume averaged 0.71, while the figure was 1.25 when bids at the marginal rate are taken into ac-

[20] A detailed description on the bidder behaviour in ECB open market operations from January 1999 until mid-June 2001 can be found in ECB (2001), and Nyborg et al. (2002) analyze the bidding and the performance in first 53 variable rate mainrefinancing operations based on a dataset that includes all bid submitted in these tenders.

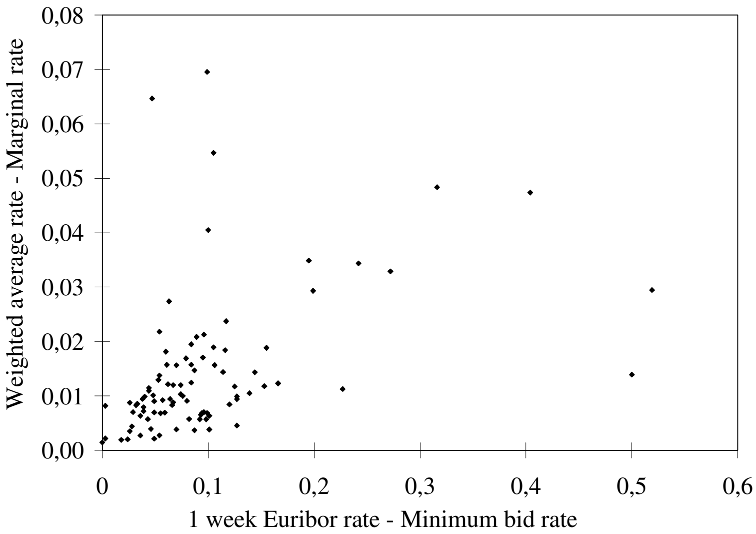

Figure 7: The effect of the interest rate expectations on the spread between the weighted average and the marginal rate

count. Therefore, the ECB would (on average) have had to scaled the allotment volume back by 29% to increase the marginal rate by one basis point or to increase it by 25% to cut the rate by one bp. To change the marginal rate by two basis points the allotment volume would have had to be cut down by 65% or increased by 55%. These figures, together with the fact that the bid volume at the marginal rate was some 54% of the allotted amount, indicate that the aggregate bid was fairly large at rates around the marginal rate; ie the demand curve was relatively flat. Furthermore, the average spread between the weighted average rate of the accepted bids and the marginal rate was 1.4 basis points in the period from 27 June 2001 and the end of March 2002. This also reflects the flatness of the demand schedule in these tenders. However, this spread seems to have depended positively on the spread between eg the 1-week Euribor rate and the minimum bid rate. This relation is illustrated in figure 7, that plots the spread between the weighted average of the rates of the accepted bids and the marginal rate against the spread between the 1-week Euribor rate and the minimum bid rate. We interpret this dependence as an indication of the uncertainty over the coming marginal rate being increased when the spread between the market rate and the minimum bid rate increased, and hence, (at least some of) the banks seem to have responded to this uncertainty by supplying bids at higher rates or at different prices.

A positive spread between the average accepted rate and the marginal rate can be rationalized by assuming two types of banks participating in the central bank tenders. Furthermore, it can be shown that in such a case the spread might be increasing in interest rate uncertainty. In the case of the Euro money market, there is a bunch of small banks that acquire their refinancing only through the central bank operations, ie they participate the central bank main refinancing operations, but they do not trade liquidity (actively) at the interbank market. From now on, we refer to these as "small banks", whereas the banks that participate both the central bank tenders and the interbank market will be called "large banks". It is shown in appendix A that it's optimal for a small bank to submit a bid (or bids) at rate(s) higher than the expected marginal rate. In this case the increasing cost of the central bank liquidity is compensated by the reduction of the uncertainty (about the amount of liquidity provided to the bank) that is associated with bidding at the marginal rate. Moreover, if the small banks have inferior information over the forthcoming market rate (and thus also on the marginal rate of the allotment) they are expected to bid at the higher rates (relative to the expected marginal rate) the larger the uncertainty is. Normally, the higher uncertainty is associated with more intense interest rate hike expectations. Consequently, when there are counterparties that do not trade in the interbank market, the aggregate demand schedule is (weakly) downward sloping instead of being flat. However, the assumption of heterogeneous banks does not otherwise alter the results we derived for heterogeneous banks; the marginal rate is still determined by the bids of large competitive counterparties. From money market point of view, the central bank reserves provided to small institutions should not be taken into account in the money market liquidity.

Another explanation to the positive spread between the weighted average of the rates of the accepted bids and the marginal rate of the allotment could be derived by assuming the banks to be risk averse and possessing asymmetric information about the coming marginal rate. Wang and Zender (2002) show in a recent paper on multi-unit auctions, that the equilibrium bid schedules for risk averse bidders that have private information are downward sloping, ie demand reduction is present in those cases for banks to avoid the winner's curse. However, in a recent paper by Nyborg et al. (2002) analyses the bidding in the ECB main refinancing operations with microdata. They find that private information and the winner's curse are not important in these auctions.

6 Summary and conclusion

In this paper we have constructed an equilibrium model for the short-term money markets, when the central bank provides liquidity through variable rate tenders. The reserve maintenance period in the model is assumed to last for two days, in order to keep the model as tractable as possible while still maintaining the effect of the interest rate expectations. The relation between market rate of interest and liquidity for both days of the reserves maintenance period is derived from single bank's profit maximization problem at the interbank market. The central bank decides on the intended liquidity supply by minimizing a quadratic loss function that contains both the deviations of the expected market rate from the target rate of the central bank and differences between the liquidity supply and the target liquidity. This means that the central bank aims at holding the market rate of interest close to a target value, but it also wants to stabilize the liquidity over the reserves maintenance period. The banks are assumed to observe symmetric signals over the coming market rate, while preparing their bids. In the chapter on the banks bid behaviour, we analyzed bidding under both the multiple rate and the single rate procedures.

We show that the central bank can meet its target both for the expected interest rate and expected liquidity in the final day, if it does not use the standing facilities as independent signalling device. In this case, the location of the interest rate corridor (set by the standing facilities) around the target rate depends on the distribution of the liquidity shocks; as long as the shock distribution is symmetric, the central bank targets can be met by setting the corridor such that the target rate is in the mid-point of it (ie a symmetric corridor). However, if the rates of the standing facilities were set independently of the target rate, the difference between the expected market rate and the target rate would depend on the asymmetry of both the shock distribution and interest rate corridor.

We also showed that in the first day of the reserves maintenance period under static interest rate expectations, the central bank will provide the markets with liquidity that equals the target liquidity, and the expected market rate of interest will be at the level targeted by the central bank regardless of the preferences weighting parameter when the shock distribution is symmetric. However, if the shock distribution was asymmetric, the liquidity supply would depend on the preference weighting of the central bank. The higher the relative weight set on the liquidity deviations, the more the interest rate would differ from its target and the closer the equilibrium liquidity would be to the reserve requirement.

The determination of optimal liquidity supply becomes more complicated when the banks expect the target rate to be changes between the two days of the reserves maintenance period. In such a case, as long as the central bank pays any attention to the money market interest rates, the central bank will provide more liquidity the higher the expected future interest rate is. We show, that the expected value of the market rate for first day will be above the target rate when an interest rate hike is expected. However, the simultaneous effect of both the expected interest rate hike and the increasing liquidity supply on the expected market rate for today is not necessarily monotonic.

In the analysis of the bid behaviour, we showed that under multiple rate procedure and when a reserve price (in case of the ECB the minimum bid rate) is not applied, the aggregate demand schedule the banks submit is flat at the expected market rate at least up to the amount the central bank is aiming at providing. The introduction of a minimum bid rate alters the bid behaviour; it was shown that when the minimum bid rate is effective, the equilibrium on the money market is determined as in the case of fixed rate tenders and central bank accepting all bids submitted. However, in this case the market liquidity will be below, and the expected market rate will be above the level preferred by the central bank.[21] The minimum bid rate is likely to become effective when the target rate is expected be cut, the more so the smaller is the difference between the current target and the minimum bid rate.

[21] This is a feature that is different from the equilibrium with fixed rate tenders with 100% acceptance.

It was also shown that 'underbidding' is a phenomenon that results from the minimum bid rate becoming effective. Moreover, we show that when the maturities of consecutive tenders overlap, the underbidding is enhanced to the extent that the expected market rate for the first period will rise above the prevailing target rate. Thus, in a framework that includes the combination of overlapping tenders and a reserve price, the short-term market rate will increase from the level of the target rate always when the banks expect the central bank to change its target within the ongoing reserves maintenance period – no matter in which direction the target is expected to be moved.

In the analysis of single rate tenders, we assumed the central bank to scale the supply of liquidity back from the intended level, if 'collusive seeming' bid behaviour was detected. This kind of behaviour would punish the banks by forcing them to use more the marginal lending facility. Hence, in the equilibrium the banks bid for the liquidity the central bank is willing to submit at the rate expected to realize with such liquidity supply. The punishment strategy is needed to deter the banks from submitting very steep demand curves, for which any interest rate between the minimum bid rate and the expected market rate (with the intended liquidity) could be maintained as a symmetric pure strategy equilibrium (as showed by Back and Zender (1993)).

We also studied the liquidity supply of the ECB, and bid behaviour of the banks in the ECB variable rate tenders. We found that the ECB seems to have put some weight both for interest rate and liquidity considerations in its allotment decisions. However, the effect the interest rate expectations[22] on the actual liquidity provision was found to be such low that we are willing to see ECB as a very liquidity-oriented central bank. That is, the liquidity provision closely followed the benchmark allotment, according to which the bank reserves are held stable within each reserves maintenance period. Furthermore, there were four cases detected when underbidding was obvious. Based on a closer analysis of these cases, we claimed that basic reason for underbidding was the combination of interest rate cut expectations and the minimum bid rate, and it was further enhanced by the overlapping nature of the maturities of consecutive tenders. However, the large widening of the spread between the market rate and the main refinancing rate, that followed some of the underbidding episodes, seems to have reflected more the increasing probability of the ECB 'punishing' the underbidding by lower liquidity provision in the subsequent operation, than the normal increase in market rate due to the decrease in liquidity.

Finally, we analyzed the banks bidding in the ECB main refinancing operations, and found the demand schedules to have been fairly flat at the rates close to the marginal rate of the allotments. We found some evidence according to which the uncertainty over the coming marginal rate of the allotment affects the bidding so that the demand schedules submitted are steeper.

[22] The interest rate expectations were measured as the spread between the one week Euribor rate and the minimum bid rate.

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A Heterogeneous banks

In this section, we will deviate from the basic model set up by letting some of the banks to be such that they will not participate the interbank market. In the case of the Euro money market, there is a bunch of small banks that acquire their refinancing only through the central bank operations, ie they participate the central bank main refinancing operations, but they do not trade liquidity (actively) at the interbank market. From now on, we refer to these as "small banks", whereas the banks that participate both the central bank tenders and the interbank market will be called "large banks". Small banks bid for liquidity only in order to minimize the cost of holding reserves; they do not aim at making profits between the price difference of the auctioned liquidity and the interbank liquidity. Thus, in a variable rate tender a small bank bids to minimize the following:

$$
\begin{aligned}
\max_{b_i} \Pi &= pv_i\left(q_i\right) - \sum_{j=1}^{10} q_{i,j,t} r_{i,j}^T \\
s.t.\ q_{i,j} &= \begin{cases} b_{i,j}, \text{ if } B_{r_{i,j}^T} \leq m^* \\ \frac{m^* - B_{j-1}}{B_j} b_{i,j}, \text{ if } B_{j-1} < m^* < B_{r_{i,j}^T} \\ 0, \text{ if } B_{j-1} \geq m^* \end{cases},
\end{aligned}
$$

where $pv_i(q_i)$ denotes the private value of q_i units of liquidity to bank i. The functional form of it depends on whether there is still averaging provision left, ie whether we are already on the final day of the reserves maintenance period or not.[23] The private value is increasing in liquidity, but its marginal growth

[23] The private value of the liquidity held by bank i at the last day of reserves maintenance period is:

$$
\begin{aligned}
pv_i\left(q_i\right) &= r_2^m \left[\int_{-\infty}^{-l_{i,2}+rdb_{i,2}-b_{i,2}} (l_{i,2} - rdb_{i,2} + b_{i,2} + \varepsilon_2) f(\varepsilon_2) d\varepsilon_2 \right] \\
&\quad + r_2^d \left[\int_{-l_{i,2}+rdb_{i,2}-b_{i,2}}^{\infty} (l_{i,2} - rdb_{i,2} + b_{i,2} + \varepsilon_2) f(\varepsilon_2) d\varepsilon_2 \right], \qquad (27)
\end{aligned}
$$

and for the first day of the period it's given by:

$$
\begin{aligned}
pv_i\left(q_i\right) &= r_1^m \left[\int_{-\infty}^{-l_{i,1}-b_{i,1}} (l_{i,1} + b_{i,1} + \varepsilon_1) f(\varepsilon_1) d\varepsilon_1 \right] \\
&\quad + \mathrm{E}_1\left[r_2\right] \left[\int_{-l_{i,1}-b_{i,1}}^{\infty} (l_{i,1} + b_{i,1} + \varepsilon_1) f(\varepsilon_1) d\varepsilon_1 \right] \\
&\quad + \left(r_1^d - \mathrm{E}_1\left[r_2\right]\right) \left[\int_{2R_{i,1}-l_{i,1}-b_{i,1}}^{\infty} (l_{i,1} + b_{i,1} - 2R_{i,1} + \varepsilon_1) f(\varepsilon_1) d\varepsilon_1 \right]. \qquad (28)
\end{aligned}
$$

decreases with liquidity.[24] Thus, the optimal bid for bank i must be such that with it $\partial pv_i/\partial q_i = r^T_{i,j}$, ie with this bid the marginal increase in the private value just equals the bid rate. Now, according to the model derived in the previous sections, the marginal rate of the allotment should equal the expected market rate of interest (ie r^{marginal} =E[$r\left(m^*\right)$]), therefore, while preparing its bid *in a multiple rate auction*, the bank has to consider that increasing the bid rate up from r^{marginal} is costly. However, the optimal cumulative aggregate bid $B^*_{r^{\text{marginal}}}$ is not a unique amount; in order for bank i to make an optimal bid at the marginal rate, it should be able to estimate the proportion to which the pro rata rationing is used at the marginal rate (ie the optimal bid would be implicitly given by $\left(\partial pv_i/\partial q_i\right)\left(m^* - B_{j-1}\right)/B_j = r^{\text{marginal}}$). Errors made in estimating the allotment ratio is costly for bank i, as it will move the amount of liquidity provided to it away from the optimal level. As a consequence, we expect the cost of placing a bid at a rate slightly above the marginal rate to be profitable for the small banks, as the increase in the cost of central bank liquidity is well compensated by getting rid of the uncertainty associated with bidding at the marginal rate. Now, if we assume that, in addition to (or as a consequence of) not participating in the interbank market, small banks have inferior information over the forthcoming market rate of interest (and thus also on the marginal rate of the allotment), we expect these banks to increase the spread between the bid rate and the expected marginal rate whenever the central bank is expected to increase its target in the near future. This comes from the fact that when the target rate is expected to be changed, the uncertainty related to the marginal rate increases. Thus, to avoid bidding at rates below the marginal rate the small banks are willing to bear higher costs from acquiring liquidity from the central bank. Consequently, if the banks are heterogeneous the demand schedule is not expected to be flat, the more so when the heterogeneous banks also possess asymmetric information.

Note that, this problem for small banks arises only in the multiple rate auction format. The inframarginal bid at rates above the marginal rate are not costly, thus, in a single rate auction the small banks can supply the central bank with a demand curve that reflects their true demand for liquidity arising

[24]The change in private value when the liquidity changes in the last period is given by:

$$\partial pv_i/\partial q_i = r_2^d + (r_2^m - r_2^d)F\left(-l_{i,2} + rdb_{i,2} - q_{i,2}\right), \tag{29}$$

thus,

$$\partial pv_i^2/\partial^2 q_i = -(r_2^m - r_2^d)f\left(-l_{i,2} + rdb_{i,2} - q_{i,2}\right), \tag{30}$$

and the same for the first period are:

$$\partial pv_i/\partial q_i = \mathrm{E}_1\left[r_2\right] + \left(r_1^m - \mathrm{E}_1\left[r_2\right]\right)F(-l_{i,1} - q_{i,1}) + \left(r_1^d - \mathrm{E}_1\left[r_2\right]\right)\left[1 - F(2R_{i,1} - l_{i,1} - q_{i,1})\right]. \tag{31}$$

Thus,

$$\partial pv_i/\partial q_i = \mathrm{E}_1\left[r_2\right] + \left(r_1^m - \mathrm{E}_1\left[r_2\right]\right)F(-l_{i,1} - q_{i,1}) + \left(r_1^d - \mathrm{E}_1\left[r_2\right]\right)\left[1 - F(2R_{i,1} - l_{i,1} - q_{i,1})\right]. \tag{32}$$

from the reserve requirement. Therefore, they are allotted the liquidity that is optimal for them at the marginal rate.

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