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Simulating the Effects of Imperfect Credibility: How does the Peso Problem Affect the Real Economy?

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Abstract

In this paper we analyse the macroeconomic effects of peso problems by simulating numerically a small-scale rational expectations macromodel. The model is a conventional IS-LM-AS model of an open economy under floating exchange rates. The peso problem has been incorporated in the model by assuming that the money supply process entails a small but nonzero probability of a sizable discrete shift in the money supply. In addition, the severity of a peso problem can vary over time. The procedure used in solving our model is more complicated than the standard solution methods for rational expectation models in that there are two dates at which expectations are formed.

Both deterministic and stochastic simulations were used in the analysis. Results from the deterministic simulations suggest that the presence of the peso problem leads to an overvalued real exchange rate and a higher ex ante real interest rate, which results in output losses. In the stochastic simulations, the values of the IS, AS and monetary disturbances vary along with the severity of the peso problem. The simulations show that the presence of a variable peso problem affects the correlations between macroeconomic variables, especially between the ex post yield differential and either the real exchange rate or the output gap. In the case of conventional (non-autocorrelated) IS, AS and monetary disturbances, these correlation coefficients are equal to zero. The inclusion of a variable peso problem in the simulation model changes these results: the ex post yield differential is now correlated with the real exchange rate and the output gap.

In the empirical part of the paper we demonstrate the applicability of our simulation results using Canada and the United Kingdom as examples.

Keywords: peso problem, credibility, simulation

Epätäydellisen uskottavuuden vaikutusten simulointi: Mitkä ovat peso-ongelman reaalitaloudelliset vaikutukset?

Suomen Pankin keskustelualoitteita 24/98

Veli-Matti Mattila
Tutkimusosasto

Tiivistelmä

Keskustelualoitteessa tarkastellaan ns. peso-ongelman makrotaloudellisia vaikutuksia numeeristen simulointien avulla. Simuloitava malli on tavanomainen pienen avotalouden IS-AS-LM-malli, jossa valuutta kelluu ja odotusten oletetaan muodostuvan rationaalisesti. Peso-ongelman läsnäolo näkyy rahan tarjontaprosessissa: on olemassa pieni, nollasta poikkeava todennäköisyys, että rahan tarjonta kasvaa hyppäyksenomaisesti. Lisäksi peso-ongelman vakavuus (odotettu rahan tarjonnan muutos) voi vaihdella ajassa. Mallin numeerinen simulointi vaatii tavanomaisista rationaalisten odotusten ratkaisumenetelmistä poikkeavan ratkaisutavan, sillä malli sisältää kahtena eri ajankohtana muodostettavat odotukset endogeenisten muuttujien tulevista arvoista.

Yksinkertaisen, deterministisen simuloinnin mukaan peso-ongelman olemassaolo johtaa reaalisen valuuttakurssin vahvistumiseen ja odotetun reaalikoron nousuun, mikä heijastuu reaalisina tuotantomenetyksinä. Stokastisten simulointien perusteella (ajassa) varioivan peso-ongelman olemassaolo vaikuttaa makrotaloudellisten suureiden, erityisesti toteutuneen tuottoeron ja reaalisen valuuttakurssin sekä tuottoeron ja tuotantokuilun väliseen korrelaatioon. Tavanomaisten (autokorreloimattomien) IS-, AS- ja rahasokkien vallitessa näiden muuttujien välillä ei ole korrelaatiota. Sen sijaan varioivan peso-ongelman tapauksessa korrelaatiokertoimet poikkeavat nollasta.

Työn empiirisessä osassa esitellään simulointitulosten soveltamista Kanadan ja Ison-Britannian aineistoissa.

Asiasanat: peso-ongelma, uskottavuus, simulointi

Contents

1 Introduction

The concept "*peso problem*" refers to the situation where economic agents have rationally formed expectations about discrete shifts or jumps in the values of some important economic variables. These expectations may be a reflection of the poor credibility of economic policymakers or they may be based e.g. on the anticipated outcome of future parliamentary election that can lead to substantial changes in the general economic policy. Since asset prices (like exchange rates) are based on the expected future paths of these economic variables, the possibility of discrete changes directly affects asset price behaviour. In addition, it can induce asset price movements that *ex post* seem to contradict the conventional rational expectations assumptions. As the discrete shifts are usually thought to be rare events – i.e. the probability of occurrence is low – the observation of such a shift is unlikely in a small sample of data. However, because this shift may be very substantial (implying sizable losses to investors etc.) the possibility of this outcome affects decisions made by economic agents and thus shows up in the data sample: as a result e.g. market forecasts of exchange rates may appear to be biased.[1]

In the analyses of peso problems the main emphasis has been on the theoretical and empirical implications of peso problems on asset pricing, i.e. how the presence of peso problems affects price formation in the financial markets.[2] However, the formal analysis of the effects of peso problems on the real side of the economy is scarce as Vilmunen (1998) has pointed out. This is somewhat surprising as peso problems should be especially important from the point of view of economic policy. In his paper Vilmunen (op. cit.) tries to fill this deficiency by analysing the implications of peso problems on macroeconomic variables like the output gap, the real interest rate and the terms of trade.[3] The analysis is carried out in a standard IS-LM model of an open economy under floating exchange rates and an expectations augmented Phillips curve. The peso problem is incorporated in the money supply process either as infrequent future discrete jumps generated by a Poisson process or as a sizable once and for all future shift (regime switch). Using this framework Vilmunen shows that the presence of a peso problem raises the ex ante real interest rate and leads to an appreciation of the real exchange rate implying lower aggregate output relative to the potential one. In addition, the aggregate volatility also increases because of the peso problem. As regards the nominal variables, the presence of a peso problem increases the nominal interest rate as well as the price level. However, the effect on the nominal exchange rate

[1] The case of the Mexican peso – which is also the origin of the concept "peso problem" – is the standard example used in this context. The spot exchange rate of the Mexican peso was fixed at 12.5 pesos per one US dollar from April 1954 to August 1976. At the same time the peso deposit rates were systematically above the dollar deposit rates over this period implying a forward rate that was over the materialized spot rate. Therefore, the ex post rate of return on holding Mexican peso deposits was systematically positive, i.e. there were excess returns. Under the assumption of risk neutrality this behaviour contradicts with the assumption of rational expectations since it implies that the market's forecast errors were biased. However, the existence of the interest rate differential can be explained by the market's persistent belief that the peso will be devalued. Subsequently, in August 1976, these expectations became justified as the peso was allowed to float and fell to 20 pesos per dollar, implying a devaluation of about 40 %. (Lewis 1992; Evans 1995.)

[2] See e.g. Evans (1995).

[3] Danthine & Donaldson (1998) also touch this topic in their analysis of the effects of peso problems on asset pricing in the context of a dynamic general equilibrium model.

appears to be somewhat ambiguous and depends on the values of the model parameters: a sufficient condition for a depreciation of the nominal exchange rate is that aggregate demand is sufficiently responsive to changes in the real exchange rate and the real interest rate.

In this paper we extend the macroeconomic analysis of peso problems presented in Vilmunen (op. cit.) by simulating numerically the effects of various peso problems with a small-scale rational expectations macromodel. Our motive is twofold. Firstly, we want to demonstrate how numerical analysis of peso problems can be done in a practicable way. And secondly, we want to provide additional information on the effects and significance of peso problems. For instance, what are the consequences of variation in the "severity" of a peso problem? Or how does the presence of a peso problem affect the correlations between macroeconomic variables like the output gap, the real exchange rate, the ex post real interest rate and the ex post yield differential? Especially the latter question is important as it may give us some hints that can be utilised in the empirical analysis of peso problems.

The paper has the following structure. In chapter 2 we present the macroeconomic model that is used in the analysis of peso problems. Special attention is given to the specification of the money supply process which contains the seed of the peso problem. Chapter 3 deals with the method used in the numerical simulation of the model. We discuss briefly the general aspects of solving rational expectations models and present a solution method for a model with two viewpoint dates (i.e. a model with two sets of expectations). The results of the simulations are presented in chapter 4. We start from simple deterministic simulations of peso problems and then move to stochastic simulations where aggregate demand, aggregate supply and money supply disturbances as well a variable peso problem are added to the analysis. In chapter 5 we compare these results with some empirical observations. Finally, chapter 6 concludes.

2 The model

The model that we use in the simulations of the macroeconomic effects of peso problems resembles closely the model used by Vilmunen (1998) in his theoretical analysis of peso problems. This model is an IS-LM-AS model of an open economy under floating exchange rate regime and rationally formed expectations. The choice of this particular model can be defended by the fact that it is a fairly simple and easily manageable tool for analysis and its properties and limitations are well understood. Compared to Vilmunen (op. cit.) we have made some modifications to the money supply process and incorporated conventional aggregate demand and supply disturbance terms in the model.

The IS equation of the model is given by

$$y_t - y^n = \delta[s_t + p_t^f - p_t] - \sigma[i_t - E_t(p_{t+1} - p_t)] + u_t \tag{1}$$

where y_t is domestic output, y^n is potential output, s_t is nominal exchange rate (measured as the price of foreign currency in domestic currency), p_t is domestic price level, p_t^f is foreign price level and i_t is the domestic nominal interest rate at period t. E_t is the conditional expectations operator based on the model and on the information available in period t. Finally, u_t is a demand-side disturbance term which is assumed to be normally and independently distributed with zero mean. All the variables are in logarithms except for the interest rate. According to equation (1), the deviation of output from potential is a function of the real exchange rate, $s_t + p_t^f - p_t$, and of the ex ante real interest rate, $i_t - E_t(p_{t+1} - p_t)$, with parameters δ and σ acting as elasticity and semielasticity, respectively, of aggregate demand with respect to these variables. Other elements of aggregate demand, like public expenditure, have been omitted for simplicity.

The supply side of the economy, the AS curve, is modelled using an expectations augmented Phillips curve:

$$p_t - p_{t-1} = E_{t-1}(p_t - p_{t-1}) + \theta[y_t - y^n] - \theta w_t \tag{2}$$

where θ is the slope of the Phillips curve and w_t is a supply-side disturbance term which is assumed to be normally and independently distributed with zero mean.[4] Equation (2) can be given a contract interpretation as in Vilmunen (1998, p. 12): the nominal wage rate for period t is set at the end of the previous period at the level where labour market is expected to clear, so that deviations of actual output from the expected (potential) level occur when the price level differs from the corresponding equilibrium level.

The LM equation is of the standard form

$$m_t - p_t = y_t - \lambda i_t \tag{3}$$

where m_t is (the log of) money and λ is the semielasticity of money demand with respect to nominal interest rate. For simplicity, we have assumed that the income elasticity of demand for money is one. The domestic and foreign interest rates are assumed to be connected via the uncovered interest parity, i.e.

$$i_t = i_t^f + E_t(s_{t+1} - s_t) \tag{4}$$

where i_t^f is the foreign nominal interest rate.

Finally, we specify the money supply process as follows:

$$m_t = m_t^B + \sum_{k=1}^{t} n_k \Omega_k \tag{5}$$

In equation (5), the supply of money in period t is defined as a sum of two components. Part of the money supply is generated by a random walk, i.e. $m_t^B = m_{t-1}^B + v_t$, where v_t is a normally and independently distributed disturbance term with zero mean. In addition to this stochastic process, which captures the "conventional" shocks affecting money supply, the amount of money in period t is also determined by the sum of discrete jumps or shifts in money supply that have occurred in the past and during the current time period t. Using the definition of the money supply in period t-1 we can rewrite equation (5) as

[4] Equation (2) can also be written in the form $y_t = y^n + (1/\theta)[p_t - E_{t-1}p_t] + w_t$. The disturbance term w_t is thus directly linked to the level of aggregate output.

$$m_t = m_{t-1} + v_t + n_t\Omega_t \tag{6}$$

The third term on the RHS of (6), $n_t\Omega_t$, is the origin of the peso problem and describes the possibility that there may occur a substantial discrete shift in the money supply in period t. Whether this jump occurs or not is determined by the Bernoulli random variable n_t, which takes value 1 with probability q and 0 with probability 1–q. The probability of a jump, q, is assumed to be small in the very spirit of the peso problem: the occurence of a jump is a rare event and it may not happen even during a long time span. However, this jump possibility – though minor – exists and it affects expectation formation in the economy and thus the macroeconomic equilibrium.

If a jump occurs in period t, i.e. $n_t = 1$, then money supply will increase by Ω_t. Again, in the very spirit of the peso problem, the size of the jump is assumed to be substantial: the occurence of the jump is some kind of a disaster state which brings remarkable disutility to economic agents. In this model this "disaster" is a substantial increase in the amount of money circulating in the economy with repercussions on the general price level, nominal exchange rate etc.

The essence of the peso problem can be seen by taking conditional expectation on the money supply in period t, conditional on the information set available at time t–1:

$$E_{t-1}(m_t) = m_{t-1} + E_{t-1}(v_t) + E_{t-1}(n_t\Omega_t) = m_{t-1} + E_{t-1}(n_t)E_{t-1}(\Omega_t) \tag{7}$$

In equation (7) we have assumed for simplicity that the occurence and the size of a jump are independent. If a peso problem exists, the latter term on the RHS of (7) is positive and the expected future supply of money exceeds the current level of money supply. The higher the probability of a jump and/or the larger the expected size of the jump, the larger is the expected change in the money supply. Peso problems are not necessarily constant in time: on the contrary, we can easily imagine that they tend to vary according to e.g. election results, statements made by politicians or by other changes in the information set that is available for the economic agents when they make investment or other decisions and form their expectations on the future values of variables. This time-varying nature of the peso problem could be included in our model by assuming that either the probability of a jump or the size of a jump (or even both) varies in time. The first alternative could have been the most natural in describing this variability of the peso problem but for simplicity[5] we have chosen to work with the assumption that the size of a jump, Ω_t, is time-varying. Thus it is assumed that Ω_t evolves in time according to the following stochastic process:

$$\Omega_t = \Omega_{t-1}\exp(\varepsilon_t) \tag{8}$$

[5] A time-varying probability q_t would be somewhat more problematic to model because we should restrict the value of this variable to be in the range [0, 1]. We could use some transformation but the derivation of the (conditional) expectation of this probability may become troublesome.

where ε_t is a disturbance term.[6] If we further assume that ε_t is normally and independently distributed (with mean μ and variance σ_ε^2), we can derive the (conditional) expectation of Ω_t by utilising the moment-generating function of the normal distribution[7]:

$$E_{t-1}(\Omega_t) = \Omega_{t-1}\exp(\mu + \tfrac{1}{2}\sigma_\varepsilon^2) \tag{9}$$

The time-varying nature of the jump size has important implications for the formation of expectations: as time goes by and we move from period t–1 to period t, new information comes which alters not only the jump size of current period t (which may or may not occur) but also economic agents' perceptions about future jump sizes. Thus the expected change in the money supply in the future, i.e. the "severity" of the peso problem, varies in time as a function of newest available information. For example, the outcome of parliamentary election in period t could increase the size ("looseness") of a possible change in monetary policy, implying larger Ω_t in our model (i.e. $\varepsilon_t > 0$). This policy change may not occur in period t but the increased threat affects expectations and thus the equilibrium of the economy.

Noting that $E_{t-1}n_t = q$ and using (9) we can rewrite equation (7), i.e. the expected money supply in period t (based on the information available in period t–1), as

$$E_{t-1}(m_t) = m_{t-1} + q\Omega_{t-1}\exp(\mu + \tfrac{1}{2}\sigma_\varepsilon^2) \tag{10}$$

Similarly, the expected money supply in period t+k (based on the information available in period t–1) is

$$E_{t-1}(m_{t+k}) = m_{t-1} + \sum_{i=0}^{k} q\Omega_{t-1}(\exp(\mu + \tfrac{1}{2}\sigma_\varepsilon^2))^{i+1} \tag{11}$$

If $\mu + \tfrac{1}{2}\sigma_\varepsilon^2 > 0$ (as we assume) the expected change in the money supply per period (due to the peso problem) *increases* as a function of time.[8]

Equations (1)–(4), (6) and (8) form our model of an open economy under floating exchange rate regime. The endogenous variables are the domestic output, the nominal exchange rate, the domestic interest rate and the price level while we

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[6] We have chosen this kind of a functional form for Ω_t in order to restrict the size of a jump to be positive in any circumstances. This is in the spirit of the peso problem. Equation (8) can be derived from the following formulation: $\Omega_t = \exp(x_t)$, where $x_t = x_{t-1} + \varepsilon_t$ and ε_t is some stationary disturbance term.

[7] The moment-generating function of a normally distributed random variable x is $\psi_x(t) = E(\exp(tx)) = \exp(\mu t + \tfrac{1}{2}\sigma^2 t^2)$ for every real number t. See e.g. Dudewicz & Mishra (1988, p. 255–258).

[8] The solution of the model reveals that the values of the endogenous variables like the nominal exchange rate are determined inter alia by the discounted sums of the expected future money supplies and of the changes in these expectations (see e.g. Vilmunen (1998)). These sums are finite only if $\exp(\mu + \tfrac{1}{2}\sigma_\varepsilon^2) < (1+\lambda)/\lambda$. This condition holds in the simulations presented in this paper.

take as given the values of the foreign variables, the level of potential output as well as the process driving the money supply.

Our model resembles closely the model used by Vilmunen (1998, chapter 3) in the theoretical analysis of the effects of a peso problem on macroeconomic variables. Differences can be found in the specification of the money supply process and in the inclusion of stochastic demand- and supply-side disturbances in our model. In Vilmunen's model[9] the money supply process is defined as the sum of a random walk component and a Poisson process with jump intensity (per unit time) α:

$$m_t = m_t^B + N_t \tag{12}$$

where the Poisson process (or counter) N_t registers the number of jumps occured in the past and during the current time period. The size of a jump has been scaled to 1 (i.e. 100 % log-percent) and the expected number of jumps during one time period is α. As was shown above in our model we have changed the specification of the money supply process so that the number of jumps per period follows a Bernoulli distribution and the size of the jump may vary in time. We think that this formulation is more intuitive as it restricts the possible number of jumps per period to 0 or 1 (a peso jump either occurs or not) and allows the severity of the peso problem vary along with the inflow of new information. In addition, the probability of a peso jump enters the model directly as one of its parameters (q) and corresponds to the expected number of jumps per period. This is useful in the simulation of the model as we can vary the severity of the peso problem either using the size or the probability of a jump.

What is the role of the "credibility" of monetary policy in our analysis? Peso problems as such are closely related to the lack of credibility: the existence of persistent expectations of a future monetary expansion (or a devaluation of the currency in a regime of fixed exchange rates) is often a symptom of the poor credibility of the policymakers. In our model, however, we have not specified the preferences of the monetary authorities or the political decision-makers. Thus the money supply process – or what can be considered as economic agents' perception of the process driving the money supply – is purely exogenous without a specific explanation for the existence of the peso problem.

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[9] Vilmunen (1998) analyses peso problems using two models, the first of which is the basis of our model. In the second model the money supply process is defined differently: instead of infrequent jumps the money supply may undergo a once and for all discrete shift in the future, i.e. a permanent regime switch in monetary policy is possible. This kind of a money supply specification follows Obstfeld (1987) where peso problems are analysed using the monetary approach to exchange rate determination.

3 Solving the model numerically

3.1 Some methodological remarks

The model outlined in chapter 2 for the analysis of the peso problem is a dynamic rational expectations model, i.e. it is assumed that expectations of the future values of endogenous variables are equal to the conditional forecasts based on the model itself. A rational expectations model can be solved numerically for a certain time period t using e.g. the solution method presented in Fair & Taylor (1983, 1990). In short, the solution prodedure starts by choosing the values of the model parameters and the exogenous variables (both past and [expected] future values) and setting the disturbance or error terms equal to their expected values. Then an arbitrary initial set of values for the expected endogenous variables is chosen and the model is solved. This solution provides new values for the expectations which can be used for solving the model again. This iterative process is continued until the differences between the newly solved values and the solution values of the previous iteration are within a prescribed tolerance level. When the convergence has been attained we have a numerical solution for period t and rationally formed expectations of the future values of the endogenous variables.

The procedure used in solving our rational expectations model is somewhat more complicated than the standard method. The model is reproduced in (13):

$$
\begin{aligned}
&y_t - y^n = \delta[s_t + p_t^f - p_t] - \sigma[i_t - E_t(p_{t+1} - p_t)] + u_t \\
&p_t - p_{t-1} = E_{t-1}(p_t - p_{t-1}) + \theta[y_t - y^n] - \theta w_t \\
&m_t - p_t = y_t - \lambda i_t \\
&i_t = i_t^f + E_t(s_{t+1} - s_t) \\
&m_t = m_{t-1} + v_t + n_t\Omega_t
\end{aligned}
\qquad (13)
$$

The special property of our model is that there are *two viewpoint dates*, t–1 and t, in which expectations are formed. Thus the solution of the model for a certain time period t depends, firstly, on the conditional expectations of the values of future endogenous variables that are formed during period t and are based on the information set available at that time (E_t). This information set covers all the relevant information including the current realisations of the disturbance terms u_t, v_t and w_t as well as the peso problem variables n_t and Ω_t. *In addition*, the solution depends also on those expectations that were formed during the previous time period t–1 and were based on the information set available at that time (E_{t-1}), including u_{t-1}, v_{t-1}, w_{t-1}, n_{t-1} and Ω_{t-1}.

The existence of two viewpoint dates in our model can be interpreted as reflecting the idea that nominal wage contracts for time period t are signed in period t–1 and are based e.g. on expectations of period t price level formed by information available in period t–1. Given these wage contracts (and the corresponding price level expectations $E_{t-1}p_t$) the equilibrium for period t is solved after new values of the exogenous disturbance terms and peso problem variables have materialised and economic agents in e.g. the financial markets have formed new expectations of the values of the future endogenous variables on the basis of the revised information set.

The existence of two viewpoint dates means that when we solve the model numerically for a certain time period t using the conventional solution procedures described above we should have at our disposal rationally formed expectations of the values of the future endogenous variables conditional on the information set t–1. A quick survey of the literature on the solution methods of rational expectations models revealed that – quite surprisingly – this kind of a problem has not been dealt with, at least not explicitly.[10] Our solution method[11] is based on the observation that the above-mentioned conditional expectations from period t–1 are the solution of the following system of equations:

$$\begin{aligned}
&E_{t-1}(y_t - y^n) = \delta[E_{t-1}(s_t + p_t^f - p_t)] - \sigma[E_{t-1}(i_t) - E_{t-1}(p_{t+1} - p_t)] + E_{t-1}(u_t) \\
&E_{t-1}(p_t - p_{t-1}) = E_{t-1}(p_t - p_{t-1}) + \theta[E_{t-1}(y_t - y^n)] - \theta E_{t-1}(w_t) \\
&E_{t-1}(m_t) - E_{t-1}(p_t) = E_{t-1}(y_t) - \lambda E_{t-1}(i_t) \\
&E_{t-1}(i_t) = E_{t-1}(i_t^f) + E_{t-1}(s_{t+1} - s_t) \\
&E_{t-1}(m_t) = m_{t-1} + E_{t-1}(v_t) + E_{t-1}(n_t\Omega_t)
\end{aligned} \qquad (14)$$

where we have utilised the law of iterated expectations, i.e. $E_s(E_t(x_{t+k})) = E_s(x_{t+k})$, $s < t$, in writing the equations. Now, *systems in (13) and (14) can be combined and solved simultaneously* using the standard solution methods for rational expectations models with a single viewpoint. We simply change the notation so that $E_{t-1}(a_{t+j}) = \tilde{a}_{t+j}$, $j = 0,1$, and augment the original model in (13) with these new equations and variables which determine the expected solution of the model for period t based on information set available in period t–1. When the augmented model (see Appendix 1) is solved the conditional expectation of the period t price level p_t, $E_{t-1}p_t$, is determined simultaneously with the overall solution for period t and used in the derivation of the latter.

3.2 Some practical aspects of solving the model

The macroeconomic model presented in systems (13) and (14) is solved and simulated numerically using the SIMPC software.[12] The solving of the model – i.e. a single simulation – gives us the numerical solution (equilibrium values) of

[10] Fair & Taylor (1983) come close to this kind of a problem in their solution method for a model with serially correlated errors. However, this method was subsequently discarded (see Fair & Taylor 1990).

[11] This method was suggested by Juha Tarkka.

[12] We have used the SIMPC version 3.86 (Don Econometrics, 1995). The rational expectations solution procedure in SIMPC is based on the method presented by Fair & Taylor (1983).The SIMULATE-command runs the Type I and Type II iterations defined in Fair & Taylor (op. cit.). Type III iterations should be operated by the user. In short, Type I iterations are used to determine the solutions for single periods using Newton- or Gauss-Seidel-type algorithm. These solutions form an inner loop in the Type II iterations which are rounds over the entire time horizon (period t plus the forecast horizon) in order to achieve convergence of endogenous lead variables. Type III iterations are used to test the sensitivity of the solution with respect to the lenght of the forecast horizon.

the endogenous variables for period t as well as the conditional expectations of the values of these variables for the forecast horizon. In our simulations the solution period t is normally labelled "1998" and the forecast horizon is 1999–2038, i.e. 40 years. The lenght of the forecast horizon has been determined on the basis of sensitivity analysis. We have used the SIMPC default settings for testing the convergence of the values of the endogenous variables.

In our simulations the past, current and (expected) future values of the exogenous variables are purely hypothetical. For simplicity we have assumed that the foreign price level p^f as well as the foreign nominal interest rate i^f are constant (and they are also expected to be constant in the future) which implies that the foreign real interest rate is also constant. Likewise, the domestic potential or full employment output is assumed to be constant.

In every simulation the values of the disturbance terms over the forecast horizon are set equal to their expected values while the "actual" values of these disturbances for period t ("1998") may be drawn stochastically from a known distribution.[13] As was mentioned in chapter 2, the stochastic disturbance terms in the IS, AS and money supply equations (u_t, w_t, v_t and ε_t) are assumed to be normally and independently distributed with zero mean. Thus the expectations concerning the values of the future (additive) disturbance terms u_t, w_t and v_t are set equal to zero in the simulations. However, because the term ε_t is involved as a part of a function ($\exp(\varepsilon_t)$) the expected value of that function is used instead (see equation (9)).

Finally, we have tried to choose the values of the parameters of our model in a realistic way. The semi-elasticity of the demand for money with respect to nominal interest rate, λ, is assumed to be 4.0. If the nominal interest rate is equal to 5 % (which is the assumed level of the foreign interest rate) the interest elasticity of money demand is 0.2 (see the baseline simulation in chapter 4). The elasticity of the aggregate demand with respect to real exchange rate, δ, is assumed to be 1.0 and the semi-elasticity of the aggregate demand with respect to ex ante real interest rate, σ, to be 3.0. The latter implies that the corresponding elasticity is 0.15 if the ex ante real interest rate is 5 % (see again chapter 4). The value of the parameter θ, which measures the slope of the expectations augmented Phillips curve in the elasticity form, is assumed to be 0.25.

[13] This is the procedure used with the expectations of the values of the future disturbances formed in period t (i.e. E_t). In the case of the expectations formed in period t–1 (i.e. E_{t-1}) also the values for period t are set equal to their expected values.

4 Simulation results

4.1 Basic properties of the model

In order to demonstrate the basic properties of our small-scale macromodel we present some results from simulations where the peso problem doesn't exist, i.e. we have set the probability of a jump, q, equal to zero.

The baseline simulation of the model is done by setting the period 1998 disturbance terms u, w and v equal to their means (zero).[14] Because these values correspond to the expectations of the disturbances formed in the previous period 1997 the expected values of the endogenous variables, especially the expected the price level in 1998, are "correct". Thus in the macroeconomic equilibrium of period 1998 the actual price level equals the expected one and the actual output is equal to the potential output. As the money supply is expected to remain unchanged in the future the price level as well as the nominal exchange rate are expected to be stable at the level where they are in 1998 (which implies a stable expected real exchange rate). In addition, the domestic nominal and (ex ante) real interest rates equal the foreign ones and are both 5 % in 1998.

The next simulation experiment includes a monetary shock in the period 1998: it is assumed that money supply increases permanently by 5 % (i.e. $v_{1998} = 0.05$). As the price level expectations for period 1998 and onwards were formed assuming $E_{1997}(v_{1998}) = 0$, the actual price level in 1998 as well as the new expectations for years 1999 and onwards exceed the older expected values and there is a substantial increase in the level of output over the potential one in 1998. This is achieved through a fall in the real exchange rate as well as in the ex ante real interest rate. The former is created by the fact that the monetary shock results in a depreciation of the nominal exchange rate in 1998 which exceeds the increase in the price level (i.e. sluggish adjustment of prices). The latter reflects the expectation that the price level is going to adjust fully to the money supply shock in the next period. This implies a high inflation rate in 1999. The real effects of the monetary shock are expected to disappear after 1998 and output is expected to return back to the potential level. Figure 1 summarizes the results of this simulation.[15]

An interesting charisteristic of the model is that the nominal exchange rate undershoots its final (expected) level: the monetary shock leads to a depreciation of the exchange rate but this fall is only partial relative to the size of the shock. As a result the exchange rate is expected to achieve its final level only in period 1999. Because of the uncovered interest parity assumption the expected depreciation is reflected in the positive interest differential between domestic and foreign countries. The under-shooting property of our model results from the chosen parameter values: it can be shown that there would be a over-shooting of the exchange rate if the sum of IS curve elasticity parameters is less than one, i.e. if $\delta + \sigma < 1$.

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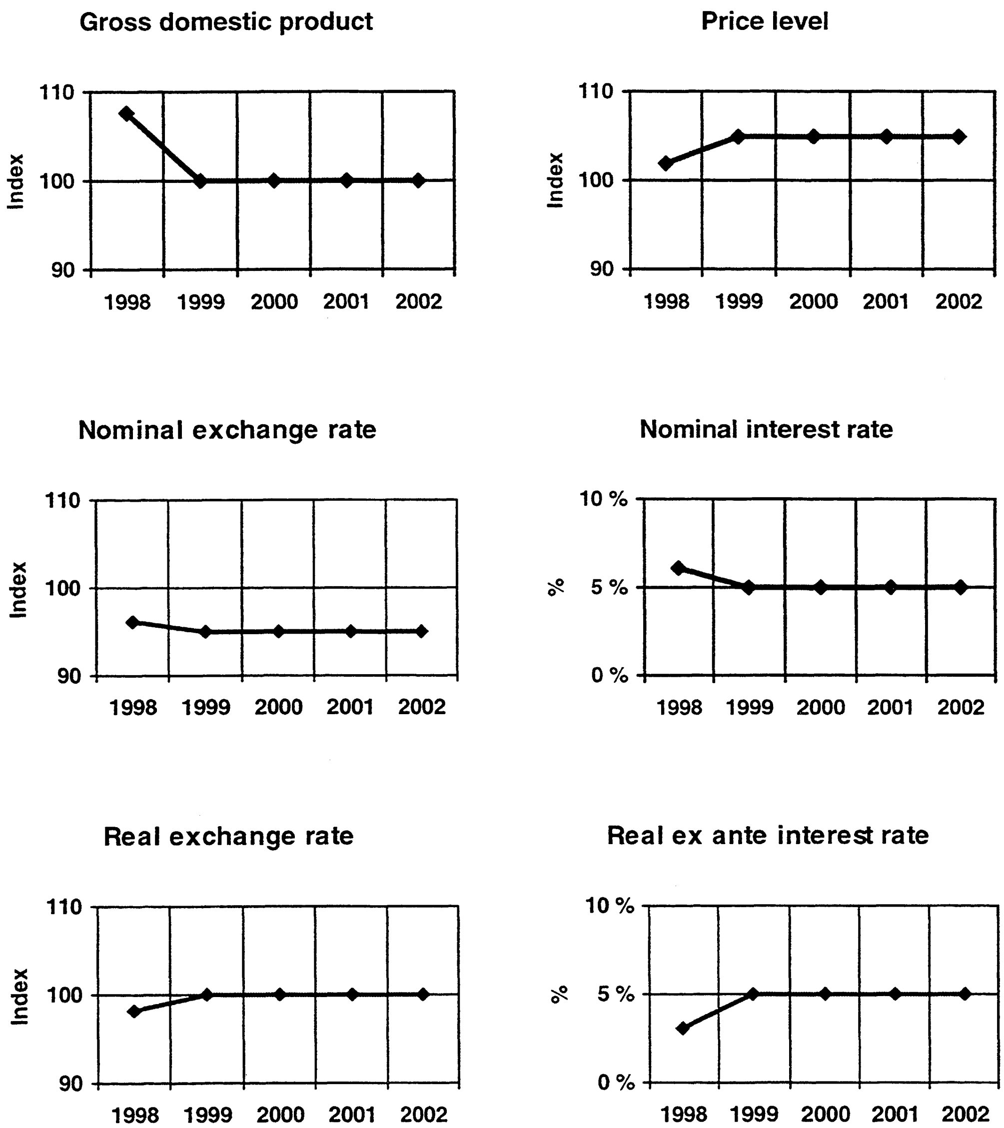
[14] We neglect here the existence of the disturbance term ε in our model because of the assumption of no peso problem.

[15] It should be noted that in figure 1 as well as in figures 2 and 3 the materialized macroeconomic equilibrium for 1998 is presented together with the *expected* values of the variables for years 1999–2002 (where the expectations are based on the information set available in period 1998).

Figure 1. **The effects of a monetary shock: the macroeconomic equilibrium for 1998 and expectations for 1999–2002 based on the information available in period 1998.**

The values of output, price level and exchange rates are presented relative to the baseline simulation (= 100). A falling exchange rate implies depreciation.



4.2 Some simple simulations of peso problems

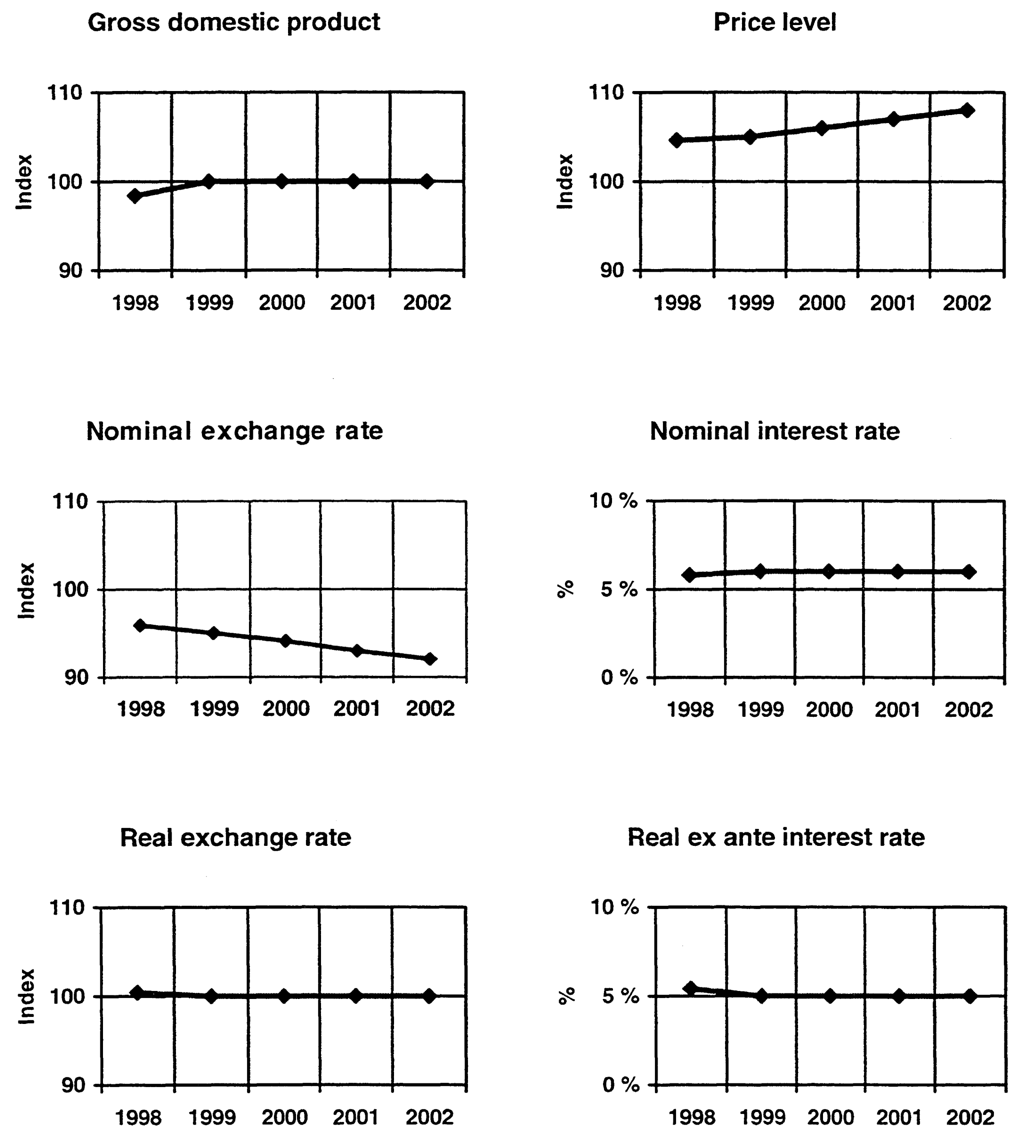
According to Vilmunen (1998) the existence of a peso problem has clear macroeconomic effects. The real exchange rate appreciates and the real ex ante interest rate rises compared to the situation where there is no peso problem. As a result the real output falls relative to the potential one: the peso problem imposes real costs on the economy. On the nominal side, the peso problem increases the nominal interest rate as well as the price level. However, the effect on the nominal exchange rate appears to be somewhat ambiguous and depends on the values of the model parameters: a sufficient condition for a depreciation of the nominal exchange rate is that aggregate demand is sufficiently responsive to changes in real exchange rate and real interest rate.

We will now proceed in our numerical analysis and include a peso problem in our model. First we will run a simple deterministic simulation where the peso problem is constant and there are no IS, AS or monetary shocks in 1998. This enables us to compare our results with those derived by Vilmunen (1998). In the following sections we will run stochastic simulations where the values of the IS, AS and monetary disturbance terms as well as the severity of the peso problem may vary.

In the next simulation it is assumed that the probability of a discrete shift in the money supply, q, is 0.05. The (expected) size of this shift is set constant and equal to 20 % of the money supply (i.e. $\Omega_t = \Omega = 0.20$). Thus the expected jump in the money supply is 1 % in every period. The results of the simulation are presented in figure 2. It should be emphasized that this simulation – like all the simulations in the following sections – has been done assuming that the peso jump *doesn't materialize* in period 1998 as we are investigating the effects of the presence of the peso problem, not the actual realization of the jump. If the jump happened, the results for period 1998 would naturally look very different, resembling those of a substantial monetary expansion.

Figure 2. **The effects of a constant peso problem: the macroeconomic equilibrium for 1998 and expectations for 1999–2002 based on the information available in period 1998.**

The values of output, price level and exchange rates are presented relative to the baseline simulation (= 100).
A falling exchange rate implies depreciation.



As can be seen from figure 2, the results correspond with those derived by Vilmunen (1998): the presence of the peso problem leads to a higher ex ante real interest rate and a stronger real exchange rate and hence to lower output in 1998. In our numerical example the expectation of a one percent jump in the money supply raises the ex ante real interest rate by 0.4 percentage points and strenghtens the real exchange rate by 0.4 percent. This lowers GDP by 1.6 percent. The price level as well as the nominal interest rate are both higher and the nominal exchange rate weaker than in a situation where the peso problem doesn't exist. The real effects are due to the expectations of a money supply jump formed in the previous period (i.e. 1997) that ex post *seem* to be false (because the money supply didn't jump): the price level in 1998 was expected to be higher and the nominal exchange rate weaker than the actual realizations. This corresponds to the effects of a negative monetary shock. As the price level adjustment is sluggish, the deviation of the nominal exchange rate from the expected level exceeds the deviation of the price level from its expected level causing the real exchange rate to appreciate. In addition, because the expectations formed in 1998 for 1999 correspond to those that were formed in 1997 for 1998 (as the expected money supply is the same in both cases), the expected depreciation of the nominal exchange rate from 1998 to 1999 is larger than the expected rise in the price level. As the interest rate reflects the expected change in the exchange rate the ex ante real interest rate rises.

The new expectations for the periods after 1999 (formed in period 1998) show that the nominal exchange rate is expected to weaken along with the rising price level: the driving force behind these movements is the expectation that money supply increases by 1 % per period. The expected real exchange rate as well as the expected real ex ante interest rate for period 1999 and onwards are constant: the higher nominal interest rate which reflects the expected depreciation of the currency is compensated by a corresponding rise in the price level. Thus real output is expected to return to its potential and stay on that level in the coming periods. But, if the expected changes in the money supply do not materialize the economic situation of the period 1998 will recur also in the future.

The previous example demonstrated the effects of the peso problem in a situation where the peso problem has been present for a longer time: the expectations formed in 1997 for period 1998 and onwards were based on the knowledge that a given discrete shift in the money supply was possible (with a constant probability). But what are the effects of a sudden emergence or disappearance of a peso problem? In the context of our model the former means that in period 1998 new information comes which alters economic agents' perception of the relevant money supply process: the possibility of a jump which in period 1997 was seen to be zero (for 1998 and onwards) now becomes non-zero. The results of this kind of a simulation are presented in figure 3. In the simulation it is assumed that in period 1998 the probability of a future 20 % jump in the money supply increases from zero to 0.05.[16]

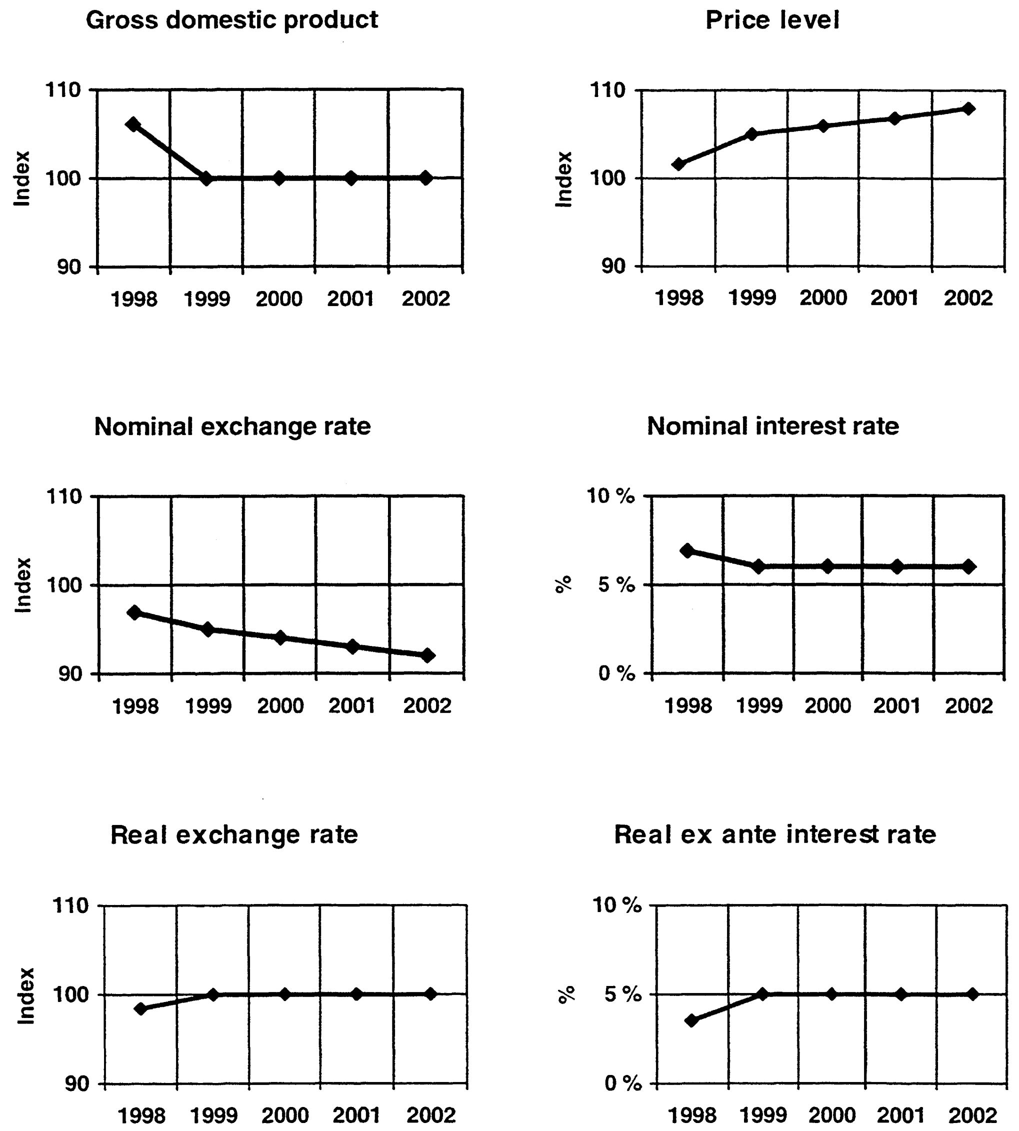
The emergence of a peso problem increases the expected future money supply resulting in a rise in the price level in 1998 and a sizable upward revision of expected future price levels together with a clear depreciation of the spot and

[16] The problem with this simulation is that we have not specified how the expectations concerning the probability of a jump are formed. It is simply assumed that the new information received in 1998 alters the expected probability of a future jump permanently. In the following section we will address the question of a time-varying peso problem in a more satisfactory way.

expected future exchange rates. Thus the expectations formed in 1997 for 1998 and onwards – like the price level expectations – appear to be misaligned. As the price level adjustment is sluggish compared to the nominal exchange rate the real exchange rate in 1998 depreciates and the ex ante real interest rate falls leading to a temporary increase in the real output. The costs of the peso problem show up in the subsequent periods if the expected changes in the money supply do not materialize. Similar results can be obtained if the probability of a jump increases, i.e. there is a worsening of an existing peso problem.

Figure 3. **The effects of the emergence of a peso problem: the macroeconomic equilibrium for 1998 and expectations for 1999-2002 based on the information available in period 1998.**

The values of output, price level and exchange rates are presented relative to the baseline simulation (= 100).
A falling exchange rate implies depreciation.



If an existing peso problem disappears (or becomes less severe) the direct effects are the opposite. The real exchange rate appreciates and the ex ante real interest rate rises compared to the situation where the severity of the peso problem is unchanged. As a result the real output in period 1998 falls. The positive effects of the easening of the peso problem will show up only in the subsequent periods.

4.3 Stochastic simulations of peso problems

In the previous section the macroeconomic consequences of a peso problem were analysed concentrating on the level effects, i.e. how the presence of a peso problem affects the levels of certain economic variables. It was found that the peso problem leads to an overvalued real exchange rate and a high real ex ante interest rate. As a result the real output of the economy is lower. From the point of view of empirical research these findings are usually of limited use: if we want to assess the presence of a (potentially time-varying) peso problem in macroeconomic time series we need more information on its effects. For instance, how does the presence of a peso problem influence the *correlations of macroeconomic variables* like real output, real exchange rate, ex post real interest rate and ex post difference in the yields of investments in different currencies? And how do these correlations differ from those created by the "normal" monetary, demand and supply shocks affecting the values of these variables?[17]

In order to answer these questions we need to run *stochastic simulations* with our model. In stochastic simulations the model is solved repeatedly along the lines described in section 3.1 but each time a new set of values for the stochastic disturbance terms for period t is drawn from specified distributions. Thus each simulation produces a different macroeconomic equilibrium for period t depending on the drawn values of the disturbance terms. This enables the calculation of variances as well as correlations of macroeconomic variables. The complicating factor in our simulations is that we are not only interested in the variables like real output (or actually the deviation of output from potential) and real exchange rate that can be calculated using the results for period t (i.e. 1998). Because the data observed e.g. by an econometrician is *ex post* we are also interested in the ex post real interest rate and the ex post yield differential in period t. The calculation of the latter variables requires information on the macroeconomic equilibrium in period t+1 (i.e. 1999). Thus we must solve the model stochastically for two consecutive periods, 1998 and 1999. This means that each simulation consists of two phases. In the first phase a set of values for the disturbance terms is drawn and the model is solved for period 1998. This gives us the macroeconomic equilibrium in period 1998 and the expectations for period 1999 and onwards based on information available in period 1998.[18] In the second phase a new set of disturbance terms is drawn and the model is solved for 1999 given the money supply in 1998 and the expectations formed in 1998. In each

[17] A more sophisticated way to investigate the effects of peso problems on macrovariables would be to use spectral analysis of time series generated by simulation experiments. See e.g. Naylor et al. (1969).

[18] As we described earlier our solution method produces also the expectations formed in 1997 on the basis of information available in that time period.

stochastic simulation experiment this procedure is repeated 250 times, i.e. the correlation coefficients are calculated from the results of 250 simulations.

A. Correlations based on ordinary IS, AS and money supply shocks

For the sake of comparison we first make the simulations without the presence of peso problems: the probability of a discrete shift in the money supply is set equal to zero. The model now includes only the ordinary monetary, aggregate demand and aggregate supply disturbance terms – v_t, u_t and w_t – which are all assumed to be normally and independently distributed with zero means and standard deviations equal to 0.02. The results from this simulation experiment are presented in table 1 (with an accuracy of one decimal). Because of the linearity of our model the means of the various macrovariables calculated from the simulations should correspond with the values of these variables in the (deterministic) baseline simulation presented in section 4.1. In addition, the means of the ex ante and ex post real interest rates as well as of the ex ante and ex post yield differentials should be the same. These conditions hold reasonably well in our stochastic simulations.

Table 1. **Correlation coefficients calculated from simulations with stochastic IS, AS and money supply shocks.**

	Output gap	Real exchange rate	Ex post real interest rate	Ex post yield differential
Output gap	1	0.8	–0.6	0.0
Real exchange rate		1	–0.7	0.0
Ex post real interest rate			1	0.6
Ex post yield differential				1

Note: the variables are $y_t - y^n$ (output gap), $s_t + p^f - p_t$ (real exchange rate), $i_t - [p_{t+1} - p_t]$ (ex post real interest rate) and $i_t - [i^f + (s_{t+1} - s_t)]$ (ex post yield differential). A rise in the real exchange rate implies real depreciation.

The correlations found among the macroeconomic variables depend on the relative "strength" of the IS, AS and money supply disturbances. This in turn depends on the values of the model parameters and on the distributions of the disturbances. In the following stochastic simulations where peso problems are analysed we have used the same parameter values and distributions as here in order to be able to concentrate strictly on the effects of peso problems.[19]

Aggregate supply (AS) and money supply shocks tend to create similar correlations among the variables (see Appendix 2). On the other hand the correlations caused by IS shocks differ substantially from those of the

[19] In fact, we have even kept the numerical values of the IS, AS and monetary disturbances the same in the peso problem simulations in order to avoid the effects of sampling on the results.

aforementioned shocks. As a whole the joint effect of the former shocks dominates the latter in the results presented in table 1. For instance, the strongly positive correlation of the output gap and the real exchange rate is the net result of perfect positive correlation in the case of AS and monetary shocks and a perfect negative correlation in the case of IS shocks.

A noteworthy observation is that the correlation of the ex post yield differential with either the output gap or the real exchange rate is zero. This is also the case when the effects of the disturbances are analysed separately. Because of the assumption of uncovered interest parity the *ex ante* yield differential is always zero: the interest differential reflects the expected change in the exchange rate. As the deviation of the actual exchange rate from its expected value depends only on the current values of the disturbances the *ex post* yield differential may be negative or positive, no matter what has been the size of the output gap or the value of the real exchange rate in the previous period. Later it is shown that the existence of a variable peso problem may cause these correlations to become non-zero.[20]

B. Correlations based on the variation of the jump probability

The correlation coefficients presented in Table 1 reflected correlations among macroeconomic variables in a situation where there is no peso problem present. The inclusion of *a constant peso problem* – i.e. a peso problem with a constant probability and a constant size of a jump – does not change those results: it only affects the means of the variables around which the variation occurs. If, for instance, we set the jump probability equal to 0.05 and the jump size equal to 0.2 (20 %) the resulting macroeconomic equilibrium for period 1998 (where the equilibrium values are calculated as the means of 250 simulations) corresponds with that presented in Figure 2. In addition, we are now able to calculate the average ex post real interest rate and ex post yield differential for period 1998. As the peso jump doesn't materialize in period 1999 the price level is on the average lower and the the nominal exchange rate stronger than what was expected given the information available in period 1998. So the (mean of the) ex post real interest rate exceeds the (mean of the) ex ante real interest rate and there exists a positive (mean of the) ex post yield differential. In our numerical example the ex post real interest rate is 0.4 percentage points higher than the ex ante real interest rate. Compared to the situation where the peso problem doesn't exist the ex post real interest rate is 0.8 percentage points higher than ex ante or ex post real interest rates. The ex post yield differential is of the same size.

A simple way to analyse the effects of *a variable peso problem* is to let the probability of a peso jump (q) vary in simulations together with the IS, AS and monetary disturbances. In each simulation new values for the disturbances as well as for the probability is drawn and the model is solved according to the principles presented above. It should be noted that the probability of a jump is kept constant during the two phases of a single simulation: the expectations concerning future changes in the money supply due to the peso problem are the same in period 1998

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[20] Strictly speaking, the correlation coefficients presented in Table 1 are conditional (E_{t-1}) correlation coefficients as the simulations have been done taking as given the situation in period t–1 ("1997"). However, as the disturbances are assumed to be independent and non-autocorrelated these coefficients are the same as the corresponding unconditional correlation coefficients.

and 1999. Thus the severity of the peso problem doesn't change when we move from period 1998 to period 1999 during a single simulation. This kind of a stochastic simulation experiment could perhaps be used to depict a situation where we have a cross-section of countries with different (but constant) peso problems.

In Table 2 we present results from simulations where the jump probability varies from simulation to simulation together with the values of the IS, AS and money supply disturbances. It is assumed that the probability q follows a uniform distribution with mean 0.05 and that the size of a possible jump is 0.2 (= 20 %).

Table 2. **Correlation coefficients calculated from simulations with stochastic IS, AS and money supply shocks and a variable jump probability q.**

	Output gap	Real exchange rate	Ex post real interest rate	Ex post yield differential
Output gap	1	0.8	–0.6	0.0
Real exchange rate		1	–0.7	0.0
Ex post real interest rate			1	0.6
Ex post yield differential				1

Note: the definitions of the variables were presented in Table 1. The size of the peso jump (Ω) is assumed to be 0.2 (= 20 %) and q follows a uniform distribution in the range [0,0.1).

The inclusion of a variable jump probability doesn't change the correlation coefficients compared to Table 1 (when the results are reported with an accuracy of one decimal). However, to some extent this seems to reflect the strong influence of money supply shocks (v_t) in our model. If we analyse the effects of a variable jump probability separately (i.e. without IS, AS or money supply disturbances) the correlation coefficients do look different. Especially, the coefficient of the ex post yield differential with either the output gap or the real exchange rate becomes *equal to minus one*: there is a perfect negative relationship between these variables (see Table 3 below). For instance, if the jump probability is small the output gap in period 1998 is negative but minor (i.e. larger than the [negative] mean) and the expected depreciation of the exchange rate is small as well. As this expected depreciation doesn't materialize in period 1999 (i.e. the money supply doesn't change) the resulting ex post yield differential is small. Thus there is a perfect negative correlation between these variables. As the size of the output gap varies together with the real exchange rate the correlation between the real exchange rate and the yield differential is also negative.

Table 3. **Correlation coefficients calculated from simulations with a variable jump probability q.**

	Output gap	Real exchange rate	Ex post real interest rate	Ex post yield differential
Output gap	1	1	−1	−1
Real exchange rate		1	−1	−1
Ex post real interest rate			1	1
Ex post yield differential				1

Note: the definitions of the variables were presented in Table 1. The size of the peso jump (Ω) is assumed to be 0.2 (= 20 %) and q follows a uniform distribution in the range [0,0.1).

If the variable jump probability is combined with IS or AS shocks (or both) these aforementioned correlation coefficients still stay clearly negative: the influence of the former dominates (given the distributions of the disturbances). By contrast, combining the variable jump probability with the money supply shock makes these coefficients quite small in absolute value.[21] Thus the influence of the variable jump probability fades away. This result is even clearer when all the different types of shocks are included as was seen in Table 2. However, it should be noted that even if the results seem to be scanty they give some hints of the possible effects of peso problems on the correlations of macroeconomic variables. These hints can be useful in the empirical work.[22]

C. Correlations based on time-varying peso problems

The way in which a variable peso problem was included in the model in the previous section gives only a partial view on the subject. Especially, it doesn't take into account the possibility that the severity of the peso problem might change from period to period. In a simulation consisting of two phases as above this kind of a time-varying peso problem means that the solution for period t is based on different expectations about future money supply jumps than the solution for period t+1. This happens if new information comes in period t+1 that alters economic agents' perception of the future jumps. The change in the expectations naturally affects the equilibrium values of the macroeconomic variables and may also have an effect on the correlations between these variables.

[21] The expected change of the money supply that is due to the peso problem is uniformly distributed in the range (0, 0.02). The money supply disturbance term v_t is normally distributed with zero mean and standard deviation equal to 0.02. Thus the latter has a strong influence on the results.

[22] The correlation coefficients presented in Tables 2 and 3 may be best interpreted as unconditional correlation coefficients given the "cross-sectional" nature of the simulations (and the assumed independence and non-autocorrelation of the disturbances in Table 2).

As was explained in chapter 2 the time-varying peso problem is included in our model by assuming that the size of a possible money supply jump, Ω_t, varies from period to period (during a single two-phase simulation) according to the following stochastic process: $\Omega_t = \Omega_{t-1}\exp(\varepsilon_t)$, where ε_t is a normally and independently distributed disturbance term. As we move from period t–1 to period t new information comes in the form of ε_t which affects the jump size of the current period (which may or may not materialize). *In addition*, it also affects the expectations concerning future jumps sizes and the expected changes in the money supply due to the peso problem (given the fixed probability of a jump, q).

In Table 4 we present results from simulations with a time-varying jump size and the normal IS, AS and money supply disturbances. The distributions (and values) of these disturbances are the same as in the previous simulations. In each simulation it is assumed that $\Omega_{1997} = 0.2$ and that $\varepsilon_t \sim N(0, 0.01)$. Given that the probability of a jump is 0.05, the expected change in the money supply for period 1998 is approximately one percent on the basis of information available in period 1997. For period 1999 and onwards the size of the expected money supply change increases (see equations (9)–(11) in chapter 2). When the values of the disturbances for period 1998 are drawn a new jump size Ω_{1998} materializes. The model is then solved for 1998. Finally, a new set of values for the disturbances is drawn and the model is solved for period 1999 with a new jump size Ω_{1999} and revised expectations of money supply jumps.

Table 4. **Correlation coefficients calculated from simulations with stochastic IS, AS and money supply shocks and a time-varying jump size.**

	Output gap	Real exchange rate	Ex post real interest rate	Ex post yield differential
Output gap	1	0.8	–0.6	0.1
Real exchange rate		1	–0.7	0.1
Ex post real interest rate			1	0.6
Ex post yield differential				1

Note: the definitions of the variables were presented in Table 1. The jump size varies according to the following process: $\Omega_t = \Omega_{t-1}\exp(\varepsilon_t)$ where ε_t is normally and independently distributed with zero-mean and variance equal to 0.1^2. In addition, it is assumed that $\Omega_{1997} = 0.2$. The probability of a jump (q) is assumed to be constant and equal to 0.05.

The inclusion of a time-varying jump size doesn't change the values of the correlation coefficients significantly compared to Table 1: the only differences are the correlations between the ex post yield differential with either the output gap or the real exchange rate which now become slightly positive. If we analyse the effects of a variable jump size separately (i.e. without IS, AS or money supply disturbances) the differences are clearer: the values of the aforementioned correlation coefficients now become fairly *positive* (see Table 5 below). The positive correlation reflects the fact that the information coming in every period changes both the macroeconomic equilibrium for the current period as well as the expected values of the variables in the future periods.

For instance, if the new information in period 1998 increases the size of a possible future money supply jump (i.e. a worsening of the peso problem from the original level) the direct effect of this is expansionary: as the future money stock is now perceived to be larger the real exchange rate depreciates and the ex ante real interest rate falls. As a result the output gap in 1998 is smaller (in absolute terms) than what would have been the case without new information. This corresponds with the results derived in section 4.2. In addition, at the same time the expected equilibrium of the next period changes. As the peso problem has become worse the expected change in the money supply in period 1999 has increased leading to an expectation of a clearly weaker exchange rate. As we have assumed that the uncovered interest parity holds this is reflected in a larger interest rate differential. Now, if the expected change in the money supply in period 1999 doesn't materialize and if the severity of the peso problem stays the same or eases (i.e. the expected size of a future jump shrinks) the nominal exchange rate in 1999 is clearly stronger than what was expected: thus the ex post yield differential is high. It is even possible that the peso problem worsens somewhat in period 1999 but the yield differential still stays on a high level.

Thus the correlation coefficient of the output gap and the ex post yield differential is positive: the expansionary first-round effect of a worsening peso problem is more probably followed by the materialization of the true nature of peso problem – high excess yields, low output – as the difference between the expected and the actual money supply is large.[23] The similar correlation between the real exchange rate and the yield differential follows from the fact that output movements are (in part) caused by corresponding changes in the real exchange rate.

Table 5. **Correlation coefficients calculated from simulations with a time-varying jump size Ω_t.**

	Output gap	Real exchange rate	Ex post real interest rate	Ex post yield differential
Output gap	1	1	–0.6	0.3
Real exchange rate		1	–0.6	0.3
Ex post real interest rate			1	0.6
Ex post yield differential				1

Note: the definitions of the variables were presented in Table 1. The jump size varies according to the following process: $\Omega_t = \Omega_{t-1}\exp(\varepsilon_t)$ where ε_t is normally and independently distributed with mean zero and variance equal to 0.1^2. In addition, it is assumed that $\Omega_{1997} = 0.2$. The probability of a jump (q) is assumed to be constant and equal to 0.05.

[23] The values of the correlation coefficients depend e.g. on the assumed variance of the disturbance term ε_t. An increase in the variance – i.e. greater variation in the peso problem – reduces the values of the aforementioned coefficients. For instance, if the variance is equal to 0.3^2 both the coefficients are about 0.25.

The current results concerning the aforementioned correlation coefficients differ clearly from those obtained in the previous section B where the values of the coefficients were equal to minus one, i.e. there was a perfect negative correlation between the ex post yield differential and both the output gap and the real exchange rate. However, there is no contradiction: these two sets of simulations describe different kinds of situations. The simulations in section B can be thought to represent correlations found e.g. in a cross-sectional analysis of countries with different but constant peso problems: countries with severe peso problems (large output losses) have higher excess yields than countries where the peso problem is less severe or non-existent. On the other hand, the simulations presented in this section can be thought to describe correlations of a country's macrovariables in a certain time period, i.e. it is a time-series view on the peso problem in one country. The variation in the severity of the peso problem produces first-round output and other real effects as well as changes in the expectations which have repercussions on the ex post yields of investments if the expectations fail to materialize.[24]

Summary of the results of the stochastic simulations

Our simulations show that the presence of a variable peso problem affects the correlations between certain macroeconomic variables, especially those between the ex post yield differential and either the real exchange rate or the output gap. In the case of conventional (non-autocorrelated) IS, AS and monetary shocks these correlations are equal to zero. The inclusion of a variable peso problem in the simulation model changes these results: ex post yield differentials are now correlated with real exchange rates and output gaps. This is quite natural as the existence of excess yields is closely connected to the presence of peso problems. Severe peso problems show up in the form of large excess yields and sizeable output losses while both the excess yields and output losses are smaller in the case of minor peso problems. This holds e.g. for cross-sectional data of countries or for a single country with episodes of different peso problems in its history.

The immediate effects of changes in the severity of a peso problem are interesting: the worsening of a peso problem has typically expansionary effects while the easening is contractionary. A worsening peso problem (with smaller output gap in the short run) also tends to be followed by a high excess yield more often than by a small excess yield: A worsened peso problem means larger expected changes in the exchange rate and thus larger interest rate differentials. And when these larger expected changes do not materialize, the ex post yield differentials are high (unless the peso problem worsens again so much that there is a clear depreciation of the currency). As a result the real costs of the peso problem for the economy are large.

[24] The coefficients in Table 5 are conditional correlation coefficients, i.e. they describe correlations between variables in a certain time period conditional on the situation in period t–1. As the variables are non-stationary (due to the formulation of the time-varying peso problem) these coefficients do not in general equal the corresponding unconditional correlation coefficients. The comparison of the coefficients in Table 5 to the those presented in Table 1 can be done because the conditional and unconditional correlation coefficients are the same when we have only the conventional (non-autocorrelated) IS, AS and monetary disturbances in our simulation model.

5 Empirical correlations between macroeconomic variables

5.1 Data description

In order to demonstrate how the results of our simulations may be applied we have done some small-scale empirical analysis with real data. As examples we have chosen the cases of Canada and the United Kingdom during 1980–1997 using the Unites States as a point of comparison for the former and Germany for the latter. Thus the real exchange rates as well as the ex post yield differentials are bilateral in nature. Apart from the short-lived visit of the pound sterling to the European exchange rate mechanism, the ERM, at the beginning of the 1990's, the currencies of Canada and the UK have been floating during the sample period. In addition, although the external value of the Canadian dollar has shown quite large swings in the past there doesn't seem to have occured clear peso-type jumps.[25] This corresponds to the setting of our simulations. In the case of the UK this assumption is more uncertain, especially regarding the situation in the fall 1992 when the pound sterling left the ERM. However, as the aim of our empirical analysis is mainly illustrative we do not consider this as a serious problem.

Our analysis has been done using quarterly data from periods 1980:1–1997:4.[26] The variables are the same as in our simulations, i.e. the output gap, the real exchange rate, the ex post real interest rate and the ex post yield differential. A detailed description of the calculation of the variables is given in the Appendix 3. As was already mentioned, the real exchange rates and yield differentials are bilateral: in the case of UK these variables are measured vis-à-vis Germany, and in the case of Canada we have used the United States as a point of comparison. The interest rates used in the calculation are 3-month rates and prices are measured by consumer price indices. The quarterly output gaps have been derived by smoothing the (seasonally adjusted) output series with the Hodrick-Prescott (HP) filter and then calculating the deviation of the actual output from the trend.

Our data series for the output gaps, the ex post real interest rates as well as for the ex post yield differentials passed the unit-root tests (see the Appendix 3). On the contrary, the bilateral real exchange rate series turned out to be strongly non-stationary (as was expected). As the stationarity of the series was essential for our analysis, we used the HP filter to calculate the trends of the real exchange rates and then approximated real exchange rates by the deviation of the actual real exchange rate from the trend. This method – though primitive – gave us series which are stationary and which can be thought to describe the developments of the real exchange rates around their equilibrium paths.

[25] For simplicity, we look at the development of the nominal exchange rate for peso jumps instead of the money supply as the latter may e.g. have gone through various structural changes in the course of time.

[26] Quarterly data may not be very appropriate as there tends to be lags between e.g. changes in the real exchange rate and the real output. However, the use of quarterly data is useful in the VAR analysis.

5.2 Analysis and results

We start the analysis by calculating the values of the conventional correlation coefficients between the aforementioned variables. The results are presented in Table 6 below. As the simulation results in chapter 4 pointed out, we can try to detect the presence of a (variable) peso problem from the correlation coefficients between the ex post yield differential and either the output gap or the real exchange rate. According to Table 6 these correlation coefficients seem to be slightly positive. In the case of the United Kingdom a simple test reveals that the coefficients are even significant at 5 % level. Naturally, it would be tempting to conclude that this may hint to the existence of a peso problem. Unfortunately our simulation results concerning time-varying peso problems – which indicated positive correlations between the aforementioned variables – are not directly applicable to this situation as they were derived as conditional correlation coefficients. Thus all we can say is that the correlation coefficients of the ex post yield differential with either the output gap or the real exchange rate presented in Table 6 seem to differ from those caused by the conventional (non-autocorrelated) IS, AS and monetary shocks, at least in the case of the United Kingdom. The values of the other correlation coefficients seem in general to be closer to the correlations caused by AS and monetary shocks than by IS shocks (see Appendix 2). However, the evidence is mixed.

Table 6. **The values of the correlation coefficients of macroeconomic variables in Canada (Can) and the United Kingdom (UK), calculated from data covering 1980:1–1997:4.**

	Output gap	Real exchange rate	Ex post real interest rate	Ex post yield differential
Output gap	1	Can: 0.31** UK: 0.19	Can: 0.0 UK: –0.19	Can: 0.16 UK: 0.29*
Real exchange rate		1	Can: –0.12 UK: –0.27*	Can: 0.17 UK: 0.28*
Ex post real interest rate			1	Can: 0.32** UK: –0.21
Ex post yield differential				1

Note: The description of the variables is presented in Appendix 3.
* Significant at 5 % level
** Significant at 1 % level

In order to be able to analyse more closely the possible presence of peso problems we proceed to vector autoregressive (VAR) modelling. A VAR model consists in regressing each current variable in the model on all the model variables lagged a certain number of times, i.e.

$$Y_t = \sum_{i=1}^{n} A_i Y_{t-i} + u_t \qquad (15)$$

where Y_s is an kx1 vector of variables in period s, A_i is the kxk coefficient matrix for variables lagged by i periods, and u_t is a kx1 vector of zero-mean disturbance terms. Given the information available at time t–1 (i.e. Y_{t-1}, Y_{t-2}, Y_{t-3}, ...) the conditional expectation of Y_t is

$$E_{t-1}(Y_t) = \sum_{i=1}^{n} A_i Y_{t-i} \qquad (16)$$

As the autoregressive part of each equation gives us the conditional t–1 expectation of the dependent variable in period t (or the joint influence of the past values of the model variables on the dependent variable), the disturbance term incorporates all the effects of shocks and new information coming in period t including the effects of the changes in the severity of a possible peso problem. Thus by calculating the correlation coefficients of these disturbance terms (or actually the corresponding residuals) we get some idea of the conditional correlations of the dependent variables, with conditionality defined in terms of the past values of the variables in the model.[27]

In both cases – Canada and the UK – we have chosen to work with a VAR(2) model with seasonal dummies, i.e. there are two lags of each variable in the equations. The choice of the lag lenght has been made by checking the overall significance of each regressor in turn with F-tests and then testing the deletion of lags with an overall F-test. In the case of the UK our model seems to be quite good as there are no striking signs of mis-specification. The model for Canada doesn't work as well: the residuals of the equation for the ex post yield differential have problems with autocorrelation and there are signs of heteroskedasticity in the real exchange rate equation. This calls for caution when interpriting the results. The estimation results as well as the summaries of the various tests of the residuals have been relegated to the Appendix 3.[28] The values of the correlation coefficients of the VAR residuals are presented in Table 7.

[27] This is just one possible (and not necessarily the best) way to use VAR models in the analysis of peso problems. Another could be the use of impulse responses.

[28] We have also checked the dynamic properties of the models. In both cases the companion matrix shows no roots outside the unit circle indicating that the systems should be stable. However, the impulse response analysis hints that there might be some problems in certain equations. An interesting detail of the dynamic analysis is that the impulse responses of our models are quite similar to each other.

Table 7. **The correlation coefficients of the VAR residuals for Canada (Can) and the United Kingdom (UK). Estimation period 1980:3–1997:4. The residuals are named after the dependent variables of the estimated equations.**

	Output gap	Real exchange rate	Ex post real interest rate	Ex post yield differential
Output gap	1	Can: 0.30* UK: 0.16	Can: –0.27* UK: –0.10	Can: –0.24* UK: –0.03
Real exchange rate		1	Can: –0.07 UK: –0.24*	Can: 0.12 UK: 0.05
Ex post real interest rate			1	Can: 0.39** UK: 0.23
Ex post yield differential				1

* Significant at 5 % level
** Significant at 1 % level

The estimated conditional correlation coefficients are in many cases quite close to the values of the unconditional coefficients. Some notable differences arise, however. Inter alia the values of the correlation coefficients between the ex post yield differential and either the output gap or the real exchange rate differ from those presented in Table 6. In the case of the UK they are now about zero: this corresponds with the correlations found in the simulations with conventional IS, AS and monetary disturbances and points to the conclusion that there doesn't seem to be evidence of the presence of a time-varying peso problem. This is somewhat disappointing as the results presented in Table 6 above hinted that there could be also other forces than the conventional shocks affecting the UK data. In the case of Canada, the results are mixed: the aforementioned coefficients are larger than in the UK case but their signs differ. This makes the interpretation of the results problematic and calls for further analysis on the subject. As far as the other correlation coefficients are concerned, the values presented in Table 7 seem in general to be closer to the correlations caused by AS and monetary shocks than by IS shocks. Compared to Table 6 this result is now more robust.[29]

[29] As our models do not include any foreign explanatory variables (describing the state of economic activity in other countries etc.) we may be missing out some important factors affecting the development of our model variables. In order to check this we incorporated into both models additional exogenous variables, namely the lagged output gap of the United States (in the Canadian model) and the lagged output gap of West Germany (in the UK model). The derivation of these variables was analogous to the derivation of the other output gap variables.

These new variables seemed to have some explanatory power in our models, especially in the output gap equations of the UK and Canada. However, the results concerning correlations between macrovariables didn't change substantially. In the case of Canada the signs of the aforementioned correlation coefficients still differed from each other like in the original model. In the case of the UK the values of these correlation coefficients increased somewhat and were slightly positive. Especially, the correlation between the yield differential and the output gap was about 0.1. This result is naturally encouraging. However, the coefficients are still so close to zero that it is difficult to say whether they could support the notion of a time-varying peso problem.

6 Concluding remarks

In this paper we have presented some further evidence on the effects of peso problems on the real side of the economy by simulating numerically a small-scale rational expectations macromodel. The model – which draws heavily from Vilmunen (1998) – is a conventional IS-LM-AS model of an open economy under floating exchange rates. The peso problem has been incorporated in the model by assuming that the process driving the money supply in the economy contains a minor but non-zero possibility for a sizable discrete shift or jump in the money supply. In addition, the severity of the peso problem may vary in time. We have not specified the reason for the existence of this jump possibility but it can be interpreted e.g. as representing the imperfect credibility of the monetary authorities.

A rational expectations model can normally be solved and simulated numerically using e.g. the solution method presented by Fair & Taylor (1983, 1990). The procedure used in solving our model is to some extent more complicated than the standard method as there are two viewpoint dates in which expectations are formed, i.e. there are two sets of expectations of the values of the future endogenous variables affecting the macroeconomic equilibrium in a certain time period t. The existence of two viewpoint dates is connected to the presence of an expectations-augmented Phillips curve in our model and can be interpreted as reflecting the idea that nominal wage contracts for period t are signed at the end of period t–1 (using information available at that time) while e.g. participants in the financial markets base their decisions and expectations on information available in period t. We have solved the problem caused by the two viewpoint dates by augmenting the original model with additional equations and variables which determine the expectations based on the information set available in period t–1 and then solving the augmented model. Thus the expectations formed in period t–1 are derived simultaneously with the overall solution for period t (including expectations formed in period t).

We have investigated peso problems using both deterministic and stochastic simulations. As we are interested in the effects of the peso problem itself, it has been assumed in each simulation experiment that the peso jump doesn't materialize during the simulation: thus it's a question of the effects of unfullfilled expectations of changes in the money supply. According to the deterministic simulations the presence of a peso problem leads to an overvalued real exchange rate and a higher ex ante real interest rate which together cause output losses. This corresponds with the results derived by Vilmunen (1998). In addition, it was shown that the emergence of (or the worsening of an existing) peso problem has temporary expansionary effects while the direct effects of a disappearance (or easening) of an existing peso problem are the opposite.

In order to get a deeper insight into the effects of peso problems we have run some stochastic simulations with our model. In these simulations we let the values of the IS, AS and monetary disturbances vary together with the severity of the peso problem, i.e. we have a (time-)varying peso problem. The output of stochastic simulations allows us to calculate variances and correlations of different variables. Especially, we have been interested in the effects of peso problems on the correlations between macroeconomic variables like the output gap, the real exchange rate, the ex post real interest rate and the ex post yield differential. Our simulations show that the presence of a variable peso problem does affect the correlations between macroeconomic variables. The most important result concerns the correlations between the ex post yield differential and either the real exchange rate or the output gap. In the case of conventional (non-autocorrelated) IS, AS and monetary disturbances these correlation coefficients (both conditional and unconditional) are equal to zero. The inclusion of a variable peso problem in the simulation model changes these results: the ex post yield differential is now correlated with the real exchange rate and the output gap. When the severity of the peso problem varies from one simulation to another – which can perhaps be used to describe e.g. the situation in a cross-sectional analysis of countries with different (but constant) peso problems – these correlations are strictly negative. This result reflects the fact that severe peso problems tend to show up in the form of large excess yields and sizable output losses. However, whether this correlation shows up in real data depends on the effects of the IS, AS and monetary disturbances which may dominate the effect of variation in the peso problem.

When the severity of the peso problem is let to vary during a single simulation – i.e. we have a time-varying peso problem in a single country with expectations changing along with the inflow of new information – the (conditional) correlation coefficients between the ex post yield differential and either the real exchange rate or the output gap appear to be slightly positive. A worsening peso problem with a smaller output gap (= smaller output losses) and a weaker real exchange rate in the short run tends to be followed by a high excess yield more often than a low excess yield: A worsened peso problem means larger expected changes in the money supply and in the nominal exchange rate and thus larger interest rate differentials. And if these larger expected changes do not materialize, the excess yields are also higher (unless the peso problem worsens again clearly).

The results of our numerical simulations are directly applicable in the empirical analysis of peso problems. Our illustrative example dealt with the cases of Canada and the United Kingdom. The ordinary correlation coefficients hinted that in the latter case there could have been something else than just the conventional IS, AS and monetary shocks affecting the data. However, the VAR analysis which was used to produce approximations of the conditional correlation coefficients didn't give much support to the hypothesis of a time-varying peso problem. As the data used in the empirical work was far from being satisfactory the case is still open for further analysis.

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Appendix 1

The simulation model

The simulations have been carried out using the following model:

$$y_t - y^n = \delta[s_t + p^f - p_t] - \sigma[i_t - p_{t+1} + p_t] + u_t$$
$$p_t = \tilde{p}_t + \theta[y_t - y^n] - \theta w_t$$
$$m_t - p_t = y_t - \lambda i_t$$
$$i_t = i^f + s_{t+1} - s_t$$
$$m_t = m_{t-1} + v_t + n_t \Omega_t$$
$$\tilde{y}_t - y^n = \delta[\tilde{s}_t + p^f - \tilde{p}_t] - \sigma[\tilde{i}_t - \tilde{p}_{t+1} + \tilde{p}_t] + \tilde{u}_t$$
$$\tilde{y}_t - y^n = \tilde{w}_t$$
$$\tilde{m}_t - \tilde{p}_t = \tilde{y}_t - \lambda \tilde{i}_t$$
$$\tilde{i}_t = i^f + \tilde{s}_{t+1} - \tilde{s}_t$$
$$\tilde{m}_t = m_{t-1} + \tilde{v}_t + \tilde{n}_t \tilde{\Omega}_t$$

The variables equipped with a tilde denote the expectations of future variables formed in period t–1, i.e. $\tilde{n}_t = E_{t-1}(n_t)$, and the lead variables without a tilde denote the expectations formed in period t, i.e. $p_{t+1} = E_t(p_{t+1})$. For simplicity, the foreign price level p^f, the foreign nominal interest rate i^f as well as the level of the domestic potential output y^n are assumed to be constant.

In the simulations we have used the following values for the model parameters: $\lambda = 4.0$, $\sigma = 3.0$, $\delta = 1.0$ and $\theta = 0.25$. The disturbance terms u_t, w_t and v_t are assumed to be normally and independently distributed with means zero and variances equal to 0.02^2. The properties of the peso problem variables, n_t and Ω_t, are discussed in the text.

Appendix 2

Tables of correlation coefficients

Table A1. **Correlation coefficients calculated from simulations where the IS disturbance term u_t varies (no peso problem).**

	Output gap	Real exchange rate	Ex post real interest rate	Ex post yield differential
Output gap	1	–1	0.9	0.0
Real exchange rate		1	–0.9	0.0
Ex post real interest rate			1	–0.4
Ex post yield differential				1

Note: the variables are $y_t - y^n$ (output gap), $s_t + p^f - p_t$ (real exchange rate), $i_t - [p_{t+1} - p_t]$ (ex post real interest rate) and $i_t - [i^f + (s_{t+1} - s_t)]$ (ex post yield differential). A rise in the real exchange rate implies real depreciation.
In the simulations the IS disturbance term u_t is assumed to be normally and independently distributed with zero mean and variance equal to 0.02^2.

Table A2. **Correlation coefficients calculated from simulations where the AS disturbance term w_t varies (no peso problem).**

	Output gap	Real exchange rate	Ex post real interest rate	Ex post yield differential[30]
Output gap	1	1	–0.6	–0.1
Real exchange rate		1	–0.6	–0.1
Ex post real interest rate			1	0.9
Ex post yield differential				1

Note: the definitions of the variables were presented in Table A1.
In the simulations the AS disturbance term w_t is assumed to be normally and independently distributed with zero mean and variance equal to 0.02^2.

[30] The correlation coefficients between the ex post yield differential and either the output gap or the real exchange rate should be zero given the assumptions that the disturbances are independent with each other and serially uncorrelated. The slight deviation from zero reflects the fact that the sample distributions of the disturbance terms do not fully obey these assumptions in this case.

Table A3. **Correlation coefficients calculated from simulations where the money supply disturbance term v_t varies (no peso problem).**

	Output gap	Real exchange rate	Ex post real interest rate	Ex post yield differential
Output gap	1	1	–0.7	0.0
Real exchange rate		1	–0.7	0.0
Ex post real interest rate			1	0.7
Ex post yield differential				1

Note: the definitions of the variables were presented in Table A1.
In the simulations the LM disturbance term v_t is assumed to be normally and independently distributed with zero mean and variance equal to 0.02^2.

Appendix 3

The results of the unit-root tests and the VAR estimations[31]

1 The data series

1.1 The calculation of the variables

Sample: 1980:1–1997:4.

The bilateral real exchange rates have been calculated using the quarterly averages of the GBP/DEM and CAD/USD series and the corresponding consumer price indices. In order to get rid of non-stationarity we have calculated the trend of the series using the Hodrick-Prescott filter with parameter value 1600 (the "industry standard") and then recalculated the real exchange rate series as the deviation of the actual value from the trend.

The output gaps have been derived from seasonally adjusted real GDP series by filtering these series using the HP filter (with parameter value 1600) and then defining the output gap as the deviation of the actual value from the trend. As the HP filter method has an end-point problem we have filtered the series starting from 1978:1 so that the beginning and the end of the data set reflect about similar points in the cycle (see e.g. Giorno et al. 1995.)

The ex post real interest rates and the ex post yield differentials have been calculated using the quarterly averages of the 3-month interest rates and the aforementioned consumer price indices and nominal exchange rate series.

1.2 Unit-root tests

The series have been tested for unit-roots using the Dickey-Fuller (DF) and the Augmented Dickey-Fuller (ADF) tests and checking the properties of the test residuals. The results are presented in Table A4.

[31] The empirical part of this paper has been carried out using PcGive 9.0 and PcFiml 9.0.

Table A4. **The results of the unit-root tests. H_0: the series has a unit root.**

Variable	Test (number of lagged differences)	Test statistic
Canada		
Output gap	ADF(1), no constant	–3.380**
Real exchange rate	ADF(3), no constant	–3.291**
Ex post real interest rate	DF	–4.873**
Ex post yield differential	ADF(2), no constant	–2.192*
United Kingdom		
Output gap	ADF(1), no constant	–2.103*
Real exchange rate	ADF(1), no constant	–3.697**
Ex post real interest rate	ADF(4)	–3.589**
Ex post yield differential	DF, no constant	–5.962**

* Significant at 5 % level
** Significant at 1 % level

2 Results of the VAR estimations

The abbreviations of the variables are the following:
Gap = Output gap
RealER = Real exchange rate
ExpRir = Ex post real interest rate
ExcR = Ex post yield differential (excess return)

2.1 The United Kingdom

```
Estimating the unrestricted reduced form by OLS
The present sample is:  1980 (3) to 1997 (4)

URF Equation 1 for Gap
Variable      Coefficient    Std.Error  t-value  t-prob
Gap_1              1.1524      0.11313   10.187  0.0000
Gap_2            -0.26659      0.11035   -2.416  0.0189
RealER_1          0.16896      0.11106    1.521  0.1336
RealER_2         -0.15120      0.10991   -1.376  0.1742
ExpRir_1         -0.15485      0.11918   -1.299  0.1990
ExpRir_2         -0.11877      0.13868   -0.856  0.3953
ExcR_1          -0.043706     0.017332   -2.522  0.0144
ExcR_2            0.16021      0.10556    1.518  0.1345
Constant        0.0027670    0.0023705    1.167  0.2479
Seasonal        0.0029417    0.0017696    1.662  0.1018
Seasonal_1     -0.0021666    0.0022297   -0.972  0.3352
Seasonal_2      0.0021983    0.0025408    0.865  0.3905

\sigma = 0.00492065   RSS = 0.00140434502
```

URF Equation 2 for RealER

Variable	Coefficient	Std.Error	t-value	t-prob
Gap_1	0.052948	0.11774	0.450	0.6546
Gap_2	0.00096885	0.11485	0.008	0.9933
RealER_1	1.6233	0.11559	14.044	0.0000
RealER_2	-0.61246	0.11439	-5.354	0.0000
ExpRir_1	0.78195	0.12404	6.304	0.0000
ExpRir_2	-0.41829	0.14433	-2.898	0.0053
ExcR_1	-0.96103	0.018038	-53.278	0.0000
ExcR_2	0.61787	0.10986	5.624	0.0000
Constant	-0.0048879	0.0024671	-1.981	0.0523
Seasonal	0.0064864	0.0018417	3.522	0.0008
Seasonal_1	-0.0056797	0.0023206	-2.448	0.0174
Seasonal_2	0.0015796	0.0026444	0.597	0.5526

\sigma = 0.00512118   RSS = 0.001521137538

URF Equation 3 for ExpRir

Variable	Coefficient	Std.Error	t-value	t-prob
Gap_1	0.0067456	0.13030	0.052	0.9589
Gap_2	0.059668	0.12709	0.469	0.6405
RealER_1	-0.22207	0.12791	-1.736	0.0878
RealER_2	0.19917	0.12658	1.573	0.1211
ExpRir_1	0.35173	0.13727	2.562	0.0130
ExpRir_2	0.16187	0.15972	1.013	0.3151
ExcR_1	-0.010969	0.019961	-0.550	0.5847
ExcR_2	-0.20004	0.12158	-1.645	0.1053
Constant	0.010019	0.0027301	3.670	0.0005
Seasonal	-0.015664	0.0020380	-7.686	0.0000
Seasonal_1	0.0068453	0.0025680	2.666	0.0099
Seasonal_2	-0.0038919	0.0029263	-1.330	0.1887

\sigma = 0.00566719   RSS = 0.001862788694

URF Equation 4 for ExcR

Variable	Coefficient	Std.Error	t-value	t-prob
Gap_1	-0.61285	0.86921	-0.705	0.4836
Gap_2	0.76408	0.84784	0.901	0.3712
RealER_1	1.6627	0.85329	1.949	0.0562
RealER_2	-1.4449	0.84443	-1.711	0.0924
ExpRir_1	-0.52053	0.91570	-0.568	0.5719
ExpRir_2	-1.5907	1.0655	-1.493	0.1409
ExcR_1	0.15111	0.13316	1.135	0.2611
ExcR_2	1.3612	0.81105	1.678	0.0987
Constant	0.038951	0.018212	2.139	0.0367
Seasonal	0.0098554	0.013596	0.725	0.4714
Seasonal_1	-0.022649	0.017131	-1.322	0.1913
Seasonal_2	-0.020624	0.019521	-1.056	0.2951

\sigma = 0.0378059   RSS = 0.08289857734

correlation of URF residuals

	Gap	RealER	ExpRir	ExcR
Gap	1.0000			
RealER	0.16358	1.0000		
ExpRir	-0.10305	-0.24334	1.0000	
ExcR	-0.026291	0.047573	0.22813	1.0000

standard deviations of URF residuals

Gap	RealER	ExpRir	ExcR
0.0049207	0.0051212	0.0056672	0.037806

loglik = 1364.4614 log|\Omega| = -38.9846 |\Omega| = 1.17273e-017 T = 70
log|Y'Y/T| = -30.4898
R^2(LR) = 0.999795 R^2(LM) = 0.608878

F-test on all regressors except unrestricted, F(32,204) = 57.552 [0.0000] **
variables entered unrestricted:
Constant Seasonal Seasonal_1 Seasonal_2

```
F-tests on retained regressors, F(4, 55)
      Gap_1      25.2156 [0.0000] **      Gap_2      1.58563 [0.1911]
   RealER_1      47.7577 [0.0000] **   RealER_2      7.46756 [0.0001] **
   ExpRir_1      15.6458 [0.0000] **   ExpRir_2      2.57219 [0.0477] *
     ExcR_1      738.963 [0.0000] **     ExcR_2      8.16323 [0.0000] **

correlation of actual and fitted
         Gap       RealER       ExpRir        ExcR
     0.95340      0.99764      0.81340     0.60311

Gap      :Portmanteau  8 lags=     9.89
RealER   :Portmanteau  8 lags=   16.911
ExpRir   :Portmanteau  8 lags=   13.534
ExcR     :Portmanteau  8 lags=   3.2915
Gap      :AR 1- 5 F( 5, 53) =    0.76815 [0.5769]
RealER   :AR 1- 5 F( 5, 53) =     1.4922 [0.2080]
ExpRir   :AR 1- 5 F( 5, 53) =    0.96286 [0.4489]
ExcR     :AR 1- 5 F( 5, 53) =    0.42942 [0.8261]
Gap      :Normality Chi^2(2)=     2.0267 [0.3630]
RealER   :Normality Chi^2(2)=     3.4709 [0.1763]
ExpRir   :Normality Chi^2(2)=     5.3512 [0.0689]
ExcR     :Normality Chi^2(2)=    0.53393 [0.7657]
Gap      :ARCH 4   F( 4, 50) =   0.36085 [0.8353]
RealER   :ARCH 4   F( 4, 50) =    1.3638 [0.2598]
ExpRir   :ARCH 4   F( 4, 50) =   0.47342 [0.7550]
ExcR     :ARCH 4   F( 4, 50) =   0.17275 [0.9513]
Gap      :Xi^2     F(16, 41) =   0.26451 [0.9971]
RealER   :Xi^2     F(16, 41) =    1.7532 [0.0744]
ExpRir   :Xi^2     F(16, 41) =   0.52487 [0.9182]
ExcR     :Xi^2     F(16, 41) =   0.43577 [0.9626]
Gap      :Xi*Xj    F(44, 13) =    0.1654 [1.0000]
RealER   :Xi*Xj    F(44, 13) =    1.1954 [0.3791]
ExpRir   :Xi*Xj    F(44, 13) =   0.71824 [0.7985]
ExcR     :Xi*Xj    F(44, 13) =   0.41622 [0.9847]

Vector portmanteau  8 lags=   100.29
Vector AR 1-5  F(80,140) =     1.2715 [0.1077]
Vector normality Chi^2( 8)=    9.3905 [0.3104]
Vector  Xi^2   F(160,292) =    0.81888 [0.9202]
Vector  Xi*Xj  F(440, 69) =    0.36856 [1.0000]
```

2.2 Canada

```
Estimating the unrestricted reduced form by OLS
The present sample is:  1980 (3) to 1997 (4)

URF Equation 1 for Gap
Variable      Coefficient     Std.Error  t-value  t-prob
Gap_1              1.3455       0.11154   12.064  0.0000
Gap_2            -0.54610       0.10865   -5.026  0.0000
RealER_1         0.096127       0.13207    0.728  0.4696
RealER_2        -0.074992       0.12634   -0.594  0.5551
ExpRir_1          0.21840       0.16731    1.305  0.1969
ExpRir_2         -0.54972       0.18148   -3.029  0.0037
ExcR_1         -0.0019651      0.055117   -0.036  0.9717
ExcR_2            0.16257       0.12511    1.299  0.1990
Constant        0.0034557     0.0027400    1.261  0.2123
Seasonal        0.0044485     0.0022494    1.978  0.0527
Seasonal_1   -2.2805e-005     0.0022237   -0.010  0.9919
Seasonal_2     -0.0014607     0.0021388   -0.683  0.4973

\sigma = 0.00626232    RSS = 0.002274562395
```

URF Equation 2 for RealER

Variable	Coefficient	Std.Error	t-value	t-prob
Gap_1	0.022997	0.084003	0.274	0.7852
Gap_2	-0.048151	0.081832	-0.588	0.5585
RealER_1	1.7001	0.099468	17.092	0.0000
RealER_2	-0.68862	0.095153	-7.237	0.0000
ExpRir_1	0.66306	0.12601	5.262	0.0000
ExpRir_2	-0.37467	0.13668	-2.741	0.0081
ExcR_1	-0.98211	0.041511	-23.659	0.0000
ExcR_2	0.79600	0.094229	8.448	0.0000
Constant	-0.0044767	0.0020636	-2.169	0.0342
Seasonal	-0.00084748	0.0016941	-0.500	0.6188
Seasonal_1	0.0026071	0.0016747	1.557	0.1250
Seasonal_2	0.00057319	0.0016108	0.356	0.7233

\sigma = 0.00471644   RSS = 0.001290198766

URF Equation 3 for ExpRir

Variable	Coefficient	Std.Error	t-value	t-prob
Gap_1	0.099296	0.089771	1.106	0.2733
Gap_2	-0.039393	0.087452	-0.450	0.6541
RealER_1	0.050981	0.10630	0.480	0.6333
RealER_2	-0.080011	0.10169	-0.787	0.4346
ExpRir_1	0.50343	0.13466	3.738	0.0004
ExpRir_2	0.0086072	0.14607	0.059	0.9532
ExcR_1	-0.014737	0.044362	-0.332	0.7409
ExcR_2	-0.0029173	0.10070	-0.029	0.9770
Constant	0.0041883	0.0022054	1.899	0.0625
Seasonal	0.0022467	0.0018104	1.241	0.2196
Seasonal_1	0.0028696	0.0017897	1.603	0.1143
Seasonal_2	0.0040829	0.0017214	2.372	0.0210

\sigma = 0.00504033   RSS = 0.001473484511

URF Equation 4 for ExcR

Variable	Coefficient	Std.Error	t-value	t-prob
Gap_1	-0.26446	0.29279	-0.903	0.3701
Gap_2	0.36121	0.28522	1.266	0.2104
RealER_1	1.0797	0.34669	3.114	0.0029
RealER_2	-0.96062	0.33165	-2.896	0.0053
ExpRir_1	0.33610	0.43920	0.765	0.4472
ExpRir_2	-0.48014	0.47639	-1.008	0.3177
ExcR_1	-0.027569	0.14469	-0.191	0.8495
ExcR_2	1.0134	0.32843	3.086	0.0031
Constant	-0.0037566	0.0071927	-0.522	0.6035
Seasonal	0.0051454	0.0059047	0.871	0.3871
Seasonal_1	0.011131	0.0058372	1.907	0.0615
Seasonal_2	0.0051911	0.0056144	0.925	0.3590

\sigma = 0.0164389   RSS = 0.01567386214

correlation of URF residuals

	Gap	RealER	ExpRir	ExcR
Gap	1.0000			
RealER	0.30176	1.0000		
ExpRir	-0.26842	-0.071898	1.0000	
ExcR	-0.23950	0.11891	0.38661	1.0000

standard deviations of URF residuals

Gap	RealER	ExpRir	ExcR
0.0062623	0.0047164	0.0050403	0.016439

loglik = 1428.4414   log|\Omega| = -40.8126 |\Omega| = 1.88499e-018   T = 70
log|Y'Y/T| = -33.6247
R^2(LR) = 0.999244   R^2(LM) = 0.615762

F-test on all regressors except unrestricted, F(32,204) = 38.473 [0.0000] **
variables entered unrestricted:
Constant   Seasonal   Seasonal_1   Seasonal_2

```
F-tests on retained regressors, F(4, 55)
      Gap_1      42.7724 [0.0000] **      Gap_2      7.12172 [0.0001] **
   RealER_1      74.5006 [0.0000] **   RealER_2      13.7409 [0.0000] **
   ExpRir_1      11.2937 [0.0000] **   ExpRir_2      3.54330 [0.0121] *
     ExcR_1      152.791 [0.0000] **     ExcR_2      18.1251 [0.0000] **

correlation of actual and fitted
          Gap       RealER      ExpRir        ExcR
      0.94273      0.99184     0.62262     0.54069

Gap       :Portmanteau  8 lags=   9.9291
RealER    :Portmanteau  8 lags=   16.767
ExpRir    :Portmanteau  8 lags=   6.4114
ExcR      :Portmanteau  8 lags=   22.967
Gap       :AR 1- 5 F( 5, 53) =    0.63299 [0.6754]
RealER    :AR 1- 5 F( 5, 53) =     2.2461 [0.0630]
ExpRir    :AR 1- 5 F( 5, 53) =    0.15973 [0.9761]
ExcR      :AR 1- 5 F( 5, 53) =     4.2929 [0.0024] **
Gap       :Normality Chi^2(2)=    0.51557 [0.7728]
RealER    :Normality Chi^2(2)=     1.5683 [0.4565]
ExpRir    :Normality Chi^2(2)=     3.9031 [0.1421]
ExcR      :Normality Chi^2(2)=    0.95628 [0.6199]
Gap       :ARCH 4  F( 4, 50) =    0.90024 [0.4710]
RealER    :ARCH 4  F( 4, 50) =     1.6605 [0.1739]
ExpRir    :ARCH 4  F( 4, 50) =    0.51305 [0.7264]
ExcR      :ARCH 4  F( 4, 50) =     1.2943 [0.2850]
Gap       :Xi^2    F(16, 41) =      1.801 [0.0652]
RealER    :Xi^2    F(16, 41) =     3.0292 [0.0021] **
ExpRir    :Xi^2    F(16, 41) =     1.0396 [0.4389]
ExcR      :Xi^2    F(16, 41) =    0.76404 [0.7140]
Gap       :Xi*Xj   F(44, 13) =    0.99551 [0.5364]
RealER    :Xi*Xj   F(44, 13) =     1.4212 [0.2506]
ExpRir    :Xi*Xj   F(44, 13) =    0.47113 [0.9681]
ExcR      :Xi*Xj   F(44, 13) =    0.39711 [0.9887]
Vector portmanteau  8 lags=   112.54
Vector AR 1-5  F(80,140) =      1.248 [0.1264]
Vector normality Chi^2( 8)=    8.5631 [0.3805]
Vector  Xi^2   F(160,292) =     0.9187 [0.7233]
Vector  Xi*Xj  F(440, 69) =    0.45808 [1.0000]
```

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