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Should new or rapidly growing banks have more equity?

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The views expressed are those of the author and do not necessarily reflect the views of the Bank of Finland.

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Should new or rapidly growing banks have more equity?

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Research Department

Abstract

There is substantial evidence that new banks and rapidly growing banks are risk prone. We study this problem by designing a relationship-lending model in which a bank operates as a financial intermediary and centralised monitor. In the absence of deposit insurance, the bank's limited liability option creates an incentive problem between the bank and its depositors, the likely outcome of which is a reduction in the amounts of resources allocated to monitoring its borrowers. Hence, the bank must signal its safety to depositors by maintaining the equity ratio held. The optimal equity ratio is dynamic, ie new banks need relatively more equity than established banks, which enjoy profitable old lending relationships – charter value – that reduce the incentive problem. However, if an established bank grows rapidly, its share of old relationships also decreases and the bank will have to raise its equity ratio. With deposit insurance, regulators should set higher equity requirements for new banks and rapidly growing banks than for those in a more established position. The results of the model can be extended to more general inter-firm control of credit institutions.

Key words: financial intermediation, relationship banking, financial fragility, bank regulation, deposit insurance, moral hazard, product quality

JEL classification numbers: G11, G21, G28

Pitäisikö uusille ja nopeasti kasvaville pankeille asettaa tavallista tiukemmat vakavaraisuus-vaatimukset?

Suomen Pankin keskustelualoitteita 16/2001

Juha-Pekka Niinimäki
Tutkimusosasto

Tiivistelmä

Empiiriset havainnot osoittavat, että uusilla ja nopeasti kasvavilla pankeilla on suurempi konkurssiriskin kuin muilla pankeilla. Tässä keskustelualoitteessa ilmiötä tutkitaan laatimalla pankin luottosuhteita kuvaava malli, jossa pankki toimii sekä rahoituksen välittäjänä että luotonhakijoiden luottokelpoisuuden arvioijana. Jos talletukset ovat vakuuttamattomia, pankin ja tallettajien välille syntyy kannustinongelma: pankki ei käytä riittävästi voimavaroja luottoasiakkaiden arviointiin. Kyetäkseen todistamaan luotettavuutensa tallettajille pankin täytyy säilyttää riittävä vakavaraisuusaste. Tarvittavan vakavaraisuusasteen osoitetaan tässä tutkimuksessa riippuvan pankin iästä. Uusien pankkien täytyy ylläpitää korkeampaa vakavaraisuusastetta kuin asemansa jo vakiinnuttaneiden pankkien, joilla on kannustinongelmaa pienentäviä pitkäaikaisia luottosuhteita (tulevia voittoja). Jos asemansa vakiinnuttanut pankki toisaalta kasvaa hyvin nopeasti, vanhojen luottosuhteiden prosenttiosuus pankin luottosalkussa pienenee, kannustinongelma pahenee ja pankin täytyy nostaa vakavaraisuusastettaan todistaakseen luotettavuutensa. Jos talletukset ovat vakuutettuja, viranomaisten tulisi asettaa sekä uusille että nopeasti kasvaville pankeille tiukemmat vakavaraisuusvaatimukset kuin muille pankeille. Tulokset ovat laajemminkin yleistettävissä luotonantaja-yrityssuhteisiin.

Asiasanat: pankit, luottosuhteet, pankkitoiminnan sääntely, talletusvakuutus, kannustinongelma

JEL luokittelu: G11, G21, G28

Contents

1 Introduction

Recent banking crises in the US, Scandinavia, Japan, and numerous transition economies have revived interest in ways to improve bank regulation regimes. Setting equity requirements is a fairly effective regulatory approach, but because equity is relatively expensive for banks, equity requirements must be carefully designed to economize on the use of equity.[1] For example, the 1988 Basle Capital Accord calls for equity capital equal to 8% of risk-adjusted assets and assigns on-balance assets to one of four risk buckets (0%, 20%, 50%, and 100%). In this paper, we design a model that recommends high equity ratios for new (*de novo*) banks and rapidly growing banks. Empirical evidence supports this recommendation.

There exist a lot of evidence that de novo banks are risky. For example, Gunther (1990) explores the experiences of Texas banks during the 1980s, where 39% of de novo banks failed in comparison to 21% for established banks. Many de novo banks invested in high-risk assets as soon as they opened their doors. Studying New England bank failures 1989–1992, Randal (1993) finds that 28% of de novo banks failed, compared to only 14% of established commercial banks and 12% of established saving banks. De novo banks characteristically grew rapidly. DeYoung et al. (1999) examine de novo banks that opened their doors during 1984–1985. Over 23% failed by 1999, compared to only 6.6% of established banks. In addition, 43.5% of de novo banks were acquired. Rapid growth increased the failure probability of both de novo and established banks. De novo failures are foreseeable under a “life-cycle” theory. De novo banks start out heavily capitalized with unseasoned loan portfolios, so initially they appear safe. Subsequent rapid asset growth, low profitability, and declining loan quality all erode the equity ratio. Within five years de novo banks are as likely to fail as established banks, and after seven years they are twice as likely to fail. DeYoung (2000) finds further evidence affirming this life-cycle theory.[2] Studying banks chartered between 1980 and 1985, he finds that during a 14-year period 23% of de novo banks failed, compared to only 8% of established banks. Hunter & Srinivasan (1990), DeYoung & Hasan (1998) and Shaffer (1998) also note the profitability problems of de novo banks.

Further evidence of the financial fragility of rapidly growing banks is provided by White (1991), who finds that US thrift groups liquidated or acquired during 1986–1989 grew by 101% in period 1982–1985, whereas the remainder of

[1] See Calomiris & Wilson (1998) for both theoretical and empirical evidence of the high costs of bank equity.

[2] De Young compares 1,664 de novo commercial banks to a benchmark sample of 2,047 small, established banks. On average, de novo banks were initially less likely to fail than established banks. After about four years they became more likely to fail than established banks, and after about eight years became twice as likely to fail as the average established bank. De novo banks eventually mature and their failure at rates fall into line with established banks.

the industry grew only by 49% during those years.[3] Hunter et al. (1996) examine de novo S&Ls chartered between 1980 and 1986. De novo bank groups with annual growth rates above 100% had a failure rate of 60%, compared to a 32% failure rate of their lower growth counterparts. An investigation Finland's banking crisis by Solttila & Vihriälä (1994) finds that lending growth was the major determinant of a bank's later non-performing assets (i.e. the faster a Finnish bank grew in 1986–1989, the more problem assets it had on its books in 1993). For savings banks, growth was the sole significant factor in explaining bank level variation in the share of nonperforming loans. Rapid growth often coincides with financial bubbles (see e.g. Kindleberger (1978), Kaminsky & Reinhart (1999)) and gambling-for-resurrection problems.

Here, we develop a dynamic model to explain the high failure risk of de novo banks and rapidly growing banks. We assume that the bank serves as both a financial intermediary and centralized monitor.[4] In the loan market, we look at the relationship-lending models of Sharpe (1990) and Dell'Ariccia et al. (1999).[5] In these models, a bank's lending to an entrepreneur creates a relationship that allows the bank to learn more about the borrower than other banks. This proprietary information creates ex post monopoly power for the bank, and allows it to profit on its infor-mationally captured old borrowers. On the other hand, competition forces the bank to sacrifice profits because it must offer lower loan interest to bring in new borrowers. We extend these models to include deposit markets, overlapping generations of borrowers, and the bank's option to monitor its borrowers (which is how the bank obtains proprietary information). Put simply, established banks have profitable old borrowers, whereas de novo banks only capture new, less profitable, borrowers.[6]

In a deposit market without deposit insurance, the bank's limited liability option creates an incentive problem between the bank and its depositors (who cannot observe the bank's actions) – the bank will not monitor its borrowers. Therefore, the bank must signal its safety to depositors by raising relatively expensive equity. The optimal equity ratio is dynamic: a de novo bank needs more equity than an established bank, which has profitable old lending relationships –

[3] Some banks grew very rapidly. White (1991, p. 101): "The 'champions' of these 'flameouts' were Diversified American Saving Bank of Lodi, California, which grew from $11 million in assets in 1982 to $978 million in assets in 1985, and a de novo thrift, Bloomfield Savings and Loan in Birmingham, Michigan, which grew from $2 million in assets in assets in 1982 to $676 million in 1985." Esty (1997) also provides an interesting case study of a thrift that adopted a strategy of high-risk investment and aggressive growth in 1983. The thrift grew 828% in just two years. Although this strategy ultimately led to its failure, rapid growth was profitable for thrift's shareholders. Their gain, however, was small in compared to that FSLIC spent resolving the failure.

[4] Diamond (1984), Ramakrishnan & Thakor (1984) and Holmström & Tirole (1997) examine financial intermediation and centralized monitoring.

[5] See also Rajan (1992), Boot & Thakor (2000) and Chan et al. (1986). Rajan (1992) compares the costs and benefits of relationship-bank debt and arm's-length debt. Boot & Thakor (2000) examine whether relationship lending can survive competition against transaction lending and capital market lending. Chan et al. (1986) study reusability of borrower-specific information. Our framework is related to their analysis since the bank can reuse borrower-specific information.

[6] There is empirical evidence that banks have long-term lending relationships with borrowers and that the relationships effect both on the availability of loan and the loan terms, e.g. Petersen & Rajan (1994, 1995), Elsas & Krahnen (1998), Harloff & Körtig (1998), Shaffer (1998) and Degryse & Van Cayseele (2000). Boot (2000) surveys this literature.

charter value – that reduces the incentive problem.[7] However, if an established bank grows very rapidly, the relative share of profitable old lending relationships decreases in the loan portfolio, and the bank must also raise its equity ratio. The optimal equity ratio is derived in the context of the bank's optimal growth path. In theory, the bank can grow forever and its optimal size can approach infinity. Here, the equity ratio approaches zero and the incentive problem vanishes at no cost even if loan risks are *fully correlated*. This result extends Diamond's (1984) finding that the incentive problem vanishes at no cost when bank's size approaches to infinity with *independent* loan risks.

Interestingly, with deposit insurance, the deposit insurance option creates an almost identical incentive problem to that of the limited liability option. The regulator, therefore, should set high equity requirement for both de novo banks and rapidly growing banks under deposit insurance.

De novo banks and rapidly growing banks have received little attention in the theoretical literature. Dell'Ariccia et al. (1999) find that potential de novo banks suffer an adverse selection effect due to their inability to determine whether an applicant borrower is an average-quality borrower or a low-quality borrower rejected by his previous bank. Moreover, Keeton (2000) notes that loan growth can rise for three reasons: supply shifts, demand shifts, or shifts in productivity. Rapid loan growth increases credit losses only when the source of increased lending is shift in supply. Indeed, he finds supply shifts appear to account from much of the variation in loan growth, and posits that this may explain why rapid loan growth is often followed by credit losses.[8] We also borrow on the finding of one of my recent papers, Niinimäki (2001), whereby both new banks and rapidly growing banks need additional equity.[9] The framework here is different, however. In my other paper, the bank is a monopoly, loans are long-term, and the bank's size is essentially fixed. Here, the bank sector is fully competitive, loans are short-term, and banks can grow forever.

Benston et al. (1991, p. 308) note that in 1985, the rapid growth of S&L associations was limited by a regulation imposing higher equity requirements on institutions that grew their total liabilities by more than 15% a year. Moreover, growth at an annual rate of 25% of liabilities was prohibited without prior approval. DeYoung et al. (1999, p. 6) remind us that the FDIC now requires de novo banks to maintain an 8% Tier 1 equity-capital-to-risk-based-assets ratio for their first three years, whereas the Federal Reserve requires new state chartered Fed member banks to keep this ratio above 9%. The Tier 1 requirement for established banks is only 4%.

This paper proceeds as follows. Section 2 shows that bank monitoring is needed to prevent borrowers' effort aversion. Section 3 characterizes equilibrium loan rates for new and old borrowers. Section 4 reveals how limited liability

[7] Empirical evidence proves that banks with high charter value take less risks, e.g. Keeley (1990), Demsetz et al. (1996) and Galloway et al. (1997). There is some theoretical analysis on charter values. For example, Acharya (1986) examines optimal bank closure/reorganization policies, Hellman et al. (2000) focus on risk-shifting incentives and Hyytinen & Takalo (2000) show that excessive transparency may decrease charter value, and hence, increase risk-taking.

[8] For loan growth see also Minsky (1977) and Guttentag & Herring (1984).

[9] My model, Niinimäki (2001), assumes two banks, Bank A and Bank B. Bank A grants liquid short-time loans, while Bank B grants illiquid 30-year mortgages. Bank A can reinvest its whole loan portfolio every day, but Bank B can reinvest only 3.3% of its funds every year. Obviously, Bank B is easier to supervise than Bank A due to the glacial changes in its loan portfolio, so Bank B is entitled to a lower equity ratio than Bank A.

creates an incentive problem between the bank and depositors. Sections 5–9 present the main results including a discussion of the optimal size and optimal growth path for a bank. Section 10 concludes with an overview of the results and their regulatory implications.

2 Loan markets

2.1 Monitoring with a positive NPV

In this section, we design a framework in which only monitored loans (bank loans) are socially profitable due to the borrowers' effort aversion. Consider a risk-neutral entrepreneur seeking outside finance for a project that requires a unit of investment at the beginning of a period and yields a stochastic, commonly observable output at the end of the period. The output is Y units if the project succeeds and 0 if it fails. If the first project succeeds, the entrepreneur can undertake a second project in the next period with probability α. The entrepreneur immediately consumes any net output earned on the first project, and thus cannot invest collateral for the second project. The entrepreneur can influence on the success probability of his projects by exerting effort. With effort, a project will succeed with probability q, while without effort the probability of success is qp. Hence, q represents the unavoidable risk. Effort exertion incurs a fixed cost c to the entrepreneur, while zero effort is without cost. With effort, a project is assumed to have a positive net present value (NPV)

$$qY - r_f - c > 0, \tag{2.1}$$

where r_f is the risk-free interest rate. Without effort, the project is assumed to have negative NPV

$$qpY < r_f. \tag{2.2}$$

The entrepreneur can finance his project either directly from lenders or from a bank. Lenders are assumed to be incapable of determining whether the entrepreneur exerts effort or not, while the bank has the ability to monitor. We first focus on direct lending. Lenders are ready to finance a project at a competitive loan rate R such that $qR = r_f$, if the entrepreneur exerts effort. With effort, the entrepreneur's expected earnings are

$$q(y - R) - c + \delta\alpha q\left[q(Y - R) - c\right], \tag{2.3}$$

where δ denotes a discount rate. Here q(Y–R)–c represents the returns of the first project. The entrepreneur can undertake the second project only if the first project succeeds (with probability q) and if he encounters a second investment alternative (with probability α). The second project yields the same expected income as the first project, q(Y–R)–c. Without effort, the entrepreneur's expected earnings are

$$qp(Y - R) + \delta\alpha qp\left[qp(Y - R)\right]. \tag{2.4}$$

We assume an *effort aversion problem*, so that in the absence of monitoring the entrepreneur avoids effort

$$(1+\delta\alpha q)[q(Y-R)-c]<(1+\delta\alpha qp)[qp(Y-R)]. \tag{2.5}$$

Since a project has negative NPV without effort, effort aversion makes uninformed lending unprofitable. We now turn to bank finance, beginning with the following assumption.

Assumption 1 (Expensive equity): *Equity capital is more expensive to a bank than deposits, since each unit of equity incurs an extra cost ε to a bank.*

Assumption 1 motivates the bank to minimize its equity ratio. Hence, when the bank has a fixed amount of equity, it attempts to maximize the amount of deposits and thus the size of the bank. The bank must have some equity, however, to signal its safety to depositors.

The bank funds loans by raising equity (a share e_t of funds in period t) and deposits (a share $1–e_t$ of funds). It pays risk-free interest r_f on deposits and $r_f+\varepsilon$ on equity. Hence, the total cost of a fund unit is $r_f+e_t\varepsilon$. In addition, the banker bears a personal cost of monitoring which has a monetary value m.[10] Given these costs and the success probability q, the break-even single-period loan rate R_2^t is

$$R_2^t=\frac{1}{q}(m+r_f+e_t\varepsilon). \tag{2.6}$$

In R_2^t, t denotes the lending period. The loan rate depends on the period's equity ratio, e_t. A monitored loan is assumed to be profitable even if the bank is fully equity financed, $e_t=1$, and even in a single-period lending relationship, $q(Y-R_2^t)-c>0$, or

$$qY-m-r_f-c-\varepsilon>0. \tag{2.7}$$

Hence, monitored projects have positive NPV. We make the following assumption.

Assumption 2 (Monitoring): *The bank that monitored the entrepreneur's first project can monitor his second project at no cost. The bank cannot commit to monitoring that is unobservable to outsiders.*

Assumption 2 states that the bank learns to know its borrower so well during the first project that it can monitor his second project at no cost.[11] Hence, the bank has

[10] For a small bank, m represents the banker's personal mental exertion (non-monetary cost). For a larger bank, m may represent the wage costs of bank managers hired to monitor borrowers.

[11] In the first project, the bank carefully monitors the entrepreneur's books and accounts and learns to know him, his workers, production facilities, etc. In the second project much less extra monitoring is needed, so the monitoring cost of the second project is lower. For simplification, the second monitoring is assumed to be costless.

propriety information during the second project. The latter part of the assumption creates an incentive problem between the bank and its depositors, who cannot observe whether or not the bank is monitoring. In addition, the later part of the assumption simplifies loan rate competition.

2.2 Equilibrium loan interest rates

In this section, we solve for the equilibrium loan rates for the first and second project. We obtain the same results as Sharpe (1990) and Dell'Ariccia et al. (1999), i.e. new borrowers are unprofitable for a bank, but old borrowers are profitable. At time t, $t \in \{0,...,\infty\}$, there exist varying types of entrepreneurs and borrowers.

– *New entrepreneurs* undertake their first projects. A fraction q of them will succeed and a fraction αq of them will be able to undertake a second project at t+1.
– *Old entrepreneurs* undertook their first projects in the previous period, succeeded and can now undertake a second project.
– The bank's *new borrowers* are new entrepreneurs.
– The bank's *old borrowers* consist of those old entrepreneurs who received a loan in the previous period. If the bank monitored during that loan period, it now has proprietary information on its old borrowers.

Pre-commitments to the two-period contracts are assumed to be unenforceable, and thus the bank only grants standard single-period loans. The time line of the bank competition is following.

i) Each bank announces a loan rate offer for new entrepreneurs and a loan rate offer for old entrepreneurs. The latter rate is the same for the bank's old borrowers and other old entrepreneurs.
ii) Banks observe the offered loan rates.
iii) Given the offers, the bank recognizes whether or not its offer is competitive. It has the option to make a new (lower) offer to its old borrowers.[12]
iv) The entrepreneur accepts the lowest interest rate offer. If a new entrepreneur receives identical offers, he chooses a bank randomly. An old entrepreneur, however, prefers his previous bank.

Let us start by assuming that banks monitor. We will first examine old entrepreneurs and a representative bank. Given Assumption 2, the bank has an information advantage on its old borrowers, since it can monitor their second projects at no cost. From (2.6) we see the single-period break-even loan rate R_2^t other banks are ready to offer to the bank's old borrowers is

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[12] This assumption is based on relationship lending. Given the special relationship between the bank and its old borrowers, the bank observes when old borrowers are leaving and makes them a new offer. Moreover, an old borrower optimally contacts his original bank again. He knows that the bank has proprietary information on him and thus should be able to make the lowest offer. The old borrower collects offers from other banks to get his original bank to drop its loan rate offer to a competitive level.

$$R_2^t = \frac{1}{q}(m + r_f + e_t\varepsilon).$$

Here, R_k^t denotes loan rate for project number k, k∈{1,2}, at time t, t∈{1,...,∞}. Note that R_2^t is relatively high, since it must cover the monitoring cost. The bank correctly anticipates that other banks will offer R_2^t at stage i. Hence, the bank must offer R_2^t at stage i or iii to retain its old borrowers. If a loan offer incurs any cost to the bank, it optimally offers R_2^t already at stage i. The bank can then keep its old borrowers that provide a positive profit

$$qR_2^t - r_f - e_t\varepsilon = m > 0. \tag{2.8}$$

Here, qR_2^t represents expected loan interest income and $r_f - e_t\varepsilon$ represents payments to depositors and shareholders. Note that the profit is equal to the bank's information advantage over other banks, m. Because banks are identical, each bank offers R_2^t to old entrepreneurs at stage i.

Now consider new entrepreneurs. The bank rationally recognizes that a fraction αq of its new borrowers will be profitable old borrowers in the next period, t+1. Thus, the bank is willing to drop its loan rate for the first project R_1^t to where it earns zero expected returns over the lifetime of the average borrower relationship

$$qR_1^t - m - r_f - e_t\varepsilon + \delta\alpha q(qR_2^{t+1} - r_f - e_{t+1}\varepsilon) = 0. \tag{2.9}$$

The returns on the first project, $qR_1^t - m - r_f - e_t\varepsilon$, consist of expected loan interest income qR_1, monitoring costs m and payments to depositors and shareholders r_f–eε. Since the bank earns a positive profit m from the second project, the returns on the first project must be negative

$$qR_1^t - m - r_f - e_t\varepsilon < 0 \tag{2.10}$$

($qR_1^t - r_f - e_t\varepsilon > 0$ because interest income covers payments to depositors and shareholders even if it is insufficient to cover the banker's monitoring efforts, m.)

The pair R_1^t, R_2^t form an equilibrium. To see this, let us first examine new entrepreneurs. The bank cannot attract them if its offer exceeds R_1^t, but it is unwilling to underbid R_1^t, since, given its limited size, the offer R_1^t attracts enough new entrepreneurs. Further, underbidding with monitoring is unprofitable. We now turn to the old entrepreneurs. The bank knows that the other banks offer R_2^t. Hence, it can retain old borrowers only by offering R_2^t either at stage i or iii. If making the offer incurs any costs, the bank offers R_2^t already at stage i. Alternatively, the bank may attempt to attract old entrepreneurs from other banks by underbidding R_2^t. Such an offer $\hat{R}_2$ $(\hat{R}_2 < R_2^t)$ is, however, unprofitable. First,

$\hat{R}_2$ with monitoring will not cover the bank's costs. Second, the previous banks of these entrepreneurs will offer $\hat{R}_2$ at stage iii and keep their profitable old borrowers. Third, by offering $\hat{R}_2$ at stage i, the bank also decreases its returns from its own old borrowers. Thus, the pair R_1^t, R_2^t is an equilibrium. If the bank is going to shirk monitoring, it offers the same loan rates, since the bank cannot commit that it will not monitor later.[13]

Period 0 is a special case. Half of the population consists of new entrepreneurs and another half of the population represents *one-period entrepreneurs*, who live only for a period. The entrepreneur types are assumed to be observable. Banks offer loan rate R_1^0 to new entrepreneurs (see (2.10)). One-period entrepreneurs receive an offer R_2^0 that provides a zero profit in a single-period relationship (see (2.6)). We summarize this subsection as follows.

Lemma 1 (Loan rates): *Banks offer loan rate R_1^t to new entrepreneurs and R_2^t to old entrepreneurs. The banks then earn negative returns on their new borrowers and positive returns on their old borrowers. At time 0, the one-period entrepreneurs receive a break-even offer R_2^0.*

This simple framework borrows on the same key idea as the models of Sharpe (1990) and Dell'Ariccia et al. (1999): new borrowers are unprofitable, but old borrowers are profitable. While our framework may be less realistic, it recognizes the bank's active monitoring role. This role makes it possible to extend the analysis to the incentive problem between the bank its depositors. Who monitors the monitor?

3 The incentive problem and incentive constraint

In this section, we specify the *incentive problem* – the bank does not invest in costly monitoring – and the *incentive constraint*, i.e. monitoring must be at least as profitable than non-monitoring. The bank sector consists of identical, competitive banks which are formed at t=0 and which will operate forever.[14]

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[13] We emphasize two points. First, a bank which fails to monitor its new borrowers in the previous period can in this period charge relatively high loan rate R_2^t from these (now old) borrowers, since old borrowers are relatively unprofitable to the other banks. However, loan rate R_2^t does not represent proprietary information. To see this, notice that old non-monitored borrowers provide expected income qpR_2^t to the bank without a monitoring cost. In contrast, a bank that monitored its borrowers in the previous period, now earns expected income qR_2^t from these borrowers without a monitoring cost. Only in this latter case the bank has proprietary information. Second, notice that an entrepreneur always accepts bank's decision to shirk monitoring since he also earns higher expected returns without monitoring.

[14] Since banks are formed simultaneously, all follow the same growth path. This raises the problem of an eventual shortage of borrowers if all banks are allowed to grow forever. We can assume that the amount of borrowers increases simultaneously with banks, or alternatively, that some banks leave the bank sector, e.g. by merging with another banks. Here, we focus on a bank that can operate forever.

Banks finance overlapping generations of borrowers. Each bank has a fixed amount of equity E and a continuum of borrowers $[0,S_t]$ where S_t denotes banks' size is period t, $t \in \{1,...,\infty\}$. We make the following assumptions.

Assumption 3 (Nonstochastic α, q): *The law of large numbers ensures that the fractions α and q in bank's loan portfolio are nonstochastic.*

Assumption 3 implies that if the bank monitors its S_t borrowers, a fixed amount qS_t of them will succeed, and the fixed amount αqS_t of them will undertake the second project. The bank is thus risk-free. Assumption 3 simplifies the bank's growth function. It will later be relaxed.

Assumption 4 (Completely correlated risk): *Without monitoring, the entire loan portfolio fails with probability p.*

Assumptions 3 and 4 state that if a bank has S_t non-monitored borrowers, qS_t of them will succeed with probability p and all borrowers will fail with probability 1–p. Assumption 4 generates the most difficult incentive problem between the bank and depositors, since the problem cannot be reduced through diversification as in Diamond (1984). If we can design a scheme that removes the incentive problem under complete correlation, the scheme removes the problem under imperfect correlation.

More precisely, depositors are unable to observe whether the bank monitors or not. This, along with the bank's limited liability, generates an incentive problem, whereby the bank refuses to invest in costly monitoring. Given the negative NPV of non-monitored projects, no bank will be formed until the bank can convince depositors that it monitors. Depositors can infer a monitoring strategy by keeping track of the bank's growth and its equity ratio. The time-line can now be extended as follows.

v) Depositors observe loan rate offers. In addition, parameters α,q,p,m,E,ε and the sizes of the previous periods $S_0,S_1,...,S_{t-1}$ are commonly known. These sizes must have been such that the incentive constraint has been satisfied in every period. To attract deposits in the current period, the bank again selects its size S_t so that the incentive constraint is satisfied. Thereafter, the bank announces how many deposits S_t–E it will allow.[15]
vi) Depositors observe the incentive compatible S_t and they make their deposits.
vii) The bank raises the amount S_t–E of deposits and grants S_t loans.
viii) Loan returns materialize. Given Assumption 3, the bank is risk-free. It pays interest r_f on deposits and bank's owners then receive the rest of the returns.

We specify the bank's optimal actions by four claims.

Claim 1 (Maximal size): *Each bank attempts to maximize its size in every period.*

[15] If monitoring is observable, the bank will only be funded by deposits, since equity is more costly than deposits.

A fully competitive banking sector minimizes the loan rate. Since equity incurs a fixed cost Eε to the bank, the funding costs of the loan unit (payments to depositors and shareholders) $r_f + E\varepsilon/S_t$ are minimized when the bank's size S_t is maximized. If the bank fails to maximize its size, it needs to charge higher loan rates than other banks and cannot attract borrowers. *Hence, the bank maximizes its size in every period to be able to minimize its current lending rate.* Moreover, as we see later in detail, by maximizing its size in the previous period, the bank simultaneously maximizes the amount of profitable old borrowers in the current period. This increases growth possibilities and decreases the lending rate in the current period. Moreover, *by maximizing its size in the current period, the bank maximizes its growth possibilities and minimizes its lending rate in the next periods.* Therefore, the bank maximizes its size in every period.

Claim 2 (Binding incentive constraint): *The incentive constraint is binding in every period.*

The binding incentive constraint determines the bank's optimal size in every period. If the incentive constraint is non-binding, the bank can grow. Note that since the funding costs of a loan unit $r_f + E\varepsilon/S_t$ are minimized when size S_t is maximized, the bank has increasing returns of scale and its optimal size would be infinite without the incentive constraint. Hence, the incentive constraint is binding. The expected returns of the monitoring strategy π_t^m and non-monitoring strategy π_t^{nm} are equal in every period t

$$\sum_{i=0}^{\infty} \delta^i \pi_{t+i}^m = \sum_{i=0}^{\infty} (\delta p)^i \pi_{t+i}^{nm} \qquad \forall t, t \ni \{0,1,2,...\} \tag{3.1}$$

Even if every bank earns zero expected profits during its lifetime

$$\sum_{i=0}^{\infty} \delta^i \pi_i^m = 0 = \sum_{i=0}^{\infty} (\delta p)^i \pi_i^{nm} \tag{3.2}$$

an established bank with profitable old borrowers may enjoy positive future profits. The incentive constraint of time t, (3.1), can be written as

$$\pi_t^m + \delta \sum_{i=1}^{\infty} \delta^{i-1} \pi_{t+i}^m = \pi_t^{nm} + \delta p \sum_{i=1}^{\infty} (\delta p)^{i-1} \pi_{t+i}^{nm} \tag{3.3}$$

We also know that the binding incentive constraint at t+1 will be

$$\sum_{i=1}^{\infty} \delta^{i-1} \pi_{t+i}^m = \sum_{i=1}^{\infty} (\delta p)^{i-1} \pi_{t+i}^{nm}.$$

By inserting this in (3.3), we rewrite the incentive constraint of time t as

$$\pi_t^m + \delta(1-p) \sum_{i=1}^{\infty} \delta^{i-1} \pi_{t+i}^m = \pi_t^{nm}. \tag{3.4}$$

The first term on the left-hand side represents the returns of the monitoring strategy at time t. The second term represents the difference in future profits from time t+1 on. If the bank chooses a non-monitoring strategy, it risks failure and loss of these profits. Future profits are relatively larger for banks that monitor. On the right-hand side, the term π_t^{nm} represents the returns of the non-monitoring strategy at t. The exact magnitude of the future profits is solved in Appendix A.

Claim 3 (Future profits): *Future profits are* $\delta\sum_{i=1}^{\infty}\delta^{i-1}\pi_{t+i}^{m} = \alpha q\delta L_t(qR_2^{t+1} - r_f - E\varepsilon/S_{t+1}) = \alpha q\delta L_t m$. *Hence, the incentive constraint can be rewritten as* $\pi_t^{m} + \delta(1-p)\alpha q L_t\left(qR_2^{t+1} - r_f - E\varepsilon/S_{t+1}\right) = \pi_t^{nm}$.

Here, L_t denotes the amount of new borrowers at time t. Intuitively, at time t the bank invests in its L_t new borrowers by monitoring them. A fraction αq of them will undertake a second project at t+1 and then produce a positive profit $\alpha q\delta L_t m$ for the bank. Hence, the future profits from t+1 on, $\alpha q\delta L_t m$, arise from the profitable old borrowers at t+1, and these profits are based on monitoring at time t. Future profits thus originate from the monitoring investment in the current period.

Finally, we consider the case of the bank that monitored its L_{t-1} new borrowers in the previous period t–1. Since at time t the bank can now monitor its now old borrowers at no cost, we assume that the bank monitors them whether it chooses to monitor its new borrowers or not. The old borrowers then certainly provide returns at t. What happens if the bank does not monitor its L_t new borrowers and these loans fail? The bank has then both successful loans (old borrowers) and failed loans (new borrowers). Depositors observe the bank's credit losses and attempt to withdraw their deposits. Do bank's returns on old borrowers cover deposit payments? We make the following claim.

Claim 4 (Limited liability): *A bank's optimal size (the amount of new borrowers) is so big that if a risk realizes when the bank has both monitored old borrowers and non-monitored new borrowers, the bank cannot pay off all deposits. The bank will then fail and its owners will receive no returns.*

The technical proof is omitted here for reasons of brevity. Intuitively, the incentive constraint can bind if $\pi_t^{m} \leq \pi_t^{nm}$. This is possible only if the bank benefits from limited liability when it does not monitor.

4 Bank's optimal size in periods 0,1,2, …

4.1 The de novo bank

In this section we will solve bank's optimal (maximal) size at time 0, S_0. Given Claims 1–3, the bank maximizes its size so that the incentive constraint at date 0 is binding, i.e.

$$\pi_0^m + \delta(1-p)\alpha q L_0(qR_2^1 - r_f - E\varepsilon/S_1) = \pi_0^{nm}, \tag{4.1}$$

where $qR_2^1 - r_f - E\varepsilon/S_1 = m$. The incentive constraint includes the following terms.

– The expected returns of the monitoring strategy at date 0 are

$$\pi_0^m = \tfrac{1}{2}S_0(qR_2^0 - r_f - m - E\varepsilon/S_0) + \tfrac{1}{2}S_0(qR_1^0 - r_f - m - E\varepsilon/S_0). \tag{4.2}$$

Recall that at t=0, half of the borrowers are one-period entrepreneurs who live only for a period. The first term represents bank's zero returns on them, $qR_2^0 - r_f - m - E\varepsilon/S_0 = 0$ (see Lemma 1, (2.6)). The second term represents another half of the borrowers. These new borrowers provide negative returns to the bank, $qR_1^0 - r_f - m - E\varepsilon/S_0 < 0$ (see (2.10)).

– The expected returns of the non-monitoring strategy in period 0 are

$$\pi_0^{nm} = p\left[\tfrac{1}{2}S_0(qR_2^0 - r_f - E\varepsilon/S_0) + \tfrac{1}{2}S_0(qR_1^0 - r_f - E\varepsilon/S_0)\right] - (1-p)E(r_f + \varepsilon).$$

By inserting the monitoring cost into the square brackets, we rewrite as

$$\begin{aligned}\pi_0^{nm} = {} & p\left[\tfrac{1}{2}S_0(qR_2^0 - r_f - m - E\varepsilon/S_0) + \tfrac{1}{2}S_0(qR_1^0 - r_f - m - E\varepsilon/S_0)\right] \\ & + pmS_0 - (1-p)E(r_f + \varepsilon)\end{aligned} \tag{4.3}$$

Note that the term in square brackets is now equal to (4.2). The non-monitoring strategy provides this return only when the loans succeed (with probability p). The non-monitoring strategy is then more profitable than the monitoring strategy, since monitoring costs are avoided, pmS_0. However, when the loans fail, the bank loses its equity, $-(1-p)E(r_f + \varepsilon)$.

– The third term of the incentive constraint, $\delta(1-p)\alpha q L_0(qR_2^1 - r_f - E\varepsilon/S_1)$, represents future profits. We know that the amount of new borrowers at t=0, L_0, is equal to $\tfrac{1}{2}S_0$. By inserting $L_0 = \tfrac{1}{2}S_0$, (4.2) and (4.3) into (4.1), we rewrite the incentive constraint as

$$\begin{aligned}& \tfrac{1}{2}S_0(1-p)\left[qR_1^0 - r_f - m - E\varepsilon/S_0 + \alpha q\delta(qR_2^1 - r_f - E\varepsilon/S_1)\right] \\ & = pmS_0 - (1-p)E(r_f + \varepsilon)\end{aligned} \tag{4.4}$$

The term in square brackets is zero, since an average borrower provides zero expected returns to the bank during his lifetime (Recall (2.9) and that $E/S_t = e_t$). The incentive constraint simplifies to

$$0 = pmS_0 - (1-p)E(r_f + \varepsilon) \tag{4.5}$$

The first term represents the benefits of shirking monitoring. With probability p the bank earns higher returns than with monitoring since it avoids the costs of monitoring, mS_0. The second term represents the cost of shirking, with probability 1–p that the bank loses its equity. Here, we emphasize a few points. First, the bank does not monitor without equity, since $0<pmS_0$. Hence, no bank can be formed without equity, because limited liability makes the non-monitoring strategy preferable. Second, a fully equity financed bank (S_0=E) certainly monitors. To see this, we rewrite the right-hand side of (4.5) as

$$E[pm - (1-p)(r_f + \varepsilon)],$$

where $pm - (1-p)r_f = p(m + r_f) - r_f < pqY - r_f < 0$. Here, the first inequality comes from (2.7), $qY - m - r_f - c - \varepsilon > 0$, and the second inequality comes from (2.2), $qpY < r_f$. Hence, monitoring provides higher returns than non-monitoring, $0 > E[pm - (1-p)(r_f + \varepsilon)]$. Third, there always exist a size big enough that the bank does not monitor, $0 < pmS_0 - (1-p)E(r_f + \varepsilon)$. Thus, given the fixed amount of equity, the bank's maximal size can be solved from (4.5)

$$S_0 = (1-p)E(r_f + \varepsilon)/pm. \tag{4.6}$$

If the bank was bigger, the incentive constraint would not be binding, and the bank would not monitor. Rational depositors would avoid the bank. If the bank was smaller, it would have to charge higher interest on loans. The bank would not attract borrowers. Hence, the optimal size is S_0. We summarize this section as follows.

Lemma 2 (De novo bank): *At time 0, the bank has no profitable old borrowers and monitoring incentives are created entirely by equity. Without equity the bank would not monitor. Given the fixed amount of equity, the bank's optimal size is S_0.*

4.2 The established bank

In this section, we solve for the optimal size at time t when the bank has followed a monitoring strategy. At time t the bank has αqL_{t-1} old borrowers and

$$L_t = S_t - \alpha qL_{t-1}, \quad t \in \{1,2,...\}, \qquad L_0 = \tfrac{1}{2}S_0 \tag{4.7}$$

new borrowers, for example $L_0 = \frac{1}{2}S_0, L_1 = S_1 - \alpha qL_0, L_2 = S_2 - \alpha qL_1,...$. Hence, the amount of new borrowers is equal to the difference between bank's size and the amount of old borrowers (new borrowers in the previous period). The incentive constraint of time t is

$$\pi_t^m + \delta(1-p)\alpha q L_t (qR_2^{t+1} - r_f - E\varepsilon / S_{t+1}) = \pi_t^{nm}, \tag{4.8}$$

where $qR_2^{t+1} - r_f - E\varepsilon/S_{t+1} = m$. The incentive constraint consists of the following terms.

– The returns of the monitoring strategy in period t are

$$\pi_t^m = (S_t - \alpha q L_{t-1})(qR_1^t - r_f - m - E\varepsilon / S_t) + \alpha q L_{t-1}(qR_2^t - r_f - E\varepsilon / S_t). \tag{4.9}$$

The first term represents bank's (negative) returns from its new borrowers; the second term represents bank's (positive) returns from its old borrowers.

– The returns of the non-monitoring strategy in period t are

$$\pi_1^{nm} = p\left[(S_t - \alpha q L_{t-1})(qR_1^t - r_f - E\varepsilon / S_t) + \alpha q L_{t-1}(qR_2^t - r_f - E\varepsilon / S_t)\right] - (1-p)E(r_f + \varepsilon)$$

In the square brackets, the first term represents returns from new borrowers and the second term represents returns from old borrowers. Outside the square brackets, the term represents the risk of losing equity when the risk materializes. Inserting the monitoring cost in the square brackets, we get

$$\begin{aligned}\pi_1^{nm} = {} & p\left[\left(S_t - \alpha q L_{t-1}\right)\left(qR_1^t - r_f - m - E\varepsilon / S_t\right) + \alpha q L_{t-1}\left(qR_2^t - r_f - E\varepsilon / S_t\right)\right] \\ & + pm\left(S_t - \alpha q L_{t-1}\right) - \left(1-p\right)E(r_f + \varepsilon).\end{aligned} \tag{4.10}$$

The term in the square brackets equals (4.9). Outside the square brackets, the first term represents the bank's extra returns from shirking monitoring. The second term is the cost of equity.

– The third term of the incentive constraint, $\delta(1-p)\alpha q L_t (qR_2^{t+1} - r_f - E\varepsilon / S_{t+1})$, is future profits.

Inserting $L_t = S_t - \alpha q L_{t-1}$, (4.9) and (4.10) into (4.8), we rewrite the incentive constraint as

$$\begin{aligned}&(S_t - \alpha q L_{t-1})(1-p)\left[qR_1^t - r_f - m - E\varepsilon / S_t + \alpha q \delta (qR_2^{t+1} - r_f - E\varepsilon / S_{t+1})\right] = \\ &(S_t - \alpha q L_{t-1})pm - (1-p)\alpha q L_{t-1}(qR_2^t - r_f - E\varepsilon / S_t) - (1-p)E(r_f + \varepsilon)\end{aligned} \tag{4.11}$$

The term in the square brackets is equal to zero, since an average borrower provides zero expected returns to the bank over his lifetime (Recall (2.9)). The incentive constraint simplifies to

$$0 = (S_t - \alpha q L_{t-1})pm - (1-p)\alpha q L_{t-1}(qR_2^t - r_f - E\varepsilon / S_t) - (1-p)E(r_f + \varepsilon) \tag{4.12}$$

The first term represents those extra returns that a bank can earn by avoiding the costs of monitoring. These returns depend on the amount of new borrowers, $S_t - \alpha q L_{t-1}$. The second term represents profits from old borrowers. Without monitoring, the bank risks failure and loss of these profits. The third term represents the risk of losing equity without monitoring. Thus, the first term decreases monitoring incentives, while the last two terms increase these incentives. Note that old borrowers $\alpha q L_{t-1}$ restrict the incentive problem in two ways. The first term reveals that old borrowers decrease the need to invest in monitoring. The second term reveals that old borrowers provide positive profits. Recalling that $qR_2^t - r_f - E\varepsilon / S_t = m$ (see (2.8)), the optimal size can be solved from (4.12)

$$\begin{aligned} S_t &= \frac{1}{pm}\left[(1-p)E(r_f + \varepsilon) + \alpha q L_{t-1} m\right] \\ &= S_0 + \alpha q L_{t-1} / p. \qquad (4.13) \\ &= S_0 + \tfrac{\alpha q}{p}(S_{t-1} - \alpha q S_{t-2} + (\alpha q)^2 L_{t-3}). \end{aligned}$$

The bank's size increases along with the amount of old borrowers $\alpha q L_{t-1}$ and hence it also in-creases growth relative to the previous period, $S_{t-1} - S_{t-2}$. We summarize our findings as follows.

Lemma 3 (Established bank): *In periods 1,...,∞, both equity and old lending relationships motivate the bank to monitor. Old relationships decrease monitoring costs and provide profits.*

We solve for the bank's optimal size in period 1 using (4.13)

$$S_1 = S_0 + \tfrac{1}{2}\alpha q S_0 / p, \qquad (4.14)$$

since $L_0 = \tfrac{1}{2} S_0$. The bank thus grows from period 0 to period 1.[16] Unfortunately, equation (4.13) gets impracticable in later periods. For example, in period 100 the size is $S_0 + \alpha q L_{99} / p$. This is rather uninformative when the amount of old borrowers L_{99} is unknown. Fortunately, the optimal size can be solved as a function of the bank's initial size, S_0. The following proposition is proved in Appendix B by induction.

Proposition 1 (Bank's optimal size): *A bank's optimal size in period t, t={1,2,...}, is*

[16] Size S_0 ensures that the bank monitors at t=0. Through monitoring, the bank creates profitable lending relationships for period 1. At t=1, both these old relationships and equity encourage the bank to monitor. Hence, the bank can grow. Size S_1 ensures that the bank monitors at t=1. By monitoring at date 1, the bank creates profitable lending relationships for date 2. At t=2, both the old relationships from date 1 and equity encourage further monitoring. The bank can grow further. This process continues from one period to the next – forever.

$$S_t = S_0\left[1+\frac{\alpha q}{p}\sum_{i=1}^{t-1} G^{i-1} + \frac{\alpha q}{2p} G^{t-1}\right], \text{ in which } G = \alpha q(\frac{1}{p}-1). \tag{4.15}$$

5 Optimal rate of growth

It is easy to obtain the following result from Proposition 1.

Proposition 2 (Optimal growth): *A bank's optimal rate of growth is*

$$S_{t+1} - S_t = \frac{\alpha q S_0}{2p}(1+G)G^{t-1} \tag{5.1}$$

Proof: We obtain from (4.1)

$$\begin{aligned} S_{t+1} - S_t &= S_0\left[1+\frac{\alpha q}{p}\sum_{i=1}^{t} G^{i-1} + \frac{\alpha q}{2p} G^{t}\right] - S_0\left[1+\frac{\alpha q}{p}\sum_{i=1}^{t-1} G^{i-1} + \frac{\alpha q}{2p} G^{t-1}\right] \\ &= S_0\left[\frac{\alpha q}{p}\left(1+G+G^2+\ldots+G^{t-2}+G^{t-1}\right)+\frac{\alpha q}{2p}G^t\right. \\ &\quad \left. -\frac{\alpha q}{p}\left(1+G+G^2+\ldots+G^{t-2}\right)-\frac{\alpha q}{2p}G^{t-1}\right] \\ &= \frac{\alpha q S_0}{2p}\left(1+G\right)G^{t-1}. \end{aligned} \tag{5.2}$$

Using proposition 2, we get the following results.

Corollary 1 (Growth factors): *a) A bank monotonously grows if $\alpha>0$. b) The rate of growth increases with α and q, but slackens with p. c) If G>1, the rate of growth increases with t and approaches eternity in infinity. If G<1, the rate of growth slackens with t and approaches zero.*

The proof is found in Appendix C. Recall that q denotes the probability that a monitored loan succeeds and α denotes a share of successful borrowers who can undertake the second project. Intuitively, a) when α=0, an entrepreneur can undertake only one project and no long-term lending relationships occurs. Only equity then motivates the bank to monitor and thus it cannot grow. When α>0, profitable long-term relationships are created. These relationships increase monitoring incentives and the bank can grow. By growing, the bank creates more lending relationships, which increase monitoring incentives and help it grow further. This process can go on forever. b) The larger α and q, the greater the number of long-term lending relationships the bank has. Monitoring incentives are high and the bank can grow rapidly. In contrast, the greater p is, the higher the probability of success – and the profitability – of the non-monitoring strategy. This decreases monitoring incentives and slows growth. c) It is possible to rewrite

G>1 as $\alpha q(\frac{1}{p}-1)>1$.[17] Hence, the rate of growth increases with t if αq is high (many old lending relationships) and if p is small (the non-monitoring strategy succeeds with small probability). Through growth, the bank creates many profitable lending relationships, allowing it to grow even more rapidly in the next period. When G<1, growth slows. Growth creates profitable lending relationships, but reduces the equity ratio. In contrast to the case G>1, the value of lending relationships is insufficient to fully compensate for the reducing equity ratio. Hence, the incentive problem worsens and growth slackens.

6 Optimal size in eternity and optimal equity ratio

We can solve for a bank's optimal size in eternity from Proposition 1.

Proposition 3 (Size in eternity): *If G≥1, the bank's optimal size in eternity approaches infinity. If G<1, bank's optimal size in eternity approaches the steady-state level*

$$S_{ss} = S_0\left(1+\frac{\alpha q}{p}\frac{1}{1-G}\right), \text{ in which } G=\alpha q(\frac{1}{p}-1). \tag{6.1}$$

Proof. We rewrite the bank's optimal size (5.2) as

$$S_t = S_0\left[1+\frac{\alpha q}{p}(1+G+G^2+...+G^{t-2})+\frac{\alpha q}{2p}G^{t-1}\right], \quad t\geq 2. \tag{6.2}$$

When G≥1, S_t exceeds $S_0(1+G+G^2+...+G^{t-2})\alpha q/p$. We know that $1+G+G^2+...+G^{t-2}\geq t-1$ since G≥1. Here, t–1 approaches infinity when t grows without bound. Thus, both $S_0(1+G+G^2+...+G^{t-2})\alpha q/p$ and S_t must approach infinity when t grows without bound. When G<1, the third term in square brackets, $\alpha q/2p\, G^{t-1}$, approaches zero when t grows without bound. The first term in square brackets is fixed and the second term can be expressed as a sum of an infinite geometric series $\alpha q/p(1-G)$. The bank's optimal size is thus $S_0[1+\alpha q/p(1-G)]$. Q.E.D

Given Corollary 1, the intuition is obvious. When the rate of growth approaches infinity in eternity, the optimal size must approach infinity, too. When the rate of growth slows to zero in eternity, the optimal size settles to a steady-state level. This level increases with α and q, but decreases with p (the intuition is the same as in Corollary 1). Propositions 1 and 2 lead us to the following result.

[17] The case G<1 seems more practical than the case G>1 for several reasons. First, the case G>1 requires that the success probability of the non-monitoring strategy is smaller than ½. Second, when G>1 the bank's optimal equity ratio approaches zero in eternity. This seems unrealistic. Third, when G>1, the bank sector consists of a single monopoly bank in eternity. This is a bit problematic, since the bank sector is assumed to be competitive. Nevertheless, we also examine the case G>1 in the following.

Corollary 2 (Equity ratio): *When $G \geq 1$, the bank's equity ratio E/S_t approaches zero in eternity. The intensive cost per loan unit $E\varepsilon/S_t$ also approaches zero. When $G<1$, the equity ratio approaches the steady-state level E/S_{ss} in eternity. In both cases, the equity ratio decreases. Established banks have lower equity ratios than de novo banks.*

The result is trivial, given Propositions 1 and 2. Intuitively, a de novo bank must signal its safety to depositors with a higher equity ratio. Depositors rationally recognize that an established bank has profitable old lending relationships. Hence, it will monitor even if it has lower equity ratio.

Diamond (1984) assumes that borrower's returns are independent and identically distributed. When the bank (number of borrowers) grows without bound, the risk of returns is diversified away and the bank's returns are governed by the law of large numbers. The incentive cost per loan unit then approaches zero. Our results are quite similar. When $G \geq 1$, the bank grows without bound and the incentive cost per loan approaches zero. However, our mechanism is completely different and the loan risks may be *fully correlated* (when $q=1$, all non-monitored loans either succeed simultaneously with probability p, or fail simultaneously with probability $1-p$).

Corollary 3 (Future profits): *When $G>1$, the bank's future profits approach infinity even in a fully competitive banking sector.*

Proof: Claim 1 expresses the bank's future profits $\alpha q\delta m L_t$. Inserting $L_t = S_t - \alpha q L_{t-1}$ from (4.7) into $\alpha q\delta m L_t$ we obtain $\alpha q\delta m(S_t - \alpha q L_{t-1})$. Using (4.7) again, we get $L_{t-1} = S_{t-1} - \alpha q L_{t-2}$. Inserting this into $\alpha q\delta m(S_t - \alpha q L_{t-1})$, we obtain $\alpha q\delta m(S_t - \alpha q(S_{t-1} - \alpha q L_{t-2}))$ or

$$\alpha q\delta m(S_t - \alpha q S_{t-1} + (\alpha q)^2 L_{t-2}). \tag{6.3}$$

This approaches infinity when t grows without bound, since $\alpha q\delta m>0$, $(\alpha q)^2 L_{t-2}>0$ and $S_t - S_{t-1}$ approaches infinity when $G>1$ (recall Corollary 1). Q.E.D

Intuitively, future profits are based on monitoring. When the magnitude of the monitoring investment approaches infinity today, future profits also approach infinity. Keeley (1990) argues that in earlier decades various anti-competitive restrictions endowed banks with market power and created positive charter values. These charter values restricted banks' risk-taking. Increasing competition later decreased charter values and made risk-taking more profitable. We fully agree. However, as shown, the bank may enjoy substantial future profits even when the bank system is fully competitive. Even when $G<1$, future profits may be very high.

7 Rapid growth

Suppose a banker gains new wealth he can invest in the bank. This new equity capital incites the banker to expand the bank more rapidly than the optimal growth path allows. The desired size is denoted by $\hat{S}$. We solve for the required equity ratio when the bank grows rapidly. Recall that (4.13) implies that the bank's maximal size with the given amounts of equity and old borrowers (S_t is replaced by $\hat{S}$)

$$\hat{S} = \frac{1}{pm}\left[(1-p)\hat{E}(r_f + \varepsilon) + \alpha q L_{t-1} m\right], \tag{7.1}$$

where $\hat{E}$ is the new amount of equity. The required amount of equity can be solved from (7.1) as

$$\hat{E} = \frac{pm\hat{S} - \alpha q L_{t-1} m}{(1-p)(r_f + \varepsilon)}. \tag{7.2}$$

Dividing by $\hat{S}$, we obtain the required equity ratio

$$\hat{e} = \frac{pm - \dfrac{\alpha q L_{t-1} m}{\hat{S}}}{(1-p)(r_f + \varepsilon)}. \tag{7.3}$$

From (7.3), we see that the required equity ratio increases with the desired size $\hat{S}$. When a bank grows without bound, i.e. $\hat{S}$ approaches infinity, the required equity ratio approaches

$$\hat{e} = \frac{pm}{(1-p)(r_f + \varepsilon)}. \tag{7.4}$$

Hence, the equity ratio of the rapidly growing bank is equal to the equity ratio of a de novo bank at t=0, E/S_0 (This is easy to verify from (4.6)). Therefore, a bank that has followed the optimal growth path up to time t, and has thus monotonously reduced its equity ratio, must raise its equity ratio back to the initial high level. Intuitively, when a bank grows rapidly, the share of profitable old lending relationships $\alpha q L_{t-1} m/\hat{S}$ decreases in bank's loan portfolio (see (7.3)). In the extreme case, this share approaches zero when the bank's size $\hat{S}$ grows without bound. The positive incentive effect of old lending relationships then vanishes and monitoring incentives must be created exclusively with equity. This case is identical to that of the de novo bank – the bank must signal its safety by raising its equity ratio to the initial high level, E/S_0.

Proposition 4 (Rapid growth): *If an established bank plans to grow rapidly, it must raise its equity ratio.*

8 Stochastic case

In this section, Assumption 3 is relaxed. We begin by assuming that α remains fixed, but q is stochastic, $\tilde{q}$, and fluctuates from time to time. The expected value of $\tilde{q}$ is q. For simplicity's sake, $\tilde{q}$ can only have two realized values q_L or q_H, $q_L<q_H$, where q_L is assumed to be so high that the bank does not fail when it realizes. Banks and depositors observe the realized value of $\tilde{q}$ ex post by following credit loss information. The optimal size of the bank is (recall (4.13))

$$S_t = S_0 + \alpha q_j L_{t-1}/p, \quad j \in \{L, H\}. \tag{8.1}$$

Hence, the optimal size is smaller if q_L realizes than if g_H realizes. If q_L happens, i.e. few borrowers succeed and the bank suffers from credit losses at t–1, the bank's optimal size at time t is relatively small. It is often argued that credit losses provide only backward-looking information that cannot be used to predict the future. In our setting, credit losses also provide forward-looking information. The more credit losses a bank suffered yesterday, the less profitable lending relationships – charter value – it has today and the worse its incentive problem today. The bank's size must thus be smaller, and its equity ratio higher, than if fewer credit losses had occurred.

Let us now assume that α is stochastic, $\tilde{\alpha}$, but q is fixed. The realized value of $\tilde{\alpha}$ is denoted by $\tilde{\alpha}$. The bank observes $\tilde{\alpha}$ at stage iv of the time line and commits to the realized share $\tilde{\alpha}q$ of old borrowers when it raises deposits. The optimal size of the bank is (recall (4.13))

$$S_t = S_0 + \hat{\alpha} q L_{t-1}/p. \tag{8.2}$$

The bank's size thus increases with the fraction of old entrepreneurs that can take a second loan, $\tilde{\alpha}q$.

Proposition 5 (Stochastic case): *The bank's size increases with the realized values of* $\tilde{\alpha}$ *and* $\tilde{q}$.

Intuitively, the higher the realized values of $\tilde{\alpha}$ and $\tilde{q}$, the more profitable the old lending relationships of the bank. This reduces the incentive problem, so the bank can grant plenty of new loans.

9 Learning by doing

Recall from (4.13), the bank's optimal size in period t

$$S_t = \frac{1}{pm}\left[(1-p)E(r_f + \varepsilon) + \alpha q L_{t-1} m\right].$$

Suppose that an entrepreneur undertakes only one project, $\alpha=0$, and that the monitoring cost m depends on time; m is replaced by m_t. Hence, the bank's optimal size is now

$$S_t = \frac{(1-p)E(r_f + \varepsilon)}{pm_t}. \tag{9.1}$$

Moreover, we assume a learning-by-doing effect, whereby the monitoring cost decreases over time, $dm_t/dt<0$. It is apparent from (9.1) that the bank can be allowed to grow when it matures. The incentive constraint will be satisfied in later periods even when the bank grows, since the decreased monitoring cost makes monitoring more profitable. The bank's equity ratio decreases monotonously. The learning-by-doing effect reduces the equity ratio in the same way as old profitable lending relationships.

10 Conclusions

This paper studied the incentive problem facing banks granting relationship loans in a fully competitive economy. Relationship lending typically generates propriety information for the bank about its customers. This confers on the bank market power over its informationally captured old borrowers. Old borrowers thus yield positive profits for the bank, while new borrowers are unprofitable (at least until they come back for a second loan). The paper shows that both a bank's equity capital and profitable old lending relationships restrict the incentive problem. A key idea is that a de novo bank must maintain a relatively high equity ratio, whereas an established bank needs less equity since it has profitable old lending relationships – charter value – that reduce its incentive problem. The de novo bank has unprofitable new borrowers, no charter value, and incentives created exclusively by high equity ratio. Moreover, an established bank that grows rapidly can see the relative magnitude of its old lending relationships decrease, which again forces the bank to raise its equity ratio in order to signal safety to existing and potential depositors.

John et al. (1991) point out that the incentive problem of limited liability is identical to the incentive problem of deposit insurance.[18] In our setting, deposit insurance eliminates the motives of banks to signal their safety and the motives of depositors to monitor banks. The task of monitoring the bank thus is transferred to the bank regulator, who may attempt to regulate the bank by setting equity requirements. Since the incentive problem of deposit insurance is identical to the incentive problem of limited liability, the incentive compatible equity levels are also identical. The regulator optimally sets high equity requirement for de novo banks and rapidly growing banks.

We end this paper with five caveats. First, if the number of old lending relationships varies in the bank's loan portfolio, the realized number is difficult to evaluate. Second, even when the number of the old relationships is known, their monetary value may, in practice, be unknown. Third, this paper is likely a bit overly optimistic towards slowly growing banks. Such a bank may operate in a

[18] See also Niinimäki (2000).

declining local economy and may carry a high risk of failure. Fourth, this paper definitely overly optimistic towards renewed loans of old borrowers, since such loans can be used to hide credit losses. Fifth, we were probably too pessimistic on the subject of rapid growth for those situations where the bank has market power and operates in a prospering local economy.

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Appendix A

We can rewrite the expected returns of the safe strategy from t+1 on as follows

$$
\begin{aligned}
&\sum_{i=1}^{\infty}\delta^{i-1}\pi_{t+i}^{m} = \\
&\alpha q L_t (qR_2^{t+1} - r_f - \varepsilon e_{t+1}) + (S_{t+1} - \alpha q L_t)(qR_1^{t+1} - r_f - m - \varepsilon e_{t+1}) \ + \\
&\delta\big[\alpha q L_{t+1}(qR_2^{t+2} - r_f - \varepsilon e_{t+2}) + (S_{t+2} - \alpha q L_{t+1})(qR_1^{t+2} - r_f - m - \varepsilon e_{t+2})\ \big] + \\
&\delta^2\big[\alpha q L_{t+2}(qR_2^{t+3} - r_f - \varepsilon e_{t+3}) + (S_{t+3} - \alpha q L_{t+2})(qR_1^{t+3} - r_f - m - \varepsilon e_{t+3})\big] + \\
&\delta^3\big[\alpha q L_{t+3}(qR_2^{t+4} - r_f - \varepsilon e_{t+4}) + \ldots \ \big] + \ldots.
\end{aligned}
\tag{A.1}
$$

Some manipulation gives

$$
\begin{aligned}
&\alpha q L_t (R_2^{t+1} - r_f - \varepsilon e_{t+1}) + \\
&(S_{t+1} - \alpha q L_t)(qR_1^{t+1} - r_f - m - \varepsilon e_{t+1}) + \alpha q \delta L_{t+1}(qR_2^{t+2} - r_f - \varepsilon e_{t+2}) \ + \\
&\delta\big[(S_{t+2} - \alpha q L_{t+1})(qR_1^{t+2} - r_f - m - \varepsilon e_{t+2}) + \ \alpha q \delta L_{t+2}(qR_2^{t+3} - r_f - \varepsilon e_{t+3}) \ \big] + \\
&\delta^2\big[(S_{t+3} - \alpha q L_{t+2})(qR_1^{t+3} - r_f - m - \varepsilon e_{t+3}) + \ \alpha q \delta L_{t+3}(qR_2^{t+4} - r_f - \varepsilon e_{t+4}) \ \big] + \ldots
\end{aligned}
\tag{A.2}
$$

Since $L_{j+1} = S_{j+1} - \alpha q L_j \ \ \forall j, \ j \ni \{1,2,\ldots\}$ we get

$$
\begin{aligned}
&\alpha q L_t (R_2^{t+1} - r_f - \varepsilon e_{t+1}) + \\
&(S_{t+1} - \alpha q L_t)\big[qR_1^{t+1} - m - r_f - \varepsilon e_{t+1} + \alpha q \delta(qR_2^{t+2} - r_f - \varepsilon e_{t+2}) \ \big] + \\
&\delta(S_{t+2} - \alpha q L_{t+1})\big[qR_1^{t+2} - m - r_f - \varepsilon e_{t+2} + \alpha q \delta(qR_2^{t+3} - r_f - \varepsilon e_{t+3}) \ \big] + \\
&\delta^2(S_{t+3} - \alpha q L_{t+2})\big[qR_1^{t+3} - m - r_f - \varepsilon e_{t+3} + \alpha q \delta(qR_2^{t+4} - r_f - \varepsilon e_{t+4})\big] + \ldots
\end{aligned}
\tag{A.3}
$$

Given (2.9), $qR_1^j - m - r_f - e_j\varepsilon \ + \alpha q \delta(R_2^{j+1} - r_f - e_{j+1}\varepsilon) = 0, \ \forall j$, all sums inside the square brackets are equal to zero. Hence, (A.3) simplifies to

$$
\alpha q L_t (R_2^{t+1} - r_f - \varepsilon e_{t+1}) = \alpha q L_t m. \quad \text{Q.E.D.} \tag{A.4}
$$

Appendix B

Proposition 1 says that bank's optimal size in period t is

$$S_t = S_0\left[1+\frac{\alpha q}{p}\sum_{i=1}^{t-1}G^{i-1}+\frac{\alpha q}{2p}G^{t-1}\right], \qquad G=\frac{\alpha q(1-p)}{p}. \tag{B.1}$$

We show this is true through induction. The proof has two parts. First, we show that when the equation (B.1) is true in period t, it will be true also in period t+1. We know from (4.13), that a bank's optimal size in period t can also be expressed as

$$S_{t+1} = S_0+\frac{\alpha q}{p}L_t. \tag{B.2}$$

Recalling from (4.7) that $L_t = S_t - \alpha q L_{t-1}$, we obtain

$$S_{t+1} = S_0+\frac{\alpha q}{p}S_t-\frac{(\alpha q)^2}{p}L_{t-1}. \tag{B.3}$$

or

$$S_{t+1} = GS_t+S_0+\alpha q S_t-\frac{(\alpha q)^2}{p}L_{t-1}. \tag{B.4}$$

Recalling from (4.13) that $S_t = S_0+\alpha q L_{t-1}/p$, we obtain from (B.4)

$$S_{t+1}=S_0(1+\alpha q)+GS_t. \tag{B.5}$$

Inserting S_t from (B.1) to (B.5), we get

$$\begin{aligned}
S_{t+1} &= S_0\left\{1+\alpha q+G\left[1+\frac{\alpha q}{p}\sum_{i=1}^{t-1}G^{i-1}+\frac{\alpha q}{2p}G^{t-1}\right]\right\}\\
&= S_0\left\{1+\frac{\alpha q}{p}\left[p+\frac{pG}{\alpha q}+\sum_{i=1}^{t-1}G^{i}\right]+\frac{\alpha q}{2p}G^{(t+1)-1}\right\}\\
&= S_0\left\{1+\frac{\alpha q}{p}\left[1+\sum_{i=1}^{t-1}G^{i}\right]+\frac{\alpha q}{2p}G^{(t+1)-1}\right\}\\
&= S_0\left\{1+\frac{\alpha q}{p}\sum_{i=1}^{(t+1)-1}G^{i-1}+\frac{\alpha q}{2p}G^{(t+1)-1}\right\}.
\end{aligned} \tag{B.6}$$

This is fully in accord with optimal condition (B.1). Hence, we have shown that is when (B.1) is true at time t, it is also true at t+1. In the second part of the proof, we show that the optimal condition (B.1) is true at times 0 and 1. When t=1, (B.1) simplifies to

$$S_1 = S_0\left[1+\frac{\alpha q}{2p}\right]. \tag{B.7}$$

This is exactly the same as in (4.14). Hence, (B.1) is true at t=1. Consequently, we have shown that (B.1) states the bank's optimal size correctly in each period t, $t\in\{1,2,...\}$. Q.E.D

Appendix C

We see the bank's optimal growth from Proposition 2

$$S_{t+1} - S_t = \frac{\alpha q S_0}{2p}(1+G)G^{t-1}, \; G = \alpha q(\tfrac{1}{p} - 1). \tag{C.1}$$

a) We must show that a bank grows when α>0. We see from (C.1) that growth is zero if α=0 and that it is positive if α>0, since q, S_0, p>0.
b) We show that the rate of growth increases with α and q, but slackens with p.

$$\begin{aligned}\frac{d}{d\alpha}\left(S_{t+1} - S_t\right) &= \frac{d}{d\alpha}\left[\frac{\alpha q S_0}{2p}(1+G)G^{t-1}\right] \\ &= \frac{qS_0}{2p}(1+G)G^{t-1} + \frac{\alpha q S_0}{2p}G^{t-1}\frac{dG}{d\alpha} \\ &\quad + \frac{\alpha q S_0}{2p}(1+G)(t-1)G^{t-2}\frac{dG}{d\alpha} \; > \; 0,\end{aligned} \tag{C.2}$$

in which all three terms are positive since $dG/d\alpha>0$.

$$\begin{aligned}\frac{d}{dq}\left(S_{t+1} - S_t\right) &= \frac{d}{dq}\left[\frac{\alpha q S_0}{2p}(1+G)G^{t-1}\right] \\ &= \frac{\alpha S_0}{2p}(1+G)G^{t-1} + \frac{\alpha q S_0}{2p}G^{t-1}\frac{dG}{dq} \\ &\quad + \frac{\alpha q S_0}{2p}(1+G)(t-1)G^{t-2}\frac{dG}{dq} \; > \; 0,\end{aligned} \tag{C.3}$$

in which all three terms are positive, since $dG/dq>0$.

$$\begin{aligned}\frac{d}{dp}\left(S_{t+1} - S_t\right) &= \frac{d}{dp}\left[\frac{\alpha q S_0}{2p}(1+G)G^{t-1}\right] \\ &= \frac{-\alpha q S_0}{2p^2}(1+G)G^{t-1} + \frac{\alpha q S_0}{2p}G^{t-1}\frac{dG}{dp} \\ &\quad + \frac{\alpha q S_0}{2p}(1+G)(t-1)G^{t-2}\frac{dG}{dp} \; < \; 0,\end{aligned} \tag{C.4}$$

in which all three terms are negative, since $dG/dp<0$.

c) We get from (C.1)

$$\frac{d}{dt}\left(S_{t+1} - S_t\right) = \frac{d}{dt}\left[\frac{\alpha q S_0}{2p}(1+G)G^{t-1}\right] = \frac{\alpha q S_0}{2p}(1+G)G^{t-1}\ln G. \tag{C.5}$$

Hence, the rate of growth increases with t if $\ln G > 0$, and decreases with t if $\ln G < 0$. Consequently, the rate of growth increases with t if G>1 and decreases with t if G<1. Q.E.D

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