Mikael Bask – Carina Selander

Robust Taylor rules in an open economy with heterogeneous expectations and least squares learning

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Abstract

The aim of this paper is threefold: (i) to investigate if there is a unique rational expectations equilibrium (REE) in the small open economy in Galí and Monacelli (2005) that is augmented with technical trading in the foreign exchange market; (ii) to investigate if the unique REE is adaptively learnable in a recursive least squares sense; and (iii) to investigate if the unique and adaptively learnable REE is desirable in an inflation rate targeting regime in the sense that a low and not too variable CPI inflation rate in equilibrium is achieved. The monetary authority is using a Taylor rule when setting the nominal interest rate, and we investigate numerically the properties of the model developed. A main conclusion is that the monetary authority should increase (decrease) the interest rate when the CPI inflation rate increases (decreases) and when the currency gets stronger (weaker) to have a desirable rule that is robust with respect to the degree of technical trading in the foreign exchange market. Thus, the value of the currency is a better response variable than the output gap in the most desirable parametrizations of the interest rate rule.

Key words: determinacy, foreign exchange, inflation rate targeting regime, interest rate rule, robust monetary policy, technical trading

JEL classification numbers: E52, F31

Heterogeeniset odotukset, uskomusten päivitys ja avoimen talouden robustit rahapolitiikan korkosäännöt

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Mikael Bask – Carina Selander
Rahapolitiikka- ja tutkimusosasto

Tiivistelmä

Tässä työssä tarkastellaan lähinnä kolmea kysymystä. Ensiksi tutkitaan, onko teknisellä valuuttamarkkinakaupalla täydennetyssä pienen avotalouden dynaamisessa jäykkien hintojen makromallissa löydettävissä yksikäsitteinen rationaalisten odotusten tasapaino. Tätä yksikäsitteisen tasapainon olemassaoloa koskevaa kysymystä täydennetään seuraavaksi tasapainon adaptiiviseen opittavuuteen liittyvillä tarkasteluilla. Taloudenpitäjät päivittävät talouden liikelakia koskevia uskomuksiaan tilastollisin menetelmin, ja tasapainon opittavuudella tarkoitetaan tässä yhteydessä uskomusten yhtäläistymistä rationaalisten odotusten kanssa. Kolmanneksi työssä tarkastellaan, onko yksikäsitteisellä ja adaptiivisesti opittavissa olevalla rationaalisten odotusten tasapainolla inflaatiotavoitteeseen perustuvan rahapolitiikan kannalta toivottavia ominaisuuksia eli, tarkemmin ilmaisten, onko hidas ja vähän vaihteleva kuluttajahintainflaatio sopusoinnussa tällaisen tasapainon kanssa. Keskuspankki käyttää nimelliskorkoa asettaessaan korkosääntöä, ja työssä käytetyn mallin ominaisuuksia tarkastellaan numeerisin menetelmin. Tutkimuksen päätuloksen mukaan keskuspankin tulisi nostaa korkotasoa, kun kuluttajahintainflaatio kiihtyy ja kotimaan valuutta vahvistuu, jotta keskuspankin käyttämä korkosääntö toimisi tehokkaasti teknisen valuuttamarkkinakaupan yleistyessä. Tästä seuraa, että normaalin tuotantokuilun sijasta keskuspankin tulisi käyttää valuuttakurssia korkosäännössään koron muutospaineiden indikaattorina.

Avainsanat: määrittyneisyys, ulkomaan valuutta, inflaatiotavoite, korkosääntö, robusti rahapolitiikka, tekninen kaupankäynti

JEL-luokittelu: E52, F31

Contents

1 Introduction

Bakground During the last two decades, a new paradigm in monetary policy has evolved. This paradigm concerns independent central banks, openness and inflation rate targeting. In other words, monetary policy is conducted by the central bank without political influence, with the purpose of creating price-stability and credibility to evade the time-inconsistency problem. Further on, monetary policy is conducted through interest rate managing with an explicit target for the inflation rate. This new paradigm has been developed almost without any guidance from the academic literature (see p. 3 in Woodford, 2003). However, since this practise nowadays is established among central banks of the industrialized countries, the literature within this area is flourishing.

In 1993, John B Taylor (1993) demonstrated that the monetary policy of the Federal Reserve could be described by the following interest rate rule:

$$r_t = 0.04 + 1.5\left(\pi_t - 0.02\right) + 0.5\left(y_t - \overline{y}\right), \tag{1.1}$$

where r_t is the Federal Reserve's operating target for the funds rate, π_t is the inflation rate according to the GDP deflator, y_t is the logarithm of real GDP, and $\overline{y}$ is the logarithm of potential real GDP. This kind of rule has been the center of attention within the monetary policy literature since it was presented and is often referred to as a Taylor rule. In particular, the Taylor rule in (1.1) prescribes setting an operating interest rate target in response to the inflation rate and the (output) gap between the logarithm of real GDP and the logarithm of potential real GDP.

The key question in the literature is whether this type of interest rate rule, which does not incorporate a target path for the monetary aggregates, can control the price level and create price-stability. In other words, the success of this type of monetary policy rule hinges on the central bank's ability to shape market expectations of future interest rates, inflation rates and income levels. It is, therefore, important for the central bank to commit to the rule, be as transparent as possible in its decision making, and make the correct policy-decisions as often as possible. Taylor (1999) also argues that since the interest rate rule in (1.1) describes the Federal Reserve's policy during a successful period, one should adopt a rule like this in policy-making in which the interest rate is set in response to the inflation rate and the output gap.

However, since most countries trade extensively with other countries, and, therefore, should be considered as open economies, one might ask whether some exchange rate index also should be included in the monetary policy rule. Taylor (2001) does not think so, and the reason is that

> "... rules that react directly to the exchange rate ... sometimes work *worse* than policy rules that do not react directly to the exchange rate" (p. 267, italics added).

Instead, Taylor (2001) argues that the indirect effect that exchange rates have on monetary policy, via its effect on the inflation rate and the output gap, is to prefer since it results in fewer and less erratic changes in the interest rate.

Our model In this paper, two types of Taylor rules are embedded in a theoretical framework consisting of a dynamic IS-type equation, a new Keynesian Phillips curve, and a parity condition at the international asset market. The first rule is a contemporaneous data specification of the output gap, the inflation rate and the change in an exchange rate index (that, in the analysis below, consists of a single exchange rate), whereas the second rule is a contemporaneous expectations specification of the same variables. Further on, technical trading is incorporated into the foreign exchange market in the form of extrapolation of trends in the exchange rate index, and the reason is that several questionnaire surveys made at currency markets around the world confirm that technical trading, or chartism, is extensively used in currency trade.

Examples of questionnaire surveys include Cheung and Chinn (2001), who conducted a survey at the US market; Lui and Mole (1998), who conducted a survey at the Hong Kong market; Menkhoff (1997) and (1998), who conducted a survey at the German market; Oberlechner (2001), who conducted surveys at the markets in Frankfurt, London, Vienna and Zurich; and Taylor and Allen (1992), who conducted a survey at the London market. An extensive exploration of the trading behavior at the foreign exchange market is also found in Oberlechner (2004) that is based on surveys conducted at the European and the North American markets.

Thus, we include the change in an exchange rate index into the monetary authority's interest rate rule, even though Taylor (2001) claims that this kind of rule might worsen the outcome of monetary policy. However, as also is argued in Taylor (2001), more research is needed to investigate whether this claim holds in all types of models, and our contribution to the literature is to examine to what extent monetary policy *is* and *should be* affected when currency trade is partly driven by chartism.

Our approach It is well-known that models in economics and finance, in which agents have rational expectations regarding some of the variables in the model, may exhibit a multiplicity of rational expectations equilibria (REE). This is problematic. For instance, without imposing additional restrictions into such a model, it is not known in advance which of the REE that the agents will coordinate on, if there will be any coordination at all. To give an example, the effects of monetary policy is not known beforehand: is it the case that the agents will coordinate on an equilibrium that has undesirable properties, like a too high inflation rate, or an equilibrium with a low inflation rate?

Therefore, after augmenting the small open economy in Galí and Monacelli (2005) with technical trading in the foreign exchange market, we explore for which parameter values we have Taylor rules that give rise to determinacy, ie, a unique REE. Further on, which is a self-evident fact, but often neglected in the literature, is that a unique REE is not the same as a desirable REE. For this reason, we check whether the REE is desirable in an inflation rate targeting regime. In other words, is the unique inflation rate low enough and not too variable in equilibrium?

In between the questions on determinacy and the desirability of the inflation rate in equilibrium, we investigate if the REE is adaptively learnable in recursive least squares sense. The reason is that rational expectations is a rather strong assumption since it assumes that agents often have an outstanding capacity when it comes to deriving equilibrium outcomes of the variables in a model. This assumption has, therefore, in the more recent literature, been complemented by an analysis of the possible convergence to the REE (see Evans and Honkapohja (2001) for an introduction to this literature).

To be more precise, it is assumed that expectations are formed by a correctly specified model, ie, a model that nests the REE, but without having perfect knowledge about the parameter values in the model. However, using past and current values of all variables in the model, the parameter values are learned over time since the beliefs are revised as new information is gained. Thus, the question in focus is this: will the agents learn the parameter values in the model that corresponds to the unique REE?

Even though questionnaire surveys made at foreign exchange markets around the world demonstrate that technical trading techniques are used extensively in currency trade, it is not obvious to what extent these techniques are used at each moment in time. Clearly, the aforementioned surveys reveal an inverse relationship between the extent of chartism and the time horizon in currency trade, but the exact proportion of technical trading is still not known when conducting monetary policy. Therefore, to find robust parametrizations of the Taylor rules, the desirable properties of a rule should be relatively unaffected by the degree of technical trading in the foreign exchange market.

Finally, since the model developed is too large for theoretical analysis, we have to illustrate our findings numerically.[1] Specifically, we use calibrated values of the structural parameters in our model that are found in other papers within this research area (see Bullard and Mitra, 2002, and references therein).

Relation to the literature To slightly simplify the picture, there are two strands of literature that explore the effects of monetary policy in the new Keynesian framework. In the first strand of literature, an optimal policy rule for the monetary authority is derived via optimization of a welfare function, but the conditions for determinacy and adaptive learnability of the REE are often neglected (see, eg, Galí and Monacelli, 2005). In the second strand of literature, the focus is on finding parametrizations of Taylor rules that give rise to a unique REE that also is adaptively learnable in recursive least squares sense. However, the interest rate rules that satisfy these criteria are not evaluated using a welfare function as the metric (see, eg, Bullard and Mitra, 2002).

Our paper fills the gap between the two aforementioned papers since we, like Bullard and Mitra (2002), search for Taylor rules that are associated with a unique and an adaptively learnable inflation rate in equilibrium, but also, like Galí and Monacelli (2005), evaluate this equilibrium using a loss-function. However, a discrepancy between our paper and papers in which optimal monetary policy rules are derived is that we restrict the search for the most desirable rules among those rules that give rise to determinacy and adaptive learnability of the REE. The loss-function that we make use of in the analysis concerns the expected inflation rate and the conditional volatility of the inflation rate in equilibrium.

[1] MATLAB routines for this purpose are available on request from the authors.

In addition to our paper, Bullard and Schaling (2006) also fill the gap between the two aforementioned strands of literature, but in a two-country setting. They, however, do not include an exchange rate index in any of the Taylor rules investigated. Llosa and Tuesta (2006) explore the determinacy and learnability requirements for monetary policy using the small open economy in Galí and Monacelli (2005), and they include the change in the nominal exchange rate in some of the examined interest rate rules. All these authors, however, disregard from technical trading in the foreign exchange market, like all authors within this area do. A closely related exception is Honkapohja and Mitra (2006) who investigate the case with heterogeneous agents in a closed economy.

Two surveys of the literature that focus on determinacy and learnability requirements for monetary policy in a closed economy are Bullard (2006) and Evans and Honkapohja (2003), and they also discuss optimal monetary policy rules.

If we do not restrict our discussion to monetary policy in the new Keynesian framework, Frankel and Froot (1986) implemented fundamental and technical analyses into an exchange rate model, and they were among the first that utilized this setup when focusing at the foreign exchange market. Brock and Hommes (1997) provide a model with an evolutionary switching between a costly but sophisticated forecasting strategy, and a free but simple rule of thumb strategy. We make use of Frankel and Froot's (1986) setup in our model, but postpone the incorporation of an evolutionary switching between strategies for future research.

For a literature survey on heterogeneous agent models in economics and finance, see Hommes (2006), and for an introduction to exchange rate determination in a behavioral finance framework, see De Grauwe and Grimaldi (2006).

Our main finding Contrary to what Taylor (2001) claims, we find parametrizations of interest rate rules with robust and desirable properties that include the change in an exchange rate index. Further on, these rules do not include the output gap, which might be an advantage since it comes closer to the reality of central banking. To be more specific, due to data revisions, it is often the case that policy-makers do not have the correct information on a variable such as real GDP when needed. This is even more true when it comes to a variable such as potential real GDP.

Of course, one should not take our finding that the Taylor rule should include the change in an exchange rate index to be robust and desirable too literally since this result relies on calibrated values of the structural parameters in the model. Instead, our message is this, if we travesty the quote by Taylor (2001):

> Monetary policy rules that react directly to the exchange rate, or an exchange rate index, sometimes work *better* than policy rules that do not react directly to such quantities.

It is self-evident that future research should explore the robustness of our finding.

A caveat There are not too many papers that incorporate chartism in a foreign exchange model, and we believe there are two reasons for this. The first reason is that many researchers do not believe that currency traders using technical analysis can survive in the market, and the second reason is that even if some researchers are aware of the use of chartism in currency trade, most of them argue that it is of uttermost importance to explain *why* these traders survive in the market.

We are sympathetic to this standpoint, which can be traced back to Friedman (1953), but we also believe that this may be a hindrance to a better understanding of the *effects* of technical trading in the foreign exchange market since it is not easy to develop a theoretical model that satisfactorily explain human behavior at the currency market or at any financial asset market.

De Long et al (1990) is an example that contradicts Friedman's (1953) claim that non-rational traders cannot survive in the market in the long-run. In their model, noise traders, having erroneous beliefs, will bear more risk than risk averse and rational traders, meaning that the former traders may earn more money than the latter traders, and, therefore, may survive in the market.

Outline of the paper The theoretical framework is outlined in Section 2, whereas the search for robust Taylor rules with desirable properties is in focus in Section 3. The paper is concluded in Section 4, and the Appendix contains technical details.

2 Theoretical framework

Our theoretical framework consists of three parts: (i) the small open economy in Galí and Monacelli (2005), which is our baseline model; (ii) equations that describe the trading behavior at the foreign exchange market; and (iii) a Taylor rule for the monetary authority. Due to the findings in Bullard and Mitra (2002), two types of Taylor rules are investigated: (i) a contemporaneous data specification; and (ii) a contemporaneous expectations specification. Specifically, these two types of Taylor rules have appealing properties in a closed economy, and our aim is to investigate if these rules still have appealing properties in an open economy. The three parts are outlined in Sections 2.1–2.3, respectively.

2.1 Baseline model

Basically, the Galí and Monacelli (2005) model is a dynamic stochastic general equilibrium model with imperfect competition and nominal rigidities. In their model, the world economy is represented by a continuum of infinitely small economies, meaning that since each economy is of measure zero, its policy decisions do not have any impact on the rest of the world. Consequently, there is no room for strategic behavior in monetary policy-making. It is also assumed that the economies share identical household preferences, firm technology and market structure, while different economies are subject to correlated productivity shocks. Finally, firms set prices in a staggered fashion as in Calvo (1983).

After extensive derivations, the Galí and Monacelli (2005) model can be reduced to a dynamic IS-type equation and a new Keynesian Phillips curve

$$\begin{cases} x_t = x^e_{t+1} - \alpha\left(r_t - \pi^e_{H,t+1} - \overline{rr}_t\right) \\ \pi_{H,t} = \beta\pi^e_{H,t+1} + \gamma x_t \end{cases}, \tag{2.1}$$

where x_t is the output gap, r_t is the nominal interest rate, $\pi_{H,t}$ is the domestic inflation rate, and $\overline{rr}_t$ is the natural rate of interest. To be more specific, the output gap is the deviation of output from its natural level, where the latter is output in the absence of nominal rigidities. The domestic inflation rate is the rate of change in the index of domestic goods prices, and the natural rate of interest is the real interest rate that is consistent with output's natural level. Finally, the superscript e denotes expectations. (In Section 2.2, we will discuss how expectations are formed in the model.)

For our purpose, (2.1) is not in an appropriate form since there are no expected exchange rate terms in the equations. These terms are necessary when modeling the trading behavior at the foreign exchange market. It is, however, possible to use the following equations, which are derived in Galí and Monacelli (2005), to rewrite (2.1) into a suitable form

$$\begin{cases} \pi_t = \pi_{H,t} + \delta\Delta s_t \\ s_t = e_t + p^*_t - p_{H,t} \end{cases}, \tag{2.2}$$

where π_t is the CPI inflation rate, s_t is the terms of trade, e_t is the nominal exchange rate (or, more broadly, an exchange rate index), p^*_t is the index of foreign goods prices, and $p_{H,t}$ is the index of domestic goods prices. Specifically, the terms of trade is the relative price of the home country's import goods in terms of its domestically produced goods, and the CPI inflation rate is the rate of change in the index of goods prices. Thus, the difference between the two measures of the inflation rate, $\pi_{H,t}$ and π_t, is that the former measure is based on all prices for domestically produced goods, whereas the latter measure is based on all prices within the home country, imported goods included. CPI is also an abbreviation for consumer price index. Finally, the asterisk denotes a foreign quantity.

Now, if we rewrite the equations in (2.1) with help of those in (2.2), we get two of the equations that form our baseline model[2]

$$\begin{cases} x_t = x^e_{t+1} - \alpha\left(r_t - \frac{1}{1-\delta}\cdot\left(\pi^e_{t+1} - \delta\left(\Delta e^{e,m}_{t+1} + \pi^{e,*}_{t+1}\right)\right) - \overline{rr}_t\right) \\ \pi_t = \beta\pi^e_{t+1} + \gamma\left(1-\delta\right)x_t + \delta\left(\Delta e_t - \beta\Delta e^{e,m}_{t+1} + \pi^*_t - \beta\pi^{e,*}_{t+1}\right) \end{cases}, \tag{2.3}$$

where the superscript e,m denotes (aggregated) expectations at the foreign exchange market. The third equation in the baseline model, which also is

[2]See the Appendix for the derivation of (2.3).

derived in Galí and Monacelli (2005), is the condition for uncovered interest rate parity (UIP)

$$r_t - r_t^* = \Delta e_{t+1}^{e,m}. \tag{2.4}$$

Thus, (2.4) is a parity condition at the international asset market. Finally, we assume that the natural rate of interest is governed by the following stochastic process

$$\overline{rr}_t = \rho \overline{rr}_{t-1} + \varepsilon_t, \tag{2.5}$$

where $0 \leq \rho < 1$ is the serial correlation in the process, and $\varepsilon_t \in IID\,(0, \sigma_\varepsilon^2)$. To sum up, (2.3)–(2.5) is the complete baseline model that will be augmented with equations that describe the trading behavior at the foreign exchange market as well as a Taylor rule for the monetary authority. Note that the stochastic process in (2.5) also is assumed to hold in Bullard and Mitra (2002).

At this stage, let us say a few words about the structural parameters in our baseline model. $\beta > 0$ is the discount factor that is used when the representative household in the home country maximizes a discounted sum of instantaneous utilities derived from consumption and leisure. $\delta \in [0, 1]$ is the share of consumption in the home country allocated to imported goods, meaning that δ is an index of openness of the economy. For example, the equations in (2.3) reduces to those in Bullard and Mitra (2002) when $\delta = 0$ since the home country is a closed economy in this case.

The other two parameters in the model, α and γ, are not that easy to interpret since they are functions of structural parameters in the Galí and Monacelli (2005) model. Shortly, α depends on four parameters: (i) the openness index, δ; (ii) the intertemporal elasticity of substitution in consumption; (iii) the elasticity of substitution between domestic and foreign goods in consumption; and (iv) the elasticity of substitution between foreign goods in consumption. Moreover, γ depends on α as well as three other parameters: (i) the discount factor, β; (ii) the intertemporal elasticity of substitution in labor supply; and (iii) the share of firms that set (new) prices in each time period (see Calvo, 1983).

Since we investigate the properties of the model developed numerically, we do not need to emphasize the exact relationships between the structural parameters in our baseline model and the structural parameters in the Galí and Monacelli (2005) model. Of course, to fully grasp the micro-foundations in the baseline model and their relationships with the dynamic IS-type equation and the new Keynesian Phillips curve in (2.3) as well as the UIP condition in (2.4), it is necessary to consult Galí and Monacelli (2005).

2.2 Trading behavior at the foreign exchange market

There are two types of traders in the foreign exchange market; (i) agents who use chartism, or technical analysis, in their trade, meaning that they utilize past exchange rates to detect patterns that are extrapolated into the future; and (ii) agents who use fundamental analysis in their trade, meaning that they have rational expectations regarding the next time period's exchange rate, or, as in our model, the next time period's change in the exchange rate. Thus, these agents know that there are agents who use technical trading techniques in currency trade, and they take this into account when forming their exchange rate expectations.

In this paper, we assume that the chartists use a simple technical trading technique

$$\Delta e_{t+1}^{e,c} = \Delta e_t, \tag{2.6}$$

ie, the chartists expect that the nominal exchange rate will continue to increase (decrease) in the next time period, if it has increased (decreased) in the current time period. To be more specific, if the exchange rate increased (decreased) between time periods $t-1$ and t, the chartists believe that the exchange rate also will increase (decrease) between time periods t and $t+1$. Moreover, to keep the structural parameters in the model developed as few as possible, it is assumed that these two consecutive increases (decreases) in the exchange rate are of the same size. Finally, the superscript e, c denotes chartist expectations.

Then, if we move on to the fundamentalists, it is assumed that they have rational expectations regarding the change in the nominal exchange rate

$$\Delta e_{t+1}^{e,f} = \Delta e_{t+1}^{e}, \tag{2.7}$$

which means that the expected change in the exchange rate is equal to the mathematically expected change in the exchange rate, conditioned on all information available to this type of currency trader. This information includes the structure of the complete model as well as past and current values of all variables in the model, meaning that the dating of expectations is time period t. (However, as will be discussed in Section 3.2.2, when a contemporaneous expectations specification of the Taylor rule is used by the monetary authority, we assume that the dating of expectations is time period $t-1$.) Finally, the superscript e, f denotes fundamentalist expectations.

The expected exchange rate terms that appear in (2.3)–(2.4) are aggregated expectations at the foreign exchange market. Specifically, these expectations are a weighted average of chartist and fundamentalist expectations

$$\begin{aligned} \Delta e_{t+1}^{e,m} &= \omega \Delta e_{t+1}^{e,c} + (1-\omega)\, \Delta e_{t+1}^{e,f} \\ &= \omega \Delta e_t + (1-\omega)\, \Delta e_{t+1}^{e}, \end{aligned} \tag{2.8}$$

where $\omega \in [0,1]$ is the proportion of chartists in currency trade. Thus, aggregated expectations are a weighted average of the current change in the exchange rate and the next time period's mathematically expected change in the exchange rate. Consequently, as long as there are chartists present in the foreign exchange market, aggregated expectations do not coincide with rational expectations.

2.3 Taylor rules

We will investigate the properties of the complete model using two specifications of the Taylor rule: (i) a contemporaneous data specification of the rule; and (ii) a contemporaneous expectations specification of the rule. Moreover, since the nominal exchange rate or the change in this exchange rate may affect the economy's outcome in equilibrium, a term including the latter variable is included in both types of rules

$$r_t = \zeta_c + \zeta_x x_t + \zeta_\pi \pi_t + \zeta_e \Delta e_t, \tag{2.9}$$

and

$$r_t = \zeta_c + \zeta_x x_t^e + \zeta_\pi \pi_t^e + \zeta_e \Delta e_t^{e,f}, \tag{2.10}$$

where also a constant has been added. (In Section 3.3.1, it will be shown that this constant is equal to the foreign nominal interest rate.) Thus, in the Taylor rule in (2.10), the monetary authority has rational expectations.

3 A unique and desirable REE that is learnable?

Now, after having completed the description of our theoretical framework, we will investigate the properties of the model developed: (i) is there a unique REE in the model?; (ii) is the unique REE characterized by recursive least squares learnability?; and (iii) is the unique and adaptively learnable REE desirable in an inflation rate targeting regime in the sense that the inflation rate is low enough and not too variable in equilibrium? All three questions will be answered, for both specifications of the Taylor rule in (2.9)–(2.10), in Sections 3.1–3.3, respectively.

3.1 Determinacy

Let us begin with the question if there are any parametrizations of the Taylor rules in (2.9)–(2.10) that give rise to a unique CPI inflation rate in equilibrium.

3.1.1 Contemporaneous data in the Taylor rule

If the Taylor rule in (2.9) is used when the monetary authority is setting the nominal interest rate, meaning that they respond to current data of the output gap, the CPI inflation rate and change in the nominal exchange rate, the complete model in (2.3)–(2.5) and (2.8)–(2.9) can be written in matrix form as follows[3]

$$\begin{bmatrix} 1+\alpha\zeta_x & \alpha\zeta_\pi & \alpha\left(\frac{\delta\omega}{1-\delta}+\zeta_e\right) \\ \gamma(\delta-1) & 1 & \delta(\beta\omega-1) \\ \zeta_x & \zeta_\pi & \zeta_e-\omega \end{bmatrix} \begin{bmatrix} x_t \\ \pi_t \\ \Delta e_t \end{bmatrix} \tag{3.1}$$

$$= \begin{bmatrix} -\alpha\zeta_c - \frac{\alpha\delta}{1-\delta}\cdot\pi_{t+1}^{e,*} \\ \delta\left(\pi_t^* - \beta\pi_{t+1}^{e,*}\right) \\ r_t^* - \zeta_c \end{bmatrix} +$$

$$\begin{bmatrix} 1 & \frac{\alpha}{1-\delta} & \frac{\alpha\delta(\omega-1)}{1-\delta} \\ 0 & \beta & \beta\delta(\omega-1) \\ 0 & 0 & 1-\omega \end{bmatrix} \begin{bmatrix} x_{t+1}^e \\ \pi_{t+1}^e \\ \Delta e_{t+1}^e \end{bmatrix} + \begin{bmatrix} \alpha \\ 0 \\ 0 \end{bmatrix} \cdot \overline{rr}_t.$$

Thus, to have a unique and stable REE, all three eigenvalues of the following coefficient matrix must be inside the unit circle since x_t, π_t and Δe_t are free (see, eg, Blanchard and Kahn, 1980)

$$\mathbf{\Gamma} = \begin{bmatrix} 1+\alpha\zeta_x & \alpha\zeta_\pi & \alpha\left(\frac{\delta\omega}{1-\delta}+\zeta_e\right) \\ \gamma(\delta-1) & 1 & \delta(\beta\omega-1) \\ \zeta_x & \zeta_\pi & \zeta_e-\omega \end{bmatrix}^{-1} \times \begin{bmatrix} 1 & \frac{\alpha}{1-\delta} & \frac{\alpha\delta(\omega-1)}{1-\delta} \\ 0 & \beta & \beta\delta(\omega-1) \\ 0 & 0 & 1-\omega \end{bmatrix}. \tag{3.2}$$

However, deriving necessary and sufficient conditions for determinacy is not meaningful for practical reasons since these expressions would be too large and cumbersome to interpret. Consequently, we adopt the strategy in Bullard and Mitra (2002), and illustrate our findings for determinacy using calibrated values of the structural parameters.

To be more specific, the following parameter values, or range of values, are used in the analysis that are the same values as in Bullard and Mitra (2002)

$$\begin{cases} \alpha = \frac{1}{0.157}, & \beta = 0.99, & \gamma = 0.024, & \delta = 0.2, \\ \rho = 0.35, & 0 \le \zeta_x \le 4, & 0 \le \zeta_\pi \le 10, & -5 \le \zeta_e \le 5. \end{cases} \tag{3.3}$$

Of course, the parameters δ, ω and ζ_e do not appear in Bullard and Mitra (2002) since their model is for a closed economy. The index of openness of the economy is $\delta = 0.2$. However, to perform a sensitivity analysis of the numerical findings, we will also investigate the case when this index is $\delta = 0.4$.[4] In the former case, the index is slightly larger than the import/GDP ratio in the US, and in the latter case, which is the parameter setting in Galí and Monacelli (2005), the index corresponds roughly to the import/GDP ratio in Canada and Sweden.

In all figures below, the regions in the parameter space of $(\omega, \zeta_x, \zeta_\pi, \zeta_e)$ for which we have a unique REE are shown. Specifically, since ω and ζ_x are given, it is the combinations of ζ_π and ζ_e that are in the light areas in the figures that give rise to determinacy. In *Figure 1*, there is no technical trading

[3] See the Appendix for the derivation of (3.1).

[4] Detailed results are available on request from the authors.

in the foreign exchange market (ie, $\omega = 0$), meaning that all currency trade is guided by fundamental analysis, and the monetary authority does not take into account the output gap when setting the interest rate (ie, $\zeta_x = 0$).[5]

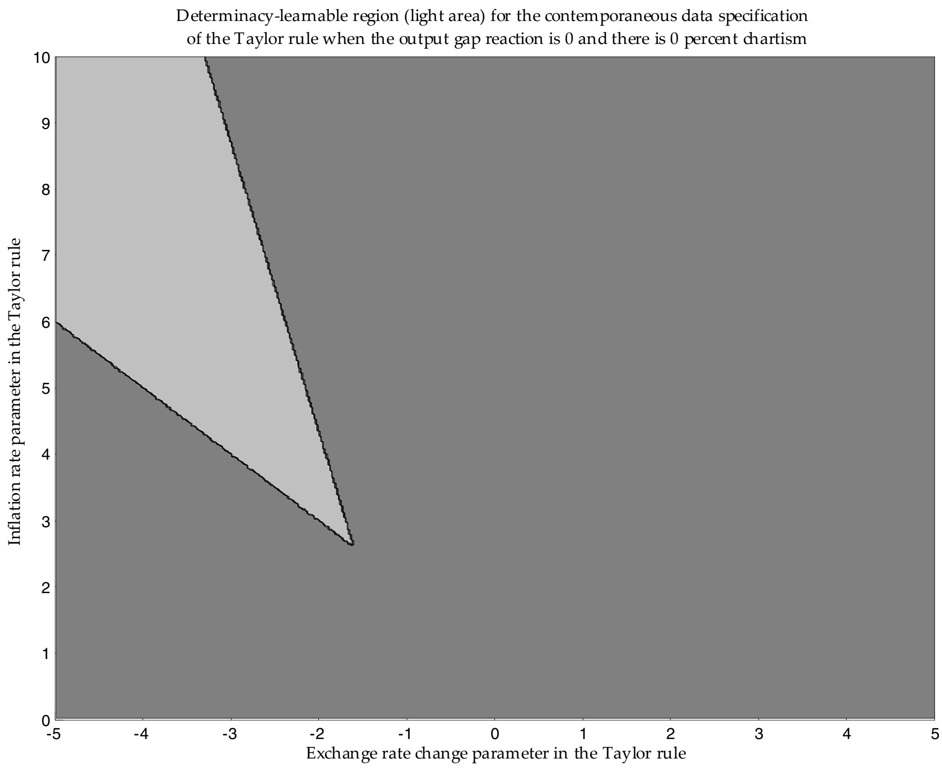

Figure 1

In *Figures 2a–b*, the proportion of chartists in currency trade has increased to 25 per cent, meaning that 75 per cent of the trade is guided by fundamental analysis, and the parameter value in the Taylor rule that describes the output gap reaction is $\zeta_x = 0$ and $\zeta_x = 2$, respectively.

[5] To keep the number of figures in the paper at a minimum, the regions in the figures are not only the regions for determinacy, but also the regions for adaptive learnability. Thus, as also will be clear in Section 3.2.1, when there is unique REE, the agents that use fundamental analysis, which also includes the monetary authority, will learn this REE.

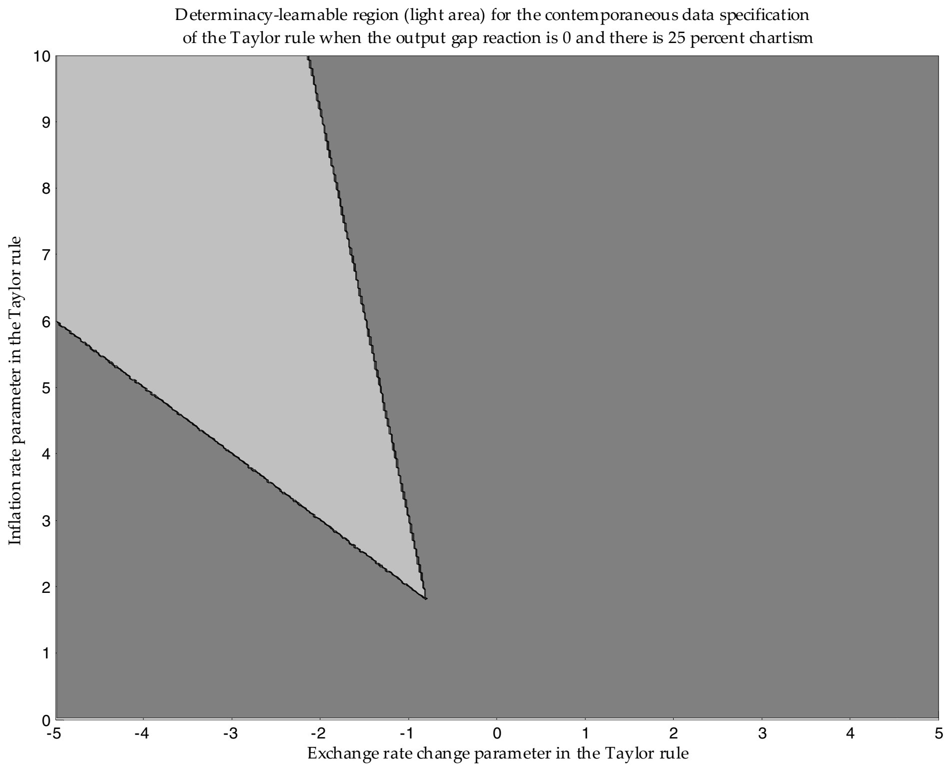

Figure 2a

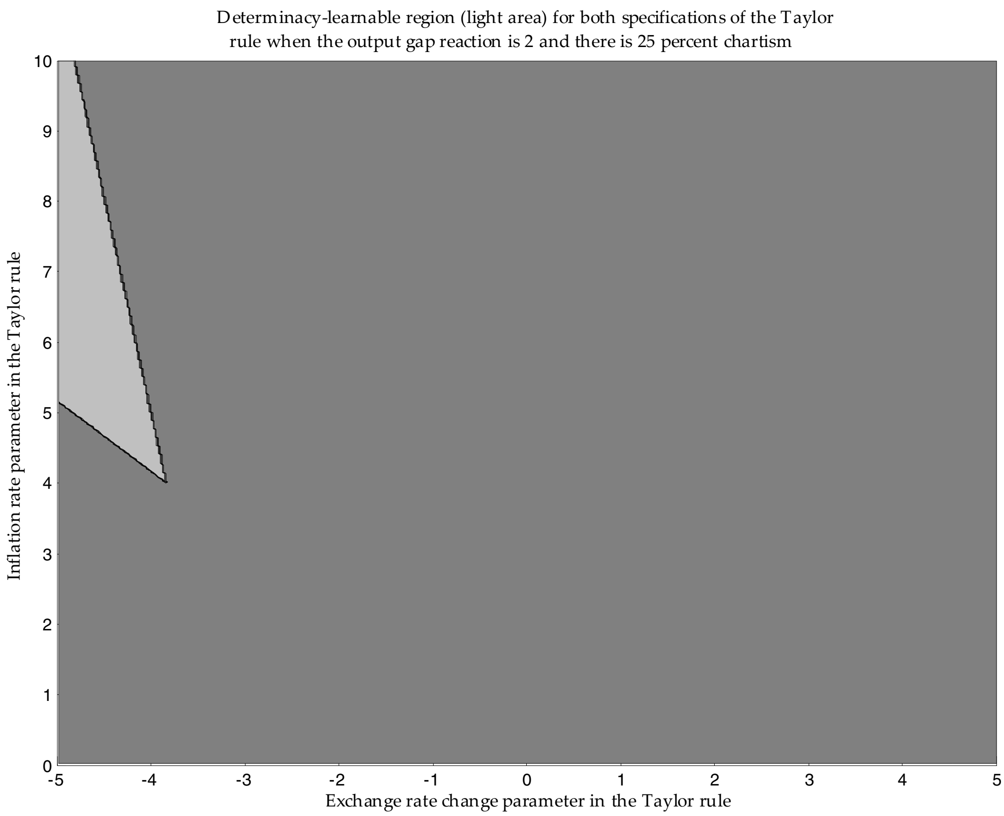

Figure 2b

A visible result in *Figures 2a–b* is that the region for a unique REE decreases when the monetary authority reacts stronger to the output gap. This is also true when there is no chartism in currency trade, even though we do not show this result explicitly (since there are no determinacy regions for $\zeta_x \geq 2$, which also is true for $\zeta_x = 4$ when $\omega = 0.25$). Moreover, if we compare *Figure 1* and *Figure 2a*, the determinacy region is larger when there is technical trading in the foreign exchange market.

In *Figures 3a–c*, half of the trade in the foreign exchange market is driven by technical analysis, whereas in *Figures 4a–c*, chartism is used in 75 per cent of the trade. Finally, in *Figures 5a–c*, all trade in foreign exchange is based on technical analysis, meaning that no trade is guided by fundamental analysis. In all these figures, the parameter value that describes the output gap reaction is $\zeta_x = 0$, $\zeta_x = 2$ and $\zeta_x = 4$, respectively.

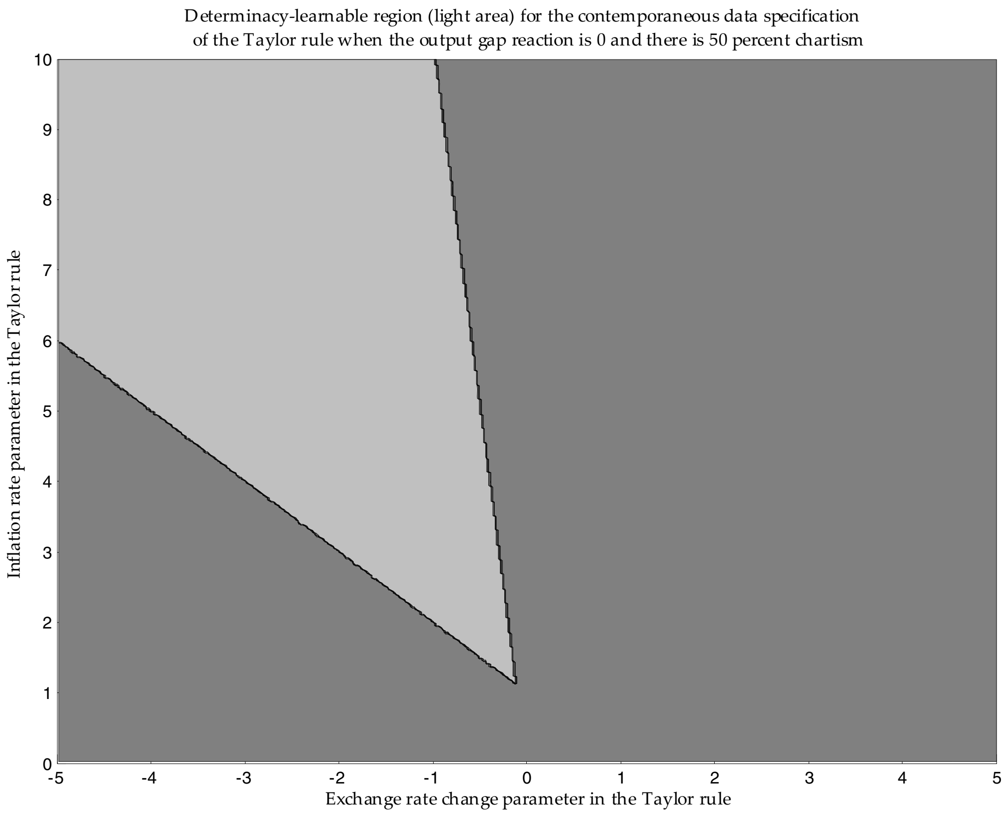

Figure 3a

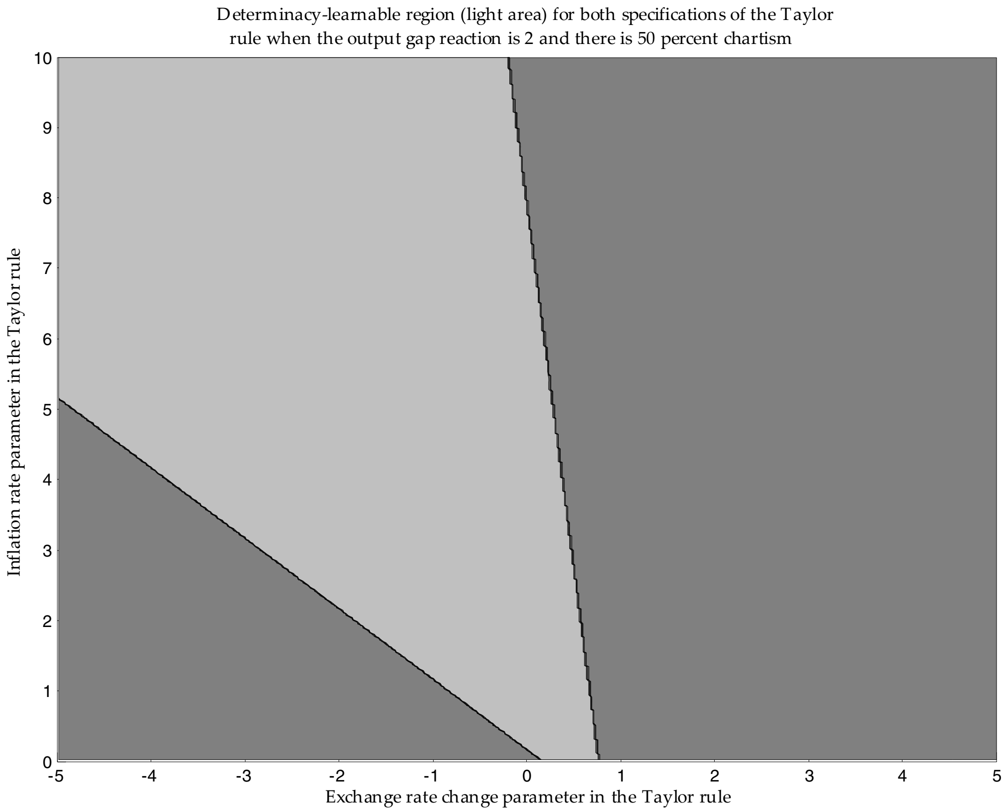

Figure 3b

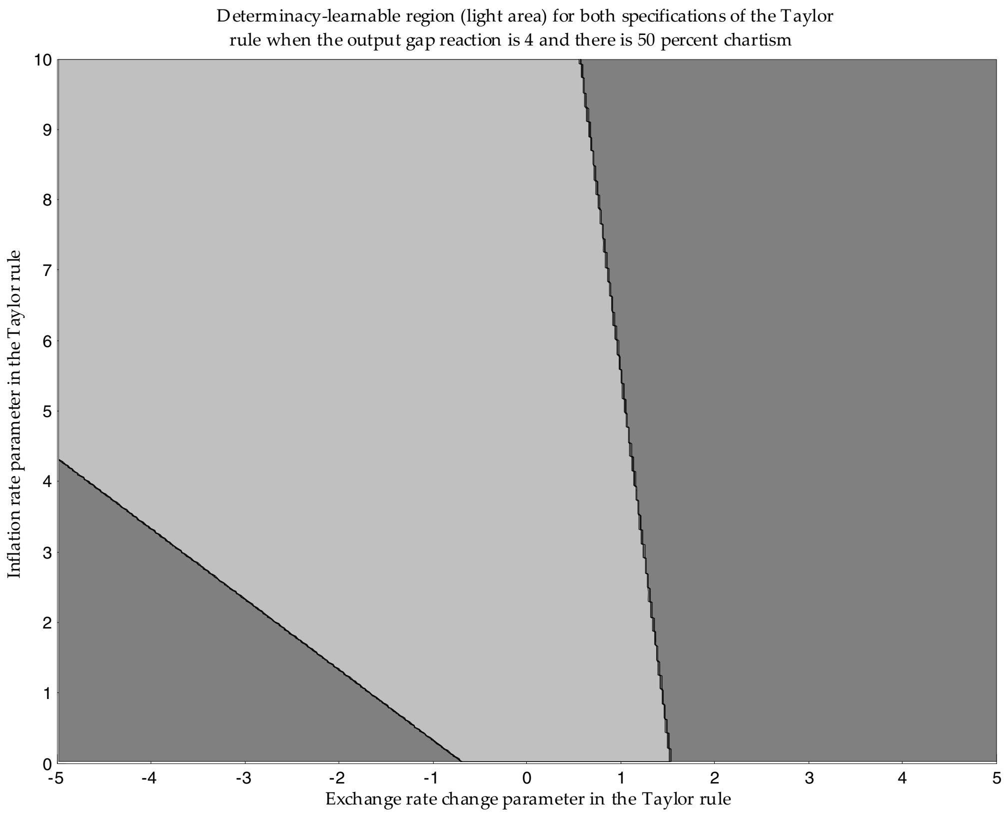

Figure 3c

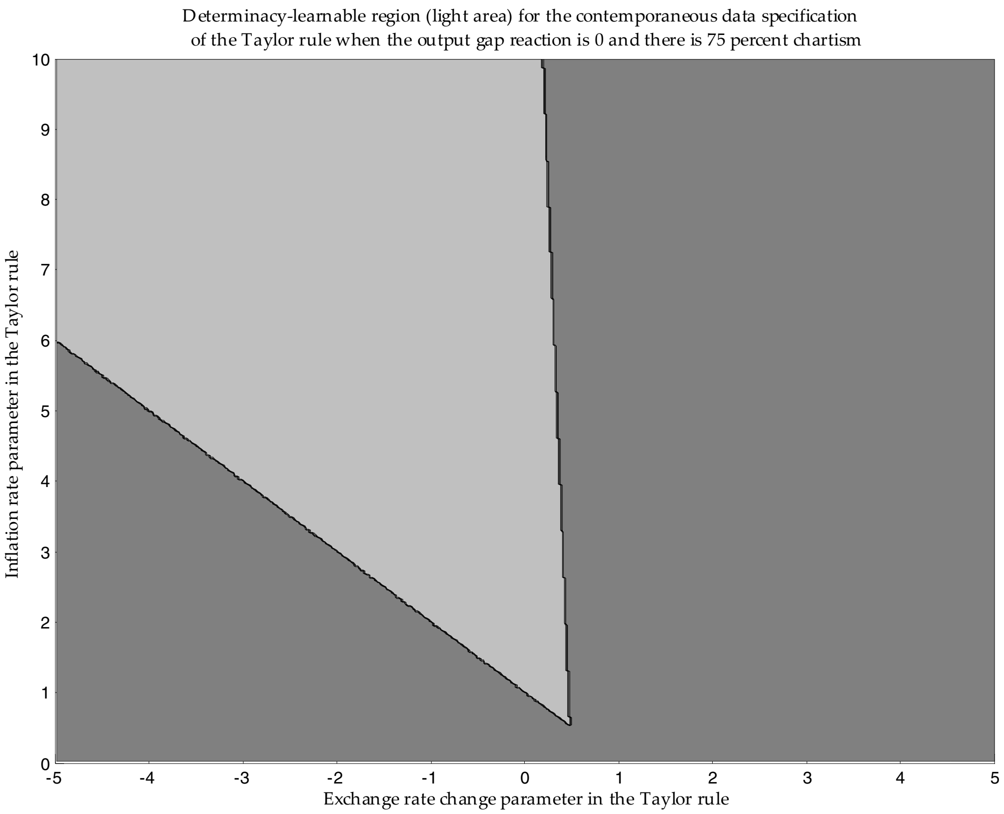

Figure 4a

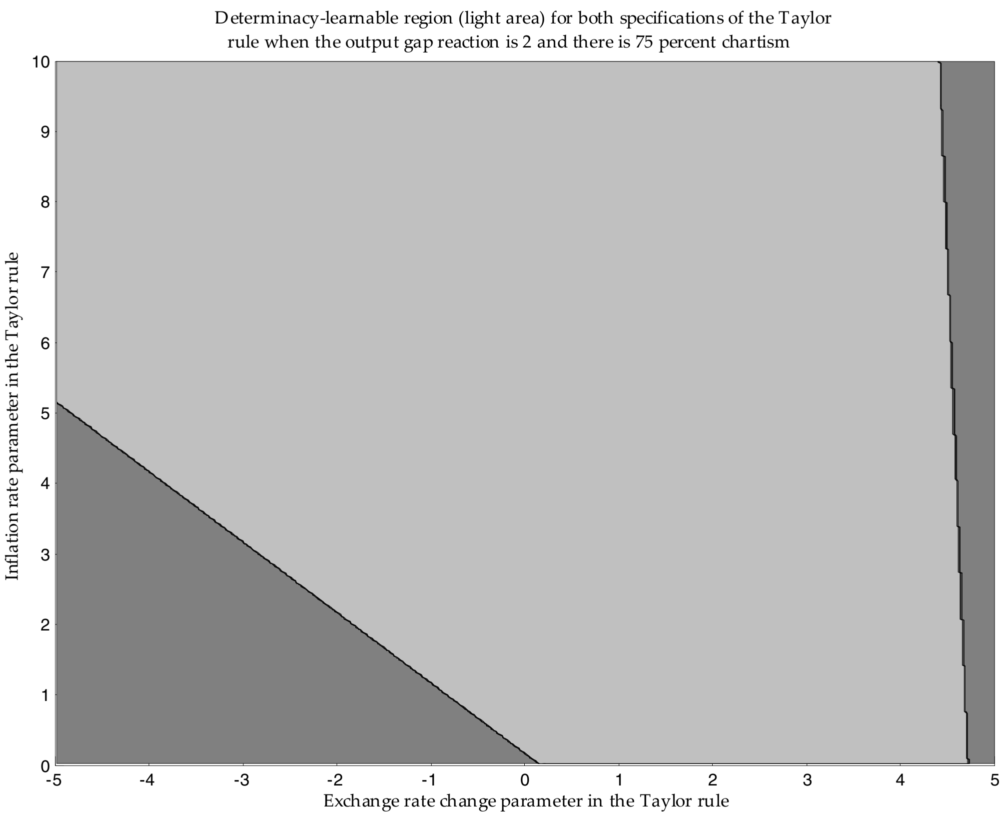

Figure 4b

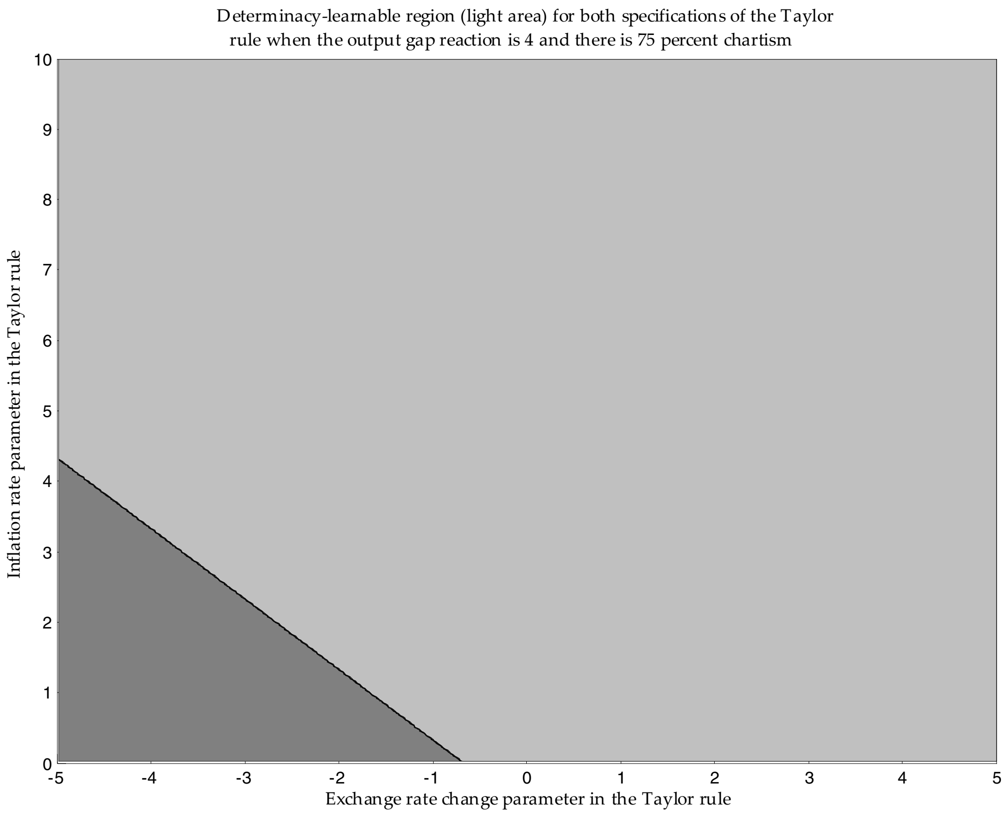

Figure 4c

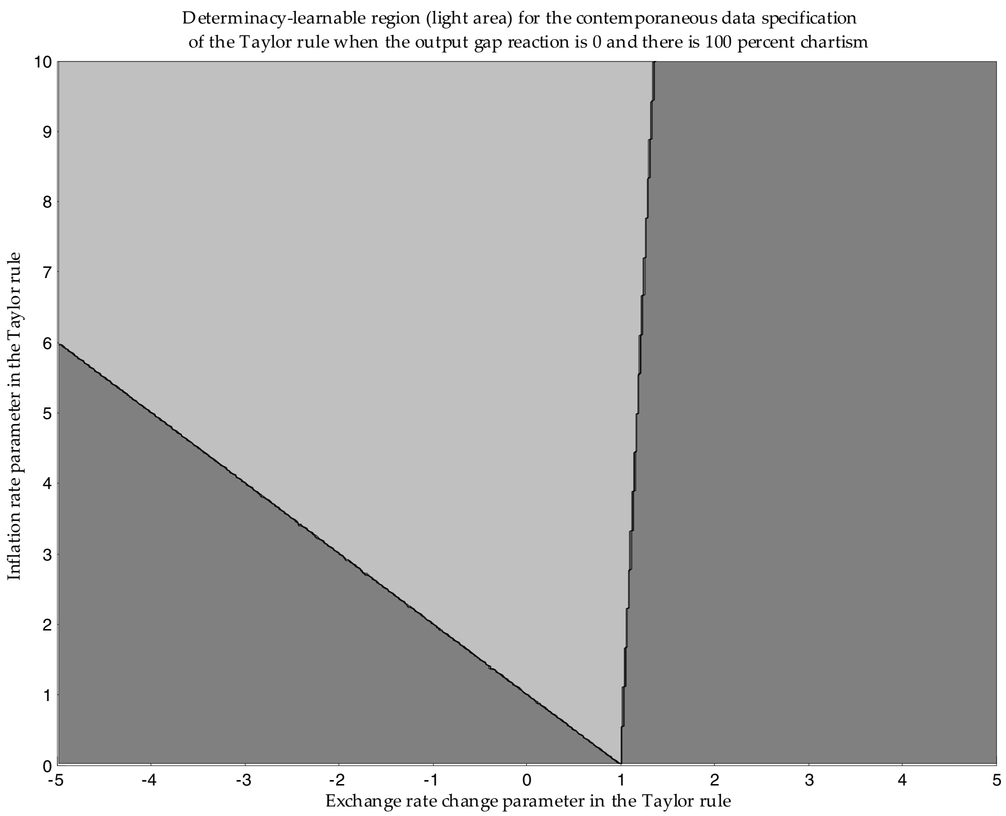

Figure 5a

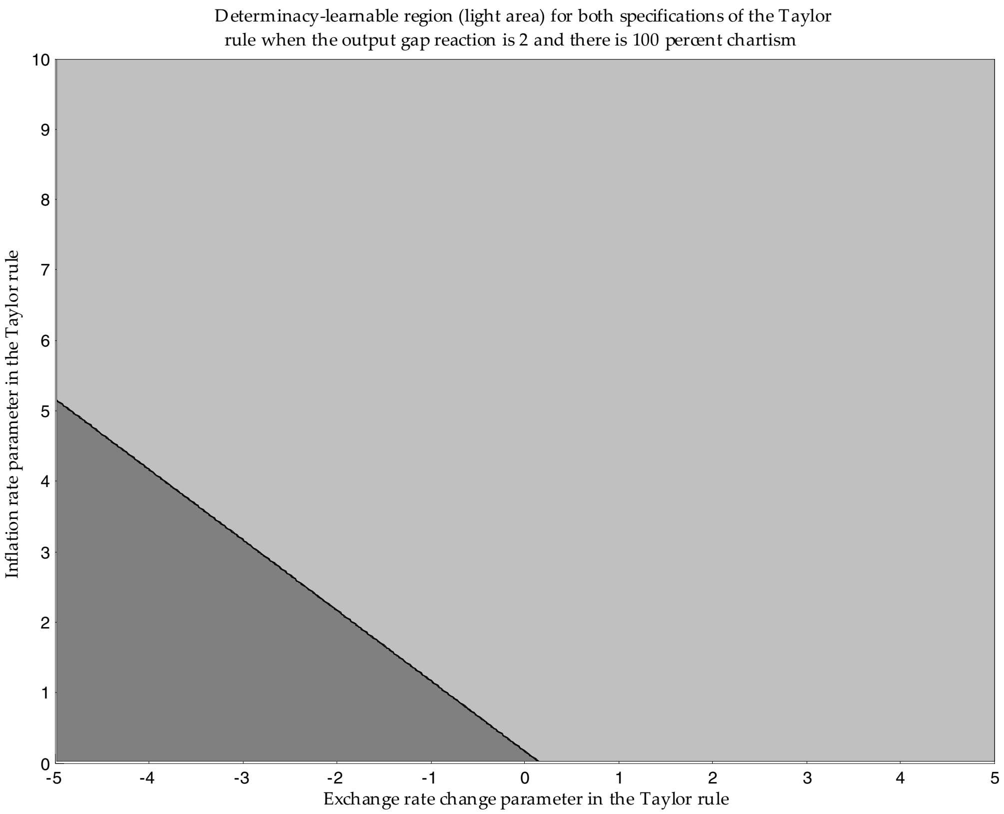

Figure 5b

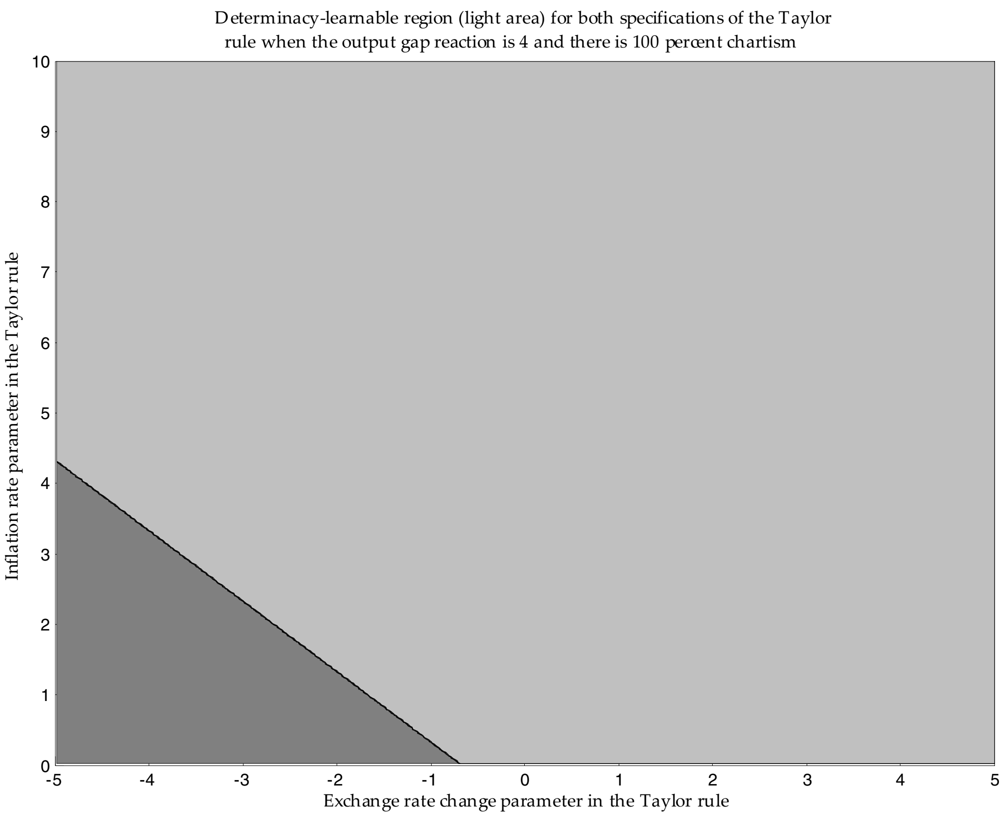

Figure 5c

When at least half of the trade in the foreign exchange market is driven by technical analysis, the region for a unique REE increases when the monetary authority reacts stronger to the output gap, which is in contrast with the result in *Figures 2a–b*. Moreover, given the output gap reaction in the Taylor rule, the determinacy region gets larger when the proportion of chartists in currency trade increases (even if it seems that the size of the regions are the same in *Figure 4c* and *Figure 5c*).

What is the intuition behind the result that an increase in technical trading induces a larger determinacy region? There is a similar result in Bullard and Mitra (2006) who investigate the conditions for determinacy (and learnability that we will discuss in Section 3.2.1) in a closed economy like the one in Bullard and Mitra (2002), where the monetary authority uses a Taylor rule that is augmented with a term that includes the previous time period's nominal interest rate to have policy inertia. Bullard and Mitra (2006) conclude that policy inertia helps to alleviate the problem with a multiplicity of REE, and the similarity with our model is that an increase in chartism is a form of increased inertia since there is a larger emphasize on the current exchange rate change instead of the next time period's (mathematically) expected exchange rate change.

We will restrict our discussion about the findings in the figures to the results previously mentioned, and the reason is that we save the conclusions till after we have investigated the robustness and desirability of a specific REE in the perspective of an inflation rate targeting regime. It might, for example, be tempting to conclude that the monetary authority should not react to exchange rate changes, if the reaction to the output gap is strong enough and at least half of the trade in foreign exchange is based on chartism (that is a reliable assumption according to questionnaire surveys). However, as will be clear in Section 3.3.1, it is not a favorable approach to restrict the parameter ζ_e in the Taylor rule to 0 since there are several parametrizations of the rule that give rise to a better outcome in equilibrium in terms of the expected inflation rate and the conditional volatility of the inflation rate when $\zeta_e < 0$.

Sensitivity analysis When the index of openness of the economy increases from $\delta = 0.2$ to $\delta = 0.4$, none of the findings are affected. That is, when at least half of the trade in the currency market is driven by technical analysis, the determinacy region increases when the monetary authority reacts stronger to the output gap, whereas the opposite is true when less than half of the trade is driven by chartism. Moreover, given the output gap reaction in the Taylor rule, the determinacy region gets larger when the proportion of chartists in currency trade increases. Finally, since we will learn in Section 3.3.1 that robust and desirable Taylor rules do not include a reaction to the output gap, we observe that the determinacy region is smaller when the economy is more open when $\zeta_x = 0$.

3.1.2 Contemporaneous expectations in the Taylor rule

If the Taylor rule in (2.10) is used when the monetary authority is setting the nominal interest rate, meaning that they respond to current expectations of the output gap, the CPI inflation rate and change in the nominal exchange rate, the complete model in (2.3)–(2.5), (2.8) and (2.10) can be written in matrix form as follows[6]

$$\begin{bmatrix} 1 & 0 & \frac{\alpha\delta\omega}{1-\delta} \\ \gamma(\delta-1) & 1 & \delta(\beta\omega-1) \\ 0 & 0 & -\omega \end{bmatrix} \begin{bmatrix} x_t \\ \pi_t \\ \Delta e_t \end{bmatrix} \tag{3.4}$$
$$= \begin{bmatrix} -\alpha\zeta_c - \frac{\alpha\delta}{1-\delta}\cdot\pi_{t+1}^{e,*} \\ \delta\left(\pi_t^* - \beta\pi_{t+1}^{e,*}\right) \\ r_t^* - \zeta_c \end{bmatrix} + \begin{bmatrix} -\alpha\zeta_x & -\alpha\zeta_\pi & -\alpha\zeta_e \\ 0 & 0 & 0 \\ -\zeta_x & -\zeta_\pi & -\zeta_e \end{bmatrix} \begin{bmatrix} x_t^e \\ \pi_t^e \\ \Delta e_t^{e,f} \end{bmatrix}$$
$$\begin{bmatrix} 1 & \frac{\alpha}{1-\delta} & \frac{\alpha\delta(\omega-1)}{1-\delta} \\ 0 & \beta & \beta\delta(\omega-1) \\ 0 & 0 & 1-\omega \end{bmatrix} \begin{bmatrix} x_{t+1}^e \\ \pi_{t+1}^e \\ \Delta e_{t+1}^e \end{bmatrix} + \begin{bmatrix} \alpha \\ 0 \\ 0 \end{bmatrix} \cdot \overline{rr}_t.$$

The dating of current expectations in (3.4) is time period $t-1$, meaning that the monetary authority has rational expectations regarding the variables in the interest rate rule, conditioned on all information available in the previous time period.

When deriving conditions for determinacy, we do the following substitution

$$\left[x_t^e, \pi_t^e, \Delta e_t^{e,f}\right]' = \left[x_t, \pi_t, \Delta e_t\right]' + \boldsymbol{\epsilon}_t. \tag{3.5}$$

where $\boldsymbol{\epsilon}_t$ is a vector with error terms. Consequently, to have a unique and stable REE, all three eigenvalues of the same coefficient matrix as in Section 3.1.1 must be inside the unit circle since x_t, π_t and Δe_t are free (see (3.2)).[7] Therefore, we refer to *Figures 1-5* and the discussion around them for the regions in the parameter space of $(\omega, \zeta_x, \zeta_\pi, \zeta_e)$ for which we have a unique inflation rate in equilibrium.[8]

Sensitivity analysis Obviously, we get exactly the same results when increasing the index of openness of the economy from $\delta = 0.2$ to $\delta = 0.4$ as when increasing the same index when a contemporaneous data specification of the Taylor rule is used by the monetary authority. This is because the same coefficient matrix determines the conditions for determinacy for both types of interest rate rules (see (3.2)).

[6] See the Appendix for the derivation of (3.4).

[7] See the Appendix for the derivation of this result.

[8] That is, the regions in *Figures 1–5* are the determinacy regions when a contemporaneous expectations specification of the Taylor rule is used, even if it is written 'contemporaneous data specification of the Taylor rule' in some of the figures (see *Figure 1*, *Figure 2a*, *Figure 3a*, *Figure 4a* and *Figure 5a*). In fact, it will turn out in Section 3.2.2 that there are parametrizations of the contemporaneous expectations specification of the interest rate rule that give rise to a unique REE that is not adaptively learnable.

3.2 Least squares learning

The assumption in (2.7) is that when fundamental analysis is used in currency trade, the agents have rational expectations in the sense that the expected change in the exchange rate is equal to the mathematically expected change in the exchange rate, conditioned on all information available to the currency trader. Thus, since this information not only includes past and current values of all variables in the model, but also a perfect knowledge about the structure of the model, rational expectations is a rather strong assumption. This assumption has, therefore, in the more recent literature, been complemented by an analysis of the possible convergence to the REE.

It is assumed that expectations are formed by a correctly specified model, ie, a model that nests the REE, but without having perfect knowledge about the parameter values in the model. However, using past and (depending on the dating of expectations) current values of all variables in the model, the parameter values are learned over time since the beliefs are revised as new information is gained. To be more precise, we will examine if the unique REE is characterized by recursive least squares learnability. But since expectational stability, or E-stability, implies learnability (see, eg, Evans and Honkapohja, 2001), the focus in the analysis will be on E-stability. This is because the latter concept is easier to handle mathematically.

When there is a unique REE in the model, we make use of the minimal state variable (MSV) solution, which is the solution of a linear difference equation that depends linearly on a set of variables such that there does not exist a solution that depends linearly on a smaller set of variables (see McCallum, 1983). This is also the approach taken in Bullard and Mitra (2002).

Finally, recall that the agents that use technical analysis do not learn anything since they use a mechanical rule in their trade in foreign exchange.

3.2.1 Contemporaneous data in the Taylor rule

Let us start with the contemporaneous data specification of the interest rate rule as it is presented in (2.9).

First, using matrices and vectors, the model in (3.1) can be written as follows

$$\mathbf{\Xi} \cdot \mathbf{y}_t = \mathbf{\Pi} + \mathbf{\Sigma} \cdot \mathbf{y}^e_{t+1} + \mathbf{\Upsilon} \cdot \overline{rr}_t, \tag{3.6}$$

where $\mathbf{y}_t = [x_t, \pi_t, \Delta e_t]'$ is the state of the economy. A suggested MSV solution of the model in (3.6) is, therefore

$$\mathbf{y}_t = \widehat{\mathbf{\Theta}} + \widehat{\mathbf{\Lambda}} \cdot \overline{rr}_t, \tag{3.7}$$

where $\widehat{\mathbf{\Theta}}$ and $\widehat{\mathbf{\Lambda}}$ are parameter vectors to be determined with the method of undetermined coefficients. Hence, calculate the mathematically expected state of the economy in the next time period

$$\begin{aligned} \mathbf{y}^e_{t+1} &= \widehat{\mathbf{\Theta}} + \widehat{\mathbf{\Lambda}} \cdot \overline{rr}^e_{t+1} \\ &= \widehat{\mathbf{\Theta}} + \widehat{\mathbf{\Lambda}} \cdot \rho \overline{rr}_t, \end{aligned} \tag{3.8}$$

where (2.5) is used in the second step in (3.8), and the dating of expectations is time period t. Thereafter, substitute (3.8) into the model in (3.6)

$$\mathbf{\Xi} \cdot \mathbf{y}_t = \mathbf{\Pi} + \mathbf{\Sigma} \cdot \left(\widehat{\mathbf{\Theta}} + \widehat{\mathbf{\Lambda}} \cdot \rho \overline{rr}_t \right) + \mathbf{\Upsilon} \cdot \overline{rr}_t, \tag{3.9}$$

or, if solved for the contemporaneous values of the model's variables

$$\begin{aligned} \mathbf{y}_t &= \mathbf{\Xi}^{-1} \cdot \mathbf{\Pi} + \mathbf{\Xi}^{-1} \cdot \mathbf{\Sigma} \cdot \left(\widehat{\mathbf{\Theta}} + \widehat{\mathbf{\Lambda}} \cdot \rho \overline{rr}_t \right) + \mathbf{\Xi}^{-1} \cdot \mathbf{\Upsilon} \cdot \overline{rr}_t \\ &= \mathbf{\Xi}^{-1} \cdot \mathbf{\Pi} + \mathbf{\Gamma} \cdot \left(\widehat{\mathbf{\Theta}} + \widehat{\mathbf{\Lambda}} \cdot \rho \overline{rr}_t \right) + \mathbf{\Xi}^{-1} \cdot \mathbf{\Upsilon} \cdot \overline{rr}_t \\ &= \mathbf{\Xi}^{-1} \cdot \mathbf{\Pi} + \mathbf{\Gamma} \cdot \widehat{\mathbf{\Theta}} + \left(\mathbf{\Gamma} \cdot \widehat{\mathbf{\Lambda}} \cdot \rho + \mathbf{\Xi}^{-1} \cdot \mathbf{\Upsilon} \right) \cdot \overline{rr}_t, \end{aligned} \tag{3.10}$$

where $\mathbf{\Gamma} = \mathbf{\Xi}^{-1} \cdot \mathbf{\Sigma}$. Finally, by comparing the parameters in (3.7) and (3.10), we can solve for the MSV solution

$$\mathbf{y}_t = (\mathbf{I} - \mathbf{\Gamma})^{-1} \cdot \mathbf{\Xi}^{-1} \cdot \mathbf{\Pi} + (\mathbf{I} - \mathbf{\Gamma} \cdot \rho)^{-1} \cdot \mathbf{\Xi}^{-1} \cdot \mathbf{\Upsilon} \cdot \overline{rr}_t, \tag{3.11}$$

since

$$\begin{cases} \widehat{\mathbf{\Theta}} = \mathbf{\Xi}^{-1} \cdot \mathbf{\Pi} + \mathbf{\Gamma} \cdot \widehat{\mathbf{\Theta}} \\ \widehat{\mathbf{\Lambda}} = \mathbf{\Gamma} \cdot \widehat{\mathbf{\Lambda}} \cdot \rho + \mathbf{\Xi}^{-1} \cdot \mathbf{\Upsilon} \end{cases}, \tag{3.12}$$

where $\mathbf{I}$ is the identity matrix.

Now, is the MSV solution in (3.11) characterized by recursive least squares learnability? To have a REE that is learnable, the parameter values in the perceived law of motion (PLM) of the economy have to converge to the parameter values in the economy's actual law of motion (ALM) (see, eg, Evans and Honkapohja, 2001). In fact, the suggested MSV solution in (3.7) is also the PLM of the economy (which is emphasized by the 'hat'-symbol since $\mathbf{\Theta}$ and $\mathbf{\Lambda}$ are parameter vectors that are estimated), and the solution in (3.10) is the ALM of the economy.

To be more precise, to have the ALM of the economy, a possibly non-rational forecast of the next time period's state of the economy should be substituted into the model in (3.6) allowing for non-rational expectations. However, since the mathematical expression in (3.10) would not be affected by this substitution, (3.10) is also the ALM of the economy. (In Section 3.2.2, when a contemporaneous expectations specification of the Taylor rule is used by the monetary authority, we will partly focus the presentation on the derivation of the economy's ALM.)

Observe that there is a mapping from the parameter values in the PLM to the parameter values in the ALM

$$\mathbf{M}_{MSV} \begin{pmatrix} \widehat{\mathbf{\Theta}} \\ \widehat{\mathbf{\Lambda}} \end{pmatrix} = \begin{pmatrix} \mathbf{\Xi}^{-1} \cdot \mathbf{\Pi} + \mathbf{\Gamma} \cdot \widehat{\mathbf{\Theta}} \\ \mathbf{\Gamma} \cdot \widehat{\mathbf{\Lambda}} \cdot \rho + \mathbf{\Xi}^{-1} \cdot \mathbf{\Upsilon} \end{pmatrix}, \tag{3.13}$$

and consider the matrix differential equation

$$\begin{aligned} \frac{\partial}{\partial \tau} \begin{pmatrix} \widehat{\mathbf{\Theta}} \\ \widehat{\mathbf{\Lambda}} \end{pmatrix} &= \mathbf{M}_{MSV} \begin{pmatrix} \widehat{\mathbf{\Theta}} \\ \widehat{\mathbf{\Lambda}} \end{pmatrix} - \begin{pmatrix} \widehat{\mathbf{\Theta}} \\ \widehat{\mathbf{\Lambda}} \end{pmatrix} \\ &= \begin{pmatrix} \mathbf{\Xi}^{-1} \cdot \mathbf{\Pi} + \mathbf{\Gamma} \cdot \widehat{\mathbf{\Theta}} \\ \mathbf{\Gamma} \cdot \widehat{\mathbf{\Lambda}} \cdot \rho + \mathbf{\Xi}^{-1} \cdot \mathbf{\Upsilon} \end{pmatrix} - \begin{pmatrix} \widehat{\mathbf{\Theta}} \\ \widehat{\mathbf{\Lambda}} \end{pmatrix}, \end{aligned} \tag{3.14}$$

where τ is 'artificial' time. Then, the MSV solution in (3.11) is E-stable, if the parameter vectors $\widehat{\mathbf{\Theta}}$ and $\widehat{\mathbf{\Lambda}}$ are locally asymptotically stable under (3.14). This is also the case if all eigenvalues of the following matrixes have negative real parts (see, eg, Evans and Honkapohja, 2001)

$$\begin{cases} \partial\left(\frac{\partial\widehat{\mathbf{\Theta}}}{\partial\tau}\right)/\partial\widehat{\mathbf{\Theta}} = \mathbf{\Gamma} - \mathbf{I} \\ \partial\left(\frac{\partial\widehat{\mathbf{\Lambda}}}{\partial\tau}\right)/\partial\widehat{\mathbf{\Lambda}} = \mathbf{\Gamma}\cdot\rho - \mathbf{I} \end{cases}. \tag{3.15}$$

Since $0 \leq \rho < 1$, we can limit our attention to the first row in (3.15).

It is clear that when there is a unique REE and the state of the economy is in the neighborhood of the REE, the agents that use fundamental analysis, which also includes the monetary authority, will learn this REE. To see this result explicitly, note that the characteristic equation for the determinacy problem is (see Section 3.1.1)

$$|\mathbf{\Gamma} - \lambda_d \cdot \mathbf{I}| = 0, \tag{3.16}$$

where λ_d is the eigenvalue (that has three solutions), and that the characteristic equation for the learnability problem is

$$\begin{aligned} |\mathbf{\Gamma} - \mathbf{I} - \lambda_l \cdot \mathbf{I}| &= \\ |\mathbf{\Gamma} - (1 + \lambda_l) \cdot \mathbf{I}| &= 0, \end{aligned} \tag{3.17}$$

where λ_l is the eigenvalue (that also has three solutions). Thus

$$\mathrm{Re}\,(\lambda_l) = \mathrm{Re}\,(\lambda_d - 1)\,, \tag{3.18}$$

which means that when λ_d is inside the unit circle, λ_l has a negative real part. Therefore, we refer to *Figures 1–5* and the discussion around them for the regions in the parameter space of $(\omega, \zeta_x, \zeta_\pi, \zeta_e)$ for which we have a unique and an adaptively learnable inflation rate in equilibrium. Be aware that even though there is a REE that is adaptively learnable in recursive least squares sense, this REE does not have to be unique.

It is not easy to give the intuition behind the result that an increase in technical trading induces a larger learnability region. However, there is a similar result in Bullard and Mitra (2006) that we discussed in Section 3.1.1. Specifically, policy inertia not only induces a larger determinacy region in their model, it also induces a larger learnability region. Thus, since chartism is a form of inertia, it is reasonable to expect a larger learnability region when the degree of technical trading in the foreign exchange market increases.

Sensitivity analysis Since a unique REE always is adaptively learnable, we get the same general results when the index of openness of the economy increases from $\delta = 0.2$ to $\delta = 0.4$ as when increasing the same index when we investigated the conditions for determinacy.

3.2.2 Contemporaneous expectations in the Taylor rule

Let us continue with the contemporaneous expectations specification of the interest rate rule as it is presented in (2.10).

First, using matrices and vectors, the model in (3.4) can be written as follows

$$\mathbf{\Xi}_0 \cdot \mathbf{y}_t = \mathbf{\Pi} + \mathbf{\Xi}_1 \cdot \mathbf{y}_t^e + \mathbf{\Sigma} \cdot \mathbf{y}_{t+1}^e + \mathbf{\Upsilon} \cdot \overline{rr}_t, \tag{3.19}$$

where $\mathbf{\Xi}_0 - \mathbf{\Xi}_1 = \mathbf{\Xi}$. Therefore, and guided by structure of the model's MSV solution, we assume that the PLM of the economy is

$$\mathbf{y}_t = \widehat{\mathbf{\Theta}} + \widehat{\mathbf{\Lambda}} \cdot \overline{rr}_{t-1} + \mathbf{\Phi} \cdot \varepsilon_t, \tag{3.20}$$

from which we calculate the mathematically expected state of the economy in the current time period

$$\mathbf{y}_t^e = \widehat{\mathbf{\Theta}} + \widehat{\mathbf{\Lambda}} \cdot \overline{rr}_{t-1}, \tag{3.21}$$

as well as in the next time period

$$\begin{aligned} \mathbf{y}_{t+1}^e &= \widehat{\mathbf{\Theta}} + \widehat{\mathbf{\Lambda}} \cdot \overline{rr}_t^e \\ &= \widehat{\mathbf{\Theta}} + \widehat{\mathbf{\Lambda}} \cdot \rho \overline{rr}_{t-1}. \end{aligned} \tag{3.22}$$

Recall that the dating of expectations in (3.21) is time period $t-1$. Moreover, to have an exact correspondence with Bullard and Mitra (2002), we assume that the dating of expectations in (3.22) is also time period $t-1$. (Recall that the dating of expectations when contemporaneous data are used in the Taylor rule is time period t.)

Then, if we substitute the expected states of the economy in (3.21)–(3.22) into the PLM of the economy in (3.20), we get the economy's ALM

$$\begin{aligned} \mathbf{\Xi}_0 \cdot \mathbf{y}_t &= \mathbf{\Pi} + \mathbf{\Xi}_1 \cdot \left(\widehat{\mathbf{\Theta}} + \widehat{\mathbf{\Lambda}} \cdot \overline{rr}_{t-1} \right) + \\ &\quad \mathbf{\Sigma} \cdot \left(\widehat{\mathbf{\Theta}} + \widehat{\mathbf{\Lambda}} \cdot \rho \overline{rr}_{t-1} \right) + \mathbf{\Upsilon} \cdot \overline{rr}_t \\ &= \mathbf{\Pi} + \mathbf{\Xi}_1 \cdot \left(\widehat{\mathbf{\Theta}} + \widehat{\mathbf{\Lambda}} \cdot \overline{rr}_{t-1} \right) + \\ &\quad \mathbf{\Sigma} \cdot \left(\widehat{\mathbf{\Theta}} + \widehat{\mathbf{\Lambda}} \cdot \rho \overline{rr}_{t-1} \right) + \mathbf{\Upsilon} \cdot \left(\rho \overline{rr}_{t-1} + \varepsilon_t \right), \end{aligned} \tag{3.23}$$

or, if solved for the contemporaneous values of the model's variables,

$$\begin{aligned} \mathbf{y}_t &= \mathbf{\Xi}_0^{-1} \cdot \mathbf{\Pi} + \mathbf{\Xi}_0^{-1} \cdot \mathbf{\Xi}_1 \cdot \left(\widehat{\mathbf{\Theta}} + \widehat{\mathbf{\Lambda}} \cdot \overline{rr}_{t-1} \right) + \\ &\quad \mathbf{\Xi}_0^{-1} \cdot \mathbf{\Sigma} \cdot \left(\widehat{\mathbf{\Theta}} + \widehat{\mathbf{\Lambda}} \cdot \rho \overline{rr}_{t-1} \right) + \mathbf{\Xi}_0^{-1} \cdot \mathbf{\Upsilon} \cdot \left(\rho \overline{rr}_{t-1} + \varepsilon_t \right) \\ &= \mathbf{\Xi}_0^{-1} \cdot \left(\mathbf{\Pi} + \mathbf{\Xi}_1 \cdot \widehat{\mathbf{\Theta}} + \mathbf{\Sigma} \cdot \widehat{\mathbf{\Theta}} \right) + \\ &\quad \mathbf{\Xi}_0^{-1} \cdot \left(\mathbf{\Xi}_1 \cdot \widehat{\mathbf{\Lambda}} + \mathbf{\Sigma} \cdot \widehat{\mathbf{\Lambda}} \cdot \rho + \mathbf{\Upsilon} \cdot \rho \right) \cdot \overline{rr}_{t-1} + \mathbf{\Xi}_0^{-1} \cdot \mathbf{\Upsilon} \cdot \varepsilon_t. \end{aligned} \tag{3.24}$$

Observe again that there is a mapping from the parameter values in the PLM to the parameter values in the ALM

$$\mathbf{M}_{MSV} \begin{pmatrix} \widehat{\mathbf{\Theta}} \\ \widehat{\mathbf{\Lambda}} \end{pmatrix} = \begin{pmatrix} \mathbf{\Xi}_0^{-1} \cdot \left(\mathbf{\Pi} + \mathbf{\Xi}_1 \cdot \widehat{\mathbf{\Theta}} + \mathbf{\Sigma} \cdot \widehat{\mathbf{\Theta}} \right) \\ \mathbf{\Xi}_0^{-1} \cdot \left(\mathbf{\Xi}_1 \cdot \widehat{\mathbf{\Lambda}} + \mathbf{\Sigma} \cdot \widehat{\mathbf{\Lambda}} \cdot \rho + \mathbf{\Upsilon} \cdot \rho \right) \end{pmatrix}, \tag{3.25}$$

and consider the matrix differential equation

$$\frac{\partial}{\partial\tau}\begin{pmatrix}\widehat{\mathbf{\Theta}}\\ \widehat{\mathbf{\Lambda}}\end{pmatrix} = \mathbf{M}_{MSV}\begin{pmatrix}\widehat{\mathbf{\Theta}}\\ \widehat{\mathbf{\Lambda}}\end{pmatrix} - \begin{pmatrix}\widehat{\mathbf{\Theta}}\\ \widehat{\mathbf{\Lambda}}\end{pmatrix} \tag{3.26}$$
$$= \begin{pmatrix}\mathbf{\Xi}_0^{-1}\cdot\left(\mathbf{\Pi}+\mathbf{\Xi}_1\cdot\widehat{\mathbf{\Theta}}+\mathbf{\Sigma}\cdot\widehat{\mathbf{\Theta}}\right)\\ \mathbf{\Xi}_0^{-1}\cdot\left(\mathbf{\Xi}_1\cdot\widehat{\mathbf{\Lambda}}+\mathbf{\Sigma}\cdot\widehat{\mathbf{\Lambda}}\cdot\rho+\mathbf{\Upsilon}\cdot\rho\right)\end{pmatrix} - \begin{pmatrix}\widehat{\mathbf{\Theta}}\\ \widehat{\mathbf{\Lambda}}\end{pmatrix},$$

where the equation's fix point is the MSV solution of the model in (3.19). Hence, if the parameter vectors $\widehat{\mathbf{\Theta}}$ and $\widehat{\mathbf{\Lambda}}$ are locally asymptotically stable under (3.26), the MSV solution is E-stable, which is the case when all eigenvalues of the following matrixes have negative real parts

$$\begin{cases}\partial\left(\frac{\partial\widehat{\mathbf{\Theta}}}{\partial\tau}\right)/\partial\widehat{\mathbf{\Theta}} = \mathbf{\Xi}_0^{-1}\cdot(\mathbf{\Xi}_1+\mathbf{\Sigma})-\mathbf{I}\\ \partial\left(\frac{\partial\widehat{\mathbf{\Lambda}}}{\partial\tau}\right)/\partial\widehat{\mathbf{\Lambda}} = \mathbf{\Xi}_0^{-1}\cdot(\mathbf{\Xi}_1+\mathbf{\Sigma}\cdot\rho)-\mathbf{I}\end{cases}. \tag{3.27}$$

Due to the fact that $0 \leq \rho < 1$, we can limit our attention to the first row in (3.27), meaning that the relevant characteristic equation is

$$\left|\mathbf{\Xi}_0^{-1}\cdot(\mathbf{\Xi}_1+\mathbf{\Sigma})-\mathbf{I}-\lambda_l\cdot\mathbf{I}\right| = 0, \tag{3.28}$$

where λ_l is the eigenvalue (that has three solutions).

It turns out that the regions in the parameter space of $(\omega, \zeta_x, \zeta_\pi, \zeta_e)$ for which we have a unique and an adaptively learnable inflation rate in equilibrium are not the same as when a contemporaneous data specification of the Taylor rule is used by the monetary authority. See *Figures 2–9.*

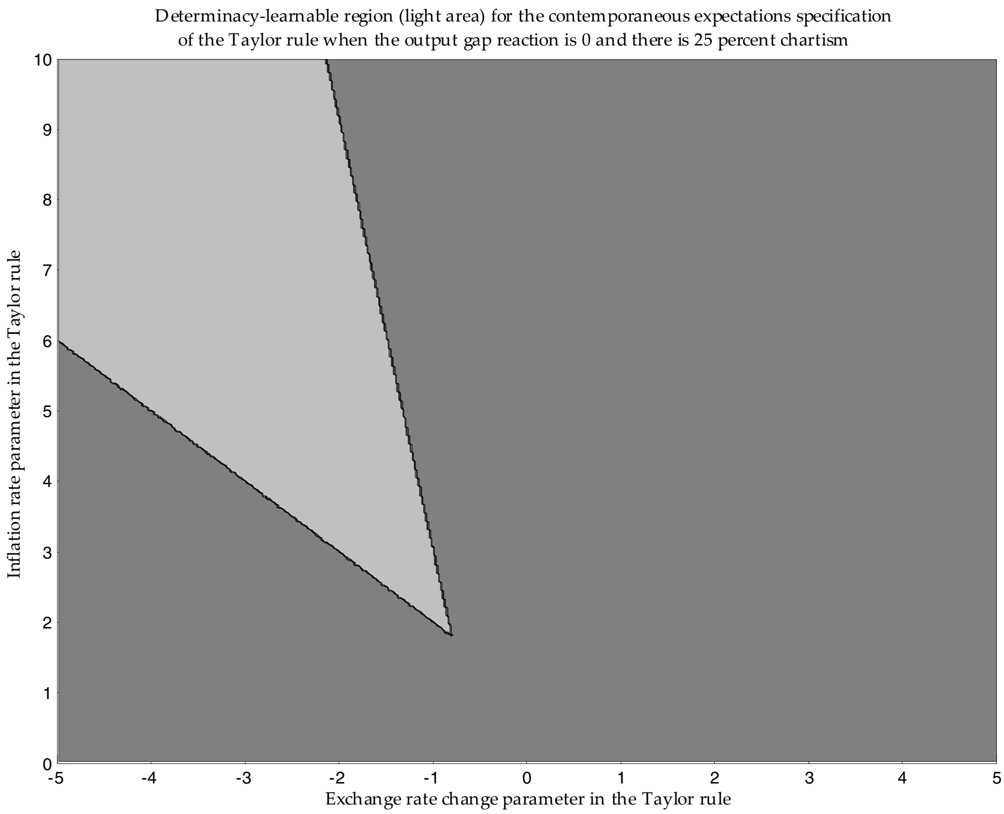

Figure 6

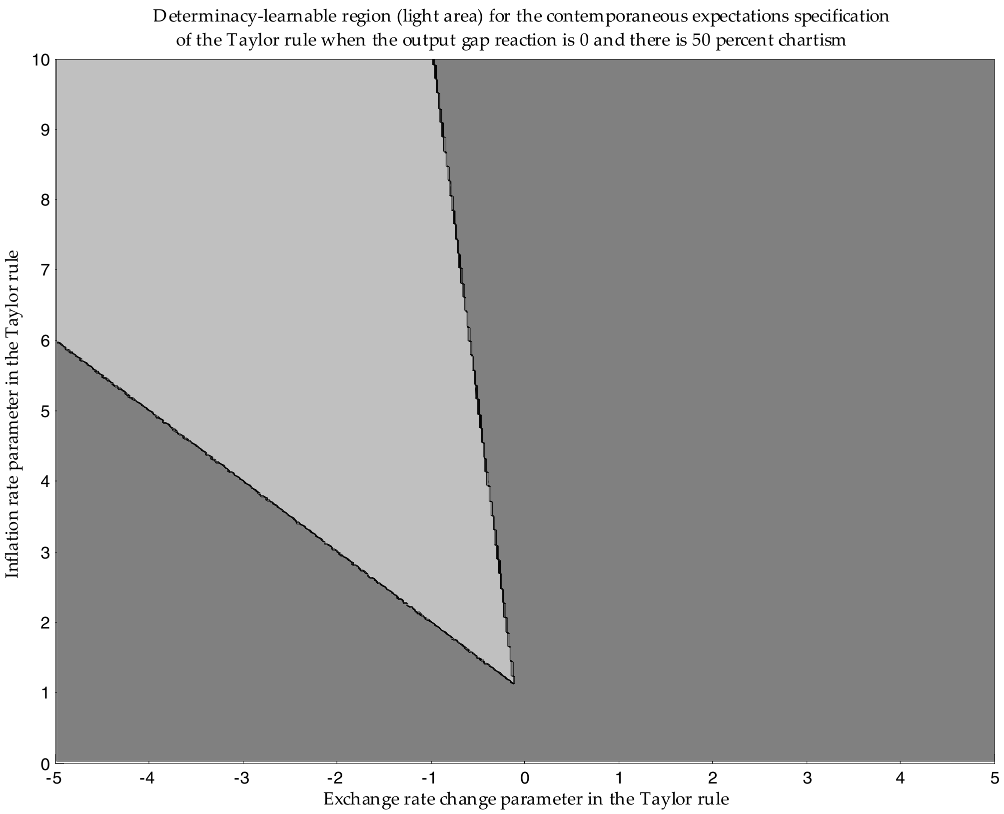

Figure 7

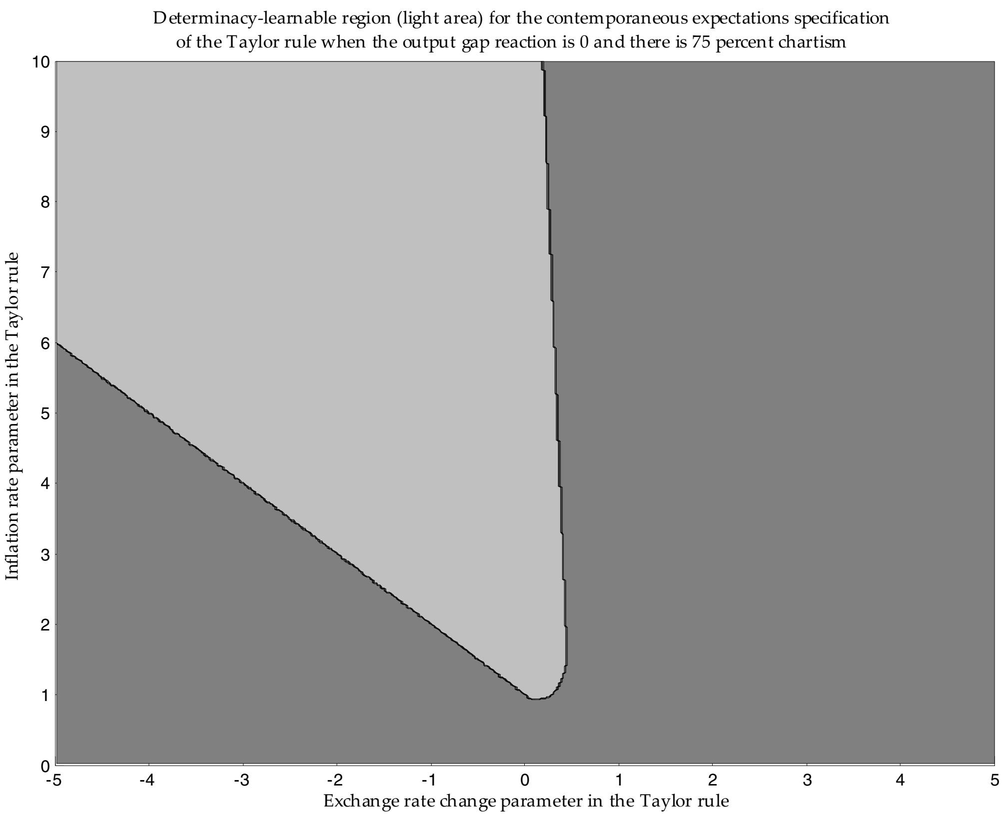

Figure 8

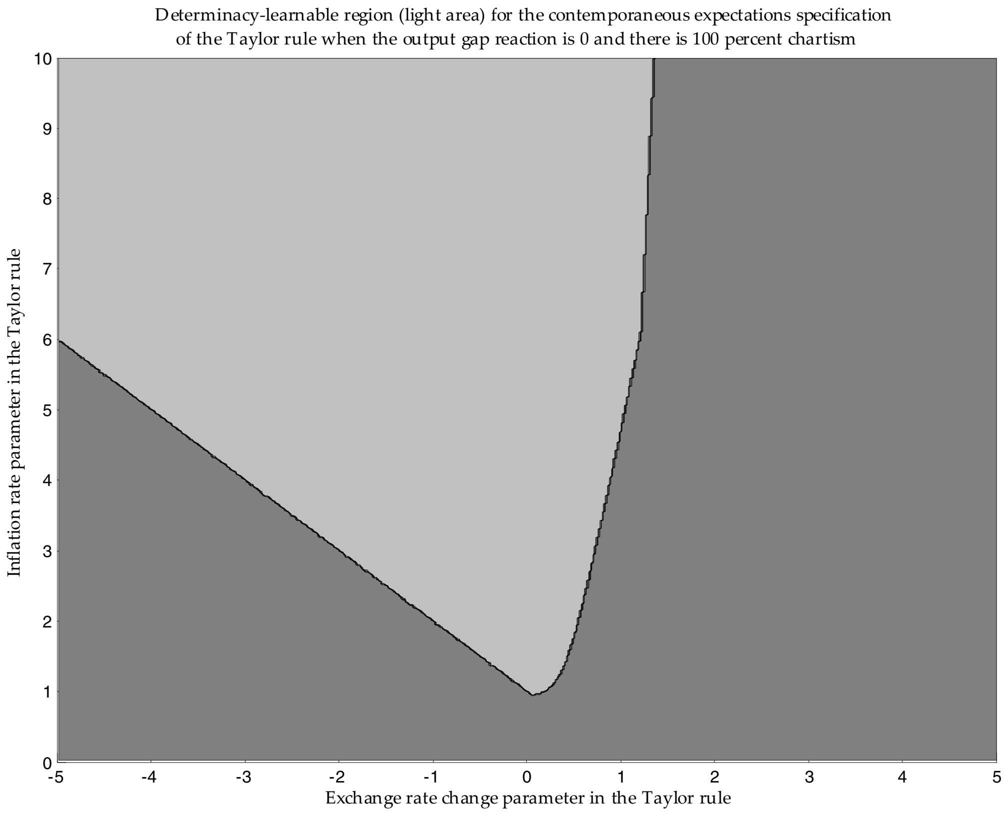

Figure 9

To be more precise, the learnability regions are exactly the same for both types of rules when $\zeta_x > 0$, but slightly different when $\zeta_x = 0$. In the latter case, the regions are slightly smaller when contemporaneous expectations of the variables are used in the interest rate rule than when contemporaneous data are used in the rule (eg, compare *Figure 4a* and *Figure 8* as well as *Figure 5a* and *Figure 9*). Since the determinacy regions for both types of rules are exactly the same, this means that there are parametrizations of the contemporaneous expectations specification of the interest rate rule that give rise to a unique REE that is not adaptively learnable. Note that a factor in common for most of these parametrizations is that $\zeta_e > 0$.

Sensitivity analysis When increasing the index of openness of the economy from $\delta = 0.2$ to $\delta = 0.4$, none of the findings are affected. For both parameter settings, this means that when at least half of the trade in the currency market is driven by technical analysis, the learnability region increases when the monetary authority reacts stronger to the output gap, whereas the opposite is true when less than half of the trade is driven by chartism. Moreover, given the output gap reaction in the Taylor rule, the learnability region gets larger when the proportion of chartists in currency trade increases. Finally, when $\zeta_x = 0$, the learnability region is smaller when the economy is more open.

3.3 Robust and desirable Taylor rules

As already discussed in Section 1, our paper fills the gap between papers that derive optimal policy rules in the new Keynesian framework and papers that focus on determinacy and adaptive learnability of the REE in the same framework. For this task, we use a loss-function as our metric that takes the expected CPI inflation rate and the conditional volatility of the CPI inflation rate in equilibrium as arguments.

Moreover, to find robust parametrizations of the Taylor rules, the properties of a rule should be relatively unaffected by the degree of technical trading in the foreign exchange market. That is, the rule should give rise to determinacy, adaptive learnability of the REE, and a desirable outcome according to the loss-function for most proportions of chartism in currency trade.

3.3.1 Contemporaneous data in the Taylor rule

Starting with the contemporaneous data specification of the interest rate rule in (2.9), the expected CPI inflation rate in equilibrium is, according to the MSV solution in (3.11)

$$E_t(\pi_t) = \left[(\mathbf{I} - \mathbf{\Gamma})^{-1} \cdot \mathbf{\Xi}^{-1} \cdot \mathbf{\Pi} + (\mathbf{I} - \mathbf{\Gamma} \cdot \rho)^{-1} \cdot \mathbf{\Xi}^{-1} \cdot \mathbf{\Upsilon} \cdot E_t(\overline{rr}_t) \right]_{(2)}, \quad (3.29)$$

and the conditional volatility of the CPI inflation rate in equilibrium is

$$var_t(\pi_t) = \left[\left((\mathbf{I} - \mathbf{\Gamma} \cdot \rho)^{-1} \cdot \mathbf{\Xi}^{-1} \cdot \mathbf{\Upsilon} \right) \cdot \left((\mathbf{I} - \mathbf{\Gamma} \cdot \rho)^{-1} \cdot \mathbf{\Xi}^{-1} \cdot \mathbf{\Upsilon} \right)' \right]_{(2,2)} \cdot \sigma_\varepsilon^2, \quad (3.30)$$

where (2) and $(2,2)$ refer to the second element in the vector and the second element along the diagonal in the matrix, respectively. Thereafter, substitute the assumed values of the structural parameters and the exogenous variables in the model into the expressions in (3.29)–(3.30). Thus, we make use of the parameter values in (3.3) in the evaluation of the model's outcome in equilibrium.

When it comes to the exogenous variables in the model, these are π_t^*, $\pi_{t+1}^{e,*}$ and r_t^*. In addition, we treat $\overline{rr}_t$ as an exogenous variable. To make the analysis as simple as possible, we set $\pi_t^* = \pi_{t+1}^{e,*} = 0$. Moreover, when the variables in the Taylor rule are at their target values, i.e., $x_t = \pi_t = \Delta e_t = 0$ in (2.9), and the economy is in a stationary equilibrium, the domestic interest rate is equal to the foreign interest rate due to the parity condition in (2.4) that holds at the international asset market. Thus, the constant ζ_c in the Taylor rule must be equal to the foreign interest rate. Finally, we set $E_t(\overline{rr}_t) = r_t^*$, because the natural rate of interest is, in a stationary equilibrium, equal to the nominal interest rate due to the first equation in (2.1), which, in turn, is equal to the foreign interest rate due to the aforementioned parity condition.

The loss-function that we use as our metric to evaluate the desirability of a specific REE is formulated as follows

$$L = H(|E_t(\pi_t)| - 0.01) + H\left(var_t(\pi_t) - 0.2\sigma_\varepsilon^2\right), \quad (3.31)$$

where $H(\cdot)$ is the Heaviside step function.[9] Thus, the loss-function in (3.31) is minimized and equal to 0 when the expected CPI inflation rate is within $\pm$ 0.01, and the conditional volatility of the CPI inflation rate is at most $0.2\sigma_{\varepsilon}^{2}$. The motivation of the limits for a desirable inflation rate is that they are typical in established inflation rate targeting regimes, whereas the choice of the limit for the variability of the inflation rate is somewhat arbitrary. As an example of an established inflation rate targeting regime, the Swedish Riksbank has defined price-stability as an increase in the CPI of two per cent, but with a tolerance margin of plus/minus one percentage point around this target.

Typically, the loss-function in the optimal monetary policy literature may take the following form

$$L = E_t \left(\sum_{i=0}^{\infty} \beta^i \left(\pi_{t+i}^2 + \eta x_{t+i}^2 \right) \right), \tag{3.32}$$

where η is the relative weight placed by the monetary authority on the output gap target. This type of loss-function is often called flexible inflation rate targeting (see Svensson, 1999), where $\eta = 0$ is strict inflation rate targeting, but it can also be viewed as a quadratic approximation of the welfare function of a representative household (see Woodford, 2003).

Since we, in this paper, disregard from reputational matters in monetary policy, we do not take into account the future paths for the inflation rate and the output gap when evaluating the outcome of monetary policy. Moreover, we are interested in strict inflation rate targeting. Thus, our loss-function in (3.31) differs from the loss-function $L = E_t(\pi_t^2)$ in two respects: (i) we care about the two first moments of the inflation rate; and (ii) in an inflation rate targeting regime, we do not discriminate between an inflation rate at the middle of the band and an inflation rate that is within the band but not at the middle. Of course, we could formulate the loss-function in (3.31) in more general terms.

Needless to say, the interest rate set by the monetary authority must be non-negative. Moreover, we are searching for robust and desirable parametrizations of the Taylor rule in the sense that the desirable properties of the rule should be relatively unaffected by the degree of technical trading in the foreign exchange market, which implies that a robust parametrization of the rule should satisfy $L = 0$ for a range of values of ω. This is because the exact proportion of chartists in currency trade is not known when conducting monetary policy.

In *Tables 1a–c*, the degree of technical trading in the foreign exchange market is 25, 35, 45, 55, 65, 75, 85 and 95 per cent, and the interest rate abroad is 0.01, 0.02 and 0.03, respectively. Moreover, we are performing a grid search for desirable parametrizations of the Taylor rule in which the parameter values in the rule are whole numbers. (Throughout this section, the choices of sets of ω in the grid search are to avoid matrices that are singular.)

[9] The Heaviside step function has the following property: $H(x) = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases}$.

Table 1a

The Taylor rules below satisfy the criteria, in terms of the expected CPI inflation rate in equilibrium and the conditional volatility of the CPI inflation rate in equilibrium, to be desirable rules. Moreover, the rules are associated with a unique equilibrium that is adaptively learnable in least squares sense, and the nominal interest rate in equilibrium is non-negative.

The nominal interest rate abroad and the expected natural rate of interest are both equal to 0.01. Consequently, the constant in the Taylor rule is also 0.01. The index of openness of the economy is 0.2.

Expected CPI inflation rate:	within +/- 0.01
Conditional volatility of CPI inflation rate:	at most 0.2 times the conditional volatility of the stochastic process governing the natural rate of interest
Unique and learnable equilibrium:	when 25, 35, 45, 55, 65, 75, 85 and 95 per cent chartism in currency trade

The parameter values in the rules are whole numbers.

Parameter values in the Taylor rule	Nominal interest rate	Expected CPI inflation rate	Conditional volatility of CPI inflation rate
(output gap, inflation rate, exchange rate change, per cent chartism)			
(0, 8, -2, 95)	0.0014	-0.0032	0.0262
(0, 8, -2, 25)	0.0020	-0.0019	0.0861
(0, 9, -2, 95)	0.0025	-0.0025	0.0232
(0, 9, -2, 25)	0.0033	-0.0011	0.0838
(0, 10, -3, 95)	7.5059e-4	-0.0037	0.0280
(0, 10, -3, 25)	9.9770e-4	-0.0024	0.0881
(0, 10, -3, 15)[1]	0.0011	-0.0019	0.1156
(0, 10, -4, 95)	7.5506e-4	-0.0067	0.0064
(0, 10, -4, 25)	0.0010	-0.0047	0.0785

[1] The Taylor rule is also desirable when 15 per cent chartism in currency trade.

Table 1b

The Taylor rules below satisfy the criteria, in terms of the expected CPI inflation rate in equilibrium and the conditional volatility of the CPI inflation rate in equilibrium, to be desirable rules. Moreover, the rules are associated with a unique equilibrium that is adaptively learnable in least squares sense, and the nominal interest rate in equilibrium is non-negative.

The nominal interest rate abroad and the expected natural rate of interest are both equal to 0.02. Consequently, the constant in the Taylor rule is also 0.02. The index of openness of the economy is 0.2.

Expected CPI inflation rate:	within +/- 0.01
Conditional volatility of CPI inflation rate:	at most 0.2 times the conditional volatility of the stochastic process governing the natural rate of interest
Unique and learnable equilibrium:	when 25, 35, 45, 55, 65, 75, 85 and 95 per cent chartism in currency trade

The parameter values in the rules are whole numbers.

Parameter values in the Taylor rule	Nominal interest rate	Expected CPI inflation rate	Conditional volatility of CPI inflation rate
(output gap, inflation rate, exchange rate change, per cent chartism)			
(0, 8, -2, 95)	0.0028	-0.0064	0.0262
(0, 8, -2, 25)	0.0040	-0.0037	0.0861
(0, 9, -2, 95)	0.0049	-0.0050	0.0232
(0, 9, -2, 25)	0.0066	-0.0022	0.0838
(0, 10, -3, 95)	0.0015	-0.0073	0.0280
(0, 10, -3, 25)	0.0020	-0.0047	0.0881
(0, 10, -3, 15)[1]	0.0022	-0.0039	0.1156

[1] The Taylor rule is also desirable when 15 per cent chartism in currency trade.

Table 1c

The Taylor rules below satisfy the criteria, in terms of the expected CPI inflation rate in equilibrium and the conditional volatility of the CPI inflation rate in equilibrium, to be desirable rules. Moreover, the rules are associated with a unique equilibrium that is adaptively learnable in least squares sense, and the nominal interest rate in equilibrium is non-negative.

The nominal interest rate abroad and the expected natural rate of interest are both equal to 0.03. Consequently, the constant in the Taylor rule is also 0.03. The index of openness of the economy is 0.2.

Expected CPI inflation rate:	within +/- 0.01
Conditional volatility of CPI inflation rate:	at most 0.2 times the conditional volatility of the stochastic process governing the natural rate of interest
Unique and learnable equilibrium:	when 25, 35, 45, 55, 65, 75, 85 and 95 per cent chartism in currency trade

The parameter values in the rules are whole numbers.

Parameter values in the Taylor rule	Nominal interest rate	Expected CPI inflation rate	Conditional volatility of CPI inflation rate
(output gap, inflation rate, exchange rate change, per cent chartism)			
(0, 8, -2, 95)	0.0043	-0.0095	0.0262
(0, 8, -2, 25)	0.0060	-0.0056	0.0861
(0, 9, -2, 95)	0.0074	-0.0074	0.0232
(0, 9, -2, 25)	0.0099	-0.0033	0.0838

Two results are found in the tables: (i) the monetary authority should increase (decrease) the interest rate when the CPI inflation rate increases (decreases) and when the currency gets stronger (weaker), but not care about the output gap, to have a desirable rule that also is robust; and (ii) the number of interest rate rules with these properties decreases with increases in the foreign interest rate.

In *Tables 2a–c*, we have repeated the same procedure with the exception that the parameter ζ_e in the Taylor rule is restricted to 0. As a consequence, the proportion of chartists in currency trade is limited to 55, 65, 75, 85 and 95 per cent since there are no desirable parametrizations of the rule when the proportion is 45 per cent or lower, having restricted the parameter values in the interest rate rule to whole numbers.

As before, the number of desirable interest rate rules that are robust decreases with increases in the foreign interest rate. Further on, which is true irrespective of whether the parameter ζ_e in the Taylor rule is restricted to 0 or not, the monetary authority should react strongly to changes in the inflation rate to have an outcome that is desirable in terms of the expected inflation rate and the conditional volatility of the inflation rate in equilibrium.

In *Tables 3a–c*, the grid search for desirable Taylor rules has been refined in the sense that the parameter values in the rules are multiples of 0.1. This also means that we restrict the presentation to the share of rules that satisfy $L = 0$ for different sets of ω.

Three results are found in the tables: (i) the number of desirable rules decreases when the range of values of ω increases, and irrespective of whether the value of ζ_e is restricted to 0 or not; (ii) the number of robust and desirable rules decreases with increases in the foreign interest rate, and also irrespective of the value of ζ_e (as also noted above); and (iii) the number of rules that give rise to a unique and adaptively learnable REE, but not restricted to $L = 0$, decreases when the range of values of ω increases. Of course, that the share of rules that are associated with determinacy and learnability is not affected by the foreign interest rate is not surprising since this variable is not part of the coefficient matrix in (3.2).

But, then, which parametrization of the Taylor rule is the best rule? When the interest rate abroad is 0.01 and 0.02, respectively, it is the same 16 rules that satisfy $L = 0$ for the set of ω that includes 5, 15, 25, 35, 45, 55, 65, 75, 85 and 95 per cent chartism in currency trade. Of these 16 parametrizations of the interest rate rule, the following rule is desirable down to 2 percent technical trading in the foreign exchange market

$$r_t = r_t^* + 9.9\pi_t - 3.2\Delta e_t. \tag{3.33}$$

The Taylor rule in (3.33) fails to be desirable when the interest rate abroad is 0.03, and this is because the expected inflation rate is not within $\pm$ 0.01 when a large proportion of currency trade is driven by technical analysis. Concerning the other 15 parametrizations of the interest rate rule, the parameters belong to the sets $\zeta_x \in [0, 0.1]$, $\zeta_\pi \in [9.2, 10]$ and $\zeta_e \in [-3.4, -2.9]$. Thus, in principle, the monetary authority should not care about the output gap when setting the interest rate since the value of the currency is a better substitute.

Table 2a

The Taylor rules below satisfy the criteria, in terms of the expected CPI inflation rate in equilibrium and the conditional volatility of the CPI inflation rate in equilibrium, to be desirable rules. Moreover, the rules are associated with a unique equilibrium that is adaptively learnable in least squares sense, and the nominal interest rate in equilibrium is non-negative.

The nominal interest rate abroad and the expected natural rate of interest are both equal to 0.01. Consequently, the constant in the Taylor rule is also 0.01. The index of openness of the economy is 0.2.

Expected CPI inflation rate:	within +/- 0.01
Conditional volatility of CPI inflation rate:	at most 0.2 times the conditional volatility of the stochastic process governing the natural rate of interest
Unique and learnable equilibrium:	when 55, 65, 75, 85 and 95 per cent chartism in currency trade[1]

The parameter values in the rules are whole numbers. Finally, since there are 37 rules that satisfy the criteria, only the 10 best rules according to the conditional volatility of the CPI inflation rate are listed.

Parameter values in the Taylor rule	Nominal interest rate	Expected CPI inflation rate	Conditional volatility of CPI inflation rate
(output gap, inflation rate, exchange rate change, per cent chartism)			
(1, 8, 0, 95)	0.0085	-9.2669e-4	6.9865e-4
(1, 8, 0, 55)	0.0092	-1.0997e-4	0.0117
(1, 9, 0, 95)	0.0086	-7.9084e-4	6.6905e-4
(1, 9, 0, 55)	0.0094	1.4530e-5	0.0113
(2, 8, 0, 95)	0.0086	-0.0010	3.8413e-4
(2, 8, 0, 55)	0.0092	-1.5080e-4	0.0117
(2, 9, 0, 95)	0.0087	-9.0074e-4	3.7480e-4
(2, 9, 0, 55)	0.0093	-2.0571e-5	0.0115
(2, 10, 0, 95)	0.0088	-7.9179e-4	3.6582e-4
(2, 10, 0, 55)	0.0095	8.1409e-5	0.0113
(3, 8, 0, 95)	0.0086	-0.0011	2.8590e-4
(3, 8, 0, 55)	0.0091	-1.8865e-4	0.0118
(3, 9, 0, 95)	0.0088	-9.6749e-4	2.8104e-4
(3, 9, 0, 55)	0.0093	-5.7619e-5	0.0116
(3, 10, 0, 95)	0.0089	-8.5817e-4	2.7630e-4
(3, 10, 0, 55)	0.0094	4.6640e-5	0.0115
(4, 9, 0, 95)	0.0087	-0.0010	2.3618e-4
(4, 9, 0, 55)	0.0092	-9.2913e-5	0.0117
(4, 10, 0, 95)	0.0089	-9.0982e-4	2.3311e-4
(4, 10, 0, 55)	0.0094	1.2024e-5	0.0115

[1] None of the Taylor rules below are desirable when 45 per cent chartism in currency trade or lower.

Table 2b

The Taylor rules below satisfy the criteria, in terms of the expected CPI inflation rate in equilibrium and the conditional volatility of the CPI inflation rate in equilibrium, to be desirable rules. Moreover, the rules are associated with a unique equilibrium that is adaptively learnable in least squares sense, and the nominal interest rate in equilibrium is non-negative.

The nominal interest rate abroad and the expected natural rate of interest are both equal to 0.02. Consequently, the constant in the Taylor rule is also 0.02. The index of openness of the economy is 0.2.

Expected CPI inflation rate:	within +/- 0.01
Conditional volatility of CPI inflation rate:	at most 0.2 times the conditional volatility of the stochastic process governing the natural rate of interest
Unique and learnable equilibrium:	when 55, 65, 75, 85 and 95 per cent chartism in currency trade[1]

The parameter values in the rules are whole numbers. Finally, since there are 33 rules that satisfy the criteria, only the 10 best rules according to the conditional volatility of the CPI inflation rate are listed.

Parameter values in the Taylor rule	Nominal interest rate	Expected CPI inflation rate	Conditional volatility of CPI inflation rate
(output gap, inflation rate, exchange rate change, per cent chartism)			
(1, 8, 0, 95)	0.0169	-0.0019	6.9865e-4
(1, 8, 0, 55)	0.0184	-2.1994e-4	0.0117
(1, 9, 0, 95)	0.0172	-0.0016	6.6905e-4
(1, 9, 0, 55)	0.0188	2.9060e-5	0.0113
(2, 8, 0, 95)	0.0172	-0.0021	3.8413e-4
(2, 8, 0, 55)	0.0183	-3.0160e-4	0.0117
(2, 9, 0, 95)	0.0175	-0.0018	3.7480e-4
(2, 9, 0, 55)	0.0187	-4.1142e-5	0.0115
(2, 10, 0, 95)	0.0177	-0.0016	3.6582e-4
(2, 10, 0, 55)	0.0190	1.6282e-4	0.0113
(3, 8, 0, 95)	0.0172	-0.0022	2.8590e-4
(3, 8, 0, 55)	0.0182	-3.7729e-4	0.0118
(3, 9, 0, 95)	0.0175	-0.0019	2.8104e-4
(3, 9, 0, 55)	0.0186	-1.1524e-4	0.0116
(3, 10, 0, 95)	0.0177	-0.0017	2.7630e-4
(3, 10, 0, 55)	0.0188	9.3280e-5	0.0115
(4, 9, 0, 95)	0.0175	-0.0020	2.3618e-4
(4, 9, 0, 55)	0.0185	-1.8583e-4	0.0117
(4, 10, 0, 95)	0.0177	-0.0018	2.3311e-4
(4, 10, 0, 55)	0.0187	2.4048e-5	0.0115

[1] None of the Taylor rules below are desirable when 45 per cent chartism in currency trade or lower.

Table 2c

The Taylor rules below satisfy the criteria, in terms of the expected CPI inflation rate in equilibrium and the conditional volatility of the CPI inflation rate in equilibrium, to be desirable rules. Moreover, the rules are associated with a unique equilibrium that is adaptively learnable in least squares sense, and the nominal interest rate in equilibrium is non-negative.

The nominal interest rate abroad and the expected natural rate of interest are both equal to 0.03. Consequently, the constant in the Taylor rule is also 0.03. The index of openness of the economy is 0.2.

Expected CPI inflation rate:	within +/- 0.01
Conditional volatility of CPI inflation rate:	at most 0.2 times the conditional volatility of the stochastic process governing the natural rate of interest
Unique and learnable equilibrium:	when 55, 65, 75, 85 and 95 per cent chartism in currency trade[1]

The parameter values in the rules are whole numbers. Finally, since there are 30 rules that satisfy the criteria, only the 10 best rules according to the conditional volatility of the CPI inflation rate are listed.

Parameter values in the Taylor rule	Nominal interest rate	Expected CPI inflation rate	Conditional volatility of CPI inflation rate
(output gap, inflation rate, exchange rate change, per cent chartism)			
(1, 8, 0, 95)	0.0254	-0.0028	6.9865e-4
(1, 8, 0, 55)	0.0276	-3.2991e-4	0.0117
(1, 9, 0, 95)	0.0258	-0.0024	6.6905e-4
(1, 9, 0, 55)	0.0282	4.3591e-5	0.0113
(2, 8, 0, 95)	0.0257	-0.0031	3.8413e-4
(2, 8, 0, 55)	0.0275	-4.5240e-4	0.0117
(2, 9, 0, 95)	0.0262	-0.0027	3.7480e-4
(2, 9, 0, 55)	0.0280	-6.1713e-5	0.0115
(2, 10, 0, 95)	0.0265	-0.0024	3.6582e-4
(2, 10, 0, 55)	0.0284	2.4423e-4	0.0113
(3, 8, 0, 95)	0.0258	-0.0033	2.8590e-4
(3, 8, 0, 55)	0.0273	-5.6594e-4	0.0118
(3, 9, 0, 95)	0.0263	-0.0029	2.8104e-4
(3, 9, 0, 55)	0.0278	-1.7286e-4	0.0116
(3, 10, 0, 95)	0.0266	-0.0026	2.7630e-4
(3, 10, 0, 55)	0.0282	1.3992e-4	0.0115
(4, 9, 0, 95)	0.0262	-0.0031	2.3618e-4
(4, 9, 0, 55)	0.0277	-2.7874e-4	0.0117
(4, 10, 0, 95)	0.0266	-0.0027	2.3311e-4
(4, 10, 0, 55)	0.0281	3.6072e-5	0.0115

[1] None of the Taylor rules below are desirable when 45 per cent chartism in currency trade or lower.

Table 3a

The number of Taylor rules that satisfy the criteria, in terms of the expected CPI inflation rate in equilibrium and the conditional volatility of the CPI inflation rate in equilibrium, to be desirable rules. Moreover, the rules are associated with a unique equilibrium that is adaptively learnable in least squares sense, and the nominal interest rate in equilibrium is non-negative.

The nominal interest rate abroad and the expected natural rate of interest are both equal to 0.01. Consequently, the constant in the Taylor rule is also 0.01. The index of openness of the economy is 0.2.

Expected CPI inflation rate:	within +/- 0.01
Conditional volatility of CPI inflation rate:	at most 0.2 times the conditional volatility of the stochastic process governing the natural rate of interest

The parameter values in the rules are multiples of 0.1.

Unique and learnable equilibrium:	when 5, 15, 25, 35, 45, 55, 65, 75, 85 and 95 per cent chartism in currency trade
Number of rules with a unique and learnable equilibrium:	7 122 out of 418 241, ie, 1.7028 per cent of the rules
Number of desirable rules:	16 out of 418 241, ie, 0.0038 per cent of the rules
Number of desirable rules when no exchange rate change reaction:	0 out of 4 141

Unique and learnable equilibrium:	when 15, 25, 35, 45, 55, 65, 75, 85 and 95 percent chartism in currency trade
Number of rules with a unique and learnable equilibrium:	12 790 out of 418 241, ie, 3.0580 per cent of the rules
Number of desirable rules:	404 out of 418 241, ie, 0.0966 per cent of the rules
Number of desirable rules when no exchange rate change reaction:	0 out of 4 141

Unique and learnable equilibrium:	when 25, 35, 45, 55, 65, 75, 85 and 95 per cent chartism in currency trade
Number of rules with a unique and learnable equilibrium:	25 591 out of 418 241, ie, 6.1187 per cent of the rules
Number of desirable rules:	2 482 out of 418 241, ie, 0.5934 per cent of the rules
Number of desirable rules when no exchange rate change reaction:	0 out of 4 141

Unique and learnable equilibrium:	when 35, 45, 55, 65, 75, 85 and 95 per cent chartism in currency trade
Number of rules with a unique and learnable equilibrium:	65 085 out of 418 241, ie, 15.5616 per cent of the rules
Number of desirable rules:	14 669 out of 418 241, ie, 3.5073 per cent of the rules
Number of desirable rules when no exchange rate change reaction:	0 out of 4 141

Unique and learnable equilibrium:	when 45, 55, 65, 75, 85 and 95 per cent chartism in currency trade
Number of rules with a unique and learnable equilibrium:	128 699 out of 418 241, ie, 30.7715 per cent of the rules
Number of desirable rules:	60 574 out of 418 241, ie, 14.4830 per cent of the rules
Number of desirable rules when no exchange rate change reaction:	1 out of 4 141, ie, 0.0241 per cent of the rules

Unique and learnable equilibrium:	when 55, 65, 75, 85 and 95 percent chartism in currency trade
Number of rules with a unique and learnable equilibrium:	199 165 out of 418 241, ie, 47.6197 percent of the rules
Number of desirable rules:	129 365 out of 418 241, ie, 30.9307 percent of the rules
Number of desirable rules when no exchange rate change reaction:	3 152 out of 4 141, ie, 76.1169 percent of the rules

Table 3b

The number of Taylor rules that satisfy the criteria, in terms of the expected CPI inflation rate in equilibrium and the conditional volatility of the CPI inflation rate in equilibrium, to be desirable rules. Moreover, the rules are associated with a unique equilibrium that is adaptively learnable in least squares sense, and the nominal interest rate in equilibrium is non-negative.

The nominal interest rate abroad and the expected natural rate of interest are both equal to 0.02. Consequently, the constant in the Taylor rule is also 0.02. The index of openness of the economy is 0.2.

Expected CPI inflation rate:	within +/- 0.01
Conditional volatility of CPI inflation rate:	at most 0.2 times the conditional volatility of the stochastic process governing the natural rate of interest

The parameter values in the rules are multiples of 0.1.

Unique and learnable equilibrium:	when 5, 15, 25, 35, 45, 55, 65, 75, 85 and 95 per cent chartism in currency trade
Number of rules with a unique and learnable equilibrium:	7 122 out of 418 241, ie, 1.7028 per cent of the rules
Number of desirable rules:	16 out of 418 241, ie, 0.0038 per cent of the rules
Number of desirable rules when no exchange rate change reaction:	0 out of 4 141

Unique and learnable equilibrium:	when 15, 25, 35, 45, 55, 65, 75, 85 and 95 percent chartism in currency trade
Number of rules with a unique and learnable equilibrium:	12 790 out of 418 241, ie, 3.0580 per cent of the rules
Number of desirable rules:	241 out of 418 241, ie, 0.0576 per cent of the rules
Number of desirable rules when no exchange rate change reaction:	0 out of 4 141

Unique and learnable equilibrium:	when 25, 35, 45, 55, 65, 75, 85 and 95 per cent chartism in currency trade
Number of rules with a unique and learnable equilibrium:	25 591 out of 418 241, ie, 6.1187 per cent of the rules
Number of desirable rules:	1 034 out of 418 241, ie, 0.2472 per cent of the rules
Number of desirable rules when no exchange rate change reaction:	0 out of 4 141

Unique and learnable equilibrium:	when 35, 45, 55, 65, 75, 85 and 95 per cent chartism in currency trade
Number of rules with a unique and learnable equilibrium:	65 085 out of 418 241, ie, 15.5616 per cent of the rules
Number of desirable rules:	4 234 out of 418 241, ie, 1.0123 per cent of the rules
Number of desirable rules when no exchange rate change reaction:	0 out of 4 141

Unique and learnable equilibrium:	when 45, 55, 65, 75, 85 and 95 per cent chartism in currency trade
Number of rules with a unique and learnable equilibrium:	128 699 out of 418 241, ie, 30.7715 per cent of the rules
Number of desirable rules:	32 809 out of 418 241, ie, 7.8445 per cent of the rules
Number of desirable rules when no exchange rate change reaction:	0 out of 4 141

Unique and learnable equilibrium:	when 55, 65, 75, 85 and 95 per cent chartism in currency trade
Number of rules with a unique and learnable equilibrium:	199 165 out of 418 241, ie, 47.6197 per cent of the rules
Number of desirable rules:	99 213 out of 418 241, ie, 23.7215 per cent of the rules
Number of desirable rules when no exchange rate change reaction:	2 819 out of 4 141, ie, 68.0753 per cent of the rules

Table 3c

The number of Taylor rules that satisfy the criteria, in terms of the expected CPI inflation rate in equilibrium and the conditional volatility of the CPI inflation rate in equilibrium, to be desirable rules. Moreover, the rules are associated with a unique equilibrium that is adaptively learnable in least squares sense, and the nominal interest rate in equilibrium is non-negative.

The nominal interest rate abroad and the expected natural rate of interest are both equal to 0.03. Consequently, the constant in the Taylor rule is also 0.03. The index of openness of the economy is 0.2.

Expected CPI inflation rate:	within +/- 0.01
Conditional volatility of CPI inflation rate:	at most 0.2 times the conditional volatility of the stochastic process governing the natural rate of interest

The parameter values in the rules are multiples of 0.1.

Unique and learnable equilibrium:	when 5, 15, 25, 35, 45, 55, 65, 75, 85 and 95 per cent chartism in currency trade
Number of rules with a unique and learnable equilibrium:	7 122 out of 418 241, ie, 1.7028 per cent of the rules
Number of desirable rules:	0 out of 418 241
Number of desirable rules when no exchange rate change reaction:	0 out of 4 141

Unique and learnable equilibrium:	when 15, 25, 35, 45, 55, 65, 75, 85 and 95 percent chartism in currency trade
Number of rules with a unique and learnable equilibrium:	12 790 out of 418 241, ie, 3.0580 per cent of the rules
Number of desirable rules:	16 out of 418 241, ie, 0.0038 per cent of the rules
Number of desirable rules when no exchange rate change reaction:	0 out of 4 141

Unique and learnable equilibrium:	when 25, 35, 45, 55, 65, 75, 85 and 95 per cent chartism in currency trade
Number of rules with a unique and learnable equilibrium:	25 591 out of 418 241, ie, 6.1187 per cent of the rules
Number of desirable rules:	175 out of 418 241, ie, 0.0418 per cent of the rules
Number of desirable rules when no exchange rate change reaction:	0 out of 4 141

Unique and learnable equilibrium:	when 35, 45, 55, 65, 75, 85 and 95 per cent chartism in currency trade
Number of rules with a unique and learnable equilibrium:	65 085 out of 418 241, ie, 15.5616 per cent of the rules
Number of desirable rules:	949 out of 418 241, ie, 0.2269 per cent of the rules
Number of desirable rules when no exchange rate change reaction:	0 out of 4 141

Unique and learnable equilibrium:	when 45, 55, 65, 75, 85 and 95 per cent chartism in currency trade
Number of rules with a unique and learnable equilibrium:	128 699 out of 418 241, ie, 30.7715 per cent of the rules
Number of desirable rules:	14 528 out of 418 241, ie, 3.4736 per cent of the rules
Number of desirable rules when no exchange rate change reaction:	0 out of 4 141

Unique and learnable equilibrium:	when 55, 65, 75, 85 and 95 per cent chartism in currency trade
Number of rules with a unique and learnable equilibrium:	199 165 out of 418 241, ie, 47.6197 per cent of the rules
Number of desirable rules:	77 188 out of 418 241, ie, 18.4554 per cent of the rules
Number of desirable rules when no exchange rate change reaction:	2 481 out of 4 141, ie, 59.9131 per cent of the rules

Sensitivity analysis We have again performed a sensitivity analysis of the numerical findings in which the index of openness of the economy has been increased from $\delta = 0.2$ to $\delta = 0.4$. Basically, the findings that we reported when this index was equal to 0.2 are not affected. However, in comparison, the share of rules that give rise to a unique and adaptively learnable REE is larger when the smallest proportions of chartism in currency trade is excluded in the grid search, whereas the opposite is true when the smallest proportions of technical trading is included. We also found the same results in our search for desirable parametrizations of the Taylor rule.

3.3.2 Contemporaneous expectations in the Taylor rule

Continuing with the contemporaneous expectations specification of the interest rate rule in (2.10), the expected CPI inflation rate in equilibrium is[10]

$$E_{t-1}(\pi_t) = \left[(\mathbf{I}-\mathbf{\Gamma})^{-1}\cdot\mathbf{\Xi}^{-1}\cdot\mathbf{\Pi}+(\mathbf{I}-\mathbf{\Gamma}\cdot\rho)^{-1}\cdot\mathbf{\Xi}^{-1}\cdot\mathbf{\Upsilon}\cdot E_{t-1}(\overline{rr}_t)\right]_{(2)}, \tag{3.34}$$

and the conditional volatility of the CPI inflation rate in equilibrium is

$$\begin{aligned} & var_{t-1}(\pi_t) \qquad\qquad (3.35) \\ = \; & \rho^2\cdot\left[\left((\mathbf{I}-\mathbf{\Gamma}\cdot\rho)^{-1}\cdot\mathbf{\Xi}^{-1}\cdot\mathbf{\Upsilon}\right)\cdot\left((\mathbf{I}-\mathbf{\Gamma}\cdot\rho)^{-1}\cdot\mathbf{\Xi}^{-1}\cdot\mathbf{\Upsilon}\right)'\right]_{(2,2)}\cdot\sigma_\varepsilon^2 + \\ & \left[\left(\mathbf{\Xi}_0^{-1}\cdot\mathbf{\Upsilon}\right)\cdot\left(\mathbf{\Xi}_0^{-1}\cdot\mathbf{\Upsilon}\right)'\right]_{(2,2)}\cdot\sigma_\varepsilon^2. \end{aligned}$$

Except for a difference in the dating of expectations, the expected CPI inflation rate in equilibrium in (3.34) is the same as in (3.29). This means that it does not matter if contemporaneous data are used in the Taylor rule or if contemporaneous expectations, formed in the previous time period, of the variables are used in the Taylor rule, the expected CPI inflation rate in equilibrium is exactly the same.

Even though the conditional volatility of the CPI inflation rate in equilibrium in (3.35) is not the same as in (3.30), there is a linear relationship between them. To be more precise, the former quantity can be written as $\rho^2\sigma_\pi^2 + \alpha^2\gamma(1-\delta)\sigma_\varepsilon^2 = 0.1225\sigma_\pi^2 + 0.77894\sigma_\varepsilon^2$, where σ_π^2 is the latter quantity, and where we have substituted the parameter values in (3.3) into the expression.[11] This means that the ordering from the best interest rate rule to the worst rule is the same, irrespective of whether the specification of the rule includes contemporaneous data or contemporaneous expectations of the included variables.

However, none of the parametrizations of the Taylor rule are satisfactory from the point of view of the variability of the CPI inflation rate. This is because the conditional volatility of the CPI inflation rate in equilibrium always is larger than (the somewhat arbitrary limit) $0.2\sigma_\varepsilon^2$. Specifically, the variability

[10] See the Appendix for the derivations of (3.34)–(3.35).

[11] See the Appendix for the derivation of this result.

of the CPI inflation rate can never be below $0.77894\sigma_{\varepsilon}^{2}$. Of course, this does not mean that one should never adopt a contemporaneous expectations specification of the Taylor rule in monetary policy-making. The reason is that it may be the case that the contemporaneous data specification of the Taylor rule is not accessible due to data revisions. Consequently, one is forced to use a rule that includes contemporaneous expectations of the variables, which also means that one must accept a higher conditional volatility of the CPI inflation rate in equilibrium.

Sensitivity analysis Obviously, the ordering of the Taylor rules are not affected when the index of openness of the economy increases from $\delta = 0.2$ to $\delta = 0.4$. It is also still true that none of the parametrizations of the Taylor rule are satisfactory from the point of view of the variability of the CPI inflation rate, because this variability can never be below $\alpha^{2}\gamma\left(1-\delta\right)\sigma_{\varepsilon}^{2} = 0.5842\sigma_{\varepsilon}^{2}$.

4 Concluding discussion

We do not repeat our findings in this discussion. Instead, we conclude with a few remarks on the model developed and shortly discuss the claims in Taylor (2001) that the monetary authority's interest rate rule should not include a reaction to an exchange rate index to be favorable, which is in contrast with our finding.

Our model Firstly, a few words about the technical trading technique in (2.6) are in place. It is clear that questionnaire surveys made at currency markets around the world not only confirm that chartism is extensively used in currency trade, but they also confirm that some variant of a moving average technique is the most commonly used technical trading technique. This means that exchange rates in the more distant past also should affect the decision to trade, and not only the exchange rates in time periods t and $t-1$. Thus, a model with bounded memory could be in place (see Honkapohja and Mitra, 2003; for adaptive learning with bounded memory).

In Bask (2006), an asset pricing model for the exchange rate is developed in which the current rate is affected by an exponentially weighted moving average of all past exchange rates. When analyzing the effects of changes in monetary fundamentals, it is clear that the exchange rate in time period $t-1$ has a first-order effect on the current rate, while rates in the more distant past have a second-order effect on the current exchange rate. Encouraged be this finding, we restricted the technique in (2.6) to only include the exchange rates in time periods t and $t-1$. An advantage of this restriction is that the complete model would, otherwise, be too cumbersome to analyze, even numerically. This is because we would have to work with extremely large matrixes when investigating if a certain parametrization of a Taylor rule is associated with a unique, adaptively learnable and desirable inflation rate in equilibrium.

Secondly, we could formulate the interest rate rules in (2.9)–(2.10) in terms of the level of the nominal exchange rate; the actual level of the exchange rate in (2.9), and the mathematically expected level of the exchange rate in (2.10).

However, having in mind that there have been several monetary arrangements throughout history aiming at achieving less variable exchange rates, we stick with the formulations of the interest rate rules in (2.9)–(2.10) and focus on the change in the nominal exchange rate. Of course, it is part of future research to search for robust and desirable parametrizations of the Taylor rules that take current and past levels of the exchange rate as arguments.

Thirdly, the dating of expectations might be important for the findings in this paper. Recall that when contemporaneous data are used in the Taylor rule as in (2.9), the dating of expectations is time period t, whereas when contemporaneous expectations of the variables are used in the Taylor rule as in (2.10), the dating of expectations is time period $t-1$. As was explained in Section 3.2.2, the reason for the latter assumption is that we would like to have an exact correspondence with Bullard and Mitra (2002). For the same reason, one should also investigate the case when the Taylor rule is (2.9) and the dating of expectations is time period $t-1$. In Bullard and Mitra (2002), the findings are not affected by this change of dating of expectations.

Finally, the recursive least squares learning algorithm that is used by the fundamentalists is a decreasing gain algorithm. It would, therefore, be interesting to complement the analysis in this paper with the case in which the learning algorithm is a constant gain algorithm, especially when the fundamentalists, including the monetary authority, is using a PLM of the economy that does not include any REE. This is because it might open up for so-called escape dynamics in the inflation rate from a self-confirming equilibrium (see, eg, Cho et al, 2002, and Williams, 2004, for an introduction to this recent literature, Bullard and Cho, 2005, for an example of escape dynamics in a closed economy like the one in Bullard and Mitra, 2002), and Milani (2006) for an example in which the constant gain parameter is estimated).

Taylor's (2001) claim he vigilant reader might object that we, in this paper, are not really meeting the claim in Taylor (2001). This is because we investigate the properties of the model developed using specifications of the Taylor rule that include the change in the nominal exchange rate, while Taylor (2001) is discussing interest rate rules that include the current and past levels of the real exchange rate. To be more specific, Taylor (2001) is discussing the following rule

$$r_t = \zeta_c + \zeta_x x_t + \zeta_\pi \pi_t + \zeta_q q_t + \zeta_{q'} q_{t-1}, \tag{4.1}$$

where q_t is the real exchange rate.

In fact, the investigation in this paper is adequate, and there are two reasons for this. Firstly, by assuming that $\zeta_{q'} = -\zeta_q$, we turn our focus from levels of the real exchange rate to the change in the real exchange rate. This also means, since the real exchange rate is $q_t = e_t + p_t^* - p_t$, where p_t is the CPI, that the Taylor rule in (4.1) can be written as follows:

$$r_t = \zeta_c + \zeta_x x_t + \left(\zeta_\pi - \zeta_q\right) \pi_t + \zeta_q \Delta e_t + \zeta_q \pi_t^*. \tag{4.2}$$

Secondly, since we assume that $\pi_t^* = 0$ in the numerical analysis, the interest rate rule in (4.2) is, in principle, exactly the same as the rule in (2.9). Note that the assumption $\zeta_{q'} = -\zeta_q$ is necessary to transform the Taylor rule in (4.1) to the rule in (4.2).

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Appendix

Derivation of (2.3)

Firstly, shift the first equation in (2.2) one time period forward in time, and rearrange terms

$$\pi^e_{H,t+1} = \pi^e_{t+1} - \delta \Delta s^e_{t+1}. \tag{A.1}$$

Secondly, shift the second equation in (2.2) one time period forward in time, and take differences

$$\begin{aligned} \Delta s^e_{t+1} &= \Delta e^{e,m}_{t+1} + \Delta p^{e,*}_{t+1} - \Delta p^e_{H,t+1} \\ &= \Delta e^{e,m}_{t+1} + \pi^{e,*}_{t+1} - \pi^e_{H,t+1}. \end{aligned} \tag{A.2}$$

Thirdly, substitute (A.2) into (A.1), and solve for $\pi^e_{H,t+1}$

$$\pi^e_{H,t+1} = \frac{1}{1-\delta} \cdot \left(\pi^e_{t+1} - \delta \left(\Delta e^{e,m}_{t+1} + \pi^{e,*}_{t+1}\right)\right). \tag{A.3}$$

Fourthly, shift (A.3) one time period backward in time

$$\pi_{H,t} = \frac{1}{1-\delta} \cdot \left(\pi_t - \delta \left(\Delta e_t + \pi^*_t\right)\right). \tag{A.4}$$

Fifthly, substitute (A.3) into the first equation in (2.1), and the first equation in (2.3) is derived. Finally, substitute (A.3)–(A.4) into the second equation in (2.1), solve for π_t, and the second equation in (2.3) is derived.

Derivation of (3.1) Firstly, substitute aggregated expectations at the foreign exchange market in (2.8) and the Taylor rule in (2.9) into the dynamic IS-type equation in (2.3), and rearrange terms

$$\begin{aligned} & (1+\alpha\zeta_x)\, x_t + \alpha\zeta_\pi \pi_t + \alpha \left(\frac{\delta\omega}{1-\delta} + \zeta_e\right) \Delta e_t \\ = \; & x^e_{t+1} + \frac{\alpha}{1-\delta} \cdot \pi^e_{t+1} + \frac{\alpha\delta(\omega-1)}{1-\delta} \cdot \Delta e^e_{t+1} \\ & -\alpha\zeta_c - \frac{\alpha\delta}{1-\delta} \cdot \pi^{e,*}_{t+1} + \alpha \overline{rr}_t. \end{aligned} \tag{A.5}$$

Secondly, substitute aggregated expectations at the foreign exchange market in (2.8) into the new Keynesian Phillips curve in (2.3), and rearrange terms

$$\begin{aligned} & \gamma(\delta-1)\, x_t + \pi_t + \delta(\beta\omega - 1)\, \Delta e_t \\ = \; & \beta\pi^e_{t+1} + \beta\delta(\omega-1)\, \Delta e^e_{t+1} + \delta\pi^*_t - \beta\delta\pi^{e,*}_{t+1}. \end{aligned} \tag{A.6}$$

Thirdly, substitute aggregated expectations at the foreign exchange market in (2.8) and the Taylor rule in (2.9) into the UIP condition in (2.4), and rearrange terms

$$\zeta_x x_t + \zeta_\pi \pi_t + (\zeta_e - \omega)\, \Delta e_t = (1-\omega)\, \Delta e^e_{t+1} - \zeta_c + r^*_t. \tag{A.7}$$

Finally, put (A.5)–(A.7) into matrix form, and (3.1) is derived.

Derivation of (3.4) Firstly, substitute aggregated expectations at the foreign exchange market in (2.8) and the Taylor rule in (2.10) into the dynamic IS-type equation in (2.3), and rearrange terms

$$\begin{aligned} & x_t + \frac{\alpha\delta\omega}{1-\delta} \cdot \Delta e_t \\ = & -\alpha\zeta_x x_t^e - \alpha\zeta_\pi \pi_t^e - \alpha\zeta_e \Delta e_t^{e,f} + \\ & x_{t+1}^e + \frac{\alpha}{1-\delta} \cdot \pi_{t+1}^e + \frac{\alpha\delta(\omega-1)}{1-\delta} \cdot \Delta e_{t+1}^e \\ & -\alpha\zeta_c - \frac{\alpha\delta}{1-\delta} \cdot \pi_{t+1}^{e,*} + \alpha\overline{rr}_t. \end{aligned} \tag{A.8}$$

Secondly, substitute aggregated expectations at the foreign exchange market in (2.8) and the Taylor rule in (2.10) into the UIP condition in (2.4), and rearrange terms

$$-\omega\Delta e_t = -\zeta_x x_t^e - \zeta_\pi \pi_t^e - \zeta_e \Delta e_t^{e,f} + (1-\omega)\Delta e_{t+1}^e - \zeta_c + r_t^*. \tag{A.9}$$

Finally, put (A.6) and (A.8)–(A.9) into matrix form, and (3.4) is derived. Note that (A.6) is unaffected by the type of Taylor rule that is used by the monetary authority.

Derivation of (3.2) when the Taylor rule is (2.10) Substitute (3.5) into (3.4), and note that the coefficient matrix for the vector $[x_t, \pi_t, \Delta e_t]'$ at the left-hand side of (3.4) is now

$$\begin{aligned} & \begin{bmatrix} 1 & 0 & \frac{\alpha\delta\omega}{1-\delta} \\ \gamma(\delta-1) & 1 & \delta(\beta\omega-1) \\ 0 & 0 & -\omega \end{bmatrix} - \\ & \begin{bmatrix} -\alpha\zeta_x & -\alpha\zeta_\pi & -\alpha\zeta_e \\ 0 & 0 & 0 \\ -\zeta_x & -\zeta_\pi & -\zeta_e \end{bmatrix} \\ = & \begin{bmatrix} 1+\alpha\zeta_x & \alpha\zeta_\pi & \alpha\left(\frac{\delta\omega}{1-\delta}+\zeta_e\right) \\ \gamma(\delta-1) & 1 & \delta(\beta\omega-1) \\ \zeta_x & \zeta_\pi & \zeta_e-\omega \end{bmatrix}, \end{aligned} \tag{A.10}$$

which is the same coefficient matrix for the vector $[x_t, \pi_t, \Delta e_t]'$ at the left-hand side of (3.1). Consequently, the relevant coefficient matrix when deriving the conditions for determinacy is (3.2).

Derivations of (3.34)–(3.35) The fix point in (3.26) is the MSV solution of the model in (3.19). Hence

$$\begin{cases} \widehat{\mathbf{\Theta}} = \mathbf{\Xi}_0^{-1} \cdot \left(\mathbf{\Pi} + \mathbf{\Xi}_1 \cdot \widehat{\mathbf{\Theta}} + \mathbf{\Sigma} \cdot \widehat{\mathbf{\Theta}}\right) \\ \widehat{\mathbf{\Lambda}} = \mathbf{\Xi}_0^{-1} \cdot \left(\mathbf{\Xi}_1 \cdot \widehat{\mathbf{\Lambda}} + \mathbf{\Sigma} \cdot \widehat{\mathbf{\Lambda}} \cdot \rho + \mathbf{\Upsilon} \cdot \rho\right) \end{cases} \tag{A.11}$$

which means that the MSV solution is

$$\begin{aligned} \mathbf{y}_t &= \widehat{\mathbf{\Theta}} + \widehat{\mathbf{\Lambda}} \cdot \overline{rr}_{t-1} + \mathbf{\Phi} \cdot \varepsilon_t \\ &= (\mathbf{I} - \mathbf{\Gamma})^{-1} \cdot \mathbf{\Xi}^{-1} \cdot \mathbf{\Pi} + \\ & \quad (\mathbf{I} - \mathbf{\Gamma} \cdot \rho)^{-1} \cdot \mathbf{\Xi}^{-1} \cdot \mathbf{\Upsilon} \cdot \rho\overline{rr}_{t-1} + \mathbf{\Xi}_0^{-1} \cdot \mathbf{\Upsilon} \cdot \varepsilon_t \end{aligned} \tag{A.12}$$

where $\mathbf{\Phi} = \mathbf{\Xi}_0^{-1} \cdot \mathbf{\Upsilon}$ follows from comparing the PLM in (3.20) with the ALM in (3.24) since the PLM of the economy is guided by the structure of the model's MSV solution. Recall that $\mathbf{\Gamma} = \mathbf{\Xi}^{-1} \cdot \mathbf{\Sigma}$ and $\mathbf{\Xi}_0 - \mathbf{\Xi}_1 = \mathbf{\Xi}$. Thereafter, take the conditional expectations and volatility of (A.12), note that $E_{t-1}\left(\rho \overline{rr}_{t-1}\right) = E_{t-1}\left(\overline{rr}_t\right)$ due to the stochastic process for the natural rate of interest in (2.5), and (3.34)–(3.35) follows.

Derivation of the relationship between (3.30) and (3.35) Since

$$
\begin{aligned}
\mathbf{\Xi}_0^{-1} \cdot \mathbf{\Upsilon} &= \begin{bmatrix} 1 & 0 & \frac{\alpha\delta\omega}{1-\delta} \\ \gamma\left(\delta - 1\right) & 1 & \delta\left(\beta\omega - 1\right) \\ 0 & 0 & -\omega \end{bmatrix}^{-1} \begin{bmatrix} \alpha \\ 0 \\ 0 \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0 & \frac{\alpha\delta}{1-\delta} \\ \gamma\left(1 - \delta\right) & 1 & \frac{\delta(\alpha\gamma\omega+\beta\omega-1)}{\omega} \\ 0 & 0 & -\frac{1}{\omega} \end{bmatrix}^{-1} \begin{bmatrix} \alpha \\ 0 \\ 0 \end{bmatrix} \\
&= \begin{bmatrix} \alpha \\ \alpha\gamma\left(1 - \delta\right) \\ 0 \end{bmatrix}
\end{aligned} \tag{A.13}
$$

it follows that

$$
\begin{aligned}
&\left[\left(\mathbf{\Xi}_0^{-1} \cdot \mathbf{\Upsilon}\right) \cdot \left(\mathbf{\Xi}_0^{-1} \cdot \mathbf{\Upsilon}\right)'\right]_{(2,2)} \\
&= \left[\begin{bmatrix} \alpha \\ \alpha\gamma\left(1 - \delta\right) \\ 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \alpha\gamma\left(1 - \delta\right) \\ 0 \end{bmatrix}'\right]_{(2,2)} \\
&= \begin{bmatrix} \alpha^2 & \alpha^2 & 0 \\ \alpha^2\gamma\left(1 - \delta\right) & \alpha^2\gamma\left(1 - \delta\right) & 0 \\ 0 & 0 & 0 \end{bmatrix}_{(2,2)} \\
&= \alpha^2\gamma\left(1 - \delta\right),
\end{aligned} \tag{A.14}
$$

and the postulated linear relationship between (3.30) and (3.35) follows.

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