Antti J Tanskanen – Petri Niininen – Kari Vatanen

# Risk-based classification of financial instruments in the Finnish statutory pension scheme TyEL

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The views expressed in this paper are those of the author and do not necessarily reflect the views of the Bank of Finland.

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# Risk-based classification of financial instruments in the Finnish statutory pension scheme TyEL

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## Abstract

Sufficient solvency of a pension insurance company responsible for defined-benefit pensions guarantees that the pensions are paid regardless of turbulence in the financial market. In the Finnish occupational pension system TyEL, the required level of solvency capital (solvency limit) and its computation are specified in the statutes. Before the solvency limit can be determined, financial instruments must be classified into the five statutory asset classes based on risk. The solvency limit is computed on the basis of this classification and the average return, volatility and correlation parameters defined in the statutes. The solvency limit framework is formulated in the spirit of Markowitz portfolio theory and implicitly assumes that returns follow Gaussian distributions. This, however, is not actually the case with many – if not most – financial instruments. Similarly, it is not obvious how to handle illiquid assets, those with short time series, and which collection of financial instruments can be combined into a single asset (portfoliocation) for the purpose of classification. In this study, we propose two methods of handling these issues: (1) a decision tree-based method; and (2) a Bayesian method. We show how fat tails of return distributions are taken into account in the classification process, and how qualitative assessment of risks is combined with quantitative classification of financial assets. Coupled with suitable data transformations, both proposed methods provide efficient and suitable bases for asset classification in the TyEL pension scheme.

Keywords: Bayesian methods, classification, solvency, non-Gaussian return distributions, TyEL occupational pension scheme

JEL classification numbers: C11, G22, G23, G28, G32

# Sijoitusten riskiperusteinen luokittelu Suomen lakisääteisessä työeläkejärjestelmässä

## Suomen Pankin keskustelualoitteita 9/2010

Antti J. Tanskanen – Petri Niininen – Kari Vatanen
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## Tiivistelmä

Eläkevakuutusyhtiön riittävä vakavaraisuus takaa eläkkeiden maksun myös rahoitusmarkkinoiden kriisitilanteissa. Suomalaisessa työeläkejärjestelmässä (TyEL) vaadittu toimintapääoman taso, vakavaraisuusraja, ja sen laskenta määritellään laissa. Vakavaraisuusrajan määrittämiseksi rahoitusinstrumentit täytyy luokitella riskin mukaan yhteen tai tarvittaessa useampaan laissa määritellyistä viidestä luokasta. Vakavaraisuusraja lasketaan tämän jälkeen luokittelun perusteella luokkien keskituottojen, volatiliteettien ja korrelaatioparametrien avulla.

Vakavaraisuuskehikko perustuu oleellisesti Markowitzin portfolioteoriaan, ja vakavaraisuusrajan laskennassa oletetaan sijoitustuottojen jakautuneen gaussisen tuottojakauman mukaisesti. Useiden sijoitusinstrumenttien tapauksessa tämä oletus ei kuitenkaan päde. Lisäksi epälikvidien sijoitusten luokittelu, lyhyen tuottohistorian käsittely ja yksittäisten sijoituskohteiden yhdistely (portfoliointi) tuovat omat haasteensa luokitteluun. Tässä tutkimuksessa ehdotetaan ratkaisuksi kahta menetelmää: 1) päätöksenteon puumalli ja 2) bayesilainen metodi. Kummassakin näistä luokittelumenetelmistä otetaan huomioon tuottojakauman ”paksuhäntäisyys” ja osoitetaan, miten kvalitatiivinen riskinäkemys yhdistetään tarvittaessa kvantitatiiviseen luokitteluprosessiin. Yhdistettynä sopiviin datamuunnoksiin kumpikin tutkituista menetelmistä osoittautui tehokkaaksi luokittelumenetelmäksi TyEL-järjestelmässä.

Avainsanat: bayesilaiset menetelmät, luokittelu, vakavaraisuus, ei-gaussiset tuottojakaumat, työeläkejärjestelmä, TyEL

JEL-luokittelu: C11, G22, G23, G28, G32

# Contents

# 1 Introduction

Robust risk measuring methods are necessary for appropriate risk management of an investor. Such measures also form the basis of solvency situation determination for an insurance company. The recent financial crisis has generated unanticipated market movements of market risk factors not seen since the Great Depression at this scale. Consequently, the risk models based on the past return history have not been able to predict changes in the economic conditions, and novel methods and models are actively investigated.

The Finnish statutory occupational pension scheme known as TyEL is an earnings-related pension scheme. While a number of pension laws for different sectors in the Finnish economy exist, TyEL pension scheme covers most wage earners in the private sector. The administration of TyEL pension scheme is decentralized to seven private insurance companies and about 27 funds or foundations, each responsible for its share of the prefunded pensions.

TyEL pension scheme is partially prefunded with a rate ~36% (Biström et al, 2008), which makes the pension reserves considerable in the national context. These prefunded pensions are shown as liabilities in the balance sheet of a pension insurance company. Liabilities are computed from the expected pension payments using a fixed 3% discount rate. This however is not the required rate of return for the liabilities: 90% of the required rate of return depends on solvency of TyEL pension insurance companies capped to 15% for each company, and 10% of the required rate of return depends on average return on listed equity investments of the pension companies. The required rate of return is mostly used to increase the prefunded part of future pensions.

A pension insurance company responsible for prefunded defined-benefit pensions has solvency capital to absorb variability in the market value of assets (eg Tuomikoski, 2000). Since asset allocation largely determines the risk level of investments, sufficient amount of solvency capital should depend on the asset allocation of the company.

## 1.1 Statutory classification of financial instruments

In TyEL pension scheme, the required level of solvency capital (solvency limit in the following) is a regulatory limit that determines the maximum risk level a pension insurance company can take. When the solvency of the company is below the solvency limit, the company is not allowed to pay any bonuses to its customers, which will significantly affect the company’s competitive situation.

The solvency limit depends on the pension insurance company’s asset allocation. To determine the solvency limit, financial instruments are classified to five major asset classes based on the best estimate of risk associated with the financial instrument, irrespective of the apparent form of the instrument. It should be noted that assets are also classified according to their legal form, however, the risk-based classification is primary for the purpose of solvency limit computation, and it provides the basis for the computation of the solvency limit, which is based on the well-known Markowitz (1952) portfolio theory (for details, see Chapter 2 and Appendix A) and Value-At-Risk estimation (VaR; see eg Jorion, 2000).

As is common with any estimation based on financial time-series, a number of complications are present in the classification: shortness of available return data, autocorrelation of return series reflecting illiquidity, non-Gaussianity of return distributions, and, occasionally, unrepresentativeness of the data. Since these issues have a more general bearing than merely to the Finnish pension scheme, we believe that the issues with asset classification, and their possible resolutions, merit scientific analysis.

## 1.2 This study

In this study, we propose two systematic asset classification methods suitable for TyEL pension scheme: (1) a decision tree-based method; and (2) a Bayesian method. The ultimate aim of the classification is to provide a realistic picture of the solvency situation of a pension insurance company within the solvency framework. This requires that the classification of financial assets reflects the risks associated with the investments, and therefore that the classification method is able to address the issues with the financial data as well as to tease out the characteristics of financial time-series.

The proposed decision tree-based method enables rigorous classification of financial instruments based on the observed return time-series. The proposed Bayesian method integrates qualitative judgment of risk (expert view) in the classification process. Both methods take non-Gaussianity of returns into account in the classification, and when coupled with suitable data transformations they provide robust approaches to the classification of financial instruments.

The study is organized as follows. Chapter 2 reviews the Finnish solvency framework. In Chapter 3, we present the decision tree based methodology for classification of financial assets, and in Chapter 4 we propose the Bayesian method, which enables classification of assets with short available return time-series. To elucidate the presented classification methods and to compare the resulting classifications, we investigate example classifications of financial instruments (Chapter 5). In Chapter 6, we employ copula methods to assess whether the proposed methods improve VaR estimates at the portfolio level. To conclude the article, we discuss the method, results and directions for future research (Chapter 7).

# 2 Finnish statutory solvency framework

In TyEL pension scheme, the primary classification of a financial asset is its classification to one (or in certain cases, to several) of the five statutory asset classes: (I) money market instruments, (II) bonds, (III) real estate, (IV) equities, (V) other investments. To allow for a more fine-grained classification, each asset class is further divided into 3–5 subclasses, each of which has volatility and the expected return defined. Parameters associated with each class are given in Table 1.

To compute the solvency limit, a complete classification of all financial instruments to major classes and subclasses is required. The classification is used to compute weight $\beta_{i,j}$ of assets in subclass j of major class i, from which the total weight $\beta_i$ of each major class i is computed by summing, that is, $\beta_i = \Sigma_j\beta_{i,j}$ for $i \varepsilon \{1,2,3,5\}$. Case i=4 is discussed in the following.

Ratio $\lambda$ of the required return of liabilities (currently $\lambda$ is set at 10%) depends on average return on listed equity investments of TyEL companies. This collectively carried equity risk leaves tracking error to each insurance company from ratio $\lambda$ of investment portfolio and thereby it improves the risk-bearing capability of an insurance company. The common equity risk is deducted from the weight of Class IV, that is, the weight used is $\beta_4 = \Sigma_j\beta_{4,j} - \lambda$.

The expected return and volatility of each major class is computed by $m_i = \Sigma_j\beta_{i,j}m_{i,j}$ and $s_i = \Sigma_j\beta_{i,j}s_{i,j}$, where $m_{i,j}$ and $s_{i,j}$ are the expected return and volatility of each subclass defined in the statutes (see Table 1A). Since volatility of a major class is computed as a weighted average of volatilities of subclasses, all subclasses are assumed to be fully correlated with every subclass within an asset class. Finally, the solvency limit is given by

$$[-(\Sigma_i\beta_i m_i - t) + a\sqrt{(\Sigma_{i,j}\beta_i\beta_j s_i s_j r_{ij} + \lambda^2 S^2)}]/100 \qquad (2.1)$$

where correlation matrix $r_{ij}$ is defined in Table 1B. Term S (the statutes give it numerical value 4.5%) describes tracking error of equity investments of an insurance company left after the deduction of the common equity risk. Parameter t defines the technical interest rate, which depends on solvency of TyEL pension insurance companies capped to 15% for each company.

Table 1.

(A) Major classes, subclasses and the associated parameters: the expected annual return (mean) and the standard deviation of annual returns (volatility). $VaR_{97.5\%}$ is given here for completeness, and it is computed assuming Gaussian distribution for the returns.
(B) Correlation matrix between the major asset classes (denoted by Roman numerals).

(A)

| Solvency class and subclass | Mean (%) | Volatility (%) | $VaR_{97.5\%}$ |
|---|---|---|---|
| I Money market instruments | | | |
| 1 Sovereign notes (ETA/OECD) | 3.0 | 0.8 | -1.4 |
| 2 Bank certif. of deposit (ETA/OECD) | 3.5 | 1.5 | -0.6 |
| 3 Commercial papers (ETA/OECD) | 4.0 | 2.5 | 0.9 |
| 4 Other money market instruments | 3.5 | 3.0 | 2.4 |
| II Bonds | | | |
| 1 Pension fund bonds | 4.5 | 2.0 | -0.6 |
| 2 Sovereign bonds (ETA/OECD) | 5.0 | 5.0 | 4.8 |
| 3 Sovereign bonds (non-OECD) | 6.0 | 6.0 | 5.8 |
| 4 Corporate bonds (ETA/OECD) | 6.0 | 6.0 | 5.8 |
| 5 Other bonds | 7.0 | 9.0 | 10.6 |
| III Real estate | | | |
| 1 Residential property (ETA/OECD) | 6.0 | 7.0 | 7.7 |
| 2 Commercial property (ETA/OECD) | 7.0 | 10.0 | 12.6 |
| 3 Other property (ETA/OECD) | 7.0 | 11.0 | 14.6 |
| 4 Other property (non-OECD) | 8.5 | 15.0 | 20.9 |
| IV Equities | | | |
| 1 Listed equities (ETA/OECD) | 8.0 | 18.0 | 27.3 |
| 2 Unlisted equities (ETA/OECD) | 10.0 | 24.0 | 37.0 |
| 3 Other equities | 11.0 | 28.0 | 43.9 |
| V Other investments | | | |
| 1 Non-Euro denom. money market instr. | 4.0 | 4.5 | 4.8 |
| 2 Non-Euro denominated bonds | 6.5 | 7.5 | 8.2 |
| 3 Commodities | 8.0 | 20.0 | 31.2 |
| 4 Other investments | 12.0 | 34.0 | 54.6 |

(B)

| | I | II | III | IV | V |
|---|---|---|---|---|---|
| I | 1 | 0.3 | 0 | 0 | 0.2 |
| II | 0.3 | 1 | 0 | 0 | 0.2 |
| III | 0 | 0 | 1 | 0.4 | 0 |
| IV | 0 | 0 | 0.4 | 1 | 0 |
| V | 0.2 | 0.2 | 0 | 0 | 1 |

Equation (2.1) gives the ratio of capital relative to liabilities required if the company aims at providing bonuses for its customers; otherwise bonuses are forbidden. The solvency limit is defined to ensure that with 97.5% probability the insurance company is considered solvent after one year, when returns are distributed according to a multinormal distribution (see Appendix for derivation). Hence, factor a is given numerical value 1.96, which corresponds to 97.5th percentile in the Gaussian distribution. Since no allocation changes are assumed during the one-year period, equation (2.1) describes $VaR_{97.5\%}$ with respect to return in excess of the required return. Unlike commonly used VaR models, the solvency framework has constant risk parameters.

Two special cases are listed in the statutes: excessive concentration of the portfolio requires more solvency capital; financial derivatives which are classified either as risk-reducing or other derivatives depending on their delta-adjusted risk exposure. Risk-reducing derivative instruments must be classified *together* with the underlying financial instruments, while other derivatives are classified to the class that the underlying financial instrument would be classified to.

# 3 Decision tree classification

Risk characteristics of asset classes are defined in the statutes via the expected return and volatility parameters (Table 1) corresponding to those observed for well-diversified portfolios. Since the parameters do not uniquely define asset classes, we assume that the characteristic features of the statutory asset classes are captured by certain benchmark time-series consisting of returns from well-diversified portfolios of homogeneous instruments.

Under these assumptions, the classification of assets to major classes can be based on a measure of similarity (eg correlation) between the portfolio of homogeneous assets and the market index describing the major class. Asset classification consists of two phases: (1) financial instruments are classified to major classes; (2) instruments are classified to one of the subclasses. These two phases can be combined into a single phase, however, we have kept them separate for simplicity.

Before classification, it is of importance to check that the analyzed data is appropriate for the purpose, and whether any data transformations are required.

## 3.1 Data transformations

Any analysis of investment returns should be based on reliable data. To ensure that this is the case, certain data transformations may be needed (Brooks and Kat, 2002) due to, eg differences in liquidity (observable via autocorrelation, and via bid-ask spreads). Firstly, we base the following risk analysis on one-month log-returns as is common in the literature (eg Jorion, 2000). Secondly, we remove the first order autocorrelation from the benchmark time-series and from the classified assets using equation

$$R = \frac{(A_{k+1} - rA_k)}{(1 - r)} \tag{3.1}$$

which reassigns autocorrelated part of returns to the 'correct' point in time-series (Geltner, 1993, Brooks and Kat, 2002). Autocorrelation removal induces an increase in volatility, an increase in correlation with more liquid indices, and, occasionally, a slight reduction in arithmetic returns (Brooks and Kat, 2002). In most cases, it suffices to remove the first order autocorrelation.

To evaluate which time-series incorporate significant autocorrelation, we employ the Ljung-Box (1978) and Breusch-Godfrey (Godfrey, 1978) tests at 5% significance level. If either of these tests showed autocorrelation at 1 lag, we employed equation (2) for the autocorrelation correction. After these corrections, we assume that the data is of sufficient quality for classification.

A further obstacle to time-series analysis is the length of the available return data. Long investment horizon of a pension company requires that risk-estimates are based on long-term behavior of assets. This suggests that time-series describing an asset should cover at least one business cycle. According to the National Bureau of Economic Research (NBER, 2003), average post-1945 duration of a business cycle is 67 months. According to (NBER, 2008), the latest business cycle (March 2001 – Nov 2007) lasted 81 months, which we assume to define the sufficient length of data for classification.

## 3.2 Classification to major classes

The major asset classes are defined in the statutes, however, the statutes do not define how an instrument should be classified based on the investment risk associated with the instrument. We assume that assets can be classified to the major classes based on the similarity of instrument's return data to the benchmark time-series representing the major class. While the measure of dependence between the instrument and the benchmark used for the classification is not given in the statutes, it is assumed in the statutes that correlations between major asset classes define their mutual dependence structure. Consistently with this, we assume that a correlation measure is the most appropriate measure of dependence between a financial instrument and a major asset class.

More precisely, we assume that the classification is based on the Spearman's correlation between the returns of an instrument and the benchmark time-series. Unlike the more commonly-used Pearson's linear correlation, Spearman's rank-correlation does not depend on the shape of return distributions, only on their mutual dependence structure (that is, Spearman's rank-correlation is unchanged in monotonic transformations of the time-series, see Nelson, 1999). In practice, the use of Spearman's correlation enables more reliable risk assessment of, eg derivative-like instruments (Lhabitant, 2004, Getmansky, 2005).

Table 2. **Correlation ranges for classification used in the Decision tree model.**

Notation r(equity) refers to Spearman's correlation against the equity benchmark, r(bond) to the highest of Spearman's correlation against bond benchmarks, and r(Euribor) Spearman's correlation against 3 month Euribor.

| Class | r(equity) | r(bond) | r(Euribor) |
|---|---|---|---|
| I (money market) | [-1, 1] | [-1, 1] | (0.5, 1] |
| II (bonds) | (-0.2, 0.2) | (0.5, 1] | [-1, 1] |
| III (real estate) | [0.2, 0.7] | [-0.5, 0.7] | [-1, 1] |
| IV (equity) | (0.7, 1] | [-1, 1] | [-1, 1] |
| V (other) | any other combination of correlations | | |

In Decision tree classification, we assign a financial instrument to a major class based on Spearman's rank-correlation as follows: If correlation of the benchmark time-series and the instrument is within the pre-defined major class' correlation ranges at 2.5% confidence level (tested via the z-transformation), the instrument is assigned to the major class corresponding to the benchmark time-series. Testing proceeds sequentially from the high risk class downwards: IV, III, II, I. Hence, this method is called Decision tree method. If correlation of an instrument is not sufficient for classification to any of classes I–IV, the instrument is assigned to Class V. The correlation ranges for each class are given in Table 2.

## 3.3 Classification to subclasses

A subclass is in the statutory framework defined by the expected return and volatility (Table 1), which suggests that volatility should be the defining statistic for subclass selection. However, it is not uncommon that distributions of investment returns are more skewed and fat-tailed than the normal distribution. Thus, volatility is not the most appropriate measure of risk.

In the solvency framework, portfolio risk is measured as the one-year $VaR_{97.5\%}$ point. More precisely, the solvency requirement is computed as the 97.5th percentile of excess return distribution via Gaussian approximation $1.96\sigma-\mu$, where $\mu$ is expected excess return and $\sigma$ is volatility. Consistently with this, we base classification to subclasses on $VaR_{97.5\%}$ in Decision tree method. To use one-month data for classification, statutory parameters are scaled to compute one-month VaR estimates (Table 3; ranges are defined as midpoints between subclasses consistently with the assumption that subclasses are fully correlated within each major class).

An appropriate subclass is chosen based on the empirically estimated $VaR_{97.5\%}$ of the instrument. When long time-series of reliable data is available, it suffices to compute $VaR_{97.5\%}$ from the data as a sample estimate. For short time-series, it is not obvious how to estimate 97.5th percentile of return time-series, but certain approximations can be used. In Gaussian approximation, $VaR_{97.5\%}$ is given by $1.96\sigma-\mu$, while the lower bound for an unimodal return distribution is given by Vysochinskij-Petunin's formula $3\sigma-\mu$ (Pukelsheim, 1994). Having estimated $VaR_{97.5\%}$, we can classify each financial instrument to one of the subclasses.

# 4 Bayesian method

Inclusion of other sources of information besides the return data is desirable in risk-based classification. Bayesian methods enable integration of, eg qualitative assessment of the expected fund performance with observed return time-series, which is important when data is scarce or it is suspected that the available data does not represent future returns of the fund.

## 4.1 Classification to major classes

The major classification of financial instruments depends on correlations between the benchmark time-series and the financial instrument. Prior qualitative assessment of correlation $r_{prior}$ is coupled with the sample estimate $r_{sample}$ to obtain a posterior estimate of correlation $r_{posterior}$ using equation (Schisterman et al, 2003)

$$u_{posterior} = \sigma^2 (n_{prior} \tanh^{-1}(r_{prior}) + n_{sample} \tanh^{-1}(r_{sample})) \quad (4.1)$$

$$r_{posterior} = \tanh(u_{posterior}) \quad (4.2)$$

where $n_{prior}$ is weight of prior estimate (loosely, the number of prior observations); and $n_{sample}$ is the number of data points in the sample. Variance $\sigma^2$ of the posterior estimate is given by $(n_{prior} + n_{sample})^{-1}$. The posterior distribution of correlation is given by $N(u_{posterior},\sigma)$ (Schisterman et al, 2003), which enables us to compute credible intervals for posterior estimates used in the classification.

In Bayesian modeling, the posterior distribution of parameters contains all information. Using the posterior probability distributions of asset correlations against the benchmark time-series, we can infer the major class for the instrument. To obtain probability that an asset belongs to, eg Class I (Money Market), we must first define appropriate correlation ranges and, second, compute the probability that the correlation of the asset against the benchmark time series are within these ranges based on the posterior probability distribution of correlations. Classification to one of the major classes I–V depends on a combination of equity, money market and bond correlation.

Table 4 defines the probabilities that an asset belongs to a major class. It is worth noting that multiple non-overlapping ranges are used in classes II, IV, and V. We choose the major class with the highest probability as the suitable major class for an instrument.

For example, the probability than an asset belongs to Class I is $P(\{|Corr(equity)| < 0.2 \text{ and } Corr(money\ market) > 0.5\})$, that is, the probability that the asset is a money market instrument (in the classification) depends on the posterior probability that its returns have correlation against the money market benchmark over 50% and, simultaneously, that its returns have correlation against the equity benchmark less than 20%.

Table 3. **$VaR_{97.5\%}$ ranges for each major class (denoted by I–V) and subclass (denoted by 1–5)**

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| I | <0.38% | [0.38%, 0.82%) | [0.82%, 1.2%) | >1.24% | |
| II | <1.58% | [1.58%, 2.65%) | [2.65%, 2.89%) | [2.89%, 3.70%) | >3.70% |
| III | <4.27% | [4.27%, 5.36%) | [5.36%, 6.71%) | >6.71% | |
| IV | <11.13% | [11.13%, 13.84%) | >13.84% | | |
| V | <2.96% | [2.96%, 7.18%) | [7.18%, 14.44%) | >14.44% | |

## 4.2 Classification to subclasses

Classification of an instrument to a subclass requires estimate of $VaR_{97.5\%}$. To obtain a Bayesian estimate of $VaR_{97.5\%}$, we must first choose probability distribution (eg Gaussian distribution) that describes return time-series. Then we need prior distributions for parameters (eg the expected return is normally distributed and volatility is gamma-distributed), which are obtained by expert-view. This information can be coupled with data using the Bayes' rule, which yields posterior probability distribution of parameters. From this information, the distribution of $VaR_{97.5\%}$ can be computed, and the appropriate subclass can be inferred.

In this study, we employ two hierarchical probability models: (1) the normal model defined by

$$x \sim N(\mu,\sigma), \mu \sim N(m,s), \sigma \sim \text{Gamma}(a,b) \quad (4.3)$$

and (2) the generalized extreme value (GEV; see eg McNeil et al, 2004) model defined by

$$x \sim \text{GEV}(\mu,\sigma,\xi), \quad \mu \sim N(m,s) \quad \sigma \sim \text{Gamma}(a,b), \quad \xi \sim N(C,D) \quad (4.4)$$

Hyperparameters m, s, a, b, C, D$\varepsilon$R define the prior information: the expected return $\mu$ has normal distribution (mean m, standard deviation s), volatility $\sigma$ has gamma distribution (mean a, shape b), and the GEV shape parameter $\xi$ has normal distribution (mean C, standard deviation D).

A practical way to analyze the models is to employ Markov Chain Monte Carlo-methods (MCMC; see eg Rachev et al, 2006, and Puustelli et al, 2007), as realized by OpenBUGS (Spiegelhalter et al, 2008). The end result of analysis is the posterior probability distribution of parameters from which we can compute the probability distribution of $VaR_\alpha$ in the Normal model using equation

$$\Phi^{-1}(\alpha)\sigma - \mu \quad (4.5)$$

where $\Phi^{-1}$ is inverse of cdf of the normal distribution, and in the GEV model using equation

$$\sigma\frac{[-\log(\alpha)^{-\xi} - 1]}{\xi - \mu} \quad (4.6)$$

where the initial data is assumed to describe losses. Hence, we obtain probability distribution of $VaR_{97.5\%}$ which incorporates both expert view (if present), its uncertainty (typically present), and data. It is worth noting that equations (4.2) and (4.3) are both of form A$\sigma$–$\mu$. A typical value of $\xi$ is -0.1, and hence the GEV distribution yields a more conservative risk estimate (3.08$\sigma$–$\mu$) than the normal distribution (1.96$\sigma$–$\mu$), when all other things are kept equal. It remains to use these distributions to infer the subclass.

Table 4. **Definition of the probability of association to a major class in the Bayesian method.**

Each row defines the probability that an asset belongs to the particular major class. Probabilities are computed based on the posterior probability distribution of parameters of the used probability model.

| | |
|---|---|
| Class I: | P({\|Corr(equity)\| < 0.3 and Corr(money market) > 0.5}) |
| Class II: | P({\|Corr(equity)\| < 0.3 and Corr(bonds) in [0.7,0.9]}U{Corr(bonds) > 0.9}) |
| Class III: | P({Corr(equity) in [0.2, 0.7] and Corr(bonds) in [0.2,0.8]}) |
| Class IV: | P({Corr(bonds) in [-0.4,0.4] and Corr(equity) in [0.6,0.8]}U{Corr(equity) > 0.8}) |
| Class V: | P({\|Corr(bonds)\|<0.3 and \|Corr(equity)\|<0.2 and \|Corr(money market)\|<0.5 }) |

The subclass is chosen based on $VaR_{97.5\%}$ analogously to the major class selection. $VaR_{97.5\%}$ is computed based on the posterior probability distribution of the model parameters, which yields the posterior probability distribution of $VaR_{97.5\%}$. This posterior probability distribution gives the probability of each subclass, when the $VaR_{97.5\%}$ probability distribution is binned to ranges defined in Table 3. The subclass with the most probability mass attached to it is chosen as the suitable subclass. To assess the applicability and usability of the developed methods, we now turn to example classifications.

# 5 Example classifications

## 5.1 Decision tree classification

To make the proposed methodology more concrete, we present an example classification. Table 5 shows statistics of the assets chosen. The data consists of financial instruments typically found in a well-diversified portfolio of an institutional investor, and they span the common asset classes. First, the two stock market indices represent the stock portfolios of an institution: Dow Jones Stoxx 600 Return (henceforth abbreviated STOXX 600) and S&P500 Total Return (SP500) illustrate well-diversified equity portfolios. The skewness and kurtosis of these two indices are typical of share portfolios: the distribution has negative skewness and significant excess kurtosis, which show that the returns are not normally distributed.

DJ EuroStoxx 50 leveraged index (ES50LEV) has been included as a proxy for a portfolio with risk increasing derivatives. Other examples of non-linear returns and alternative assets are depicted by two hedge fund indices by CS/Tremont (Global index, HFGlobal, and fixed income arbitrage index, HFarb) and the Finnish hedge fund with the longest history (Danske Omega) to demonstrate the classification framework in more complex cases. Again, the skewness and kurtosis indicate non-normal returns, with kurtosis being even more pronounced than with stock indices.

The fixed income portion of a portfolio is exemplified by two bond funds. Nordea Pro Euro (Nordea Bond) and Goldman Sachs Global High Yield (GS HY global) are both directed to institutional investors and have long history of returns. The former invests primarily in European government bonds while the latter invests in high yield fixed income securities in developed countries. For money market instruments, Seligson & Co money market AAA fund (Seligson MM) represents short maturities with minimal spread risk. As expected, the fixed income fund dealing with sovereign bonds has normally distributed returns. The high yield bond fund has similar distribution characteristics to the equity indices.

The indices and assets in the sample represent the typical asset classes found in an institutional investor's portfolio. These data provide the basis against which the correlations are analyzed.

As benchmarks, we have selected five widely used indices: MSCI World index for stock market; two indices for fixed income markets (iBoxx corporate and sovereign bonds); and 3 month Euribor for the money market investments. The correlation of a financial instrument against the benchmark determines the major solvency class of the instrument. Since two benchmark series are defined for bonds, the highest correlation against the two fixed income benchmarks is used. As discussed above, $VaR_{97.5\%}$ is used to determine the subclass.

Table 5 shows that the proposed Decision tree classification produces intuitively reasonable results. All stock index based assets fall in solvency class IV (Equities). As expected, hedge funds are less homogenous and belong to either group III (Real Estate) or in group IV (Equities). Government bond fund falls into class II (fixed income) while the riskier high yield bond fund is classified to III due to high stock market correlation. Finally, money market assets are classified to solvency class I (money market).

Table 5. **Data and benchmark indices (August 2001 – December 2008), and their associated statistics**

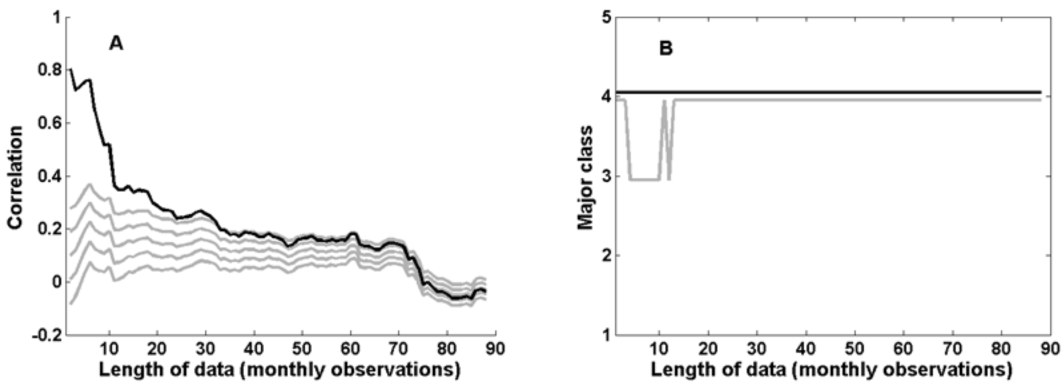
| Instrument | SP500 | STOXX | ES50LEV | HFGlobal | HFARB | Danske Omega | Nordea Bond | GS HY global | Seligson MM |
|---|---|---|---|---|---|---|---|---|---|
| Ljung-Box, prob. | 0.01† | 0.03† | 0.16 | 0.00† | 0.00† | 0.16 | 0.07 | 0.04† | 0.00† |
| Breusch-Godfrey, prob. | 0.30 | 0.12 | 0.06 | 0.70 | 0.00† | 0.00† | 0.13 | 0.10 | 0.00† |
| Jarque-Bera*, prob. | 0.0% | 0.1% | 0.00 | 0.0% | 0.0% | 0.0% | 83.7% | 0.0% | 0.0% |
| skewness | -0.9 | -0.8 | -0.93 | -0.3 | -4.9 | -4.4 | 0.0 | -1.9 | 0.9 |
| kurtosis | 2.2 | 1.1 | 1.9 | 2.5 | 33.5 | 32.2 | 0.0 | 6.7 | 1.5 |
| VaR-based volatility p.a. | 17.6% | 23.1% | 48.2% | 11.5% | 10.8% | 2.2% | 3.4% | 18.1% | 0.7% |
| Spearman rank correlations vs. benchmarks: | | | | | | | | | |
| MSCI World | 93.0% | 77.8% | 77.0% | 74.2% | 28.4% | 18.8% | -26.2% | 49.6% | -18.3% |
| iBoxx € corporate oa tr | 2.3% | -7.5% | -8.0% | 11.5% | 31.0% | 7.3% | 74.9% | 23.7% | 46.9% |
| iBoxx € sovereign oa tr | -27.3% | -36.1% | -38.1% | -18.7% | 4.4% | -2.1% | 96.9% | -7.1% | 46.8% |
| Solvency class** | IV.1 | IV.2 | IV.3 | IV.1 | III.3 | III.1 | II.2 | III.4 | I.1 |

The last row (Solvency class) shows classification in the Decision tree method.
† H0 (no autocorrelation) rejected at 5% confidence level either by Ljung-Box or Breusch-Godfrey statistic.
* If autocorrelation has been detected, these figures have been calculated from an autocorrelation corrected time series. The same applies to benchmark indices.
** For Class II, correlation in one of the benchmark bond indices is sufficient if it falls in the given range (Table 2). For Class III and IV, all of the bond benchmarks should fall in the given range.

## 5.2 Bayesian classification

Bayesian classification methods enable a reliable way to classify financial instruments with short and/or non-reliable time-series. Figure 1A shows that shortness of a return time-series may produce spurious small-sample correlation estimates, as shown by the fact that estimated correlation is close to 1 based on short time-series while longer time-series yield correlation close to zero. This issue can be addressed either by requiring that a certain number of observations must be available or by using a Bayesian method with a reasonable prior distribution. The second method is preferable because it enables the use of even the little information contained in the short time-series.

Figure 1. **Impact of expert view on classification**




(A) Comparison of correlation between MSCI World equity index and Nordea Pro Obligaatio fund using the sample correlation (black solid curve) and a Bayesian method for the correlation computation (solid light gray curves) using equally spaced priors within range [-20%, 20%] with 20 observations. Correlation is plotted as a function of the number of monthly observations; (B) Classifi-cation of SP500 index as a function of the number of monthly points using the Bayesian method (solid black line) and on sample correlation (solid light gray line), as a function of the length of return data.

Bayesian correlation estimation improves the estimation of association between assets and the benchmark time-series. The closer the prior is to the true correlation, the bigger the improvement in correlation estimate. In Figure 1A, any initial estimate within range [-20%, 20%] of the long-term correlation improves the correlation estimate. In our tests, the Bayesian method provided most improvement when the number of data points was less than 20. When this result is converted to an actual classification (Figure 1B), we see that classification converges significantly more rapidly to the appropriate class.

Table 6 demonstrates that Bayesian classification and Decision tree classification produce similar results despite the differences in approaches, when sufficient amount of data is available. The main difference is the classification of Danske Omega, which is classified to Class III in Decision tree classification and to Class V in the Bayesian approach. Decision tree approach proceeds from the class with the highest risk to the class with the lowest risk, and Danske Omega ends up in Class III due to its low correlation. In the Bayesian method, the most probable class is chosen for each asset, and for Danske Omega Class V is more probable than Class III.

When the data set is truncated to 24 months of data, spurious classifications may be produced: in this case, Seligson MM does not show sufficient correlation against Euribor to be classified to Class I (Money market) and hence it is classified to Class V (Other investments). When longer time-series is used, Seligson MM is appropriately classified in Class I (Money market). This kind of issues can be addressed by using appropriate Bayesian priors, as shown by the last row in Table 6. Hence, the results of Table 6 support the view that a judiciously chosen prior and the use of Bayes' rule improve classification when little reliable data is available.

Table 7 shows uncertainty associated with classification: equity indices SP500, STOXX600 and ES50LEV clearly belong to Class IV (Equities), while Hedge Funds (HFGlobal; HFarb; Danske Omega) are more ambivalent. Similarly, GS HY global has a significant equity-like component, but the best classification for it is Class III (Real estate). The results confirm the common wisdom that hedge funds may be equity-like, bond-like or uncorrelated to either bonds or equities.

The length of available, reliable data influences classification to subclasses. Table 8 shows subclass selection based on 6 methods: (1) historical volatility; (2) Historical $VaR_{97.5\%}$; (3) Bayesian non-informative normal model; (4) Bayesian informative normal model; (5) Bayesian non-informative GEV model; and (6) Bayesian informative GEV model. Table 8A bases the minor classification on the longest publicly available time series; Table 8B on 89 month subset 8/2001–12/2008; and Table 8C on 12 month subset 6/2006–5/2007 during bull period. In Bayesian informative models, prior distributions of parameters are assume to have reasonable expected values in long-term, and in each case uncertainty of parameters is chosen so that the prior has little influence on classification based on long time-series. In Bayesian non-informative models, non-informative priors are used.

Table 6. **The impact of the choice of correlation measure on the major classification (major classes denoted by Roman numerals)**

| Method/Basis for classification | SP500 | STOXX 600 | ES50LEV | HFGlobal | HFarb | Danske Omega | Nordea Bond | GS HY global | Seligson MM |
|---|---|---|---|---|---|---|---|---|---|
| Decision tree classification | IV | IV | IV | IV | III | III | II | III | I |
| Non-informative Bayesian | IV | IV | IV | IV | III | V | II | III | I |
| Bayesian classification | IV | IV | IV | IV | III | V | II | III | I |
| Non-informative Bayesian (24m) | IV | IV | IV | IV | III | V | II | III | V |
| Bayesian classification (24m) | IV | IV | IV | IV | III | V | II | III | I |

Pearson's correlation and Spearman's rank-correlation suggest similar classifications, except for Danske Omega fund for which Spearman's rank-correlation finds less association with the benchmark time-series than Pearson's correlation. When short time-series of 24 month data (24m) is used, spurious correlations may be found.

Table 7. **Probability of classification to major classes in the Bayesian method**

| Method/Basis for classification | SP500 | STOXX 600 | ES50LEV | HFGlobal | HFarb | Danske Omega | Nordea Bond | GS HY global | Seligson MM |
|---|---|---|---|---|---|---|---|---|---|
| Class I | 0.0% | 0.0% | 0.0% | 0.1% | 2.4% | 1.0% | 4.9% | 0.1% | 59.9% |
| Class II | 0.0% | 0.0% | 0.0% | 0.0% | 0.4% | 0.0% | 67.7% | 0.0% | 0.4% |
| Class III | 5.2% | 10.8% | 9.6% | 25.1% | 43.2% | 16.1% | 0.9% | 43.1% | 7.4% |
| Class IV | 88.4% | 79.2% | 81.7% | 48.9% | 4.0% | 2.1% | 0.0% | 22.7% | 0.1% |
| Class V | 0.0% | 0.1% | 0.1% | 1.2% | 20.7% | 47.8% | 0.0% | 10.0% | 9.4% |

The data used consists of 89 monthly returns. Probabilities do not add up to one due to overlapping definitions of class correlations.

Table 8.

Classification of 9 assets using six different methods with (A) the data set of maximum length; (B) with the 89 point data set; and (C) with 12 point data set from a low-volatility regime (2006/6–2007/5). See text for the description of methods. The major asset classes are denoted by Roman numerals, and subclasses are denoted by Arabic numerals.

**(A) Long data series**

| Method/Basis for classification | SP500 | STOXX 600 | ES50LEV | HFGlobal | HFarb | Danske Omega | Nordea Bond | GS HY global | Seligson MM |
|---|---|---|---|---|---|---|---|---|---|
| Historical volatility | IV.1 | IV.1 | IV.3 | IV.1 | III.3 | V.1 | II.2 | III.4 | I.1 |
| Historical VaR 97.5% | IV.1 | IV.2 | IV.3 | IV.1 | III.3 | V.1 | II.2 | III.4 | I.1 |
| Bayes diffuse normal | IV.1 | IV.1 | IV.3 | IV.1 | III.3 | V.1 | II.2 | III.4 | I.1 |
| Bayes informative normal | IV.1 | IV.2 | IV.3 | IV.1 | III.4 | V.1 | II.2 | III.4 | I.1 |
| Bayes diffuse GEV | IV.1 | IV.2 | IV.3 | IV.1 | III.4 | V.1 | II.2 | III.4 | I.1 |
| Bayes informative GEV | IV.1 | IV.2 | IV.3 | IV.1 | III.4 | V.1 | II.2 | III.4 | I.1 |

**(B) Data from 2001–2008**

| Method/Basis for classification | SP500 | STOXX 600 | ES50LEV | HFGlobal | HFarb | Danske Omega | Nordea Bond | GS HY global | Seligson MM |
|---|---|---|---|---|---|---|---|---|---|
| Historical volatility | IV.1 | IV.1 | IV.3 | IV.1 | III.4 | V.1 | II.2 | III.4 | I.1 |
| Historical VaR 97.5% | IV.1 | IV.3 | IV.3 | IV.1 | III.4 | V.1 | II.2 | III.4 | I.1 |
| Bayes diffuse normal | IV.1 | IV.2 | IV.3 | IV.1 | III.4 | V.1 | II.2 | III.4 | I.1 |
| Bayes informative normal | IV.1 | IV.2 | IV.3 | IV.1 | III.4 | V.1 | II.2 | III.4 | I.1 |
| Bayes diffuse GEV | IV.1 | IV.2 | IV.3 | IV.1 | III.4 | V.1 | II.2 | III.4 | I.1 |
| Bayes informative GEV | IV.1 | IV.2 | IV.3 | IV.1 | III.4 | V.1 | II.2 | III.4 | I.1 |

continues ...

...Table 8 continued

**(C) Data from 2006–2007**

| Method/Basis for classification | SP500 | STOXX 600 | ES50LEV | HFGlobal | HFarb | Danske Omega | Nordea Bond | GS HY global | Seligson MM |
|---|---|---|---|---|---|---|---|---|---|
| Historical volatility | IV.1 | IV.1 | IV.1 | IV.1 | III.1 | V.1 | II.2 | III.2 | I.1 |
| Historical VaR 97.5% | IV.1 | IV.1 | IV.1 | IV.1 | III.1 | V.1 | II.2 | III.2 | I.1 |
| Bayes diffuse normal | IV.1 | IV.1 | IV.1 | IV.1 | III.1 | V.1 | II.2 | III.2 | I.1 |
| Bayes informative normal | IV.1 | IV.2 | IV.3 | IV.1 | III.4 | V.1 | II.2 | III.4 | I.1 |
| Bayes diffuse GEV | IV.1 | IV.1 | IV.1 | IV.1 | III.1 | V.1 | II.2 | III.4 | I.1 |
| Bayes informative GEV | IV.1 | IV.2 | IV.3 | IV.1 | III.4 | V.1 | II.2 | III.4 | I.1 |

As shown by Table 8A, the choice of the classification method has a little influence on the result even when long time-series are used: The risk-estimates based on $VaR_{97.5\%}$ are higher than those based on volatility and a Gaussian distribution. The GEV models closely correspond to classification based on historical $VaR_{97.5\%}$ due to long data used, while the normal models better correspond to classification based on historical volatility. Table 8B shows results based on 89 data points, and the main difference between the results of Tables 8A–B is that in Table 8B returns from year 2008 have higher weight, which results higher risk estimates.

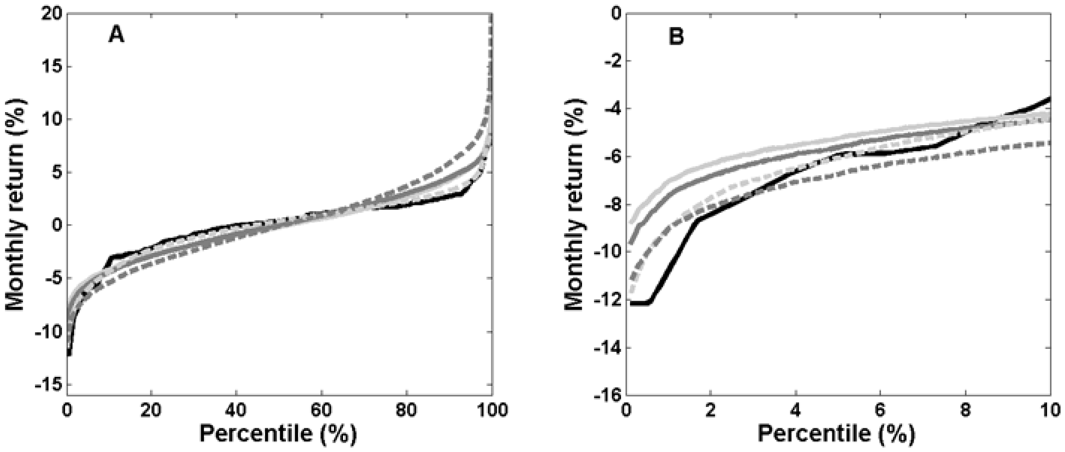
Table 8C shows the classification in the short time-series case, when the available data is truncated from 89 data points (Table 6) to only 12 data points. The data used in Table 8C is deliberately taken from a low-volatility regime (2006/6–2007/5). In this case, prior brings more information to the estimation, and, consequently, the two Bayesian informative models provide the best risk estimates. This shows that the use of short time-series may significantly distort the risk estimates.

Data taken from bull market period shows that little risk is associated with equity investments, while periods containing year 2008 (Table 8B) show that equity risk is at the high end of the scale. It is quite clear that neither of these viewpoints captures the essence of investment targets of a long-term investor. The Bayesian method enables improve risk estimates when scarce data is available, as demonstrated by Table 8A.

# 6 Validation

In previous chapters, we have defined various ways to improve risk-based classification of financial instruments. The next step is to investigate, how and whether these improvements influence risk estimates at the portfolio level. For this purpose, we investigate how the underlying models perform relative to each other by comparing the portfolio-level risk-estimates obtained by coupling estimated univariate time-series models via the Gaussian copula.

Figure 2.




Comparison of percentiles of portfolio returns in the Gaussian model (solid light gray line), the GEV model (solid dark gray line), the VaR-modified Gaussian model (dashed dark gray line); and the kernel density based model (dashed light gray line). Black solid line shows the empirical returns. Panel (A) shows range 0–100% in the cumulative density function, while Panel (B) only shows range 0-10% of the cumulative density function.

Firstly, we fit four univariate time-series models to the 9 time-series analyzed previously: (1) in Gaussian model the expected return and volatility define each univariate model; (2) the GEV distribution provides an example of non-normal model, and it is fitted to each time series using the maximum likelihood methods; (3) in the 'VaR-modified Gaussian model' volatility of each time series is chosen in such a way that $VaR_{97.5\%}$ is reproduced by the model; and (4) a kernel density model enables assessment of the impact of non-Gaussianity of return time-series.

Secondly, we construct portfolio return time-series from return time-series of individual funds using the Gaussian copula (based on the observed correlation matrix). The use of a gaussian copula with fixed correlation matrix for the entire estimation period corresponds to the fixed correlation matrix used in the statutory solvency framework employed in TyEL pension scheme.

Thirdly, portfolio-level risk estimates are assessed by a comparison of the modeled results with those from an equally-weighted monthly-rebalanced portfolio constructed from the empirically observed return data.

Figure 2 compares percentiles of portfolio returns in the four models and in empirical sample. The GEV model slightly improves the fit at the portfolio level compared to the normal model. The VaR-modified Gaussian model provides the worst fit to the data for percentiles above 8%, however, below this threshold, it significantly improves risk estimates compared to the normal model and the GEV model. The data is best reproduced by the kernel density model, as can be expected.

This result shows that taking non-gaussianity and tail risk into account in fitting models does improve risk measures at the portfolio level. Since these models underlie the classification methods presented in previous chapters, it can be expected that these results are relevant to the classification. More precisely, the Gaussian model corresponds to classification of assets based on volatility. In the VaR-modified Gaussian model, volatility of each marginal distribution is calibrated to reproduce the observed $VaR_{97.5\%}$, and it therefore corresponds to classification according to $VaR_{97.5\%}$ instead of volatility. Figure 2 shows that the VaR-modified Gaussian model significantly improves $VaR_{97.5\%}$ estimate compared to the Gaussian model. Hence, classification that takes into account fat tails of return time-series does improve risk assessment at the portfolio level.

# 7 Discussion

## 7.1 This study

In this study, we have presented two methods to classify financial instruments in the solvency framework of the Finnish occupational pension system TyEL. Of the two methods, Decision tree method provides a more streamlined, computationally-light classification method, while the Bayesian method enables a more fine-graded classification but it is computationally more demanding. The two proposed methods possess a number of important characteristics. Firstly, the methods demonstrate that the statutory solvency framework can reasonably well accommodate financial instruments with fat-tailed return distributions. Secondly, each method provides an efficient, rigorous framework for classification of assets consistently with the statutory solvency framework, while at the same time resolving a number of issues with the classification process.

A fully mechanistic classification of assets does not capture the true risk associated with a financial investment (eg after a structural break in data), and be believe that any financial instrument classification must incorporate subjective assessment. The Bayesian method enables a rigorous way to quantify subjective information about the financial instrument via the choice of prior distributions. The importance of qualitative assessment is underlined under the all-too-common conditions where the available data only covers a small portion of a business cycle. Bayesian methods also regularize statistics in the sense that estimates typically do not vary wildly when data is scarce.

The choice of a proper asset class is not always as well-defined as one would like, in particular for instruments that contain features from several major classes, eg high yield bonds and hedge funds. For these instruments, relatively minor changes may tip the scale toward another major class, which suggests that the scope of the solvency framework is too narrow. However, a hedge fund is more a strategy than an asset class, and therefore the result on the complexity of hedge fund classification is consistent with observation.

## 7.2 Solvency is not all about prudency

It can be argued that classification should entirely be based on precautiousness, that is, to excessive prudency. This, however, is not a view that we can share. The main task of a pension insurance company is to take care of pensions. In TyEL pension scheme, one (perhaps, the best) way to guarantee future pensions is to obtain good investment performance, which requires that appropriate amount of risk is taken (not too little, not too much). This requires that assessment of the solvency situation is neither overly optimistic nor overly pessimistic.

Tuomikoski (2000) argues that excessive short-term prudency is not desirable for pension investments. We fully agree with Tuomikoski (2000) on the importance of this point, which in particular requires that financial instruments should not be classified punitively. An example of such punitive classification would be the case in which hedge funds are automatically classified as equity (Class IV) regardless of their characteristics (see also Table 7).

The solvency framework should be transparent and efficient, but not overly simplistic. It is a tool for the regulator, not a tool for making the actual investment decisions. It is of importance that the classification method in the solvency framework is based on the actual risk associated with a financial instrument, not merely on its legal form. This enables better assessment of the risks associated with financial instruments, and thereby it enables better use of capital administered by a pension company. The Finnish law wisely and explicitly requires that financial instrument are classified according to their actual risk.

## 7.3 Comparison with previous studies

The issues investigated in this study are not unique to the Finnish pension system, eg the use of VaR-like risk measures is almost universal in the financial industry. Despite this, the use of Bayesian methods in the estimation of VaR is not as widespread as one would expect. This, however, may change in the future (see eg Rachev et al, 2006). Lambrigger et al (2007) show that Bayesian methodology integrating expert opinion with observed data can significantly improve estimation of operational risk. This result is similar to our observation that expert view can significantly improve risk estimates and risk-based classification of financial instruments.

Tuomikoski (2000) briefly describes the pre-2007 classification in TyEL pension scheme, in which classification was based on the legal form of the investment. In 2007, the classification of financial instruments in TyEL pension scheme was revised to be based on actual risk associated with the instrument. We have described two methods to classify financial assets based the actual risk, and therefore our work provides an update on Tuomikoski (2000).

While the exposure was limited to the Finnish set-up, the issues considered in this study have also bearing on other insurance schemes. For example, we believe that TyEL pension scheme that allows allocation to hedge funds when solvency is suitably high provides a more appropriate assessment of risks associated with hedge fund investments than a strict 5 per cent limit imposed on pension funds in Spain, Greece, Portugal and on 'pensionkasse' in Germany (Stewart, 2007). The risk-taking of a pension investor should reflect the nature of liabilities, and be limited by the solvency, not by excessively prudent statutory limits.

Solvency II forces insurance companies to use VaR as their risk measure of choice. TyEL pension insurance companies do not fall under Solvency II, but nevertheless VaR has been the risk measure of choice for gauging the appropriate level for the minimum solvency capital in TyEL pension scheme. Solvency limit in TyEL pension scheme is computed as $VaR_{97.5\%}$ at 1-year horizon, while Solvency II uses the 'soft' 99.5% level (at 1-year horizon) Solvency capital requirement and 'hard' 80–90% level (at 1-year horizon) Minimum capital requirement.

VaR based solvency models commonly suffer from pro-cyclical behavior due to dynamic risk parameters. This may result significantly over-estimation or under-estimation of the risks associated with investment, in particular when only short time-series are available. This is not the case with the solvency framework in TyEL pension scheme, because the risk parameters are static. The impetus behind static parameters is that a pension company has long term liabilities, and hence the investment strategy should be aimed at durations of tens of years. Risk-based classification may bring a slight pro-cyclicality in the solvency framework. Since the solvency limit depends on the classification, pro-cyclical classification of financial assets may lead to pro-cyclicality of the solvency limit. This is not in the interest of the pension system, which has a long investment horizon.

## 7.4 Issues with the Finnish solvency framework

The solvency framework is not appropriate for classification of individual instruments, because the measure of similarity between a single asset and the market index does not necessarily have adequate statistical co-relation. Thus, it is not always obvious which set of assets can be combined to a portfolio that can be classified as a single unit in the solvency framework. For example, the constituent stocks that together form the Eurostoxx 50 index have relatively low correlations against the Eurostoxx 50 index (typically within range 40–85%). Hence, it is not obvious that all of these stocks would be treated as equity when classified individually. This issue is particularly important with respect to alternative investments, eg hedge funds.

While the solvency limit is often interpreted as the capital requirement that ensures that the company is solvent with 97.5% probability after one year, this is not strictly the case. The reason for this is that the solvency limit is computed relative to the liabilities, not to the assets. Hence, there is a leverage effect when the sum of assets exceeds liabilities and the solvency limit slightly underestimates the required solvency capital to ensure solvency at one-year horizon. A typical magnitude of the leverage effect is of the order of 1% of liability when the solvency capital is 10% of liability.

Negative correlation between two assets reduces risk associated with the portfolio. For example, a fund with only short equity position has equity risk (essentially by definition), however, when coupled with a long equity fund, the short fund actually reduces equity risk. Hence, we believe that funds with strong negative correlations should be classified in the same way as risk-reducing derivatives. This, however, is not how they are classified at the moment and the statutory classification scheme thus does not reflect the risk characteristics of a portfolio containing a short fund.

We have argued that classification of assets based on VaR improves the risk assessment of a portfolio in the computation of solvency limit. Hence, we believe that it would be appropriate to define $VaR_{97.5\%}$ points for each subclass in the statutory framework for the purpose of classification in addition to the now-defined representative subclass volatilities.

# References


Biström, P – Elo, K – Klaavo, T – Risku, I – Sihvonen, H (2008) **Statutory pensions in Finland. Long-term projections 2007.** Finnish Centre for Pensions, Reports 2008, 1.

Brooks, C – Kat, H M (2002) **The statistical properties of hedge fund index returns and their implications for investors.** J. Alternative Investments 5, 25–44.

Geltner, D (1993) **Estimating market values from appraised values without assuming an efficient market.** J. Real Estate Research 8, 325–345.

Getmansky, M (2005) **The life cycle of hedge funds: fund flows, size and performance.** UMASS, Amherst working paper, Amherst, MA.

Godfrey, L (1978) **Testing for higher order serial correlation in regression equations when the regressors in-clude lagged dependent variables.** Econometrica 46, 1303–1310.

Jorion, P (2000) **Value at Risk: The new benchmark for Managing Financial Risk.** 2nd edition, McGraw-Hill, New York.

Lambrigger, D D – Shevchenko, P V – Wüthrich, M V (2007) **The quantification of operational risk using in-ternal data, relevant external data and expert opinions.** J. Operational Risk 2, 3–27.

Ljung, G – Box, G (1978) **On a measure of lack of fit in time series models.** Biometrika 65, 297–303.

Lhabitant, F-S (2004) **Hedge funds: quantitative insights.** John Wiley & Sons, New Jersey.

Markowitz, H (1952) **Portfolio selection.** J. Finance 7, 77–91.

National Bureau of Economic Research NBER (2003) **Memo of business cycle dating committee.** Oct 21, 2003 (http://www.nber.org/cycles/recessions.pdf).

National Bureau of Economic Research NBER (2008) **Determination of the december 2007 peak in economic activity.** Dec 11, 2008 (http://www.nber.org/cycles/dec2008.pdf).

Nelson, R B (1999) **An introduction to copulas.** Springer, New York.

Pukelsheim, F (1994) **The three sigma rule.** Amer. Statist. 48, 88–91.

Puustelli, A – Koskinen, L – Luoma, A (2008) **Bayesian modelling of financial guarantee insurance.** Insurance Math. Econom. 43, 245–254.

Rachev, S T – Hsu, J S J – Bagasheva, B S – Fabozzi, F J (2006) **Bayesian method in finance.** John Wiley & sons, New Jersey.

Schisterman, E F – Moysich, K B – England, L J – Rao, M (2003) **Estimation of the correlation coefficient using the Bayesian approach and its applications for epidemiologic research.** BMC medical research methodology 3, 5.

Spiegelhalter, D J– Thomas, A – Best, N – Lunn, D (2008) **OpenBUGS version 3.02.** user manual.

Stewart, F (2007) **Pension fund investment in hedge funds.** OECD working papers on insurance and private pensions, 12. OECD Publishing. doi:10.1787/086456868358.

Tuomikoski, J (2000) **Financial solidity of pension insurance companies and pension funds within the Finnish employment pension system.** Transactions of the 10th international AFIR colloquium, Tromsø.

Zangari, P (1996) **Riskmetrics technical document.** Riskmetrics.

# Appendix

## Derivation of the solvency limit

Given the somewhat obscure form of equation (2.1), it is not self-evident that equation (2.1) is based on the Markowitz portfolio theory. In the following, we show that equation (2.1) can be derived as $VaR_{97.5\%}$ point of a portfolio in the Markowitz portfolio theory, if we assume that equity tracking error is the sixth asset class.

Assume that correlation matrix is given in Table A1, volatility of class TE is S, and the expected return of class TE is zero, all other parameters are as in Table 1. Equation (2.1) can be derived as the 97.5th percentile of portfolio return distribution. Portfolio variance is $w^T\Sigma w$, while the expected excess return of a portfolio is $\mu$–t, and hence $VaR_{97.5\%}$ point is

$$-(\mu - t) + \Phi^{-1}(0.975)\sqrt{(w^T\Sigma w)} \tag{A1.1}$$

where $\Sigma$ is the covariance matrix defined by correlation matrix in Table A1 and by volatilities computed as in Chapter 2, t is the required return, $\mu$ is the expected return, w is the vector of portfolio weights. Function $\Phi^{-1}$ is the inverse of cdf of the normal distribution

When portion $\lambda$ of assets is allocated to asset class TE and the scaling of parameters in Table 1 is taken into account, the solvency limit is defined as the excess ratio of assets to liabilities needed for the company to be solvent after one year at 97.5 per cent probability, that is

$$[-(\Sigma_i \beta_i m_i - t) + a\sqrt{(\Sigma_{ij}\beta_i\beta_j s_i s_j r_{ij} + \lambda^2 S^2)}]/100 \tag{A1.2}$$

which coincides with equation (2.1). In equation (A1.2), it is denoted $a = \Phi^{-1}(0.975)$. It should be noted that it is assumed here that the assets in excess of the solvency limit are invested in risk-free zero-return instruments, since equation (A1.2) yields a value relative to the liabilities.

Table A1.

The expanded correlation matrix with equity tracking error (TE) as a class. The major asset classes are denoted by Roman numerals.

| | I | II | III | IV | V | TE |
|---|---|---|---|---|---|---|
| I | 1 | 0.3 | 0 | 0 | 0.2 | 0 |
| II | 0.3 | 1 | 0 | 0 | 0.2 | 0 |
| III | 0 | 0 | 1 | 0.4 | 0 | 0 |
| IV | 0 | 0 | 0.4 | 1 | 0 | 0 |
| V | 0.2 | 0.2 | 0 | 0 | 1 | 0 |
| TE | 0 | 0 | 0 | 0 | 0 | 1 |

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