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Research Department

22.9.1994

## Risk Sharing in the Pricing of Payment Services by Banks

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# Risk Sharing in the Pricing of Payment Services by Banks

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## Abstract

The banking industry has traditionally covered a large part of its operating costs by net interest earnings, based on the spread between deposit and lending rates. This reflects the common practice of underpricing various services provided to customers, especially depositors. The purpose of this paper is to present an explanation to this phenomenon by analyzing the pricing of transaction deposit accounts as arrangements for pooling transaction cost uncertainty among depositors. It turns out that, when transactions are stochastic, and depositors are risk averse, there is an incentive to minimize explicit transaction charges. Moral hazard may explain why some service charges are applied, however.

## Tiivistelmä

Pankit ovat perinteisesti kattaneet merkittävän osan toimintakustannuksistaan korkokatteen avulla ja vastaavasti alihinnoitelleet palveluja, joita ne tarjoavat asiakkailleen, erityisesti tallettajille. Tässä tutkimuksessa osoitetaan, että tämä ilmiö on selitettävissä tarkastelemalla käyttelytilien hinnoittelua vakuutusjärjestelmänä transaktiokustannusten epävarmuutta vastaan. Tällöin kilpailullisilla markkinoilla pankit pyrkivät minimoimaan transaktiopalvelujen suoraa hinnoittelua. Moral hazard-tekijä (joka tässä merkitsee palvelujen alihinnoittelun vaikutusta niiden käyttöön) voi kuitenkin selittää, miksi palvelumaksuja jonkin verran käytetään.

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# 1 Introduction

The banking industry has traditionally covered a large part of its operating costs by net interest earnings, based on the spread between deposit and lending rates. This reflects the common practice of underpricing various services provided to customers, especially depositors. In recent years, banks in many countries have attempted to move towards "direct pricing", meaning more reliance on activity-based service charges. As a result, the pricing of transaction deposits,<sup>1</sup> in particular, has gradually become more sophisticated in many banking markets, but interest rate spreads and underpricing of transaction services have remain ubiquitous (see Vittas et al. (1988) for an international survey). Both the special features of traditional bank service pricing and the recent changes call for a better understanding of the nature of the transaction deposit markets.

The problem has in fact both a positive and a normative dimension. From a positive point of view, one may ask why banks often seem to "cross-subsidize" transaction services provided to depositors (these services include cheque processing, giro transfers, card-activated payments etc.). Put in another way, it can be asked why depositors seem to accept "implicit interest" in the form of free or underpriced services instead of requiring explicit, pecuniary return for their funds. From a normative point of view, in turn, the question is what the observed pricing patterns reveal about the operation of the market for bank services, and what kind of changes, if any, are advisable in the public policies influencing bank pricing.

The standard conjecture in the literature is that in perfectly competitive, frictionless markets, competition among banks would establish an equilibrium in which the explicit interest on transaction deposits would equal the marginal (opportunity) cost of funds to the bank – perhaps the money market rate. Parallel to that, transactions services provided by banks would be priced according to their marginal factor cost (see Fischer, (1983) and Saving (1979), for example). This kind of bank pricing system has been labelled "the Johnson norm", after Harry G. Johnson who advocated it from the efficiency perspective (Johnson, 1968). According to this view, then, any deviations from the "Johnson norm" type of pricing must be caused by regulation or other market imperfections disturbing the competitive price mechanism.

It is of course obvious that if authorities impose ceilings on deposit interest rates, competition for deposits will bring about "implicit interest" (Startz, (1979, 1983)). Other types of intervention have also been considered: Walsh (1983) has pointed out that, if explicit interest is taxable, it may be optimal for banks to substitute implicit for explicit interest even in a competitive environment without deposit rate ceilings. The tax argument is generalized further by Tarkka (1992). In the tax based models, the marginal tax rate of interest income is balanced with the marginal efficiency loss of rewarding depositors in kind (with free, non-marketable services).

The purpose of this paper is to present an alternative explanation of the stylized facts of deposit pricing. This explanation does not rely on distortions

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<sup>1</sup> "Transaction deposits" is a generic term for all deposits which can be used as means of payment. These include cheque accounts, and various other types of salary accounts with names varying between countries.

created by regulation, taxes, or less than perfect competition. Instead, it is shown that implicit interest may serve as a device which reduces the exposure of depositors to the uncertainty regarding their transaction needs and the costs invoked by those needs. If banks are risk-neutral, or at least able to pool away the transactions uncertainty faced by individual depositors, even the competitive equilibrium may well include "below-cost" pricing of transactions services, and significant interest rate spreads set to compensate for the revenue foregone in the transactions service business.

Previously, demand deposit contracts have been analyzed from the risk sharing perspective by Diamond and Dybvig (1983), Smith (1984) and Jacklin (1987). These contributions focus on the "early withdrawal" characteristic of the deposits, resulting from the banks' capacity to perform maturity transformation. Essentially, the maturity transformation analysis provides a rigorous analysis of the effect of reserve holding costs on the deposit interest rates. The resource costs of payment services and the pricing of these services has been almost entirely overlooked in the risk sharing literature, however. In Tarkka (1989), the sharing of transaction cost risk is used to explain the viability of the implicit interest in competitive deposit markets. However, that analysis disregards moral hazard. In the present study, this limitation is avoided by applying the optimal contract (principal-agent) framework to the deposit pricing problem.

The problem at hand can be seen as a part of a fundamental issue in monetary theory, concerning the determination of the rate of return on money in a free competitive equilibrium. The question why people voluntarily hold low-yielding, monetary-like assets was presented by Hicks (1935) and it has been a subject of controversy in the recent years, too. According to the widely quoted "legal restrictions hypothesis", equilibria in which money yields a lower rate of return than other assets is possible only because government intervention restricts free competition in the supply of means of payment (see Black (1970) and Wallace (1983)). According to the "transactions cost hypothesis", by contrast, various accounting and administrative costs may be used to explain why assets which are inferior to others in terms of their yield are voluntarily held and used as means of exchange (White, (1987)). The present study contributes to the legal restrictions controversy by suggesting that the uncertainty of transaction costs may well be the basic reason why a low-yielding means of payment may be part of a competitive equilibrium.

The structure of the paper is as follows. In section 2, a simple benchmark model is considered in which competition forces banks to provide transactions services "free of charge", and to cover the costs involved by lowering the rate paid on deposits (indirect pricing). In section 3, the model is generalized by allowing moral hazard effects in the demand for transaction services, which explains the mixed use of direct and indirect pricing of transaction services. The results on optimal prices are derived by assuming representative agents; the problem of heterogeneous depositors is taken up in section 4, where some sufficient conditions are given for the results of the previous section to hold also when there is heterogeneity. Section 5 summarizes and discusses the results.

## 2 Uncertain Transaction Costs and Risk Sharing

We consider a competitive market for transaction accounts. These accounts provide deposits with a cheque, giro, or electronic transfer facility so that the account may be used as a store for transactions balances. Such an account can be viewed as vehicle for joint delivery of two distinct bank services, i.e. depository and transactions services. Obviously, the customer uses the depository service whenever she keeps some funds in the account. Transactions services, on the other hand, are used whenever the funds in the account are used as means of payment. To execute payments, the bank performs operations such as cheque clearings or payment transfers.

The demand for bank services is derived from a very simple transactions framework, which is a variant of the "cash in advance" models. We consider a single planning period, so confining the analysis to static equilibria. There is a large number of customers. During the period of analysis, each customer spends a given amount of money. The customers are different from each other with respect to the amount of money they spend, however. The key feature of the model is that the number of transactions in which the money is spent is random. This randomness is idiosyncratic in nature. In order to focus on the effects of uncertain transaction costs, other sources of uncertainty are disregarded. So, for instance, the value of income which is spent during each period is assumed to be known by the customers at the beginning of the period.

More precisely, the representative customer receives an endowment  $Y$  at the beginning of each period, holds it in the form of deposits during the period and spends it at the end of the period. Spending takes place in a random number of transactions, each paid separately. The uncertainty regarding the number of transactions is modelled by assuming that the average size of the transactions is a random variable  $v$ . Let us denote the demand for deposit balances by  $M$  and the number of transactions required by  $N$ . Then, the customer's demand for bank services is given by the equations

$$M = Y \tag{1}$$

$$N = \tilde{v} \cdot Y \tag{2}$$

$$P(\tilde{v} \leq v) = F(v) \tag{3}$$

The size-of-transactions variable  $\tilde{v}$  is independently and identically distributed across customers. In this section the probability distribution  $F(v)$  is taken to be exogenous. Moral hazard is therefore not present. This assumption will be relaxed in a later section of this paper, however.

The customer chooses between different banks by maximizing the expected value of a convex utility function

$$W = E(U(Y - T)) \quad (4)$$

where  $E$  is the expected value operator and  $T$  denotes the net payments from the depositor to the bank, i.e. service charges net of deposit interest. If service charges are greater than deposit interest payments,  $T$  is positive; negative values are also possible, of course. Generally, banks will determine  $T$  as function of the depositor's characteristics and her conduct. This kind of functions, which may be nonlinear, are often called tariff functions (see Wilson (1993)).

Here we assume that banks can make the tariff  $T$  conditional on the use of bank services only. Thus, no external characteristics of customers can be used as a basis of bonuses or discounts. This implies that the pricing policies of banks can be characterized by tariff functions of the type

$$T = T(M, N) \quad (5)$$

The partial derivatives of the tariff function define the marginal interest rate on deposits  $i_m$  and the marginal service charge  $p_m$  on transactions in the following way:

$$i_m(M, N) = -\frac{\partial T(M, N)}{\partial M} \quad (6a)$$

$$p_m(M, N) = \frac{\partial T(M, N)}{\partial N} \quad (6b)$$

The exclusion of the customers' characteristics from the tariff function could imply serious informational problems. In the present model, however, the consumers' characteristics are completely revealed by their conduct, and reflected in quantities which can enter the tariff function. More precisely, the banks can condition their tariffs on an exact indicator  $M$  of the customer's characteristic  $Y$ , even though the latter is "hidden" in principle. Therefore, the deposit market is in the end not distorted by information asymmetry.

The competition in the deposit market is assumed to work as follows. In the first stage, each bank posts a tariff function  $T(M, N)$ , thus fixing the terms it offers to its depositors. In designing this function, the banks have only aggregative information on the depositors. On the basis of the distribution of  $v$ , the depositors compare the banks, choosing one of the banks promising highest expected utility  $W$ .

In this kind of market, with free and costless entry, competition ensures that the only tariffs which can survive in the market are those which give the depositors highest possible utility, subject to the constraint that the expected profit from each deposit constraint must be zero. By symmetry, of course, all banks post the same tariff function in the equilibrium. This function can be derived by maximizing  $W$  subject to the break-even constraint.

Now we turn to describe the nature of the break-even constraint. The profit of a representative bank from a representative deposit relationship is

$$\pi = T + r \cdot M - c \cdot N \quad (7)$$

where  $c$  is the unit cost of transaction services and  $r$  is the rate of return the bank earns on funds. Both  $c$  and  $r$  are assumed to be known constants.

Competition ensures that the  $E(\pi) = 0$ . So, in equilibrium,

$$E(T) = c \cdot E(N) - r \cdot E(M). \quad (8)$$

Note that  $E(N) = E(v) \cdot E(Y)$  and  $E(M) = E(Y)$ . So, in principle,

$$E(T) = [c \cdot E(v) - r] \cdot E(Y). \quad (9)$$

The bank does not observe  $Y$  directly and is unable to use it as an argument of its tariff function. However, the present model has the convenient property that  $M = Y$  and consequently a customer's  $Y$  can be inferred from her deposits. Thus, any equilibrium tariff must satisfy the following property:

$$E(T) = [c \cdot E(v) - r] \cdot M \quad (10)$$

The actual form of the equilibrium tariff  $T(M, N)$  can now be found by maximizing the utility of the representative customer subject to the constraint (10). Formally, this problem can be presented in the form of the following program:

$$\max_T W = \int f(v) U(Y - T) dv \quad (11)$$

$$\text{s.t.} \quad \int f(v) [cv - r] Y dv = \int f(v) T dv$$

which is equivalent to

$$\max_T W = \int f(v) \{ U(Y - T) dv + \lambda [T - (cv - r) Y] \} dv \quad (12)$$

Now, the first-order necessary condition for the solution to this program is simply

$$\frac{\partial U(Y - T)}{\partial T} = -\lambda \quad (13)$$

where  $\lambda$  is the Lagrange multiplier associated with the zero-profit constraint. The result (13) implies for the marginal utility of income

$$\frac{\partial U(Y-T)}{\partial Y} = \lambda \quad (14)$$

This formula has a simple, but crucial interpretation. In this simple case without moral hazard, the optimal deposit contract makes the depositor's marginal utility a constant, i.e. independent of the transactions cost uncertainty. In the optimum, insurance is thus complete. In particular,

$$T(M, N) = E[T(M, N)] \quad (15)$$

i.e. the tariff is based only on certain variables. By (10), the equilibrium tariff must be the following:

$$T(M, N) = [c \cdot E(v) - r] \cdot M \quad (16)$$

Now, due to the linearity of the tariff function just derived, the marginal deposit rate and the marginal service charge are constants. They are defined as

$$i_m = -\frac{\partial T(M, N)}{\partial M} = r - c E(v) \quad (17)$$

$$s_m = \frac{\partial T(M, N)}{\partial N} = 0 \quad (18)$$

The equilibrium deposit rate equals the banks' opportunity cost of funds, less the cost (per one unit deposited funds) of providing the depositor with the required transaction services. The service charge is zero.

These results are straightforward implications of the basic theory of competitive insurance markets without moral hazard. The simplicity of the optimal tariff results from the fact that in this model, the parameters of the probability distribution of transaction costs are completely revealed by the customers' deposit balances. By pricing the transaction account exclusively on the basis of deposit balances, i.e. through the interest rate spread, the bank is able to provide complete and fairly priced insurance to all deposit customers.

This result is interesting because it constitutes an example in which "free" transaction services are not necessarily due to any frictions or distortions in the price mechanism. Rather, in the simple model just presented, the underpricing of transaction services and the accompanying interest rate spread result from a first-best, competitive insurance arrangement. While obviously not proving anything about actual deposit markets, this demonstrates that "the user pays principle" and efficiency should not be casually equated, also not in banking.

There are two obvious caveats, however. Both have to do with distortions which may prevent the first-best optimum such as described above from being realized. First, what happens if the deposit balances do not accurately reveal the probability distribution of transaction costs? Does the problem of adverse

selection arise in that case? Second, what if the use of transactions services is endogenous, so that the "free" services characteristic of the complete insurance solution induces the customer to increase the number of transactions? How is the optimal pricing system changed by the ensuing moral hazard problem? These questions will be addressed in the next section.

### 3 The Case with Moral Hazard

We now turn to the problem of extending the simple model of the previous section to cover cases in which there is a greater role for endogenous behaviour. Essentially, the changes to the model are two. First, the simplistic, mechanical identity between the average deposit balances and the depositor's expenditure is relaxed; second, the expected number of transactions required for a given expenditure flow is made dependent on the depositor's behaviour, and hence on the incentives provided by the bank. With such extensions, the risk sharing model can be used to analyze more complex deposit pricing schemes than the one described above. In particular, it turns out that the levying of a service charge on transaction services can now be explained.

The analysis will be carried out under the restriction that the banks apply a linear tariff. Hence, the representative customer faces a given deposit interest rate and a given service charge (fee) for each transaction. The banks' tariff design problem then simplifies into the task of determining the parameters  $i$  (the deposit rate) and  $p$  (the service charge) of the tariff function

$$T = p \cdot N - i \cdot M. \quad (19)$$

The linear tariff is not rare in banking, although more complex tariffs have been gaining in popularity (see Vitas (1988) for example). Still, the main reasons for analyzing the linear case here are tractability and robustness. As is well known, only very weak results can be derived from the theory of (nonlinear) optimal risk sharing contracts except in special cases or unless extremely stylized models are used. Often, almost all that can be said is that an optimal contract must provide partial insurance to the risk averse party. More serious, even the existence of a well-defined optimum contract requires rather stringent conditions as regards the stochastic specification of the problem (cf. Mirrlees (1974), Holmström (1979), Rogerson (1985), Jewitt (1988)).<sup>2</sup> The linear case, by contrast, is applicable to wider set of problems (see Varian (1980) for an example).

Another simplifying assumption is that the analysis will be carried out using the representative agent approach. Hence, the consequences of customer heterogeneity are mostly disregarded. This issue is, however, discussed briefly in a later section.

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<sup>2</sup> On the other hand, linear contracts can be shown to be optimal in certain cases (Holmström and Milgrom (1993)).

### 3.1 The Transactions Framework

The simplest way to introduce some flexibility into the transactions framework is to allow the consumer-depositor to choose between two alternative means of payment, e.g. deposits and currency. We adopt this approach, introducing a choice parameter  $\theta$  which governs the fraction of expenditure  $\theta Y$  which is spent through bank deposits. The remaining part of the expenditure flow  $(1-\theta)Y$  is assumed to be spent through currency. Retaining the cash-in-advance framework in other respects, the demand for deposits can now be written as

$$M = \theta \cdot Y \quad (20)$$

The number of payment transactions has an additive random component:

$$N = k \cdot \theta^\rho \cdot Y + e \quad (21)$$

It is expected that  $\rho > 1$ , implying that if the depositor increases the share of her expenditure which is paid with deposits, the number of transactions increases relatively faster than the deposit holdings. This phenomenon could result from the fact that the average size of currency transactions is smaller than the average size of deposit transactions.<sup>3</sup> Hence, channelling more of the expenditure through the bank account, will decrease the average size of the transaction.

The stochastic term  $e$  in the transactions equation is assumed to have a zero mean and a well-behaved frequency distribution. It should be noted that in the present formulation, the number of payment transactions is approximated by a continuous variable. This does not, however, appear to have any important consequences in terms of the results obtained below.

In this specification, the use of transaction services is endogenous, and the expected transaction costs cannot be unambiguously estimated from the stock of deposit balances. In these respects, the formulation is more general than the simple benchmark case of section two.

### 3.2 The Depositor's Problem

The individual utility functions of depositors are written in a fashion which has become common in the principal-agent (optimal contract) literature:

$$W = U(Y - T) + g(\theta). \quad (22)$$

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<sup>3</sup> Whitesell (1989, 1992) has analyzed models of bank customer behaviour which have these properties.

The utility function consists of two additive terms. The first one includes the utility derived from income, net of charges  $T$  to the bank. The function  $U(\cdot)$  is assumed to be continuous, twice differentiable and to exhibit the following standard properties:

$$U'(Y-T) > 0 \quad (\text{monotonicity})$$

$$U''(Y-T) < 0 \quad (\text{strict concavity})$$

$$U'''(Y-T) \geq 0 \quad (\text{nonincreasing absolute risk aversion})$$

where ' and '' and ''' denote first, second and third derivatives, respectively. The concavity of the utility function with respect to  $T$  implies risk aversion which plays a crucial part in the model. The assumption that absolute risk aversion is nonincreasing is fairly standard in the literature on decision making under uncertainty.

The second term in (22) captures the direct utility effects of using the transaction services of a bank. The larger part of one's expenditure is paid from the account, the less one's own effort is required for the execution of the transactions. The  $g(\cdot)$  function is assumed to be continuous, twice differentiable and to exhibit the following properties:

$$g'(\theta) > 0 \quad \text{and}$$

$$g''(\theta) > 0.$$

Under the above made assumptions on the form of the tariff and on the specification of the transactions technology, we have

$$T = p(k \cdot \theta^\rho \cdot Y + e) - i \cdot \theta \cdot Y \quad (23)$$

Observing this, the problem of the representative depositor becomes:

$$\max_s E(W) = E[U(Y - p \cdot e - p \cdot k \theta^\rho \cdot Y + i \cdot \theta \cdot Y)] + g(\theta). \quad (24)$$

From this, the following first-order necessary condition for maximum can be derived:

$$E(W') = -E[U'(Y - p \cdot e - p \cdot k \theta^\rho \cdot Y + i \cdot \theta \cdot Y)](p \cdot k \rho \theta^{\rho-1} - i) \cdot Y + g'(\theta) = 0, \quad (25)$$

or, for short,

$$E(U') \cdot (p \cdot k \rho \theta^{\rho-1} - i) \cdot Y = g'(\theta). \quad (26)$$

The first-order condition implicitly defines  $\theta$  as a function of  $p$ ,  $i$ , and other parameters of the depositor's problem, provided that the problem is well-behaved; the latter may be checked by inspecting the concavity of the objective function with respect to  $\theta$ . The depositor's second-order condition, which ensures concavity, may be expressed as the following inequality:

$$\frac{dE(W')}{d\theta} = A - B + q''(\theta) < 0, \quad (27)$$

$$\text{where } A = E(U'') \cdot (pk\rho\theta^{\rho-1} - i)^2 \cdot Y^2$$

$$B = E(U') \cdot pYk\rho(\rho-1)\theta^{\rho-2}$$

In the inequality, the term  $A$  is known to be negative by the concavity of the utility function, except in the special case in which  $i = pk\rho\theta^{\rho-1}$  and  $A = 0$  will result. The term  $B$  is unambiguously positive, if  $p$  is positive; and finally, the term  $g''(\theta)$  is negative by assumption. All in all, to ensure the validity of the second-order condition, it suffices to show that  $p$  is not negative. Below, it will be established that in a competitive deposit market, it will always be the case that  $p \geq 0$  and thus the second-order condition holds.

### 3.3 The Bank's Pricing Problem

With a linear tariff, the bank's profit from the representative deposit relationship can be written as

$$\pi = (p - c) \cdot N + (r - i) \cdot M \quad (28)$$

Taking into account the transactions framework specified above, this can be written as

$$\pi = (p - c) \cdot (\theta^\rho Y + e) + (r - i) \cdot \theta Y \quad (29)$$

As in section 2 above, the assumptions of perfect competition and free entry with the same information available to all banks exclude the possibility that the expected profit from any deposit relationship could be different from zero. Therefore, the pricing parameters in the tariff function must be such that  $E(\pi) = 0$ , which implies

$$i = r + (p - c) \cdot \theta^{\rho-1} \quad (30)$$

In the competitive deposit market, the bank must offer depositors the tariff which maximizes each depositor's ex ante utility subject to the break-even constraint, as given by the expression (30).

Formally, the bank's problem is then

$$\max_p E(W) = E[U(Y - p \cdot e - p \cdot k\theta^p \cdot Y + i \cdot \theta \cdot Y)] + g(\theta). \quad (31)$$

so that  $i = r - (c - p) \cdot \theta^{p-1}$  (break-even constraint)

and  $\theta = \theta(p)$  (incentive constraint)

The function  $\theta = \theta(p)$  is implicitly defined by the depositor's first order optimality condition, taking into account the break-even constraint to eliminate  $i$ .<sup>4</sup>

Substituting the break-even constraint and the incentive constraint into the maximand (i.e. into the expression for  $E(W)$  in (31)), the bank's tariff problem may be converted to an unconstrained maximization problem:

$$\max_p E(W) = E[U(Y - pe - Y \cdot c \cdot (\theta(p))^p + Y \cdot r \cdot \theta(p))] + g(\theta(p)). \quad (32)$$

The first-order necessary condition for maximum reads now

$$E[U'(Y - T)(r \cdot Y \cdot \theta' - c \cdot Y \rho \theta^{p-1} \theta' - e)] + g'(\theta) \cdot \theta' = 0 \quad (33)$$

Here the symbol  $\theta'$  denotes the derivative of  $\theta$  with respect to  $p$  when  $i$  is allowed to change in a way specified by the bank's break-even constraint. It is possible to show that  $\theta' < 0$ , meaning that higher service charges must lead to a reduction both in deposit holdings and in the expected number of transactions. This is proven in Appendix 1.

The bank's first order condition may be simplified further to yield

$$E(U' \cdot e) - E(U') \cdot (r - c\rho\theta^{p-1}) \cdot Y \cdot \theta' = g' \cdot \theta' \quad (34)$$

Using the property that  $\theta' < 0$ , the depositor's first-order condition (32) may be multiplied by  $\theta'$  to yield the equation

$$E(U') \cdot (pk\rho\theta^{p-1} - i) \cdot Y \cdot \theta' = g' \cdot \theta'. \quad (35)$$

Next, this equation (35) may be combined with the bank's first order condition (34), enabling us to eliminate  $g' \cdot \theta'$ :

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<sup>4</sup> To be accurate, the function  $\theta(p)$  includes also  $Y$  and  $\rho$  among its arguments. These depositor characteristics are, however, dropped here to simplify notation. This should not cause any loss of clarity in the present section where the representative agent assumption (identical depositors) is used.

$$E(U' \cdot e) - E(U') \cdot (r - c\rho\theta^{p-1}) \cdot Y \cdot \theta' = E(U') \cdot (pk\rho\theta^{p-1} - i) \cdot Y \cdot \theta' \quad (36)$$

This yields, upon simplification,

$$E(U' \cdot e) = E(U') \cdot [r - i - (c - p) \cdot k\rho\theta^{p-1}] \cdot Y \cdot \theta' \quad (37)$$

Finally, substituting the break-even constraint for  $(c-p)$ , we get

$$E(U' \cdot e) = E(U') \cdot (r - i)(1 - \rho) \cdot Y \cdot \theta' \quad (38)$$

This formula defines the optimal interest rate spread as predicted by the model. If the model works properly, the spread should be positive, of course. To study whether this is the case, it is useful to note that  $E(U' \cdot e) = \text{Cov}(U', e)$  which is due to the assumption that  $E(e) = 0$ . Now,  $d(U')/de = -p \cdot U''$  which has the same sign as  $p$  and is zero if  $p = 0$ .

On the basis of this result, it is easy to check that the optimal interest rate spread is indeed generally positive. This may be done by showing that a zero spread  $r-i = 0$  and a negative spread  $r-i < 0$  both would lead to a contradiction.

Consider the possibility of a zero spread first. Under the assumption that the utility function  $U(Y-T)$  and the incentive function  $\theta(p)$  are continuous and everywhere differentiable, a zero spread would imply that the right hand side of the equation (38) would be equal to zero. By the break even constraint, a zero spread also implies  $p = c$ . However, the left hand side of the equation is zero only if  $p = 0$ , which leads to a contradiction.

A negative spread would make the right hand side of the equation negative. With a negative spread,  $p = c - (r-i)/\theta^{p-1} > c$ . However,  $p > c$  is not possible, because the left hand side is negative only if  $p < 0$ . Thus the negative spread also leads to a contradiction. It can be concluded that  $(r-i) > 0$ .

The result that the equilibrium interest rate spread is positive implies that the equilibrium service charge  $p$  is lower than the marginal cost of transaction services,  $c$ . This is easily seen from the break-even constraint, e.g. in the form  $p-c = -(r-i)/\theta^{p-1}$ . The service charge will generally be positive, however. Negative service charges would make the covariance term on the left hand side of (38) negative, leading into contradiction with the positive right hand side. Finally, zero service charges would imply  $E(U' \cdot e) = 0$ , which is also in contradiction with the positive right hand side of (38). It can thus be concluded that  $c < p < 0$ .

Some insights into the effects of various obstacles for risk sharing in deposit markets can also be derived from the formula for optimal interest rate spread. One pertains to the role of moral hazard. This is captured by the derivative  $\theta'$  which measures the effects of the bank's pricing policy on depositor behaviour. The greater is that derivative in absolute terms, the bigger obstacle moral hazard should constitute for the insurance function of deposit banks. This intuition is confirmed by inspection of the formula (38). If  $\theta'$  is increased in absolute terms, then, the interest rate spread must diminish. As the interest rate spread is reduced, the service charge  $p$  will rise towards the

marginal cost  $c$  of transaction services. This increases the covariance term on the left hand side of the equation, thus reinforcing the effect of  $\theta'$  on  $(r-i)$ .

Another insight can be developed on the effects of the properties of the transaction framework. Call the ratio  $N/M$  the "velocity" of the deposit account. Our assumptions on the transaction framework imply that we can write  $E(N/M) = \theta^{\rho-1}$ . We see that the parameter  $\rho$  governs the extent to which depositors' behaviour influences the (expected) velocity of deposits. We assumed above that  $\rho > 1$ . The larger  $\rho$ , the more influence depositors have on velocity; if  $\rho$  decreases, approaching 1, the velocity will become totally exogenous in the limit. If other factors in the equation (38) are held constant, changes in  $\rho$  will have an effect on the interest rate spread and the service charge. The more behaviour influences velocity, the smaller is the interest rate spread (and the higher is the service charge). The more exogenous the velocity becomes, the larger is the interest rate spread and the more full is the insurance given against liquidity cost uncertainty.

## 4 A Case of Heterogeneous Depositors

The analysis presented in the previous section was conducted within the representative agent framework. This approach is obviously limited in the sense that the results may not be valid in situations where agents are in fact heterogeneous with respect to some relevant characteristic or characteristics. The interesting kind of heterogeneity is with respect to a variable which cannot be used as a basis for first-degree price discrimination, either because the customer-specific characteristics are unobservable or because price discrimination is not allowed on the basis of characteristics in question.

In the context of the deposit market, differences in income levels probably constitute an important source of heterogeneity, and it would be desirable to be able to analyze the effects of this heterogeneity. The task is nontrivial, however, since the analysis of optimal contracts is extremely difficult in situations combining moral hazard (hidden action) and adverse selection (hidden information) problems. It is therefore necessary to focus on some tractable special cases and try to derive some general insights from them.

Presently, one simple case is given in which the heterogeneity of depositors with respect to income can be allowed without disturbing the results derived in the previous section. The case amounts to making a specific assumption on the type of the utility function and the nature of transactions uncertainty.

First, assume that the depositors' utility functions are logarithmic with respect to income (this case obviously satisfies the assumptions used in section 3 on the shape of the utility function):

$$W = \text{Log}(Y - T) + g(\theta) \tag{39}$$

Second, assume that the transactions uncertainty is proportional to income. This means that the customer-specific random variable  $e$  can be presented as  $e = w \cdot Y$  where  $w$  is a random variable which has a finite support and is i.i.d across all depositors.

Under these assumptions, we have

$$U'(Y-T) = \frac{1}{(Y-T)} \quad (40)$$

and

$$T = (p \cdot k\theta^p + p \cdot w - i \cdot \theta) \cdot Y \quad (41)$$

so that

$$U' = \frac{1}{[Y \cdot (1 - p \cdot k\theta^p - p \cdot w + i \cdot \theta)]} \quad (42)$$

Note that for the marginal utility to be well defined for all  $w$ , the variable  $w$  must never obtain too large positive values. This puts some limits on the support of that variable.

It is easy to check that the depositor's first order condition (25) now becomes

$$(p \cdot k\rho\theta^{p-1} - i)E\left[\frac{1}{(1 - p \cdot k\theta^p + p \cdot w - i \cdot \theta)}\right] = g'(\theta). \quad (43)$$

The choice or "effort" parameter  $\theta$  is seen to be determined independently of  $Y$ . In equilibrium, all depositors therefore choose the same value for  $\theta$ . Of course, its derivative  $\theta'$  is now independent of  $Y$  too. This is important for the determination of the optimal tariff.

The first order condition for the optimal tariff (38) may now be written as follows:

$$E\left[\frac{w}{(1 - p \cdot k\theta^p - p \cdot w + i \cdot \theta)}\right] = (r - i)(1 - \rho) \cdot \theta \cdot E\left(\frac{1}{(1 - p \cdot k\theta^p - p \cdot w + i \cdot \theta)}\right) \quad (44)$$

This expression, together with the break even constraint, defines the optimal tariff in the present special case. The optimal tariff will of course have the same general properties which were derived in the more general case of section 3. There is, however, an interesting additional feature which stems from the fact that  $Y$  is no longer present in these optimality conditions. Therefore, in this special case, the equilibrium tariff is the same for all depositors, regardless of their income level.

## 5 Discussion

Transaction needs of deposit account holders are not entirely predictable. Therefore, deposit pricing according to the "Johnson norm", amounting to full marginal cost-based activity charges on transactions, leaves the depositor as the "residual claimant" of the transactions cost uncertainty. This would be justified if the depositors were risk neutral. On the other hand, risk averse depositors would prefer contracts which reduce the costs of surprise transactions. In a competitive banking industry, banks would be forced to take this into account and develop deposit pricing mechanisms which satisfy depositor needs better than the full cost-based pricing of services.

The analysis presented in this paper has demonstrated how banks can use underpricing of transaction (payment) services to offer their customers insurance against uncertainty related to transactions. In the simple case, in which uncertainty regarding transaction needs is the only source of risk, no moral hazard is present and the expected transaction costs are proportional to the deposit balances, full insurance is possible. Competitive banking industry will then provide payment services to depositors without explicit charges. The costs of producing these services will be covered by setting a broad enough spread between deposit and lending rates.

This simple pricing system may no longer be viable if expected transaction costs incurred by the bank from servicing a deposit account endogenous and not perfectly revealed by the deposit balance. By "endogenous" it is meant that transaction needs are influenced by the depositor's behaviour and ultimately by the price incentives given to her. This feature brings the element of "moral hazard" into the problem. In section 3 above, it was shown that when moral hazard is present in a tractable linear tariff framework, only incomplete insurance will be provided by competitive banking industry. More precisely, some service charges on transaction services will have to be instituted, but there will remain a "cross subsidy" on transactions in any case, in the sense that the service charges will be below marginal costs of producing transactions. This "subsidy" is financed from the interest margin.

These results are, in fact, quite contrary to the standard view of the nature and causes of the "implicit interest phenomenon" which is the term commonly used to describe the provision of underpriced transaction services to depositors. Usually, economists have seen the "user pays" system with no cross subsidy on transactions as the one which would prevail in competitive, undistorted markets. By contrast, in the models analyzed in this paper, the "first-best" solution would be to apply no service charges; but various frictions may alter the situation so that some service charges will be applied. Here, market imperfections are needed to explain the presence of service charges, whereas usually imperfections or regulation has been needed to explain the absence (or lowness) of direct service charges.

In risk sharing problems, moral hazard (hidden action) is not the only one potentially relevant source of frictions. In many instances, adverse selection (hidden information) is considered to be of primary importance. This has been considered in deposit pricing by Shaffer (1984), for instance. However, there are reasons why adverse selection (of depositors) may not be as great a problem in transaction deposit markets as in some other markets. These reasons follow

from the fact that the deposit relationship involves a continuous service relationship between the bank and the customer, and it is possible to recontract as new information on the other party is uncovered. Further, banks are in a good position to obtain information on the different characteristics of their deposit customers. This means that if depositor heterogeneity is important for the determination of service charges and depositor interest rates, the banks would probably be able to avoid the adverse selection problem by price differentiation.

The analysis presented in section 4 of this paper suggests, however, that the need for price differentiation is not necessarily implied by customer heterogeneity, not even when the customers are different with respect to income which implies heterogeneity in transaction needs, deposit holdings, and risk aversion. In that section an example was presented in which the same equilibrium tariff was valid for all depositors, regardless of their income. This special case, requiring a particular type of utility function, could perhaps serve as a starting point to future extensions of the models presented in this paper aiming to cover the case of heterogeneous depositors in a more general, yet tractable way.

One broad conclusion emerging from the analysis is that "low" or "inferior" rates of return on assets which help to shield their owner from uncertainty with respect to transaction costs may be compatible with free, competitive equilibrium. Applied to banking problems, this insight warns from casually equating interest rate spreads and "underpriced" services with inefficiencies and lack of price competition. In the models presented in this paper, low-yielding assets emerge as a part of an insurance arrangement between banks and their customers. The novelty here is in the nature of the uncertainty: previous research has focused on idiosyncratic risks in income, or time preference, but has not considered transaction cost uncertainty although this could prove to be the crucial element in understanding the nature of monetary services provided by banks.

The idea that banks may actually compete in providing transaction services below their actual cost gives new content to the concept of "liquidity creation" and may even suggest an avenue for further research attempting to give better foundations to the "transaction cost" theory of money and liquidity. In particular, this line of research could help to integrate monetary theory with the theory of banking, which theories have developed more and more independently of each other since the Diamond and Dybvig (1983) model and the legal restrictions hypothesis have begun to dominate their respective fields.

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## Appendix 1.

### Service Charges and Depositor Behaviour

In this appendix, the response of depositors to the pricing policy of banks is analyzed. Consider an increase in the service charge  $p$ , accompanied by such an adjustment in the deposit rate  $i$  as to keep the expected profits of the bank at the previous (zero) level. This implies that when  $p$  is increased, the deposit rate  $i$  must increase as well, as specified by the constraint  $i = r + (p-c)\theta^{\rho-1}$ . It will be proven that this kind of increase in  $p$  will lead to a decrease in the choice parameter  $\theta$ , and ultimately a reduction in deposit balances  $M$  and the expected number of payments  $E(N)$ .

By the implicit function theorem,

$$\theta' = -\frac{[dE(W')/dp]}{[dE(W')/d\theta]}, \quad (A1)$$

where  $E(W')$  denotes the partial derivative of the depositor's objective function with respect to  $\theta$ . Here, in taking the derivative with respect to  $p$ ,  $i$  must be allowed to vary to maintain  $E(\pi) = 0$ . Now, to ascertain that  $\theta' < 0$  it is sufficient to establish that  $dE(W')/dp < 0$ , for the denominator in (A1) is known to be negative. (This property is equivalent to the concavity of the objective function in  $\theta$  which was demonstrated in the body of the text for nonnegative values of  $p$ .)

What remains is to evaluate  $dE(W')/dp$  at the depositor's optimum. The starting point is the depositor's first order condition (31). Substituting the break-even constraint for  $i$  yields

$$E(W') = -E[U'(Y-T)] \cdot G \cdot Y + g'(\theta) = 0, \quad (A2)$$

where  $G = [p(\rho-1) + c]k\theta^{\rho-1} - r$ .

Note that the factor  $G$  must be positive for the first order condition to hold. After the application of the break-even constraint the tariff  $T$  is now

$$T = p \cdot e + c \cdot k\theta^{\rho} \cdot Y - r \cdot \theta \cdot Y \quad (A3)$$

Differentiating (A1) with respect to  $p$  gives

$$\frac{dE(W')}{dp} = E[U''(Y-T) \cdot e]G \cdot Y - E[U'(Y-T)] \cdot (\rho-1)k\theta^{\rho-1} < 0 \quad (A4)$$

The negativity of this expression can be demonstrated in the following way. Due to the obvious negativity of the second term, it suffices to show that the first term is nonpositive. Now, clearly  $E[U''(Y-T) \cdot e] = \text{Cov}[U'', e]$  and the

nonpositivity of the covariance results from the fact that  $d(U'')/de = -(U''')p$  which is nonpositive for all  $p \geq 0$  by the assumption made on the shape of the utility function (nonincreasing absolute risk aversion).

We have thus shown that  $\theta' < 0$ . Since  $M = \theta Y$  and  $E(N) = k \cdot \theta^p Y$  are increasing in  $Y$ , we have  $dM/dp < 0$  and  $dE(N)/dp < 0$ .

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