BANK OF FINLAND DISCUSSION PAPERS

3 • 2002

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Performance of monetary policy with internal central bank forecasting

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Finlands Banks diskussionsunderlag

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Support from the Academy of Finland, Bank of Finland, Yrjö Jahnsson Foundation and Nokia Group is gratefully acknowledged.

*This work was in part done when the first author was a visiting researcher to the European Central Bank and the second author a visiting scholar at the Bank of Finland. We thank these banks for research support and hospitality.

Suomen Pankin keskustelualoitteita
Finlands Banks diskussionsunderlag

http://www.bof.fi

ISBN 951-686-767-7
ISSN 0785-3572
(print)

ISBN 951-686-768-5
ISSN 1456-6184
(online)

Suomen Pankin monistuskeskus
Helsinki 2002

Performance of monetary policy with internal central bank forecasting

Bank of Finland Discussion Papers 3/2002

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Research Department

Abstract

Recent models of monetary policy have analysed the desirability of different optimal and *ad hoc* interest-rate rules under the restrictive assumption that forecasts of the private sector and central bank are homogeneous. In this paper, we study from a learning perspective the implications of heterogeneity across forecasts by the central bank and private agents for the performance of interest-rate rules.

Key words: adaptive learning, stability, heterogeneity, monetary policy

JEL classification numbers: E52, E31, D84

Rahapolitiikka ja keskuspankin talousennusteet

Suomen Pankin keskustelualoitteita 3/2002

Seppo Honkapohja – Kaushik Mitra
Tutkimusosasto

Tiivistelmä

Viimeaikaisessa rahapolitiikkaa koskevassa kirjallisuudessa on tarkasteltu erilaisten korkopoliittisten sääntöjen käyttökelpoisuutta olettaen, että keskuspankin ja yksityisen sektorin ennusteet ovat samanlaiset. Tässä tutkimuksessa tarkastellaan, mitä seurauksia on siitä, että keskuspankin ja yksityisten sektorin taloudenpitäjien ennusteet poikkeavat toisistaan. Näkökulmana tutkimuksessa on oppimiseen perustuva yksityisen sektorin odostustenmuodostus.

Asiasanat: oppiminen, stabiilius, heterogeenisuus, rahapolitiikka

JEL-luokittelu: E52, E31, D84

Contents

1 Introduction

The question whether monetary policy should be forward-looking, ie based on forecasts of future inflation and other variables, has raised debates in the recent research into monetary policy making. On one hand, empirical evidence on Germany, Japan and the US since 1979 provided by (Clarida, Gali, and Gertler 1998) suggests that central banks are forward looking in practice. More general discussions also pose the question whether central banks should focus attention to economic fundamentals or "follow the markets", which "sometimes stray far from fundamentals", see p. 60–61 of (Blinder 1998). Bank of England Inflation Reports, see (Bank of England 2001), discuss private sector forecasts while the June and December Issues of the Monthly Bulletin of the European Central Bank, see (European Central Bank 2001b), present both internal macroeconomic projections and forecasts by other instutions. However, the precise role of these forecasts in the decision making of these central banks is not revealed.[1]

On the other hand, theoretical studies have shown that the conduct of optimal monetary policy on the part of the bank can lead to a choice of the instrument, the short-term nominal interest rate, which reacts to the next period forecast of inflation and/or output gap, see (Clarida, Gali, and Gertler 1999) for a survey of the recent literature. This conclusion can nevertheless be problematic as forward looking monetary policy rules, both some formulations of optimal setting of the instrument as well as Taylor rules based on forecasts of inflation and/or output gap, can lead to indeterminacy of equilibria, see. eg (Clarida, Gali, and Gertler 1999), (Bernanke and Woodford 1997), (Bullard and Mitra 2001b) and (Evans and Honkapohja 2000). However, indeterminacy need not arise if the forward-looking interest rate rule is carefully designed, see (Bullard and Mitra 2001b) and (Evans and Honkapohja 2000).

In the literature just cited, the forecasts refer to those of the private sector, see eg (Hall and Mankiw 1994) for a discussion of targeting of private forecasts. (Evans and Honkapohja 2000) forcefully make a case for incorporating private forecasts of inflation and output gap into the interest rate rule as the reaction function of the optimal central bank behavior under discretion.[2] Naturally, for such a proposal to make sense it is required that the private sector forecasts are observable.

(Evans and Honkapohja 2000) show that small measurement errors would not lead to large deviations from optimality. However, (Orphanides 2000) and others have argued that there are large errors in private forecasts. While private forecasts by different institutions are regularly published, it is not self-evident that these published numbers accurately represent the expectations of the private sector that are relevant for the key private sector economic decisions. Thus the observability problems might in fact be more serious than they appear at first sight. Moreover, if it becomes known that the decisions of the monetary policy maker depend significantly on the forecasts by private

[1] Earlier (Hall 1984), p. 146, proposed that the "Fed's internal procedure" should place some weight on "reliable outside forecasts."

[2] (Evans and Honkapohja 2001b) extend the results of (Evans and Honkapohja 2000) to the case of commitment.

institutions, these institutions might alter their forecasts in a strategic way so as to influence the decisions about the conducted monetary policy.

These arguments suggest that a case can be made for the use of internal forecasts by the central bank in the decision making on monetary policy. Moreover, it seems likely that internal forecasts, rather than those of other institutions, play the central role in actual monetary policy decisions and the recent literature by (Bernanke and Woodford 1997), (Svensson 1997), (Svensson 1999a), (Svensson and Woodford 1999) and (Svensson 2001) incorporates internal forecasts by the central bank in models of monetary policy. All these reasons justify the assumption that the central bank and private sector may have potentially different forecasts of endogenous variables like inflation and output.

The implications of different forecasts by the central bank and the private sector can potentially shed some light on the issue of transparency in the formulation of monetary policy, which has been widely debated in both academic and policy circles. It is frequently argued that central banks should be as transparent as possible, so that the actions of central bankers become credible and the public comes to understand this. (It is suggested that this would make it easier to support a low inflation regime.) The case where both the bank and the private sector have identical forecasts can conceptually be thought to describe a transparent central bank so that the private agents have adopted the forecasts of the bank. In contrast, the forecasts of the bank and private agents can easily differ if the central bank is not transparent about its decision making process (as some authors like (Svensson 1999b) claim is the case with the European Central Bank). Our analysis can, therefore, provide some insight into the desirability of transparency on the part of the central bank.

From an analytical viewpoint, the distinction between private sector and central bank forecasts of inflation and output gap is not relevant in a rational expectations equilibrium (REE). This is because, in an REE, the expectations of different agents are identical unless there are asymmetries in the information sets. However, even if the information sets of different agents are the same, distinguishing between private sector and central bank forecasts makes sense in a learning framework, where the expectations of different agents are usually heterogenous when the economy is (at least transitorily) outside REE. The learning approach to modelling expectations formation has gained popularity in the recent literature and we will take this view point in our study.[3]

In models of adaptive learning the economic agents are assumed to use forecast functions that depend on some parameters and, at any moment of time, the economic decisions are made on the basis of expectations/forecasts obtained from these functions. The values of the parameters in the forecast functions and the expectations of the agents are adjusted over time as new data becomes available. Parameter updating is often assumed to be done using standard econometric methods such as recursive least squares (RLS)

[3] There has recently been extensive research into the learning approach to macroeconomics, see (Evans and Honkapohja 2001a) for a systematic treatise. Overviews and surveys are provided eg by (Evans and Honkapohja 1999) , (Marimon 1997), (Sargent 1993) and (Sargent 1999).

estimation. A key issue of interest is whether this kind of adaptive learning behavior converges to REE over time. If this is the case, then eventually the forecast functions of the agents are those associated with the REE.

Taking the learning viewpoint to expectations formation and forecasts, we analyze the implications of heterogeneity in private sector and central bank forecasts for the performance of forward-looking interest rate rules. The earlier literature on learning and monetary policy has largely employed the simplifying assumption that only private forecasts affect the economy (or equivalently that forecasts of private agents and the policy maker are identical).[4] Our objective is to study how the conditions for learnability of equilibrium, ie stability of equilibrium under adaptive learning, are affected by heterogeneities in expectations and learning rules. The model we use is standard in the recent literature on monetary policy conducted with interest rates rules, see the surveys by (Clarida, Gali, and Gertler 1999), (Woodford 1999) and (McCallum 1999).

The heterogeneity in expectations and learning can naturally take different forms. The first and simplest possibility we study is that both private sector and central bank forecast functions have the same parametric form and the updating of these forecast functions is done using the same learning algorithm. (We specifically assume the RLS algorithm that has been widely used in the literature.) Heterogeneity in expectations is then solely due to differences in initial beliefs.

The second step we consider relaxes the assumption of identical estimation algorithms. One subcase here is that the updating algorithms are in the same class, but the strength of reaction to forecast errors in parameter updating differs between the private sector and the central bank. Another subcase arises when the algorithms used by the private agents and the policy maker are different. For example, the private sector might use the stochastic gradient (SG) algorithm that is simpler to implement than RLS.[5]

Finally, we analyze the case of asymmetric information in forecasting between the private sector and the central bank. For brevity, we can only take up a simple case. One agent, say the private sector, is assumed to have superior (full) information, as it can observe both of the two shocks, while the other agent with the limited information sees only one shock. (Alternatively, one may assume that the central bank has full and the private sector limited information.) We develop the analytical techniques and convergence conditions in this particular setting, but the methods are more generally applicable to other situations of asymmetric information. *Restricted perceptions equilibrium*, proposed by (Evans and Honkapohja 2001a), is an appropriate equilibrium concept for this situation. In this equilibrium the corresponding forecast function of

[4] See (Bullard and Mitra 2001b), (Bullard and Mitra 2001a), (Evans and Honkapohja 2000), (Honkapohja and Mitra 2001a), (Evans and Honkapohja 2001b) and (Mitra 2001). (Carlstrom and Fuerst 2001) study the standard model of monetary policy under the assumption that private sector has rational expectations and the Central Bank tries to learn.

[5] These forms of heterogeneity in learning are studied in (Honkapohja and Mitra 2001b) for general frameworks with heterogeneity. In independent work (Giannitsarou 2001) considers similar cases under the more restrictive assumption that the economy depends on the average expectations of the agents.

the agent with limited information is misspecified relative to the symmetric information REE even though the forecasting is optimal relative to the restricted information set.[6]

The analysis of learning dynamics in the context of monetary policies provides a very natural example of settings, where adaptive learning takes place under *structural heterogeneity*. Structural heterogeneity means that the expectations of different agents differ and these expectations enter the economic model in different ways. In the model of monetary policy the private sector expectations influence the economy directly through aggregate demand and the new Phillips curves, while the central bank forecasts enter through the interest rate. The earlier literature on adaptive learning with heterogenous expectations has typically made the simplifying assumption that the economic structure is nevertheless homogenous so that, for example, the economy might depend just on the average expectations of the private agents.[7] Our analysis is based on the results for general forward-looking multivariate linear models with structural heterogeneity, derived in the companion paper (Honkapohja and Mitra 2001b).

The general message from our analysis is that the learnability restrictions for interest rate rules derived under the assumption of homogenous expectations/forecasts continue to be important when heterogeneity is present. They are a necessary condition for convergence of adaptive learning to equilibrium with heterogenous forecasts and learning rules. However, these conditions need not be sufficient for learnability under the structural heterogeneity due to the differential effects of the central bank and the private sector on the actual outcome of the economy. Additional conditions are often required for convergence of learning to take place. The nature of these additional conditions depends on the prevailing type of heterogeneity in the learning, and we will provide specific results for the different cases listed above. Interestingly, these results have natural interpretations as suggestions concerning the forecasting activity of the central bank.

2 Analytical framework

The analysis will be conducted using a standard model with a representative consumer and monopolistic competition in markets for differentiated products. It is assumed that firms face restrictions on price changes, so that only a fraction of firms can change its price in any given period. Real balances enter the utility function of the consumer, who can also make savings in the form of government bonds. In the formal treatment we employ directly the log-linearized model, and we thus adopt the framework that is formally as outlined

[6](Sargent 1999) and (Cho, Williams, and Sargent 2001) study a model of the natural rate hypothesis and asymmetric information in which the central bank has a misspecified model.

[7]See (Honkapohja and Mitra 2001b) for further discussion of and references to the literature on heterogenous expectations and learning.

in Section 2 of the survey paper by (Clarida, Gali, and Gertler 1999).[8] We clearly need a specific model for analytical reasons and we emphasize that our approach is applicable to the very similar frameworks that have been used in the recent literature.

The structural model consists of two equations:

$$z_t = -\varphi(i_t - \hat{E}_t^P \pi_{t+1}) + \hat{E}_t^P z_{t+1} + g_t, \quad (1)$$
$$\pi_t = \lambda z_t + \beta \hat{E}_t^P \pi_{t+1} + u_t, \quad (2)$$

where z_t is the "output gap", ie the difference between actual and potential output, π_t is the inflation rate, ie the proportional rate of change in the price level from $t-1$ to t, and i_t is the nominal interest rate. $\hat{E}_t^P \pi_{t+1}$ and $\hat{E}_t^P z_{t+1}$ denote *private sector* expectations of inflation and output gap next period. We will use the same notation without the "^" and superscript P to denote RE of the private sector. All the parameters in (1) and (2) are positive. $0 < \beta < 1$ is the discount rate of the representative firm.

(1) is a dynamic "IS" curve that can be derived from the Euler equation associated with the household's savings decision. (2) is a "new Phillips curve" that can be derived from optimal pricing decisions of monopolistically competitive firms facing constraints on the frequency of future price changes. The essence of the new Phillips curve is the forward-looking character of the inflation expectations.

u_t and g_t denote observable shocks following first order autoregressive processes:

$$\begin{pmatrix} u_t \\ g_t \end{pmatrix} = \begin{pmatrix} \rho & 0 \\ 0 & \mu \end{pmatrix} \begin{pmatrix} u_{t-1} \\ g_{t-1} \end{pmatrix} + \begin{pmatrix} \hat{u}_t \\ \hat{g}_t \end{pmatrix}, \quad (3)$$

where $0 < \mu < 1, 0 < \rho < 1$ and $\hat{g}_t \sim iid(0, \sigma_g^2), \hat{u}_t \sim iid(0, \sigma_u^2)$. The demand shock g_t may be rationalized as a preference shock or as expected changes in government purchases relative to expected changes in potential output. The "cost push" shock u_t captures features that might affect expected marginal costs other than those entering through z_t.

We supplement equations (1) and (2) with monetary policy that is conducted by means of control of the nominal interest rate i_t.[9] We focus on rules where the interest rate is adjusted in accordance with the central bank expectations of output gap and inflation next period and possibly the exogenous shocks. Then

$$i_t = \chi_0 + \chi_\pi \hat{E}_t^{CB} \pi_{t+1} + \chi_z \hat{E}_t^{CB} z_{t+1} + \chi_g g_t + \chi_u u_t. \quad (4)$$

Again the same notation without the "^" and superscript CB will denote RE (of the central bank).

[8] See eg (Woodford 1996) for the nonlinear model and its log-linearized version. As noted by (Clarida, Gali, and Gertler 1999), the same framework is used in a number of other papers.

[9] It should be noted that we have left out explicit consideration of the intertemporal government budget constraint. This is appropriate only if fiscal policy in the form of lump-sum taxes is passively adjusted in the sense of (Leeper 1991), so that taxes are set to ensure fulfillment of the intertemporal government budget constraint.

Written this way, the rule (4) seems like an *ad hoc* Taylor (or instrument) rule considered, for instance, in (Bullard and Mitra 2001b). However, formally similar rules can also arise from *optimal policy* on the part of the central bank as we now show. For example, postulating a standard quadratic objective function

$$\min \frac{1}{2} E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[\alpha z_{t+i}^2 + \pi_{t+i}^2 \right] \right\} \tag{5}$$

that describes flexible inflation targeting, we can consider optimal monetary policy under discretion. In the objective function (5) α is the relative weight for output deviations and β is the discount rate. The policy maker is assumed to discount future at the same rate as the private sector. (If desired, one could allow for a possible deviation of socially optimal output from potential output and a non-zero target value for the inflation rate.)

Following (Evans and Honkapohja 2000) we consider optimal discretionary policy by minimizing (5) subject to general private sector expectations (ie even outside REE) and (2). The first order condition is

$$\lambda \pi_t + \alpha z_t = 0 \tag{6}$$

that, together with (1) and (2), implies an interest rate rule like (4) that would depend on private expectations. As there are possibly large errors in measuring private expectations, a plausible procedure would use the internal forecasts by the central bank in place of private sector expectations. This is in the spirit of the approach of (Svensson 2001), section 5.3, and (Svensson and Woodford 1999), sections 3.1–3.2, according to which the interest rate is determined to satisfy (6), (1) and (2) with given central bank forecasts.[10] These considerations lead to a rule

$$\begin{aligned} i_t &= [1 + (\lambda^2 + \alpha)^{-1} \varphi^{-1} \lambda \beta] \hat{E}_t^{CB} \pi_{t+1} + \varphi^{-1} \hat{E}_t^{CB} z_{t+1} \\ &\quad + \varphi^{-1} g_t + (\lambda^2 + \alpha)^{-1} \varphi^{-1} \lambda u_t, \end{aligned} \tag{7}$$

which is of the form (4). We will refer to (7) as the *expectations based (EB-) optimal rule.*

The optimal interest rate under discretion can be characterized in different ways, as pointed out in (Clarida, Gali, and Gertler 1999). Assuming that the economy is in the fundamental REE, (6) implies that the interest rate can be written as

$$i_t = (1 + \frac{(1-\rho)\lambda}{\rho \alpha \varphi}) \hat{E}_t^{CB} \pi_{t+1} + \varphi^{-1} g_t, \tag{8}$$

which is a special case of the rule (4) where in the REE $\hat{E}_t^{CB} \pi_{t+1} = E_t \pi_{t+1}$. We therefore think of (8) as a specified interest rate rule where internal forecasts by the central bank are used for the inflation expectations. We will refer to (8)

[10] (Svensson 2001) and (Svensson and Woodford 1999) have a somewhat different model and also discuss other aspects of policy making. However, they do not tell how the forecasts of the central bank are determined.

as the *rational expectations (RE-) optimal rule* as in (Evans and Honkapohja 2000).

As mentioned before, in the earlier literature (Bullard and Mitra 2001b) assume identical forecasts and learning algorithms (versions of RLS) for the private sector and the central bank to derive conditions for (local) stability of fundamental or minimal state variable (MSV) solutions under learning dynamics. Under these assumptions, the conditions for stability are given by E-stability conditions, and Bullard and Mitra found that the *Taylor principle* (see (Woodford 2000)) completely characterized learnability of the MSV solution.[11] In particular, interest rate rules satisfying the Taylor principle are learnable while rules violating this principle are unlearnable. Regarding optimal rules, (Evans and Honkapohja 2000) found that, with private expectations, the rule (7) yields both stability under learning and determinacy whereas rule (8) is stable but leads to indeterminacy in some parameter domains.

Our purpose in this paper is to analyze the robustness of these results to the heterogeneity in forecasts and/or learning algorithms for the private sector and the central bank. Our model, therefore, comprises of equations (1), (2), and (3) supplemented with the interest rate rule (4), of which (7) and (8) are special cases. We thus substitute equation (4) into (1) and reduce our system to

$$\begin{aligned}\begin{pmatrix} z_t \\ \pi_t \end{pmatrix} &= \begin{pmatrix} -\varphi \\ -\lambda\varphi \end{pmatrix}\chi_0 + \begin{pmatrix} 1 & \varphi \\ \lambda & \beta+\lambda\varphi \end{pmatrix}\begin{pmatrix} \hat{E}_t^P z_{t+1} \\ \hat{E}_t^P \pi_{t+1} \end{pmatrix} + \\ &\quad \begin{pmatrix} -\varphi\chi_z & -\varphi\chi_\pi \\ -\lambda\varphi\chi_z & -\lambda\varphi\chi_\pi \end{pmatrix}\begin{pmatrix} \hat{E}_t^{CB} z_{t+1} \\ \hat{E}_t^{CB} \pi_{t+1} \end{pmatrix} + \\ &\quad \begin{pmatrix} -\varphi\chi_u & 1-\varphi\chi_g \\ 1-\lambda\varphi\chi_u & \lambda(1-\varphi\chi_g) \end{pmatrix}\begin{pmatrix} u_t \\ g_t \end{pmatrix}.\end{aligned} \tag{9}$$

For future reference, we write the above system in a general form

$$\begin{aligned} y_t &= D + A^P\hat{E}_t^P y_{t+1} + A^{CB}\hat{E}_t^{CB} y_{t+1} + Bw_t, \\ w_t &= Fw_{t-1} + v_t. \end{aligned} \tag{10}$$

where $y_t = (z_t, \pi_t)'$, $w_t = (u_t, g_t)'$ and A^P, A^{CB}, B denote the right hand matrices in (9), and F is the (diagonal) matrix appearing in (3), namely

$$F = \begin{pmatrix} \rho & 0 \\ 0 & \mu \end{pmatrix}. \tag{11}$$

We assume $\chi_z \geq 0$ and $\chi_\pi \geq 0$ throughout the paper.

We will consider learnability of the minimal state variable (MSV) solution for the model (10). It takes the form

$$y_t = a + bw_t, \tag{12}$$

where a, b are to be computed in terms of the structural parameters of the model.[12] The MSV solution is generically unique, see (Honkapohja and Mitra 2001b) for the proof:

[11] Intuitively, the Taylor principle means that in the event of a 1% rise in expected inflation, the nominal interest rate rises by more than 1% in the long run.

[12] There are other stationary REE in addition to the MSV solution if the model is indeterminate. See (Honkapohja and Mitra 2001a) for a detailed analysis of indeterminacy in this model of monetary policy.

Proposition 1 *The model of monetary policy (9), or (10), has a unique MSV solution of the form (12) if the matrices $I-(A^P+A^{CB})$ and $I-F'\otimes(A^P+A^{CB})$ are invertible.*

The MSV solution $\bar{a}, \bar{b}$ can be obtained by solving the following system of linear equations

$$\begin{aligned} a &= D+(A^P+A^{CB})a \\ b &= (A^P+A^{CB})bF+B. \end{aligned}$$

The latter equation is matrix-valued, but it can be vectorized.

3 Heterogenous forecasts under homogenous learning rules

In this section we assume that the central bank and the private sector have different forecasts although their forecast functions take the same general form and they use asymptotically identical learning algorithms in updating the parameter values of their forecast functions.

It has been observed for a wide variety of different models (with homogenous forecasts and learning) that convergence to the REE obtains if and only if certain stability conditions, known as expectational stability (or E-stability) conditions, are satisfied, see eg (Evans and Honkapohja 2001a). In this section we obtain the E-stability conditions that govern convergence of the economy to the REE under real time learning as long as the bank and private agents use asymptotically identical versions of RLS.[13] The assumed learning rules do, however, allow the bank and the private sector to have different initial beliefs about the parameters they are estimating, so that their forecasts in general differ for finite time periods.

The formulation of E-stability (and learning) starts from the perceptions of the agents. The agents are assumed to have perceptions about the stochastic process that the endogenous variables of the economy follow. These are called the *perceived law of motion* (PLM) and they are assumed to have the same parametric form (12) as the REE of interest. For given values of the parameters of the PLM and the current values of the exogenous variables, the agents use the estimated PLM to make forecasts about the values of the endogenous variables next period. This step yields the forecast functions of the agents. This formulation of forecasting by the private agents and the central bank is a natural first approach, since the forecast functions correspond, under specific parameter values, to the equilibrium forecast functions. However, we do acknowledge that they represent a greatly simplified view of actual forecasting practices.[14]

The next step is to insert these forecasts into the model (10) and compute the temporary equilibrium of the economy, also called the *actual law of motion*

[13] This result is discussed in detail in the companion paper (Honkapohja and Mitra 2001b).

[14] For example, we do not make a distinction between conditional and unconditional forecasts that is important in practice, cf. eg (European Central Bank 2001a).

(ALM). The ALM turns out to have the same parametric form as the PLMs of the agents. E-stability is then determined by the differential equation in which the parameters partially adjust (in virtual time) in the direction of the ALM parameter values.

Formally, we assume that the private sector and the central bank, respectively, have PLMs of the form that corresponds to the MSV solutions (12) but they have different parameter values. The PLMs are

$$\begin{aligned} y_t &= a^P + b^P w_t = (\phi^P)' x_t & (13) \\ y_t &= a^{CB} + b^{CB} w_t = (\phi^{CB})' x_t & (14) \end{aligned}$$

with corresponding forecast functions[15]

$$\begin{aligned} \hat{E}_t^P y_{t+1} &= a^P + b^P F w_t, & (15) \\ \hat{E}_t^{CB} y_{t+1} &= a^{CB} + b^{CB} F w_t, & (16) \end{aligned}$$

where $x_t = (1, w_t)'$ is a vector of variables relevant in forecasting and $(\phi^i)' = (a^i, b^i)$, with a^i a 2−dimensional vector and b^i an 2×2 matrix for $i = P, CB$.

Inserting these forecasts into the model (10), one obtains the ALM followed by inflation and output as

$$\begin{aligned} y_t &= D + A^P a^P + A^{CB} a^{CB} + [(A^P b^P + A^{CB} b^{CB})F + B] w_t \\ &= [D + A^P a^P + A^{CB} a^{CB}, (A^P b^P + A^{CB} b^{CB})F + B] \begin{bmatrix} 1 \\ w_t \end{bmatrix} \\ &\equiv T(\varphi_1', \varphi_2') x_t. \end{aligned}$$

Written explicitly, the mapping from the PLMs to the ALM, called the T map, takes the form

$$\begin{aligned} a^P &\to D + A^P a^P + A^{CB} a^{CB}, \\ a^{CB} &\to D + A^P a^P + A^{CB} a^{CB}, \\ b^P &\to (A^P b^P + A^{CB} b^{CB})F + B, \\ b^{CB} &\to (A^P b^P + A^{CB} b^{CB})F + B. \end{aligned}$$

We look at E-stability of the REE in which the bank and the private sector have identical forecasts, that is when $a^P = a^{CB} = \bar{a}$ and $b^P = b^{CB} = \bar{b}$. The REE is said to be *E-stable* if it is a locally asymptotically stable fixed point under differential equation (I denotes the identity matrix)

$$\begin{aligned} da^P/d\tau &= D + (A^P - I) a^P + A^{CB} a^{CB}, \\ db^P/d\tau &= A^P b^P F - b^P + A^{CB} b^{CB} F + B, \\ da^{CB}/d\tau &= D + A^P a^P + (A^{CB} - I) a^{CB}, \\ db^{CB}/d\tau &= A^P b^P F + A^{CB} b^{CB} F - b^{CB} + B. \end{aligned}$$

[15]In this formulation parameter estimates will be assumed to depend on data up to $t-1$ but the current observation on exogenous variables is used in the forecasts. This is commonly done in the literature.

This system is linear and the equations for (a^P, a^{CB}) and (b^P, b^{CB}) are independent from each other. For the subsystem involving a^P, a^{CB} stability is determined by the matrix

$$\begin{pmatrix} A^P - I & A^{CB} \\ A^P & A^{CB} - I \end{pmatrix}.$$

The companion paper (Honkapohja and Mitra 2001b) shows that two of the eigenvalues of this system are -1 and the other two are those of $A^P + A^{CB} - I$, so that this system is stable provided the eigenvalues of $A^P + A^{CB}$ have real parts less than one. As for the (b^P, b^{CB}) subsystem, (Honkapohja and Mitra 2001b) show that stability requires that the matrix

$$\begin{pmatrix} F \otimes A^P - I & F \otimes A^{CB} \\ F \otimes A^P & F \otimes A^{CB} - I \end{pmatrix}$$

has eigenvalues with negative real parts. The eigenvalues of this matrix are either equal to -1 or correspond to those of $F \otimes (A^P + A^{CB}) - I$. Consequently, the necessary and sufficient conditions for E-stability are that the eigenvalues of

$$A^P + A^{CB} - I \text{ and } F \otimes (A^P + A^{CB}) - I$$

have negative real parts.

These requirements for stability are exactly the E-stability condition when the bank and the private sector have identical forecasts. In this case $\hat{E}_t^P y_{t+1} = \hat{E}_t^{CB} y_{t+1}$ and the matrix in front of the common expectations $\hat{E}_t^P y_{t+1}$ in (10) becomes $A^P + A^{CB}$. The conclusion then follows by applying the results in (Evans and Honkapohja 2001a). Since the matrix F in (11) is diagonal and has positive elements (ie $\mu > 0$ and $\rho > 0$), the necessary and sufficient conditions for E-stability in fact reduce to the condition that the eigenvalues of $A^P + A^{CB} - I$ have negative real parts, as shown in (Bullard and Mitra 2001b).

We have thus verified:

Proposition 2 *The MSV solution to the model of monetary policy (9) is E-stable under heterogenous forecasts if and only if the corresponding model with homogenous expectations is E-stable. The E-stability condition is that the eigenvalues of the matrix $A^P + A^{CB} - I$ have negative real parts.*

As already mentioned above, the stability of the system under learning dynamics obtains if and only if the E-stability conditions are satisfied. In actual real time learning the central bank and private sector use versions of RLS in their updating of estimates of parameters that are relevant to their forecasting. However, the learning rules can start with different initial beliefs about the parameters, so that they differ along the actual time path. The E-stability conditions, therefore, govern convergence to REE even when we allow this form of heterogeneity. We also note that under some (mild) regularity conditions, the RLS algorithm will converge to an E-unstable symmetric (MSV) solution with probability zero. See (Evans and Honkapohja 2001a) for these details.

Proposition 2 shows that the stability conditions obtained in the homogenous case in (Bullard and Mitra 2001b) and (Evans and Honkapohja 2000) are not as restrictive as they might seem. The assumption that the bank and private sector have the same type of forecast functions and same learning algorithms serves as a good first approximation.

It follows that interest rules that satisfy the *Taylor principle* can be determinate as well as learnable, as in (Bullard and Mitra 2001b), even with this form of heterogeneity. It is heartening to note that the Taylor principle continues to dictate principles of good monetary policy. Similarly, if the bank uses its own internal forecasts in the expectations based policy rules for its conduct of optimal monetary policy, as in the rules (7) or (8), convergence to the REE continues to obtain as in (Evans and Honkapohja 2000). As we have emphasized, these results are important since in practice the central bank probably does not observe the private sector expectations accurately and it seems more reasonable to assume that the bank forms its own internal forecasts of inflation and output.

4 Heterogenous learning rules

The preceding section allowed for different forecasts for the central bank and private agents but assumed that the bank and private sector use learning rules that were asymptotically identical (even though these rules differed along the transition to REE). A greater degree of heterogeneity would allow for learning rules that differ even asymptotically or for altogether different learning algorithms. Here we take up these two further forms of heterogeneity.

The first subcase assumes that both the bank and the private agents use versions of RLS in their updating schemes but they differ in the degree of adaption to forecast errors. This allows for inertia in the formation of expectations as well as various weighting schemes for data in later periods relative to early ones, see eg (Ljung and Söderström 1983), (Marcet and Sargent 1989b) and (Evans, Honkapohja, and Marimon 2001) for different possibilities.[16]

The second subcase considers a scenario where the agents use different learning rules. The rules we consider are RLS and stochastic gradient (SG) type algorithms. These algorithms involve a trade-off between simplicity and efficiency. The RLS algorithm is statistically efficient but computationally expensive, whereas the SG algorithm is relatively easy to compute but lacks some of the good statistical properties of RLS (see Section 4.2 for further discussion).

[16] See also (Evans and Honkapohja 2001a), Chapter 15, Section 2, for a discussion of alternative gain sequences.

4.1 RLS Learning with different gain sequences

We continue to assume that the private sector and the bank use forecast functions (15)–(16) in forming their forecasts of inflation and output. Consequently, the analysis of E-stability is identical to that in Section 3. However, the analysis of real time learning is different since we now allow the bank and the private sector to display different speeds of adaption in their updating of estimates of parameters required for their forecasting. Versions of the RLS algorithm take the form

$$\begin{aligned} (\phi_t^i)' &= (\phi_{t-1}^i)' + \gamma_{i,t}(R_t^i)^{-1}x_{t-1}(y_{t-1} - \phi_{t-1}^i x_{t-1})' \\ R_t^i &= R_{t-1}^i + \gamma_{i,t}[x_{t-1}(x_{t-1})' - R_{t-1}^i], \end{aligned} \tag{17}$$

for $i = P, CB$, where we have used the notation in equation (13) from the preceding section.[17] (Here prime denotes transpose.) The first equation describes the updating of the parameters of the PLM of the private sector and the central bank, while the second equation updates the matrix of second moments of x_t that is needed in the updating of the PLM parameters. (We note that in this formulation the estimation of parameters for time t is again based on information available in $t-1$ but forecasts use current data. This is a common assumption in the literature.)

The sequences $\gamma_{i,t}, i = P, CB$ are known as the sequence of gains. The gain sequences can differ even asymptotically, and our interest is in the implications of heterogeneity in the form of different gain sequences of the central bank and private agents. The gain sequence indicates how much weight, say, the private agent puts on forecast errors $y_{t-1} - \phi_{t-1}^P x_{t-1}$. For standard RLS it is given by $\gamma_{i,t} = t^{-1}$. Modifications to standard RLS can be obtained by permitting greater or smaller response than t^{-1} to the forecast errors, adjusted for by the matrix of second moments and the state of exogenous variables. It is possible to include various weighting schemes, inertia in updating of forecast rules and even independent random fluctuations in adaption speeds, see the companion paper (Honkapohja and Mitra 2001b) and the references cited above for detailed discussion. Inertia in the formation of expectations is observed in experimental data, see for instance (Marimon and Sunder 1993) and (Evans, Honkapohja, and Marimon 2001).

We write the (possibly random) gain sequences in the form

$$\gamma_{P,t} = \gamma_t(\gamma_{P,t}\gamma_t^{-1}) \text{ and } \gamma_{CB,t} = \gamma_t(\gamma_{CB,t}\gamma_t^{-1}),$$

where γ_t is an exogenously given, nonincreasing deterministic sequence satisfying certain properties. The different asymptotics are captured by assuming that

$$E(\gamma_{P,t}\gamma_t^{-1}) \to \delta_P \text{ and } E(\gamma_{CB,t}\gamma_t^{-1}) \to \delta_{CB}, \text{ as } t \to \infty \text{ with } \delta_P \neq \delta_{CB}.$$

Here the mathematical expectations are taken over the possible independent randomness in the individual gains.

[17] See eg Section 3 of (Evans and Honkapohja 1998) for the derivation of the formal details.

The results in (Honkapohja and Mitra 2001b) imply that local convergence of learning under these algorithms is determined by the following two matrices

$$\begin{pmatrix} \delta_P(A^P - I) & \delta_P A^{CB} \\ \delta_{CB} A^P & \delta_{CB}(A^{CB} - I) \end{pmatrix} \tag{18}$$
$$= \begin{pmatrix} \delta_P I & 0 \\ 0 & \delta_{CB} I \end{pmatrix} \begin{pmatrix} A^P - I & A^{CB} \\ A^P & A^{CB} - I \end{pmatrix}$$

and

$$\begin{pmatrix} \delta_P(F \otimes A^P - I) & \delta_P F \otimes A^{CB} \\ \delta_{CB} F \otimes A^P & \delta_{CB}(F \otimes A^{CB} - I) \end{pmatrix} \tag{19}$$
$$= \begin{pmatrix} \delta_P I & 0 \\ 0 & \delta_{CB} I \end{pmatrix} \begin{pmatrix} F \otimes A^P - I & F \otimes A^{CB} \\ F \otimes A^P & F \otimes A^{CB} - I \end{pmatrix},$$

and we state the formal convergence result:

Proposition 3 *If the private sector and the central bank use modified RLS learning algorithms with different gain sequences, then learning converges locally if the matrices (18) and (19) have eigenvalues with negative real parts.*

We first note that if the gain sequences are the same asymptotically, ie $\delta_P = \delta_{CB}$, the asymptotic behavior of the system is identical to that in the preceding section. In other words, the necessary and sufficient condition for convergence to the MSV solutions is given by the E-stability conditions. However, the situation is quite different if $\delta_P \neq \delta_{CB}$. The stability conditions are in general affected by the relative size of the gain parameters, though the earlier E-stability condition is still relevant, as we now show.

4.1.1 Stability and instability conditions for interest rate rules

The first substantive result that follows from Proposition 3 concerns the general form of interest rate policies (4). We first provide some necessary conditions that must be satisfied for an equilibrium to be locally stable under learning. The next corollary is proved in Appendix A.1.

Corollary 4 *Consider model (9) and assume that the private sector and the central bank use modified RLS learning algorithms with different gain sequences. The two conditions*

$$(1-\beta)\chi_z + \lambda(\chi_\pi - 1) \ > \ 0, \tag{20}$$
$$\chi_z + \lambda\chi_\pi - \delta_P \delta_{CB}^{-1}[\lambda - \varphi^{-1}(1 - \beta + \delta_P^{-1}\delta_{CB})] \ > \ 0. \tag{21}$$

are necessary for local stability of the symmetric equilibrium under learning,

Note that (20) is precisely the Taylor principle that completely characterized stability under learning for the homogenous case considered in (Bullard and Mitra 2001b). Corollary 4 shows the continued importance of the Taylor principle in the presence of heterogenous rules. In particular, it shows that rules violating this principle continue to be unstable as is the case under homogenous forecasts.

In general, the Taylor principle need not suffice for stability under learning since condition (21) depends on $\delta_P^{-1}\delta_{CB}$. The interest rule may require a stronger response to inflation and/or output (via larger χ_π or χ_z) than what is dictated by the Taylor principle, especially for small values of $\delta_P^{-1}\delta_{CB}$. To illustrate this, assume that $\beta + \lambda\varphi > 1$.[18] Then, if $\chi_z = 0$, the necessary condition (21) requires $\chi_\pi > \delta_P\delta_{CB}^{-1}[1 - \lambda^{-1}\varphi^{-1}(1 - \beta + \delta_P^{-1}\delta_{CB})]$, which is strictly more than 1, for any $\delta_P^{-1}\delta_{CB} < (\beta + \lambda\varphi - 1)(1 + \lambda\varphi)^{-1}$. Consequently, the necessary condition is stronger than the Taylor principle (20).

However, one can show that the Taylor principle is necessary and sufficient for stability when $\delta_P^{-1}\delta_{CB} \geq 1$. Intuitively, this last requirement means that the central bank should put at least as much weight on incoming information about the economy when revising its parameter estimates as does the private sector. The next corollary is also proved in Appendix A.1.

Corollary 5 *Consider model (9) when the private sector and the central bank use modified RLS learning algorithms with different gain sequences and assume that*

$$\delta_P^{-1}\delta_{CB} \geq 1.$$

The dynamics of the economy is then locally stable under learning if and only if

$$(1 - \beta)\chi_z + \lambda(\chi_\pi - 1) > 0.$$

A common theme that emerged from (Bullard and Mitra 2001b) for a variety of interest rate rules is that a strong enough response towards inflation or output always resulted in stability under learning dynamics. Corollary 5 shows this to be true when $\delta_P^{-1}\delta_{CB} \geq 1$. In fact, it can be shown that for any $\delta_P^{-1}\delta_{CB}$ (and values of the structural parameters), if the central bank is aggressive enough by choosing large values of χ_z and χ_π, the symmetric equilibrium becomes necessarily stable.[19]

On the other hand, Corollary 4 has hinted that policies satisfying the Taylor principle can sometimes lead to instability. This can indeed happen if the bank puts much less weight on incoming information about the economy than the private sector (ie if δ_{CB} is much smaller than δ_P). This can be seen informally by examining the left-hand side of (18). We can write

$$\begin{pmatrix} \delta_P(A^P - I) & \delta_P A^{CB} \\ \delta_{CB}A^P & \delta_{CB}(A^{CB} - I) \end{pmatrix} \quad (22)$$
$$= \delta_P \begin{pmatrix} A^P - I & A^{CB} \\ \delta_P^{-1}\delta_{CB}A^P & \delta_P^{-1}\delta_{CB}(A^{CB} - I) \end{pmatrix},$$

[18] This parameter restriction will typically be satisfied since the discount factor β is assumed to be close to 1. For example, the restriction is satisfied for the calibrated values in both (Woodford 1999) and (Clarida, Gali, and Gertler 2000).

[19] For brevity, we do not formally develop this result.

so that, if $\delta_P^{-1}\delta_{CB}$ is sufficiently small, half of the eigenvalues of the matrix (22) are approximately equal to zero while the other half are approximately the eigenvalues of $A^P - I$. The latter set contains an eigenvalue with positive real part. This intuition is made more rigorous below in the result:

Corollary 6 *Consider model (9) when the private sector and the central bank use modified RLS learning algorithms with different gain sequences and assume that $\beta + \lambda\varphi > 1$. If*

$$\delta_P^{-1}\delta_{CB} < \frac{\beta + \lambda\varphi - 1}{1 + \varphi(\chi_z + \lambda\chi_\pi)}, \tag{23}$$

the dynamics of the economy is locally unstable under learning.[20]

As mentioned before, the parameter restriction $\beta+\lambda\varphi > 1$ is very often satisfied since β is close to 1. Corollary 6 points to the danger of instability even for interest rules satisfying the Taylor principle when the central bank does not put enough weight on the forecast errors while revising its parameter estimates. The general intuition for the result is as follows.

One observes from the model (9) that, while the central bank has a stabilizing effect, the private sector has a de-stabilizing influence on the economy. (9) makes it clear that if private sector expectations of inflation (or output) deviate upward from the RE value, then actual inflation (and output) increase, which leads, *ceteris paribus*, to upward revisions of both $\hat{E}_t^P \pi_{t+1}$ and $\hat{E}_t^P z_{t+1}$ (note that all the entries of A^P are positive). On the other hand, if the central bank's expectations $\hat{E}_t^{CB}\pi_{t+1}$ or $\hat{E}_t^{CB} z_{t+1}$ deviate upwards from the RE value, π_t and z_t fall, which tends to guide the bank's non-rational expectations towards the RE values as all the entries of A^{CB} are negative. More formally, one observes that the eigenvalues of A^{CB} are non-positive, while the eigenvalues of A^P are positive and one of them exceeds 1. The eigenvalue exceeding one is the key to understanding our instability results under heterogenous forecasts and learning.[21]

Under homogenous forecasts, it is the sum of the matrices A^P and A^{CB} that determined stability under learning dynamics. Pursuit of the Taylor principle by the bank is then able to guide non-rational expectations of the private sector towards RE. However, under heterogenous forecasts, this is no longer sufficient because of the differential effects of the different forecasts via A^P and A^{CB} and the different weights in parameter updating on π_t and z_t. It now becomes very important for the bank to put sufficient weight on new data about the exogenous observables while revising its forecasts of inflation and output, so that, in conjuction with the Taylor principle, its stabilizing influence outweighs the de-stabilizing influence of the private sector to render the symmetric REE stable. This makes intuitive sense since, after all, these shocks are indicative of inflationary pressures in the economy.

The results indicate the degree to which the results in (Bullard and Mitra 2001b) and (Evans and Honkapohja 2000) are affected by the use of

[20] The proof is immediate by making a strict reversal of condition (21).

[21] This intuition continues to be true when we examine different types of learning rules in Section 4.2.

differential gains by the bank and the private sector. If $\delta_P^{-1}\delta_{CB}$ satisfies the condition in Corollary 6, then the Taylor principle no longer suffices to guarantee convergence. Moreover, even if the central bank behaves optimally by following the rules (7) or (8), convergence to the equilibrium may not take place unless the bank attaches sufficient weight to current data in the updating of the PLM parameters. We illustrate this further in the next subsection.

4.1.2 Robustness of the rules to gain differentials

Corollary 6 provides only sufficient conditions for the learning dynamics to be unstable. We now use numerical techniques to study to what extent the differences in learning actually influence stability for plausible values of structural parameters. As a useful by-product, we also consider the desirability of different optimal interest rate rules (7) and (8) advocated under the assumption of homogenous forecasts in the previous literature.

We first look at variants of optimal policies considered in Section 2. The two variants are the EB-optimal rule, equation (7), and the RE-optimal rule, namely equation (8). (Evans and Honkapohja 2000) recommend the EB-optimal rule in part on grounds of determinacy: the rule (7) is always determinate under RE, the rule (8) can become indeterminate for values of ρ close to zero. Regarding learnability, (Evans and Honkapohja 2000) show that under homogenous forecasts and learning, both the rules (7) and (8) are stable under learning. We now consider the implications of heterogeneity in learning rules for these results by means of numerical analysis.

The calibrated parameters values in (Woodford 1999) are used in this discussion:

Calibrated Example: $\varphi = (.157)^{-1}, \lambda = .024$, and $\beta = .99$.[22]

We allow α to range in the interval $(0, 1]$, which captures the scenarios ranging from strict inflation targeting (α close to 0) to that of flexible inflation targeting. The ratio $\delta_P^{-1}\delta_{CB}$ is allowed to range in the interval $(0, 2]$.[23]

Figures 1 and 2 illustrate the stability region for the EB-optimal rule (7) and the RE-optimal rule (8), respectively, with the value $\rho = .9$ for the persistence parameter of the u_t shock, which was used in (Clarida, Gali, and Gertler 2000). The shaded and unshaded regions mean stability and instability, respectively. For the EB-optimal rule, we find that for all $\alpha \in (0, 1]$, the symmetric equilibrium is stable under learning dynamics whenever $\delta_P^{-1}\delta_{CB} \geq 0.2$. Instability can only arise for $\delta_P^{-1}\delta_{CB} < 0.2$. However, for the RE-optimal rule stability is guaranteed only when $\delta_P^{-1}\delta_{CB} \geq 1$ whereas most values of $\delta_P^{-1}\delta_{CB} < 1$ lead to instability for all $\alpha \in (0, 1]$. We, therefore, find that the EB-optimal rule performs better than the RE-optimal rule as it yields more robustly stability with differences in the gain parameters of the learning rules.

Differences in gain sequences also affect the stability of *ad hoc* Taylor type rules like (4) considered in (Bullard and Mitra 2001b). Figure 3 plots the sta-

[22] We have found that our results are in fact robust to the calibrated values in (Clarida, Gali, and Gertler 2000) who use $\varphi = 1$, $\lambda = .3$ and the same β.

[23] Both rules are always stable for all $\alpha \in (0, 1)$ when $\delta_P^{-1}\delta_{CB} > 2$.

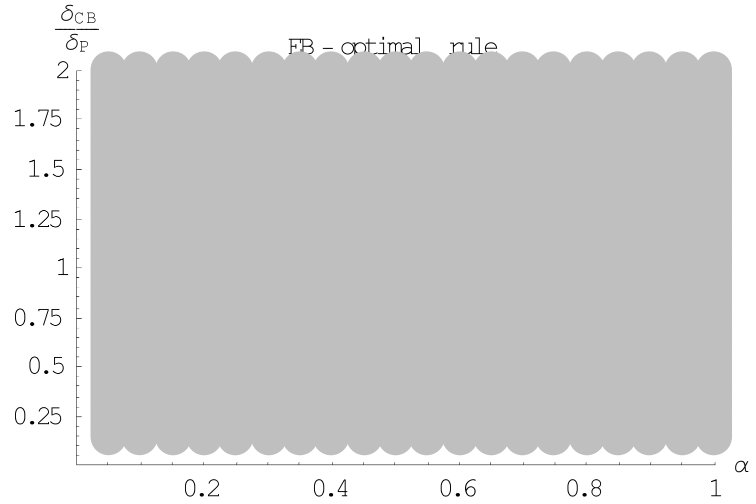

Figure 1: The EB-optimal rule for the calibrated example in the space of $(\alpha,\ \delta_{CB}/\delta_P)$ with $\rho = 0.9$. The shaded region is stable. Note that almost the whole space is now stable.

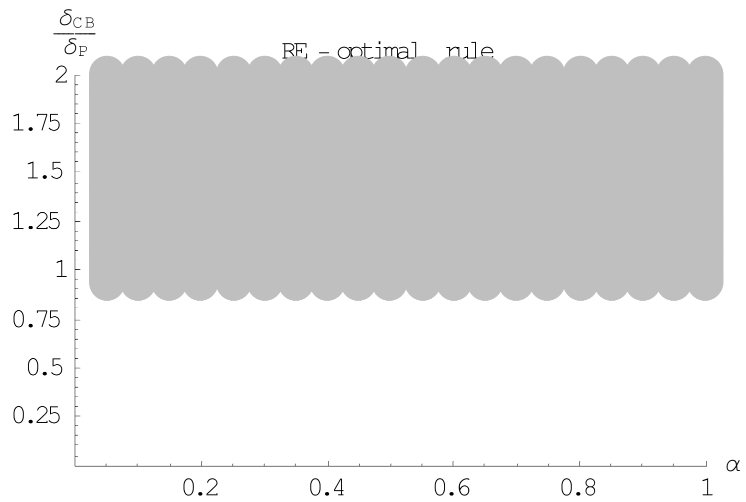

Figure 2: The RE-optimal rule for the calibrated example in the space of $(\alpha,\ \delta_{CB}/\delta_P)$ with $\rho = 0.9$. The shaded region is stable. Note that for δ_{CB}/δ_P less than 1 we usually have instability.

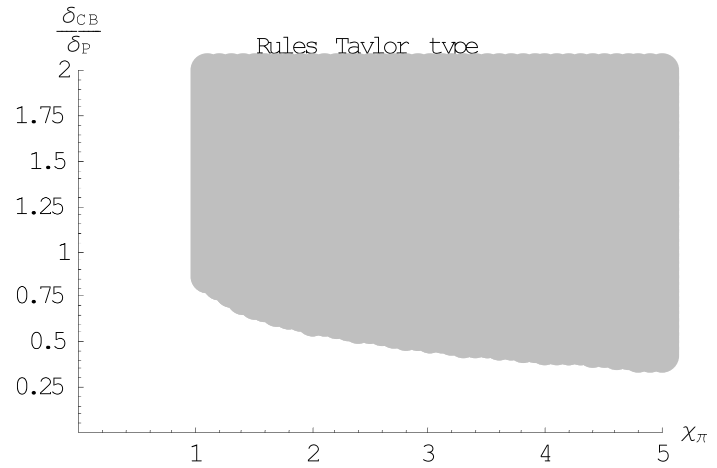

Figure 3: Taylor rules for the calibrated example in the space of (χ_π, δ_{CB}/δ_P) with $\chi_\pi = 0$ and $\mu = 0.35$. The shaded region is stable and the blank region unstable. Note that for δ_{CB}/δ_P less than 1 we often have instability even with χ_π more than one.

bility region of the rule (4) with $\chi_z = 0$ and the above calibration. We assume that there is no cost-push shock u_t and that $\mu = 0.35$ as in (Woodford 1999). The horizontal axis indicates values for χ_π while the vertical axis indicates the values for $\delta_P^{-1}\delta_{CB} \in (0, 2]$. In the analysis of (Bullard and Mitra 2001b), all rules with $\chi_\pi > 1$ are stable under learning. However, for values of $\delta_P^{-1}\delta_{CB} < 1$ even rules with $\chi_\pi > 1$ can now lead to instability. Stability is guaranteed for rules satisfying the Taylor principle (ie $\chi_\pi > 1$) only when $\delta_P^{-1}\delta_{CB} \geq 1$, as shown in Corollary 5. Of course, rules with $\chi_\pi < 1$ continue to deliver unstable dynamics, as shown in Corollary 4. The general message is that, with a Taylor rule in place, the central bank should put enough weight on incoming information to ensure stability of the economy.

4.2 RLS learning and SG learning

In this section we consider a different form of heterogeneity in the learning rules of the private agents and the bank. The broad aim is consider settings where the bank is using a learning algorithm that is either more or less sophisticated than the algorithm of the private sector. Central banks usually devote a large amount of resources in forming forecasts of economy-wide variables, see (Romer and Romer 2001). Our analysis in this section provides an analytical answer to the question whether such actions on the part of the central bank are justified.

Specifically, we assume that there are two possible types of learning algorithms, the RLS and the stochastic gradient (SG) algorithms that the private agents or the central bank might use. The RLS algorithm is more common than SG in the literature. The SG algorithm is computationally much simpler than the RLS algorithm; however the latter is more efficient from an econometric viewpoint since it uses information on the second moments of the variables. More precisely, for parameter estimation of fixed exogenous stochastic processes, both the RLS and SG algorithms yield consistent estimates of parameters but the former in addition possesses some optimality properties. For instance, if the underlying shock process is *iid* normal, then the RLS estimator is minimum variance unbiased.[24]

When the central bank uses the SG algorithm, parameter updating takes the form

$$(\phi_t^{CB})' = (\phi_{t-1}^{CB})' + \gamma_{CB,t} x_{t-1}(y_{t-1} - \phi_{t-1}^{CB} x_{t-1})', \tag{24}$$

where we have used the notation in equation (14) from Section 3. The main difference from the RLS algorithm (17) is that (24) does not involve the matrix of second moments. The private sector is assumed to use the RLS algorithm. For simplicity, it is assumed that there are no differential in gain sequences, ie $\delta_P = \delta_{CB} = 1$ in the notation of Section 4.1.

The results in the companion paper (Honkapohja and Mitra 2001b) show that local convergence under learning is determined by the following two matrices

$$\begin{pmatrix} A^P - I & A^{CB} \\ A^P & A^{CB} - I \end{pmatrix}, \tag{25}$$

$$\begin{pmatrix} F \otimes A^P - I & F \otimes A^{CB} \\ M_w F \otimes A^P & M_w F \otimes A^{CB} - M_w \otimes I \end{pmatrix}, \tag{26}$$

when the private sector uses RLS and the central bank uses the SG algorithm. Here

$$M_w = \lim_{t\to\infty} E(w_t w_t') = \begin{pmatrix} (1-\rho^2)^{-1}\sigma_u^2 & 0 \\ 0 & (1-\mu^2)^{-1}\sigma_g^2 \end{pmatrix}$$

with $\sigma_u^2 = var(\hat{u}_t)$ and $\sigma_g^2 = var(\hat{g}_t)$. Formally, we state:

Proposition 7 *If the private sector uses RLS and the central bank the SG algorithm, then learning converges locally if the matrices (25) and (26) have eigenvalues with negative real parts.*

Analogous conditions may be obtained when the private agents use SG learning and the bank uses RLS by inter-changing the roles of A^P and A^{CB} in (25) and (26).

We observe from Proposition 7 that the E-stability conditions continue to be necessary for stability. An application of the results in (Bullard and Mitra 2001b) immediately yields the following necessary condition:

Corollary 8 *Consider model (9). If the private sector uses RLS and the central bank the SG algorithm in their learning rules (or vice versa), the dynamics of the economy is stable under learning only if* $\lambda(\chi_\pi - 1) + (1-\beta)\chi_z > 0$.

[24] See (Evans and Honkapohja 2001a), Section 3.5 for a discussion and references to SG learning. We note that these properties refer to the usual statistical analysis that involves parameter estimation for exogenous processes.

We, therefore, see that the conclusion that interest rules violating the Taylor principle are undesirable is quite robust. In particular, such "passive" rules lead to instability irrespective of whether the central bank uses a sophisticated algorithm like RLS or a simple algorithm like SG.

However, as is clear from Proposition 7, the E-stability conditions are no longer sufficient for convergence of learning. The learnability conditions are now influenced by the persistence in the shocks (ρ and μ) and their variances (σ_u^2 and σ_g^2) via the matrix (26). Next, we study in more detail, first, the situation when the central bank uses the SG algorithm in its updating equations while the private sector uses RLS and, second, the converse situation when the bank uses RLS and the private agents SG in the following section.

Before proceeding we note one result, which is easily seen from (26): If ρ and μ are small enough, the E-stability conditions are sufficient for learning stability irrespective of whether the private agents use RLS or the SG algorithm.

4.2.1 Bank uses SG and private agents use RLS

We now turn to the situation when the central bank uses the SG algorithm in its updating equations and the private sector uses RLS. As observed above, the matrices (25) and (26) need to have eigenvalues with negative real parts for stability.

Since stability of (25) is equivalent to the Taylor principle, we concentrate on the matrix (26). It can be shown (eg using Mathematica) that the characteristic polynomial of the matrix (26) is symmetric in the quantities (ρ, σ_u^2) and (μ, σ_g^2) of the two shocks u_t and g_t, so that w.l.o.g. the relevant stability or instability conditions can be obtained by considering the case of a single shock, say, g_t. Thus, we may formally assume that F is a scalar μ when examining the eigenvalues of (26). When we obtain stability or instability conditions in terms of μ and σ_g^2, it should be kept in mind that the same conditions are required also for ρ and σ_u^2.

The next thing to note is that (26) is exactly the same matrix (19) (or (36)) which appears in Section 4.1, if we replace $\delta_P^{-1}\delta_{CB}$ by (now the scalar) M_w (compare (26) and (36)). In other words, with this identification, (26) will have eigenvalues with negative real parts if and only if (19) has so, given that both $\delta_P^{-1}\delta_{CB}$ and M_w are positive. This observation is useful since we are able to directly apply most of the results of Section 4.1. Henceforth, for the general interest rate rules (4), we confine ourselves to the one shock case, g_t, although, as noted, the symmetric conditions in u_t are also needed in the case of two shocks.

By the above arguments, Corollary 5 implies the following:

Corollary 9 *Consider model (9). Assume that $(1-\mu^2)^{-1}\sigma_g^2 \geq 1$ and that the central bank uses the SG algorithm while the private sector uses the RLS algorithm in their learning rules. The dynamics of the economy is then locally stable under learning if and only if $\lambda(\chi_\pi - 1) + (1-\beta)\chi_z > 0$.*

We note that the condition $\sigma_g^2 + \mu^2 \geq 1$ in Corollary 9 is likely to be easily satisfied for plausible values of these parameters. For example, the calibrated values in (Woodford 1999) satisfy this condition since $\mu = .35$ and $\sigma_g^2 = 3.72$.

In addition, large enough values of χ_z and χ_π continue to make the symmetric equilibrium necessarily stable. However, as in Corollary 6, the Taylor principle does not guarantee stability under learning and we consider this further. The following Corollary is proved in Appendix A.2.

Corollary 10 *Consider model (9). Assume that the central bank uses the SG algorithm and the private sector uses the RLS algorithm in their learning rules. Then for all* $\mu > \bar{\mu} \equiv 2(1 + \beta + \lambda\varphi)^{-1}$,[25] *the dynamics of the economy is unstable under learning if*

$$(1 - \mu^2)^{-1}\sigma_g^2 < \frac{\mu(1 + \beta + \lambda\varphi) - 2}{1 + \mu\varphi(\chi_z + \lambda\chi_\pi)}. \tag{27}$$

Roughly, the intuition for Corollaries 9 and 10 is as follows. While the Taylor principle suffices for stability of (25), it does not for (26). From (26) one observes that a large σ_g^2 enhances the stabilizing influence of the bank (recall that A^{CB} has negative entries only), which provides intuition for Corollary 9. On the other hand, a small σ_g^2 effectively works towards eliminating the stabilizing influence of the bank and a large μ (in conjunction with this) enhances the destabilizing influence of the private sector via (26) since A^P has an eigenvalue more than 1. This provides some intuition for Corollary 10. More formally, the latter is also evident from the fact that in this case half the eigenvalues of (26) are approximately zero and the other half are approximately the eigenvalues of $F \otimes A^P - I$ and that A^P has an eigenvalue more than 1.

Even though Corollary 10 gives theoretical conditions for instability, these conditions will in general be hard to satisfy for plausible values of parameters. It, therefore, seems that stability under learning is very likely to obtain when the bank uses the SG algorithm and subscribes to the Taylor principle in view of Corollary 9.

These results are also borne out by the numerical results for the optimal and Taylor rules.[26] Generally, we find that both the RE-optimal and the EB-optimal rules lead to stability under learning provided that σ_g^2 is not very small. However, instability arises for small enough values of σ_g^2 and relatively large values of μ, as expected from the discussion above. Nevertheless, the EB-optimal rule continues to be more robust than the RE-optimal rule in the sense that it delivers a stable economy for a larger domain of values of μ and σ_g^2. In addition, rules fulfilling the Taylor principle yield stability as long as σ_g^2 (or σ_u^2, by symmetry) are not too small.

[25] We note that $\bar{\mu} \approx 0.93$ with the Woodford values and is approximately .87 with the Clarida *et al* values.

[26] We do not provide details, which are available on request from the second author.

4.2.2 Bank uses RLS and private agents use SG

We now consider the converse situation when the central bank uses RLS and the private sector the SG algorithm. In this case we need (25) and

$$\begin{pmatrix} F \otimes A^{CB} - I & F \otimes A^{P} \\ M_w F \otimes A^{CB} & M_w F \otimes A^{P} - M_w \otimes I \end{pmatrix} \tag{28}$$

to have eigenvalues with negative real parts for stability.

We first show that the Taylor principle continues to completely characterize stability for interest rules under certain conditions as shown in the next corollary, which is proved in Appendix A.3.

Corollary 11 *Consider model (9). Let the central bank use RLS and the private sector the SG algorithm in their learning rules. Assume the following two conditions*

$$\begin{aligned} 1 &\leq (1-\mu^2)^{-1}\sigma_g^2, \\ \mu &\leq 2^{-1}\bar{\mu} = (1+\beta+\lambda\varphi)^{-1}, \end{aligned}$$

with $\bar{\mu}$ as in Corollary 10. Then the symmetric equilibrium is locally stable under learning if and only if $\lambda(\chi_\pi - 1) + (1-\beta)\chi_z > 0$ for the interest rule (4).

Thus, stability obtains if μ (and ρ) is small enough and σ_g^2 (and σ_u^2) is not too small in the sense made precise in the Corollary. Woodford's calibrated values satisfy the conditions of Corollary 11 since $\mu = .35$ and $\sigma_g^2 = 3.72$ so that, with these values, the Taylor principle completely characterizes stability.

However, instability may arise when μ is large. The next corollary, which is also proved in Appendix A.3, shows that the symmetric equilibrium may be rendered unstable in this case. Before stating the result we define the following expressions

$$v_1 \equiv \frac{2-\mu(1+\beta+\lambda\varphi)+\mu\varphi[(2-\beta\mu)\chi_z+2\lambda\chi_\pi]}{\mu\lambda\varphi-(1-\mu)(1-\beta\mu)}, \tag{29}$$

$$v_2 \equiv \frac{1+\mu\varphi(\chi_z+\lambda\chi_\pi)}{\mu(1+\beta+\lambda\varphi)-2}. \tag{30}$$

Corollary 12 *Consider the model (9) with the interest rule (4). Assume that the central bank uses RLS and the private sector uses the SG algorithm in their learning rules. Let $\bar{\mu}$ be as in Corollary 10 and define*

$$\tilde{\mu} \equiv (2\beta)^{-1}[1+\beta+\lambda\varphi-\sqrt{(1+\beta+\lambda\varphi)^2-4\beta}].$$

We have $\bar{\mu} > \tilde{\mu}$.[27] The dynamics of the economy is locally unstable under learning if
(a) $(1-\mu^2)^{-1}\sigma_g^2 > v_1$ when $\mu > \tilde{\mu}$
or
(b) $(1-\mu^2)^{-1}\sigma_g^2 > \min[v_1, v_2]$ when $\mu > \bar{\mu}$,
where v_1, v_2 are defined in (29)–(30).

[27] $\tilde{\mu} \approx .68$ with the Woodford values and is approximately .58 with the Clarida *et al* values.

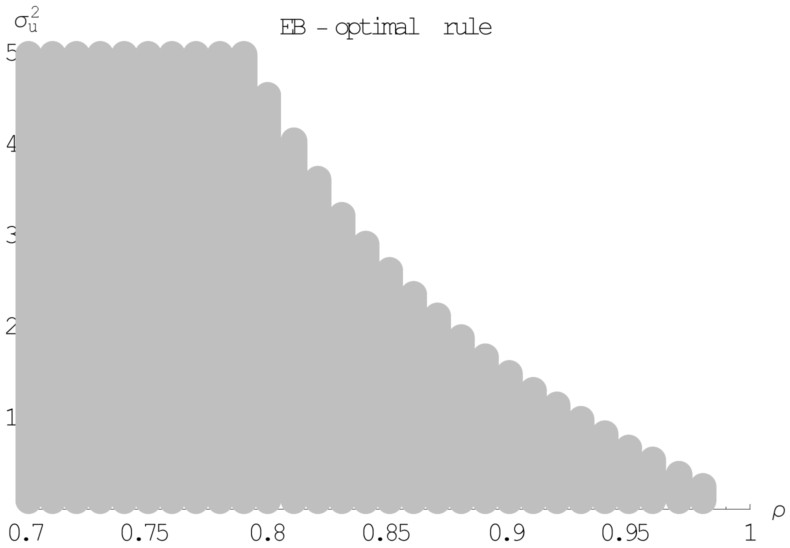

Figure 4: The EB-optimal rule for the calibrated example in the space of $(\rho,\ \sigma_u^2)$ with $\alpha = 0.1$ when the private sector uses the SG algorithm and the central bank uses RLS. The shaded region is stable.

Some intuition for Corollaries 11 and 12 follows from our discussion in the previous section. A small enough value of μ always contributes to stability in (28), which explains Corollary 11. On the other hand, a large value of μ (together with a large σ_g^2) increases the possibility of instability arising from the behavior of the private sector (since A^P has an eigenvalue more than 1) when it uses the SG algorithm, which provides some intuition for Corollary 12, see (28).

Regarding policy response, we note that υ_1, υ_2 in Corollary 12 are increasing in χ_π (and χ_z) so that if the central bank reacts aggressively enough to inflation, the inequalities (a) and (b) will be violated. In general, the central bank can continue to contribute towards stability in the learning dynamics by choosing large values of χ_π (and χ_z).

For further analysis we revert to numerics for the calibrated example. When considering the performance of the two optimal rules, we note that the rules fully neutralize the g_t shock, and so we must state the conditions in terms of ρ and σ_u^2. We have stability for small values of ρ with both versions of the optimal rules, as might be expected from Corollary 11. However, instability arises with either rule when ρ and σ_u^2 are large enough (as expected from Corollary 12). Nevertheless, the EB-optimal rule continues to yield stability under learning for a larger range of values of ρ and σ_u^2 than the RE-optimal rule. Figures 4-5 illustrate this phenomenon in the (ρ, σ_u^2) space with $\alpha = .1$, $\rho \in [0.7, 1)$ and $\sigma_u^2 \in (0, 5]$.[28]

A similar picture emerges with the Taylor rules. Rules fulfilling the Taylor principle lead to instability for large enough values of ρ or μ (as expected from the corollary above). This is illustrated in Figure 6, where we have assumed

[28] For values of $\rho \in (0, 0.7)$ and $\sigma_u^2 \in (0, 5]$, we have stability with either version of the optimal rule.

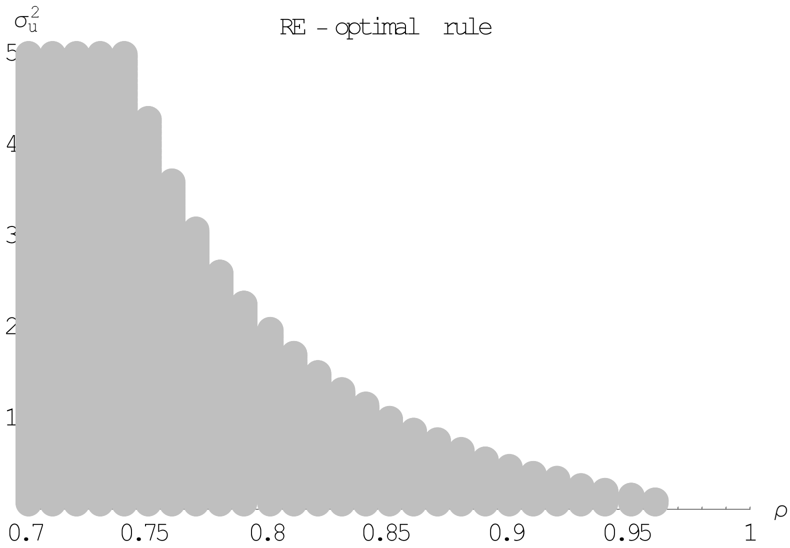

Figure 5: The RE-optimal rule for the calibrated example in the space of $(\rho,\ \sigma_u^2)$ with $\alpha = 0.1$ when the private sector uses the SG algorithm and the central bank uses RLS. The shaded region is stable.

that there is no cost push shock as in (Woodford 1999). In Figure 6 we set $\sigma_g^2 = 3.72$ (the calibrated value in (Woodford 1999)) and $\chi_z = 0$. Note that rules with $\chi_\pi > 1$ imply instability for values of μ close to 1. Figure 7 also (re)emphasizes this instability. It plots Taylor rules in the (χ_π, σ_g^2) space with $\chi_z = 0$ and $\mu = .9$. Note that most of the space associated with rules satisfying the Taylor principle yields now instability.[29]

5 Asymmetric information between the private sector and central bank

Another and quite different form of heterogeneity in forecasting arises if there are differences in the information sets of the private agents and the monetary policy maker. Clearly, these differences can take various forms and a systematic study of the consequences of differential information for learning and monetary policy must be left for future. Here we take up only one case in which the assumption is that one party observes only one of shocks while the other sees both of them. We develop the formal analysis in the case where the central bank does not have knowledge of the shocks to marginal costs that the private sector experiences (and observes), but we will comment on the other possibilities at the end of the section.

If the private sector observes both shocks, its PLM is as before, ie it has

[29] On the other hand, with $\mu = .35$ as in (Woodford 1999) rules fulfilling the Taylor principle yield stability under learning illustrating that the problem of instability arises only for μ close to 1.

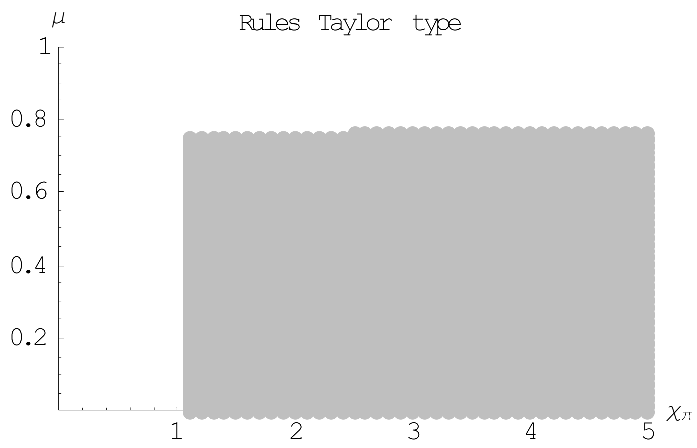

Figure 6: Taylor rules for the calibrated example in the space of $(\chi_\pi,\ \mu)$ with $\chi_\pi = 0$ and $\sigma_g^2 = 3.72$ when the private sector uses the SG algorithm and the central bank uses RLS The shaded region is stable and the blank region unstable. Note that for μ values close to 1 we have instability even with χ_π more than one.

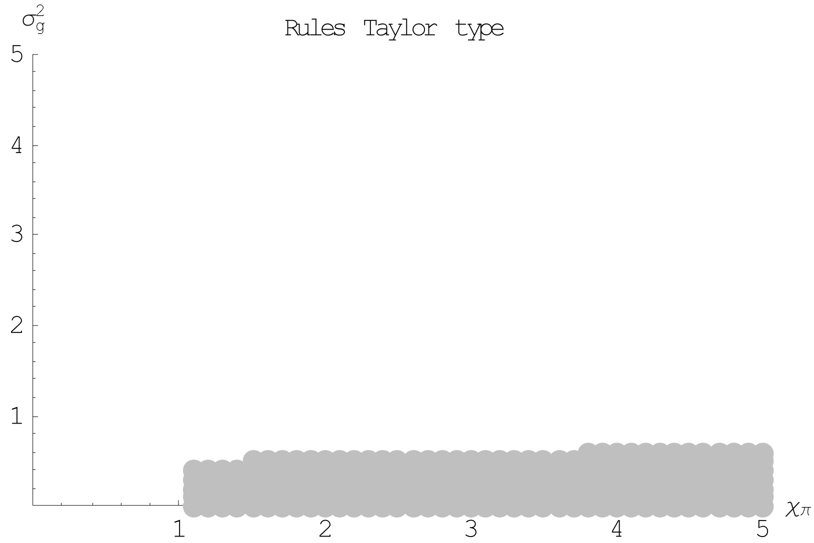

Figure 7: Taylor rules for the calibrated example in the space of $\left(\chi_\pi,\ \sigma_g^2\right)$ with $\chi_\pi = 0$ and $\mu = 0.9$ when the private sector uses the SG algorithm and the central bank uses RLS The shaded region is stable and the blank region unstable. Note that a large portion of the parameter space is unstable now even with χ_π more than one.

the form

$$\begin{pmatrix} z_t \\ \pi_t \end{pmatrix} = a^P + b^P \begin{pmatrix} g_t \\ u_t \end{pmatrix}, \text{ where}$$
$$a^P = \begin{pmatrix} a_z^P \\ a_\pi^P \end{pmatrix} \text{ and } b^P = \begin{pmatrix} b_{zg}^P & b_{zu}^P \\ b_{\pi g}^P & b_{\pi u}^P \end{pmatrix}.$$

As before, the forecasts of the private sector are obtained by advancing the PLM one period, so that

$$\hat{E}_t^P \begin{pmatrix} z_{t+1} \\ \pi_{t+1} \end{pmatrix} = a^P + b^P \begin{pmatrix} \mu & 0 \\ 0 & \rho \end{pmatrix} \begin{pmatrix} g_t \\ u_t \end{pmatrix}.$$

It is now assumed that the central bank does not observe the shock to the marginal costs u_t and, moreover, does not have a good signal about it. Thus the central bank guesses that the values of the endogenous variables depend just on the aggregate demand shock g_t, which it is taken to observe. Its PLM thus has the form

$$\begin{pmatrix} z_t \\ \pi_t \end{pmatrix} = a^{CB} + b^{CB} g_t, \text{ where}$$
$$a^{CB} = \begin{pmatrix} a_z^{CB} \\ a_\pi^{CB} \end{pmatrix} \text{ and } b^{CB} = \begin{pmatrix} b_z^{CB} \\ b_\pi^{CB} \end{pmatrix},$$

and the forecast function of the central bank is

$$\hat{E}_t^{CB} \begin{pmatrix} z_{t+1} \\ \pi_{t+1} \end{pmatrix} = a^{CB} + b^{CB} \mu g_t.$$

It must be emphasized that the forecast function of the central bank does not nest the REE studied previously, ie the PLM of the central bank is misspecified even asymptotically. However, it can still be asked whether the economy will converge to some equilibrium that is rational in a limited information sense. In such a *restricted perceptions equilibrium* (RPE) the forecasts are optimal relative to the restricted information of the central bank. These equilibria are studied eg in Chapter 13 of (Evans and Honkapohja 2001a).[30]

Given these PLMs, the temporary equilibrium of the economy, ie the ALM, is obtained by substituting the resulting forecast functions into the basic model (9). Moreover, the restriction $\chi_u = 0$ must be introduced, since the central bank does not observe u_t and so the interest rate rule cannot be based on it (we also assume for notational simplicity that $\chi_0 = 0$ in the rule (4) since this does not affect the analysis). The ALM is now

$$\begin{aligned} y_t = & \begin{pmatrix} 1 & \varphi \\ \lambda & \beta + \lambda\varphi \end{pmatrix} a^P + \begin{pmatrix} -\varphi\chi_z & -\varphi\chi_\pi \\ -\lambda\varphi\chi_z & -\lambda\varphi\chi_\pi \end{pmatrix} a^{CB} + \\ & \left[\begin{pmatrix} 1 & \varphi \\ \lambda & \beta + \lambda\varphi \end{pmatrix} b_g^P + \begin{pmatrix} -\varphi\chi_z & -\varphi\chi_\pi \\ -\lambda\varphi\chi_z & -\lambda\varphi\chi_\pi \end{pmatrix} b^{CB} \right] \mu g_t + \\ & \begin{pmatrix} 1 & \varphi \\ \lambda & \beta + \lambda\varphi \end{pmatrix} b_u^P \rho u_t + \begin{pmatrix} 1 - \varphi\chi_g & 0 \\ \lambda(1 - \varphi\chi_g) & 1 \end{pmatrix} \begin{pmatrix} g_t \\ u_t \end{pmatrix}, \end{aligned}$$

[30] Related concepts of "limited information REE" and "self-confirming equilibrium" are considered in (Marcet and Sargent 1989a), (Sargent 1991) and (Sargent 1999).

where b_g^P and b_u^P are, respectively, the 1st and 2nd columns of matrix b^P. Using the notation from Section 2 we can write this formally as

$$\begin{aligned} y_t &= A^P a^P + A^{CB} a^{CB} + \left[\mu\left(A^P b_g^P + A^{CB} b^{CB}\right) + B_g\right] g_t \\ &\quad + (\rho A^P b_u^P + B_u) u_t, \end{aligned} \tag{31}$$

where

$$(B_g, B_u) = \begin{pmatrix} 1-\varphi\chi_g & 0 \\ \lambda(1-\varphi\chi_g) & 1 \end{pmatrix}.$$

For both PLMs the parameters are assumed to be updated by recursive least squares. (For simplicity, we bypass the issues that arise when different agents use different algorithms, which were discussed in Section 4.) The parameter updating algorithm for the private sector is as in Section 4.1 with $\gamma_{P,t} = t^{-1}$, ie equation (17). Introducing the notation $\xi_t^{CB} = (a_t^{CB}, b_t^{CB})$ and $(x_t^{CB})' = (1, g_t)$, the estimation algorithm for the central bank takes the form

$$\begin{aligned} (\xi_t^{CB})' &= (\xi_{t-1}^{CB})' + t^{-1}(R_t^{CB})^{-1} x_{t-1}^{CB}(y_{t-1} - \xi_{t-1}^{CB} x_{t-1}^{CB})', \\ R_t^{CB} &= R_{t-1}^{CB} + t^{-1}[x_{t-1}^{CB}(x_{t-1}^{CB})' - R_{t-1}^{CB}]. \end{aligned} \tag{32}$$

We note that this formulation is similar to that of the algorithm of the private sector, except that it incorporates the imperfect observability of the exogenous shocks by the monetary policy maker. u_t is not observable for the policy maker and, therefore, u_t does not appear in the state variables x_t^{CB}.

The RPE is given by the solution to the equations

$$\begin{aligned} a^P &= A^P a^P + A^{CB} a^{CB} \\ a^{CB} &= A^P a^P + A^{CB} a^{CB} \end{aligned}$$

for the constant terms,

$$b^P = \left[\mu(A^P b_g^P + A^{CB} b^{CB}) + B_g, \rho A^P b_u^P + B_u\right] \tag{33}$$

for the parameters of the second term in the PLM of the private sector, and

$$b^{CB} = \mu(A^P b_g^P + A^{CB} b^{CB}) + B_g$$

for the parameters of the second term in the PLM of the central bank.

Comparing these to the full information REE discussed in Section 3, it is seen that the constant terms a^P and a^{CB}, as well as the terms b_g^P and b^{CB}, are, respectively, equal and have the same values in both the RPE and the REE. Regarding the dependence of the forecast functions on the cost-push shock, in the RPE the forecast function of the central bank does not depend on u_t whereas the private sector forecast function does by assumption. This is seen from the equation $b_u^P = \rho A^P b_u^P + B_u$ for the parameters of the u_t dependence.

The condition for local convergence of learning with rules (17) and (32) is given in the following proposition:

Proposition 13 *The RPE is locally stable under learning if all eigenvalues of the following two matrices*

$$\begin{pmatrix} A^P - I & A^{CB} \\ A^P & A^{CB} - I \end{pmatrix}, \rho A^P - I.$$

have negative real parts.

The proof is given in Appendix A.4.

The condition for the first matrix in Proposition 13 is simply the E-stability requirement for the full information REE. The second condition on the matrix $\rho A^P - I$ requires the autocorrelation coefficient ρ in the cost push shocks to be sufficiently small since A^P has an eigenvalue bigger than one (the other one being between 0 and 1). If one computes the eigenvalues of A^P, the following corollary is immediate.

Corollary 14 *The RPE is locally stable under learning iff*

$$\begin{aligned} (1-\beta)\chi_z + \lambda(\chi_\pi - 1) &> 0, \\ (2\beta)^{-1}[1 + \beta + \lambda\varphi - \sqrt{(1+\beta+\lambda\varphi)^2 - 4\beta}] &\equiv \tilde{\rho} \geq \rho. \end{aligned}$$

In the corollary $\tilde{\rho}$ is the same value as $\tilde{\mu}$ that appears in Corollary 12. For example, the value of $\rho = .9$ used in (Clarida, Gali, and Gertler 2000) violates this condition for the parameter values used there. We also note that if instead the central bank observes (only) the u_t shock but not the g_t shock, then the condition for stability is the same as above with μ replacing ρ in Corollary 14.[31]

The results are quite different if, in contrast, the private sector observes less than the central bank. The stability condition reduces to the E-stability requirement (in either case of non-observability of g_t or u_t by the private sector). This follows by interchanging the roles of A^P and A^{CB} in Proposition 13 and noting that the eigenvalues of A^{CB} are non-positive.

This analysis, therefore, supports the idea that the central bank should spend enough resources in acquiring good information about the shocks hitting the economy. In fact, there is recent empirical evidence that the Federal Reserve appears to possess information about the current and future state of the economy that is not known to commercial forecasters, see (Romer and Romer 2001).

6 Discussion and concluding remarks

In this paper we have considered the argument that the use of central bank internal forecasts in monetary policy making might be a source of instability of the economy. For the analysis we employed a standard forward looking

[31] This scenario may also be plausible since the g_t shock can represent shocks to household preferences. Finally, if the CB observes neither shock, then the stability condition in Corollary 14 will also include the same upper bound for μ.

model that is currently the workhorse for studies of monetary policy and is being used to give advice to policy makers. In reality one would think that the practice of using internal forecasts will in general create significant differences between central bank forecasts and those of the private sector and that these differences can have potentially important effects on the economy. We studied the consequences of the use of internal forecasts for the stability of the economy by means of the learning approach to expectations formation, in which agents may at least temporarily have non-rational forecast functions that are corrected over time. Such an approach is necessary since in any framework with symmetric information the distinction between private sector and central forecasts is irrelevant if RE modelling is used.

The paper has looked at the properties of both some optimal policies and Taylor rules. We have analyzed different cases of heterogenous forecasts and learning. In general, heterogeneity in forecasting and learning leads to further constraints on good policy in addition to the learnability constraints derived under homogenous learning rules. Looking at specific policies, the forecast based rule with internal central bank forecasts, recently suggested by (Evans and Honkapohja 2000), performed well more robustly than some other formulations of optimal discretionary policy. However, that policy – as well as learnable Taylor rules – may not be stable under heterogenous learning for some parameter configurations.

Based on our analysis, we can make the following general suggestions for the conduct of good monetary policy on the part of the central bank.

First, the interest rate rule should satisfy the Taylor principle. Our analysis supports this suggestion since, with forward looking rules, the Taylor principle is equivalent to E-stability of the equilibrium and it is always a necessary condition for convergence of the economy under heterogenous learning.

Second, the central bank should take incoming information about the economy seriously and put sufficient weight on these indicators while setting its interest rate rule. This suggestion is supported by our analysis of the differences in the degree of responsiveness when forecast functions are updated (Section 4.1).

Third, the bank should spend sufficient resources in obtaining large amount of information about the exogenous shocks. This suggestion is supported by the analysis of Section 5 and in part by that in Section 4.2.

Fourth, the central bank should be transparent about its forecasts. If the central bank forecasts are made public, it is possible that the bank and the private agents will have the same forecasts. If so, the equilibrium will be stable under weaker conditions than when they have different forecasts, a situation more likely to arise when the bank is not transparent.

Informal discussions of monetary policy do tend to support all of these suggestions. Our contribution has been to lend weight to these informal discussions in an analytical treatment of monetary policy.

A Appendix

A.1 Proof of corollaries 4 and 5

To shorten notation, define $\delta \equiv \delta_P^{-1}\delta_{CB}$. Consider the matrix on the right hand side of (22). Ignoring the scalar $\delta_P > 0$ (which does not affect the sign of the real parts of the eigenvalues of (22)), it can be checked that one of the eigenvalues of the matrix within parentheses on the right hand side of (22) is $-\delta$ and the remaining 3 eigenvalues are given by the characteristic polynomial

$$\begin{aligned} p(m) &= m^3 + a_1 m^2 + a_2 m + a_3, \text{ where} \\ a_1 &= 1 - \beta + \delta + \delta\varphi[\chi_z + \lambda(\chi_\pi - \delta^{-1})] \equiv 1 - \beta + \delta + \zeta_1, \\ a_2 &= \delta(1-\beta) + \delta\varphi[(2-\beta)\chi_z + \lambda(2\chi_\pi - \delta^{-1} - 1)] \equiv \delta(1-\beta) + \zeta_2, \\ a_3 &= \delta\varphi[(1-\beta)\chi_z + \lambda(\chi_\pi - 1)]. \end{aligned} \tag{34}$$

where the definitions of ζ_1, ζ_2 are introduced to shorten notation and should be obvious from above. The necessary and sufficient conditions for the eigenvalues of $p(m)$ to have negative real parts (the Routh conditions) are $a_1 > 0$, $a_3 > 0$, and $a_1 a_2 > a_3$. These conditions imply that $a_2 > 0$ also.

Note that $a_3 > 0$ if and only if $(1-\beta)\chi_z + \lambda(\chi_\pi - 1) > 0$, which provides the necessary condition (20) in Corollary 4. Second, $a_1 > 0$ if and only if condition (21) in Corollary 4 is satisfied. This completes the proof of Corollary 4.

To prove Corollary 5 we first note that, when $\delta > 1$, the Taylor principle suffices to make $a_1 > 0$. We still need to show that $a_1 a_2 > a_3$.

Note that $\zeta_2 = \zeta_1 + a_3$, which we use below. In addition, ζ_1 and ζ_2 are positive when $\delta > 1$. Now

$$\begin{aligned} a_1 a_2 - a_3 &= \delta(1-\beta)(1-\beta+\delta) + \delta(1-\beta)\zeta_1 + (1-\beta+\delta)\zeta_2 \\ &\quad + \zeta_1\zeta_2 - a_3 \qquad (35) \\ &= \delta(1-\beta)(1-\beta+\delta) + \delta(1-\beta)\zeta_1 + (1-\beta+\delta)(\zeta_1 + a_3) \\ &\quad + \zeta_1(\zeta_1 + a_3) - a_3 \\ &= \delta(1-\beta)(1-\beta+\delta) + [(\delta+1)(1-\beta)+\delta]\zeta_1 + \zeta_1^2 \\ &\quad + [(1-\beta+\delta) + \zeta_1 - 1]a_3 \\ &= \delta(1-\beta)(1-\beta+\delta) + [(\delta+1)(1-\beta)+\delta]\zeta_1 + \\ &\quad \zeta_1^2 + [\delta - \beta + \zeta_1]a_3. \end{aligned}$$

$\delta > 1$ suffices to make the coefficient of a_3 in the final line above positive since $\zeta_1 > 0$, which in turn implies $a_1 a_2 - a_3 > 0$.

We still need to check the matrix (19) for stability. As in (22), rewrite matrix (19) as (by pulling out δ_P)

$$\delta_P \begin{pmatrix} F \otimes A^P - I & F \otimes A^{CB} \\ \delta_P^{-1}\delta_{CB} F \otimes A^P & \delta_P^{-1}\delta_{CB}(F \otimes A^{CB} - I) \end{pmatrix} \tag{36}$$

and we examine the eigenvalues of the matrix within parentheses in (36). By eg using Mathematica, we first note that the characteristic polynomial of this (8×8) matrix is symmetric in the shocks ρ and μ, so that we may consider only the shock ρ and the resulting 4th degree polynomial. One eigenvalue of this polynomial is $-\delta$ and the remaining 3 eigenvalues are given by the polynomial

$$
\begin{aligned}
q(m) &= m^3 + b_1 m^2 + b_2 m + b_3, \text{ where} \\
b_1 &= 2 - \rho(1+\beta) + \delta + \delta\rho\varphi[\chi_z + \lambda(\chi_\pi - \delta^{-1})] \\
&\equiv 2 - \rho(1+\beta) + \delta + \tau_1, \\
b_2 &= (1-\rho)(1-\beta\rho) + \delta[2 - \rho(1+\beta)] + \delta\rho\varphi[(2-\beta\rho)\chi_z \\
&\quad + \lambda(2\chi_\pi - \delta^{-1} - 1)] \\
&\equiv (1-\rho)(1-\beta\rho) + \delta[2 - \rho(1+\beta)] + \tau_2, \\
b_3 &= \delta(1-\rho)(1-\beta\rho) + \delta\rho\varphi[(1-\beta\rho)\chi_z + \lambda(\chi_\pi - 1)] \\
&\equiv \delta(1-\rho)(1-\beta\rho) + \tau_3,
\end{aligned}
\tag{37}
$$

where the definitions of τ_i $(i = 1, 2, 3)$ should again be obvious from above.

Note that $\tau_2 = (\tau_1 + \tau_3)$, and that $\tau_i > 0$ by the Taylor principle and the assumption that $\delta > 1$. Next we observe that $b_1 > 0$ and $b_3 > 0$ also by the Taylor principle and the assumptions $0 < \rho < 1$ and $0 < \beta < 1$. We now need to determine the sign of $b_1 b_2 - b_3$ for which we use $\tau_2 = (\tau_1 + \tau_3)$ below.

$$
\begin{aligned}
b_1 b_2 - b_3 &= [2 - \rho(1+\beta) + \delta + \tau_1][(1-\rho)(1-\beta\rho) + \delta\{2 - \rho(1+\beta)\} + \tau_2] \\
&\quad - \delta(1-\rho)(1-\beta\rho) - \tau_3 \\
&= [2 - \rho(1+\beta) + \delta][(1-\rho)(1-\beta\rho) + \delta\{2 - \rho(1+\beta)\}] + \\
&\quad \tau_1[(1-\rho)(1-\beta\rho) + \delta\{2 - \rho(1+\beta)\}] + \tau_2[2 - \rho(1+\beta) + \delta] \\
&\quad + \tau_1\tau_2 - \delta(1-\rho)(1-\beta\rho) - \tau_3 \\
&= [2 - \rho(1+\beta)][(1-\rho)(1-\beta\rho) + \delta\{2 - \rho(1+\beta)\}] \\
&\quad + \delta(1-\rho)(1-\beta\rho) + \delta^2[2 - \rho(1+\beta)] + \tau_1[(1-\rho)(1-\beta\rho) \\
&\quad + \delta\{2 - \rho(1+\beta)\}] \\
&\quad + \tau_2[2 - \rho(1+\beta) + \delta] + \tau_1\tau_2 - \delta(1-\rho)(1-\beta\rho) - \tau_3 \\
&= [2 - \rho(1+\beta)][(1-\rho)(1-\beta\rho) + \delta\{2 - \rho(1+\beta)\}] \\
&\quad + \delta^2[2 - \rho(1+\beta)] + \tau_1[(1-\rho)(1-\beta\rho) + \delta\{2 - \rho(1+\beta)\}] \\
&\quad + (\tau_1 + \tau_3)[2 - \rho(1+\beta) + \delta] + \tau_1(\tau_1 + \tau_3) - \tau_3 \\
&= [2 - \rho(1+\beta)][(1-\rho)(1-\beta\rho) + \delta\{2 - \rho(1+\beta)\}] \\
&\quad + \delta^2[2 - \rho(1+\beta)] + \tau_1[(1-\rho)(1-\beta\rho) + \delta\{2 - \rho(1+\beta)\}] \\
&\quad + \tau_1[2 - \rho(1+\beta) + \delta + \tau_1] + \tau_3[1 + \delta - \rho(1+\beta) + \tau_1].
\end{aligned}
$$

Note that in the final line above all terms are positive since $0 < \rho < 1$, $0 < \beta < 1$ and $\delta > 1$. Hence, $b_1 b_2 - b_3 > 0$ when $\delta > 1$.

Finally, when $\delta = 1$ the stability matrices are the same as in the homogenous case so that E-stability follows from the Taylor principle.

A.2 Proof of corollary 10

As mentioned in the text, the conditions for stability of (26) when $F \equiv \mu$ are identical to that of (19) or (36). Therefore, the characteristic polynomial (37), in Appendix A.1, determines stability in this case after making the substitution $\delta \equiv (1-\mu^2)^{-1}\sigma_g^2$, so that it takes the form

$$\begin{aligned}
\hat{p}(m) &= m^3 + \hat{b}_1 m^2 + \hat{b}_2 m + \hat{b}_3, \text{ where} \\
\hat{b}_1 &= 2 - \mu(1+\beta+\lambda\varphi) + (1-\mu^2)^{-1}\sigma_g^2[1+\mu\varphi(\chi_z+\lambda\chi_\pi)], \\
\hat{b}_2 &= (1-\mu)(1-\beta\mu) - \mu\lambda\varphi \\
&\quad +(1-\mu^2)^{-1}\sigma_g^2[2-\mu(1+\beta)+\mu\varphi\{(2-\beta\mu)\chi_z+\lambda(2\chi_\pi-1)\}], \\
\hat{b}_3 &= (1-\mu^2)^{-1}\sigma_g^2[(1-\mu)(1-\beta\mu)+\mu\varphi\{(1-\beta\mu)\chi_z+\lambda(\chi_\pi-1)\}].
\end{aligned}$$

in (37). The necessary and sufficient conditions for the eigenvalues of $\hat{p}(m)$ to have negative real parts are given by $\hat{b}_1 > 0, \hat{b}_3 > 0$ and $\hat{b}_1\hat{b}_2 > \hat{b}_3$.

The instability condition in Corollary 10 is simply $\hat{b}_1 < 0$. We note that for $\hat{b}_1 < 0$ it is necessary that $\mu > \bar{\mu} \equiv 2(1+\beta+\lambda\varphi)^{-1}$.

A.3 Proof of corollaries 11 and 12

We first prove the inequalities

$$\bar{\mu} > \tilde{\mu} > 2^{-1}\bar{\mu}. \tag{38}$$

Here $\bar{\mu}$ and $\tilde{\mu}$ are defined in Corollaries 10 and 12, respectively.

Now,

$$\bar{\mu} > \tilde{\mu} \Leftrightarrow \frac{2}{(1+\beta+\lambda\varphi)} > \frac{1+\beta+\lambda\varphi-\sqrt{(1+\beta+\lambda\varphi)^2-4\beta}}{2\beta}.$$

Rearranging and squaring, we require

$$(1+\beta+\lambda\varphi)^2 - 4\beta > (1+\beta+\lambda\varphi)^2 + \frac{16\beta^2}{(1+\beta+\lambda\varphi)^2} - 8\beta$$

or

$$(1+\beta+\lambda\varphi)^2 > 4\beta$$

or

$$(1-\beta)^2 + \lambda^2\varphi^2 + 2\beta\lambda\varphi + 2\lambda\varphi > 0,$$

which is true, given that the parameters are all positive.

In a similar way we show that

$$2^{-1}\bar{\mu} < \tilde{\mu} \Leftrightarrow \frac{1}{(1+\beta+\lambda\varphi)} < \frac{1+\beta+\lambda\varphi-\sqrt{(1+\beta+\lambda\varphi)^2-4\beta}}{2\beta}.$$

Rearranging and squaring, we require

$$(1+\beta+\lambda\varphi)^2 - 4\beta < (1+\beta+\lambda\varphi)^2 + \frac{4\beta^2}{(1+\beta+\lambda\varphi)^2} - 4\beta$$

or

$$4\beta^2(1+\beta+\lambda\varphi)^{-2} > 0,$$

which is true.

To prove Corollary 11, we first note that necessity of the Taylor principle $(1-\beta)\chi_z + \lambda(\chi_\pi - 1) > 0$ follows from the necessity of E-stability through the matrix (25). Thus assume that the Taylor principle holds and consider the characteristic polynomial of the (8×8) matrix (28). It is again symmetric in the shocks ρ and μ, so that we may consider only the shock μ and the resultant 4th degree polynomial. One eigenvalue of this polynomial is -1 and the remaining 3 eigenvalues are given by the polynomial

$$\begin{aligned} r(m) &= m^3 + c_1 m^2 + c_2 m + c_3, \qquad (39)\\ c_1 &= 1 + \mu\varphi(\chi_z + \lambda\chi_\pi) + \{2 - \mu(1+\beta+\lambda\varphi)\}(1-\mu^2)^{-1}\sigma_g^2,\\ c_2 &= (1-\mu^2)^{-1}\sigma_g^2[2 - \mu(1+\beta+\lambda\varphi) + \mu\varphi\{(2-\beta\mu)\chi_z + 2\lambda\chi_\pi\} +\\ &\quad (1-\mu^2)^{-1}\sigma_g^2\{(1-\mu)(1-\beta\mu) - \mu\varphi\lambda\}],\\ c_3 &= (1-\mu^2)^{-2}(\sigma_g^2)^2[\mu\varphi\{(1-\beta\mu)\chi_z + \lambda\chi_\pi\} + \{(1-\mu)(1-\beta\mu) - \mu\varphi\lambda\}]. \end{aligned}$$

Clearly, $c_1 > 0$ and $c_3 > 0$ since $\mu < 2^{-1}\bar{\mu}$, $0 < \beta$, $\mu < 1$ and the Taylor principle holds. We still need to show that $c_1c_2 - c_3 > 0$. We introduce the notation $\delta \equiv (1-\mu^2)^{-1}\sigma_g^2$ below. We first write

$$\begin{aligned} c_1c_2 &= \delta\{1 + \mu\varphi(\chi_z + \lambda\chi_\pi)\}\{2 - (1+\beta+\lambda\varphi)\mu\} + \delta^2\{2 - \mu(1+\beta+\lambda\varphi)\}^2 +\\ &\quad \delta\mu\varphi\{1 + \mu\varphi(\chi_z + \lambda\chi_\pi)\}\{(2-\beta\mu)\chi_z + 2\lambda\chi_\pi\} +\\ &\quad \delta^2\mu\varphi\{2 - \mu(1+\beta+\lambda\varphi)\}\{(2-\beta\mu)\chi_z + 2\lambda\chi_\pi\} +\\ &\quad \delta^2\{1 + \mu\varphi(\chi_z + \lambda\chi_\pi)\}\{(1-\mu)(1-\beta\mu) - \mu\varphi\lambda\} +\\ &\quad \delta^3\{2 - \mu(1+\beta+\lambda\varphi)\}\{(1-\mu)(1-\beta\mu) - \mu\varphi\lambda\} \end{aligned}$$

In this expression $(1-\mu)(1-\beta\mu) - \mu\varphi\lambda$ is positive for all $\mu < \tilde{\mu}$ and hence for all $\mu < 2^{-1}\bar{\mu}$ (using (38)) since the expression $\mu\lambda\varphi - (1-\mu)(1-\beta\mu)$ is increasing in μ and is zero when $\mu = \tilde{\mu}$ so that it is negative for all $\mu < \tilde{\mu}$.

In computing $c_1c_2 - c_3$ we ignore first, second, third and fifth terms of the preceding expression for c_1c_2 and (to economize on space) denote them only by .. while keeping in mind that these terms are all positive. We retain only the fourth and final (sixth) term and obtain

$$\begin{aligned} c_1c_2 - c_3 &= \delta^2\mu\varphi\{2 - \mu(1+\beta+\lambda\varphi)\}\{(2-\beta\mu)\chi_z + 2\lambda\chi_\pi\} +\\ &\quad \delta^3\{2 - \mu(1+\beta+\lambda\varphi)\}\{(1-\mu)(1-\beta\mu) - \mu\varphi\lambda\}\\ &\quad -\delta^2[\mu\varphi\{(1-\beta\mu)\chi_z + \lambda\chi_\pi\} + \{(1-\mu)(1-\beta\mu) - \mu\varphi\lambda\}] + ..\\ &= \delta^2\mu\varphi\{2 - \mu(1+\beta+\lambda\varphi)\}[\{(1-\beta\mu)\chi_z + \lambda\chi_\pi\} + \{\chi_z + \lambda\chi_\pi\}] +\\ &\quad \delta^3\{2 - \mu(1+\beta+\lambda\varphi)\}\{(1-\mu)(1-\beta\mu) - \mu\varphi\lambda\} -\\ &\quad \delta^2[\mu\varphi\{(1-\beta\mu)\chi_z + \lambda\chi_\pi\} + \{(1-\mu)(1-\beta\mu) - \mu\varphi\lambda\}] + ..\\ &= \delta^2\mu\varphi\{(1-\beta\mu)\chi_z + \lambda\chi_\pi\}\{1 - \mu(1+\beta+\lambda\varphi)\} +\\ &\quad \delta^2\mu\varphi\{2 - \mu(1+\beta+\lambda\varphi)\}\{\chi_z + \lambda\chi_\pi\} +\\ &\quad \delta^2\{(1-\mu)(1-\beta\mu) - \mu\varphi\lambda\}[\delta\{2 - \mu(1+\beta+\lambda\varphi)\} - 1] + .. \end{aligned}$$

The first two terms in the final expression are positive since $\mu < 2^{-1}\bar{\mu}$. Regarding the final explicit term, it was shown above that $(1-\mu)(1-\beta\mu) - \mu\varphi\lambda > 0$ for all $\mu < 2^{-1}\bar{\mu}$. Moreover, since $\delta \geq 1$ and $\mu < 2^{-1}\bar{\mu}$,

$$\begin{aligned} \delta\{2 - \mu(1 + \beta + \lambda\varphi)\} - 1 &\geq 2 - \mu(1 + \beta + \lambda\varphi) - 1 \\ &= 1 - \mu(1 + \beta + \lambda\varphi) > 0, \end{aligned}$$

which proves that the final term is positive. Thus $c_1c_2 - c_3 > 0$ (recall that the .. terms in the expression for $c_1c_2 - c_3$ above are all positive).

To prove Corollary 12, consider the characteristic polynomial (39). Assume that the Taylor principle holds since otherwise we immediately have instability. We note that for $c_1 < 0$, it is necessary that $\mu > \bar{\mu}$ and for $c_2 < 0$, it is necessary that $\mu\lambda\varphi - (1-\mu)(1-\beta\mu) > 0$. The rest follows, since $c_1 < 0$ or $c_2 < 0$ suffices for instability.

A.4 Proof of proposition 13

We follow the methodology in Section 13.1.1 of (Evans and Honkapohja 2001a). To obtain the associated differential equation for the algorithms (17) (with $\gamma_{P,t} = t^{-1}$) and (32), we compute

$$\begin{aligned} & E(R^P)^{-1}x_{t-1}(y_{t-1} - \phi^P x_{t-1})' \\ = \; & E(R^P)^{-1}\begin{pmatrix} 1 \\ g_{t-1} \\ u_{t-1} \end{pmatrix}\left[(z_{t-1}, \pi_{t-1}) - (1, g_{t-1}, u_{t-1})(\phi^P)'\right] \\ = \; & E(R^P)^{-1}\begin{pmatrix} 1 \\ g_{t-1} \\ u_{t-1} \end{pmatrix}\left[(1, g_{t-1}, u_{t-1})\left[\begin{pmatrix} \tilde{a}_z & \tilde{a}_\pi \\ \tilde{A}_{zg} & \tilde{A}_{\pi g} \\ \tilde{A}_{zu} & \tilde{A}_{\pi u} \end{pmatrix} - (\phi^P)'\right]\right], \end{aligned}$$

where the temporary notation $\tilde{a}_z$, $\tilde{A}_{zg}$ etc. is obtained from the transposed form of the (31), ie

$$(z_{t-1}, \pi_{t-1}) = (1, g_{t-1}, u_{t-1})\begin{pmatrix} \tilde{a}_z & \tilde{a}_\pi \\ \tilde{A}_{zg} & \tilde{A}_{\pi g} \\ \tilde{A}_{zu} & \tilde{A}_{\pi u} \end{pmatrix}.$$

Taking expectations and limits the associated differential equation for the private sector algorithm becomes

$$\begin{aligned} d\phi^P/d\tau &= (R^P)^{-1}(Exx')[T^P(\phi^P, \phi^{CB}) - \phi^P] \\ dR^P/d\tau &= Exx' - R^P, \end{aligned} \tag{40}$$

where $Exx' = \lim_t Ex_t x_t'$ and

$$T^P(\phi^P, \phi^{CB}) = (A^P a^P + A^{CB} a^{CB}, \mu(A^P b_g^P + A^{CB} b^{CB}) + B_g, \rho A^P b_u^P + B_u).$$

The associated differential equation for the algorithm of the central bank is obtained analogously by computing

$$\begin{aligned} & E(R^{CB})^{-1}x_{t-1}^{CB}(y_{t-1}-\phi^{CB}x_{t-1}^{CB})' \\ = \;& E(R^{CB})^{-1}\begin{pmatrix} 1 \\ g_{t-1} \end{pmatrix}(1,g_{t-1})\left[\begin{pmatrix} \tilde{a}_z & \tilde{a}_\pi \\ \tilde{A}_{zg} & \tilde{A}_{\pi g} \end{pmatrix}-(\phi^{CB})'\right] \\ & +E(R^{CB})^{-1}\begin{pmatrix} 1 \\ g_{t-1} \end{pmatrix}u_{t-1}(\tilde{A}_{zu},\tilde{A}_{\pi u}). \end{aligned}$$

Since g_t and u_t are uncorrelated and have zero means, the second term in this expression is zero. The associated differential equation for the central bank is then

$$\begin{aligned} d\phi^{CB}/d\tau &= (R^{CB})^{-1}(Ex^{CB}(x^{CB})')[T^{CB}(\phi^P,\phi^{CB})-\phi^{CB}] \\ dR^{CB}/d\tau &= Ex^{CB}(x^{CB})'-R^{CB}, \end{aligned} \tag{41}$$

where $Ex^{CB}(x^{CB})' = \lim_t Ex_t^{CB}(x_t^{CB})'$ and

$$T^{CB}(\phi^P,\phi^{CB}) = (A^P a^P + A^{CB}a^{CB}, \mu(A^P b_g^P + A^{CB}b^{CB}) + B_g).$$

As is well known, the local stability of the associated differential equations (40) and (41) is governed by the local stability of the "small" differential equations

$$\begin{aligned} d\phi^P/d\tau &= T^P(\phi^P,\phi^{CB})-\phi^P \\ d\phi^{CB}/d\tau &= T^{CB}(\phi^P,\phi^{CB})-\phi^{CB}, \end{aligned}$$

which are the modified E-stability differential equations. Inspecting the T^P and T^{CB} mappings it is seen that for constant terms a^P and a^{CB}, as well as for the terms b_g^P and b^{CB}, we can just repeat the E-stability arguments for homogenous case in Section 3. The E-stability equation for b_u^P is simply

$$db_u^P/d\tau = (\rho A^P - I)b_u^P + B_u,$$

which completes the proof.

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