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Abstract

This paper studies the cyclical properties of optimal emission taxes and emissions using a real business cycle model with a stock pollutant. We derive conditions for the procyclicality of optimal emission tax and show that the tax is in typical conditions procyclical. The possibility of a countercyclical behavior of the emission tax increases if 1) the pollution is short-lived and the emission transfer into environmental damages rapidly 2) emissions are countercyclical, 3) marginal damages are strongly increasing and 4), in disutility case, the marginal utility of consumption increases with the increase in the intensity of the harmful environmental process. In the climate change context we show that the optimal carbon tax is procyclical irrespectively on the production technology. Instead, the technology is a key determinant of the cyclicality of the emissions. The optimal carbon tax correlates almost fully with the consumption and as a rule-of thumb, it could be indexed to the consumption level of the economy. The relative scale of tax deviations relative to the consumption deviations is determined by the inverse of the intertemporal elasticity of substitution. Comparison between the optimal emission tax and an optimally set constant emission tax shows that the constant tax leads to very slightly higher emissions but the general economic effects are next to negligible.

JEL classification: E32, Q54, Q58

Keywords: optimal emission tax, cyclical properties

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1 Introduction

This paper contributes to the understanding of cyclical behavior of optimal emission tax and emissions. Since the seminal paper by Weitzman (1974), the question of optimal environmental policy under uncertainty has received a wide attention. Typically, the analysis has focused on imperfect knowledge on costs and benefits of a given economic activity, i.e. a sort of second best setting of policy planning. A related question is the first best optimal adaptation of policy when the economic circumstances vary over time. Recently, the global concern on climatic change support introduction of policies that have economy-wide effects. Such policies with large economic impacts pose an increasing need to know how the policies should be adjusted over business cycles. Without adjustment the policy could, for example, have unwanted adverse effects on the economy during economic cycles.

We adapt a standard RBC model framework with a pollution stock segment in lines with Golosov *et al.* (2011). The pollution stock is accumulated through the use of polluting input and the pollution stock sustains adverse environmental effects that can both decrease the productivity of the economy and cause direct utility loss on households. The analysis presented is general and can be used in describing any stock pollutant. However, to fix ideas, we concentrate on the case of climate change and carbon dioxide emissions. In this context the polluting input describes the use of fossil fuels in the energy sector. We do not follow the approach of explicit abatement costs similar to (Nordhaus, 1993), but the emission reduction possibilities of the economy are based on substitution between polluting input and other inputs of production as well as decrease in overall level of energy use and in production. In our model, the energy services are provided through aggregation of the use of polluting input and a special form of nonpolluting energy sector capital. We vary the substitutability between the polluting input and energy capital.[1] The case of low substitutability describes the current energy sector, where majority of existing capital supports the use of fossil fuels. The high substitutability case presents a possible future economy, where is more clean technology energy capital allowing for non-polluting energy generation, such as wind turbines and photovoltaics. In practice, the elasticity of substitution between the two energy inputs determines the costs of emission abatement.

The economic analysis of the climate change problem has typically been studied using integrated assessment models (IAM) (e.g. Nordhaus, 1993; Stern, 2007; Nordhaus, 2007; Golosov *et al.*, 2011). These IAMs concentrate on the core phenomena of global warming, i.e. economic growth, carbon cycle and environmental damages contributing to the loss of economic productivity. These phenomena are slow and the effects are long-lasting. Therefore, the models focus on time-scales of hundreds of years. The models yield the optimal time path of carbon tax in cost-benefit analysis. In addition, explicit pollution or temperature limits can be studied with the models. The time path of carbon tax is the core of the optimal climate policy. However, these models abstract away from shorter term stochastic

[1] This approach is comparable to the view taken by Acemoglu *et al.* (2012) who study the effect of substitutability between polluting and clean inputs on climate change mitigation. In our study the clean input is a stock variable whereas in their model the clean input is an unspecified instantaneously adaptable variable.

fluctuations in the economy. Therefore, the IAMs can say little about the optimal policy structure related to the random changes in the time scales of the business cycles. The recent collapse of carbon price in European emission trading system suggests that climate policy may need to be adjusted over economic cycles. The hesitation of the policy makers of Europe in adjusting the policy scheme indicates that the connection between optimal climate policy and business cycles is not clearly understood.

We construct a model for analysis using the general time-lag structure for environmental damages, introduced by Gerlagh & Liski (2012) and show that there is a strong inclination for pro-cyclical emission taxes. For rapidly transmitted pollution, such as sulfur oxides, the tax fluctuation is governed by the production level whereas with slow processes the optimal tax follows the consumption fluctuations. Especially, in the case of climate change the size of atmospheric carbon stock dwarfs the business cycle changes in carbon emissions and the tax level is driven by consumption alone. Both the analytic model and the numerical simulation suggest that the optimal carbon tax could be indexed to consumption level in the economy as the correlation between carbon tax and consumption is nearly perfect. The case of optimal emissions is different. Our numerical simulations show that cyclical properties of the emissions depend strongly on the production technology: High substitutability between polluting input and associated capital leads to counter-cyclical emissions whereas low substitutability leads to pro-cyclical emissions.

Most of the earlier studies on the effects of stochastic shocks on emission control have focused on the second best optimal setup of Weitzman (1974) and the relative performance of price and quantity regulation. Newell & Pizer (2003) generalize the analysis to the case of stock pollutants and show that in the case of climate change, the emission tax seems preferable to the quantity regulation. A contrary result was derived by Kelly (2005) with a static model where risk preferences of the representative consumer were incorporated into the model. Reasonable level of risk aversion seems to overcome the usual slope arguments between marginal costs and benefits and favor the quantity regulation over taxes. Quite recently, Newell & Pizer (2008) have generalized the Weitzman's setting to adapt regulation where quantity caps are indexed to other economic variables. Their analysis shows that in the case of climate change the indexing of the cap might be preferable in some countries but neither quantity policy outperforms the other systematically. Fischer & Springborn (2011) were, to our knowledge, first to apply RBC model in studying the macroeconomic effects of emission control. They study the relative properties of emission cap, tax and an intensity target policies. Their study shows that emission tax increases the size of business cycle fluctuations of the economic variables, whereas emission cap decreases them and intensity target is neutral. However, they model a case of exogenous emission target. Analogously, the studied policies are in a sense limited, not trying to enforce the first best optimum.

A study closest to ours is one by Heutel (2012) where optimal adjustment of CO_2 tax and emissions over business cycle is studied. He calibrates and numerically simulates a RBC model with carbon stock related environmental damages and finds that both the optimal tax and emissions are pro-cyclical.

Our study deviates from this approach in many perspectives and, therefore, conveys additional insight on the question. First, we do not have explicit abatement effort in the model but the economy needs to reorganize input use for climate change mitigation. Second, we derive an analytical formulation for the optimal tax and use it to analyze the problem. Third, we allow for more general production technology and assess the future prospects of mitigation. Fourth, our carbon cycle and temperature model allows for more reasonable description of time-lags appearing in the climate system.

The rest of paper is organized as follows. Section 2 describes the model framework in a general form and Section 3 presents the optimal policy and analyzes the cyclicality of the emission tax. In Section 4 we calibrate the model to the carbon dioxide emissions and presents the behavior of carbon emissions and optimal carbon dioxide tax over the business cycle. Section 5 concludes.

2 The model

2.1 Planner solution

We build a business cycle model in which the social planner maximizes the expected net present value of representative household's utility streams. We assume that the utility is derived from private consumption, C_t, and disutility through hours devoted to labor, H_t. The production of final good demands energy which in turn is produced by using a polluting input, X_t. The input use causes emissions that accumulate into the stock of pollution, S_t. The pollution stock contributes to the natural phenomena that are harmful to the society in general. The strength of these natural phenomena are denoted by D_t and we assume that the negative effects of the phenomena arise through direct periodic disutility as well as through decrease in global productivity. In the climate policy context D_t denotes quadratic temperature change from the preindustrial temperature level.

In the analytical model we use a linear multi-box description for the pollution stock and the natural phenomena. However, we use the framework introduced by Gerlagh & Liski (2012) and work directly with vector of past emissions $\chi_t = (X_{t-1}, X_{t-2}, \ldots)$.[2] In practice, it proves useful to define a auxiliary function that yields the contribution of the periodic emissions on the strength of the environmental phenomena

$$D(\chi_t) := \sum_{j=1}^{\infty} \omega_j X_{t-j}. \tag{1}$$

Thus, further analysis can be performed with a general lag-structure $\boldsymbol{\omega} = (\omega_1, \omega_2, \ldots)$, where ω_js are constant marginal contributions of emission increments on the harmful environmental phenomenon. See Appendix A for details.

Since our focus is on business cycles we omit all the economic growth features from our model. Instead,

[2]These lagged input uses could be linked to the nonrenewable resource stocks but this is an option we do not use here (cf. Golosov *et al.*, 2011).

we assume that there is a constant amount of households supplying labor force, H_t, that can work both in the sector producing the aggregate final good as well as in the energy sector producing and processing the polluting input. Besides labor, L_t, the production of the final good needs energy, E_t, and capital, K_t^{fin}, but is also affected by the environmental conditions, summarized here by $D(\chi_t)$. Formally the production function for final good is

$$Y_t = f(L_t, K_t^{fin}, E_t, D(\chi_t); A_t). \tag{2}$$

Productivity in the final good sector, A_t follows an exogenous stochastic process with positive support and variance. The unconditional expectation $\mathbb{E}A_t$ is equal to unity. The stochastic process has autocorrelation $\rho_A \geq 0$. Throughout the paper, we assume that the production function of final good satisfies the Inada conditions and has constant returns to scale. Thus, the optimization leads to interior solution for all the inputs.

The energy input is generated through polluting input and energy capital K_t^{ene}

$$E_t = g(X_t, K_t^{ene}) \tag{3}$$

and polluting input production needs labor inputs

$$X_t = h(H_t - L_t). \tag{4}$$

Our model does not contain explicit abatement possibilities (cf. Nordhaus, 1993, 2008; Heutel, 2012). Instead, the emission control needs to be through labor allocation and substitution between polluting input and capital in the energy production. Both final good and energy sector capital stocks are assumed to depreciate with a capital specific constant rate δ_i, where $i \in \{fin, ene\}$. The equations of motion for capital stocks are standard

$$K_{t+1}^i = (1-\delta)K_t^i + I_t^i. \tag{5}$$

As we analyze a closed economy, the sum of investments I_t^i is equal to the savings, i.e.

$$I_t^{fin} + I_t^{ene} = Y_t - C_t. \tag{6}$$

The optimization problem of the social planner can be presented as a stationary dynamic program, where value function V is determined through optimization problem

$$V(K_t^{fin}, K_t^{ene}, A_t, \chi_t) = \max_{\{H_t, L_t, I_t^{fin}, I_t^{ene}\}} u(C_t, H_t, D(\chi_t)) + \beta \mathbb{E}_t V(K_{t+1}^{fin}, K_{t+1}^{ene}, A_{t+1}, \chi_{t+1}) \tag{7}$$

subject to constraints given by the equations (2) – (6). Since labor supply and labor use in final good production uniquely determine the polluting input use, and the saving decision determines the level of consumption, the optimization is performed through labor supply, labor use in final good sector and the investment variables. Solving the problem yields optimality conditions (see Appendix B)

$$u_{Ct} = \beta \mathbb{E}_t \left[(1 - \delta_{fin} + f_{K,t+1}) u_{C,t+1} \right], \tag{8}$$

$$u_{Ct} = \beta \mathbb{E}_t \left[(1 - \delta_{ene} + f_{E,t+1} g_{K,t+1}) u_{C,t+1} \right], \tag{9}$$

which are the standard Euler equations for consumption smoothing,

$$f_{Lt} u_{Ct} + u_{Ht} = 0, \tag{10}$$

which determines the labor supply and

$$(f_{Lt} - f_{Et} g_{Xt} h_{\tilde{L}t}) u_{Ct} = h_{\tilde{L}t} \mathbb{E}_t \sum_{s=1}^{\infty} \beta^s \omega_s \left[u_{C,t+s} f_{D,t+s} + u_{D,t+s} \right], \tag{11}$$

which pins down the optimal allocation of labor between the two sectors.[3] The labor allocation term is augmented with terms that present the environmental harm caused by the polluting input. The LHS of equation (11) denotes the difference between marginal utility derived from marginal productivities of labor in the final good sector and in polluting input sector. The latter term represents the utility generated by the input use in the final good production. The RHS contains the net present value of marginal environmental effects of polluting input use in utility terms. Without environmental consequences LHS should equate to zero. However, the use of polluting input causes harmful emissions ($f_4 < 0$ and $u_2 < 0$) and, to compensate this detriment the marginal productivity of labor in polluting input production has to be larger than in the final good production ($f_E g_X h_{\tilde{L}} > f_L$) in utility terms. The labor allocation equation can be written in monetary terms simply by

$$f_{Lt} - f_{Et} g_{Xt} h_{\tilde{L}t} = h_{\tilde{L}t} u_{Ct}^{-1} \mathbb{E}_t \sum_{s=1}^{\infty} \beta^s \omega_s \left[u_{C,t+s} f_{D,t+s} + u_{D,t+s} \right]. \tag{12}$$

This formulation proves most useful when studying the cyclical properties of the optimal emission tax.

2.2 Market solution

In the market economy, the agents do not take into account the externalities their decisions cause. With current model there is only one externality, namely the environmental effects of the emissions from polluting input use. Anticipating the optimal policy, given the social planner solution above, we pose a carbon tax on the polluting input use. The aim is to derive the first best optimal emission tax through comparison between the market solution and the planner solution.

Household has two capital goods, $\mathbf{K} := (K^{fin}, K^{ene})$, for investing their savings in. In addition to savings decision, the household makes a labor supply decision, H_t. Thus, the dynamic optimization problem of consumption smoothing and labor supply is

$$v(\mathbf{K}_t) = \max_{\{H_t, \mathbf{K}_{t+1}\}} u(C(H_t, \mathbf{K}_{t+1}), H_t, D(\chi_t)) + \beta \mathbb{E}_t v(\mathbf{K}_{t+1}), \tag{13}$$

[3]Subscript $\tilde{L}$ in function h denotes the partial derivative of the function relative to the labor used as an input in polluting input production.

where consumption is defined through an auxiliary function

$$C(H_t, \mathbf{K}_{t+1}) := w_t H_t + R_t + \sum_{i \in \{con, ene\}} \left[(1 + r_t^i - \delta_i) K_t^i - K_{t+1}^i \right]. \tag{14}$$

The final good is chosen to be the numeraire good. Variable w_t is the income from labor markets, r_t is the gross interest paid by the firm for its capital and R_t a lump-sum tax rebate to the consumers. The real interest rate in the asset markets is $r^i - \delta^i$. In market equilibrium with positive amounts of investments in both capital assets, the expected value of real interest rate has to coincide. Solving the optimization problem leads to the standard Euler equations

$$u_{Ct} = \beta \mathbb{E}_t \left[(1 - \delta_{fin} + r_{t+1}^{fin}) u_{C,t+1} \right] \tag{15}$$

and

$$u_{Ct} = \beta \mathbb{E}_t \left[(1 - \delta_{ene} + r_{t+1}^{ene}) u_{C,t+1} \right], \tag{16}$$

which are counterparts of equations (8) and (9) in the planner solution. The optimal labor supply decision is given by

$$w_t u_{Ct} + u_{Ht} = 0. \tag{17}$$

The optimization problem of the firm in final good sector is static. Profit maximization problem of the firm producing the final good is defined as

$$\max_{\{L_t, K_t^{fin}, E_t\}} f(L_t, K_t^{fin}, E_t, D(\chi_t)) - w_t L_t - r_t^{fin} K_t^{fin} - p_t E_t. \tag{18}$$

Variable p_t denotes the energy price. Given the simplifying assumption above, the first order conditions for the optimum are

$$f_{Lt} = w_t, \tag{19}$$

$$f_{Kt} = r_t^{fin} \tag{20}$$

and

$$f_{Et} = p_t. \tag{21}$$

Since the production technology has constant returns to scale, the equilibrium profits are equal to zero in every period.

The energy sector firms use the polluting input in the energy production. Firms do not internalize the externality from the emissions unless the government sets a emission tax on polluting input. This tax is in general time-dependent and we denote it by τ_t.[4] The profit maximization problem is static and formulated as

$$\max_{\{X_t, K_t^{ene}\}} p_t g(X_t, K_t^{ene}) - (q_t + \tau_t) X_t - r_t^{ene} K_t^{ene}, \tag{22}$$

[4]Since we measure the polluting input use in units of emission, the unit of tax is equal to the unit of price of the input, q_t, i.e. USD/t_{CO_2}.

where q_t is the price of polluting input. The production function g satisfies Inada conditions and, therefore, we have interior solution described by the necessary first order conditions

$$p_t g_{Xt} = q_t + \tau_t \tag{23}$$

$$p_t g_{Kt} = r_t^{ene}. \tag{24}$$

Thus, the positive emission tax reduces the use of polluting input compared with the no policy market equilibrium. The firms' ability to switch between polluting input and non-polluting capital depends on the rate of substitution between the two inputs.

Finally, there are firms producing and processing the polluting input. The firms in the polluting input sector face a static profit maximization problem. Given the Inada conditions on the production technologies in the consumer good and energy sectors, in the equilibrium there is always demand for polluting input. Thus, we focus here on interior solution. The maximization problem is simply

$$\max_{\tilde{L}_t} q_t h(\tilde{L}_t) - w_t \tilde{L}_t, \tag{25}$$

which leads to the necessary first order condition of optimality

$$q_t h_{\tilde{L}}(\tilde{L}_t) = w_t. \tag{26}$$

From the market equilibrium point of view it would not make a difference if the emission tax would have been imposed on producer of the input. The tax incidence would remain the same.[5]

The market equilibrium described by Euler equations and the first order conditions can be summarized in four equations. The first two equations describe the optimal consumption smoothing and investment decisions by the households

$$u_{Ct} = \beta \mathbb{E}_t \left[(1 - \delta_{fin} + f_{K,t+1}) u_{C,t+1} \right], \tag{27}$$

and

$$u_{Ct} = \beta \mathbb{E}_t \left[(1 - \delta_{ene} + f_{E,t+1} g_{K,t+1}) u_{C,t+1} \right], \tag{28}$$

where interest rate has been determined through marginal productivity of capital in the two capital using sectors. The third equation represents the labor supply decision, i.e. equation (17), rewritten here for convenience,

$$f_{Lt} u_{Ct} + u_{Ht} = 0. \tag{29}$$

Three equations above are equal with the equilibrium equations of the planner solution (8) – (10). The fourth equation determines the labor allocation between the final good and polluting input sectors[6]

$$f_{Lt} - f_{Et} g_{Xt} h_{Lt} = -h_{Lt} \tau_t. \tag{30}$$

[5]Since the fossil fuel trade is international, it is typical to set national climate policies on domestic sectors, such as energy sector.

[6]Equation is derived from observation that in the equilibrium the wage has to be equal between the labor using sectors.

Without tax, labor is allocated in a manner that equates marginal products of labor in both sectors. However, the emission tax penalizes use of polluting input and in the equilibrium the marginal product of labor in polluting input sector needs to be higher than in the final good sector since $h_L > 0$ and $\tau \geq 0$. If the tax level is optimally chosen, the RHS of equation (30) can be made to coincide with the planner solution (11). In that case all the equilibrium determining equations are equal between market and planner solutions indicating that the tax level enforces the planner solution in the market equilibrium.

3 Optimal emission tax

We study the cyclical properties of the optimal emission tax without informational constraints. Thus, the policy administrator can adjust the policy after the productivity shock is realized. By direct comparison of equations (12) and (30) one observes that the optimal tax level is

$$\tau_t = -u_{Ct}^{-1}\mathbb{E}_t \sum_{s=1}^{\infty} \beta^s \omega_s \left[u_{C,t+s} f_{D,t+s} + u_{D,t+s} \right]. \tag{31}$$

The tax is composed of two factors. First, the sum factor gives the expected net present value of marginal disutility caused by the present emissions. The first term in the parenthesis is the value of the marginal production damage and the second term represents direct disutility effects. This factor is the natural driver of the emission taxation. The parameter values ω_s present the transmission of emissions into harmful environmental processes (Appendix A). Jointly with the discount factor, β, it determines the relative contributions of different time lags on the present tax level.

The other factor in product (31) is the inverse of marginal utility of consumption transforms the environmental harm, measured in utility terms, into monetary terms. Since the utility function is concave, the higher the present consumption level, the higher the emission tax in monetary terms, other things being equal. This second term, presenting a sort of present wealth effect, may have notable contribution on the cyclical adjustment of the optimal emission tax, regardless of harmful environmental processes studied.

3.1 Fluctuations around the steady-state

We study the business cycle fluctuations of the optimal emission tax through relative deviations around the steady-state level. By direct observation the deterministic no-growth steady-state of the optimal tax level (31) is

$$\bar{\tau} = \left[|\bar{f}_D| + \frac{|\bar{u}_D|}{\bar{u}_C} \right] \Omega, \tag{32}$$

where bar denotes for steady-state values. The parameter $\Omega := \sum_t \beta^t \omega_t$ aggregates the future marginal damage flows caused by the last unit of emissions into present value terms. The aggregator is a scalar product of sequence of discount factors and the lag-structure between emissions and resulting harmful environmental phenomena.To study the behavior of optimal emission tax over the business cycle, we present equation (31) in a log-linearized form around the steady-state value $\bar{\tau}$ above

$$\hat{\tau}_t = -\hat{u}_{Ct} + \sum_{s=1}^{\infty} w_s \mathbb{E}_t \left[(\hat{u}_{C,t+s} + \hat{f}_{D,t+s}) W_Y + \hat{u}_{D,t+s} W_U \right]. \tag{33}$$

The parameter $w_s := \Omega^{-1}\beta^s \omega_s$ denotes the weight of each time lag on tax fluctuation. Thus, the second term is a weighted average of relative changes in marginal environmental harm in the future periods. In the sum, the parameters $W_Y := \Omega|\bar{f}_D|/\bar{\tau}$ and $W_U := \Omega|\bar{u}_D|/(\bar{u}_C\bar{\tau})$ are the weights of the pollution loss and direct disutility harms in the steady state tax level, respectively.[7]

This provides us an important tool to judge the cyclicality of emission tax. If equation (33) turns out to be positive after expansionary technology shock, then the emission tax is procyclical. Using the general functional forms $u_t(C_t, H_t, D_t)$ and $f_t(L_t, X_t, K_t, D_t; A_t)$, for the utility and production, respectively, we can rewrite equation (33) in the terms of percentage deviations of variables from their steady-state values

$$\begin{aligned} \hat{\tau}_t \quad = \quad & -\varepsilon_{u_C C}\hat{C}_t + W_Y \sum_{s=1}^{\infty} w_s \mathbb{E}_t \left[\varepsilon_{u_C C}\hat{C}_{t+s} + \varepsilon_{u_C D}\hat{D}_{t+s} \right] \\ & + W_Y \sum_{s=1}^{\infty} w_s \mathbb{E}_t \left[\varepsilon_{f_D L}\hat{L}_{t+s} + \varepsilon_{f_D X}\hat{X}_{t+s} + \varepsilon_{f_D K}\hat{K}_{t+s} + \varepsilon_{f_D D}\hat{D}_{t+s} + \varepsilon_{f_D A}\hat{A}_{t+s} \right] \\ & + W_U \sum_{s=1}^{\infty} w_s \mathbb{E}_t \left[\varepsilon_{u_D C}\hat{C}_{t+s} + \varepsilon_{u_D D}\hat{D}_{t+s} \right], \end{aligned} \tag{34}$$

where $\varepsilon_{\varphi_x y} := y\varphi_x^{-1}\varphi_{xy}$ denotes the y elasticity of marginal utility or damage φ_x.[8] The first term is the deviation of present marginal utility of consumption. The second term in the first row represents the weighted average of expected future marginal utility of consumption. The second and third rows are weighted averages of expected marginal production loss and marginal disutility of pollution, respectively.

The relative importance of the terms in equation (34) is dependent on three factors: The magnitude of related elasticity, the size of the related response and, in the case of future values, the match between lag structures of the deviation of a related variable and the environmental process. The first two factors are rather obvious, but the last factor needs explanation. In the case of future values, the magnitude of an effect is determined through product $w_s\hat{x}_s$, where x is the variable in question.[9] For the product to have large values, the both factors, w_s and $\hat{x}_s$, need to have large values, i.e. weighted average of $\hat{x}_s$ has high weights on lags where $\hat{x}_s$ has high values. If this is not the case and the weights are high when deviations in the variable are small, the variation of the variable contributes weakly to the variation of the tax.

[7]Parameters w_s are weights in the ordinary sense since $w_s \geq 0$ and $\sum_s w_s = 1$. Analogously, W_Y and W_U are weights since $W_Y \geq 0$, $W_U \geq 0$ and $W_Y + W_U = 1$ (see equation (32)).

[8]We have assumed here that consumption, C_t, and labor supply, H_t, appear in the utility function in an additively separable manner. Therefore, the labor supply fluctuations do not appear in the equation. Instead we allow for second derivative u_{CD} to differ from zero. We have also used the fact that D_t cannot change instantly.

[9]We make here a rather safe assumption that the elasticities are relatively constant over the business cycle.

For these timing reasons it is useful to divide the analysis into two cases. The first case studied is a fast environmental process, where pollution is short-lived and the emissions contribute rapidly and more strongly to the harmful environmental process. In the second case, the pollution is long-lived and the transfer of emissions into harmful environmental processes are slow. We call these cases as fast and slow environmental process, respectively. For fast processes, ω_s peaks within few years of the emissions and decays to zero in decade or two. For slow processes, ω_s peaks only after several decades and decays to zero during the next couple of centuries. Since the weights are $w_s = \beta^s \omega_s$, they peak earlier than the ω_s and decay faster.

In general terms, it can be stated that the economic decision variables as well as the productivity shock driving the fluctuations follow time paths similar to those of the fast environmental process: Deviation peaks instantly or within few years and deviation decays monotonically in a decade or two. By construction, the environmental damage variable, $\hat{D}_{t+s}$, follows the time path ω_s, which by definition correlates with w_s. Due to concavity of the optimization problem, the elasticity $\varepsilon_{u_C C}$ is negative and elasticities $\varepsilon_{u_D D}$ and $\varepsilon_{f_D D}$ are positive. The cross elasticities $\varepsilon_{u_C D}$ and $\varepsilon_{u_D C}$ are equal to zero if the utility function is additively separable. If u is not additively separable, the two elasticities are of opposite signs because $u_D < 0$ but $u_C > 0$. Under an usual assumption of multiplicative production loss damages the other elasticities, $\varepsilon_{f_D L}$, $\varepsilon_{f_D X}$, $\varepsilon_{f_D K}$ and $\varepsilon_{f_D A}$ are positive and less than or equal to unity.

3.2 The effect of the pace of polluting process on optimal taxation

Next, we analyze the cyclical properties of the optimal tax through the effects of positive productivity shocks on economy by using equation (34). In a fast environmental process case, such as the emissions of sulfur oxides, the environmental process fluctuates in the same time scale as the economic variables. Therefore, all the terms in equations above can contribute to the tax fluctuations.

The first term describes the present consumption level and due to the concavity of the utility function it is positive. The other terms are less straightforward. Let us study the case where there is only production loss damages ($W_Y = 1$ and $W_U = 0$).[10] The near future deviations of the marginal utility of consumption are typically of the same magnitude as the present one. Since the signs are opposite, these terms largely cancel each other. It is reasonable to assume that the positive technology shock increases production enough to dominate the adverse environmental effects of possible increase in environmental damages (variable D). Under this assumption, the second row in equation (34) is positive. As pollution does not affect directly utility, other terms of equation (34) are zero. Therefore, fluctuations in the output dominate indicating a procyclical optimal emission tax.

In a special case of catastrophic environmental process the economy may be on the brink of a tipping point and have a very large positive elasticity $\varepsilon_{f_D D}$. Thus, the fluctuations in marginal damages could

[10] In this case $u_D \equiv 0$.

be driven by the changes in the intensity of the environmental process. If, in addition, the technology is such that the positive productivity shock causes a decrease in emissions resulting in a weakened environmental process, D, the marginal production damages could be countercyclical. Under these rather extreme conditions, the optimal emission tax could be countercyclical too. However, these conditions seem to be rather peculiar and, perhaps, only hold in rare instances.

If there are only the direct disutility effects ($W_U = 1$ and $W_Y = 0$), the strong procyclical force of the first term in equation (34) remains. The other effects on the optimal fluctuations of the tax emerge through terms in the third row of the equation. It is rather standard to assume that consumption and pollution affect utility in an additively separable way. In this case $\varepsilon_{u_D C}$ and $\varepsilon_{u_C D}$ are equal to zero and only the changes in environmental process affect the tax rate. Since marginal disutility is increasing ($\varepsilon_{u_D D} > 0$), the third row drives tax towards countercyclicality if a positive productivity shock decreases emissions. If the marginal disutility effect is strong enough to reverse the procyclical effect of the marginal utility of consumption, as in the catastrophic environmental process case above, the optimal emission tax could be countercyclical. In most cases the conditions presented seem unlikely, and therefore we can conclude that under additively separable utility function, the optimal emission tax is typically procyclical.

Analysis changes slightly, if utility function is not additively separable, i.e $u_{CD} \neq 0$. Consider the case in which $u_{CD} > 0$, which leads to $\varepsilon_{u_D C} < 0$ and $\varepsilon_{u_C D} > 0$. This assumption would mean that the marginal damage from pollution decreases with the consumption level, i.e. wealthier households suffer less from pollution.[11] In this case the first term in the third row drives the tax towards countercyclicality and together with a countercyclical pollution stock can dominate the cyclical properties over the procyclical effects of marginal utility of consumption discussed above. Thus, suitable functional specification for the utility function increases the plausibility of a countercyclical optimal emission tax. In the case of rapidly transmitted emissions, with additive utility function and procyclical emissions, the optimal emission tax is procyclical and driven by the production level. But, with a non-additive utility function and countercyclical emissions, a countercyclical optimal emission tax is possible, if $\varepsilon_{u_D C} < 0$ and $\varepsilon_{f_D D} < 0$ are large enough in absolute terms.

For the slow environmental processes, such as carbon dioxide emissions, the mechanisms of the cyclicality of tax are different. Under slow natural processes the pollution stock is accumulated over long period of time. In that case, the short-lived variations in periodic emissions have a small effect on the pollution level and, therefore, on the actual marginal damages caused by the environmental phenomena as the changes of the variable D are very small. Due to the slow progress of the environmental process, the fluctuations in economic variables have vanished before the present emissions start to alter the environmental conditions, i.e. before weights w_s get values differing significantly from zero. In addition, these environmental effects occur after such a long lag that the discounting differentiates

[11] This might be reasonable since wealthier households have higher consumption level, which makes it is easier to shelter from environmental phenomena.

even the weights for the future deviations, w_s, from the progress of the environmental process ω_s. This lag diminishes the impacts of the fluctuations of environmental harms too as the correlation between w_s and ω_s is decreased. Therefore, under a slow environmental process, the changes in marginal damage, caused by business cycle fluctuations, are typically very small in utility terms.

Instead of changes in marginal damages, the changes in optimal taxes over the business cycle are mostly driven by a present economic situation, i.e. the first term in equation (34). Due to the concavity of consumption utility, the increase in consumption during booms lowers the marginal utility losses caused by the emission tax and, therefore, the households can afford higher emission taxes in monetary terms. As the marginal environmental damages in utility terms remain roughly constant, the monetary emission tax needs to be increased in economic boom to preserve the marginal utility losses from the emission tax. In formal terms, mainly the changes in the first term in equation (34) drive the cyclicality of the emission tax, i.e. only the present consumption level has significant impacts on the cyclical properties of the tax. Therefore, the cyclicality of optimal emission tax under slow environmental processes should follow initial consumption.

An only notable exception is the case of catastrophic environmental harm, with a very large elasticities $\varepsilon_{f_D D}$ and $\varepsilon_{u_D D}$, where even a slight deviation in D can affect the tax fluctuations. In that case the cyclical tendencies follow those of the emissions. If the emissions are countercyclical and the environmental effects strong enough, they can overcome procyclical effects of the present marginal utility of consumption. Since under a long-lasting pollution and a slow environmental process, the fluctuations in D are very modest, the countercyclicality of an optimal emission tax seems unlikely.

To summarize, changes in emissions during the economic booms have the more significant direct marginal damage effects on welfare the stronger the emissions alter the harmful environmental processes and the sooner the harms emerge after the emission. If the harmful environmental process is catastrophic and the economy is near a tipping point, the marginal damage effects may dominate the business cycle properties of the tax. Under a production technology in which positive productivity shock decreases emissions, there is a force driving the emission tax towards countercyclicality. When harms stem through direct utility effects, utility function is not additively separable and $u_{CD} > 0$ the countercyclical forces are strengthened. However, if the damages occurring from the pollution stock are less violent, the emission tax tends to be procyclical. In the case of fast process, tax is driven by the movements in the aggregate production whereas, in the case of slow process, the only term significantly affecting the cyclicality of the tax is the present marginal utility of consumption. As a rule of thumb, we can conclude that, typically, optimal emission tax follows output in the case of fast environmental processes but in the case of slow processes it follows the present consumption level. In the latter case, a small intertemporal elasticity of substitution leads to a strong tax fluctuation compared to the consumption fluctuations and vice versa.

4 Optimal CO_2 taxation over business cycles

4.1 Calibration

Our numerical specification focuses on the case of carbon dioxide emissions and their link to the anthropogenic climate change. Thus, the pollution stock in the model is the CO_2 stock in the atmosphere and the environmental phenomenon related to it is global mean temperature. We assume that the harmful effects of the global warming can be uniquely linked to the quadratic temperature difference from the preindustrial temperature level.

The calibration of the economic model relies on existing macroeconomic literature whereas the climate model is based on the literature on integrated assessment models. Our model follows a general macroeconomic approach suggested by Golosov *et al.* (2011) and, thus, we do not model explicit abatement decision but the abatement is performed through input choice decisions in the production sectors. Instead of explicit function of abatement costs, the costs of climate change mitigation depend on production technologies. In practice, the mitigation costs are determined through the parameters of the production function.

The carbon-climate model used in the numerical calculations consists of a linear one-box model for the anthropogenic CO_2 stock

$$S_t = (1 - \sigma)S_{t-1} + X_t \tag{35}$$

and a non-linear one-box model for the quadratic temperature deviation

$$D_{t+1} = D_t + \frac{1}{T}\left(\pi\left[\frac{\log(1 + S_t/S_{pre})}{\log 2}\right]^2 - D_t\right). \tag{36}$$

The parameter σ presents the decay rate of the atmospheric CO_2. Our specification for carbon dynamics abstracts away from the very long term accumulation of atmospheric CO_2 (Archer, 2005; Archer *et al.*, 2009). We justify the chosen simplification by two arguments. First, we study the business cycle effects on optimal policy. The steady-state would not be feasible with near-permanent increases in atmospheric carbon concentration. This corresponds to the omission of economic growth phenomena in economic model part. Second, the effect of long-lasting fraction of atmospheric carbon has an effect on optimal tax level, but the effects on business cycle properties are minor. We follow Heutel (2012) and set an atmospheric carbon decay rate $\sigma = 0.0021$ which represents a decay time-scale longer than hundred years (Figure 1). This reflects the relative slowness of carbon dynamics compared with the dynamics of business cycles. Although, a one-box approach is a rough simplification of the global carbon cycle, it allows us to introduce observed lag between emissions and the temperature change they cause.[12]

[12]The fine structure of carbon flows has only a modest role in business cycle assessments. The main effects occur also in the general form of the time-lag structure. For more information on global carbon cycle see e.g. (Maier-Reimer & Hasselmann, 1987; Siegenthaler & Sarmiento, 1993; Moore & Braswell, 1994).

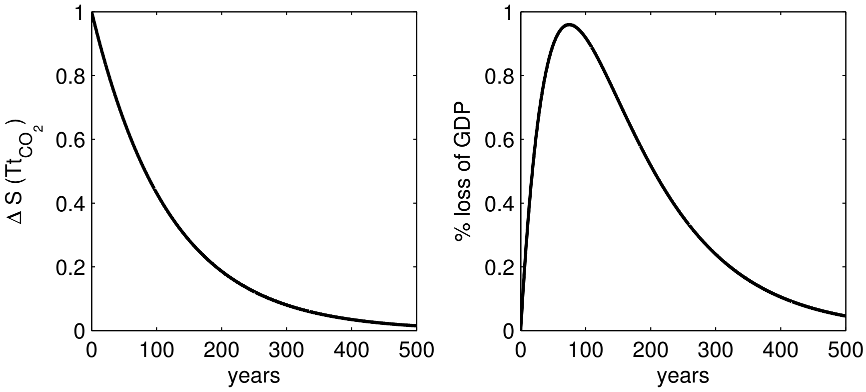

Figure 1: Development of pollution stock and GDP loss following a transient 1 $TtCO_2$ emission

The quadratic temperature dynamics follows the current understanding on radiative forcing of atmospheric CO_2, i.e. the every doubling of atmospheric CO_2 concentration increases the temperature by the magnitude of so called climate sensitivity parameter. The typical consensus value for climate sensitivity is 3 Kelvin degrees (K). Since we model directly the quadratic temperature deviation $\pi = 9$ K^2. For pre-industrial CO_2 stock we use a value 2.13 Tt_{CO_2} and set the relaxation time of temperature to $T = 50$ years. Together with slow decay of atmospheric carbon, the long relaxation time creates slow changing and long-lasting effects from a transient emission peak as indicated by the resulting impulse response in Figure 1. It is easily observed that our one-box model cannot accommodate the high level of persistence presented by Golosov *et al.* (2011) and Gerlagh & Liski (2012). However, as we focus on business cycle properties of the optimal tax, the behavior of carbon cycle after 300 hundred years plays hardly a role. Instead, our model accommodates the several decades long lag observed between carbon emissions and the peak temperature deviations. This lag in general has an important effect on business cycle properties of the carbon tax.

We set discount factor $\beta = 0.995$, which is slightly higher than used in the standard quarterly business cycle models but lower than the largest ones used in the climate change literature.[13] In setting the depreciation rate of physical capital we follow standard values in business cycle literature and assume $\delta_i = 0.025$, for both technologies $i \in \{fin, ene\}$.[14] With time periods of a quarter of a year, these roughly correspond to annual pure time preference rate of 2 percent and annual depreciation rate of 10 %.

We model the global final good production technology as a Cobb-Douglas formulation with labor, capital and energy inputs. The squared temperature deviation decreases the productivity in a multiplicative manner. We formulate the production function as

$$f(L, K^{fin}, E, D; A) = A(1 - \Delta_y D^{1+\xi}) \left(\frac{L}{L_0}\right)^{\alpha_L} \left(\frac{K^{fin}}{K_0^{fin}}\right)^{\alpha_K} \left(\frac{E}{E_0}\right)^{1-\alpha_L-\alpha_K}, \tag{37}$$

[13] For example, Stern (2007) sets β to 0.999.

[14] It might be reasonable to argue, that the energy capital is more long-lasting than the production technologies on average. However, the effect of different depreciation rates does not have qualitative effects on the results.

where A is the exogenous productivity variable, indicating global production level at calibration point normalized to unity and Δ_y and ξ are the production loss scale and elasticity parameters. The calibration point levels of the share of labor, the amount of capital and the level of energy use in final good sector are presented by L_0, K_0^{fin} and E_0, respectively. As usual, the parameters α_L and α_K are the cost shares of labor and capital in final good production. Production technology has constant returns to scale. The production losses are presented by a concave function $1 - \Delta_y D^{1+\xi}$. We set parameter values $\Delta_y = 0.003$ and $\xi = 0.1$ to approximate the damage function specified by Gerlagh & Liski (2012) for modest temperature deviations. With $\xi > 0$, the marginal damages from squared temperature deviations are increasing, starting from zero level.

The energy input is aggregated through a CES specification. The energy aggregate captures the main avenue for emission abatement, i.e. replacing fossil fuels with energy production capital that does not emit carbon dioxide. Examples of such capital are wind turbines, photovoltaics and nuclear power. Again we use a calibrated form for the production technology:

$$g(X, K^{ene}) = E_0 \left[\mu \left(\frac{X}{X_0} \right)^{\frac{\gamma-1}{\gamma}} + (1-\mu) \left(\frac{K^{ene}}{K_0^{ene}} \right)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}. \tag{38}$$

The function gives the amount of productive energy services used in production. The parameter E_0 sets the scale.[15] The remaining CES aggregate yields the relative change of the use of the energy services compared to the calibration point. We measure fossil fuel use X in units of CO_2 emissions and energy capital K^{ene} in usual monetary terms. The weight of fossil fuels in energy production is μ and the elasticity of substitution between fossil fuels and energy capital is γ. The calibrated form of CES-function proves useful when we perform sensitivity analysis on substitutability between fossil fuels and energy capital. Namely, the calibrated form enforces the steady-state solution to the initial calibration point (x_0, K_0) for all values of γ.

Finally, the production function for fossil fuel production and energy generation is assumed to be linear using only labor as an input. Specifically we set

$$h(\tilde{L}) = B\tilde{L}, \tag{39}$$

where B is a productivity parameter. The role of production function is to give an endogenous price for polluting input.

The utility function is assumed to additively consist from consumption term and the disutility of labor supply. The specification we use is

$$u(C, H) = \frac{C^{1-1/\chi} - 1}{1 - 1/\chi} - \kappa \frac{H^{1+1/\nu}}{1 + 1/\nu}, \tag{40}$$

where χ is the elasticity of intertemporal substitution and ν is the Frisch elasticity of labor supply. The scale of labor disutility, κ, is calibrated to yield one unit of labor supply in the steady state without policy intervention. Unlike in the analytical part, in the numerical part we assume that the social

[15] We do not need to specify the value of E_0 as it is canceled in the production function of final good.

Table 1: Model parameters and their values

Symbol	Explanation	Value
β	pure time preference discount factor	0.995
χ	elasticity of intertemporal substitution	1
ν	Frisch elasticity of labor supply	2
κ	scale of labor disutility	0.91
α_L	labor share of national income	0.57
α_K	capital (final good sector) share of national income	0.32
Δ_y	scale of production loss	0.003
ξ	elasticity of marginal production loss	0.1
L_0	BAU labor share in final good sector	0.89
K_0^{fin}	BAU capital in final good sector	10.08
μ	fossil fuel weight in energy sector CES	0.66
γ	elasticity of substitution between K and x	0.25, 1, 4
x_0	BAU fossil fuel use (Gt/Qtr)	8.64
K_0^{ene}	BAU capital in energy sector	1.12
B	efficiency of labor in fossil fuel sector	0.079
δ_{fin}	depreciation rate of capital in final good sector	0.025
δ_{ene}	depreciation rate of capital in energy sector	0.025
ρ_A	persistence of productivity shock A	0.95
σ	decay rate of atmospheric CO_2	0.0021
S_{pre}	pre-industrial atmospheric CO_2 stock (Gt)	2130
π	climate sensitivity parameter (K^2)	9
T	relaxation time of temperature deviation (Qtr)	200

losses from the global warming emerge only through production loss. Therefore, we do not specify direct disutility effect of global warming. This is the typical case in many of the integrated assessment models.

The calibrated parameter values are presented in Table 1. The carbon and temperature models are calibrated using data on Heutel (2012) and Gerlagh & Liski (2012). The economy is calibrated using data on cost shares of labor, capital, fossil fuels and the relative share of energy capital on the whole capital stock. In addition we calibrate the emission level to the year 2010 level. For the calculation needed in the calibration step, we utilize the above functional forms, choosing Cobb-Douglas special case ($\gamma = 1$) for the energy aggregator g. Details of calibration procedure of the economy are presented in Appendix C.

Table 2: Steady-state values of key variables.

		$\rho = 0.25$	$\rho = 1$	$\rho = 4$
	Market	Planner: percentage change		
Production	0.944	-0.3	0.6	5.4
Consumption	0.661	0.1	0.6	3.2
Capital	10.19	-0.3	0.6	5.4
Energy capital	1.13	-9.9	0.6	55.5
Energy consumption	1.00	-12.9	-12.0	-7.5
Anthropogenic CO_2 (Tt_{CO_2})[1]	4.12	-14.2	-17.9	-36.3
		Planner: absolute values		
Temperature increase (K)	4.7	4.2	4.1	3.5
Emission tax (USD/t_{CO_2})	–	29.9	29.9	29.8

[1] Steady-state level of anthropogenic carbon stock is linearly dependent on emissions as well as on polluting input use. Thus, the percentage changes are the same for anthropogenic carbon stock, emissions and polluting input use.

4.2 Steady-state properties

In order to illustrate the effects of the optimal CO_2 taxation on the economy, we present the steady-state properties of the model. Table 2 reports the values of key variables of social optimum in three cases of elasticity of substitution between energy capital and polluting input. These values are contrasted with the market solution which, due to calibration, does not depend on the value of elasticity of substitution. The market steady-state describes an economy where CO_2 emission have been kept indefinitely at the 2010 level. This results in 5.6 % loss in production compared with the case of no climate change. The introduction of CO_2 tax improves the welfare and manages to reduce the atmospheric carbon content as well as the resulting mean temperature. However, the climate change mitigation efforts and their effects depend on the elasticity of substitution. When energy capital and polluting input are complements ($\rho = 0.25$), the main avenue for mitigation is through energy saving, which leads to a minor production loss. Yet, consumption can be increased as the sustained capital stocks are lower. In the substitute case ($\rho = 4$), non-polluting energy technologies can be more easily used as a substitute for fossil fuels. Although, energy use is slightly decreased its production is notably less polluting. This sustains higher production level as environmental damages are decreased.

It is important to notice that the optimal CO_2 tax is roughly the same for all different values of elasticity of substitution. This follows from the marginal damages of atmospheric carbon being quite constant in a relatively wide interval of emission levels. This supports the earlier findings which suggest that price policies might outperform the quantity based policies in climate change mitigation (e.g Newell & Pizer, 2003; Nordhaus, 2008). For example, if the production technology is unknown or it changes in time, the level of optimal tax remains relatively invariant compared with changes in optimal emission levels. Therefore, the setting of a carbon tax needs less information on production technology than a setting of a quantity restriction policy.

Our approach can be associated with the usual abatement terminology: The present value of avoided future marginal damages and the present utility loss are the marginal benefits and the marginal abatement costs (MAC), respectively. Both the benefits and costs are measured in present monetary terms. The production technology contributes to the marginal abatement costs: The complement production technology case is connected to high level of MAC whereas the substitute case is associated with a low level of MAC. Thus, there is a relatively low level of emission reduction in the complement case. As pointed out earlier the marginal benefits from abatement are relatively flat. Therefore, the changes in MAC do not have significant effects on the optimal tax level. However, an increase in the present production and consumption levels shift the marginal benefit and marginal abatement cost curves. Marginal benefits are increased in monetary terms but the change in abatement level depends on the changes in MAC curve. These upward shift of the marginal benefit curve tends to increase resulting optimal tax level if the MAC is not flat. Therefore, the first best optimal CO_2 tax is bound to vary over the business cycles.

4.3 Carbon tax over business cycles

Figures 2 and 3 show impulse responses to a 1 percentage point positive technology shock in the planner's solution for the different elasticities of substitution between energy capital and polluting input, γ. Chosen cases are the Cobb-Douglas technology, $\gamma = 1$, the case where two inputs are complements, $\gamma = 0.25$, and a substitute case, $\gamma = 4$. From these the Cobb-Douglass specification is most comparable to the simulations of Heutel (2012). Figure 2 represents the impulse responses for household decision variables, production inputs other than polluting input and resulting production. Figure 3 represents impulse responses for optimal emission tax, emissions, i.e. polluting input use and climate model variables.

Figure 2 shows that the energy sector technology does not have a notable impact on cyclical properties of variables other than those in the energy sector. Regardless of the value of elasticity of substitution the non-polluting capital in energy sector is increased. When the substitutability between polluting input and energy sector capital is low, the use of polluting input is increased as well (Figure 3). In the case of high elasticity of substitution, emissions increase initially. However, as capital stock in the energy sector is built, emissions clearly decrease in the long run. Regardless of initial positive response of emissions in the case of high substitutability, positive technology shock decreases pollution stock and temperature in the long run.

The impulse responses of pollution stock and temperature deviation follow logically the time-path of emissions: low elasticity of substitution leads procyclical changes in pollution stock and temperature; high substitution case leads to countercyclical changes in these variables. However, with high elas-

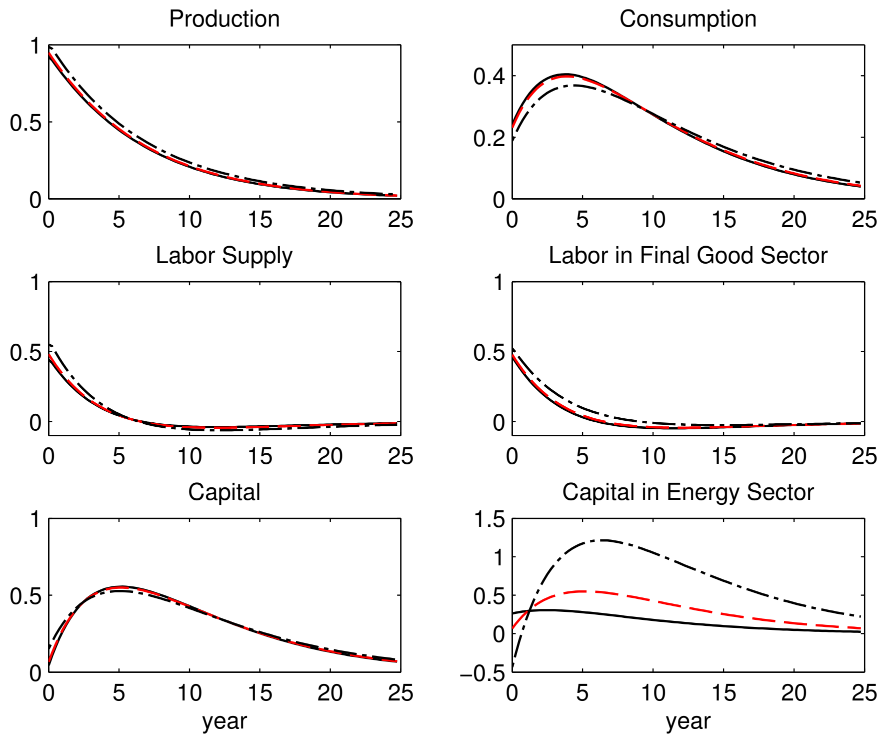

Figure 2: **Social Planner: Impulse Responses to a 1 percentage positive productivity shock.** Black solid line represents complement case (γ=0.25), red dashed line Cobb-Douglas production function (γ=1) and black dashed line with dots substitute case (γ=4).

ticity the time structure is more complicated as the pollution stock increases first before the energy generation by non-polluting capital starts to decrease emissions and, finally, pollution stock. The slow pace of environmental process is implied through long lags and the general weakness of the reaction of both the pollution stock and the temperature deviation.[16] Especially, the changes in temperature deviation are long lasting and have lags on scale of fifty years. These long lasting effects are significantly different from almost instantly emerging environmental effects studied by Heutel (2012). This difference is explained by modeling choices as our model incorporates a slow process of carbon dioxide contributing to the global warming and affecting the economy, as suggested by scientific literature. The strength of the response depends on the elasticity of substitution. For the Cobb-Douglas case the cyclical deviations are two orders of magnitude smaller than the driving productivity shock. For other cases of elasticity of substitution, the larger but still a magnitude smaller than the driving shock.

Whereas the business cycle variations of emissions depend drastically on the elasticity of substitution, the impulse response of optimal emission tax (Figure 3) is strikingly similar in all the three cases studied. These numerical results support the analytical results that in the carbon dioxide emission case, i.e. a very slow environmental process, the optimal tax follows the cyclical movements of marginal utility of consumption, i.e. the consumption level. Interestingly, a productivity shock causes lasting

[16] Note that the time axis is fourfold in the case of temperature deviation.

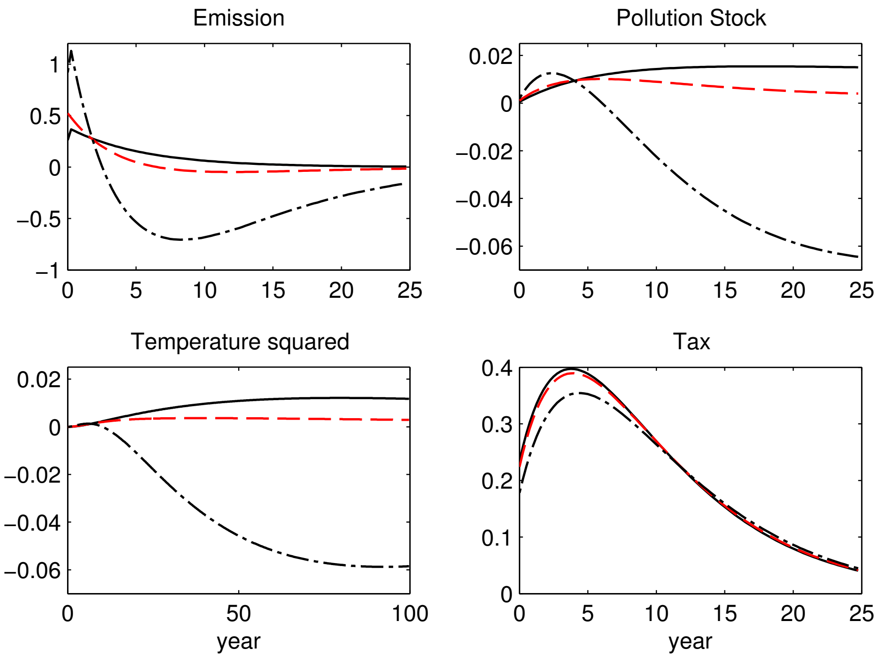

Figure 3: **Social Planner: Impulse Responses to a 1 percentage positive productivity shock.** Black solid line represents complement case (γ=0.25), red dashed line Cobb-Douglas production function (γ=1) and black dashed line with dots substitute case (γ=4).

changes in the tax level. However, the deviations in emission tax level over the business cycle are, in the case of logarithmic utility, of the same scale as the consumption deviation. Thus, the resulting tax movements are reasonably modest.

Our results stay relatively similar for the different specifications of the utility function, namely, for the different values of the elasticity of intertemporal substitution, χ. Lower value of the elasticity dampen the reaction of consumption but keeps the reaction of production and other macro variables qualitatively unchanged. However, a lower value of elasticity increases the emission tax fluctuations compared with consumption fluctuations.[17] Therefore, the fluctuation of emission tax is decreasing with respect to the elasticity of intertemporal substitution. The stronger procyclicality of emission tax due to lower elasticity of intertemporal substitution decreases relatively the use of polluting input, and can invert the response of emissions negative also for the low values of elasticity of substitution between energy capital and polluting input, γ (cf. Figure 3). In total, the chosen value of χ does not have substantial effect on the impulse responses or steady-state values but it may affect the sign of response of emissions and temperature in the case of low value of γ.

Table 3 presents correlations of emissions and optimal tax with the key macroeconomic variables for different values of the elasticity of substitution in energy sector. The correlations support the

[17] Under CES-utility formulation, the relative changes in emission tax over business cycle are equal to relative changes in consumption times the inverse of the coefficient of (inter-temporal) elasticity of substitution, i.e. the dominant term $\varepsilon_{u_C C}\hat{C} = \hat{C}/\chi$ in equation (34).

Table 3: Stochastic properties of the emission and emission tax.

	$\gamma = 0.25$	$\gamma = 1.0$	$\gamma = 4.0$
corr(τ, Y)	0.89	0.89	0.87
corr(τ, C)	1.00	1.00	1.00
corr(τ, X)	0.86	0.37	-0.65
corr(X, Y)	0.99	0.76	-0.19
corr(X, C)	0.86	0.38	-0.65

finding that the degree of substitution between energy capital and fossil fuel in the energy sector is an important determinant of the cyclical behavior of emissions. When energy capital is complement to the use of fossil fuel, react emissions procyclically but when they are substitutes the emissions turn countercyclical. Correlations also indicate the above discussed result that the relationship between consumption and optimal tax is strikingly close as their correlation is near unity in all three cases. Almost perfect correlation reflects that the changes in optimal taxes are mainly driven by the changes in present consumption level and the cyclical changes of emissions and resulting pollution harms have minor roles for the welfare. These results suggest a rule-of-thumb policy recommendation that optimal carbon dioxide emission tax could be tied to the present consumption level. Optimal reactions of emission taxes follow closely consumption and are therefore quite limited in magnitude. Recently the price of CO2 emissions in the European emission trading scheme have varied greatly from around 13 euros/ton of CO2 to around 4 euros. Our results provide evidence that such remarkable changes in the emission taxes or prices are probably excessive.

The variations in the emission tax do not determine the actual variations in the emission level. Instead, the emissions are determined by the technology in the energy sector. If the fossil fuel use and energy sector capital are strict complements, there is a very close covariation of emissions and production level. This suggests a policy where an emission cap would be tied to the production level. However, this approach suits only to the case where substitution possibilities of polluting input are limited. Thus, the close correlation of the polluting input and production may brake down in the future as the energy sector capital is directed towards utilization of renewable resources.

4.4 Comparison with constant tax rate

As the results above suggest the fluctuations of the optimal emission tax are quite limited. Therefore, the economic effects of varying tax rate may be modest. For this reason, it is reasonable to compare the optimal outcomes with the ones resulting from an optimally set, yet, constant tax rate. Figure 4 compares the effects of optimally set constant tax to the cyclically adjusted one for the technologies in which polluting input and energy capital are either substitutes or complements.

Reactions of output and consumption are very similar in these two policy schemes and therefore not

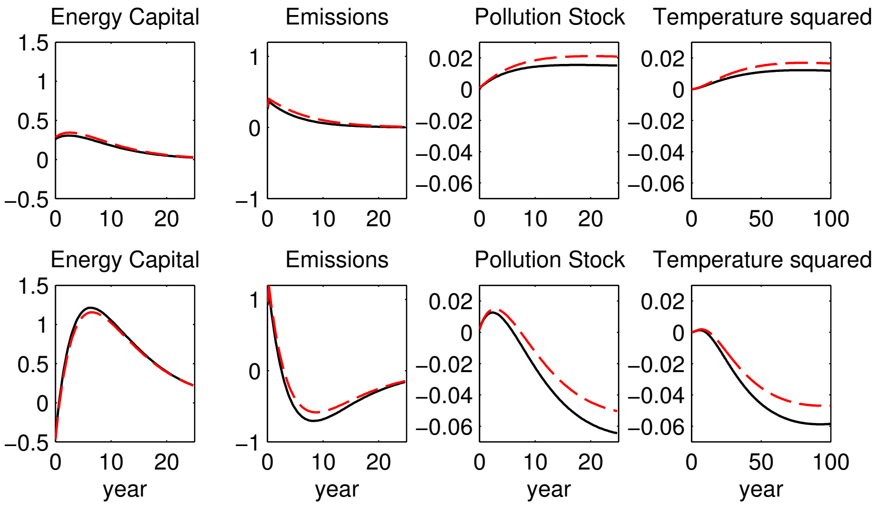

Figure 4: **Comparison of impulse responses of variables connected to the pollution process to a 1 percentage positive productivity shock for constant (red dashed line) and optimal tax rates (black solid line).** The first row presents the case where energy capital and polluting input are complements (γ=0.25) and the second row presents the substitute case (γ=4).

reported in the figure. It is slightly surprising that even though the optimal tax increases for years due to the persistent positive technology shock, the relative effects of the tax on economy are minor and dictated by the productivity changes. Because optimal tax increases in economic upturns, optimal policy generates mildly less emissions and decrease temperature compared to constant tax rate. This reaction is achieved by stricter emission control leading to smaller investments in energy capital in the complement case and higher investments in the substitute case. However, the quite minor effects of cyclical taxes reflect that the global warming is driven by long-term phenomena and administering the emissions over the business cycles does not have a significant impact on climate.

As mentioned above, the lower values of the elasticity of intertemporal substitution, χ, increase the tax fluctuations over the business cycle. Thus, it is expected that the effect of constant tax are stronger in comparison with optimal tax when χ has low values. Our numerical analysis indicates that the chosen value of the elasticity of intertemporal substitution has qualitatively important effect only on the responses of emissions and resulting pollution stock and temperature deviation. The observed differences are rather small in magnitude. However, with low values of χ the constant tax leads to procyclical emissions when the polluting input and energy sector capital are complements, although, the optimal tax would force countercyclical emissions. In general, the welfare losses related to an optimally set constant tax rate seem to be next to negligible.

5 Conclusions

We develop a real business cycle model to analyze the business cycle properties of both optimal emissions and optimal emission taxes. Our starting point is a macroeconomic model augmented with a set of pollution-environment model equations. The emission is caused by the polluting input use in the energy sector. We assume, that the main avenue of emission abatement is through investments in non-polluting capital in the energy sector. The pollution is assumed to contribute to a harmful environmental process which causes both production losses as well as direct disutility for a representative household. Relative to existing papers on cyclical fluctuations of emission taxes, we present and decompose an equation for optimal tax rate in a general form. We split our analysis to two rough cases: short-lived pollution where emissions transfer rapidly into environmental harm and long-lived pollution with slow transmission of emission into environmental harm. The analysis of the tax decomposition suggests that the emission tax tends to be procyclical in general. The possibility of a countercyclical behavior of the emission tax increases if 1) the pollution is short-lived and the emission transfer into environmental damages rapidly 2) emissions are countercyclical, 3) marginal damages are strongly increasing and 4), in disutility case, the marginal utility of consumption increases with the increase in the intensity of the harmful environmental process. If environmental damages emerge through production losses only, the fluctuations of optimal tax are mostly driven by the production level and the marginal utility of consumption for fast and slow environmental processes, respectively.

We adapt our model framework to the carbon dioxide emissions and global warming and solve the model numerically. Carbon dioxide emissions have very sluggish effects on global temperature and, thus, the negative welfare effects occur with a considerable lag. Thus, the analytical result for slow environmental processes holds and the business cycle variations of the optimal tax level follow the level of marginal utility of consumption. With the constant elasticity of intertemporal substitution the tax follows the consumption level in almost perfect correlation. The amplitude of fluctuations is, however, determined through the inverse of the elasticity of intertemporal substitution, i.e. lower the elasticity, the higher the amplitude. This result is independent of assumed production technology. The opposite holds for the optimal emissions as the technology tends to determine the business cycle fluctuations of the emissions: if the polluting input and the non-polluting energy capital are complements, the positive productivity shock leads to higher emissions compared to the case of the two inputs being substitutes. The effect of persistent productivity shock both on the optimal emission tax and on the emission level are long lasting.

We compare the optimally varying emission tax to an optimally set but constant emission tax in the case of positive productivity shock. As expected, the modest business cycle variation of the optimal emission tax does not lead to a notable difference in the outcome compared with a constant tax. However, after a positive productivity shock the constant tax leads to higher emissions and resulting temperature than the optimal tax. The lower elasticity of intertemporal substitution tends to make the case for optimally varying emission tax stronger. This follows the relatively stronger business cycle variations of the optimal tax.

Although, we study the case of first best optimum our results shed light on the relative performance of price and quantity regulation in climate change mitigation. First, the business cycle behavior of taxes is driven by the marginal utility of consumption whereas the optimal level of emissions is also affected by the production technology. Therefore, tax regulation does not need information about production technology whereas optimal quantity regulation does. Second, the results indicate that a simple way to adjust a climate policy over business cycles is to index the tax to the consumption deviation from its steady state path. The indexing of emissions to the production level is another option, but only if the technology is such that the substitution between polluting input and non-polluting energy technologies is difficult. Third, if the emission control is adjusted periodically over the business cycles, the relative properties of price and quantity regulation converge as the first best solution is enforced. Thus, the effects of price and quantity regulations on fluctuations of the macro variables is equalized. This weakens the risk aversion argument proposed by Kelly (2005), making the case for price over quantities stronger.

In general, our results imply that the changes in the carbon dioxide emission taxes should stay quite limited and much more moderate than recently observed price changes of emissions in the European emission trading scheme. As the first best social cost of carbon is reasonably stable, this observation suggests that the price fluctuations in quantity based climate policies should be stabilized too. In addition, our results show that an optimally set constant emission tax is almost as efficient policy tool as the cyclically varying tax rate. The cyclical phases have very limited effects on global temperature itself but the warming is driven by the long-term trend of using fossil fuels.

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A Linear pollution – environment model

In this section we justify the use of linear general lag structure formulation for the harmful effects of the past emissions (see also Gerlagh & Liski, 2012). First, we specify m-box model for pollution stock. The stocks are described by $(m \times 1)$ vector S_t. The stocks are driven by one-dimensional emissions X_t. The linear equation of motion is

$$S_t = \mathbf{\Sigma} S_{t-1} + \mathbf{E} X_t, \tag{41}$$

where $\mathbf{\Sigma}$ is a $(m \times m)$ matrix and $\mathbf{E}$ is a $(m \times 1)$ vector of parameters.

Similarly we can specify a n-box model for the environmental processes of which the first box measures the processes harmful to the society. The values of the boxes are collected in a $(n \times 1)$ vector D_t with equation of motion

$$D_{t+1} = \mathbf{\Delta} D_t + \mathbf{\Pi} S_t. \tag{42}$$

Here $\mathbf{\Delta}$ and $\mathbf{\Pi}$ are $(n \times n)$ and $(n \times m)$ matrices of parameters, respectively.

Using recursion, we can formulate the effect of past emissions on pollution stocks

$$S_{t+j-i} = \mathbf{\Sigma}^{j-i+1} S_{t-1} + \sum_{k=0}^{j-i} \mathbf{\Sigma}^k \mathbf{E} X_{t+j-i-k} \tag{43}$$

and the effect of pollution stocks on natural phenomena

$$D_{t+j+1} = \mathbf{\Delta}^{j+1} D_t + \sum_{i=0}^{j} \mathbf{\Delta}^i \mathbf{\Pi} S_{t+j-i}. \tag{44}$$

In order to get the linkage between X_t and D_{t+j} we need to join these two equations. This leads to equation of motion for D_{t+j+1} from initial state (D_t, S_{t-1})

$$D_{t+j+1} = \mathbf{\Delta}^{j+1} D_t + \sum_{i=0}^{j} \mathbf{\Delta}^i \mathbf{\Pi} \mathbf{\Sigma}^{j-i+1} S_{t-1} + \sum_{i=0}^{j} \sum_{k=0}^{j-i} \mathbf{\Delta}^i \mathbf{\Pi} \mathbf{\Sigma}^k \mathbf{E} X_{t+j-i-k}. \tag{45}$$

Period t initial values S_{t-1} and D_t have dwindling effect on current state of environment as j increases. In the high j limit, the state of environment is determined through past emission only.[18]

The impact of emission X_{t+l} on state of environment on period $t+j+1$ is directly obtained through partial differentiation:

$$\frac{\partial D_{t+j+1}}{\partial X_{t+l}} = \sum_{i=0}^{j-l} \mathbf{\Delta}^i \mathbf{\Pi} \mathbf{\Sigma}^{j-i-l} \mathbf{E}. \tag{46}$$

The possibly rather complicated connections between components of S_t and D_t can be summarized as a $(n \times 1)$ vector

$$\Omega_k := \sum_{i=0}^{k} \mathbf{\Delta}^i \mathbf{\Pi} \mathbf{\Sigma}^{k-i} \mathbf{E}. \tag{47}$$

The analytical model is based on general lag-structure $\boldsymbol{\omega} = (\omega_1, \omega_2, \ldots)$. Assuming, that it is the first component of D that measures the damages to the society caused by the state of the environment it holds $\omega_k = (\Omega_k)_1$.

[18]Here we assume that eigenvalues of $\mathbf{\Sigma}$ and $\mathbf{\Delta}$ all lie inside the unit circle.

B Solving the planner's problem

The optimization problem of the social planner can be presented as a stationary dynamic program, where value function V is determined through optimization problem

$$V(K_t^{con}, K_t^{ene}, A_t, \chi_t) = \max_{\{H_t, L_t, I_t^{con}, I_t^{ene}\}} u(C_t, H_t, D(\chi_t)) + \beta \mathbb{E}_t V(K_{t+1}^{con}, K_{t+1}^{ene}, A_{t+1}, \chi_{t+1}) \tag{48}$$

subject to constraints given by the equations (2) – (6) rewritten here for convenience:

$$Y_t = f(L_t, K_t^{con}, E_t, D(\chi_t))$$

$$E_t = g(X_t, K_t^{ene})$$

$$X_t = h(H_t - L_t)$$

$$K_{t+1}^i = (1-\delta)K_t^i + I_t^i$$

$$I_t^{con} + I_t^{ene} = Y_t - C_t$$

Necessary first order conditions yield for labor supply

$$u_{Ct} f_{Et} g_{Xt} h_{Lt} + u_{Ht} = 0, \tag{49}$$

labor use in final good production

$$u_{Ct} f_{Lt} - u_{Ct} f_{Et} g_{Xt} h_{Lt} - h_{Lt} \beta \mathbb{E}_t V_{4,t+1} = 0 \tag{50}$$

and for investements

$$-u_{Ct} + \beta \mathbb{E}_t V_{i,t+1} = 0, \tag{51}$$

for both technologies $\{con, ene\}$ corresponding to indices $i \in \{1, 2\}$, respectively.

Using the envelope therorem we get dynamic relationships for the value function

$$V_{1t} = u_C f_K + (1 - \delta_{con}) \beta \mathbb{E}_t V_{1,t+1} \tag{52}$$

$$V_{2t} = u_C f_E g_K + (1 - \delta_{ene}) \beta \mathbb{E}_t V_{2,t+1} \tag{53}$$

and

$$V_{3+i,t} = [u_{Ct} f_{Dt} + u_{Dt}]\omega_i + \beta \mathbb{E}_t V_{4+i,t+1}, \tag{54}$$

for all lags $i \in \{1, 2, \ldots\}$. Here we have used definition (1) for function D.

The solution of the model can be summarized into four equations of which (49) is the first describing the solution to the static labor supply optimization by the households. Second and third equations link the expected marginal productivities of capitals in the two technologies

$$u_{Ct} = \beta \mathbb{E}_t \{[f_{K,t+1} + (1 - \delta_{con})] u_{C,t+1}\} \tag{55}$$

$$u_{Ct} = \beta \mathbb{E}_t \{[f_{E,t+1} g_{K,t+1} + (1 - \delta_{ene})] u_{C,t+1}\} \tag{56}$$

and for labor allocation one gets using recursion

$$[f_{Lt} - f_{Et} g_{Xt} h_{Lt}] u_{Ct} = h_{Lt} \mathbb{E}_t \sum_{s=1}^{\infty} \beta^s \omega_s [u_{C,t+s} f_{D,t+s} + u_{D,t+s}]. \tag{57}$$

C Calibration of economy model

The economy is calibrated based on the following data: labor costs cover 64 % of global GDP, 10 % of production capital is on the energy sector, 7 % of global GDP is used on fossil fuels and quarterly global CO_2 emission in year 2010 were 8.6 Gt/Qtr. The climate model affects the productivity level and contributes to the calibration process directly. The values of climate model are taken from literature.

As a first step it is directly observed that total capital has a cost share of $1 - 0.64 = 0.36$ as there are no other basic inputs other than labor and capital. Since fossil fuel costs are labor costs, the remaining labor cost share is devoted to consumption good production. Thus, $\alpha_L = 0.64 - 0.07 = 0.57$. In order to find α_K, we calculated the marginal productivities of the two capital types in a Cobb-Douglas case of the production function g:

$$\alpha_K f_0 / K_0^{con} = \rho + \delta_{con}$$

and

$$(1 - \mu)(1 - \alpha_L - \alpha_K) f_0 / K_0^{ene} = \rho + \delta_{ene},$$

where f_0 is the calibration production level of consumption good. Using these two equations and the fact that total capital costs are divided between the two technologies, leading to equation

$$\alpha_K + (1 - \mu)(1 - \alpha_L - \alpha_K) = 0.36$$

we end up with

$$\alpha_K = \frac{0.36\, x}{K_0^{ene} / K_0^{con} + x} = 0.324.$$

Here we have used short-hand notation $x = (\rho + \delta_{con}) / (\rho + \delta_{ene})$. The knowledge of α_L and α_K with fossil fuel share of 7 % directly leads to final parameter of the production technology g, namely

$$\mu = 0.07 / (1 - \alpha_L - \alpha_K) = 0.66$$

The parameter of labor productivity in fossil fuel sector B is calculated from the equality of marginal productivities of labor in both sectors jointly with data on emission level and fixing the labor supply in calibration point to unity. Thus equal labor productivity yields

$$B L_0 = \alpha_L / \mu / (1 - \alpha_L - \alpha_K) X_0$$

and the production function of fossil fuels with unit labor supply

$$X_0 = B(1 - L_0).$$

From these two equation is easy to derive

$$B = (1 + \alpha_L / \mu / (1 - \alpha_L - \alpha_K)) X_0 = 0.079$$

with $X_0 = 8.6$ Gt/Qtr.

The final task is to find rest of the calibration point, i.e. L_0, K_0^{con} and K_0^{ene}. From knowledge of B and X_0 it is straightforward to see that $L_0 = 1 - X_0/B = 0.89$. From the marginal productivity equations we get the calibration points for the capital goods but we need the value of calibration level of production. Since at the calibration point the production function gets the value

$$f_0 = A(1 - \Delta_y D_0^{1+\xi}),$$

where $A = 1$ and $D_0 = \pi X_0/\sigma$. Thus, we can calculate the values $K_0^{con} = 10.11$ and $K_0^{con} = 1.12$ from the marginal productivity equations above. For numerical assessment of the optimal tax level, we need to adjust the production levels from unity to the correct global GDP level. We use estimate $17.3 \cdot 10^9$ USD/Qtr. Since we measure the emission in gigatonnes the numerical tax results are scaled by 17.3 in order to arrive in correct nominal value of the emission tax.

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