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Nonlinear Dynamics of Speculative Attacks on the Finnish Markka, 1987—1992

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Abstract

In this paper, I estimate nonlinear autoregressive models for Finnish short-term interest rates using daily data. The nonlinear models considered in the paper are the logistic (LSTAR) and exponential (ESTAR) autoregressive models. The estimated LSTAR model appears to capture some of the interest rate dynamics associated with the speculative attacks against the Finnish markka. The combined LSTAR-GARCH models are also estimated.

Tiivistelmä

Tässä työssä tarkastellaan empiirisesti Suomen lyhyiden korkojen dynamiikkaa epälineaaristen aikasarjamallien avulla. Aineistona ovat yhden ja kolmen kuukauden Helibor-korkojen päivähvainnot vuosilta 1987–1992. Tarkasteltavat aikasarjamallit kuuluvat ns. STAR-mallien joukkoon. Tulosten mukaan epälineaarisia malleja tarvitaan erityisesti kuvaamaan spekulatiivisten hyökkäyksiin liittyviä voimakkaita korkojen muutoksia.

I would like to thank Timo Teräsvirta and seminar participants at the FPPE seminar in International Economics, at the Workshop on Modern Time Series Analysis in Finance in University of Århus, at the XVI Symposium of Finnish Economists and at the Symposium on Computer-Aided Time Series Modeling and Neural Nets for many valuable comments on this paper.

Contents

1 Introduction

There is a large body of empirical evidence on nonlinear dependence in financial time series. Especially GARCH effects are well documented; see, for example, Bollerslev, Chou and Kroner (1992) and the many references therein. However, GARCH models are only one class of nonlinear models. Many other nonlinear models have also been developed. In many models of these nonlinearities enter through a mean rather than through a variance as in standard ARCH models. They include, among others, bilinear, threshold autoregressive and exponential autoregressive models; see, for example, Granger and Teräsvirta (1993). The evidence on nonlinearities in mean in financial time series is, however, still scarce, especially when compared with the evidence on the GARCH effects.

Recently, Hsieh (1989, 1991), Kräger and Kugler (1993) and Scheinkman and LeBaron (1989) have tested for nonlinearities using data from the foreign exchange market and stock market. Hsieh (1989, 1991) concluded that most of the nonlinearities in the data are caused by conditional heteroskedasticity. On the other hand, Kräger and Kugler (1993) found that threshold autoregression (SETAR) and GARCH models both account for most, but not all, of the nonlinearities in exchange rate changes.

This paper examines nonlinear dependence in changes in Finnish short-term interest rates. I test for linearity against well specific alternative, i.e. smooth transition autoregressive (STAR) models. The STAR model family includes the logistic STAR (LSTAR) and the exponential STAR (ESTAR) models, which can be used to characterize different types of nonlinear behaviour. In these models nonlinearities enter through a mean.

The data used in this study comprises short-term interest rates in the Finnish money market over the period 1987 to 1992. This period is an interesting one. It is characterized by repeated speculative attacks against the Finnish markka and very high and volatile short-term interest rates. It turns out that nonlinearity in the conditional mean is needed to describe the response of the interest rate process to exceptionally large shocks due to the speculative attacks. I also estimate combined LSTAR-GARCH models, which appear to capture most of the linear and nonlinear dependence in the interest rate changes.

The paper is organized as follows: Section 2 contains a brief account of the smooth transition autoregressive models. Section 3.1 describes the data used in this study and in section 3.2 I test for nonlinear dependence and estimate LSTAR models. In section 3.3 I estimate combined LSTAR-GARCH models. Conclusions are presented in section 4.

2 Testing for linearity against STAR models

2.1 STAR models

There is a plethora of different nonlinear models. A particular class of nonlinear time series models considered in this paper are the Smooth Transition Autoregression (STAR) models. STAR models are capable of generating many kinds of nonlinear processes as Teräsvirta (1994) has demonstrated. More specifically, STAR models allow the local interest rate dynamics to vary between two extreme regimes. The transition between these two regimes is generally smooth, implying that there is a continuum of states between extreme regimes. Teräsvirta (1994) has also devised a Box-Jenkins type identification scheme, which can be easily used in applications. I now give a brief account of STAR models and discuss tests for linearity against STAR models.

Consider the following STAR model of the order p

$$y_t = \pi_1' z_t + \pi_2' z_t F(y_{t-d}) + u_t, \tag{2.1}$$

where $u_t \sim \text{nid}(0,\sigma^2)$, $\pi_j = (\pi_{j1}, ..., \pi_{jp})'$, $j=1, 2$, $z_t = (1,z_t^*)$, $z_t^* = (y_{t-1}, ..., y_{t-p})'$. The term $F(y_{t-d})$ denotes a transition function, where d is the delay parameter. The transition function can have values between zero and unity. Teräsvirta (1994) and Teräsvirta and Anderson (1992) present two possible transition functions. The first one is an exponential one:

$$F(y_{t-d}) = [1 - \exp(-\gamma(y_{t-d} - c)^2)], \quad \gamma > 0. \tag{2.2}$$

The second one is the logistic function:

$$F(y_{t-d}) = [1 + \exp(-\gamma(y_{t-d} - c))]^{-1}, \quad \gamma > 0. \tag{2.3}$$

Model (2.1) with (2.2) is called an exponential STAR (ESTAR) model. When applied to interest rates, an ESTAR model implies that high positive or negative interest rate changes have similar local dynamics, whereas changes in the mid-region have different local dynamics. An ESTAR model can be used to present interest rate dynamics in which realizations return towards "normal" values in the same manner after both large negative and positive changes. The ESTAR model is a slight generalization of the exponential autoregressive model of Haggan and Ozaki (1981).

Model (2.1) with (2.3) is called a logistic STAR (LSTAR) model. In this case the local dynamics in the extreme regimes (F=0 and F=1) is different and the transition from one regime to another may be smooth. Applied to interest rates, a LSTAR model can be used to describe a situation where the local dynamics with negative or moderately positive y(t–d) differs from local dynamics with high positive y(t–d). The LSTAR model is a generalization of a two-regime threshold autoregressive model.

In specifying a STAR model I use the following steps suggested by Teräsvirta (1994). First, I specify a linear model as a base for testing linearity. Second, I test for linearity against the STAR model using the linear model under the null hypothesis. In this step I also determine the value of the delay parameter d. Finally, I choose between LSTAR and ESTAR models, keeping d fixed and using a sequence of tests of nested hypothesis. When determining the maximum lag, p, in the linear model, I use the AIC model selection criterion. It is important to capture all linear dependence, since the linearity test also has power against serially correlated errors.

Teräsvirta (1994) shows that a Lagrange multiplier (LM) type test of linearity against STAR (both LSTAR and ESTAR), assuming d is known, is equivalent to the test of

$$H_0:\ \beta_{2j}=\beta_{3j}=\beta_{4j}=0,\quad j=1...p$$

against H_1: "H_0 is not valid" in the artificial regression:

$$u_t^f=\beta_0+\beta_1' w_t+\sum_{j=1}^{p}\beta_{2j}y_{t-j}y_{t-d}+\sum_{j=1}^{p}\beta_{3j}y_{t-j}y_{t-d}^2+\sum_{j=1}^{p}\beta 4j y_{t-j}y_{t-d}^3+v_t, \tag{2.4}$$

where u_t^f is an OLS residual from the AR(p) model and v_t is an error term in the regression. The artificial regression is based on a cubic approximation of $F(y_{t-d})$ about $\gamma = 0$. I select the value of d by carrying out tests for different values of d. If I reject linearity for more than one value of d, I choose the value of d for which the p-value of the selected test is the lowest. The advantage of this test is that it is solely based on linear regression.

The third stage is to choose between LSTAR and ESTAR. This can be done by a sequence of tests within (2.4). The following sequence of hypotheses can be used to distinguish between LSTAR and ESTAR models:

$$H_{04}:\ \beta_{4j}=0,\quad j=1...p$$
$$H_{03}:\ \beta_{3j}=0\ |\ \beta_{4j}=0,\quad j=1...p$$
$$H_{02}:\ \beta_{2j}=0\ |\ \beta_{3j}=\beta_{4j}=0,\quad j=1...p$$

Teräsvirta (1994) proposes a simple decision rule. First, carry out the three tests suggested above. If the p-value of the test of H_{03} is the smallest of the three, select an ESTAR model, if not, choose a LSTAR model. The motivation for the test derives from the fact that one can interpret the β's as the parameters of the orginal model (2.1) with either (2.2) or (2.3). See Teräsvirta (1994) for a more detailed exposition.

2.2 Testing linearity in the conditional mean in the presence of ARCH

So far, the most frequently reported nonlinearities in financial time series have been ARCH effects. This raises the question as to how to exclude the possibility that the rejection of the null hypothesis in favour of STAR is due to conditional heteroskedasticity.

Luukkonen, Saikkonen and Teräsvirta (1988) have discussed the asymptotic relative efficiency of a class of nonlinearity tests. They consider, among others, LM tests for linearity against the exponential autoregressive (EAR) model and against the threshold autoregressive (TAR) model. They demonstrate that the test for ARCH has zero asymptotic relative efficiency against bilinear or EAR models.

It follows from the symmetry argument that the LM tests for EAR or TAR also have zero asymptotic relative efficiency when the true model is ARCH. Note, however, that even though a test has zero asymptotic relative efficiency, it can be globally consistent. Indeed, Luukkonen, Saikkonen and Teräsvirta (1988) demonstrate that the LM-type linearity test against the exponential autoregressive model also has power against ARCH.

In order to ensure that the (possible) rejection of the null hypothesis of linearity against nonlinearity in the conditional mean is not due to ARCH effects, I consider the test of Davidson and MacKinnon (1985). They have presented tests in the presence of unspecificied heteroskedasticity.[1] I apply their approach here. As above, the test to be robustified is a Lagrange Multiplier test.

The Davidson–MacKinnon test can be constructed using regressions as in Teräsvirta's test. As above, the first step is to regress y_t on z_t and compute the corresponding residuals, u_t. The second step is, however, different from above. The next step is to regress the elements of z_t on h_t, where h_t is the consistent estimator of $\partial f_t/\partial \psi_t$ under H_0 and f_t is in our case $(\pi_{20} + \pi_2' z_t)F(y_{t-d})$ and ψ_t is the corresponding parameter vector. In the case of the LSTAR model, the h_t can be obtained by using a cubic approximation as above. The final step is to weight the residuals by u and regress 1 on the weighted residuals. The explained sum of squared from this regression is our test statistic.

Simulation evidence on the small sample properties of the Davidson–MacKinnon was also obtained.[2] According to the results on the empirical size of the test, the test is slightly conservative. Furthermore, the Davidson–MacKinnon test has good power in detecting the STAR model in the simulated data. Overall, the test appears to behave reasonably well in the sample sizes considered in this paper.

3 Empirical Results

3.1 Data

The empirical sections of this paper are concerned with an investigation of daily short-term interest rates in the Finnish money market. The raw data in level form is graphed in Figure 1. Empirical investigations are based on differences. The period considered is from the beginning of 1987 to autumn 1992. The selection of the period is dictated by the fact that the Finnish money market did not emerge in its present form until the beginning of 1987. The sample size is 1436 observations.

The interest rates are based on Helibor rates. The Bank of Finland calculates daily Helibor rates (Helsinki Interbank Offer Rates) for 1, 2, 3, 6, 9 and 12 month maturities as the average bid rate for bank CDs quoted by the five largest banks. These CDs are issued by banks and the Bank of Finland and are the most liquid securities in the money market. In the empirical work I use the continuously

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[1] See also Granger and Teräsvirta (1993, 69–70).

[2] For brevity the results are not given here.

compounded counterparts of the Helibor rates for one- and three-month Helibor rates.

Figure 1 reveals some basic features of the data. Especially in the second half of the period the interest rate dynamics is dominated by repeated speculative attacks against the Finnish markka, which are reflected in high and volatile interest rates.

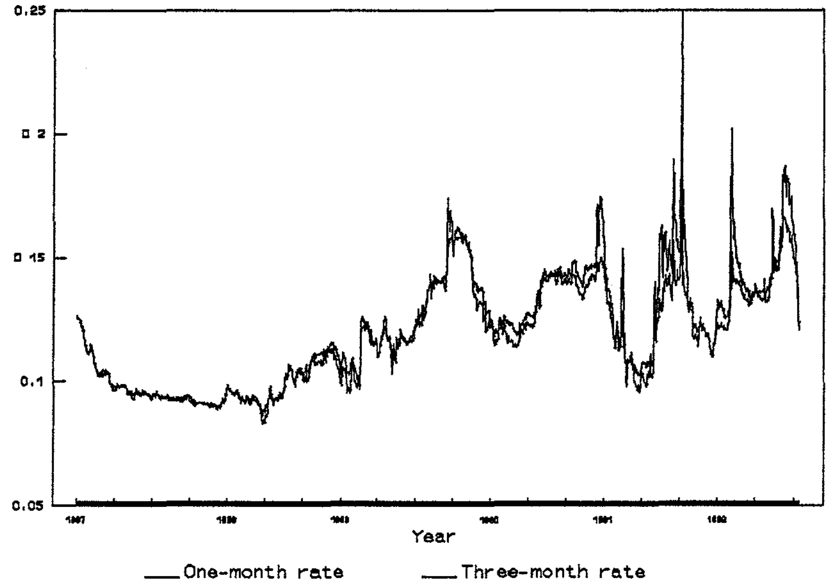
In first part of the period the Bank of Finland applied a currency basket system. The value of the markka was linked to a currency basket index and was allowed to vary within specified limits. In June 1991 the Bank of Finland linked the markka to the ecu following similar decisions by Norway and Sweden. The link was severed in September 1992 when the Bank of Finland allowed the currency to float. Sweden (November 1992) and Norway (December 1992) later followed suit. Both the currency basket and the ecu-link were target zone regimes of the type studied by Krugman (1991) and Svensson (1991), among others.

There are several clear peaks in the interest rate series. Usually, a large upward jump in rates is followed by nearly as large (or an even larger) downward movement. It can also be seen that the interest rate changes associated with the speculative attacks increased towards the end of the period. The attacks were triggered by several macroeconomic factors. The Finnish economy suffered a large exogenous shock in the 1980s as a result of a drastic decline in the volume of trade with the Soviet Union. Furthermore, both price competitiveness in the manufacturing sector and the external balance deteriorated in the 1980s. All these factors undermined the credibility of fixed-exchange rate policy.

The most turbulent period was autumn 1991, when the one-month interest rate peaked at over 27 % on 14 November. The upward jump was followed by an even bigger fall in absolute terms as the Bank of Finland decided to allow the markka to float temporarily. The markka was the devalued by 12.3 per cent.

Despite the devaluation, the foreign exchange market remained turbulent in 1992, as can be seen from the volatility in interest rates. As a result, the markka was the first European currency to be floated in early September 1992. The decision to let the markka float was followed by a decline in short-term interest rates. The period investigated ends shortly after the beginning of the float.

Figure 1. **One- and three-month interest rates: daily observations 1987–1992(autumn)**



3.2 Testing for linearity

To test linearity, I specified an AR model for daily interest rate changes. The maximum lag of the linear model was chosen using AIC was 5 for the one-month Helibor-rates and 3 for the three-month rates.

Table 1 reports summary statistics for the residuals from the linear model. According to the adjusted Box-Pierce statistics (Diebold, 1988), the linear models appear to capture all the linear dependence in interest rate changes. The squared residuals exhibit substantially more autocorrelation than the raw residuals, which points to conditional heteroskedasticity. Note that this may also indicate nonlinearity in mean. Significant skewness and kurtosis are present in both residuals.

Table 2 reports the selected values of the delay parameter and the choice between the LSTAR and the ESTAR model. I use the F version of the LM test. The null hypothesis of linearity is clearly rejected for both series with very low p-values. In both cases I select the LSTAR model.

In Table 3 I report the values of the Davidson–MacKinnon test statistics and corresponding p-values. As above, I use the F version of the test. The null hypothesis of linearity is clearly rejected in favour of the LSTAR model also in the case of the Davidson–MacKinnon test. According to this result, the rejection of linearity above is not due to heteroskedasticity. Thus I can proceed to estimate LSTAR models.

Table 1. **Summary statistics for linear model residuals**

One-month rate		
Sample size	1432	
Mean	0.0000	
Std Dev	0.0046	
Skwness	−4.82	
Kurtosis	177.96	
Adjusted BP		
BP (10)	2.19	(0.9947)
BP (20)	10.63	(0.9553)
BP (30)	16.98	(0.9728)
McLeod-Li		
ML (10)	134.78	(0.0000)
ML (20)	141.47	(0.0000)
ML (30)	142.51	(0.0000)
Three-month rate		
Sample size	1432	
Mean	0.0000	
Std Dev	0.0021	
Skwness	−5.91	
Kurtosis	151.55	
Adjusted BP		
BP (10)	7.17	(0.7093)
BP (20)	17.16	(0.6426)
BP (30)	28.75	(0.5308)
McLeod-Li		
ML (10)	36.27	(0.0000)
ML (20)	55.95	(0.0000)
ML (30)	56.89	(0.0002)

Kurtosis is the coefficient of excess kurtosis, which is zero under the null hypothesis of normal distribution. BP(n) and ML(n) denote adjusted Box-Pierce and McLeod-Li test statistics with n lags. The figures in parentheses following the values of the BP and ML tests statistics are p-values.

Table 2. **Test statistics of the linearity test for different values of the delay parameter**

Series	Value of d	Test statistic	Corresponding p-value
One-month Helibor rate			
	1	76.08	$1.91*10^{-169}$
	2	58.81	$4.89*10^{-137}$
	3	21.35	$6.69*10^{-53}$
	4	28.44	$1.53*10^{-70}$
	5	46.81	$2.24*10^{-112}$
Three-month Helibor rate			
	1	53.52	$5.54*10^{-84}$
	2	19.90	$9.90*10^{-32}$
	3	10.57	$5.56*10^{-16}$

Table 3. **Test statistics of the fourth-order, cubic and quadratic linearity tests**

Series	Value of d	Test	Test statistic	Corresponding p-value
One-month Helibor rate				
	1	F_L	76.08	$1.91*10^{-169}$
	1	F_4	14.43	$8.28*10^{-14}$
	1	F_3	48.23	$3.44*10^{-46}$
	1	F_2	133.75	$2.24*10^{-116}$
Three-month Helibor rate				
	1	F_L	53.52	$5.54*10^{-84}$
	1	F_4	53.30	$1.20*10^{-32}$
	1	F_3	82.62	$2.78*10^{-49}$
	1	F_2	95.30	$3.18*10^{-56}$

Table 4. **Testing for linearity against the LSTAR model assuming unspecified heteroskedasticity**

Series	Value of d	Test statistic	Corresponding p-value
One-month Helibor rate	1	35.12	0.0024
Three-Month Helibor rate	1	22.47	0.0075

3.3 Basic Model Results

In this section, I estimate LSTAR models by nonlinear least-squares.

Table 5 summarizes the estimation results. The parameters are estimated precisely. R^2 is 0.34 and 0.38 for the one- and three-month rates, respectively. These can be compared with the R^2's for the linear models, which were 0.05 and 0.03. The improvement is quite substantial. Furthermore, it can be seen from the values of the ratio s^2/s_L^2 that the error variances of LSTAR models are notably less than those of the AR models.

According to the estimated value of γ, the transition between extreme regimes is rapid. Thus the estimated models resemble two-regime TAR models. The estimated values of c are quite high, 0.016 and 0.01, respectively. Only a few observations in the series exceed these values. The story appears to be as follows: Most observations lie in the normal regime, where the daily changes are positive, but moderate or negative. However, there are some very large positive jumps in the interest rates which move the process into the upper regime where the transition function F=1. Figures 2 and 3 show the estimated transition functions.

Figures 2 and 3 support the interpretation that the main need for a nonlinear model in the mean arises from the devaluation attacks. The transition function only rarely assumes a non-zero value and this happens after the spot rate has risen rapidly. The large positive changes in interest rates are associated with speculative attacks against the markka. The univariate model can not, of course, predict those shocks, but when an attack occurs, the LSTAR model describes the movements in the interest rate that follow such an event.

Table 7 lists the observations where the estimated transition function for the three-month interest rate has a clear non-zero value, together with the interest rate level and change in the interest rates. The estimated value of the transition function obtained its highest value on 14 November 1993, i.e. just prior to the devaluation of the markka. The markka was allowed to float for the second time on 8 September 1992. The speculation dynamics associated with this decision is different from the previous (very short) period of floating. The daily changes in interest rates and the estimated values of the transition function are more moderate than in the previous case.

The dynamic properties of the estimated models can be better understood by investigating the roots of the characteristic polynomials associated with the estimated models. I consider the roots of the two extreme regimes (F=0 and =1), as did Teräsvirta and Anderson (1992) and Teräsvirta (1994). The most prominent roots are reported in Table 6.

Table 5. **The estimated LSTAR model**

One-month Helibor rate

$$dr(t) = \underset{(3.06)}{0.0796}dr(t-1) - \underset{(-5.30)}{0.1199}dr(t-2) + (\underset{(19.83)}{0.0690} - \underset{(-22.32)}{2.8140}dr(t-1) - \underset{(-4.66)}{0.6303}dr(t-2) +$$

$$\underset{(8.99)}{1.0934}dr(t-3)) * \{1 + \exp[-\underset{(2.67)}{1453.8382}(dr(t-1) - \underset{(43.05)}{0.0165})]\}^{-1} + u_t^f$$

$R^2 = 0.34$, $s = 0.00383$, $s^2/s_L^2 = 0.70$, sk = 3.22, ek = 48.30,

LM(10) = 5.09 (0.0000), LM(20) = 3.52 (0.0000), LM(30) = 3.37 (0.0000)

ML(10) = 539.97 (0.0000), ML(20) = 548.94 (0.0000), ML(30) = 550.91 (0.0000)

Three-month Helibor rate

$$dr(t) = \underset{(3.99)}{0.1036}dr(t-1) + \underset{(6.36)}{0.1438}dr(t-3) + (\underset{(13.25)}{0.0561} - \underset{(-14.51)}{5.1165}dr(t-1) - \underset{(-11.07)}{2.5115}dr(t-2) +$$

$$\underset{(2.34)}{0.7035}dr(t-3)) * \{1 + \exp[-\underset{(9.54)}{675.0633}(dr(t-1) - \underset{(22.22)}{0.0101})]\}^{-1} + u_t^f$$

$R^2 = 0.38$, $s = 0.00168$, $s^2/s_L^2 = 0.64$, sk = 0.73, ek = 16.10

LM(10) = 2.58 (0.0148), LM(20) = 2.81 (0.0000), LM(30) = 2.35 (0.0000)

ML(10) = 329.37 (0.0000), ML(20)=438.69 (0.0000), ML(30) = 446.55 (0.0000)

s = the residual standard deviation. s_L = the residual standard deviation of the linear AR(3) and AR(5) model for one- and three-month Helibor rates, respectively. sk = skewness. ek = excess kurtosis. LM(n) is the test for autocorrelated residuals and ML(n) is the McLeod-Li tests with n lags. The figures in parentheses following the values of the LM and ML tests statistics are p-values. The figures in parentheses under parameter estimates are the estimated t-statistics.

Table 6. **Limiting behaviour of the estimated models and the most prominent roots of the characteristic polynomials**

Serie	Lim. behaviour	Regime	Most prominent root(s)	Modulus	Period
One-month Helibor rate	USSP	F=1	−2.15	2.15	–
	$6.1*10^{-4}$	F=0	0.04 ± 0.34i	0.35	4.31
Three-month Helibor rate	USSP	F=1	−4.40	4.40	–
	0.0011	F=0	−0.23 ± 0.45i	0.51	3.07

Note: USSP = Unique stable singular point.

For both interest rate series the complex roots in the normal regimes, i.e. in the regimes where F=0, have a modulus of less than one. On the other hand in the regime where F=1, the dominant root is real and larger than the one in absolute value, so that the process is locally explosive. There is thus asymmetry of regimes. In normal times there is no tendency for the process to explode. After a large positive jump in interest rates associated with a speculative attack, the interest rate is pushed aggressively back down. However, the process does not explode because it then re-enters the locally stationary regime. In the lower regime the most prominent pair of the complex roots has a period of 4.3 and 3.1 days for one- and three-month interest rates, respectively.

The long-run properties of the estimated models were analyzed by studing the behaviour of the process using different starting values $y^0_{t-1}, ..., y^0_{t-p}$. Table 6 reveals that both the one- and three-month interest rate models have a unique stable stationary point.

Diagnostics show that some problems remain in the models. LM denotes a Lagrange multiplier test for no autocorrelation against the k-order AR residuals; see Eitrheim and Teräsvirta (1993). High values imply that there is still some autocorrelation in the LSTAR residuals. The McLeod-Li test statistics also suggest misspecification, which may be due to omitted conditional heteroskedasticity. There is also positive skewness and kurtosis present in the LSTAR residuals, although the kurtosis is much less than for their counterparts computed from the residuals of the AR models. Non-normality of the residuals is not surprising given the many large shocks in the data. Shocks have been typically positive, hence the positive skewness in the residuals. Note that the estimated residuals of the AR models were skewed to the left, not to the right as above. This is because the estimated AR models do not capture the interest rate dynamics following the large positive shocks.

Figure 2. **Values of the estimated transition function for the one-month Helibor rate**

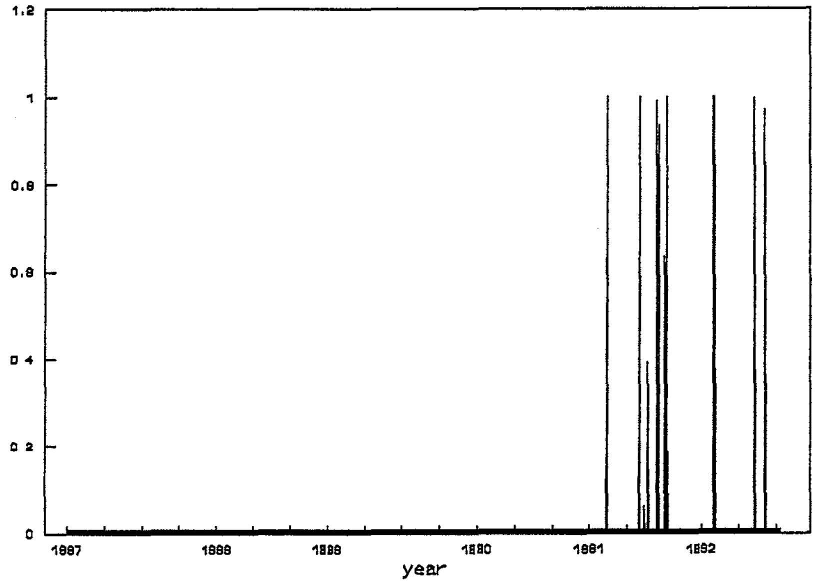


Figure 3. **Values of the estimated transition function for the three-month Helibor rate**

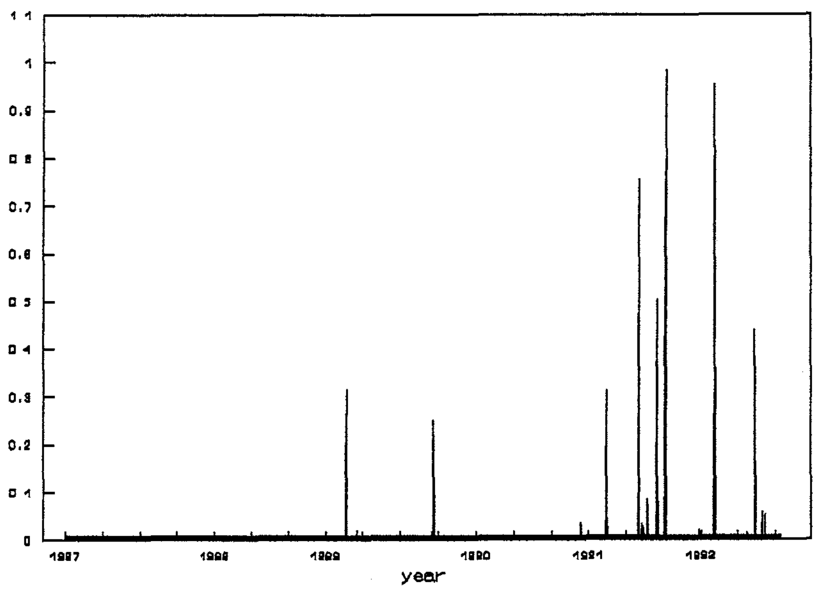
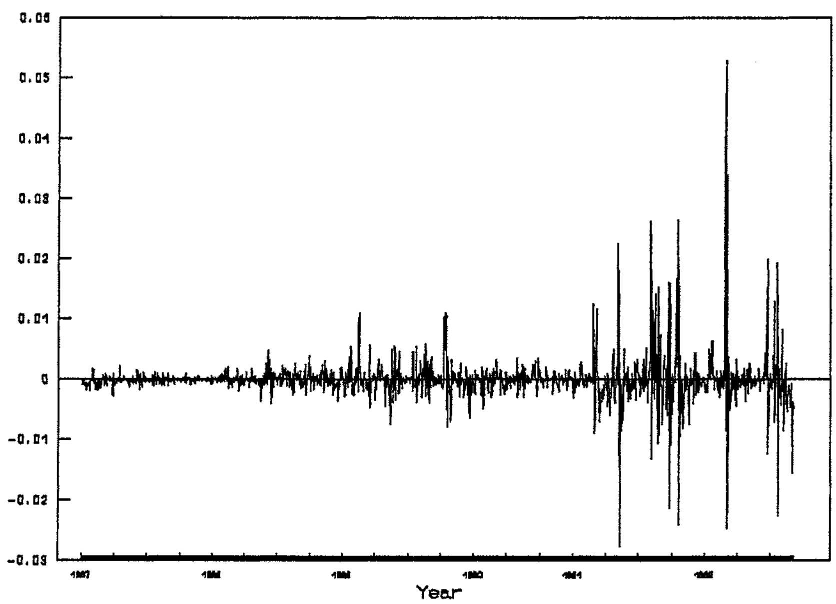


Table 7. **Three-month Helibor rates and estimated values of the transition function**

Day	r_t	Change in r_t	Estimated value of F	Note
21 Mar 1989	11.03	0.81	0.18	Revaluation of markka on 17 March
22 Mar 1989	12.23	0.28	0.32	
27 Nov 1989	16.10	0.88	0.25	
29 May 1991	12.50	0.92	0.32	
29 Aug 1991	12.43	1.21	0.75	Pressure against markka increases
18 Oct 1991	14.92	0.79	0.16	
21 Oct 1991	15.97	1.05	0.50	Overnight rate over 27 % on 22 October
24 Oct 1991	15.12	0.67	0.08	
11 Nov 1991	16.00	1.16	0.67	
13 Nov 1991	17.23	1.00	0.42	
14 Nov 1991	18.90	1.67	0.98	Markka floats temporarily
03 Apr 1992	14.40	0.96	0.09	
06 Apr 1992	13.44	0.69	0.37	
08 Apr 1992	15.51	1.51	0.95	
06 Aug 1992	15.84	1.01	0.44	
26 Aug 1992	15.72	0.62	0.06	
03 Sep 1992	16.82	0.60	0.05	Markka is allowed to float on 8 September

Note: Interest rates and changes in interest rates are expressed here as a simple interest rates, whereas the estimated value of the transition function is calculated using continuously compounded rates.

Figure 4. **LSTAR residuals: one-month interest rate**



3.4 Results from the LSTAR model with GARCH residuals

Thus the LSTAR models appear to explain most, but not all, the temporal dependence in the daily interest rate changes. In particular, it seems that some conditional heteroskedasticity remains in the LSTAR residuals. In this section I try to describe this heteroskedasticity by estimating a combined LSTAR-GARCH model.

ARCH and GARCH models due to Engle (1982) and Bollerslev (1986) are standard models for capturing time-varying conditional heteroskedasticity. In these models the conditional volatility follows an autoregressive linear process.[3] In this paper the variance specification also incorporates the lagged interest rate level as an exogenous variable. The interest rate variable captures the phenomenon that the volatility of interest rates increases with the interest rate level.

Table 8 presents results using a LSTAR model with GARCH residuals. Maximum likelihood estimates of the parameters were obtained by assuming conditional normality of errors. With only one exception all parameters are estimated quite precisely. The exception is that the estimates of γ are rather uncertain. This uncertainty is due to the fact that the parameter (γ) has a very large value. The reason for this has been explained in detail in Teräsvirta (1994). The parameter estimates of c are lower than before, but the number of observations that exceed values is still small. In this respect the results have not changed much.

In the case of three-month interest rates, the parameter estimates of the variance equation imply a non-stationary variance process. In the variance equation the parameter estimate of the interest rate level is also much smaller for the three-month interest rate than for the one-month interest rate.

The test statistics are based on the standardized residuals. The McLeod-Li test statistics do not reveal any remaining heteroskedasticity in the standardized residuals. There is still significant skewness and kurtosis present in the standardized residuals. This is not, however, surprising given the fact that the numerous positive shocks are exogenous in the univariate system and thus cannot be modelled. On the other hand, the combined LSTAR and nonlinear GARCH model explains most of the large negative changes.

[3] Recently, many authors have suggested GARCH models that allow some asymmetric dynamics in the variance term. These models include EGARCH by Nelson (1990) and TARCH by Zakoian; see Rabemananjara and Zakoian (1993), among others.

Table 8. **The estimated LSTAR model with GARCH residuals**

One-month Helibor rate

$$dr(t) = \underset{(6.36)}{0.316} dr(t-1) - \underset{(-1.86)}{0.0801} dr(t-2) + (\underset{(4.03)}{0.0167} - \underset{(-4.15)}{1.164} dr(t-1) - \underset{(-2.60)}{0.389} dr(t-2) +$$

$$\underset{(3.04)}{0.314} dr(t-3)) * \{1 + \exp[-\underset{(0.16)}{6168.303}(dr(t-1) - \underset{(29.70)}{0.0073})]\}^{-1} + u_t^f,$$

$$h_t = -\underset{(-13.57)}{1.01 * 10^{-5}} + \underset{(16.43)}{0.616} e_{t-1}^2 + \underset{(6.98)}{0.323} h_{t-1} + \underset{(13.11)}{0.00012} r(t-1) + \varepsilon_t^f$$

s_{LG} = 0.80928, sk = 4.72, ek = 61.22

ML(10) = 1.36 (0.9993), ML(20) = 1.65 (0.9999), ML(30) = 2.81 (0.9999)

Three-month Helibor rate

$$dr(t) = \underset{(6.28)}{0.186} dr(t-1) + \underset{(5.40)}{0.102} dr(t-3) + (\underset{(8.18)}{4.02 * 10^{-2}} - \underset{(-7.96)}{4.307} dr(t-1) - \underset{(-3.21)}{0.862} dr(t-2) +$$

$$\underset{(1.98)}{0.753} dr(t-3)) * \{1 + \exp[-\underset{(0.63)}{13532.15}(dr(t-1) - \underset{(183.97)}{0.0067})]\}^{-1} + u_t^f,$$

$$h_t = -\underset{(-10.29)}{2.90 * 10^{-7}} + \underset{(13.82)}{0.246} e_{t-1}^2 + \underset{(74.22)}{0.785} h_{t-1} + \underset{(10.33)}{3.21 * 10^{-6}} r(t-1) + \varepsilon_t^f$$

s_{LG} = 0.990262, sk = 0.87, ek = 10.40

ML(10) = 3.85 (0.9539), ML(20) = 6.10 (0.9988), ML(30) = 8.52 (0.9999)

s_{LG} = the residual standard deviation of the LSTAR-GARCH model for standardized residuals, sk = skewness and ek = excess kurtosis for standardized residuals. ML(n) is the McLeod-Li test for standardized residuals with n lags. The figures in parentheses following the values of the ML tests statistics are p-values. The figures in paratheses under parameter estimates are the estimated t-statistics.

Table 9 reports the most prominent roots of the characteristic polynomials for LSTAR-GARCH models. In the case of the three-month rate the results are same as before. The regime is explosive if F=1 and stationary if F=0. On the other hand, the results differ from previous ones in the case of the one-month interest rate. In this case both extreme regimes are stationary. The most prominent pair of roots in the upper regime has a modulus of 0.94 and a period of 2.8 days. In the lower regime the most prominent pair of roots has a modulus of 0.28 and a period of 6.5 days. The long-run solution of one- and three-month interest rate models has a single stationary point which equals zero.

Whereas the interest rates drop very sharply after a positive shock in the LSTAR model, the drop is less dramatic in the combined LSTAR-GARCH model. This implies, among other things, that the combined LSTAR-GARCH model clearly produces larger forecasting errors than the LSTAR model in cases where interest rates fall dramatically after the initial shock. Figure 5, which contains the graphs of the actual and fitted values for one such episode, illustrates this point. The period examined runs from 7 November to 27 November 1991. During this period daily changes in short-term interest rates were very large owing to the speculative attack against the markka and temporary period of floating; see also section 3.1. With more moderate changes in interest rates the differences between the models are not so clear.[4]

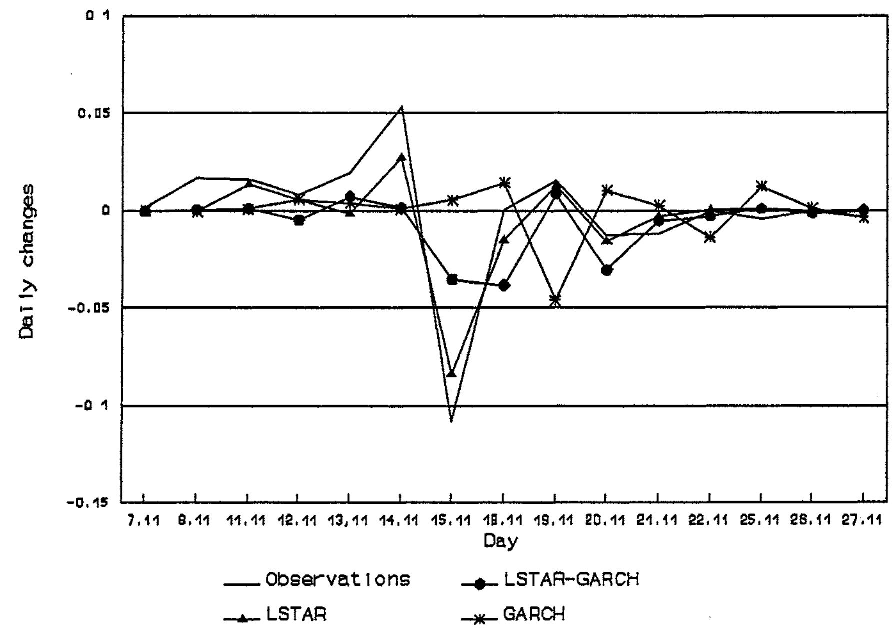
Table 9. **Limiting behaviour of the estimated models and the most prominent roots of the characteristic polynomials for LSTAR-GARCH models**

Interest rate series	Lim. behaviour	Regime	Most prominent root(s)		Modulus	Period
One-month	USSP	F=1	−0.60	± 0.73i	0.94	2.77
Helibor rate	0.0	F=0	0.16	± 0.23i	0.28	6.51
Three-month	USSP	F=1	−3.84		3.84	–
Helibor rate	0.0	F=0	0.54		0.54	–

Note: USSP = Unique stable singular point.

[4] The figure includes also fitted values of the estimated GARCH-model, which is discussed below.

Figure 5. **Actual and fitted interest rate changes for the one-month interest rate**



Normal GARCH models were also estimated. The results are presented in the Appendix. For both interest rate series, the variance was non-stationary. The sum of the coefficients for the lagged error term and conditional variance was clearly above one. When compared to the GARCH model, which has a linear conditional mean, the LSTAR-GARCH model generally yields a lower volatility estimate. In the GARCH model a large shock produces a high volatility estimate. On the other hand, the LSTAR-GARCH model handles a large shock through a nonlinear conditional mean and yields a better point prediction.

Overall, the results from the combined LSTAR-GARCH models are interesting. The combined LSTAR-GARCH models appear to capture allmost all nonlinear dependence in the data. The results are not, however, without problems. The dynamics for the one-month interest rates is problematic as both extreme regimes are stationary in contrast to the results acquired using LSTAR models. In the case of the three-month interest rates, the dynamics is basically the same as before. However, the variance specification implies a non-stationary variance.

4 Conclusions

This paper provides strong evidence on nonlinear dependence in the conditional mean in daily interest rate changes in the Finnish money market. The evidence is obtained by testing for linearity against well-specified alternatives, i.e. STAR models, and estimating logistic smooth transition autoregressive (LSTAR) models. The estimated LSTAR models describe some of the dynamics associated with repeated speculative attacks.

The LSTAR models usually have a continuum of different regimes between two extreme regimes. In the estimated models the transition between different regimes was rapid. The local interest rate dynamics is mostly determined by the "normal" regime, which is stationary. However, when a large postive shock occurs the dynamics is characterized by the other (upper) regime, which is explosive. The upper extreme regime describes how the interest rate behaves after the shock. After a large positive interest rate shock, the interest rate quickly falls back towards the initial level. In the data set these shocks are due to speculative attacks, which have typically triggered a sharp upward jump in short-term interest rates.

The results in this paper differ from those of many previous studies, where the nonlinearities enter through the variance rather than the mean. Note, however, that according to the results there is still volatility clustering in the data. GARCH models are needed to describe this. Indeed, the combined LSTAR-GARCH moel appears to capture almost all nonlinear dependence in daily interest rate changes, even though the results with the LSTAR-GARCH specification are not fully satisfactory.

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Appendix

Table A1. **The estimated GARCH models**

One-month Helibor rate

$$dr(t) = \underset{(9.57)}{0.352}dr(t-1) - \underset{(-3.12)}{0.108}dr(t-2) + \underset{(4.96)}{0.142}dr(t-5) + u_t^f$$

$$h_t = \underset{(-23.14)}{-2.56*10^{-6}} + \underset{(21.18)}{0.677}e_{t-1}^2 + \underset{(43.73)}{0.584}h_{t-1} + \underset{(22.91)}{0.00003}r(t-1) + \varepsilon_t^f$$

s_G = 0.96721, sk = 5.35, ek = 85.70

LB(10) = 15.44 (0.1168), LB(20) = 23.51 (0.2645), LB(30) = 32.53 (0.3433)

ML(10) = 0.17 (0.9999), ML(20) = 0.29 (0.9999), ML(30) = 0.78 (0.9999)

Three-month Helibor rate

$$dr(t) = \underset{(5.75)}{0.153}dr(t-1) - \underset{(-2.00)}{0.066}dr(t-2) + \underset{(2.29)}{0.040}dr(t-3) + u_t^f$$

$$h_t = \underset{(-10.45)}{-3.20*10^{-7}} + \underset{(19.18)}{0.435}e_{t-1}^2 + \underset{(66.59)}{0.687}h_{t-1} + \underset{(10.55)}{3.58*10^{-6}}r(t-1) + \varepsilon_t^f$$

s_G = 0.99029, sk = 0.89, ek = 10.36

LB(10) = 19.58 (0.0335), LB(20) = 22.77 (0.3002), LB(30) = 27.09 (0.6185)

ML(10) = 4.65 (0.91)33, ML(20) = 6.65 (0.9977), ML(30) = 8.53 (0.9999)

s_G = the residual standard deviation of the GARCH model for standardized residuals, sk = skewness and ek = excess kurtosis for standardized residuals. LB(n) is the Ljung-Box test and ML(n) is the McLeod-Li test for standardized residuals with n lags. The figures in parentheses following the values of the LB and LM tests statistics are p-values. The figures in parentheses under parameter estimates are the estimated t-statistics.

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