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Jean-Marie Viaene — Itzhak Zilcha

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Multiple Uncertainty, Forward-Futures Markets and International Trade

SUOMEN PANKIN KESKUSTELUALOITTEITA • FINLANDS BANKS DISKUSSIONSUNDERLAG

Suomen Pankki
Bank of Finland
P.O.Box 160, SF-00101 HELSINKI, Finland
☎ + 358 0 1831

Jean-Marie Viaene* — Itzhak Zilcha

Research Department
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Abstract

The optimum behavior of a competitive risk-averse international trader who supplies or demands commodities invoiced in foreign currency is examined when his profits are subject to several forms of risk: production, domestic cost, the exchange rate and the commodity price. The focus of the analysis lies in the optimality conditions for the level of trade and the extent of forward exchange and commodity futures commitments. New results on the implications of the framework for the separation and the double-hedging theorems are derived. The behavior of the same firm with and without complete markets is compared and conditions are obtained for a domestic price guarantee or a gradual introduction of missing markets to promote the level of international trade. (JEL D81, D84, F19, F31)

Tiivistelmä

Tutkimuksessa analysoidaan, millaista on perushyödykkeiden kansainvälistä kauppaa kilpailullisilla markkinoilla harjoittavan, riskejä karttavan yrittäjän optimaalinen toiminta, kun hänen myymänsä tai ostamansa hyödykkeet laskutetaan ulkomaanvaluutoissa ja hänen tuottoihinsa kohdistuu monenlaisia riskitekijöitä: tuotanto, kotimaiset kustannukset, valuuttakurssit ja hyödykkeen hinta. Tutkimuksessa keskitytään optimaaliseen kaupan volyymiin sekä termiini- ja futuurisitoumusten laajuuteen. Uusia tuloksia johdetaan "separation" ja "double-hedging" -väittämien implikaatioista. Tutkimuksessa vertaillaan saman yrityksen toimintaa sekä termiini- ja futuurimarkkinoiden vallitessa että tilanteessa, jossa näitä markkinoita ei ole, ja tulokseksi saadaan, että kansainvälisen kaupan volyymiä lisääviä tekijöitä ovat joko kotimaisten hintojen vakauttaminen tai puuttuvien termiini- ja/tai futuurimarkkinoiden asteittainen kehittäminen. (JEL D81, D84, F19, F31)

Contents

1 Introduction

The international trade in goods is subject to several forms of risk. A producer can estimate the volume of his own production plans but random shocks such as disease, spoilage, rain at harvest, strikes and technical breakdowns affect the output and export realization. Hence, he does not know his own and therefore aggregate production and cannot infer the international clearing price, that remains a stochastic variable in his decision problem. Uncertainties in the use of domestic factors and their price (cost of energy, environmental regulation, legal suits, ...) introduce randomness as well, but in the cost structure of the firm. Exchange rate volatility is an important factor in the analysis of international trade, this notion having surfaced in the seventies when exchange rates became increasingly volatile.

The received theory has regarded price and exchange rate risk either in isolation or as a co-product but ignored other forms of risk. Though convenient, this assumption seems descriptively unrealistic. Take, for example the case of a commodity producer. A convention of world trading markets is to quote commodity prices in the currency of one of the major consuming countries. If this currency is his own, the producer has receipts invoiced in his own currency. He avoids the exchange risk altogether and is left with the price, output and cost risks when evaluating his profits. Stein (1979), Newbery and Stiglitz (1981, chap. 13), Rolfo (1980), Anderson and Danthine (1983) and others present models in which the price and the supply of the underlying commodity (though not stochastic costs) are uncertain and are concerned with the precise relationship between the two. If this currency is not his own the commotity producer faces the exchange risk and has therefore to cope with the product of several random variables (price, output/cost and exchange rate) when determining the overall variabily of his revenue in domestic currency.

The aim of this paper is to examine the optimum behavior of a competitive risk-averse international trader who supplies or demands commodities invoiced in foreign currency and faces stochastic output, cost, exchange rate and commodity price. Ethier (1973), Baron (1976), Kawai (1981), Viaene and de Vries (1992) have studied the open economy firm behavior when exchange rate uncertainty is taken in isolation, with or without forward foreign exchange markets. Kawai and Zilcha (1986) come the closest to the present paper in that they present a model of exchange rate and commodity price uncertainties (though not stochastic output and cost) with forward exchange and commodity price markets. In what follows new results will be derived and further compared to the existing literature. It will be seen that most conclusions can be generated from a particular version of the model.

The paper is organized as follows. Section 2 discusses a model of the international producer. The firm's optimal behavior is analyzed under complete markets, that is when both the forward exchange and the commodity futures market exist. Section 3 discusses the optimality conditions for the level of trade and, in particular, verifies the implications of the framework for the separation theorem and the full-double hedging theorem. Section 4 introduces cost and output uncertainty one by one and shows the implication of each situation for the level of international trade. Section 5 verifies the sensitivity of the results to risk aversion. Section 6 deals with the behavior of the same firm with incomplete markets (no forward and futures markets) and derive conditions under which a domestic price quarantee promotes the level of international trade. Finally, section 7 concludes the paper.

2 The international firm's risk-bearing optimum

Consider the problem of a competitive, risk-averse exporting firm that produces one single output to be exported using one domestic primary input. The firm transforms inputs W according to a production function $Q = Q(W, \tilde{\varepsilon})$ that is stochastic because of a undiversifiable production shock $\tilde{\varepsilon}$, the tilde (~) signifying a random variable. We assume the usual concavity and Inada conditions for production:

$$Q(0,\tilde{\varepsilon})=0, Q_1(W,\tilde{\varepsilon})>0, Q_{11}(W,\tilde{\varepsilon})<0, Q_1(0,\tilde{\varepsilon})=\infty, \\ \text{and } Q_2(W,\tilde{\varepsilon})>0, \text{ for all W and } \tilde{\varepsilon}. \tag{C.1}$$

where the subscripts indicate partial differentation. Firm's income in local currency is given by $\tilde{e}\tilde{p}Q(W, \tilde{\varepsilon})$ where $\tilde{e}$ is the random spot foreign exchange rate and $\tilde{p}$ is the random foreign-currency price of the commodity. The production process adopted by the firm gives rise to a stochastic cost function $C = C(W, \tilde{\eta})$ where $\tilde{\eta}$ is a undiversifiable cost shock. We assume the cost function to be strictly convex, increasing and twice differentiable:

$$C(0,\tilde{\eta})=0, C_1(W,\tilde{\eta})>0, C_{11}(W,\tilde{\eta})>0 \text{ and } C_2(W,\tilde{\eta})>0 \text{ for all } W,\tilde{\eta} \tag{C.2}$$

The subjective probability distribution of $\tilde{e}$, $\tilde{p}$, $\tilde{\varepsilon}$ and $\tilde{\eta}$ is exogenously given.

In addition to choosing production inputs, the firm has up to two options to insure itself against the risks $\tilde{e}$ and $\tilde{p}$ that it faces. First, if a futures market exists, the firm can sell forward an amount Z of the commodity at the futures price, p_f, the transaction adding thus $\tilde{e}(p_f-\tilde{p})Z$ to the local currency receipts. Second, if a forward exchange market exists, the firm can sell an amount X of foreign exchange at the current forward rate e_f, bringing $(e_f-\tilde{e})X$ to its local currency receipts. Transactions in both forward-futures markets are assumed cost-free and the standard length of forward-futures contracts is assumed to correspond to the production lag. With both forward and futures markets available, firm's profits in domestic currency are expressed as:

$$\tilde{\Pi}=\tilde{e}\tilde{p}Q(W,\tilde{\varepsilon})-C(W,\tilde{\eta})+\tilde{e}(p_f-\tilde{p})Z+(e_f-\tilde{e})X, \tag{1}$$

where the following notation is used:

$\tilde{\Pi}$ = firm's profits in domestic currency units
$\tilde{e}$ = the spot foreign exchange rate (one period hence)
e_f = the forward foreign exchange rate (for delivery in one period)
$\tilde{p}$ = the commodity spot price (one period hence) in foreign currency

p_f = the commodity futures price (for delivery in one period) in foreign currency
W = the level of primary inputs
Z = the quantity of the commodity sold (if Z > 0) or purchased (if Z < 0) in the futures market
X = the amount of foreign exchange sold (if X > 0) or purchased (if X < 0) forward
$\tilde{\varepsilon}$ = quantity shock
$\tilde{\eta}$ = cost shock

The firm selects the choice variables (W, X, Z) so as to maximize $EU(\tilde{\Pi})$, the expected value (E is the mathematical expectations operator) of a strictly concave, increasing and differentiable von Neumann-Morgenstern utility function U(.) defined over firm's profits in local currency $\tilde{\Pi}$. The first-order conditions for this maximization problem are the following:

$$\text{For W:}\ EU'(\tilde{\Pi}^*)[\tilde{e}\tilde{p}\tilde{Q}_1^* - \tilde{C}_1^*] = 0, \tag{2}$$

$$\text{For Z:}\ EU'(\tilde{\Pi}^*)[\tilde{e}(p_f - \tilde{p})] = 0, \tag{3}$$

$$\text{For X:}\ EU'(\tilde{\Pi}^*)[e_f - \tilde{e}] = 0, \tag{4}$$

where U'(.) is the marginal utility, an asterisk (*) indicates the optimum levels when markets are complete (namely, when both futures-forward markets exist) and $\tilde{Q}_1^*$ and $\tilde{C}_1^*$ are shorthand notations for $Q_1(W^*, \tilde{\varepsilon})$ and $C_1(W^* \tilde{\eta})$ respectively. The first-order conditions (2) to (4) can be rewritten as:

$$E\tilde{e}\tilde{p}\tilde{Q}_1^* - \frac{EU'(\tilde{\Pi}^*)\tilde{C}_1^*}{EU'(\tilde{\Pi}^*)} = -\frac{Cov[U'(\tilde{\Pi}^*), \tilde{e}\tilde{p}\tilde{Q}_1^*]}{EU'(\tilde{\Pi}^*)}, \tag{5}$$

$$E\tilde{e}\tilde{p} - e_f p_f = \frac{Cov[U'(\tilde{\Pi}^*), \tilde{e}\tilde{p}]}{EU'(\tilde{\Pi}^*)}, \tag{6}$$

$$E\tilde{e} - e_f = -\frac{Cov[U'(\tilde{\Pi}^*), \tilde{e}]}{EU'(\tilde{\Pi}^*)}. \tag{7}$$

Of course, the results depend critically upon the cost and output risks and the nature of the covariances. For instance, the futures-forward markets would be individually unbiased ($E\tilde{e} = e_f$) if $cov[U'(\tilde{\Pi}^*), \tilde{e}] = 0$ and simultaneously unbiased as well if $Cov[U'(\tilde{\Pi}^*), \tilde{e}\tilde{p}] = 0$.

So far, nothing has been said about the importer's behavior. The latter can be viewed as buying an imported commodity as input for use in the production of a final good that sells on local markets only at a price of unity. This is equivalent to interpreting $C(W, \tilde{\eta})$ as the new production function, $\tilde{e}\tilde{p}Q(W, \tilde{\varepsilon})$

as the new cost function where the role of $\tilde{\varepsilon}$ and $\tilde{\eta}$ are reversed. By so doing, the importer's behavior becomes analytically equivalent.

3 Properties of the optimum with complete markets

We want to explore how, with complete markets, the optimizing decisions covered by (5) to (7) change in response to the exporter's probability belief and attitutes towards risk. In particular, the interest lies in knowing the extent by which complete markets insulate the level of output and of international trade from the various risks and in analyzing the role played by hedging in this context.

3.1 Full-double hedging theorem

Without cost and output uncertainty, the full-double hedging theorem states that the optimal forward-futures contracting from (5) to (7) is a complete double-hedge i.e. $Z^* = Q(W^*,1)$ and $X^* = p_f Z^*$ (Kawai and Zilcha (1986)). Important to the theorem are two underlying assumptions:

(A.1) The forward exchange and commodity futures markets are separately unbiased i.e. $E\tilde{e} = e_f$, $E\tilde{p} = p_f$.

(A.2) The forward-futures markets are jointly unbiased, i.e. $E\tilde{e}\tilde{p} = e_f p_f$.

The difference between separate and joint unbiasedness lies in the additional assumption, $\mathrm{Cov}(\tilde{e}, \tilde{p}) = 0$, that is required for the latter but not for the former. A consequence of assumptions (A.1) and (A.2) is that the gains from speculation vanish and it is therefore optimal for the exporter to hedge his trade transactions completely. We shall use $Q(W)$, $C(W)$ instead of $Q(W,1)$ and $C(W,1)$ when no uncertainty in production or cost exists. With cost and output uncertainty the following outcome derives.

Proposition 1. If the unbiasedness assumptions (A.1) and (A.2) hold, then: (a) Under cost uncertainty the optimal forward-futures contracting is a full double-hedge if the stochastic cost shock $\tilde{\eta}$ is independent of the exchange rate $\tilde{e}$ and of the domestic price of the commodity $\tilde{e}\tilde{p}$; (b) Under output uncertainty the full double-hedge does not hold.

Proof. Consider the case of cost uncertainty only, output uncertainty being treated by analogy and therefore relegated to the Appendix. Rewrite the profit function (1):

$$\tilde{\Pi} = \tilde{e}\tilde{p}(Q(W) - Z) + \tilde{e}(p_f Z - X) - C(W, \tilde{\eta}) + e_f X. \tag{8}$$

By assumptions (A.1) and (A.2), $e_f = E\tilde{e}$ and $e_f p_f = E\tilde{e}\tilde{p}$. From (5) and (6) these assumptions about unbiasedness imply $Cov[\tilde{e}\tilde{p}, U'(\tilde{\Pi})] = 0$ and $Cov[\tilde{e}, U'(\tilde{\Pi})] = 0$. Hence, if $Q(W^*) = Z^*$ and $p_f Z^* = X^*$ is the optimal solution, (8) turns out $\tilde{\Pi} = e_f X - C(W^*, \tilde{\eta})$. As a result, the conditions $Cov[\tilde{e}\tilde{p}, U'(e_f X - C(W^*, \tilde{\eta}))] = 0$ and $Cov[\tilde{e}, U'(e_f X^* - C(W^*, \tilde{\eta}))] = 0$ hold if and only if the distribution of $\tilde{e}$ and of $\tilde{e}\tilde{p}$ are independent of that of $\tilde{\eta}$. Under these two conditions of independence, the double-hedging theorem holds under cost uncertainty.

The general result that seems to come from the first part of Proposition 1 is that independence between $\tilde{\eta}$ and $\tilde{e}$, $\tilde{\eta}$ and $\tilde{e}\tilde{p}$, and therefore full double-hedging is more likely to result from a cost disturbance that is firm-specific than one that is economy-wide. A nation-wide cost disturbance, like a major technological advance or an overall rise in domestic factor costs are shocks that are likely to affect the distribution of $\tilde{e}$ and $\tilde{e}\tilde{p}$ and make double-hedging not the optimal contracting. The second part of the proposition could explain the empirical finding that international firms do not hedge completely (Van Nieuwkerk (1979)).

3.2 The separation theorem

Consider first a world of no cost and output uncertainty in (5) to (7). Eq. (5) would simplify to $e_f p_f Q_1(W^*) = C_1(W^*)$, which equates the value marginal product of the input to its marginal cost, i.e., the competitive input rental. The optimal demand for the single primary factor and hence, the optimal level of output and trade, would be chosen at that point and would therefore be independent of the distribution of the random variables and of the firm's attitude towards risk. This is the contribution of futures-forward markets and the essence of the separation theorem as stated by Ethier (1973), Danthine (1978), Holthausen (1979), Deder, Just and Schmitz (1980) and others. With cost and output uncertainty the outcomes are summarized by the following proposition.

Proposition 2. In the presence of both forward-futures markets, when cost and output are uncertain, the firm's optimum demand for inputs depends on the utility function and the probability distribution of the random variables. The separation theorem holds only if both marginal product and marginal cost of input are not random.

Proof. Suppose that the marginal cost of input is not random. The firm's optimum demand for inputs initially given by (5) then becomes:

$$E\tilde{e}\tilde{p}Q_1(W^*, \tilde{\varepsilon}) = -\frac{Cov[U'(\tilde{\Pi}^*), \tilde{e}\tilde{p}Q_1(W^*, \tilde{\varepsilon})]}{EU'(\tilde{\Pi}^*)} + C_1(W^*). \tag{11}$$

The expected value marginal product is higher than the certain marginal cost as long as $Cov[U'(.), \tilde{e}\tilde{p}Q_1(.)] < 0$. With $Q_1(W^*, \tilde{\varepsilon})$ random, W^* does depend on the utility function and the probability distribution of the random variables.

With a non-random marginal product, the substitution of (6) into (11) gives $e_f p_f Q_1(W^*) = C_1(W^*)$ and the separation theorem holds.

The theoretical and empirical literature has extensively used two functional forms to specify the supply randomness: (1) a multiplicative risk (e.g. $\tilde{\varepsilon}W$, $E\tilde{\varepsilon} = 1$) and (2) an additive risk (e.g. $W + \tilde{\varepsilon}$, $E\tilde{\varepsilon} = 0$). When multiplicative, the stochastic supply shocks affect a proportion of the production while, when additive, these are independent of the size of the production. In terms of our model a multiplicative shock satisfies conditions (C.1) with $\tilde{Q}_{12} > 0$, the marginal product remaining random; an additive risk requires to make conditions (C.1) less strict, $EQ(0, \tilde{\varepsilon}) = 0$ on average only, $\tilde{Q}_{11} \leq 0$ and $\tilde{Q}_{12} \geq 0$. With $\tilde{Q}_{12} = 0$ the marginal product is independent of any supply shock and is therefore no longer random. This particular version of the model is compatible with the separation theorem.

Though the focus of the literature has been on stochastic output, the cost shocks can be specified along the same lines. A difference, however, is that with cost uncertainty a semi-separation result is obtained when the random shock is multiplicative.

Proposition 3 (Semi-separation theorem). Assume that multiplicative cost uncertainty is introduced, and that the unbiasedness assumptions (A.1) and (A.2) about the futures-forward markets hold. The optimal output $Q(W^*)$ is independent of the joint distribution of $(\tilde{e}, \tilde{p})$.

Proof. Let $C(W, \tilde{\eta}) = \tilde{\eta}C(W)$ with $E\tilde{\eta} = 1$. Consider the necessary and sufficient conditions for an optimum in equations (2), (3) and (4). Due to the strict concavity of the maximand there is a unique solution (W^*, Z^*, X^*). Let W^* be the unique solution to the equation:

$$e_f p_f \frac{Q_1(W^*)}{C_1(W^*)} - 1 = \frac{\mathrm{Cov}[\tilde{\eta}, U'(e_f p_f Q(W^*) - \tilde{\eta}C(W^*))]}{EU'(e_f p_f Q(W^*) - \tilde{\eta}C(W^*))} \tag{12}$$

and define:

$$Z^* = Q(W^*) \text{ and } X^* = P_f Z^*$$

In this case, $\tilde{\Pi} = e_f X^* - \tilde{\eta}C(W^*)$ and it is easy to verify that equations (2), (3) and (4) hold. Thus this is the optimal solution. Also, it is easy to see from (12) that W^* does not depend upon the joint distribution of $(\tilde{e}, \tilde{p})$, although it depends on the utility function. Hence, with multiplicative cost uncertainty, and forward-futures markets, any two firms with identical technologies but <u>different</u> probability beliefs about the exchange rate and price will export an equal amount of output as long as their attitude toward risk is the same.

4 The effects of cost and output uncertainty

This section extends the analysis of the previous section by considering the effects of cost uncertainty first and output uncertainty later. Assuming complete markets we show how each type of randomness affects the level of international trade.

4.1 Cost uncertainty only

Throughout this section we assume separate and joint unbiasedness as given by (A.1) and (A.2). Also, $C(W, \tilde{\eta}) = \tilde{\eta}C(W)$ where $\tilde{\eta}$ is independent of $\tilde{e}$ and $\tilde{p}$. Let us add the following assumption about the utility function:

(A.3) $U'(x)$ is convex and $xU'(x)$ is strictly concave in x.

Such conditions hold, for example, for quadratic utility and for the constant relative risk aversion utility, $U(x) = x^{1-\gamma}/(1-\gamma)$ with $1 \geq \gamma \geq 0$.

Consider random cost $\tilde{\eta}C(W)$, the production function being nonrandom. Denote the optimum in this case by W^*, X^*, Z^*. Also let us consider the case where $\tilde{e}$ and $\tilde{p}$ are random but we take the "certainty equivalent" cost, i.e., consider the benchmark case, where $\eta = 1$. In this case let the optimal input be $\bar{W}$. First, it is easy to verify that under the unbiasedness assumptions (A.1) and (A.2) the optimum, under uncertain cost, is given by:

$$Z^* = Q(W^*) \qquad X^* = P_f Z^*$$

where W^* is uniquely determined by the equation:

$$E[\tilde{e}\tilde{p}Q_1(W^*) - C_1(W^*, \tilde{\eta})]U'(e_f p_f Q(W^*) - C(W^*, \tilde{\eta})) = 0 \tag{13}$$

Now we prove that,

<u>Proposition 4.</u> Under assumptions (A.1)–(A.3) introducing uncertainty about cost results in a <u>higher</u> level of international trade, i.e., $W^* > \bar{W}$.

Proof. Let $C(W, \tilde{\eta}) = \tilde{\eta}C(W)$. We have shown that full double hedging holds in this case. Equation (13) can therefore be rewritten as:

$$E_{e,p}\{E_{\eta}[\tilde{e}\tilde{p}Q_1(W^*) - \tilde{\eta}C_1(W^*)]U'(e_f p_f Q(W^*) - \tilde{\eta}C(W^*))\} = 0.$$

Since U' is convex in $\tilde{\eta}$ and $\tilde{\eta}C_1(W^*)U'(.)$ is strictly concave in $\tilde{\eta}$ (see (A.3)) by taking expectation with respect to $\tilde{\eta}$ in the previous expression we obtain:

$$E_{e,p}[\tilde{e}\tilde{p}Q_1(W^*) - C_1(W^*)]\, U'(e_f p_f Q(W^*) - C(W^*)) < 0. \qquad (14)$$

On the other hand in the benchmark case we have:

$$\bar{Z} = Q(\bar{W}), \qquad \bar{X} = P_f\bar{Z}, \qquad \Pi = e_f\bar{X} - C(\bar{W})$$

where $C(\bar{W})$ represents the cost function since $\eta = 1$ in this case. Hence, $\bar{W}$ is given by:

$$E_{e,p}\{[\tilde{e}\tilde{p}Q_1(\bar{W}) - C_1\bar{W})]\, U'(e_f p_f Q(\bar{W}) - C(\bar{W}))\} = 0. \qquad (15)$$

But our maximand is a strictly concave function in W due to our assumption that Q(.) is strictly concave while the cost function C(.) is a convex function of W. Thus from (14) and (15) we obtain that $W^* > \bar{W}$. Namely, the output for export is larger when the cost is random. Higher production levels, and therefore revenues, are necessary to offset the large losses that would be incurred under unfavourable states of nature.

4.2 Production uncertainty only

Assume now that $\eta = 1$ while $\tilde{\varepsilon}$ is random with $E\tilde{\varepsilon} = 1$. Moreover:

(A.4) The absolute risk aversion $- U''(x)/U'(x)$ is convex in x.

Assumption (A.3) and (A.4) hold, for example, for constant relative risk aversion utility $U(x) = x^{1-\gamma}/(1-\gamma)$, with $1 \geq \gamma \geq 0$.

Proposition 5. Assume that (A.3) and (A.4) hold and that $Q(W, \tilde{\varepsilon}) = \tilde{\varepsilon}Q(W)$. Uncertainty about production results in a lower output (and hence lower trade), i.e., $W^* < \bar{W}$.

The proof of this proposition is relegated to the Appendix. Thus increasing output risk has opposite effect on the trade level to that of increased cost uncertainty. To understand the reason behind these opposing directions of effects of uncertainty, we should look at the non-symmetric role that the production shock $\tilde{\varepsilon}$ and the cost shock $\tilde{\eta}$ play in the expected marginal utility. Observing carefully the function inside the expectation in equation (13) we see that under cost uncertainty, i.e., when $\tilde{\eta}$ is random (and $\varepsilon = 1$) the convexity of the function $U'(x) - xU'(x)$ is crucial for this comparison. In this case, by (A.3), it is a strictly convex function of $\tilde{\eta}$. On the other hand, under production uncertainty, i.e., when $\tilde{\varepsilon}$ is random while $\eta = 1$, the convexity in $\tilde{\varepsilon}$ of the function inside the expectation operator in equation (2) matters. In this case, the convexity of $xU'(x) - U'(x)$ is important. Since this is a strictly concave function in $\tilde{\varepsilon}$ we obtain a reversed inequality. Thus the result in this case goes in the opposite direction.

5 The degree of absolute risk aversion

Given our model let us explore the effect of increasing risk aversion upon the production level. This boils down to comparing the optimal production and trade levels of two exporting firms, which differ only in their attitude towards risk. To simplify the proof let us show separately the cases of cost uncertainty and output uncertainty.

Proposition 6. Consider the competitive exporting firm facing price and exchange rate uncertainty. Increasing risk aversion results in: (a) decreasing output in the presence of production uncertainty if $\tilde{Q}_{12} > 0$ but increasing output if $\tilde{Q}_{12} < 0$; (b) decreasing output in the presence of cost uncertainty if $\tilde{C}_{12} > 0$ but increasing output if $\tilde{C}_{12} < 0$.

Proof. To prove (a) let us assume that $\tilde{\varepsilon}$ is random while $\eta = 1$. Thus, let two firms be identical except to their von-Neumann Morgenstern utility functions U and V. Let $V(\Pi) = h(U(\Pi))$ for all Π, where $h' > 0$ and $h'' < 0$; namely, the firm with V is more risk a averse than the firm with U. Let the optimum input levels be W^*_u and W^*_v, i.e., the following equations hold, where Π^*_u and Π^*_v are the corresponding optimal profits:

$$E[\tilde{e}\tilde{p}Q_1(W^*_u, \tilde{\varepsilon}) - C_1(W^*_u, 1)]\,U'(\tilde{\Pi}^*_u) = 0 \tag{16}$$

$$E[\tilde{e}\tilde{p}Q_1(W^*_v, \tilde{\varepsilon}) - C_1(W^*_v, 1)]h'(U(\tilde{\Pi}^*_v))U'(\tilde{\Pi}^*_v) = 0 \tag{17}$$

Under our assumptions $Q(W, \varepsilon)$ and $Q_1(W, \varepsilon)$ are monotone increasing in ε. For each fixed values of e, p define $\hat{\varepsilon}$ by $epQ_1(W^*_v, \hat{\varepsilon}) - C_1(W^*_v, 1) = 0$. Equation (17) can be rewritten as:

$$\begin{aligned} E_{e,p}\{ &\int_{\varepsilon \geq \hat{\varepsilon}} [epQ_1(W^*_v, \varepsilon) - C_1(W^*_v, 1)]h'(\tilde{\Pi}^*_v)U'(\tilde{\Pi}^*_v) \\ &- \int_{\varepsilon < \hat{\varepsilon}} [-epQ_1(W^*_v, \varepsilon) + C_1(W^*_v, 1)]h'(\tilde{\Pi}^*_v)U'(\Pi^*_v)\} = 0. \end{aligned} \tag{18}$$

Note that, by Proposition 3, $\tilde{\Pi}^*_v = \tilde{e}\tilde{p}Q(W^*_v, \tilde{\varepsilon}) - C(W^*_v, 1)$. Also, that for $\varepsilon < \hat{\varepsilon}$, it implies that $epQ_1(W^*_v, \varepsilon) - C_1(W^*_v, 1) < 0$; $\tilde{\Pi}^*_v$ is increasing in ε. Hence for <u>any</u> $\varepsilon' > \hat{\varepsilon}$ and $e'' < \hat{\varepsilon}$ we have (for any e, p given):

$$\tilde{\Pi}_v(\varepsilon') = epQ(W^*_v, \varepsilon') - C(W^*_v, 1) > epQ(W^*_v, \varepsilon'') - C(W^*_v, 1) = \tilde{\Pi}_v(\varepsilon'').$$

Therefore, for any $\varepsilon' > \hat{\varepsilon}$ and $\varepsilon'' < \hat{\varepsilon}$ we have, $h'(U(\tilde{\Pi}_v(\varepsilon'))) < h'(U(\tilde{\Pi}_v(\varepsilon'')))$. Hence it is easy to verify from equation (18) that:

$$E_{e,p}\{\int_{\varepsilon \geq \hat{\varepsilon}} [epQ_1(W_v^*, \varepsilon) - C_1(W_v^*, 1)]U'(\tilde{\Pi}_v^*)$$
$$- \int_{\varepsilon < \hat{\varepsilon}} [-epQ_1(W_v^*, \varepsilon) + C_1(W_v^*, 1)]U'(\tilde{\Pi}_v^*)\} > 0. \tag{19}$$

Namely, we have demonstrated that (we are using the independence of $\tilde{e}$, $\tilde{p}$ from the random variable $\tilde{\varepsilon}$):

$$E[\tilde{e}\tilde{p}Q_1(W_v^*, \tilde{\varepsilon}) - C_1(W_v^*, 1)]U'(\tilde{\Pi}_v^*) > 0. \tag{20}$$

Since the maximand $E[\tilde{e}\tilde{p}Q(W, \tilde{\varepsilon}) - C(W, 1)]$ is srictly concave in W comparing (16) and (20) we conclude that $W_v^* < W_u^*$. Note that when $\tilde{Q}_{12} < 0$ the inequality in (19) and (20) is reversed.

Now let us consider case (b) where $\tilde{\eta}$ is random while $\varepsilon = 1$. Under random cost the same result can be shown. As the proof is very similar, we shall bring the main argument in the Appendix.

6 Incomplete trading regimes

In this section we shall consider cases where some of the risk-sharing markets are missing. If $\tilde{e}$ and $\tilde{p}$, or one of them, becomes an undiversifiable risk because of missing organized markets, the set of choice variables available to the exporter reduces to (W, X) if the futures market is missing ($Z \equiv 0$), to (W, Z) if the forward exchange market is missing ($X \equiv 0$), or to (W) if both are missing ($X = Z \equiv 0$). In our terminology, not to be confused with Arrow-Debreu's definitions, each of the last three situations is characterized as incomplete market in comparison to the situation under (1). Our aim in this section is to study the trade effects of the gradual introduction of missing markets. As there are numerous situations possible, it is important to limit the number of issues. With developing and transition economies in the background, this section limits itself to the comparison of two extreme cases: the benchmark case of complete markets and that of no market at all. The following result is obtained:

Proposition 7. Consider an exporting firm with uncertainty about cost (as well as exchange rate and price) and assume that assumption (A.3) holds. Then

(a) Eliminating the domestic price uncertainty, by introducing unbiased "joint" futures market will increase the output of the firm.

(b) Eliminating the cost uncertainty, keeping other uncertainties the same, will decrease the output.

Proof. The optimal input used by the firm in the presence of all uncertainties W^* is given by:

$$E_\eta\{E_{ep}[(\tilde{e}\tilde{p}Q_1(W^*) - \tilde{\eta}C_1(W^*))U'(\tilde{e}\tilde{p}Q(W^*) - \tilde{\eta}C(W^*))]\} = 0 \tag{21}$$

By assumption (A.3) the function $xU'(x) - U'(x)$ is a strictly <u>concave</u> function, thus inserting the expectation with respect to the joint distribution of $(\tilde{e}, \tilde{p})$ inside (21) yields, taking $E\tilde{e}\tilde{p} = e_f p_f$:

$$E_\eta\{(e_f p_f Q_1(W^*) - \tilde{\eta}C_1(W^*))U'(e_f p_f Q(W^*) - \tilde{\eta}C(W^*))\} > 0. \tag{22}$$

On the other hand when there is only cost uncertainty, i.e. taking the price as $e_f p_f$, the optimum $\hat{W}$ is given by the equation:

$$E_\eta\{(e_f p_f Q_1(\hat{W}) - \tilde{\eta}C_1(\hat{W}))U'(e_f p_f Q(\hat{W}) - \tilde{\eta}C(\hat{W}))\} = 0. \tag{23}$$

However, the maximand $EU[e_f p_f Q(\hat{W}) - \tilde{\eta}C(\hat{W})]$ is a strictly concave function in W. This implies that $W^* < \hat{W}$, which proves that without price uncertainty the output is larger. Part (b) of the proposition is shown in the Appendix.

Proposition 7 gives a justification for the introduction of complete markets as a policy to promote international trade. It should be noted, however, that any other policy that quarantees to producers a price in domestic currency equivalent to $e_f p_f$ would achieve the same level of output and trade. It is clear that the results in the proposition completely reverse if the function $[U'(x) - xU'(x)]$ is a concave function. Clearly, most well-known utility functions satisfy (A.3), but for certain concave U(.) on certain intervals the above concavity may hold. Thus, Proposition 7 does not hold without some assumption on the risk preferences of the firm.

7 Concluding remarks

This paper considered risk involved decisions of an international firm facing multiple risks. As is often the case, traders when evaluating their profits do not face isolated risks but a product of them. Among these risks, some are diversifiable like price and exchange rate uncertainty and some are not like random cost and production shocks. The aim of this paper was to uncover and characterize the firm's risk-bearing optimum that involved decisions about the demand for inputs and the hedge in forward and futures markets.

The existing literature has dealt with the optimal behavior of a risk averse international firm facing diversifiable risks at great length. The contribution of this paper is to see whether the existing propositions are robust with respect to adding cost and output uncertainty. Most existing results fail in this respect, and this for several reasons. Cost uncertainty leads to a higher volume of trade whether markets are complete or not; production uncertainty gives rise to a lower volume. The separation theorem does not hold as long as the marginal product and the marginal cost are stochastic. The full-double hedging holds under most cases of cost uncertainty but fails to hold under production uncertainty. Increasing risk aversion may increase the level of trade under certain conditions about marginal cost and output. However, a result remains robust: introducing unbiased forward-futures markets increases the volume of trade above the level when no organized market exists.

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Appendix

Proof of Proposition 1 (output uncertainty)

With output uncertainty only, the profit function (1) evaluated at the optimum is:

$$\tilde{\Pi}^* = \tilde{e}\tilde{p}(Q^* - Z^*) + \tilde{e}(p_f Z^* - X^*) - C(W^*) + e_f X^* + \tilde{e}\tilde{p}(Q(W^*, \tilde{\varepsilon}) - Q^*)$$

where we have added and substracted $Q^* \equiv Q(W^*)$. Let $e_f = E\tilde{e}$ and $e_f p_f = E\tilde{e}\tilde{p}$. From (5) and (6), unbiasedness implies $\mathrm{Cov}[\tilde{e}\tilde{p}, U'(\tilde{\Pi})] = 0$ and $\mathrm{Cov}[\tilde{e}, U'(\tilde{\Pi})] = 0$. Hence, if $Q^* = Z^*$ and $p_f Z^* = X^*$ we obtain $\mathrm{Cov}[\tilde{e}\tilde{p}, U'(e_f X^* - C(W^*) + \tilde{e}\tilde{p}(Q(W^*, \tilde{\varepsilon}) - Q^*))] = 0$ which is impossible. Thus the full double-hedging theorem under market unbiasedness does not hold under output uncertainty.

Before the proof of Proposition 5 it is necessary to show the following Lemma:

Lemma. Under assumptions (A.3) and (A.4) about the utility, the function $H(A) = \dfrac{E_\varepsilon U'(A\tilde{\varepsilon} + B)}{U'(A+B)}$ is increasing in A on $[0, \infty]$.

Note that $H(0) = 1$ and due to the convexity of U' we have $H(A) > 1$ for $A > 0$.

Proof of the Lemma. Differentiate H(A) for a given fixed B,

$$H'(A) = \frac{E\tilde{\varepsilon}U''(A\tilde{\varepsilon}+B)U'(A+B) - U''(A+B)EU'(A\tilde{\varepsilon}+B)}{U'(A+B)^2}$$

$$= -E\left[\tilde{\varepsilon}\frac{-U''(A\tilde{\varepsilon}+B)}{U'(A+B)}\right] + \frac{-U''(A+B)}{U'(A+B)}E\left[\frac{U'(A\tilde{\varepsilon}+B)}{U'(A+B)}\right]$$

$$> -E\frac{-U''(A\tilde{\varepsilon}+B)}{U'(A+B)} + \frac{-U''(A+B)}{U'(A+B)}E\left[\frac{U'(A\tilde{\varepsilon}+B)}{U'(A+B)}\right]$$

Since $\mathrm{Cov}\left[\tilde{\varepsilon}, \dfrac{-U''(\tilde{\varepsilon}A+B)}{U'(A+B)}\right] < 0$ due to a decreasing absolute risk aversion. Now,

$$H'(A) > \frac{-1}{U'} E\left[\frac{-U''(A\tilde{\varepsilon}+B)}{U'(A\tilde{\varepsilon}+B)} \frac{U'(A\tilde{\varepsilon}+B)}{1} - \frac{-U''(A+B)}{U'(A+B)} \frac{U'(A\tilde{\varepsilon}+B)}{1}\right]$$

$$= \frac{1}{U'} E\left[U'(A\tilde{\varepsilon}+B)\left(-\frac{-U''(A+B)}{U'(A+B)} + \frac{-U''(A\tilde{\varepsilon}+B)}{U'(A\tilde{\varepsilon}+B)}\right)\right]$$

$$= \frac{1}{U'} EU'()\left[E\left(-\frac{U''(A\tilde{\varepsilon}+B)}{U'(A\tilde{\varepsilon}+B)}\right) - \frac{-U''(A+B)}{U'(A+B)}\right]$$

$$-\mathrm{Cov}\left[U'(A\tilde{\varepsilon}+B), \frac{-U''(A\tilde{\varepsilon}+B)}{U'(A\tilde{\varepsilon}+B)}\right] > 0.$$

Since the covariance is negative due to Assumption (A.4) and also

$$E\left[\frac{-U''(A\tilde{\varepsilon}+B)}{U'(A\tilde{\varepsilon}+B)}\right] > \frac{-U''(A+B)}{U'(A+B)}.$$

Thus $H'(A) > 0$ for $A > 0$.

Proof of Proposition 5.

Equation (2) can be rewritten in this case as:

$$E_{e,p}\{E_\varepsilon[\tilde{e}\tilde{p}\tilde{\varepsilon}Q_1(W^*) - C_1(W^*)]U'(\tilde{e}\tilde{p}\tilde{\varepsilon}Q_1(W^*) - \tilde{e}\tilde{p}Z^*$$
$$+\tilde{e}(p_f Z^* - X^*) - C(W^*) + e_f X^*)\} = 0.$$

The term inside the expectation with respect to $\tilde{\varepsilon}$'s distribution is a concave function in $\tilde{\varepsilon}$ for each given values of $(\tilde{\varepsilon}, \tilde{p})$. This stems from our assumption that $xU'(x) - U'(x)$ is a concave function of x. Thus inserting the expectation with respect to $\tilde{\varepsilon}$ inside this expression will yield:

$$E_{e,p}\{[\tilde{e}\tilde{p}Q_1(W^*) - C_1(W^*)]U'(\tilde{e}\tilde{p}Q_1(W^*) - \tilde{e}\tilde{p}Z^*$$
$$+\tilde{e}(p_f Z^* - X^*) - C(W^*) + e_f X^*)\} > 0. \tag{2'}$$

But this type of argument can be applied to eqs (3) and (4) as well.

Consider now equation (4). Define the event $M = \{e | e \geq e_f\}$. Rewrite equation (4) as follows:

$$E_{p|e}\left\{\int_{-M} \frac{E_\varepsilon U'(\tilde{\Pi})}{U'(E_\varepsilon \tilde{\Pi})} U'(E_\varepsilon \tilde{\Pi})(e_f - \tilde{e})\right\} = E_{p|e}\left\{\int_{M} \frac{E_\varepsilon U'(\tilde{\Pi})}{U'(E_\varepsilon \tilde{\Pi})} U'(E_\varepsilon \tilde{\Pi})(\tilde{e} - e_f)\right\}$$

By the above claim for each $e \in M$ and $\hat{e} \in \sim M$ we have (note that ep increases in e):

$$\frac{E_\varepsilon U'(\tilde{\Pi}(e))}{U'(E_\varepsilon \tilde{\Pi}(e))} > \frac{E_\varepsilon U'(\tilde{\Pi}(\hat{e}))}{U'(E_\varepsilon \tilde{\Pi}(\hat{e}))}.$$

Thus from the above equation we derive:

$$E_{p|e}\left\{\int_{\sim M} U'(E_\varepsilon \tilde{\Pi})(e_f - \tilde{e})\right\} > E_{p|e}\left\{\int_{M} U'(E_\varepsilon \tilde{\Pi})(\tilde{e} - e_f)\right\}.$$

In other words we obtained:

$$E[U'(E_\varepsilon \tilde{\Pi})(e_f - \tilde{e})] > 0. \tag{4'}$$

Similarly we can show that

$$E\{U'(E_\varepsilon \tilde{\Pi})[\tilde{e}(p_f - \tilde{p})]\} > 0. \tag{3'}$$

But the function

$$EU[\tilde{e}\tilde{p}Q(W) - C(W) + \tilde{e}(p_f - \tilde{p})Z + (e_f - \tilde{e})X]$$

is strictly concave in W, Z, X. All the inequalities (2'), (3') and (4') hold as equalities for $\bar{W}$, $\bar{Z}$, $\bar{X}$ (since it is the optimum for the benchmark case). Due to the concavity of this maximand we conclude that $\bar{W} > W^*$, $\bar{Z} > Z^*$ and $\bar{X} > X^*$.

Proof of part (b) of Proposition 6.

The necessary and sufficient condition for the optimality of W_v^* in this case is:

$$E[\tilde{e}\tilde{p}Q_1(W_v^*, 1) - C_1(W_v^*, \tilde{\eta})]h'(U'(\tilde{\Pi}_v^*))U'(\tilde{\Pi}_v^*) = 0 \tag{24}$$

For a given pair (e, p), define $\hat{\eta}$ by $epQ_1(W_v^*, 1) - C_1(W_v^*, \hat{\eta}) = 0$.
For any $\eta' < \hat{\eta}$ and $\eta'' > \hat{\eta}$ we have:

$$\Pi_v(\eta') = epQ(W_v^*, 1) - C(W_v^*, \eta') > \Pi_v(\eta'') = epQ(W_v^*, 1) - C(W_v^*, \eta'')$$

while

$$epQ_1(W_v^*,1)-C_1(W_v^*,\eta')>0, \qquad epQ_1(W_v^*,1)-C_1(W_v^*,\eta'')<0, \tag{25}$$

and

$$h'(U(\Pi_v(\eta')))<h'(U(\Pi_v(\eta''))) \text{ for all } \eta'<\hat{\eta} \text{ and } \eta''>\hat{\eta}. \tag{26}$$

As before we reach from (24) to

$$E[\tilde{e}\tilde{p}Q_1(W_v^*,1)-C_1(W_v^*,\tilde{\eta})]U'(\Pi_v^*)>0. \tag{27}$$

Hence we obtain that $W_v^* < W_u^*$ since

$$E[\tilde{e}\tilde{p}Q_1(W_u^*,1)-C_1(W_u^*,\tilde{\eta})]U'(\Pi_u^*)=0.$$

Proof of part (b) of Proposition 7.

Let us rewrite the first order condition (21) as follows:

$$E_{e,p}\{E_\eta[\tilde{e}\tilde{p}Q_1(W^*)-\tilde{\eta}C_1(W^*)]U'(\tilde{e}\tilde{p}Q(W^*)-\tilde{\eta}C(W^*))\}=0.$$

By assumption (A.3) the function $U'(x) - xU'(x)$ is a convex function, thus inserting the expectation with respect to $\tilde{\eta}$ inside ($E\tilde{\eta} = 1$) we obtain:

$$E_{e,p}\{(\tilde{e}\tilde{p}Q_1(W^*)-C_1(W^*))U'(\tilde{e}\tilde{p}Q(W^*)-C(W^*))\}<0.$$

Denote by $\bar{W}$ the optimum without cost uncertainty, then $\bar{W}$ is a solution of the equation:

$$E_{e,p}\{[\tilde{e}\tilde{p}Q_1(\bar{W})-C_1(\bar{W})]U'(\tilde{e}\tilde{p}Q(\bar{W})-C(\bar{W}))\}=0.$$

Again due to the strict concavity of the maximand in W, we have have that $W^* > \bar{W}$. Thus output decreases when uncertainty about cost is eliminated, leaving other uncertainty untouched.

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