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Jukka Railavo
Research Department
13.10.2004

Monetary consequences of alternative fiscal policy rules

Suomen Pankin keskustelualoitteita
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The views expressed are those of the author and do not necessarily reflect the views of the Bank of Finland.

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Monetary consequences of alternative fiscal policy rules

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Abstract

In this paper we analyse the monetary impact of alternative fiscal policy rules using the debt and deficit, both mentioned as measures of fiscal policy performance in the Stability and Growth Pact (SGP). We use a New Keynesian model, with endogenous labour supply, distortionary taxation and no private capital. The economy is hit by two fundamental shocks: demand and supply shocks, which are orthogonal to each other. Monetary policy is conducted by an independent central bank that will optimise. Under discretionary monetary policy the size of the inflation bias depends on the fiscal policy regime. Using the timeless perspective approach to precommitment, output persistence increase compared to the discretionary case. The result holds with the alternative fiscal policy rules, and inflation and output persistence reflects the economic data. With the deficit rules, the autocorrelation of the tax rate is near unity irrespective of the monetary policy regime, and irrespective of the fiscal policy parameters and targets.

Key words: inflation, optimal monetary policy, fiscal policy, policy coordination

JEL classification numbers: E52, E31, E61, E62

Vaihtoehtoisen finanssipolitiikan sääntöjen monetaariset vaikutukset

Suomen Pankin keskustelualoitteita 20/2004

Jukka Railavo
Tutkimusosasto

Tiivistelmä

Tässä tutkimuksessa käydään läpi vaihtoehtoisten finanssipolitiikan sääntöjen monetaarisia vaikutuksia. Tutkimuksessa käytetyt finanssipolitiikan säännöt perustuvat joko julkisen sektorin velkaan tai alijäämään, jotka molemmat ovat vakaus- ja kasvusopimuksessa finanssipolitiikan tilan mittareita. Vaikutuksia arvioidaan uuskeynesiläisessä mallissa, jossa työn tarjonta määräytyy endogeenisesti ja jossa verotuksen vääristävät vaikutukset on otettu huomioon. Lisäksi talouteen vaikuttaa kaksi toisistaan riippumatonta sokkia, kysyntä- ja tarjontasokki. Riippumaton keskuspankki harjoittaa optimointiin perustuvaa itsenäistä rahapolitiikkaa. Rahapolitiikan ollessa harkinnanvaraista talouden inflaatiotaipumus riippuu harjoitetusta finanssipolitiikasta. Jos rahapolitiikka perustuu ajasta riippumattomaan sääntöön, tuotannon persistenssi taloudessa kasvaa verrattuna harkinnanvaraiseen rahapolitiikkaan. Tämä tulos on riippumaton valitusta verosäännöstä, ja sekä inflaation että tuotannon persistenssi vastaavat taloudessa havaittavia arvoja. Kun finanssipolitiikassa sovelletaan alijäämään perustuvaa sääntöä, veroasteen auto-korrelaatio on lähellä yhtä riippumatta vallitsevasta rahapolitiikasta sekä valituista finanssipolitiikan parametri- ja tavoitearvoista.

Avainsanat: inflaatio, optimaalinen rahapolitiikka, finanssipolitiikka, politiikka koordinaatio

JEL-luokittelu: E52, E31, E61, E62

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1 Introduction

There is a huge literature about optimal monetary policy. In these papers the fiscal policy is simple or even not modelled at all. The description of more detailed fiscal and monetary policy was reintroduced by Sargent and Wallace (1981) in their unpleasant monetaristic arithmetics. The government has access to a subsidy to factor inputs financed with lump-sum taxes aimed at dismantling the inefficiency introduced by imperfect competition in product markets.

The literature on monetary policy has focused on how the monetary policy can stabilise the economy under shocks, mainly technology shocks. Benhabib and Wen (2004) claim that an aggregate demand shock is able to explain the actual fluctuation in the RBC models. From a Keynesian point of view, demand shocks are thought to be important for generating business cycles because the slow adjustment in prices may cause resources to be under-utilised, making possible the expansion of output without increases in marginal costs in response to higher aggregate demand.

In this literature the society usually delegates monetary policy to an independent and conservative central bank that shares the welfare function of the society but puts more emphasis on inflation than the society does¹. By independence we mean that the central bank has full control over the monetary policy instruments and chooses how much public debt is monetised. However, as shown in Schmitt-Grohé and Uribe (2004b), with even a small degree of price stickiness optimal inflation volatility is close to zero.

In another fast-growing literature on optimal monetary and fiscal policy, the behaviour of both policy-makers is based on optimisation, and therefore the fiscal authority affects the price level determination. For example Schmitt-Grohé and Uribe (2003, 2004a and 2004b) find that fiscal policy in a model with distortionary taxation affects the determination of steady state inflation and inflation volatility. This opens another determination channel of the inflation bias, since as in Siu (2004) the fiscal policy tries to balance a spending shock by absorbing inflation benefits. We claim that the size of the inflation bias depends on the fiscal policy regime when fiscal policy follows a fiscal rule.

Siu (2004) states that an important result of the optimal policy literature is the prescription of policies that smooth tax distortions over time and across states of nature. When governments finance stochastic government spending by taxing labour income and issuing one-period debt, state-contingent returns on that debt allow tax rates to be roughly constant, as in Lucas and Stokey (1983) and also Chari et al (1991 and 1994). Siu (2004) continues that the serial correlation properties of optimal tax rates and real government debt differ in flexible and sticky price models. In contrast to Barro's (1979) random walk result, Chari et al (1991 and 1994) show that with flexible prices these variables inherit the serial correlation of the model's underlying shocks. Siu (2004) also states that with sticky prices the autocorrelations of these objects are near unity regardless of the persistence in the shock process, thus partially

¹See eg Barro-Gordon (1983), Rogoff (1985) and Svensson (1997).

reviving Barro’s random walk result. The finding is similar to Aiyagari et al (2002), who consider optimal policy in a model with incomplete markets.

In this paper we analyse the monetary impact of alternative fiscal policy rules. We do not base the fiscal policy behaviour on optimisation, since we are more interested in different fiscal policy regimes. We formulate alternative fiscal policy rules using the debt and the deficit, both mentioned as measures of fiscal policy performance in the Stability and Growth Pact (SGP). We do this in a New Keynesian model, with endogenous labour supply, distortionary taxation and no private capital. The economy is hit by two fundamental shocks: demand and supply shocks, which are orthogonal to each other. Monetary policy is conducted by an independent central bank that will optimise, but the fiscal authority has to follow a rule.

We show that under discretionary monetary policy, fiscal policy matters to inflation bias. If the central bank is able to commit, inflation bias disappears. More importantly, under the timeless perspective of monetary policy precommitment by Woodford (1999), output persistence increases significantly compared to the discretionary case. We also revive Barro’s (1979) random walk result with the deficit rule for both under commitment and discretionary monetary policy irrespective of the fiscal policy regime. With the debt rule the Barro result does not hold for the high debt to GDP target values, and the tax rate inherits the stochastic nature of underlying shocks.

The paper is organised as follows. Chapter 2 describes the economy: the behaviour of the household and the firm. It sets up the policy target for both the central bank and the fiscal authority. In Chapter 3 we set up our simulation procedure and introduce all the results. Chapter 4 concludes the discussion.

2 The model

We consider a production economy with a representative firm, an infinitely lived representative consumer and a public sector. There is a private production good c_t and a public good g_t that satisfy the resource constraint

$$y_t = c_t + g_t, \quad (2.1)$$

where y_t is the aggregate production. The available production technology is represented as a constant returns to scale production function

$$y_t = A l_t, \quad (2.2)$$

where l_t is labour input and $A = \Lambda_t e^{\zeta^* Time}$ denotes technological progress. Stochastic fluctuations around a deterministic trend in the log of productivity $z_t \equiv \ln \Lambda_t$ are given by an exogenous AR(1) process

$$z_t = \rho z_{t-1} + \nu_t, \quad |\rho| < 1, \quad \nu_t = N(0, \sigma_\nu^2). \quad (2.3)$$

A typical household maximises a utility function

$$E_t \sum_{t=0}^{\infty} \delta^t u(c_t, m_t, l_t; g_t) \quad (2.4)$$

subject to the budget constraint

$$c_t + m_t - (1 - \pi_t)m_{t-1} + b_t \leq (1 + r_{t-1})b_{t-1} + w_t l_t (1 - \tau_t) + \Pi_t, \quad (2.5)$$

where m_t is real money balances, b_t is government bonds held by the household in real terms, w_t is the real gross wage rate, τ_t is the tax rate and Π_t is the real profit from the firm the household owns². The household's discount factor is δ and E_t is the expectation operator conditional on information available in period t . We assume that the utility function $u(c_t, m_t, l_t; g_t)$ is continuous, increasing and concave.

The first order conditions are

$$u_c(c_t, m_t, l_t; g_t) - \xi_t = 0, \quad (2.6)$$

$$u_m(c_t, m_t, l_t; g_t) - \xi_t + \delta E_t [\xi_{t+1} (1 - \pi_{t+1})] = 0, \quad (2.7)$$

$$u_l(c_t, m_t, l_t; g_t) + \xi_t w_t (1 - \tau_t) = 0, \quad (2.8)$$

$$\xi_t = \delta E_t \xi_{t+1} (1 + r_t), \quad (2.9)$$

where ξ is the Lagrangean multiplier and subscripts note partial derivatives. Combining equations, the first order conditions yield

$$E_t \left[\frac{u_c(c_{t+1}, m_{t+1}, l_{t+1}; g_{t+1})}{u_c(c_t, m_t, l_t; g_t)} \right] = \frac{1}{(1 + r_t)\delta}, \quad (2.10)$$

$$u_m(c_t, m_t, l_t; g_t) = u_c(c_t, m_t, l_t; g_t) \frac{R_t}{1 + R_t}, \quad (2.11)$$

$$u_l(c_t, m_t, l_t; g_t) = -u_c(c_t, m_t, l_t; g_t) w_t (1 - \tau_t). \quad (2.12)$$

Now we assume a periodic utility function expressed in the form of $u(c_t, m_t, l_t; g_t) = \frac{c_t^{1-\sigma}}{1-\sigma} + \frac{\Gamma m_t^{1-\sigma}}{1-\sigma} - \frac{l_t^{1+\lambda}}{1+\lambda} + f(g_t)$. This is a CRRA utility function, where $\sigma \geq 0$ is the elasticity of the intertemporal substitution of consumption and Γ is a positive constant. $\lambda \geq 0$ is the inverse of the Frisch elasticity of the labour supply. Using the periodical utility function, the first order conditions can be rewritten as

$$c_t^{-\sigma} = E_t c_{t+1}^{-\sigma} (1 + r_t) \delta, \quad (2.13)$$

$$\Gamma m_t^{-\sigma} = c_t^{-\sigma} \frac{R_t}{1 + R_t}, \quad (2.14)$$

$$-l_t^\lambda = -c_t^{-\sigma} w_t (1 - \tau_t). \quad (2.15)$$

Combining (2.13) and (2.14) with the resource constraint yields³

²Inflation π is defined as $\frac{P_t - P_{t-1}}{P_t} = \pi_t$, which implies that $1 - \pi_t = \frac{P_{t-1}}{P_t}$. The nominal interest rate R_t is $1 + R_t = (1 + r_t) / (1 - \pi_{t+1})$, where r_t is the real interest rate and π_{t+1} is the ex post expected inflation rate.

³First we loglinearise the equations (2.13) and (2.14) and following Uhlig (1999). Log-linearisation of (2.1) around the steady state yields $\hat{y}_t = \frac{\bar{c}}{\bar{y}} \hat{c}_t + \frac{\bar{g}}{\bar{y}} \hat{g}_t$. Since we want to write IS and LM in (log) levels, we apply the definition of the logarithmic deviations, eg for output $\hat{y}_t = \ln\left(\frac{y_t}{\bar{y}_t}\right)$, and the steady state conditions. See Railavo (2003) for details.

$$\ln y_t = E_t \ln y_{t+1} + \frac{\bar{g}}{\bar{y}} [\ln g_t - E_t \ln g_{t+1}] - \frac{\bar{c}}{\bar{y}} \frac{1}{\sigma} r_t - \frac{\bar{c}}{\bar{y}} \frac{1}{\sigma} \ln \delta, \quad (2.16)$$

$$\ln m_t = \frac{\bar{y}}{\bar{c}} \ln y_t - \frac{\bar{g}}{\bar{c}} \ln g_t - \frac{1}{\sigma} R_t + \frac{1}{\sigma} \ln \Gamma. \quad (2.17)$$

A profit maximising firm hires labour, and produces and sells products in a monopolistically competitive goods market⁴. The firm produces a single good with labour l_t and pays the wage w_t per unit of labour. We can write the real marginal cost of the firm using the production technology (2.2) as follows

$$\frac{\partial}{\partial y_t} \left[w_t \left(\frac{y_t}{A} \right) \right] = w_t \frac{1}{A} = mc_t. \quad (2.18)$$

Substitute the equilibrium wage $w_t = c_t^\sigma \left(\frac{y_t}{A} \right)^\lambda (1 - \tau_t)^{-1}$ into the marginal cost equation to yield

$$c_t^\sigma y_t^\lambda A^{-(1+\lambda)} (1 - \tau_t)^{-1} = mc_t. \quad (2.19)$$

Taking natural logarithms of (2.19) and using the definition of technological development $A = \Lambda_t e^{\zeta * Time}$ yields

$$\lambda \ln y_t - (1 + \lambda) \ln \Lambda_t - (1 + \lambda) \zeta * Time + \sigma \ln c_t - \ln (1 - \tau_t) = \ln mc_t. \quad (2.20)$$

In a flexible price equilibrium the nominal price equals the mark-up over marginal cost⁵. The equilibrium conditions yield the following long-run supply function⁶

$$\ln y_t^f = \frac{\sigma \bar{g}}{\kappa} \ln g_t + \frac{1 + \lambda}{\kappa} \zeta * Time + \frac{1}{\kappa} \ln (1 - \tau_t) + \varepsilon_t^{y^f}, \quad (2.21)$$

where y_t^f is the level of flexible price output with a distortionary tax rate, and we denote $\kappa = (\sigma \frac{\bar{g}}{\bar{c}} + \lambda)$ and $\varepsilon_t^{y^f} = \frac{1+\lambda}{\kappa} z_t$.⁷

To find the pricing equation of the firm, we follow Rotemberg (1987). We assume that there exists costs to the firm when it changes prices. This assumption will introduce price stickiness and reflect the empirical aspect that individual price setting is lumpy. The forward-looking firm sets prices by minimising a quadratic loss function

$$\frac{1}{2} E_t \sum_{j=0}^{\infty} \beta^j \left[(\ln P_{t+j} - \ln P_{t+j-1})^2 + a (\ln P_{t+j} - \ln P_{t+j}^*)^2 \right], \quad (2.22)$$

where $\beta = \frac{1}{(1+r)}$, $r > 0$ is the discount factor and a is an adjustment cost parameter. By taking the first order conditions of (2.22), rearranging terms and using the supply function (2.21), the New Keynesian Phillips curve yields

$$\pi_t = \beta E_t \pi_{t+1} + a \kappa (\ln y_t - \ln y_t^f). \quad (2.23)$$

⁴We assume that the labour market is perfectly competitive.

⁵In real terms $mc_t = \frac{1}{\mu}$, where μ is the mark-up. In the steady state $\ln \bar{m} \bar{c} = \ln \frac{1}{\mu}$. See Railavo (2003) for details.

⁶Combine (2.20) with the loglinearised resource constraint. Using the steady state condition of (2.20) we can again convert the loglinearised equation into a (log) levels form.

⁷Note that $z_t \equiv \ln \Lambda_t$.

Public sector behaviour is characterised by a budget constraint, an expenditure path, a monetary policy delegated to a central bank and a fiscal policy rule. The intertemporal budget constraint for the policy authority links debt and policy choices. The real flow budget constraint can be written as

$$b_t + \tau_t y_t + \pi_t m_{t-1} + m_t - m_{t-1} = (1 + r_{t-1}) b_{t-1} + g_t, \quad (2.24)$$

where b_t is the government bonds, $\tau_t y_t$ is the tax revenue, m_t is the nominal money balances, r_t is the real interest rate and g_t is the public spending. The policy authority balances its budget with new debt, with taxes and seigniorage revenue ($\pi_t m_{t-1} + m_t - m_{t-1}$). The intertemporal government budget constraint, which sums up the expected budget surpluses, is given by

$$(1 + r) b_t \leq \sum \left(\frac{1}{1 + r} \right)^i (\pi_{t+i} m_{t-1+i} + m_{t+i} - m_{t-1+i} + \tau_{t+i} y_{t+i} - g_{t+i}). \quad (2.25)$$

Government expenditure is characterised by

$$g_t = \rho^g \frac{g_{t-1}}{y_{t-1}} y_t + (1 - \rho^g) \bar{\gamma} y_t + \varepsilon_t^g, \quad |\rho^g| \leq 1, \quad \varepsilon_t^g = N(0, \sigma_{\varepsilon^g}^2), \quad (2.26)$$

where $\bar{\gamma}$ is a constant public consumption to GDP ratio. Innovations σ_v^2 and $\sigma_{\varepsilon^g}^2$ of fundamental shocks are orthogonal to each other.

Monetary policy is delegated to an independent central bank following Rogoff (1985). Optimal monetary policy is based on minimising a loss function common to the central bank and society. The welfare loss of the central bank at time t is the expected discounted sum of the periodic loss functions

$$W_t \equiv E_t \left[\sum_{t=0}^{\infty} \beta^t L_t \right]. \quad (2.27)$$

The periodic loss function is a weighted sum of squared output and inflation deviations, given by

$$L_t = \frac{1}{2} [(\pi_t - \pi^*)^2 + \chi (\ln y_t - \ln y_t^*)^2], \quad (2.28)$$

where π^* is the inflation target, χ is the positive parameter that reflects the relative concern of the central bank for output stability and $\ln y_t^* = \frac{\sigma_{\bar{g}}}{\kappa} \ln g_t + \frac{1+\lambda}{\kappa} \zeta * Time + \varepsilon_t^{y^*}$ is the desired level of potential output for the central bank (see Appendix A). The central bank targets the efficient level of output in the absence of the monopolistic distortion. Also the non distorted flexible price output does not depend on the households' labour supply decisions. Rotemberg and Woodford (1998) have shown that the loss measure can be derived by approximating the expected utility of a representative household when $\chi > 0$. As mentioned in Aoki and Nikolov (2003), the analysis is valid for arbitrary values of χ .

In discretionary case the central bank minimises the discounted losses subject to the Phillips curve (2.23). Substituting the Phillips curve into the central bank's objective, we get a multiperiodic problem⁸

$$\min_{\{\pi_t, i=0,1,2,\dots\}} E_t \sum_{t=0}^{\infty} \beta^t \left\{ \left[\frac{1}{2} (\pi_t - \pi^*)^2 + \chi \left(\ln y_t^f - \ln y_t^* + \frac{1}{a\kappa} (\pi_t - \beta E_t \pi_{t+1}) \right)^2 \right] \right\}. \quad (2.29)$$

Under discretion, once expectations about future inflation $E_t \pi_{t+1}$ are formed, the central bank optimises taking them as given. Hence, we get a sequence of static minimisation problems, see eg Cukierman (1992, Chapter 3). Optimal monetary policy under discretion is

$$\pi_t = \pi^* - \frac{\chi}{a\kappa} \left(\ln y_t^n - \ln y_t^* + \frac{1}{a\kappa} (\pi_t - \beta E_t \pi_{t+1}) \right). \quad (2.30)$$

As a result, a central bank that emphasises output at all, creates an inflationary bias to the economy. Cukierman (1992) recalls the point made by Barro and Gordon (1983): under discretion the inflationary bias of the monetary policy carries over to the case in which the central bank cares about the future as well as about the present. Using (2.21) and (4.6) we can rewrite $\ln y_t^f - \ln y_t^* = \frac{1}{\kappa} \ln(1 - \tau_t)$. Substituting it into the optimal policy function (2.30) and rearranging yields

$$\pi_t = \frac{(a\kappa)^2}{(a\kappa)^2 + \chi} \pi^* + \frac{\chi\beta}{(a\kappa)^2 + \chi} E_t \pi_{t+1} - \frac{\chi a}{(a\kappa)^2 + \chi} \ln(1 - \tau_t). \quad (2.31)$$

Substitute (2.31) into the IS curve (2.16) and solve for the optimal nominal interest rate under discretion

$$R_t = \frac{1}{\beta} \pi_t + \frac{(a\kappa)^2}{\chi\beta} (\pi_t - \pi^*) - \frac{\bar{y}}{\bar{c}} \sigma (\ln y_t - E_t \ln y_{t+1}) + \frac{\bar{g}}{\bar{c}} \sigma (\ln g_t - E_t \ln g_{t+1}) + \frac{a\kappa}{\beta} (\ln y_t^n - \ln y_t^*) - \ln \delta. \quad (2.32)$$

The equation (2.32) has a resemblance to the Taylor (1993) rule. The implied interest rate rule satisfies the Taylor principle, ie the interest rate reacts more than one-for-one to an increase in inflation, in which case the monetary policy is often called active in the relevant literature.

Under commitment the central bank does not take expectations about future inflation as given. Then the central bank's objective is to pick π_{t+i} ,

⁸Under discretion, once the expectation about the future inflation $E_t \pi_{t+1}$ is formed, the central bank reoptimise taking them as given. Hence, we can treat the mimimisation problem in isolation for period t . See Chapter 3 in Cukierman(1992).

$\ln y_{t+i}$ and R_{t+i} to minimise a Lagrangian

$$\begin{aligned} \mathcal{L} = & E_t \sum_{i=0}^{\infty} \beta^i \left\{ \frac{1}{2} (\pi_{t+i} - \pi^*)^2 + \frac{\chi}{2} (\ln y_{t+i} - \ln y_{t-i}^*)^2 \right. \\ & + \theta_{t+i} \left[\ln y_{t+i} - \ln y_{t+i-1} + \frac{\bar{c}}{\bar{y}} \sigma^{-1} (R_{t+i} - \pi_{t+i+1}) - \frac{\bar{c}}{\bar{y}} \sigma^{-1} \ln \delta \right. \\ & \left. \left. - \frac{\bar{g}}{\bar{y}} (\ln g_{t+i} - \ln g_{t+i+1}) \right] \right. \\ & \left. + \varphi_{t+i} \left[\pi_{t+i} - \beta \pi_{t+i+1} - a\kappa (\ln y_{t+i} - \ln y_{t+1}^f) \right] \right\}, \end{aligned} \quad (2.33)$$

where θ_{t+i} and φ_{t+i} are Lagrangian multipliers. The first order conditions are

$$E_t [\pi_{t+i} - \pi^* + \varphi_{t+i} - \varphi_{t+i-1}] = 0, \quad (2.34)$$

$$E_t \left[(\ln y_{t+i} - \ln y_{t-i}^*) - \frac{a\kappa}{\chi} \varphi_{t+i} \right] = 0, \quad (2.35)$$

$$\frac{\bar{c}}{\bar{y}} \sigma^{-1} E_t (\theta_{t+i}) = 0. \quad (2.36)$$

From (2.36) we obtain that $E_t (\theta_{t+i}) = 0$ for all $i > 0$. As mentioned in Walsh (2003), this reflects the fact that the equation (2.16) imposes no real constraint on the central bank as long as there are no restrictions on varying the nominal interest rate.

Under commitment the central bank not only care about the future and present as suggested by Barro and Gordon (1983), but also about the past. Woodford (1999) calls such a policy optimal from a timeless perspective. Woodford (2003, Chapter 7) states that a time-invariant policy is optimal from a timeless perspective if the equilibrium evolution from any date t_0 onward is optimal, subject to the constraint that the economy's initial evolution be the one associated with the policy in question. Under a timeless perspective we form the following linear combination from (2.34)

$$\begin{aligned} & \left[1 + \beta + \frac{(a\kappa)^2}{\chi} \right] \pi_t - \beta E_t \pi_{t+1} - \pi_{t-1} \\ & = \left[1 + \beta + \frac{(a\kappa)^2}{\chi} \right] (\varphi_t - \varphi_{t-1}) - \beta E_t (\varphi_{t+1} - \varphi_t) \\ & \quad - (\varphi_{t-1} - \varphi_{t-2}) + \left[1 + \beta + \frac{(a\kappa)^2}{\chi} - \beta - 1 \right] \pi^*. \end{aligned} \quad (2.37)$$

By substituting the first order conditions (2.34) and (2.35) into the Phillips curve (2.23), we obtain a difference equation that fulfils the optimal φ

$$\left[1 + \beta + \frac{(a\kappa)^2}{\chi} \right] \varphi_t - \beta E_t \varphi_{t+1} - \varphi_{t-1} = (1 - \beta) \pi^* + a\kappa (\ln y_t^f - \ln y_t^*). \quad (2.38)$$

Combining equations (2.37) and (2.38), we have the optimal monetary policy under commitment from a timeless perspective

$$\begin{aligned} \left[1 + \beta + \frac{(a\kappa)^2}{\chi}\right] \pi_t &= \frac{(a\kappa)^2}{\chi} \pi^* + \beta E_t \pi_{t+1} + \pi_{t-1} \\ -a\kappa \left[\left(\ln y_t^f - \ln y_t^* \right) - \left(\ln y_{t-1}^f - \ln y_{t-1}^* \right) \right] &. \end{aligned} \quad (2.39)$$

Using $\ln y_t^f - \ln y_t^* = \frac{1}{\kappa} \ln(1 - \tau_t)$, we can rewrite (2.39) to yield

$$\begin{aligned} \left[1 + \beta + \frac{(a\kappa)^2}{\chi}\right] \pi_t &= \frac{(a\kappa)^2}{\chi} \pi^* \\ + \beta E_t \pi_{t+1} + \pi_{t-1} - a \left[\ln(1 - \tau_t) - \ln(1 - \tau_{t-1}) \right] &. \end{aligned} \quad (2.40)$$

Substitute (2.39) into the IS curve (2.16) and solve for the nominal interest rate to yield the optimal interest rate under commitment

$$\begin{aligned} R_t &= \left(\frac{1}{\beta} + 1 \right) \pi_t - \frac{1}{\beta} \pi_{t-1} + \frac{(a\kappa)^2}{\beta\chi} (\pi_t - \pi^*) \\ &\quad - \frac{\bar{y}}{\bar{c}} \sigma (\ln y_t - E_t \ln y_{t+1}) + \frac{\bar{g}}{\bar{c}} \sigma (\ln g_t - E_t \ln g_{t+1}) \\ &\quad + \frac{a\kappa}{\beta} \left[\left(\ln y_t^f - \ln y_t^* \right) - \left(\ln y_{t-1}^f - \ln y_{t-1}^* \right) \right] - \ln \delta. \end{aligned} \quad (2.41)$$

Fiscal policy, following Leeper (1991), is represented as a debt rule

$$\tau_t = \tau^* + \phi [b_{t-1} + m_{t-1}] / y_t - \psi_1. \quad (2.42)$$

Here, τ^* is a positive constant representing a long-run tax rate⁹, $b_{t-1} + m_{t-1}$ are the real government liabilities, $\psi > 0$ represents the debt to GDP ratio target and ϕ is the fiscal policy parameter. The higher values ϕ gets, the more weight the fiscal authority places on balancing the government budget. Railavo (2003) has shown that this type of fiscal policy rule results in a stable solution with Taylor rule type monetary policy if inflation response is more than one-for-one with a wide range of positive fiscal policy rule parameter values.

We also explore other fiscal policy rules. The government liabilities in the fiscal policy rule (2.42) can be replaced by the government primary deficit, in which case the fiscal policy rule is a deficit rule of the form

$$\tau_t = \tau^* + \Omega [g_t - \tau_t y_t + R_t b_{t-1}] / y_t - \psi_2, \quad (2.43)$$

where the primary deficit is $g_t - \tau_t y_t$ and the interest payment on the real debt outstanding is $R_t b_{t-1}$. This is the SGP definition of the deficit and conforms closely with the deficit based on the real government budget constraint. See Railavo (2004) for details.

An alternative to the Leeper (1991) way of writing a fiscal policy rule is to use the difference of the tax rate. It resembles an error-correction approach and the gradualistic specification is more realistic than the Leeper type as the

⁹ τ^* is related to the long-run tax rate, since $\frac{b_{t-1} + m_{t-1}}{y_t}$ need not be equal to zero.

tax rate movement is smoother. An error-correction debt rule can be written as follows

$$\tau_t = \tau_{t-1} + \phi [(b_{t-1} + m_{t-1}) - \psi_1 y_t] / y_t. \quad (2.44)$$

Railavo (2004) shows that (2.44) is highly unstable for a large range of positive parameter ϕ values when the monetary policy is active. Therefore we shall not study the effects of shocks under (2.44) using the stochastic simulation procedure described below. On the other hand, the corresponding fiscal policy rule with the deficit

$$\tau_t = \tau_{t-1} + \Omega [(g_t - \tau_t y_t + R_t b_{t-1}) - \psi_2 y_t] / y_t \quad (2.45)$$

produces stable solutions for a wide range of fiscal policy parameter, Ω , values as shown in Railavo (2004), and, hence, will be used in simulations.

3 Stochastic simulation

We analyse the time-series properties of inflation, the interest rate, output, the debt to GDP ratio and the tax rate as a response to the fundamental stochastic shocks. The stochastic nature of exogenous variables is given by (2.3) and (2.26). We also show the relationship with inflation, the interest rate, the debt to GDP ratio and the tax rate in the steady state. Our simulation procedure involves simulating the model given by equations (2.16), (2.17), (2.21), (2.23), (2.24) and (4.6). Monetary policy is either discretionary (2.31) or follows the commitment solution (2.39). Fiscal policy is conducted with different policy rules (2.42), (2.43) or (2.45). The initial and terminal values are set equal to the steady state values of the model.

We solve the model 2500 times to obtain a set of time series, which are then used to compute the variability and persistence statistics. In our procedure, simulations are done in a recursive manner. In the first round the model is simulated for 2500 periods, in the second round 2499 periods, etc. In each simulation round, the current period shocks ν and ε^g are drawn from $N(0, \sigma_\nu^2)$ and $N(0, \sigma_{\varepsilon^g}^2)$ distributions, but for subsequent periods their values are set for zero. We have set $\sigma_\nu^2 = 0.01$ and $\sigma_{\varepsilon^g}^2$ is set to be one percent of the GDP.

Following Cooley and Prescott (1995), we have set $\rho = 0.81$, which reflects that 95 percent of the technology shock remains after one quarter. Respectively we set $\rho^g = 0.975$ according to Blanchard and Perotti (2002), which means that 95 percent of the government spending shock still remains after 2 years. The model is calibrated to reflect the economic structure of a large economy and the key parameter values of the model are given in Table 1.

Table 2 shows the steady state results with the debt rule (2.42) and discretionary monetary policy (2.31). We let the fiscal policy rule parameter ϕ vary from 0.1 to 1.5 and the debt to GDP ratio target ψ_1 from a tight target (0) to a loose target (1.5). As concluded in Railavo (2003 and 2004), the low values of the fiscal policy rule parameter indicate active fiscal policy while the higher values refer to passive policy. As defined in Leeper (1991), the passive fiscal policy authority must generate sufficient tax revenues to balance the

σ	λ	a	Γ	δ	β
0.157	1.433	0.003	0.7	0.97	0.97
π^*	τ^*	ζ	χ	$\frac{g}{y}$	
0.02	0.24	0.018	0.05	0.4	

Table 1: The parameter values used and not altered in simulation

	ϕ	0.1	0.5	0.9	1.5
	ψ_1	<i>mean</i>	<i>mean</i>	<i>mean</i>	<i>mean</i>
Inflation	0	5.1	4.9	4.9	4.9
	0.6	5.8	5.1	5.0	4.9
	1.5	7.1	5.3	5.1	5.0
Interest rate	0	8.6	8.4	8.4	8.4
	0.6	9.3	8.6	8.5	8.4
	1.5	10.6	8.8	8.6	8.5
Debt to GDP ratio	0	155.5	20.3	6.2	-0.7
	0.6	818.1	145.5	75.3	40.6
	1.5	1812.2	333.2	179.0	102.6
Tax rate	0	40.2	39.3	39.2	39.2
	0.6	44.6	40.1	39.7	39.4
	1.5	51.3	41.4	40.4	39.8

Table 2: Discretionary monetary policy with the debt rule

budget regardless of inflation, whereas the active authority is not constrained by budgetary conditions. Monetary policy is active with both discretionary and commitment monetary policy. The steady state values of the tax rules (2.42) and (2.43) depend on the values of the fiscal policy parameter ϕ and Ω , respectively, and also on the values of the target, ψ_1 and ψ_2 , respectively. However, the steady state values of the tax rules (2.44) and (2.45) do not depend on the values of the fiscal policy parameter ϕ and Ω , respectively, but only on the values of the target, ψ_1 and ψ_2 , respectively.

We can see from Table 2 that there is inflation bias with discretionary monetary policy, as inflation is above the target value, $\pi^* = 0.02$. We also see that the size of the bias depends on the fiscal policy parameter, ϕ , values and the debt to GDP target, ψ_1 , values. Loosening the debt to GDP target increases the steady state debt to GDP ratio and the steady state inflation increases. The high tax rate is associated with the high debt to GDP ratios, which feeds into inflation. The debt to GDP ratio decreases as the fiscal policy parameter gets larger values, ie the fiscal policy authority reacts more with the tax rate. The largest changes in the steady state values happen when the fiscal policy parameter, ϕ , changes from 0.1 to 0.5. With higher values of ϕ , the change in the steady state values of inflation, the tax rate and the interest rate is small compared to the changes in the debt to GDP ratio. Also, with the $\phi = 0.1$, the change in the target parameter has the largest impact on the steady state values of inflation and the tax rate. This indicates that there is strong non-linearity with the low values of ϕ .

	Ω	0.1	0.5	0.9	1.5
	ψ_2	<i>mean</i>	<i>mean</i>	<i>mean</i>	<i>mean</i>
Inflation	0	9.0	5.4	5.1	5.1
	0.03	9.5	5.5	5.2	5.1
	0.1	10.6	5.7	5.3	5.2
Interest rate	0	12.5	8.9	8.6	8.5
	0.03	13.0	9.0	8.7	8.6
	0.1	14.0	9.2	8.8	8.6
Debt to GDP ratio	0	3120.0	443.8	228.6	128.9
	0.03	3408.4	529.8	274.3	156.1
	0.1	3976.3	712.5	378.5	218.7
Tax rate	0	60.0	42.1	40.7	40.0
	0.03	61.9	42.7	41.0	40.2
	0.1	65.7	43.9	41.7	40.6

Table 3: Discretionary monetary policy with the deficit rule

Table 3 shows the steady state ratios with the deficit rule (2.43). Now the deficit to GDP target ψ_2 gets values between zero and 0.1 as the fiscal policy rule parameter Ω runs from 0.1 to 1.5. Again, we see that increasing the target makes the debt to GDP ratio increase, which has an impact on inflation. We also see that the low values of the fiscal policy rule parameter result in an extremely high debt to GDP ratio in the steady state. The high debt levels are associated with the high tax rate and with the low fiscal policy parameter value. Overall, the debt and deficit rule results in similar steady state responses to changes in the fiscal policy parameter and target values.

Table 4 shows the steady state values under the deficit rule (2.45). Now the fiscal policy parameter Ω has an impact neither on the steady state tax rate nor on the steady state debt to GDP level. Increase in the deficit target ψ_2 increases the steady state debt to GDP ratio and inflation. However, changing the deficit target has a small effect on the level of the steady state inflation compared on the quite large impact to the debt to GDP ratio.

Tables 5 and 6 display the steady state values when the monetary policy authority is able to commit. As expected, inflation will be at the target level for all combinations of the fiscal policy parameter and the target values. With the debt rule (2.42), the debt to GDP ratio will increase as the fiscal policy parameter ϕ value decreases and the debt to GDP target ψ_1 value increases. With the error-correction deficit rule (2.45), the fiscal policy parameter does not have an effect on the steady state debt to GDP ratios. However, the debt to GDP ratio will increase as the deficit target increases, which results in a higher steady state tax rate.

Tables 7 to 11 display the variability and persistence statistics as a response to the underlying fundamental stochastic shocks. We let the fiscal policy parameters, ϕ and Ω , vary from 0.1 to 1.5 and the target parameter value changes from low (tight) to higher values (looser).

Barro (1979) claims that an optimal monetary and fiscal policy problem results in an optimal tax rate and debt will follow a random walk. Lucas and Stokey (1983) and Chari et al (1991 and 1994) show that with flexible

	Ω	0.1	0.5	0.9	1.5
	ψ_2	<i>mean</i>	<i>mean</i>	<i>mean</i>	<i>mean</i>
Inflation	0	4.9	4.9	4.9	4.9
	0.03	5.0	5.0	5.0	5.0
	0.1	5.0	5.0	5.0	5.0
Interest rate	0	8.4	8.4	8.4	8.4
	0.03	8.4	8.4	8.4	8.4
	0.1	8.5	8.5	8.5	8.5
Debt to GDP ratio	0	-11.1	-11.1	-11.1	-11.1
	0.03	28.8	28.8	28.8	28.8
	0.1	120.1	120.1	120.1	120.1
Tax rate	0	39.1	39.1	39.1	39.1
	0.03	39.4	39.4	39.4	39.4
	0.1	40.0	40.0	40.0	40.0

Table 4: Discretionary monetary policy with the error-correction deficit rule

	ϕ	0.1	0.5	0.9	1.5
	ψ_1	<i>mean</i>	<i>mean</i>	<i>mean</i>	<i>mean</i>
Inflation	0	2.0	2.0	2.0	2.0
	0.6	2.0	2.0	2.0	2.0
	1.5	2.0	2.0	2.0	2.0
Interest rate	0	5.5	5.5	5.5	5.5
	0.6	5.5	5.5	5.5	5.5
	1.5	5.5	5.5	5.5	5.5
Debt to GDP ratio	0	155.4	18.5	4.2	-2.9
	0.6	817.9	143.5	73.2	38.4
	1.5	1811.6	331.1	176.8	100.4
Tax rate	0	40.4	39.5	39.4	39.4
	0.6	44.9	40.3	39.9	39.6
	1.5	51.6	41.6	40.6	40.1

Table 5: Committed monetary policy with the debt rule

	Ω	0.1	0.5	0.9	1.5
	ψ_2	<i>mean</i>	<i>mean</i>	<i>mean</i>	<i>mean</i>
Inflation	0	2.0	2.0	2.0	2.0
	0.03	2.0	2.0	2.0	2.0
	0.1	2.0	2.0	2.0	2.0
Interest rate	0	5.5	5.5	5.5	5.5
	0.03	5.5	5.5	5.5	5.5
	0.1	5.5	5.5	5.5	5.5
Debt to GDP ratio	0	-13.4	-13.4	-13.4	-13.4
	0.03	51.0	51.0	51.0	51.0
	0.1	201.3	201.3	201.3	201.3
Tax rate	0	39.3	39.3	39.3	39.3
	0.03	39.7	39.7	39.7	39.7
	0.1	40.7	40.7	40.7	40.7

Table 6: Committed monetary policy with the error-correction deficit rule

prices Barro's result of an optimal tax rate to follow a random walk does not hold. Chari et al (1991 and 1994) also claim that the tax rate and debt inherit the serial correlation of the model's underlying shocks. Siu (2004) found that in a sticky price model, especially in the case in which government finances spending by increasing taxes, resulting in an accumulated debt, the autocorrelations of the debt to GDP ratio and the tax rate are near unity regardless of the persistence in the shock process, partially reviving Barro's random walk result.

In Table 7 we can see that the variability of inflation decreases as the parameter ϕ in the debt rule (2.42) gets larger values, but the variability of the tax rate increases. The variability of both inflation and the tax rate increases as the debt to GDP ratio target, ψ_1 , gets larger values. Inflation and the interest rate are highly autocorrelated for all the parameter values. The persistence of the debt to GDP ratio and the tax rate decreases as both or either the fiscal policy parameter and the debt to GDP ratio target get larger values. With the low target values, ie the low steady state debt to GDP ratio, the autocorrelation of the debt to GDP ratio and the tax rate are near unity supporting Barro's (1979) result. However, increasing the target values, ie making the debt to GDP ratio less restrictive, reduces the autocorrelation of the variables and supports the Chari et al (1991 and 1994) result even in a sticky price model. Output variability and persistence remain quite constant and low regardless of the changes in the parameter values.

Table 8 repeats the previous results now with the deficit rule (2.43). The overall results are similar to the previous results, but the persistence of the debt to GDP ratio and the tax rate do not decrease with the increase of the values of the Ω and ψ_2 parameters. Now we find support for Barro (1979) and Siu (2004) with all parameter value combinations. The change in fiscal policy do not affect the persistence of the tax rate. However, output persistence and volatility do not improve due to results with the debt rule.

The introduction of the error-correction deficit rule (2.45) does not change the results significantly compared with the deficit rule (2.43), as can be seen in

Table 9. The persistence of inflation, the interest rate, the debt to GDP ratio and the tax rate remains high. However, the variability of inflation decreases with the low fiscal policy parameter Ω values compared with the results of the deficit rule. This is due to the fact that the fiscal policy parameter has no impact on the debt to GDP ratio and hence on the level of inflation with the error-correction deficit rule. The variability of the debt to GDP ratio is smaller when the debt to GDP ratio level is smaller.

In Table 10 we see the results with committed monetary policy (2.39) and the debt rule (2.42). We can see that under the commitment of monetary policy output persistence increases significantly compared to the discretionary case. This demonstrates the timeless perspective of monetary policy as optimal monetary policy under commitment (2.40) displays a lagged inflation term. As the persistence increases there is a considerable increase in the variability of output. Whereas the variability of output increases under commitment, that of inflation and the interest rate falls. The variability of the tax rate and the debt to GDP ratio remain relatively similar with both discretionary and committed monetary policy. However, the persistence of the two increases somewhat, especially with the high fiscal policy and debt to GDP ratio target values. Still, the autocorrelation of the two variables gives support to the Barro finding when the target has low values. As the debt to GDP increases and the fiscal policy reacts more with taxes, the autocorrelation drops and the tax rate inherits the serial correlation of the shock as in Chari et al (1991 and 1994).

The same result shows up with the deficit rule (2.45). The results in Table 11 are similar to the results of discretionary monetary policy with the deficit rule except for output. Like in the previous case, the volatility and persistence of output has increased significantly compared with the discretionary monetary policy case. The autocorrelation of the debt to GDP ratio and the tax rate will remain high, reflecting Barro's results with all the fiscal parameter combinations.

4 Conclusions

In this paper we analysed the effects of alternative fiscal policy rules with optimal monetary policy. With discretionary monetary policy, inflation bias depends on the fiscal policy with both the debt and deficit rule. The fiscal policy parameter and the target values, hence the fiscal policy regime, affect the size of the bias. The higher values the fiscal policy parameter and the target parameter get the higher the steady state debt to GDP ratio is and inflation becomes. The target parameter changes increase inflation more evenly, but the policy parameter changes are more notable between low values than with high values.

With the error-correction deficit rule, the fiscal policy parameter has no impact on the steady state tax rate and, also, on the steady state debt to GDP level. A rise in the deficit target increases the steady state debt to GDP ratio and inflation. However, changing the deficit target has a small effect on

the level of the steady state inflation compared to the quite large impact on the debt to GDP ratio.

The stochastic simulation results show that under central bank commitment, output persistence increases compared to the discretionary case. The result is derived using the timeless perspective approach to precommitment by Woodford (1999). Under the timeless perspective, inflation and output persistence reflects the economic data. The fiscal policy is also compatible with the optimal monetary policy from timeless a perspective and the result holds also with alternative fiscal policy rules. The fiscal policy parameter and the target values do not affect the persistence of inflation and output. However, the variability of output increases compared to the discretionary case.

With the deficit rules, the autocorrelation of the tax rate is near unity irrespective of the monetary policy regime, and irrespective of the fiscal policy parameters and targets. Thus we revive Barro's (1979) random walk result with the deficit rules. With the debt rules and the high debt to GDP target values, the Barro result does not hold and the tax rate inherits the stochastic nature of underlying shocks. With the error-correction type of fiscal policy rule, the tax rate changes are smooth as the autocorrelation is near unity with all combinations of the fiscal policy parameter and the deficit to GDP target values.

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Appendix A1 Potential output without distortionary taxes

Let's rewrite the household's budget constraint with lump sum taxation

$$c_t + m_t - (1 - \pi_t)m_{t-1} + b_t \leq (1 + r_{t-1})b_{t-1} + w_t l_t - T_t, \quad (4.1)$$

where T_t is lump sum taxes. Now the household's utility maximisation using the periodic utility function $u(c_t, m_t, l_t; g_t) = \frac{c_t^{1-\sigma}}{1-\sigma} + \frac{\Gamma m_t^{1-\sigma}}{1-\sigma} - \frac{l_t^{1+\lambda}}{1+\lambda} + f(g_t)$ yields a first order condition for the labour supply

$$-l_t^\lambda = -[c_t^{-\sigma} w_t]. \quad (4.2)$$

The real marginal cost to the cost minimising firm is

$$\frac{\partial}{\partial y_t} \left[w_t \left(\frac{y_t}{A} \right) \right] = w_t \frac{1}{A} = mc_t. \quad (4.3)$$

With equilibrium wages $w_t = c_t^\sigma \left(\frac{y_t}{A} \right)^\lambda$ the real marginal cost is

$$c_t^\sigma y_t^\lambda A^{-(1+\lambda)} = mc_t. \quad (4.4)$$

In order to log-linearise (4.4), first substitute in the process for technological progress $A = \Lambda_t e^{\zeta * Time}$ and take natural logarithms. Use the definition $\hat{x}_t = \ln(x_t/\bar{x})$ and substitute in the resource constraint $\hat{c}_t = \frac{\bar{y}}{\bar{c}} \hat{y}_t - \frac{\bar{g}}{\bar{c}} \hat{g}_t$ to yield

$$\left(\sigma \frac{\bar{y}}{\bar{c}} + \lambda \right) \hat{y}_t - \sigma \frac{\bar{g}}{\bar{c}} \hat{g}_t - (1 + \lambda) \hat{\Lambda}_t = \hat{m}c_t. \quad (4.5)$$

In a flexible price equilibrium we get the long-run supply function to look like

$$\ln y_t^* = \frac{\sigma \frac{\bar{y}}{\bar{c}}}{\kappa} \ln g_t + \frac{1 + \lambda}{\kappa} \zeta * Time + \varepsilon_t^{y^*}, \quad (4.6)$$

where y_t^* is the level of flexible price output, which is the desired level of output for the central bank, $\kappa = \left(\sigma \frac{\bar{y}}{\bar{c}} + \lambda \right)$ and $\varepsilon_t^{y^*} = \frac{1+\lambda}{\kappa} z_t$.¹⁰ As we can see from (2.21) and (4.6), the long-run flexible price output and the desired level of output are both hit by the same technology shock (2.3).

¹⁰Note that $z_t \equiv \ln \Lambda_t$.

	ϕ	0.1	0.5	0.9	1.5
	ψ_1	<i>std</i> (<i>corr.</i>)	<i>std</i> (<i>corr.</i>)	<i>std</i> (<i>corr.</i>)	<i>std</i> (<i>corr.</i>)
Inflation	0	0.5047	0.4514	0.4212	0.4226
	—	(0.9878)	(0.9782)	(0.9761)	(0.9741)
	0.6	0.5553	0.4026	0.4010	0.4573
	—	(0.9887)	(0.9756)	(0.9726)	(0.9790)
	1.5	0.6715	0.4506	0.4235	0.4069
	—	(0.9902)	(0.9807)	(0.9753)	(0.9731)
Interest rate	0	0.5289	0.4514	0.4390	0.4226
	—	(0.9793)	(0.9726)	(0.9692)	(0.9652)
	0.6	0.5762	0.4221	0.4192	0.4804
	—	(0.9807)	(0.9665)	(0.9654)	(0.9683)
	1.5	0.6952	0.4714	0.4445	0.4262
	—	(0.9818)	(0.9724)	(0.9599)	(0.9667)
Output	0	1.6657	1.7173	1.6566	1.6850
	—	(0.1875)	(0.1914)	(0.1697)	(0.1593)
	0.6	1.6875	1.6535	1.6305	1.7114
	—	(0.1221)	(0.1675)	(0.1288)	(0.2262)
	1.5	1.6886	1.6862	1.6428	1.6436
	—	(0.1729)	(0.1662)	(0.1615)	(0.1647)
Debt to GDP ratio	0	45.560	10.809	6.1155	4.3282
	—	(0.9959)	(0.9904)	(0.9797)	(0.9465)
	0.6	45.808	9.5861	5.8572	4.8640
	—	(0.9240)	(0.9662)	(0.9762)	(0.9234)
	1.5	54.799	10.767	6.2774	4.4393
	—	(0.7774)	(0.8929)	(0.9746)	(0.7539)
Tax rate	0	4.3275	4.6883	4.4138	4.6699
	—	(0.9950)	(0.9870)	(0.9719)	(0.9147)
	0.6	4.4212	4.3136	4.4687	5.5601
	—	(0.9022)	(0.8768)	(0.8059)	(0.6704)
	1.5	5.4342	5.3802	5.5870	6.5240
	—	(0.7192)	(0.6182)	(0.4685)	(0.0470)

Table 7: Discretionary monetary policy with the debt rule
Note: corr. is the first-order autocorrelation coefficient.

	Ω	0.1	0.5	0.9	1.5
	ψ_2	<i>std</i> (<i>corr.</i>)	<i>std</i> (<i>corr.</i>)	<i>std</i> (<i>corr.</i>)	<i>std</i> (<i>corr.</i>)
Inflation	0	0.7794	0.5291	0.4189	0.4229
	–	(0.9932)	(0.9894)	(0.9801)	(0.9785)
	0.03	0.8580	0.5289	0.3959	0.3901
	–	(0.9910)	(0.9888)	(0.9781)	(0.9762)
	0.1	0.8019	0.5130	0.5292	0.3927
	–	(0.9834)	(0.9877)	(0.9869)	(0.9753)
Interest rate	0	0.8012	0.5523	0.4381	0.4423
	–	(0.9870)	(0.9817)	(0.9709)	(0.9708)
	0.03	0.8773	0.5535	0.4144	0.4097
	–	(0.9848)	(0.9823)	(0.9681)	(0.9660)
	0.1	0.8196	0.5366	0.5523	0.4132
	–	(0.9752)	(0.9810)	(0.9821)	(0.9594)
Output	0	1.6827	1.7118	1.6087	1.6302
	–	(0.1909)	(0.1533)	(0.1326)	(0.1341)
	0.03	1.6832	1.6855	1.6021	1.6675
	–	(0.1423)	(0.2103)	(0.1035)	(0.1663)
	0.1	1.6987	1.6764	1.6866	1.6796
	–	(0.1523)	(0.2090)	(0.2062)	(0.1312)
Debt to GDP ratio	0	148.53	74.916	37.169	24.766
	–	(0.9073)	(0.9898)	(0.9898)	(0.9915)
	0.03	114.93	64.706	32.066	22.272
	–	(0.7975)	(0.9850)	(0.9809)	(0.9867)
	0.1	84.717	62.420	39.342	19.857
	–	(0.5051)	(0.9720)	(0.9788)	(0.9680)
Tax rate	0	3.8454	4.2277	3.6168	3.9187
	–	(0.9658)	(0.9900)	(0.9851)	(0.9844)
	0.03	4.0399	4.0738	3.3538	3.6504
	–	(0.9497)	(0.9882)	(0.9809)	(0.9818)
	0.1	3.7197	4.2073	4.6444	3.7104
	–	(0.9007)	(0.9836)	(0.9874)	(0.9778)

Table 8: Discretionary monetary policy with the deficit rule
Note: corr. is the first-order autocorrelation coefficient.

	Ω	0.1	0.5	0.9	1.5
	ψ_2	<i>std</i> (<i>corr.</i>)	<i>std</i> (<i>corr.</i>)	<i>std</i> (<i>corr.</i>)	<i>std</i> (<i>corr.</i>)
Inflation	0	0.4674	0.4140	0.4352	0.4782
	–	(0.9808)	(0.9735)	(0.9757)	(0.9802)
	0.03	0.4251	0.3737	0.4376	0.4281
	–	(0.9774)	(0.9679)	(0.9755)	(0.9750)
	0.1	0.4796	0.4303	0.4100	0.4069
	–	(0.9805)	(0.9734)	(0.9708)	(0.9709)
Interest rate	0	0.4867	0.4326	0.4512	0.4982
	–	(0.9751)	(0.9678)	(0.9669)	(0.9755)
	0.03	0.4472	0.3915	0.4595	0.4487
	–	(0.9568)	(0.9599)	(0.9698)	(0.9683)
	0.1	0.5036	0.4482	0.4269	0.4271
	–	(0.9623)	(0.9661)	(0.9609)	(0.9585)
Output	0	1.6460	1.6394	1.6189	1.7167
	–	(0.1589)	(0.1340)	(0.1312)	(0.2000)
	0.03	1.6747	1.6865	1.6259	1.7191
	–	(0.1082)	(0.1575)	(0.1759)	(0.1870)
	0.1	1.6737	1.7057	1.6621	1.6883
	–	(0.2108)	(0.1813)	(0.0989)	(0.1696)
Debt to GDP ratio	0	22.773	4.9277	3.0851	2.3736
	–	(0.9937)	(0.9756)	(0.9637)	(0.9583)
	0.03	22.228	4.8715	3.1398	2.1743
	–	(0.9934)	(0.9602)	(0.9314)	(0.8753)
	0.1	24.924	7.0619	5.6452	5.4831
	–	(0.9896)	(0.9117)	(0.8615)	(0.8614)
Tax rate	0	4.7344	4.3898	4.6198	5.2316
	–	(0.9980)	(0.9947)	(0.9925)	(0.9924)
	0.03	4.3486	4.1160	4.8095	4.8055
	–	(0.9964)	(0.9933)	(0.9923)	(0.9894)
	0.1	5.1300	4.7837	4.4812	4.5675
	–	(0.9969)	(0.9936)	(0.9895)	(0.9858)

Table 9: Discretionary monetary policy with the error-correction deficit rule

Note: corr. is the first-order autocorrelation coefficient.

	ϕ	0.1	0.5	0.9	1.5
	ψ_1	<i>std</i> (<i>corr.</i>)	<i>std</i> (<i>corr.</i>)	<i>std</i> (<i>corr.</i>)	<i>std</i> (<i>corr.</i>)
Inflation	0	0.3780	0.2664	0.3368	0.4088
	—	(0.9993)	(0.9989)	(0.9993)	(0.9994)
	0.6	0.3780	0.4790	0.3899	0.2716
	—	(0.9993)	(0.9994)	(0.9991)	(0.9980)
	1.5	1.1061	0.3506	0.4140	0.4772
	—	(0.9993)	(0.9980)	(0.9985)	(0.9981)
Interest rate	0	0.3762	0.2655	0.3313	0.3914
	—	(0.9712)	(0.9505)	(0.9560)	(0.9810)
	0.6	0.3762	0.4619	0.3782	0.2649
	—	(0.9891)	(0.9789)	(0.9813)	(0.9685)
	1.5	1.0428	0.3385	0.4022	0.4595
	—	(0.9957)	(0.9804)	(0.9719)	(0.9862)
Output	0	5.3338	3.7499	4.6285	5.6589
	—	(0.9300)	(0.8590)	(0.9099)	(0.9373)
	0.6	5.3338	6.1079	5.0139	3.9712
	—	(0.9354)	(0.9473)	(0.9154)	(0.8751)
	1.5	9.2104	4.7310	5.5182	6.0799
	—	(0.9470)	(0.8995)	(0.9326)	(0.9433)
Debt to GDP ratio	0	55.867	8.8037	7.3973	4.8565
	—	(0.9967)	(0.9853)	(0.9840)	(0.9607)
	0.6	51.656	11.2068	6.4000	3.9692
	—	(0.9381)	(0.9704)	(0.9769)	(0.9080)
	1.5	79.207	10.593	6.1636	4.6603
	—	(0.7823)	(0.8837)	(0.9669)	(0.8086)
Tax rate	0	5.3254	3.8769	5.4214	5.4925
	—	(0.9965)	(0.9852)	(0.9826)	(0.9490)
	0.6	4.9912	5.1570	4.9484	4.8087
	—	(0.9251)	(0.9201)	(0.8696)	(0.6558)
	1.5	8.0805	5.3370	5.6354	6.9800
	—	(0.7601)	(0.6447)	(0.5486)	(0.2226)

Table 10: Committed monetary policy with the debt rule
Note: corr. is the first-order autocorrelation coefficient.

	Ω	0.1	0.5	0.9	1.5
	ψ_2	<i>std</i> (<i>corr.</i>)	<i>std</i> (<i>corr.</i>)	<i>std</i> (<i>corr.</i>)	<i>std</i> (<i>corr.</i>)
Inflation	0	0.4174	0.4230	0.3455	0.3468
	—	(0.9994)	(0.9995)	(0.9993)	(0.9994)
	0.03	0.3110	0.4737	0.2230	0.3328
	—	(0.9988)	(0.9996)	(0.9984)	(0.9993)
	0.1	0.3112	0.2766	0.2489	0.4017
	—	(0.9989)	(0.9988)	(0.9986)	(0.9994)
Interest rate	0	0.4103	0.4106	0.3349	0.3340
	—	(0.9767)	(0.9759)	(0.9704)	(0.9745)
	0.03	0.3134	0.4595	0.2255	0.3223
	—	(0.9678)	(0.9714)	(0.9163)	(0.9733)
	0.1	0.3124	0.2741	0.2466	0.3907
	—	(0.9628)	(0.9483)	(0.9472)	(0.9627)
Output	0	5.0627	5.2468	4.6996	4.7477
	—	(0.9248)	(0.9274)	(0.9073)	(0.9169)
	0.03	3.6744	5.8583	3.3931	4.6473
	—	(0.8536)	(0.9439)	(0.8247)	(0.9074)
	0.1	4.0660	4.3010	3.7303	5.5021
	—	(0.8660)	(0.8868)	(0.8459)	(0.9225)
Debt to GDP ratio	0	27.826	6.2676	3.5880	2.4566
	—	(0.9959)	(0.9833)	(0.9724)	(0.9615)
	0.03	27.942	9.7789	4.7642	5.0680
	—	(0.9941)	(0.9776)	(0.9330)	(0.9505)
	0.1	27.762	13.558	11.224	13.401
	—	(0.9826)	(0.9348)	(0.9192)	(0.9622)
Tax rate	0	4.5000	4.7386	4.5033	4.2570
	—	(0.9985)	(0.9949)	(0.9924)	(0.9876)
	0.03	4.6680	4.9807	3.8388	4.5649
	—	(0.9979)	(0.9950)	(0.9881)	(0.9887)
	0.1	4.2214	5.0815	3.8992	4.9495
	—	(0.9979)	(0.9943)	(0.9870)	(0.9891)

Table 11: Committed monetary policy with the error-correction deficit rule
Note: corr. is the first-order autocorrelation coefficient.

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