Kauko Karlo

The mixed oligopoly of cross-border payment systems

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The views expressed are those of the author and do not necessarily reflect the views of the Bank of Finland.

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The mixed oligopoly of cross-border payment systems

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Abstract

This paper presents a model depicting cross-border payment systems as a mixed oligopoly. A private net settlement system that maximises profit competes with the central banks' gross settlement system that maximises welfare. It may be optimal for the central bank system to encourage increased use of the private system by charging fees that exceed the marginal cost. The central bank system is not only a competitor but also an essential service provider, because central bank money is needed for net settlement of payments in the private system. In some cases the central bank system can paradoxically induce the private system to charge lower fees by making it expensive to use central bank money for settlement purposes.

Key words: payment systems, network economics, mixed oligopolies

JEL classification numbers: L13, L44, F36, G29

Maidenvälisten maksujärjestelmien sekaoligopoli

Suomen Pankin tutkimus
Keskustelualoitteita 11/2005

Karlo Kauko
Rahapolitiikka- ja tutkimusosasto

Tiivistelmä

Tutkimuksessa esitetään malli, joka kuvaa maidenvälisiä maksujärjestelmiä sekaoligopolina. Yksityinen voittoa maksimoiva nettoselvitykseen perustuva järjestelmä kilpailee keskuspankkien hyvinvointia maksimoivan bruttoselvitysjärjestelmän kanssa. Keskuspankkijärjestelmän voi olla optimaalista lisätä yksityisen järjestelmän käyttöä perimällä rajakustannuksia korkeampia hintoja. Keskuspankkijärjestelmä on paitsi kilpailija myös tärkeä palveluntuottaja, sillä keskuspankkirahaa tarvitaan maksujen nettoselvitykseen yksityisessä järjestelmässä. Perimällä korkeita maksuja keskuspankkirahan käytöstä selvitystarkoituksiin keskuspankkijärjestelmä voi joissain tapauksissa paradoksaalisesti saada yksityisen järjestelmän alentamaan hintojaan.

Avainsanat: maksujärjestelmät, verkostotaloustiede, sekaoligopolit

JEL-luokittelu: L13, L44, F36, G29

Contents

1 Introduction

1.1 High value cross-border payment systems in the euro area

The formation of a European single market highlights the importance of efficient cross-border payment systems. There are two competing yet complementary systems for high value cross-border payments. Central banks run their TARGET system. In this system, euro denominated payments are transferred between participating banks through Eurosystem central banks. A payment is made by debiting the central bank account of the payer and crediting the central bank account of the payee. It is possible to transfer payments to nearly all the major banks in the euro area. TARGET operates on real-time basis, and the execution of a payment takes place in a few minutes, or even seconds, whenever the system is open. It is a gross settlement system; each payment is processed separately, and no kind of netting of payments is used. (See BoF 2002, p. 246, for a more detailed description of the system).

The private Euro1 system is owned and managed by the Euro Banking Association. Payment orders are collected during the day, but payments are not executed continuously. The system is based on end-of-day netting with central bank money. The system has its own account with the ECB, and the net payment to be made or received by a participant is settled between this account and the account of each participant with the national central bank. Each participant either makes one payment to the system or receives one payment from the system at the end of the day, except if either there are no payments to be processed or if incoming and outgoing payments happen to exactly offset each other. Participants have credit and debit caps, and if these limits are violated, payments are not processed but put into an on-hold queue. The number of participants in this payment system is relatively limited, less than one hundred. In Finland, for instance, there were only three participants in January 2005.

TARGET and EURO1 are not perfect substitutes. TARGET is faster and presumably more reliable, but, on the other hand, it is more expensive to use. Despite of the limited number of participating banks, payments through EURO1 outnumber payments through TARGET. There are certain differences between typical payments processed through the two systems. Payments through EURO1, are, on average, somewhat smaller and probably less urgent. (BoF 2004, p. 154)

Previous academic literature on payment systems has not had a particular focus on competition between interbank payment systems. However, there is some analysis on welfare implications of payment system design. Kahn and Roberds (2001) have presented a theoretical analysis on large value gross payment systems. Their model does also include the real economy in an explicit way. The model implies that lack of intraday credit in a gross settlement system causes allocative distortions because agents' transaction possibilities are affected by their inability to make payments. Allocative distortions are not completely avoided unless credit is available with no collateral.

The Eurosystem does not grant intraday liquidity to banks without adequate collateral. The liquidity cost of payment needs, however, is alleviated by the fact that banks are required to hold a certain amount of reserves on their central bank accounts anyway, and these balances can be used as payment mediums. Because the minimum reserve requirement applies to the average of daily balances, a bank may temporarily have a central bank balance that does not satisfy the minimum reserve requirement.

1.2 Public ownership in network industries and mixed oligopolies

Despite of recent privatisations, government owned companies exist in many industries in different parts of the world. Public ownership seems to be particularly commonplace in network industries, such as telecommunications, utilities, airlines and payment systems. This may not be a mere random coincidence. In industries without network effects the allocation of resources normally approaches Pareto optimality if competition is enhanced, making it difficult to argue in favour of government intervention beyond antitrust policies. In network industries, instead, the situation is more complicated, and mere antitrust policies may not lead to socially optimal outcomes. According to Shy (2001) significant economies of scale are one of the typical characteristics of network industries, and it used to be commonplace to let a government controlled company operate as a 'natural monopoly' in such industries. In addition to this example it is possible to mention other kinds of special characteristics of network industries that make it impossible to achieve Pareto optimality simply by enhancing competition. In the presence of network externalities the number of customers using the same product directly enters the utility function of each individual customer, but a normal consumer ignores this externality while making consumption decisions.

In many cases public and private companies co-exist in the same industry. The literature on mixed oligopolies has analysed oligopolistic competition between private and public enterprises and the use of public ownership as a tool of industrial policy. Even though public ownership is particularly commonplace in network industries, there seem to be no theoretical analyses on mixed oligopolies in industries characterised by direct network externalities. In this sense, the following sections contain pioneering work.

The mixed oligopoly approach yields no interesting results unless it is assumed that a public firm is different from private companies. In their review article De Fraja and Delbono (1990) identify three special characteristics of publicly owned firms in previous literature.

1. The objective function may differ from private profit maximisers.
2. Cost efficiency may differ between public and private firms.
3. The public firm has been assumed to be either a Stackelberg leader or a Stackelberg follower.

It has been particularly commonplace to assume that the objective function of the public firm differs from the profit maximisation objective of private firms. One of the earliest contributions in this literature even assumed that the publicly owned firm would try to maximise the total quantity of the commodity produced by the whole industry (Merrill & Schneider 1966). In later contributions the objective function typically includes elements of social welfare, and many authors (de Fraja and Delbono 1989, White 1996, Nishimori & Ogawa 2002) have simply assumed that it is simply the sum of consumer and producer surpluses. Some very sceptical considerations have been presented at least by Sappington and Sidak (2003). Because public firms do not maximise profits and they are not subject to takeover threats, they might pursue expansion in order to satisfy managers' personal ambitions. This might lead to socially harmful practices, such as pricing below marginal costs and lobbying for restrictions that would hamper rivals' operations.

It has often been assumed that the public firm suffers from an exogenously given cost inefficiency. In many settings this assumption is the only way to avoid the trivial solution that the best policy is to nationalise the whole industry. This assumption has been justified by the hypothesis that the managers of publicly owned firms may receive conflicting instructions because of conflicts of interest among politicians and civil servants, and because the government objectives may change after general elections. (Aharoni 1982) Matsumura and Matsushima (2004) have suggested that a profit maximising firm could have incentives to invest excessively in cost-reducing activities because of strategic motivations. However, not all the empirical research indicates that public ownership would be automatically related to weaker efficiency (See Willner 1994 and Willner 2001, p. 734–736).

The public firm could be either a Stackelberg leader or a Stackelberg follower. De Fraja and Delbono (1989) demonstrated that if the public firm maximises social welfare in normal Cournot competition with identical products, it produces an output where the price of the good equals its marginal cost. However, if the public firm is a Stackelberg leader, its price turns out to be higher than the marginal cost. This outcome has clear analogies with some findings of the model to be presented in the following.

2 Assumptions of the basic model

The following model has to a large extent been inspired by the current situation in the euro area. There are two identical countries. Each country is inhabited by m bank customers. In each country there is a large number of competing banks. These banks have no market power. They may offer many kinds of financial services, but this paper analyses nothing but the provision of cross-border payments.

There are two cross-border payment systems. One of these systems is run by central banks trying to maximise welfare. The other one is jointly owned by all the banks in the two countries and it maximises the sum of profits made by the system itself and its shareholder banks. All the banks must be members of both systems. Both banks and payment systems use prices as decision variables. Prices cannot be negative. As in many previous contributions on mixed oligopolies, the public system is assumed to be the Stackelberg leader and to make its decisions first.

Each bank has an account with its national central bank. Banks' own payment system has an account in the system of the central banks. Banks cannot intermediate customers' cross-border payments without using either of the two payment systems. From the point of view of customers, the systems are not perfect substitutes.

Each bank has a large number of customers. The total number of customers in one country is m. Each bank is specialised in the sense that it has only one type of customers. The customers of a bank are completely identical, and all of them always have similar needs to initiate payments, and they always receive similar payments from abroad. The willingness to pay for the service varies between customers of different banks, but not between customers of the same bank. All the payments are of the same size. The sum of net utility to all the customers in both countries from using these payment services is

$$U = 2m\left[b_1(n_c + n_p) - b_2 n_c^2 - b_2 n_p^2 - a n_p n_c + a(n_p^2 + n_c^2)/2 - n_c p_c - n_p p_p\right] \quad (2.1)$$

where a, b_1 and b_2 are exogenous parameters ($a > 0$, $b_1 > 0$, $b_2 > 0$, $s > 0$), n_p (n_c) is the relative share of customers who make payments through the private (central bank) system; $0 \le n_p \le 1$; $0 \le n_c \le 1$. The fee for payments through the central bank system charged by banks is denoted p_c, and the fee for sending payments through the private system is denoted p_p. The expression in brackets in the formula (2.1) is the utility per customer, m the number of customers in one country and 2 the number of countries. Because payments through the two systems are likely to be substitutes but they cannot be complements, $a \ge 0$. However, if $n_c = n_p$, the value of a does not directly affect the level of utility.

Even though aggregate demand for payment services is known beforehand, it is not possible to say ex ante which bank has a clientele that actually makes payments, and which one is going to get payments.

One could argue that the utility function is unrealistic in the sense that in real life the payee does also benefit from the functioning of the payment system. On the other hand, whatever the transactions that create the need for payments are, it is always possible to adjust the terms of the transactions so that payees' needs are fully reflected in the demand for services. For instance, if the costs of making cross-border payments are very high, it is possible that some importers stop buying foreign goods, but if these buyers are truly important customers for sellers, they would offer discounts in order to lower the total cost for foreign customers.

The central bank system is a real time gross settlement (RTGS) system not based on netting. The operational cost of the central bank system is $2mcn_c^2$. Hence, the marginal cost of transactions is increasing. Sending a payment through it causes a liquidity cost ($\alpha \geq 0$) to the payer's bank because there is no mechanism that would match incoming and outgoing payments. Both banks need to reserve the required liquidity. This cost is mainly due to the fact that the bank must hold a large balance on its account with the central bank, acquire eligible securities to be used as collateral in order to acquire intraday liquidity and possibly acquire non-collateralised short-term funding from expensive private sources.

The private system, instead, is based on netting. If the customers of a bank both send and receive payments, outgoing and incoming payments offset each other, and the bank neither makes nor receives payments when these payments are settled. If there are outgoing payments but no incoming payments, the bank has to pay the net amount to the account of the private system through the central bank system. If, instead, there is nothing but incoming payments, the bank receives the difference from the private system through the central bank system. The operational cost of the private system is $2mcn_p^2$. Hence, the private and the public system are equally cost efficient, and the private system does also have increasing marginal costs. It would be possible to let the cost parameters differ between the public and the private system, which has been commonplace in previous models on mixed oligopolies, but this would make the analysis more complicated without major impact on results.

Both payment systems may also have some fixed costs; it might be unrealistic to assume that the systems have no other costs but the above mentioned increasing marginal cost. However, we can simply ignore these fixed costs because they do not affect optimal decisions.

Because of simplicity it is assumed that banks have no other costs than the fees charged by the two payment systems and the liquidity costs.

The sequential order is the following.

1) Central banks decide the fee per transaction (β_c) for using the central bank cross-border payment system. This decision is observed by all the agents.
2) The private system sets the fee (β_p) for sending a payment through the private system. This fee is observed by all the agents.
3) Banks set their fees for cross-border payments. These fees are observed by all the agents.
4) Customers observe prices and their transaction needs. Customers initiate payments.

No customer can initiate or receive more than one payment through the private system and another through the central bank system. It is not known ex ante who is going to get or initiate a cross-border payment, even though all the data to calculate the future magnitude of the flow of payments is observable after the stage 3.

Because the utility yielded by the use of a payments system depends above all on the use of the system itself rather than on the use of the competing system, and because in any interesting internal point solution the Pareto optimal use of both systems would be positive, it is assumed that the b parameters are large relative to other parameters of the model. To be more precise, the following limitations are imposed on parameter values; $b_2 > 3a/2$; $b_1 > a$; $b_2 > \alpha+c$; $b_2 > 2\alpha$; $b_1 > \alpha+c$. If these conditions are not satisfied, it is not possible to find a meaningful equilibrium where payment systems' second order conditions are satisfied, the private system prefers to stay in the market and customers make positive amounts of payments through both systems.

3 Solving the model

3.1 Profit maximising pricing of the private system

Customers maximise their utility (2.1)
$\partial U/\partial n_c = 0$
$\partial U/\partial n_p = 0$

$$\Rightarrow \quad \begin{aligned} n_c &= [2b_2(b_1 - p_c) - a(2b_1 - p_c - p_p)]/[4(b_2 - a)b_2] \\ n_p &= [2b_2(b_1 - p_p) - a(2b_1 - p_c - p_p)]/[4(b_2 - a)b_2] \end{aligned} \qquad (3.1)$$

Banks' pricing of payments through the central bank system is straightforward. It is simply the fee charged by central banks plus the liquidity cost of the bank itself.

$$p_c = \beta_c + \alpha \tag{3.2}$$

Pricing of payments made through the private system is more complicated. Because of netting, it is possible that the bank does not have to incur the cost of liquidity. If payments can be netted, the bank does not have to incur the liquidity cost of making a payment to the account of the private system through the central bank system. The likelihood of customers not receiving a payment through the private system is simply $(1-n_p)$ and the expected value of the respective liquidity cost is $\alpha(1-n_p)$. Because the number of customers of a bank is very large, the fee charged by the central bank for the final settlement in central bank money is negligible relative to the number of bank customers; the fee paid by a bank for the use of central bank money is now ignored. An analysis on this fee will be presented in the section 4.2. It follows that

$$p_p = \beta_p + \alpha(1-n_p) \tag{3.3}$$

Combining (3.1) and (3.3) yields

$$p_p = \frac{\left[2b_2(-\alpha b_1 + 2\alpha b_2 + 2b_2\beta_p) - a\left\{\alpha^2 + 4b_2\beta_p + \alpha(4b_2 - 2b_1 + \beta_c)\right\}\right]}{\left[a(\alpha - 4b_2) - 2(\alpha - 2b_2)b_2\right]} \tag{3.4}$$

By combining (3.1), (3.2) and (3.4), the profit of the private system can be calculated to equal

$$\Pi = 2mn_p\beta_p - 2mcn_p^2 =$$

$$= 2m\frac{\left\{(2\alpha - 2b_1 + \beta_c + \beta_p)a - 2b_2(\alpha - b_1 + \beta_p)\right\}\left[2b_2\left\{2b_2\beta_p + (\beta_p - b_1)c + \alpha(c - \beta_p)\right\} - a\left\{4b_2\beta_p - \alpha(\beta_p - 2c) + (\beta_c + \beta_p - 2b_1)c\right\}\right]}{\left[2(2b_2 - \alpha)b_2 - a(4b_2 - \alpha)\right]^2} \tag{3.5}$$

Because the owners of the private system are banks with no market power, they are unable to make any profits themselves. Therefore, the objective function of the private system reduces to its own profit. Park & Ahn (1999) have presented a detailed analysis on jointly owned upstream suppliers as a tool of collusion. The first order condition of profit maximisation ($\partial\Pi/\partial\beta_p = 0$) yields[1]

[1] Second order condition $\partial^2\Pi/\partial\beta_p^2 = -4m(2b_2-a)[(2b_2-a)(b_2+c-\alpha)+(2b_2-3a)b_2]/[2b_2(2b_2-\alpha)-a(4b_2-\alpha)]^2$; This is negative because $a \le 2b_2/3$ and $b_2 > 2\alpha$.

$$\beta_p = \frac{\left[2(b_1-\alpha)b_2 - a(2b_1 - 2\alpha - \beta_c)\right]\left[2b_2\{2b_2 + 2c - \alpha\} - a(4b_2 - \alpha + 2c)\right]}{\left[2(2b_2 - a)\{2b_2(2b_2 + c - \alpha) - a(4b_2 - \alpha + c)\}\right]} \tag{3.6}$$

This functional form has intuitive characteristics. The fee charged by the private system does not depend on the fee charged by the central bank system if the two systems are not substitutes ($a = 0 \Rightarrow \partial\beta_p/\partial\beta_c = 0$). It is possible to demonstrate that if the systems are substitutes ($a > 0$), high fees charged by the central bank system give the private system more freedom to charge high prices.

$$\frac{\partial\beta_p}{\partial\beta_c} = -\frac{\left[\dfrac{2am\{(b_2 - a)(4b_2 + 4c - 2\alpha) + a(2c - \alpha)\}}{\{2b_2(2b_2 - \alpha) - a(4b_2 - \alpha)\}^2}\right]}{\left[\dfrac{\partial^2\pi}{\partial\beta_p^2}\right]} \tag{3.7}$$

Because $(b_2 - a)(4b_2 + 4c - 2\alpha) + a(2c - \alpha) > 0$ even when a has its maximum value ($a = 2b_2/3$) and because $\partial^2\pi/\partial\beta_p^2 < 0$, it follows that in any meaningful case $\partial\beta_p/\partial\beta_c > 0$, which is intuitive and a typical result in Bertrand competition.

3.2 Optimal pricing by central banks

When the central bank system sets its fee, it understands that the fee will affect the allocation of resources in the payment systems in two different ways, directly and through its impact on the fee charged by the private system. As implied by (3.6), the central bank system can affect the pricing of the private system. Because the fee charged by the private system exceeds the marginal cost, central banks have an incentive to try to affect the use of the private system, and the only available tool is the fee charged by central banks themselves. However, as will be seen, the optimal policy is not to charge low fees in order to force the private system to cut its own fees.

The optimisation condition is

$$\frac{d\psi}{d\beta_c} = \frac{\partial\psi}{\partial\beta_c} + \left(\frac{\partial\psi}{\partial\beta_p}\right)\left(\frac{\partial\beta_p}{\partial\beta_c}\right) = 0 \tag{3.8}$$

where ψ is the sum of consumer surpluses and payment systems' profits. Unsurprisingly, if the central bank system cannot affect the behaviour of the private system because $a = 0$ the formula for β_c implies marginal cost pricing $\beta_c^* = 2n_c c = c(b_1 - \alpha)/(b_2 + c)$. Nevertheless, it is more interesting to analyse cases where the two systems are substitutes ($a > 0$). In principle, it is possible to calculate the closed form solution to the optimal value of the central bank fee. There is only one value of β_c that satisfies the optimisation conditions. The result is presented in the mathematical appendix 1. This formula for β_c is complicated, and probably not very condusive to our understanding of the optimal central bank pricing. However, the formula is of interest because it implies that β_c has a unique real root. It is possible to analyse optimal central bank pricing without using this formula.

Result; If a > 0, then a welfare maximising central bank system prices above the marginal cost.

Proof If β_p is treated as a function of β_c, the equation (3.8) can also be written as

$$\frac{d\psi}{d\beta_c} = \left(\frac{\partial\psi}{\partial n_p}\right)\left(\frac{\partial n_p}{\partial\beta_c}\right) + \left(\frac{\partial\psi}{\partial n_c}\right)\left(\frac{\partial n_c}{\partial\beta_c}\right) = 0 \tag{3.9}$$

By applying the formulas (3.6), (3.4) and (3.1), we can calculate that

$$\frac{\partial n_p}{\partial\beta_c} = \frac{a}{\left[4b_2(2b_2 + c - \alpha) - 4a\{2b_2 + (c+\alpha)/2\}\right]} > 0 \tag{3.10}$$

$$\frac{\partial\psi}{\partial n_p} = 2m\left[b_1 - an_c + an_p - 2b_2n_p - 2cn_p + \alpha(2n_p - 1)\right]$$

By applying the formulas (3.1), (3.2), (3.4) and (3.6) this can be rewritten as

$$\frac{\partial\psi}{\partial n_p} = 4m(b_2 - a)\frac{\left[2b_2(b_1 - a) - a(2b_1 - 2\alpha - \beta_c)\right]}{\left[(2b_2 - a)\{2b_2(2b_2 + c - \alpha) - a(4b_2 + c - \alpha)\}\right]}$$

Because in any meaningful case where customers prefer to use payment services of the central bank system $p_c = \beta_c + \alpha < b_1$, and because of the assumptions on parameter values, it follows that $\partial\psi/\partial n_p > 0$. Because $d\psi/\partial n_p > 0$ and because of (3.10) it follows that $(\partial\psi/\partial n_p)(\partial n_p/\partial\beta_c) > 0$.

If the public system prices at marginal cost, then $\partial\psi/\partial n_C = 0$ because there is no network externality in the public system. It follows that the formula (3.9) cannot hold because the sum of a positive term and zero cannot equal zero. Therefore, the equation (3.9) holds with a value of β_c

that is characterised by $(\partial\psi/\partial n_C)(\partial n_C/\partial\beta_c) < 0$, which cannot be satisfied unless $\partial\psi/\partial n_C > 0$, which is the case when the central bank system prices above its marginal cost.
Q.E.D.

The private profit maximising system does also charge a fee that is higher than the socially optimal fee, and by increasing its own fee the central bank system encourages the private system to increase its fee further. However, such a pricing policy can paradoxically improve social welfare. This odd result has the following explanation. Because the private system charges a fee that is higher than the socially optimal fee, customers do not use the private system as much as they should from the point of view of resource allocation ($d\psi/\partial n_p > 0$). Therefore, it is welfare improving to encourage customers to increase the use of the private system by making the public system more expensive. This would not eliminate monopoly pricing. In fact, it would make monopoly pricing worse, but its adverse consequences would be alleviated. The fee of the central bank system is probably relatively close to the marginal cost, implying that a decrease in the use of the system has a very weak marginal impact on the net welfare yielded by the public system.

This result has clear analogies with the old finding that subsidising a monopoly instead of taxing it can induce a Pareto improvement because the subsidy would encourage it to increase output. (See Guesnerie and Laffont 1978.) The analogies with the result of de Fraja and Delbono (1989) may be even clearer; in their model a welfare maximising Stackelberg leader in a Cournot oligopoly produces an amount that leads to an outcome where the price of the good is higher than the marginal cost; if the output were so large that the price were equal to the marginal cost its profit maximising rivals would produce much less than in the social optimum. As seen above, this finding is robust even if one assumes Bertrand competition.

The outcome of the model is mathematically complicated, but the values of β_p, n_c and n_p can be solved explicitly. (See mathematical appendix 2.)

3.3 Welfare maximising pricing of the private system

The profit maximising system prices according to (3.6), but socially optimal pricing can be analysed as a hypothetical case. The sum of surpluses (Ψ) equals the difference of consumer gross utility and the total costs of processing the payments, including costs to the two systems and the participating banks.

$$\Psi = 2m\big[b_1(n_c + n_p) - b_2 n_c^2 - b_2 n_p^2 - a n_p n_c + a(n_p^2 + n_c^2)/2 \\ - n_c^2 c + n_p^2 c + n_p(1 - n_p)\alpha + n_c \alpha\big] \tag{3.11}$$

When (3.1) is substituted for n_c and n_p, (3.2) for p_c, and (3.4) for p_p, in the special case where the two systems are not substitutes ($a = 0$) the first order condition of socially optimal pricing by the private system ($\partial\Psi/\partial\beta_p = 0$) implies[2]

$$\beta_p^* = \left(c - \frac{\alpha}{2}\right)\frac{(b_1 - \alpha)}{(b_2 + c - \alpha)} \tag{3.12}$$

Interestingly, the marginal cost for the private system differs from this socially optimal hypothetic fee. The marginal cost equals $2cn_p = c(b_1 - \alpha)/(b_2 - \alpha + c)$, which is greater than β_p^* whenever $\alpha > 0$. If the private system maximised social welfare, it would price below the marginal cost. If the liquidity cost effect is extreme ($\alpha \geq 2c$), the socially optimal fee to be charged by the private system would be 0. This result is due to the network externality of the liquidity cost effect. By sending payments through the private system the payer creates a positive externality that benefits the bank of the payee and its customers. Therefore, it is welfare maximising to encourage would-be customers to send payments through the private system. If there is no liquidity cost externality, this effect vanishes.

The result does also apply to cases where the two systems are substitutes ($a > 0$). When both systems have the same objective function, the sequential order of pricing decisions is irrelevant. If $\partial\Psi/\partial\beta_p = 0$ and $\partial\Psi/\partial\beta_c = 0$, it follows that[3]

$$\beta_p^* = \frac{\left[(b_1 - \alpha)(2c - \alpha)(b_2 + c - a)\right]}{\left[2(b_2 + c - \alpha)(b_2 + c) - 2a\left(b_2 + c - \frac{\alpha}{2}\right)\right]} \\ \beta_c^* = \frac{\left[2(b_1 - \alpha)(b_2 + c - a - \alpha)\right]}{2(b_2 + c - \alpha)(b_2 + c) - 2a\left(b_2 + c - \frac{\alpha}{2}\right)} \tag{3.13}$$

Because of 3.1, 3.2 and 3.4, it follows that

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[2] Second order condition $\partial^2\Psi/\partial\beta_p^2 = -4m[4b_2^2(b_2-\alpha+c)-2ab_2(3b_2+2c-2\alpha)+a^2(2b_2+2c-\alpha)]/[2b_2(2b_2-\alpha)-a(4b_2-\alpha)]^2$; This is negative because $a \leq 2b_2/3$ and $b_2 > 2\alpha$.

[3] Second order condition
$d^2\psi/d\beta_c^2 = 2m[a(2b_2-\alpha)(6b_2+4c-\alpha)-2(2b_2-\alpha)^2(b_2+c)-2a^2(2b_2+2c-\alpha)]/[2b_2(2b_2-\alpha)-a(4b_2-\alpha)]^2$
$d^2\psi/d\beta_p^2 = 4m[-4b2_2(b_2+c-\alpha)+2ab_2(3b_2+2c-2\alpha)+a^2(\alpha-2b_2-2c)]/[2b_2(2b_2-\alpha)-a(4b_2-\alpha)]^2$
Both expressions are negative because by assumption $a < 2b_2/3$ and $\alpha < b_2/2$.

$$\Rightarrow \beta_p - 2n_p c = -\frac{[\alpha(b_1 - \alpha)(b_2 + c - a)]}{\left[2(b_2 + c - \alpha) - 2a\left(b_2 + c - \frac{\alpha}{2}\right)\right]}$$

If the liquidity cost effect exists ($\alpha > 0$), it follows that $\beta_p - 2n_p c$, is negative, and the marginal cost of the private system exceeds the optimal private fee.

Hence, a welfare maximising private system would price below the marginal cost even when $a > 0$. If the private system prices according to (3.13), the central bank system, instead, prices at the marginal cost ($\beta_c = 2n_c c$), which is intuitive in the absence of the liquidity related network externality.

The result (3.13) implies that the welfare maximising public enterprise has a very challenging task in an industry characterised by network externalities. If it wants to implement the first best outcome, it should induce its profit maximising rival to price below marginal cost, which would often be impossible, not least because of the participation constraint of the private undertaking. If the private system has either constant or declining marginal costs and possibly some fixed costs, it would be nearly impossible to implement the first-best outcome because in the optimum the private rival would make losses and prefer to exit.

4 Central banks as service providers of the private system

4.1 Fees to be paid by the private system

The central bank system is not only a competitor of the private system. It is also an essential service provider. Until now, this issue has been ignored. The central bank system has the possibility to impose fees on both participating banks and the private payment system. Such fees would undoubtedly have an impact on the fees charged by the private agents. The impact of fees to be paid by the private system will be analysed in this section.

Let β_n denote the fee the private payment system has to pay for each payment it sends to a participating bank when payments through the private system are settled on net basis with central bank money. The total fee burden to the private payment system is

$$2m\beta_n(1 - n_p)n_p \tag{4.1}$$

And if the relative number of payments in the private system increases, the respective marginal cost is

$$(1-2n_p)\beta_n \tag{4.2}$$

Interestingly, this is negative whenever $n_p > ½$. Hence, this central bank fee can paradoxically lower the private marginal cost of payments in the private system whenever the use of the private system is intense. The fee would certainly increase the average cost of the private system, but the marginal cost is more relevant to pricing and economic efficiency. Hence, it is reasonable to analyse whether there can be cases where the misallocation of resources caused by monopoly pricing of the private system can be avoided by imposing a special fee for using central bank money for settlement purposes. As seen in (3.13), Pareto optimality requires that the private system prices below its marginal cost.

The profit of the private system will be

$$\Pi = 2m[n_p(\beta_p - cn_p) - n_p(1-n_p)\beta_n] \tag{4.3}$$

When p_p is determined by (3.4), p_c by (3.2) and n_c and n_p by (3.1), the first order condition of profit maximisation yields[4]

$$\frac{\partial\Pi}{\partial\beta_p} = 0 \Rightarrow$$

$$\beta_p = \frac{\begin{array}{l}[4b_2^2[\alpha^2 + 2b_2\beta_n + \alpha(\beta_n - b_1 - 2b_2 - 2c) + b_1(2b_2 - 2\beta_n + 2c)] \\ + a^2[2\alpha^2 - 4b_2\beta_c - 4b_2\beta_n + 2\beta_c\beta_n + \alpha(\beta_c + 3\beta_n - 2b_1 + 8b_2 - 4c) \\ - 2\beta_c c + b_1(8b_2 - 4\beta_n + 4c)] \\ - 2ab_2[3\alpha^2 - 2b_2\beta_c + 6b_2\beta_n + 2\beta_c\beta_n + \alpha(\beta_c + 4\beta_n - 3b_1 - 8b_2 - 6c) \\ - 2\beta_c c + b_1(8b_2 - 6\beta_n + 6c)]]\end{array}}{\left[2(2b_2 - a)\{a(\alpha - 4b_2 + \beta_n - c) - 2b_2(\alpha - 2b_2 + \beta_n - c)\}\right]} \tag{4.4}$$

Now, it is essential to know whether the central bank system can create a situation where this profit maximising fee leads to the Pareto optimum. The Pareto optimal allocation of resources is characterised by

[4] The second order condition is $\partial^2\Pi/\partial\beta_p^2 = -4(2b_2-a)[2b_2(2b_2+c-\beta_n-\alpha)-a(4b_2+c-\alpha-\beta_n)]/[2(2b_2-\alpha)-a(4b_2-\alpha)]^2$; This is negative if $\beta_n < [(2b_2-a)(2b_2+c-\alpha)-2b_2a]/(2b_2-a)$.

$$\frac{\partial\Psi}{\partial n_c} = 0 \qquad \frac{\partial\Psi}{\partial n_p} = 0$$

$\Rightarrow$

$$n_c^* = (b_1 - \alpha)\frac{(b_2 + c - a - \alpha)}{\left[2(b_2 - \alpha + c)(b_2 + c) - a(2b_2 + 2c - \alpha)\right]} \tag{4.5}$$

$$n_p^* = (b_1 - \alpha)\frac{(b_2 + c - a)}{\left[2(b_2 - \alpha + c)(b_2 + c) - a(2b_2 + 2c - \alpha)\right]}$$

If n_c and n_p are determined according to (3.1), p_c according to (3.2), p_p according to (3.4) and β_p according to (4.4), it follows that the conditions (4.5) are satisfied with only one combination of β_c and β_n, namely

$$\beta_c = 2n_c c = \frac{[2(b_1 - \alpha)(b_2 - a - \alpha + c)c]}{\left[2(b_2 + c)(b_2 + c - \alpha) - a(2b_2 + 2c - \alpha)\right]} \tag{4.6}$$

$$\beta_n = \frac{\left[4b_2(b_1 - \alpha)(b_2 - \alpha)(b_2 + c - a)\right]}{[(2b_2 - a)2(b_1 - b_2 - c)(b_2 - c) + a(2(b_2 - b_1 + c) + \alpha)]}$$

As the formula for β_c implies, this first best solution is characterised by marginal cost pricing for payments through the central bank system. The first best solution must be characterised by below marginal cost pricing by the private system, as implied by (3.12) and (3.13).

It is rather reasonable to believe that the central bank system does not subsidise the private system for sending payments through the central bank system. Therefore, meaningful cases are characterised by $\beta_n > 0$. The formula (4.6) for the fee β_n implies positive values if

$$b_1 > \frac{[2(b_2 + c)^2 + a(2b_2 + 2c + \alpha)]}{[2(b_2 + c - a)]} \tag{4.7}$$

This result is easy to understand intuitively; if the parameter b_1 has a relatively high value, it is likely that the number of payments will be large enough for the paradoxical situation that an additional fee by the central bank lowers the marginal cost of the private system and induces it to decrease its price. (See 4.2)

The optimal combination of fees (4.6) cannot be used to implement the first best outcome unless a number of criteria are satisfied. For instance, the formula (4.4) must lead to be the maximum of profits, not the maximum of losses or the minimum of profits. The combination of payment volumes characterised by (4.5) must be the maximum of social welfare, not the minimum. It is far from obvious that cases satisfying all the necessary criteria do exist, and many of the second order conditions are too complicated to be analysed. The criteria are listed and described in the mathematical appendix 3. However, it is easy to prove with numerical examples that meaningful internal point solution cases do exist.[5]

4.2 An irrelevance result

In the section 4.1 it was demonstrated that whenever more than half of the would-be payers actually send payments through the private system, a fee for using central bank money for the net settlement of these payments will paradoxically induce the private system to cut its own fee. It was assumed that these fees are paid by the private system. However, the central bank can also impose fees on participating banks. If a participating bank has to pay a fee (β_b) to the central bank system, its profit from intermediation of payments through the private system will be

$$p_p - \beta_p - (1 - n_p)(\alpha + \beta_b) = 0 \tag{4.8}$$

By applying the formulas (3.1), (3.2) and (4.8), we can calculate that the competitive equilibrium price for payments through the private system will be

$$p_p = \frac{\begin{array}{c} 2b_2\{2b_2(\beta_b + \beta_p) - \alpha(2b_2 - b_1) - b_1\beta_b\} \\ - a\{\alpha^2 - 2b_1\beta_b + \beta_b\beta_c + 4b_2(\beta_p + \beta_b) + \alpha(4b_2 + \beta_b - 2b_1 + \beta_c)\} \end{array}}{2b_2(2b_2 - \alpha - \beta_b) - a(4b_2 - \alpha - \beta_b)} \tag{4.9}$$

When the private system optimises, it sets its fee according to the condition

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[5] For instance, if $b_1 = 2.1$, $b_2 = 1$, $a = 0.1$, $c = 0.1$ and $\alpha = 0.05$, it follows that $n_p = n_p^* \approx 0.978$, $n_c = n_c^* \approx 0.930$, $\beta_c \approx 0.673$, $\beta_p \approx 0.147$ and $\beta_n \approx 1.937$; All the second order conditions are satisfied in this point $\partial^2\Pi/\partial\beta_c^2 \approx -0.009m < 0$; $\partial^2\Psi/\partial n_c^2 \approx -4.2m < 0$; $\partial^2\Psi/\partial n_p^2 \approx -4.00m < 0$; The private system will make a positive profit; $\Pi \approx 0.014m > 0$.

$$\frac{\partial \Pi}{\partial \beta_p} = 0 \Rightarrow$$

$$\beta_p = \frac{\begin{aligned}&[4b_2^2[\alpha^2 - 2b_2\beta_b + \beta_b^2 + 2b_2\beta_n + \beta_b\beta_n + \alpha(2\beta_b - b_1 - 2b_2 + \beta_n - 2c)\\ &- 2\beta_b c + b_1(2b_2 - \beta_b - 2\beta_n + 2c)]\\ &+ a^2[2\alpha^2 - 4b_2\beta_b + \beta_b^2 - 4b_2\beta_c + \beta_b\beta_c + 4b_2\beta_n + \beta_b\beta_n + 2\beta_c\beta_n\\ &+ \alpha(3\beta_b - 2b_1 - 8b_2 + \beta_c + 3\beta_n - 4c) - 2\beta_b c - 2\beta_c c\\ &+ b_1(8b_2 - 2\beta_b - 4\beta_n + 4c)]\\ &- 2ab_2[3\alpha^2 - 6b_2\beta_b + 2\beta_b^2 - 2b_2\beta_c + \beta_b\beta_c + 6b_2\beta_n + 2\beta_b\beta_n + 2\beta_c\beta_n\\ &+ \alpha(\beta_c - 3b_1 - 8b_2 + 5\beta_b + 4\beta_n - 6c) - 4\beta_b c - 2\beta_c c\\ &+ b_1(8b_2 - 3\beta_b - 6\beta_n + 6c)]]\end{aligned}}{[2(2b_2 - a)\{a(\alpha - 4b_2 + \beta_b + \beta_n - c) - 2b_2(\alpha - 2b_2 + \beta_n + \beta_b - c)\}]} \tag{4.10}$$

When p_p is determined according to (4.9) and (4.10) is substituted for β_p, it follows that

$$\frac{\partial p_p}{\partial \beta_n} = \frac{\partial p_p}{\partial \beta_b} = \frac{2(b_2 - a)b_2[2b_2(2b_2 - b_1 + c) - a(4b_2 - 2b_1 + \beta_c + c + \alpha)]}{[2b_2(2b_2 + c - \alpha - \beta_b - \beta_n) - a(4b_2 + c - \alpha - \beta_b - \beta_n)]^2} \tag{4.11}$$

Therefore, whenever the central bank system increases the fee to be paid by a bank sending payments to the private system by one cent and decreases the fee to be paid by the private system by one cent, the net impact of this on the fee paid by the end customer is zero. Thus, from the point of view of allocational efficiency, fees paid by payers' banks and fees paid by the private system are perfect substitutes in central bank decision making. Nothing but the sum of β_n and β_b matters to allocational efficiency, and all the combinations of these two fees leading to the desired total fee level $\beta_n + \beta_b$ are equally optimal.

On surface, this result may seem surprising, but the intuition is extremely simple. The fee burden on the payment flows is the same irrespective of whether the fees are paid by the payer bank or the payment system. The total fee burden is $2m(\beta_n + \beta_b)n_p(1 - n_p)$, and any given fee burden can be implemented with an extremely large number of combinations of β_n and β_b. Because banks make no profits anyway, there cannot be distributional effects between the private system and participating banks. In principle, there might be distributional effects between the private system and customers, but at least with these functional forms none exist.

5 Conclusions

This paper presents a model on a mixed duopoly of cross-border payment systems. There is a central bank gross payment system and a private system based on netting of payments. The private system needs central banks for settling payments on net basis with central bank money. The difference between gross and net payment systems is of importance because a gross payment system ties up more liquidity, which causes costs to participating banks. The central bank system differs from the private system not only because it is a gross payment system; moreover, it is a Stackelberg leader and it maximises social utility whereas the private system maximises profits.

One of the main findings is that if the only type of fee the central bank is able to charge is the fee paid by banks that intermediate customers' payments to be settled in the central bank system, it is welfare maximising to charge a fee that exceeds the marginal cost. This result is due to the fact that the profit maximising private system uses its market power and charges fees higher than the marginal cost, leading to sub-optimal use of the private system. Therefore, it is optimal for the central bank to encourage the use of the substituting private system. This result has some analogies with the classical finding that it may be socially optimal to subsidise a monopoly in order to encourage it to increase output (Guesnerie and Laffont 1978). Analogies with the result of de Fraja and Delbono (1989) are even clearer: in Cournot competition a Stackelberg leader public firm sells its products at a price higher than the marginal cost because this increases the use of the privately produced good.

Paradoxically, it is possible to construct examples where the central bank system should charge a fee to be paid either by the private system or the participating banks for the use of central bank money in the private net settlement system. Surprisingly, in these cases the fee to be paid by the end customer is lower than what it would be if the central bank system offered settlement services for free. These cases are characterised by a relatively large amount of payments through the private system, implying that netting can be used at a wide scale, and the fee charged by the central bank does not have to be paid very often. The central bank fee, however, can affect the profit maximising fee of the private system, and the latter effect may dominate. The result would probably not hold with a substantially different demand for payment services. If, for instance, the demand for payment services is very skewed during a day, and there is a strong yet price elastic demand for payments from country A to country B but hardly any demand for payments in the opposite direction, it is probably very difficult to construct examples where a high fee for using central bank money for net settlement purposes would decrease the marginal cost of the private system.

This model includes an explicit network externality; by sending payments through the private system a payer helps the bank of the payee to save on liquidity costs. In real life, another kind of network effect is probably rather relevant at least to the Euro1 system. In the above model, it was assumed that all the banks are automatically members in both systems, which is not true in the case of Euro1. Because not all the banks in the Euro area participate in the system, the positive network externality on participating banks should be taken into account when one evaluates the social benefits of the decision of a new member to join the system.

The central bank could not implement the first best solution if the private system had constant or declining marginal costs. Because in the optimum the fee charged by the private system is lower than its marginal cost, the marginal cost must be higher than the average cost; otherwise the private system would not voluntarily enter. It may not be necessary that the marginal cost curve of the private system is monotonically increasing. It should be enough if the marginal cost is higher than the average cost with the prevailing volume of payments.

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Appendix 1

The welfare maximising value of β_c, when there are no fees for the use of central bank money in the net settlement of payments of the private system, equals

$$\beta_c^* = 2(b_1 - \alpha)\frac{\begin{gathered}[a^4(c-2b_2) + 8b_2^2c(2b_2 + c - \alpha)^2 + a^3\{8b_2^2 - 13b_2c + 3(\alpha - c)c\} \\ + 4ab_2\{b_2^2 - 18b_2^2c + 13b_2(\alpha - c)c - 2(\alpha - c)^2c\} \\ + 2a^2\{-5b_2^3 + 26b_2^2c - 12b_2(\alpha - c)c + (\alpha - c)^2c\}]\end{gathered}}{\begin{gathered}[16b_2^2(b_2 + c)(2b_2 + c - \alpha)^2 - a^4\{\alpha - 2(b_2 - c)^2\} \\ - 2a^3\{\alpha^2 + 19b_2 + 18b_2c + 3c^2 - 2\alpha(5b_2 + 2c)\} \\ - 8ab_2\{20b_2^3 + 32b_2^2c + 15b_2c^2 + 2c^3 + \alpha(3b_2 + 2c) \\ - 2\alpha(8b_2^2 + 9b_2c + 2c^2)\} \\ + 4a^2\{33b_2^3 + 41b_2^2c + 13b_2c^2 + \alpha^2(3b_2 + c) \\ - \alpha(21b_2^2 + 16b_2c + 2c^2)\}]\end{gathered}}$$

The second order condition is

$$\frac{d^2\psi}{d\beta_c^2} = 2m\frac{[a(2b_2 - \alpha)(6b_2 - \alpha + 4c) - 2(2b_2 - \alpha)^2(b_2 + c) - 2a^2(2b_2 + 2c - \alpha)]}{[2b_2(2b_2 - \alpha) - a(4b_2 - \alpha)]^2}$$

This is negative if the numerator is. Where c = 0, the numerator equals

$$-2(2b_2 - a)(2b_2 - \alpha)(2b_2 - 2a - 2\alpha)m < 0$$

When the numerator is differentiated with respect to c, one gets

$$-2m[8b_2(b_2 - \alpha - a) + 2\alpha^2 + 4a^2 + 4a\alpha] < 0$$

implying that the numerator is even more negative when $c > 0$. $\Rightarrow d^2\psi / d\beta_c^2 < 0$ with any meaningful value of c.

Appendix 2

When β_c is optimised according to the appendix 1, and the private system maximises its profit according to (3.6), it follows that

$$\beta_p = \frac{\begin{aligned}&[(b_1-\alpha)(b_2+c-a)\{a^2+2a(\alpha-4b_2-c)+4b_2(2b_2-\alpha+c)\}\\ &[2b_2\{2(b_2+c)-\alpha\}+a\{\alpha-2(2b_2+c)\}]]\end{aligned}}{\begin{aligned}&[16b_2^2(b_2+c)(2b_2+c-\alpha)^2+a^4\{2(b_2+c)-\alpha\}\\ &-2a^3(\alpha^2+19b_2^2+18b_2c+3c^2-2\alpha(5b_2+2c))\\ &-8ab_2\{20b_2^3+32b_2^2c+15b_2c^2+2c^3+\alpha^2(3b_2+2c)-2\alpha(8b_2^2+9b_2c+2c^2)\}\\ &+4a^2\{33b_2^3+41b_2^2c+13b_2c^2+c^3+\alpha^2(3b_2+c)-\alpha(21b_2^2+16b_2c+2c^2)\}]\end{aligned}}$$

And the respective amounts of transactions in the two systems equal

$$n_p = \frac{[(b_1-\alpha)(2b_2-a)(a-b_2-c)\{a^2+2a(\alpha-4b_2-c)+4b_2(2b_2+c-\alpha)\}]}{\begin{aligned}&[-16b_2^2(b_2+c)(2b_2+c-\alpha)^2+a^4\{\alpha-2(b_2+c)\}\\ &+2a^3\{\alpha^2+19b_2^2+18b_2c+3c^2-2\alpha(5b_2+2c)\}\\ &+8ab_2\{20b_2^3+32b_2^2c+15b_2c^2+2c^3+\alpha^2(3b_2+2c)-2\alpha(8b_2^2+9b_2c+2c^2)\}\\ &-4a^2\{33b_2^3+41b_2^2c+13b_2c^2+c^3+\alpha^2(3b_2+c)-\alpha(21b_2^2+16b_2c+2c^2)\}]\end{aligned}}$$

$$n_c = \frac{\begin{aligned}&[(b_1-\alpha)[4ab_2(2\alpha^2-13\alpha b_2+19b_2^2-4\alpha c+13b_2c+2c^2)\\ &-2a^2\{\alpha^2+29b_2^2+12b_2c+c^2-2\alpha(6b_2+c)\}-a^4-3a^3(5b_2+c-\alpha)\\ &-8b_2^2(2b_2+c-\alpha)^2]]\end{aligned}}{\begin{aligned}&[-16b_2^2(b_2+c)(2b_2+c-\alpha)^2+a^4\{\alpha-2(b_2+c)\}\\ &+2a^3\{\alpha^2+19b_2^2+18b_2c+3c^2-2\alpha(5b_2+2c)\}\\ &+8ab_2\{20b_2^3+32b_2^2c+15b_2c^2+2c^3+\alpha^2(3b_2+2c)-2\alpha(8b_2^2+9b_2c+2c^2)\}\\ &-4a^2\{33b_2^3+41b_2^2c+13b_2c^2+c^3+\alpha^2(3b_2+c)-\alpha(21b_2^2+16b_2c+2c^2)\}]\end{aligned}}$$

Appendix 3

Necessary conditions for the existence of a meaningful equilibrium

1) Private system second order condition

$$\frac{\partial^2 \Pi}{\partial \beta_n^2} < 0$$

$$\Leftrightarrow 4(2b_2 - a)\frac{\begin{array}{l}[4b_2(b_2+c)(b_2+c-\alpha)(2b_2-b_1+c)\\ -a^2[\alpha^2+\alpha(2b_2-2b_1+c)-2\{4b_2^2+5b_2c+c^2-b_1(2b_2+c)\}]\\ +2a[\alpha^2 b_2+\alpha\{5b_2^2+5b_2c+c^2-b_1(3b_2+c)\}\\ -(b_2+c)\{8b_2^2+7b_2c+c^2-b_1(4b_2+c)\}]]\end{array}}{\begin{array}{c}[a\{(\alpha-4b_2)-2(\alpha-2b_2)b_2\}^2\\ [2(b_1-b_2-c)(b_2+c)+a\{\alpha+2(b_2-b_1+c)\}]]\end{array}}$$

2) Non-negativity of the private fee

$$\beta_p \geq 0$$

$$\Leftrightarrow \frac{\begin{array}{l}[4b_2^2[2\{b_1(b_2-\beta_n)+b_2(\beta_n+c)\}-\alpha(b_1+2b_2-\beta_n+c)+\alpha^2]\\ +a^2\{2\alpha^2+8b_1b_2-4b_2\beta_c-4b_1\beta_n+4b_2\beta_n+2\beta_c\beta_n\\ +\alpha(\beta_c+3\beta_n-c-2b_1-8b_2)+4b_2c\}\\ -2ab_2\{3\alpha^2+8b_1b_1-2b_2\beta_c-6b_1\beta_n+6b_2\beta_n+2\beta_c\beta_n\\ +\alpha(\beta_c+4\beta_n-3b_1-8b_2-2c)+6b_2c\}]\end{array}}{[2(2b_2-a)\{2b_2(2b_2-\beta_n-\alpha)-a(4b_2-\alpha-\beta_n)\}]} \geq 0$$

when β_n and β_c are determined by (4.6), this can be rewritten as

$$\frac{(b_1-\alpha)(2c-\alpha)(b_2+c-a)}{[2(b_2+c-\alpha)(b_2+c)-a(2b_2+2c-\alpha)]} \geq 0$$

3) Non-negativity of the private profit

$\Pi \geq 0$

$$\Leftrightarrow 2m(b_1-\alpha)^2(b_2+c-a)^2 \frac{\begin{array}{l}[4b_2(b_2+c-\alpha)(2b_2-b_1+c)\\ -a^2[\alpha^2+\alpha(2b_2-2b_1+c)\\ -2\{4b_2^2+5b_2c+c^2-b_1(2b_2+c)\}]\\ +2a[\alpha^2 b_2+\alpha\{5b_2^2+5b_2c+c^2-b_1(3b_2+c)\}\\ -(b_2+c)\{8b_2^2+7b_2c+c^2-4b_1b_2-cb_1\}]]\end{array}}{\begin{array}{l}[(2b_2-a)[2(b_2+c-\alpha)(b_2+c)-a(2b_2+2c-\alpha)]^2\\ [2(b_1-b_2-c)(b_2+c)+a(\alpha+2b_2-2b_1-2c)]]\end{array}} \geq 0$$

4) Cental bank second order condition for the fee β_n

$$\frac{\partial^2\psi}{\partial\beta_n^2} < 0$$

When the indirect effect through β_p is taken into account, this can be rewritten

$$\frac{\begin{array}{l}m\{-2b_2(2b_2-b_1+c)+a(\alpha-2b_1+4b_2+\beta_c+c)\}\\ \{8b_2^3\{\alpha(b_1-4b_2-c)+b_1(b_2-2\beta_n-c)+(b_2+c)(2b_2+2\beta_n+c)\}\\ +a^3[\alpha^2+\alpha(10b_2-2b_1+3\beta_c-2\beta_n-5c)\\ -2\{2b_1(b_2-\beta_n-2c)+(b_2+c)(4b_2+3\beta_c+2\beta_n+c)\}]\\ 2a^2[16b_2^3+9b_2^2\beta_c+10b_2^2\beta_n+17b_2^2c+14b_2\beta_c c+8b_2\beta_n c-2\beta_c\beta_n+4b_2c^2+2\beta_c c^2\\ -2\alpha^2(b_2+c)+2b_1\{4b_2^2+(\beta_n-c)c-b_2(5\beta_n+8c)\}\\ +\alpha\{-23b_2^2+2c(-\beta_c-\beta_n+c)+b_1(5b_2+2c)+b_2(-6\beta_c+4\beta_n+9c)\}]\\ -4ab_2[10b_2^3+3b_2^2\beta_c+8b_2^2\beta_n+12b_2^2c+7b_2\beta_c c+6b_2\beta_n c-2\beta_c\beta_n c+3b_2c^2\\ +2\beta_c c^2-\alpha^2(b_2+2c)+b_1\{5b_2^2+2(\beta_n-c)c-8b_2(\beta_n+c)\}\\ +\alpha[-17b_2^2+2b_1(2b_2+c)+2c(-\beta_n-\beta_c+c)+b_2\{-3\beta_c+2(\beta_n+c)\}]]\}\end{array}}{[a(\alpha-4b_2+\beta_n-c)+2b_2(-\alpha+2b_2-\beta_n+c)]^4} < 0$$

5) Central bank second order condition for the fee β_c

$$\frac{\partial^2\Psi}{\partial\beta_c^2} < 0$$

When the indirect effect through the private fee β_c is taken into account, this can be rewritten

$$\frac{\begin{aligned}&[-16b_2^2(\alpha-2b_2+\beta_n-c)^2(b_2+c)+a^4\{\alpha-2(b_2+c)\}\\&+2a^3\{\alpha^2+19b_2^2-8b_2^2\beta_n+\beta_n^2+18b_2c-4\beta_nc+3c^2-2\alpha(5b_2-\beta_n+2c)\}\\&-4a^2[33b_2^3+(\beta_n-c)^2c+\alpha^2(3b_2+c)+b_2^2(-20\beta_n+41c)\\&+\alpha\{-21b_2^2+2b_2(3\beta_n-8c)+2(\beta_n-c)c\}+b_2(3\beta_n^2-16\beta_nc+13c^2)]\\&+8ab_2[20b_2^3-16b_2^2(\beta_n-2c)+2(\beta_n-c)^2c+\alpha^2(3b_2+2c)+3b_2(\beta_n^2-6\beta_nc+5c^2)\\&-2\alpha\{8b_2^2-3b_2(\beta_n-3c)+2c(c-\beta_n)\}]]m\end{aligned}}{[(2b_2-a)^2\{a(\alpha-4b_2+\beta_n-c)+2b_2(-\alpha+2b_2-\beta_n+c)\}^2]}<0$$

6) Non-negativity of central bank fee β_n

$$\beta_n \geq 0 \Leftrightarrow \frac{[4b_2(b_1-\alpha)(b_2-\alpha)(b_2+c-a)]}{[(2b_2-a)\{2(b_1-b_2-c)(b_2-c)+a(2(b_2-b_1+c)+\alpha)\}]} \geq 0$$

7) Non-negativity of central bank fee

$$\beta_c \geq 0 \Leftrightarrow \frac{[2(b_1-\alpha)(b_2-a-\alpha+c)c]}{[2(b_2+c)(b_2+c-\alpha)-a(2b_2+2c-\alpha)]} \geq 0$$

8) Meaningful value of n_c

$$0 < n_c < 1 \Leftrightarrow 0 < \frac{(b_1-\alpha)(b_2+c-a-\alpha)}{[2(b_2-\alpha+c)(b_2+c)-a(2b_2+2c-\alpha)]} < 1$$

9) Meaningful value of n_p

$$0 < n_p < 1 \Leftrightarrow 0 < \frac{(b_1-\alpha)(b_2+c-a)}{[2(b_2-\alpha+c)(b_2+c)-a(2b_2+2c-\alpha)]} < 1$$

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