



Mikael Bask

# Measuring potential market risk



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Bank of Finland Research  
Discussion Papers  
20 • 2007

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The views expressed are those of the author and do not necessarily reflect the views of the Bank of Finland.

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Esa Jokivuolle is gratefully acknowledged for giving comments on an earlier draft of this paper. The usual disclaimer applies.

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<http://www.bof.fi>

ISBN 978-952-462-388-9  
ISSN 0785-3572  
(print)

ISBN 978-952-462-389-6  
ISSN 1456-6184  
(online)

Helsinki 2007

# Measuring potential market risk

Bank of Finland Research  
Discussion Papers 20/2007

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## Abstract

The difference between market risk and potential market risk is emphasized and a measure of the latter risk is proposed. Specifically, it is argued that the spectrum of smooth Lyapunov exponents can be utilized in what we call  $(\lambda, \sigma^2)$ -analysis, which is a method to monitor the aforementioned risk measures. The reason is that these exponents focus on the stability properties ( $\lambda$ ) of the stochastic dynamic system generating asset returns, while more traditional risk measures such as value-at-risk are concerned with the distribution of returns ( $\sigma^2$ ).

Keywords: market risk, potential market risk, smooth Lyapunov exponents, stochastic dynamic system, value-at-risk

JEL classification number: G11

# Menetelmä potentiaalisen markkinariskin määrittämiseksi

Suomen Pankin keskustelualoitteita 20/2007

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Rahapolitiikka- ja tutkimusosasto

## Tiivistelmä

Tutkimuksessa tarkastellaan markkinariskin ja potentiaalisen markkinariskin eroa ja tarkasteluissa esitellään keino jälkimmäisen mittaamiseksi. Täsmällisesti ottaen työssä esitetään, että sileiden Lyapunovin eksponenttien spektriä voidaan hyödyntää ns.  $(\lambda, \sigma^2)$ -analyysissä, jota menetelmää käytetään markkinariskin ja potentiaalisen markkinariskin tarkkailussa. Analyysin soveltuvuus näiden riskien seurantaan johtuu siitä, että sileät Lyapunovin eksponentit korostavat rahoitusvaateiden tuottoja synnyttävän stokastisen dynaamisen järjestelmän vakausominaisuuksia ( $\lambda$ ), kun perinteisissä riskimittareissa, kuten value-at-risk, kyse on sen sijaan tuottojen jakaumasta ( $\sigma^2$ ).

Avainsanat: markkinariski, potentiaalinen markkinariski, sileät Lyapunovin eksponentit, stokastinen dynaaminen järjestelmä, value-at-risk

JEL-luokittelu: G11

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# 1 Measuring market risk

Financial market risk reflects the chance that the actual return on an asset or a portfolio of assets may be very different than the expected return. For this reason, a measure of market risk is necessary to carry through a successful risk management.

Nowadays, financial investors often use value-at-risk to assess the market risk in their portfolio since they would like to ensure that the value of the portfolio does not fall below some minimum level that would expose the investor to insolvency. The value-at-risk is the level of loss on a portfolio that is expected to be equaled or exceeded with a given small probability. This risk measure can, therefore, be seen as a forecast of a given percentile, usually in the lower tail, of the probability distribution of returns.<sup>1</sup>

Of course, the probability distribution of returns is not constant since asset returns depend on the underlying economic structure.<sup>2</sup> To be more precise, quantities such as moneys and interest rates interact with each other through time, and, therefore, constitute a dynamic system, meaning that the stability of the system generating asset returns is crucial for the variance of these returns. That is, a less stable dynamic system is associated with more variable asset returns, meaning that an asset is *potentially* more risky than another asset, if the returns of the former asset is generated by a less stable system.

To clarify this further, let  $\sigma^2$  denote the conditional variance of asset returns, and (for reasons explained below) let  $\lambda$  denote the stability of the dynamic system generating these returns. Then,

$$\sigma^2 = \sigma^2(\lambda, \varepsilon) \tag{1.1}$$

where  $\varepsilon$  is exogenous shocks to the dynamic system, meaning that the conditional variance ( $\sigma^2$ ) is not only affected by the system's stability ( $\lambda$ ), it is also affected by shocks to the system ( $\varepsilon$ ). Specifically, the conditional variance of asset returns increases when the dynamic system is less stable, but also when the variance of the shocks increases. Thus, because of shocks to the system, there is no one-to-one correspondence between the conditional variance of asset returns and the stability of the dynamic system generating these returns, meaning that  $\lambda$  is not a measure of market risk. Instead,  $\lambda$  is a measure of *potential market risk*, while  $\sigma^2$  is a measure of market risk.

In other words, a change in an asset's potential market risk may or may not change its market risk since it depends on how much the variance of the shocks to the dynamic system generating asset returns has changed, if there has been any change at all. Thus, the variance of the shocks distinguishes

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<sup>1</sup> The importance of value-at-risk as a measure of financial market risk is emphasized by the fact that the Basel Committee on Banking Supervision at the Bank for International Settlements imposes financial institutions to meet capital requirements based on value-at-risk. The widespread use of value-at-risk as a measure of market risk also owes much to Dennis Weatherstone, former chairman of JP Morgan & Co., who demanded to know the market risk of the company at 4:15 P.M. every day. Weatherstone's request was met with a daily value-at-risk report.

<sup>2</sup> For this reason, Engle's (1982) ARCH model and subsequent developments of the model are invaluable tools since they can be used to estimate and predict conditional moments characterizing the probability distribution of returns.

between an asset's market risk and its potential market risk. Therefore, the stability of the dynamic system generating asset returns should be contrasted with the volatility of these returns, and this is accomplished in what we call  $(\lambda, \sigma^2)$ -analysis.

## 2 $\lambda$ : a measure of potential market risk

The purpose of this section is twofold: (i) to define the Lyapunov exponents of a stochastic dynamic system; and (ii) to motivate why these exponents provide a measure of a system's stability, meaning that they also provide a measure of potential market risk.

*Definition of  $\lambda$*

Bask and de Luna (2002) argue that the spectrum of smooth Lyapunov exponents can be used in the determination of the stability of a stochastic dynamic system. Specifically, assume that the dynamic system,  $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$ , generating asset returns is

$$S_{t+1} = f(S_t) + \varepsilon_{t+1}^s \quad (2.1)$$

where  $S_t$  and  $\varepsilon_t^s$  are the state of the system and a shock to the system, respectively, both at time  $t \in [1, 2, \dots, \infty]$ . For an  $n$ -dimensional system as in (2.1), there are  $n$  Lyapunov exponents that are ranked from the largest to the smallest exponent

$$\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n \quad (2.2)$$

and it is these exponents that provide information on the stability properties of the system  $f$ .

Then, how are the Lyapunov exponents defined? Assume temporarily that there are no shocks to the system  $f$ , and consider how it amplifies a small difference between the initial states  $S_0$  and  $S'_0$

$$S_j - S'_j = f^j(S_0) - f^j(S'_0) \simeq Df^j(S_0)(S_0 - S'_0) \quad (2.3)$$

where  $f^j(S_0) = f(\dots f(f(S_0))\dots)$  denotes  $j$  successive iterations of the system starting at state  $S_0$ , and  $Df$  is the Jacobian of the system

$$Df^j(S_0) = Df(S_{j-1}) Df(S_{j-2}) \dots Df(S_0) \quad (2.4)$$

Then, associated with each Lyapunov exponent,  $\lambda_i$ ,  $i \in [1, 2, \dots, n]$ , there are nested subspaces  $U^i \subset \mathbb{R}^n$  of dimension  $n + 1 - i$  with the property that

$$\lambda_i \equiv \lim_{j \rightarrow \infty} \frac{\log_e \|Df^j(S_0)\|}{j} = \lim_{j \rightarrow \infty} \frac{1}{j} \sum_{k=0}^{j-1} \log_e \|Df(S_k)\| \quad (2.5)$$

for all  $S_0 \in U^i - U^{i+1}$ . Due to Oseledec's multiplicative ergodic theorem, the limits in (2.5) exist and are independent of  $S_0$  almost surely with respect to

the measure induced by the process  $\{S_t\}_{t=1}^\infty$ .<sup>3</sup> Then, allow for shocks to the system  $f$ , meaning that the measure is induced by a stochastic process. In this case, the Lyapunov exponents have been named smooth Lyapunov exponents in the literature.

### *Motivation of $\lambda$*

The reason why the spectrum of smooth Lyapunov exponents provides information on the stability properties of a stochastic dynamic system may be seen by considering two different starting values of a system, where the difference is an exogenous shock at time  $t = 0$ . The largest smooth Lyapunov exponent,  $\lambda_1$ , measures the slowest exponential rate of convergence of two trajectories of the dynamic system starting at these different starting values at time  $t = 0$ , but with identical exogenous shocks at times  $t > 0$ .<sup>4</sup> In fact,  $\lambda_1$  measures the convergence of a shock in the direction defined by the eigenvector corresponding to this exponent. However, if the difference between the two starting values lies in another direction of  $\mathbb{R}^n$ , then the convergence is faster. Thus,  $\lambda_1$  measures a ‘worst case scenario’.

The average of the smooth Lyapunov exponents

$$\lambda \equiv \frac{1}{n} \sum_{i=1}^n \lambda_i \tag{2.6}$$

measures the exponential rate of convergence in a geometrical average direction. That is, the convergence of two trajectories of the dynamic system in the geometrical average of the directions defined by the eigenvectors corresponding to the different exponents. Thus,  $\lambda$  measures an ‘average scenario’. We can, therefore, compare the stability of two stochastic dynamic systems via the smooth Lyapunov exponents since a one-time shock has a smaller effect on the dynamic system with a smaller  $\lambda$  than for the system with a larger  $\lambda$ . Thus, since we are dealing with dissipative systems, meaning that  $\lambda < 0$  by definition, a dynamic system is more stable than another system, if  $\lambda$  is more negative.

An extensive discussion of the spectrum of smooth Lyapunov exponents as a measure of the stability of a stochastic dynamic system is provided in Bask and de Luna (2002). As an illustration, it is shown therein that the decrease in volatility of the exchange rates between the Swedish Krona and the ECU/Euro, after the launch of the Euro, is due to a decrease in the volatility of the shocks to the dynamic system generating these exchange rates and not to a more stable system. Thus, one can say that the market risk decreased, but that the potential market risk was unchanged.

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<sup>3</sup> See Guckenheimer and Holmes (1983) for a careful definition of the Lyapunov exponents and their properties.

<sup>4</sup> When  $\lambda_1 > 0$ , the trajectories diverge from each other, and for a bounded stochastic dynamic system, this is an operational definition of chaotic dynamics.

### 3 Testing for a change in $\lambda$

Contrary to risk measures like value-at-risk, potential market risk does not have a straightforward economic interpretation. However, it is not level of potential market risk that is of interest. Instead, it is the change in this risk,  $\Delta\lambda$ , since we are interested in the *potential change* in market risk.

The purpose of this section is, therefore, twofold: (i) to show how the smooth Lyapunov exponents can be estimated from time series data; and (ii) to discuss how hypothesis tests of these exponents can be constructed. In other words, the purpose is to show how an asset's potential market risk can be estimated from an asset return series, and to discuss how to test for a change in this risk.

#### *Estimation of $\lambda$*

Since the actual form of the dynamic system  $f$  is not known, it may seem like an impossible task to determine the stability of the system. However, it is possible to reconstruct the dynamics of the system using only a scalar time series, and, thereafter, to measure the stability of this reconstructed system. Therefore, associate the system  $f$  with an observer function,  $g : \mathbb{R}^n \rightarrow \mathbb{R}$ , that generates observed asset returns

$$s_t = g(S_t) + \varepsilon_t^m \quad (3.1)$$

where  $s_t \in S_t$  and  $\varepsilon_t^m$  are the asset return and a measurement error, respectively, both at time  $t$ . Thus, (3.1) means that the asset return series

$$\{s_t\}_{t=1}^N \quad (3.2)$$

is observed, which is used to reconstruct the dynamics of the system  $f$ , where  $N$  is the number of consecutive returns in the time series.

Specifically, the observations in a scalar time series, like the asset return series in (3.2), contain information about unobserved state variables that can be used to define a state in present time. Therefore, let

$$T = (T_1, T_2, \dots, T_M)' \quad (3.3)$$

be the reconstructed trajectory, where  $T_t$  is the reconstructed state at time  $t$  and  $M$  is the number of states on the reconstructed trajectory. Each  $T_t$  is given by

$$T_t = \{s_t, s_{t+1}, \dots, s_{t+m-1}\} \quad (3.4)$$

where  $m$  is the embedding dimension and time  $t \in [1, 2, \dots, N - m + 1]$ . Thus,  $T$  is an  $M \times m$  matrix and the constants  $M$ ,  $m$  and  $N$  are related as  $M = N - m + 1$ .

Takens (1981) proved that the map

$$\Phi(S_t) = \{g(f^0(S_t)), g(f^1(S_t)), \dots, g(f^{m-1}(S_t))\} \quad (3.5)$$

which maps the  $n$ -dimensional state  $S_t$  onto the  $m$ -dimensional state  $T_t$ , is an embedding if  $m > 2n$ .<sup>5</sup> This means that the map is a smooth map that performs a one-to-one coordinate transformation and has a smooth inverse. A map that is an embedding preserves topological information about the unknown dynamic system, like the smooth Lyapunov exponents, and, in particular, the map induces a function,  $h : \mathbb{R}^m \rightarrow \mathbb{R}^m$ , on the reconstructed trajectory

$$T_{t+1} = h(T_t) \quad (3.6)$$

which is topologically conjugate to the unknown system  $f$ . That is

$$h^j(T_t) = \Phi \circ f^j \circ \Phi^{-1}(T_t) \quad (3.7)$$

Thus,  $h$  is a reconstructed dynamic system that has the same smooth Lyapunov exponents as the unknown system  $f$ .<sup>6</sup>

Then, to estimate the smooth Lyapunov exponents of the system  $f$  generating asset returns, one must first estimate  $h$ . However, since

$$h : \begin{pmatrix} s_t \\ s_{t+1} \\ \vdots \\ s_{t+m-1} \end{pmatrix} \longrightarrow \begin{pmatrix} s_{t+1} \\ s_{t+2} \\ \vdots \\ v(s_t, s_{t+1}, \dots, s_{t+m-1}) \end{pmatrix} \quad (3.8)$$

---

<sup>5</sup> An intuitive explanation of Takens' (1981) embedding theorem may be in place due to its importance in the estimation of  $\lambda$ . For the sake of the argument, assume that  $\mathbf{M}_1 \subset \mathbf{M}$  and  $\mathbf{M}_2 \subset \mathbf{M}$  are two subspaces of dimension  $n_1$  and  $n_2$ , respectively, where  $\mathbf{M} \in \mathbb{R}^m$  is an  $m$ -dimensional manifold representing phase space for the reconstructed dynamic system. In general, two subspaces intersect in a subspace of dimension  $n_1 + n_2 - m$ , meaning that when this expression is negative, there is no intersection of the two subspaces. Therefore, and of greater interest, the self-intersection of an  $n$ -dimensional manifold with itself fails to occur when  $m > 2n$  (see Sauer et al, 1991, for generalizations of Takens', 1981, theorem).

A problem is that the dimension of the 'true' dynamic system is not known, meaning that the required embedding dimension is not either known. This problem can, however, be solved indirectly by making use of a generic property of a proper reconstruction, namely, that the dynamics in original phase space must be completely unfolded in reconstructed phase space. In other words, if the embedding dimension is too low, the dynamics is not completely unfolded, meaning that distant states in original phase space are close states in reconstructed phase space, and, therefore, are named false neighbors in phase space.

There are at least two methods to calculate the required embedding dimension from an observed time series: (i) false nearest neighbors; and (ii) the saturation of invariants on the reconstructed dynamics such as the saturation of the Lyapunov exponents. The first method is based on the aforementioned generic property of a proper reconstruction, meaning that by increasing the embedding dimension, the dynamics is completely unfolded when there are no false neighbors in reconstructed phase space (see Kennel et al, 1992).

The second method, the saturation of invariants on the reconstructed dynamics, is based on the fact that when the dynamics is completely unfolded, the Lyapunov exponents and other invariants such as entropy and fractal dimension are independent of the embedding dimension. If, however, the dynamics is not completely unfolded in reconstructed phase space, these invariants depend on the embedding dimension. Therefore, by increasing the embedding dimension, the dynamics is completely unfolded when the value of an invariant stops changing (see Fernández-Rodríguez et al, 2005, for an example regarding the largest Lyapunov exponent and a statistical test for chaotic dynamics).

<sup>6</sup> Since the  $m$ -dimensional system  $h$  has a larger dimension than the  $n$ -dimensional system  $f$ , the number of smooth Lyapunov exponents that are spurious is  $m - n$ . This issue is discussed in Dechert and Gencay (1996)–(2000) and Gencay and Dechert (1996).

the estimation of  $h$  reduces to the estimation of  $v$

$$s_{t+m} = v(s_t, s_{t+1}, \dots, s_{t+m-1}) \quad (3.9)$$

Moreover, since the Jacobian of  $h$  at the reconstructed state  $T_t$  is

$$Dh(T_t) = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ \frac{\partial v}{\partial s_t} & \frac{\partial v}{\partial s_{t+1}} & \frac{\partial v}{\partial s_{t+2}} & \cdots & \frac{\partial v}{\partial s_{t+m-1}} \end{pmatrix} \quad (3.10)$$

a feedforward neural network is a natural choice to estimate the above derivatives to be able to calculate the smooth Lyapunov exponents (see Dechert and Gencay, 1992, Gencay and Dechert, 1992, McCaffrey et al, 1992, and Nychka et al, 1992), and this is because Hornik et al (1990) have shown that a map and its derivatives of any unknown functional form can be approximated arbitrarily accurately by such a network.

#### *Inference of $\lambda$*

Shintani and Linton (2004) derive the asymptotic distribution of a neural network estimator of the smooth Lyapunov exponents

$$\sqrt{M} \left( \hat{\lambda}_{iM} - \lambda_i \right) \implies \mathbb{N}(0, V_i) \quad (3.11)$$

where  $\hat{\lambda}_{iM}$  is the estimator of the  $i$ :th exponent, based on the  $M$  reconstructed states on the trajectory,  $V_i$  is the variance of the  $i$ :th exponent, and  $i \in [1, 2, \dots, n]$ .<sup>7</sup> When it comes to the average of the smooth Lyapunov exponents, our *conjecture* is that asymptotic normality holds for a neural network estimator of  $\frac{1}{n} \sum_{i=1}^n \lambda_i$  since the eigenvectors corresponding to the different exponents are pairwise orthogonal

$$\sqrt{M} \left( \hat{\lambda}_{Mn} - \lambda \right) \implies \mathbb{N}(0, V_n) \quad (3.12)$$

where  $\hat{\lambda}_{Mn}$  is the estimator of  $\frac{1}{n} \sum_{i=1}^n \lambda_i$ , based on the  $M$  reconstructed states on the trajectory, and  $V_n$  is the variance of  $\frac{1}{n} \sum_{i=1}^n \lambda_i$ . If this conjecture is correct, it is possible to make inference of a change in potential market risk.

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<sup>7</sup> It is, therefore, possible to test for the presence of chaotic dynamics in an observed scalar time series since  $\lambda_1 > 0$  is an operational definition of chaos (see Bask et al, 2007, for an application using electricity prices, who find evidence of complicated dynamics).

## 4 $(\lambda, \sigma^2)$ -analysis

The origin of  $(\lambda, \sigma^2)$ -analysis is found in Bask and de Luna (2002) since it is argued therein that when the volatility of a variable modelled is of interest, one should also consider the stability properties of the *same* model. Specifically, a parametric model in the form of a polynomial autoregression on a projected space is fitted to the observed time series, which is utilized to measure the stability and volatility of the variable of interest (see Bask and de Luna, 2002, and de Luna, 1998, for details).<sup>8</sup>

However, when a successful risk management is in focus, it is necessary to measure the stability of the ‘true’ stochastic dynamic system generating asset returns, and not the stability of the model fitted to these returns. The reason is that there is no guarantee that the smooth Lyapunov exponents for the ‘true’ system and the model selected to measure volatility coincide with each other. Therefore, we argue that a non-parametric approach should be used when estimating the stability of the system, whereas any (good) volatility model may be used when estimating the volatility.<sup>9</sup>

### *Applications of $(\lambda, \sigma^2)$ -analysis*

Our belief is that  $(\lambda, \sigma^2)$ -analysis has at least two different but closely connected applications

(i) To *monitor* the evolution of an asset’s market risk ( $\sigma^2$ ) and its potential market risk ( $\lambda$ ), meaning that  $(\lambda, \sigma^2)$ -analysis is used as a tool to detect actual and potential changes in market risk.

Think of an asset with an unchanged market risk. That is, the conditional volatility of asset returns is measured in a rolling window, where it is found that there are no statistically significant changes in volatility over some period of time (see Leeves, 2007, for an application using stock prices before and after the Asian crisis). However, during the same period of time, the stability of asset returns has decreased since the average of the smooth Lyapunov exponents has become less negative, meaning that the asset’s potential market risk has increased. Thus, in this case,  $(\lambda, \sigma^2)$ -analysis gives an early warning that an increase in the asset’s market risk may soon occur.

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<sup>8</sup> A large-scale analysis of the European monetary integration, with the creation of the EMU, is carried out in Bask and de Luna (2005) using this methodology. To be more specific, changes in the stability and volatility of 16 European currencies and in the volatility of the shocks to these currencies are examined, and the results indicate that when most of the currencies became more (less) stable, a majority of them also became less (more) volatile. For example, following the agreement of the Maastricht Treaty, most currencies became more stable and less volatile, whereas they became less stable and more volatile when the Danish public voted against the treaty.

<sup>9</sup> Bask and Widerberg (2007) use this methodology when they examine how the integration process at the Nordic power market has affected the stability and volatility of electricity prices. To be more specific, the non-parametric approach outlined above is used when estimating the stability, whereas an EGARCH model is used when estimating the volatility. The results indicate that the integration process is associated with more stable electricity prices and a decrease in volatility of these prices, but without having a one-to-one correspondence between the changes in stability and volatility.

Of course, the same tool can be used to monitor changes in market risk and potential market risk of a portfolio of assets.

(i) To *compare* the market risk and potential market risk of two portfolios of assets.

Imagine an investor who is planning to make a portfolio investment, but is unsure about which asset to invest in. Of course, if this investor is using what has been named modern portfolio theory when making investments, it is clear that the potential market risk of different assets should not directly affect the composition of the portfolio. On the other hand, due to the fact that a portfolio's market risk depends on its potential market risk, we believe that one should not neglect the latter risk.

Think of a situation in which two different assets give rise to portfolios with the same risk-return profiles. We argue, in this case, that the investor should invest in the asset that gives rise to the portfolio with the smaller potential market risk since the market risk is time-varying and that it may be the case that the market risk of the portfolio with the higher potential market risk is unusually low. It is, of course, part of future research to derive a reasonable portfolio theory that supports such a claim.

## 5 Concluding remarks

The aim of this paper has been to argue in favor of  $\lambda$  as a measure of potential market risk, and to discuss how this measure can be used in what we call  $(\lambda, \sigma^2)$ -analysis, which is a method to distinguish between market risk and potential market risk. What remains is to derive the asymptotic distribution of a neural network estimator of the average of the smooth Lyapunov exponents, and, thereafter, take the proposed method to financial data to study its merits and possible weaknesses.

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