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Market Failures and the Additionality Effects of Public Support to Private R&D: Theory and Empirical Implications*

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Abstract

We extend the theoretical basis of the empirical literature on the effects of R&D subsidies by providing an estimable model of strategic interaction among subsidy applicants, and public and private sector R&D financiers. Our model incorporates fixed R&D costs and a cost of external finance. We derive the optimal support rule. At the intensive (extensive) margin the costs of external funding reduce (increase) the optimal subsidy rate. We also establish necessary and sufficient conditions for the existence of additionality. It turns out that additionality at the intensive margin is less likely with large spillovers. Our results suggest that the relationship between additionality and welfare may not be straightforward.

Keywords: R&D, entrepreneurial finance, R&D subsidies, innovation policy

JEL classification numbers: O38, O31, L32, H25, G28

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1 Introduction

It is widely acknowledged that private sector investments in innovation are crucial for the enhancement of economic growth and welfare. Nevertheless, the private sector is likely to invest sub-optimally in R&D because of appropriability problems and potential market failures in the provision of private funding to R&D. To stimulate private R&D investments, governments around the world are increasingly spending public funds in direct R&D subsidies and tax incentives. These innovation polices have a central role in virtually all developed countries.[1] For example, all OECD countries use direct R&D subsidies, and increasingly many offer some form of R&D tax incentive (Warda 2006, OECD 2011, and Busom, Corchuelo and Martinez Ros 2012). Both innovation support policies are also becoming more widespread in emerging countries: e.g., India uses both subsidies and tax credits.

A large empirical literature has contributed to our understanding of how these policies work: the R&D subsidy literature is surveyed, e.g., by David, Hall and Toole (2000), Garcia-Quevedo (2004), Cerulli (2010), and Zúñica-Vicente et al. (2012), and the R&D tax credit literature by Hall and van Reenen (2000), Parsons and Phillips (2007) and Mohnen and Lokshin (2010). The research effort has largely focused on the question of whether or not there is additionality, i.e, whether public support increases private R&D investment rather than crowds it out.

While the basic theoretical motivation for government support to private

[1]In this paper innovation or R&D support policies refer to R&D subsidies and tax incentives, although the set of innovation policies include a variety of other instruments such as intellectual property rights and prizes. See Takalo (2012) for a review of the instruments for innovation policy and their justifications.

R&D has been well understood for at least half a century (Nelson 1959, and Arrow 1962), the empirical literature is generally not based on theoretical models capturing the strategic decisions by firms, government agencies, and private sector financiers of R&D that constitute an essential part of an innovation policy environment. Takalo, Tanayama and Toivanen (2011) model the firm's decision to apply for a subsidy, the government's decision on the level of support, and the firm's subsequent R&D investment. In this paper, we extend that model to include fixed costs of R&D projects, and a possibility to tap financial markets for R&D funding at a cost. We produce new results concerning the sources and implications of additionality. These extensions and results provide new insights into both the role of different market failures in innovation policy design and the existing results in the literature, and should be helpful in new empirical investigations on innovation policies.

A generic form of the equation typically estimated in the literature is

$$g(R) = \mathbf{X}\beta + f(s)\delta + \epsilon \tag{1}$$

where the outcome variable $g(R)$ is often either directly the R&D investment (R) or its logarithm ($\ln R$), $\mathbf{X}$ is a vector of control variables with β being the associated vector of coefficients, $f(s)$ is a function of the public funding of R&D (which comes either in the form of direct subsidies or tax incentives) and s is the support (subsidy, tax credit) rate, i.e., the fraction of R&D paid from public funds, and ϵ is a stochastic error term.[2] The main interest has been in the estimation of δ in equation (1). The main challenge has

[2] One could also write $f(R(s), s)$ as some empirical applications use the monetary amount of the subsidy as an endogenous explanatory variable.

been the endogeneity of R&D support policies, which is mainly generated by nonrandom participation in R&D subsidy and tax incentive programs arising from both nonrandom assignment of government support and from self-selection into these support programs.[3]

The literature has used various ways to overcome the endogeneity problem (see Cerulli 2010 for a review of the methods). Popular ways are instrumental variables and selection models (e.g., Wallsten 2000, Busom 2000, and Hussinger 2008), differences-in-differences (e.g., Lach 2002), matching and other non-parametric methods (e.g., Almus and Czarnitzki 2003, and Czarnitzki, Hanel and Miguel Rosa 2011). Structural econometric and other theory-based models are used less often, but some exist (e.g., Bloom, Griffith and van Reenen 2002, González, Jaumandreu and Pazó 2005, Lokshin and Mohnen 2011, and Takalo, Tanayama and Toivanen 2011).

One important feature missing from the structural econometric models of innovation policies is the interaction between public and private financiers of R&D intensive firms.[4] In this paper we introduce a competitive financial sector funding R&D into the model of Takalo, Tanayama and Toivanen (2011). This is a simple way to model the costs of (private sector) external funding of R&D. We also add fixed costs of R&D, which determine the effects of R&D

[3]For example, in the Spanish data used by Busom, Corchuelo and Martinez Ros (2012), only 12% of SMEs and 20% of large firms investing in R&D use both subsidies and tax credits. 23% of SMEs and 17% of large firms invest with the help of subsidies only, and 17% of SMEs and 26% of large firms only use tax credits. The rest invest without neither form of support .

[4]Gelabert, Fosfuri and Tribó (2009) and Busom, Corchuelo and Martinez Ros (2012) study the interaction empirically, and Keuschnigg and Ribi (2010) and Takalo and Tanayama (2010) theoretically but to the best of our knowledge there exists no structural econometric model besides our ongoing work (Takalo, Tanayama and Toivanen 2010) that would incorporate both private and public sources of R&D funding.

support policies at the extensive margin where the firms decide whether or not to invest in R&D. It is widely thought that policies generate larger additionality at the extensive margin than at the intensive margin where firms conducting R&D decide how much they invest (see, e.g., Einiö 2009). We also use a more general form for the firm's profit function which allows an analysis of the effects of the firm's production technology.

We characterize the optimal subsidy policy in the presence of both fixed costs and external funding costs. We find that an increase in the fixed cost of R&D or in external financing cost may lead to lower or higher subsidies depending on parameter values. The government needs to give a higher subsidy to get the project implemented when fixed costs increase. An increase in the cost of external finance further raises the required subsidy at the extensive margin, conditional on one being granted. But the costs eventually become so high that it is better not to subsidize the project even if the project is then not executed. In addition, we find that an increase in the cost of external finance leads to a *reduction* in the optimal subsidy rate at the intensive margin as a higher cost of finance dampens the firm's response to the subsidy.

We also establish necessary and sufficient conditions for the existence of additionality and for additionality to lead to a welfare improvement. It turns out that the projects generating large spillovers which optimally receive large subsidies are less likely to generate additionality at the intensive margin. The existence of additionality is neither a necessary nor a sufficient condition for welfare improving subsidies.

We present the model in the next section. In Section 3, we solve the model and characterize equilibria. In Sections 2 and 3 we also show how to derive estimation equations from our model, some of which are familiar from the existing literature. This econometric model is summarized in Section 4. In Section 5 we briefly discuss the implications of our model for the rationales of R&D support policies, additionality, its relation to welfare, and the interpretation of additionality results of the empirical literature. Section 6 concludes.

2 The Model

We consider a four-stage game of incomplete information among a firm with an R&D project, a public agency that gives R&D subsidies, and private sector financiers offering funding for R&D. Henceforth we refer to the public agency simply as "the agency" and to the private sector financiers as "financiers" when no confusion may arise. The R&D project involves both a variable investment and a fixed cost. For brevity, we assume the firm has no funds of its own and one project per firm.

Timing of events. In stage one, the firm decides whether or not to apply for a subsidy for an R&D project. If the firm applies, in stage two the agency evaluates the proposed project, and decides the level of the subsidy, which amounts to a credible promise to reimburse *ex post* a share of the variable R&D investment costs. In stage three, financiers compete to supply the rest of the needed project funding. In stage four the firm decides the level of its R&D investment. If the firm invests, and has been granted a subsidy in stage two, it will be reimbursed accordingly. Finally, the project returns are realized, and divided according to the financing contract made in stage three.

Assumptions. Our goal is to build a model that not only delivers theoretical insights but that can also be estimated. We therefore use more specific functional forms than would be necessary from a purely theoretical point of view. Assumptions on functional forms are introduced as we proceed.

We make two key informational assumptions. First, the type of a firm is common knowledge. This avoids complexities arising from signaling games.[5] Second, the type of the public agency is unknown to the firm when it contemplates the subsidy application. The firm only knows the distribution of the agency type. As will be made more precise in Section 2.4, the agency's type is about how it values the project of the firm beyond the profits the project generates. It may be helpful to think of these benefits as spillovers.

The latter informational assumption in essence introduces uncertainty on the firm's side about the agency's valuation of its projects when contemplating an application. This ensures, in line with empirical evidence, equilibrium outcomes where a firm submits a costly subsidy application only to be turned down. Since in our model the agency cannot signal its type to a potential applicant, it is immaterial whether the type of the agency is private information or whether there is symmetric but incomplete information. We opt for the simpler and arguably more realistic assumption that the agency learns its type after receiving and screening an application, i.e., symmetric but in-

[5]See Takalo and Tanayama (2010) for a model where a subsidy decision by the agency acts as a signal about the firm's type for financiers.

complete information regarding the agency's type prevails at the application stage.

Compared to standard corporate finance models, where often a borrower's type is private information and hence unknown to a (private sector) lender, these two informational assumptions may sound unorthodox. However, the theoretical literature on public funding of private R&D is scant. To us, it is quite reasonable to think that a firm, when contemplating an application, does not exactly know the agency's objective function or how the agency values the firm's project. There is less ambiguity concerning the objective function of private sector financiers since they may be assumed to maximize profits.

These two informational assumptions have an important implication: the firm's type cannot be correlated with the agency type, as otherwise the firm could infer information about the agency type from the type of its own project. In the empirical implementation of the model, this means that the shock to spillovers generated by the firm's R&D (agency's type), internalized by the agency but not the firm nor its financier, is not correlated with the shock that affects the private profitability of R&D (firm's type). This assumption *does not* remove the endogeneity problem emphasized in the literature, since the subsidy amount (measured in monetary units) is still a function of the shock to the project's private profitability even if the subsidy rate (measured as a per cent of the firm's R&D expense) is not.

As is standard, we also assume that the firm's investment is non-verifiable to third parties and that hence neither the agency's nor financiers' funding decisions can be written contingent on the firm's investment.

We further make a number of (common-knowledge) assumptions concerning the behavior of the agency. In line with the practice of subsidy programs, the subsidy level is subject both to a maximum constraint that is strictly less than unity, and to a minimum constraint of zero, which binds if there is no application or the application is rejected. For simplicity we assume that the agency's budget constraint does not bind. We do however impose a cost of finance on the agency, and show that the agency will reject applications. We also assume that public funding cannot be extended towards fixed costs nor external financing costs. In practice, variable costs are easier to allocate to a given project than fixed costs, and therefore more likely to be accepted by the agency. For example, the Finnish agency granting R&D subsidies (Tekes) has rules on eligible expenses and regularly does not accept all types of costs included by applications. In particular, the costs of raising external finance are non-eligible.

We focus on perfect Bayesian equilibria as will specified in Section 3.

2.1 R&D technology

A firm needs to incur both a variable cost $R \geq 1$ and a fixed cost $F \geq 0$ to undertake an innovation project (unless otherwise indicated all variables are project specific). Conditional on investing both R and F, the firm's expected discounted profits, gross of variable and fixed costs of R&D, and possible costs of applying for a subsidy, are given by

$$\pi = \alpha^{1-\gamma} \left(\frac{R^{\gamma} - 1}{\gamma} \right) \tag{2}$$

where $\alpha \geq 0$. As can be seen from equation (2), the gross profit function is related to the well-known Box-Cox transformation: when $\gamma \to 1$, the gross profit function becomes linear in R and when $\gamma \to 0$, a logarithmic gross profit function emerges. The reason for the parameter α being raised to the power $1 - \gamma$ becomes clear in Section 2.3: it allows for a derivation of an estimable R&D equation where γ can be identified. While α and γ are related, it is helpful to think of the former as a measure of the quality (productivity) of the project and the latter as an inverse measure of the concavity of profits in R&D.[6] It is plausible to think that firms and projects differ both in the quality and the concavity of project returns.

To keep our model well behaved we impose the following restriction on γ:

Assumption 1. $\gamma \in \left(-\frac{1}{g-1}, 1\right)$.

Here $g > 1$ is the shadow cost of public funds, placing an upper-bound on the concavity of the profit function. For example, if $g = 1.2$, Assumption 1 implies that $\gamma \in (-5, 1)$.

2.2 Financial markets

Since the firm has no liquid funds of its own and since the public agency (at maximum) subsidizes a fraction of the investment ex post, the firm must raise funding from financial markets for the R&D investment. A financing contract between the firm and its financier stipulates that the returns from

[6]More precisely, if $\pi(R)$ denotes gross profits as a function of R&D and π' and π'' its first and second derivatives, then $\gamma = 1 + R\pi''/\pi'$ provides an inverse measure of the relative concavity of the gross profit function.

the project are split according to

$$\pi = \pi^B + \pi^E, \tag{3}$$

where π^B and π^E denote the financier's and the firm's share of project returns (superscripts B and E stand for a "bank" and an “entrepreneur”). In our setting this return sharing rule accommodates both equity and debt contracts. The financiers in this model are passive, arm's length financiers rather than active early-stage investors.[7]

The market rate of return (the opportunity cost of financiers' funds) is $\rho \geqslant 1$. Following the corporate finance literature, we assume competitive financial markets with free entry of identical financiers with unlimited supply of funds. As a result, we can look for a financing contract that maximizes the firm's payoff subject to a financier's zero profit condition. Since external finance is costly $(\rho \geqslant 1)$, the firm wants to minimize the amount of funds raised from the market. The firm thus asks the financier to provide the part of the project funding that is not covered by the public agency. Since subsidies are paid ex post, the financier must first fund the whole investment $R + F$ but then gets the subsidy, if any, granted to the firm by the agency. Thus the firm offers a contract that satisfies the financier's zero profit condition

$$\Pi^B = \pi^B - \rho\,(R + F) + sR = 0, \tag{4}$$

where $s \in [0, \overline{s}]$, $\overline{s} < 1$, is the subsidy rate provided by the agency. The

[7] In Takalo, Tanayama and Toivanen (2010) we allow financiers to have a more active role in the funding of the project.

financier's share of project returns is then

$$\pi^B = \rho\,(R + F) - sR. \tag{5}$$

Equation (5) fully characterizes the terms under which a competitive financier is willing to fund the firm's R&D investment.

2.3 R&D investment

Since the firm raises all the funds for the investment from outside investors, the firm's profits can be obtained from equation (3) as $\pi^E = \pi - \pi^B$. After substitution of equations (2) and (5) for π and π^B this can be rewritten as

$$\pi^E(R, s) = \frac{\alpha^{1-\gamma}}{\gamma}\,(R^\gamma - 1) - (\rho - s)\,R - \rho F, \tag{6}$$

where $\rho - s$ captures the marginal cost of R&D, which is strictly positive given that $\rho \geqslant 1$ and $s \leq \overline{s} < 1$. Since the firm's objective function (6) is concave in R, the first-order condition

$$R^{**}(s) := \arg\max_{R \geq 1} \pi^E(R, s) = \alpha\,(\rho - s)^{\frac{1}{\gamma - 1}} \tag{7}$$

gives the firm's optimal variable investment $R^{**}(s)$, conditional on investing, as a strictly increasing function of the subsidy rate.

Defining $\alpha := \exp(\mathbf{X}\beta + \epsilon)$ and taking the natural log of both sides of equation (7) yields

$$\ln R^{**}(s) = \mathbf{X}\beta + \frac{1}{\gamma - 1}\ln(\rho - s) + \epsilon. \tag{8}$$

Clearly, this is identical to the generic R&D equation (1) where $g(R) = \ln R^{**}(s)$, $f(s) = \ln(\rho - s)$ and $\delta = 1/(\gamma - 1)$. When estimating (8), one should note that equation (8) is defined at the project level (γ and ρ may be assumed to be the same for all projects of a firm). As project level R&D is typically only observed for firms receiving subsidies, one has to take care of the selection into that group.[8]

The profits generated by the optimal investment given by equation (7) need to be large enough to ensure that the firm's participation constraint is satisfied, i.e., $\pi^E(R^{**}(s), s) \geq 0$ must hold. Otherwise, the firm does not invest. By substituting equation (7) for (6) we can rewrite $\pi^E(R^{**}(s), s) \geq 0$ as

$$\frac{1}{\alpha\gamma}\left[(\rho - s)^{\frac{\gamma}{\gamma-1}}(1 - \gamma) - \alpha^{-\gamma}\right] - \rho F \geq 0. \tag{9}$$

In what follows we call equation (9) the firm's *investment constraint.*

We may now write the firm's optimal investment decision as

$$R^*(s) = 1\left[\pi^E(R^{**}(s), s) \geq 0\right] R^{**}(s), \tag{10}$$

where $1\left[\pi^E(R^{**}(s), s) \geq 0\right]$ is an indicator function taking value one if equation (9) holds and zero otherwise, and where $R^{**}(s)$ is given by equation (7). An inspection of equations (8) and (9) reveals that equation (9) yields

[8]For this reason Takalo, Tanayama and Toivanen (2010, 2011) estimate a sample selection model.

a discrete choice estimation equation where the error term (ϵ) enters non-additively and non-separably. The investment constraint hence needs to be estimated using a simulation estimator.

2.4 Public funding

The agency's utility (e.g. social welfare) from the applicant's project is given by

$$U\left(R(s),s\right)=vR\left(s\right)+\pi^{E}\left(R(s),s\right)+\Pi^{B}-gsR\left(s\right) \tag{11}$$

where $g>1$ is, as mentioned, the constant opportunity cost of the public funds. As the second and third term on the right-hand side of equation (11) show, the firm's and financier's profits enter the agency's objective function. The first term on the right-hand side gives the agency specific returns from the project. That is, v captures the effects of the firm's R&D on the agency beyond the firm's and financier's payoffs and beyond the direct costs of subsidy. For example, v can include standard welfare externalities of R&D investments such as consumer surplus or technological spillovers, but it can also include private benefits from funding the project to the agency's civil servants. Note that v can also be negative, e.g., due to duplication of R&D costs, business stealing effects, or negative environmental externalities of the project. In what follows, we will call v "the spillover rate".[9]

As equation (11) shows the spillover rate v is assumed to be linear in the investment level R. This greatly facilitates the empirical implementation of

[9] Naturally, parts of v may be systematic across firms. For example, Takalo, Tanayama and Toivanen (2011) find that a one grade increase in the evaluated level of technical challenger of a project increases the subsidy rate in Finland by ten percentage points.

the model. While certainly strong, similar assumptions are common in the literature on growth and R&D spillovers.[10]

The spillover rate $v \in V$ also captures the type of the agency, with V being some finite type space. The agency type is drawn from a common knowledge distribution with probability density function $\phi(v)$ and cumulative density function $\Phi(v)$. As mentioned, v is known to the agency when it makes the subsidy decision but is unknown to the firm when it contemplates applying for subsidies. In words, the potential applicant is uncertain about how the agency, after screening the project proposal, sees the project and its potential to generate spillovers, consumer surplus, or other (positive or negative) externalities.

Since equation (4) implies $\Pi^B = 0$, we can write the agency's problem in stage two of the game as

$$\max_{s \in [0,\bar{s}]} U\left(R^*(s), s\right) = vR^*\left(s\right) + \pi^E\left(R^*\left(s\right), s\right) - gsR^*\left(s\right) \tag{12}$$

subject to equations (7), (9), and to the agency's participation constraint

$$U\left(R^*(s^*), s^*\right) \geq 0. \tag{13}$$

Equation (13) implies that the agency's benefits from the applicant's project should be non-negative when it grants an optimal positive subsidy rate ($s^* > 0$); otherwise we assume that the agency does not grant a positive subsidy ($s^* = 0$).

To characterize the optimal agency decisions, let us first ignore the firm's

[10]This assumption allows the existence of a steady state in endogenous growth models.

investment constraint (9), the agency's participation constraint (13), and the maximum and minimum constraints for s. By using the envelope theorem, and equations (6) and (7), the first-order condition for the agency's unconstrained problem can be written as

$$s^{**} := \arg\max_{s\in\mathbb{R}} U(R^{**}(s), s) = \frac{v - \rho(g-1)(1-\gamma)}{1+\gamma(g-1)}. \tag{14}$$

If the firm's investment constraint (9) binds but the agency's participation constraint (13) does not bind, the optimal subsidy rate is given by

$$\widehat{s} := \rho - \left(\frac{1-\gamma}{\rho F\gamma\alpha + \alpha^{-\gamma}}\right)^{\frac{1-\gamma}{\gamma}}. \tag{15}$$

Depending on parameter values, the agency's optimal subsidy decision s^* is 0, $\hat{s}$, s^{**} or $\bar{s}$. It is useful to characterize parameters in two dimensions, F and v. In particular, from equation (15) we see that if

$$F \leq \hat{F} := \frac{1}{\rho\gamma\alpha}\left[(1-\gamma)\,\rho^{\frac{\gamma}{\gamma-1}} - \alpha^{-\gamma}\right], \tag{16}$$

then the firms' investment constraint (9) never binds and, by implication, $\widehat{s} \leq 0$, i.e., fixed costs are so small that they affect neither the agency's nor the firm's decisions. In contrast, if

$$F > \bar{F} := \frac{1}{\rho\gamma\alpha}\left[(1-\gamma)\,(\rho-\bar{s})^{\frac{\gamma}{\gamma-1}} - \alpha^{-\gamma}\right], \tag{17}$$

fixed costs are so high that the firm will not invest even with a maximum subsidy rate $\bar{s}$. If $F \in \left(\hat{F}, \bar{F}\right]$, the firm will invest only if it receives a subsidy, and granting $\hat{s}$ becomes an option to the agency.

In empirical implementation, one could define $\upsilon := \mathbf{Z}\lambda + \eta$ where $\mathbf{Z}$ is a vector of control variables (such as firm and project characteristics, and may differ from $\mathbf{X}$), λ is a vector of parameters to be estimated, and η a random shock to the spillover. Then, equation (14) immediately yields an (two-limit Tobit) estimation equation that allows the identification of the parameters of the agency's utility function if condition (16) holds (see Takalo, Tanayama and Toivanen 2011, for details for the case $\rho = 1$ and $\gamma = 0$). Although the general case where condition (16) does not necessarily hold is more complicated, equation (14) still forms the basis for the estimation of the parameters of the agency utility function.

2.5 Firm's application decision

In stage one of the game, the firm has to decide whether or not to apply for a subsidy. If the firm does not apply, its discounted profits are

$$\Pi_N^E = max\left\{0, \pi^E\left(R^{**}\left(0\right), 0\right)\right\}, \tag{18}$$

where the subscript N indicates that the firm does not apply for a subsidy. The right-hand side of equation (18) shows how the firm has an option to invest in R&D even without a subsidy. To value this option to invest, the firm must calculate its profits in case it invests without a subsidy. Naturally the investment is made only if the firm's investment constraint (9) holds for $s = 0$.

The firm's expected discounted profits in case it applies for a subsidy are given by

$$\Pi_A^E = max\left\{0, E_v\pi^E\left(R^*\left(s^*\right), s^*\right)\right\} - K, \tag{19}$$

where $K > 0$ is the cost of applying for subsidies, $E_v\pi^E\left(R\left(s^*\right), s^*\right)$ are the expected gross profits conditional on applying and investing in R&D, and subscript A indicates that the firm applies for a subsidy. That is, the firm, when contemplating an application, must take expectation over all possible types of the agency, and then calculate all possible subsidy rates resulting from those agency types. Then the firm can calculate the expected costs of private sector external financing and its expected investment levels resulting from those subsidy rates, and, ultimately, its expected discounted profits resulting from those investments and subsidy rates. The first term on the right-hand side of equation (19) gives the value of the option to invest with a non-negative subsidy rate. The right-hand side as a whole in turn gives the expected value of applying for a subsidy.

The firm then applies for a subsidy only if the application constraint

$$\Pi_A^E - \Pi_N^E \geqslant 0 \tag{20}$$

holds. The firm's optimal application decision can then be expressed as an indicator function $d^* = 1\left[\Pi_A^E - \Pi_N^E \geqslant 0\right]$.

The exact form of the application constraint (20) depends on the size of fixed costs. If condition (16) holds, the firm will launch the project even without a subsidy. Since the optimal unconstrained subsidy rate (equation

(14)) is an increasing function of the spillover rate, the firm can calculate that the minimum constraint of zero on the subsidy rate binds for sufficiently low spillover rates: $v \leq \underline{v} := \rho\,(g-1)\,(1-\gamma)$. Similarly, the maximum constraint of $\overline{s}$ binds for high enough spillover rates $v \geq \overline{v} := \rho g - (\rho - \overline{s})\,[1+\gamma\,(g-1)]$. For $v \in (\underline{v}, \overline{v},)$, the subsidy rate is s^{**}. Because now $\Pi_N^E = \pi^E\,(R^{**}(0), 0)$ and

$$E_v \pi^E\,(R^{**}(s^*), s^*) = \Phi\,(\underline{v})\,\pi^E\,(R^{**}(0), 0)$$

$$+\int_{\underline{v}}^{\overline{v}} \pi^E\,(R^{**}(s^*), s^*)\,\phi\,(v)\,dv + (1-\Phi\,(\overline{v}))\,\pi^E\,(R^{**}(\overline{s}), \overline{s})$$

the application constraint (20) can be written as

$$\int_{\underline{v}}^{\overline{v}} \pi^E\,(R^{**}(s^{**}), s^{**})\,\phi\,(v)\,dv + (1-\Phi\,(\overline{v}))\,\pi^E\,(R^{**}(\overline{s}), \overline{s}) \qquad (21)$$

$$-\,(1-\Phi\,(\underline{v}))\,\pi^E\,(R^{**}(0), 0) \geq K,$$

and the firm's optimal application decision as $d^* = 1[\Pi_A^E - \Pi_N^E \geq 0 \,\Big|\, F \leq \hat{F}]$ that takes value one if condition (21) holds and zero otherwise.

If $F \in \left(\hat{F}, \bar{F}\right]$, the firm will not launch the project without a subsidy (equation (18) becomes $\Pi_N^E = 0$). From equations (14) and (15) the firm can observe that for sufficiently high spillover rates, $v \geq \widehat{v} := \rho g - (\rho - \widehat{s})\,[1+\gamma\,(g-1)]$, the investment constraint remains irrelevant for the decision making of the agency. If $v < \widehat{v}$, the firm knows that either it will receive a zero subsidy in which case it will not invest or it will receive subsidy $\widehat{s}$ that just satisfies the firm's investment constraint, which by definition also leads to the zero profits. As clearly $\underline{v} < \widehat{v} < \overline{v}$, the application constraint

(21) simplifies now to

$$\int_{\hat{v}}^{\overline{v}} \pi^E \left(R^{**}(s^{**})\right) \phi\left(v\right) dv + \left(1 - \Phi\left(\overline{v}\right)\right) \pi^E \left(\overline{v}\right) \geq K, \tag{22}$$

and the firm's optimal application decision is $d^* = 1[\Pi_A^E - \Pi_N^E \geq 0 \left| F \in \left(\hat{F}, \bar{F}\right]\right.]$ that takes value one if condition (22) holds and zero otherwise.

In contrast, if condition (17) holds, the firm will not invest even if it received the maximum subsidy rate $\bar{s}$. Therefore $\Pi_A^E - \Pi_N^E = -K$, and the firm will not apply for a subsidy, i.e., $d^* = 1[\Pi_A^E - \Pi_N^E \geq 0 \left| F > \bar{F}\right.] = 0$.

In empirical implementation, one could specify $K := \exp(\mathbf{Y}\theta + \sigma)$ where $\mathbf{Y}$ is a vector of control variables (and may partially differ from $\mathbf{X}$ and $\mathbf{Z}$), θ is a vector of parameters to be estimated, and σ is a random shock to the application costs. The application constraint (21) can then be simplified by using equations (6) and (8) and some algebra (for example taking logs of both sides) to

$$\mathbf{X}\beta - \mathbf{Y}\theta + \ln\left[-E_v \ln(1 - s^{**})\right] - \sigma + \epsilon \geq 0. \tag{23}$$

As explained in Takalo, Tanayama and Toivanen (2011), this equation forms the first stage of a traditional sample selection model (Tobit type II) where the second stage is the firm's R&D equation (8), allowing for the identification of the estimated application cost parameters θ. A similar albeit more complicated procedure can be used to recover the application cost parameters in the general case where condition (16) does not necessarily hold.

3 Equilibria

We complete the model by characterizing perfect Bayesian equilibria (PBE). In our model the firm's and financiers' posterior beliefs concerning the agency's type $v \in V$ after observing a subsidy decision are inconsequential, so there is no need to model the updating of beliefs. [11]

A strategy for the firm prescribes i) an application decision in stage one as a function of the expected payoff to applying $d : \mathbb{R} \rightarrow \{0, 1\}$ where "1" and "0" denote the apply and do-not-apply decisions, respectively, and ii) an R&D investment decision in stage four for each application, subsidy, and funding decision made in earlier stages, $R : \{0, 1\} \times [0, \bar{s}] \times [0, \infty) \rightarrow [0, \infty)$. A strategy for the agency maps its type and the firm's application decision into a subsidy rate, $s : V \times \{0, 1\} \rightarrow [0, \bar{s}]$. A strategy for a financier maps the firm's application decision and the agency's subsidy rate into terms of funding, $\pi^B : \{0, 1\} \times [0, \bar{s}] \rightarrow [0, \infty)$.

A PBE in our model satisfies the following four standard criteria: 1) the firm's prior belief about the agency's type describes a rational assessment of how the agency values the firm's project. Such a rational prior belief is fully depicted by $\phi(v)$ and $\Phi(v)$; 2) the firm's strategy is $d^* = 1[\Pi_A^E - \Pi_N^E \geq 0]$, and $R^*(s)$ as given by equation (10); 3) the financier earns zero profits, i.e., $\pi^{B*}(s)$ is given by equation (5); and 4) if $d^* = 1$, the agency's strategy is $s^*(v) = \{0, \hat{s}, s^{**}, \bar{s}\}$ where s^{**} and $\hat{s}$ are given by equations (14) and (15), respectively, and if $d^* = 0$, $s^*(v) = 0$ for all v.

[11] Such updating would be an essential feature of a dynamic model where the firm or financiers would learn something about the agency's type when making sequential applications over time. This constitute an interesting but challenging topic for further research.

Furthermore, as mentioned in Section 2, we assume that whenever the agency rejects an application, it grants no subsidy. More formally, we impose an additional criterion on the agency's strategy: 5) for those v that make a rejection of an application optimal for the agency, $s^*(v) = 0$. [12]

Even under these five restrictions, there are multiple equilibrium outcomes depending on the parameter values. Fortunately, for a given pair of values of F and v, we have a unique equilibrium.

Proposition 1. *For given F and v there is a unique PBE with the following properties:*

i) Suppose $F \leq \hat{F}$. Then $R^*(s) = R^{**}(s)$ and $\pi^{B*}(s) = \rho(R^{**}(s) + F) - sR^{**}(s)$ for all d and v. If equation (21) holds, $d^* = 1$. Otherwise, $d^* = 0$ and $s^*(v) = 0$. If $d^* = 1$, $s^*(v) = 0$ for $v \leq \underline{v}$, $s^*(v) = s^{**}$ for $v \in (\underline{v}, \overline{v})$, and $s^{**} = \overline{s}$ for $v \geq \overline{v}$.

ii) Suppose $F \in \left(\hat{F}, \overline{F}\right]$. Then $\pi^{B*}(s) = \rho(R^*(s) + F) - sR^*(s)$ for all d and v. If equation (22) holds, $d^* = 1$. Otherwise, $d^* = 0$, $s^*(v) = 0$, and $R^*(0) = 0$. If $d^* = 1$, $s^*(v) = 0$ for $v < v^0$, $s^*(v) = \widehat{s}$ for $v \in [v^0, \widehat{v}]$, $s^*(v) = s^{**}$ for $v \in (\widehat{v}, \overline{v})$, and $s^*(v) = \overline{s}$ for $v \geq \overline{v}$. $R^*(0) = 0$ for $v < v^0$ and $R^*(s) = R^{**}(s)$ for $v \geq v^0$.

iii) Suppose $F > \overline{F}$. Then for all v, $d^* = 0$, $s^*(v) = 0$, $\pi^{B*}(0) = \rho(R^*(0) + F)$ and $R^*(0) = 0$.

[12] In theory, any $s \in [0, \hat{s})$ amounts to a rejection of an application, and the agency is indifferent among $s \in [0, \hat{s})$ whenever it finds it optimal to reject an application. Hence, without this additional restriction on the agency strategy, there would be a continuum of optimal agency decisions for such v that make a rejection optimal. Our understanding is that in practice, an agency grants a zero subsidy for applications it wants to reject. Note also that $s^*(v) = 0$ for all v if $d^* = 0$ comes from the practice; as mentioned, the agency cannot grant a positive subsidy rate unless it receives an application.

Proof: In the Appendix.

4 The Econometric Model

Let us briefly summarize the econometric model suggested by the theoretical model in Section 2 (for more details we refer the reader to Takalo, Tanayama and Toivanen 2010, 2011). As shown, our theoretical model yields three main estimation equations, defined at the project level: the firm's R&D and application equations (8) and (23), and the agency's subsidy equation (14) (with $v := \mathbf{Z}\lambda + \eta$). The primitives of this econometric model are 1) the parameters of the firm's profit function (β, γ, F), 2) the parameters of the agency's utility function (λ, g), 3) the parameters of the financiers' payoff function (ρ), 4) the parameters of the cost of application (θ) and 5) the parameters of the distributions of the shocks (ϵ, η, σ).[13]

In our model the generic R&D equation (1) is a first-order condition. The model directly suggests how to interpret the parameters and enter the subsidy rate into the estimation equation. The β vector of parameters shifts the quality of the project (α) and the error term ϵ is a shock to the quality of the project, observed by the firm, but unobserved to the econometrician, while the "additionality" parameter δ is related to the measure of the concavity of the gross profit function (γ).

Our theoretical model imposes specific functional forms on the generic R&D equation (1). However, any empirical implementation of equation (1)

[13]In addition one should take into account the process whereby the agency grades each application it receives. This grading process is irrelevant for the purposes of this paper and we have therefore for simplicity omitted it (see Takalo, Tanayama, Toivanen, 2011, for details).

needs to impose some functional form restrictions: the generic equation is not identified with unknown functions $g(R)$ and $f(s)$. Our theoretical model ensures that the functional forms are consistent with optimizing behavior. It is surely possible to build other theoretical models of public support of private R&D investments that yield different variants of equation (1).

Our two other main estimation equations are also based on optimization: The application equation assumes profit maximization on the part of the firm, and that the firm knows the decision rule of the agency, while being unsure about the spillover rate (v). A structural interpretation of the agency decision rule necessitates that the assumptions underlying the objective function of the agency are accepted, but one may remain agnostic about the interpretation of v. Taking a stand on what v captures is only needed in welfare or counterfactual analyses as in Takalo, Tanayama and Toivanen (2010).

5 Implications

Our theoretical model is simple, but nonetheless provides a number of implications for the design of R&D support policies and their econometric analysis. In this paper we focus on analyzing the rationales for subsidy policy and the question of additionality.

5.1 Rationales for Subsidy Policy

Public support to private R&D is typically justified by appropriability problems and financial market frictions. In our model these are captured by the parameters v and ρ that reflect R&D spillovers and the cost of external finance.

As equation (14) shows, the optimal unconstrained subsidy rate s^{**} is an increasing function of the spillover rate v as expected, but a *decreasing* function of the cost of external finance ρ (recall that $g > 1 > \gamma$). The explanation for the latter result comes from equation (7) which shows that the marginal effect of the subsidy rate on the firm's R&D at the intensive margin is decreasing in the cost of external funding (i.e., $\partial R^{**}(s)^2/\partial s \partial \rho < 0$). Because $R^{**}(s)$ is decreasing in ρ and increasing in s, a higher cost of external finance will also lead to a lower optimal subsidy amount $s^{**}R^{**}(s^{**})$ at the intensive margin.

At the extensive margin, however, the effect of external financing cost is the reverse: differentiation of the optimal constrained subsidy rate $\widehat{s}$ from equation (15)) with respect to ρ reveals a positive relationship.

Our model also suggests that the design of optimal subsidy policy crucially depends on the firms' production technology parameters (α, γ, F). For example, it can be shown that the optimal unconstrained subsidy rate s^{**} is an increasing function of γ.

5.2 Existence of Additionality

The central object of interest in the literature has been the question of whether or not an R&D support policy leads to additionality. We define that an increase in a subsidy rate generates additionality if and only if

$$R(s_1) - R(s_0) > s_1 R(s_1) - s_0 R(s_0) \tag{24}$$

holds. Here s_0 and s_1 are the pre-increase and post-increase subsidy rates with $s_1 > s_0$, and $R(s_0)$ and $R(s_1)$ are the corresponding levels of R&D investment. On the left-hand side of condition (24) we have an increase in private R&D generated by the increased subsidy rate. On the right-hand side we have the ensuing increase in the monetary amount of government support.

We immediately observe that there is no additionality unless the firm invests in R&D at least when obtaining s_1. In what follows we assume that this is the case. It is then clear that there is always additionality at the extensive margin as then $R(s_0) = 0$ and $R(s_1) > 0$ by definition. The size of the average additionality effect at the extensive margin depends on how many firms are able to launch an R&D project because they receive a subsidy.

We then use our model to evaluate the existence of additionality at the intensive margin. For brevity, we consider a marginal increase in the subsidy rate. Let $\Delta s := s_1 - s_0$ and rewrite condition (24) as

$$R(s_0 + \Delta s) - R(s_0) > (s_0 + \Delta s) R(s_0 + \Delta s) - s_0 R(s_0).$$

Dividing both sides of the inequality by Δs, then letting $\Delta s \to 0$ and using the definition of the derivative yields

$$R'(s) > R + sR'(s).$$

Substituting $R^{**}(s)$ and dR^{**}/ds from equation (7) for R and $R'(s)$ in the inequality and then simplifying gives

$$\gamma > \gamma^A := \frac{\rho - 1}{\rho - s}. \tag{25}$$

where the superscript A stands for additionality. Equation (25) gives the necessary and sufficient condition for the existence of additionality at the intensive margin in our model. It shows that the firm's gross profit function cannot be too concave for there to be additionality: a necessary condition for additionality is that γ is non-negative. For example, if profits are logarithmic in R&D (i.e., $\gamma = 0$) and there is a positive cost of finance (i.e., $\rho > 1$), there is necessarily some crowding out.

Condition (25) also shows that the threshold degree of concavity for additionality is an increasing function of the (endogenous) subsidy rate. In case firms get maximum subsidies (and s would be $\bar{s}$ in equation (25)), we immediately observe that a higher (maximum) subsidy rate is less likely to generate additionality. An analogous conclusion holds for the optimal unconstrained subsidy rate: substituting s^{**} from equation (14) for s in condition (25) and simplifying yields

$$\gamma > \frac{\rho - 1}{\rho - 1 + g - v}. \tag{26}$$

Because s^{**} is increasing in the spillover rate v, additionality becomes less likely with higher spillovers. [14]

The results arise because there are necessarily decreasing returns to R&D for a firm at the intensive margin: When the agency increases the subsidy rate to induce the firm to invest more (e.g. because v is large), the increase in R&D

[14] If v is large enough, the subsidy rate is given by $\bar{s}$, and condition (25) applies.

is smaller for every further increase of the subsidy rate (i.e., $\partial R^{**}(s)^2/\partial s^2 < 0$) .

5.3 Interpretation of Additionality Parameters

The standard approach of measuring additionality usually results in an estimated additionality parameter, or as, e.g., in Czarnitzki and Lopes-Bento (2012), a few additionality parameters, that are conditioned on an observable measure of firm type. Our model provides several new insights into the interpretation of the additionality parameters reported in the literature.

First, the response to a support decision at both the intensive and the extensive margin depends on the R&D production technology and is therefore heterogeneous if firms have different production technologies. Second, whenever a dummy variable for receiving a subsidy is used in estimations, the reaction of corporate R&D to the level of support cannot be separated from its reaction to getting (versus not getting) support. Such separation would however be desirable since the additionality effects of subsidies may be different at the intensive and at the extensive margin. Further, if the treatment effects at the two margins are not separated, the estimated additionality parameters are some weighted average over those firms treated at the extensive margin and those treated at the intensive margin. While such an average parameter is certainly informative, it is may be vulnerable to misinterpretation, e.g., when a large fraction of firms is at the extensive margin.

Finally, the interest in additionality arises from the idea that additionality is at least a necessary, if not a sufficient, condition for a welfare-improving subsidy policy. Our model however shows that those projects that generate higher spillovers (and therefore receive higher subsidy rates) are less likely to exhibit additionality at the intensive margin than projects generating smaller spillovers. This suggests that the relation between additionality and welfare may not be clear cut.

To study the issue further we characterize the conditions when a marginal increase in a subsidy rate increases welfare at the intensive margin in our model, and relate this to the conditions for the existence of additionality established in the previous subsection. The way to do this in our modeling framework is to reinterpret equation (**11**) as the social welfare function, and to assume that the agency is not maximizing this objective function because of divergent objectives (e.g. career objectives of the civil servants). A straightforward derivation of the function (**11**) with respect to s using the envelope theorem and equations (6) and (7) as in Section 2.4. reveals that $dU(R^{**}(s), s)/ds > 0$ if

$$\gamma > \gamma^W := \frac{s + \rho(g-1) - v}{(\rho - s)(g-1)}, \tag{27}$$

where the superscript W stands for a welfare improvement.

According to our model, finding additionality from a marginal increase in a subsidy rate is a necessary condition for a welfare improvement if $\gamma^A \leq \gamma^W$. Using equations (25) and (27) and simplifying, $\gamma^A \leq \gamma^W$ becomes

$$s \geq v + 1 - g. \tag{28}$$

Equation (28) shows that establishing whether or not additionality is a sufficient (but not a necessary) or a necessary (but not a sufficient) condition for a welfare improvement at the intensive margin requires knowledge of project level structural parameters.

The conclusion does not change even if we consider a welfare maximizing agency. Then the agency increases the subsidy rate if this is optimal. However, our results in the previous subsection suggest that such an increase does not need to lead to additionality: since the agency benefits at the margin more from R&D than the firm if there are positive spillovers, it may be willing to encourage the firm to invest more even if there were some crowding out. Thus it might be possible that no evidence for additionality can be found in a country where R&D projects create large spillovers (and where the support is therefore optimally extensive), while additionality is found in a country with small spillovers (and small subsidies) but otherwise the same R&D production technology.

6 Conclusions

Explicit modeling of R&D support policies has potential advantages in empirical work: it makes the mechanisms affecting the dependent variables clear, helps in uncovering the critical assumptions for identification of the model, and allows for identification of a number of objects of policy interest that might otherwise remain elusive. Our results in this paper also throw light on the alleged rationales for innovation support policies, and suggest that they do not always provide a foundation for more extensive government intervention: We find that in our model higher costs of external finance provide a reason to increase subsidies at the extensive margin but, contrary to what is often thought, suggest a reason to decrease subsidies at the intensive margin.

We also show that the degree of additionality is most likely firm- or even project-specific and depends on the production technology parameters. Further, the degree of additionality at the intensive margin inversely depends on the spillover rate. Therefore, additionality may not be observed for those projects which generate the largest benefits from public support. This suggests caution in interpreting the estimated additionality parameters in the received literature in terms of welfare effects of the existing policy.

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7 Appendix: The Proof of Proposition 1

Part i). When $F \leq \hat{F}$, condition (9) does not bind. The firm is able to invest in R&D in stage four even without a subsidy, i.e., from equation (10) $R^{*}(s) = R^{**}(s)$ for all v and d. The firm's best-reply function $R^{**}(s)$ as given by equation (7) is well-behaving since

$$\frac{\partial^2 \pi^E}{\partial R^2} = (\gamma - 1)\alpha^{1-\gamma} R^{\gamma-2} \tag{29}$$

is negative (recall that $\gamma < 1$ by Assumption 1). By implication, the firm is able raise external funding in stage three according to the terms given by equation (5), i.e., $\pi^{B*}(s) = \rho\left(R^{**}(s) + F\right) - sR^{**}(s)$ for all v and d.

In stage two, the agency chooses $s \in [0, \bar{s}]$ to maximize $U(R^{**}(s), s)$ conditional on its v and $d = 1$. We want to prove that for each $v \in V$, there is a unique optimal subsidy rate $s^{*}(v)$. Since $U(R^{**}(s), s)$ is continuous and we have linear constraints of minimum and maximum subsidies it suffice to show that $U(R^{**}(s), s)$ is concave when evaluated at the interior solution, $s = s^{**}$, i.e., we want to show that $d^2U(R^{**}(s), s)/ds^2 \mid_{s=s^{**}} < 0$.

Note first from equation (7) that

$$R' := \frac{dR^{**}}{ds} = \frac{\alpha\left(\rho - s\right)^{\frac{1}{\gamma-1}-1}}{1-\gamma} = \frac{R^{**}}{\left(1-\gamma\right)\left(\rho - s\right)} \tag{30}$$

and

$$R'' := \frac{d^2R^{**}}{ds^2} = \frac{\left(2-\gamma\right)\alpha\left(\rho - s\right)^{\frac{1}{\gamma-1}-2}}{\left(1-\gamma\right)^2} = \frac{\left(2-\gamma\right)R^{**}}{\left[\left(1-\gamma\right)\left(\rho - s\right)\right]^2}. \tag{31}$$

Then, we differentiate $U(R^{**}(s), s) = vR^{**}\left(s\right) + \pi^E\left(R^{**}\left(s\right), s\right) - gsR^{**}\left(s\right)$

twice with respect to s. Suppressing all function arguments for brevity, the first differentiation of U with respect to s gives

$$\frac{dU}{ds} = vR' + \frac{\partial \pi^E}{\partial R}R' + \frac{\partial \pi^E}{\partial s} - gR^{**} - gsR',$$

and the second differentiation yields

$$\frac{d^2U}{ds^2} = vR'' + \frac{\partial^2 \pi^E}{\partial^2 R}(R')^2 + \frac{\partial \pi^E}{\partial R}R'' + \frac{\partial^2 \pi^E}{\partial s^2} + 2\frac{\partial^2 \pi^E}{\partial R \partial s}R' - 2gR' - gsR''. \quad (32)$$

Now, $\partial \pi^E / \partial R = 0$ by the envelope theorem, and from (6) we get that $\partial^2 \pi^E / \partial s^2 = 0$ and $\partial^2 \pi^E / \partial R \partial s = 1$. By using these insights, equation (32) simplifies to

$$\frac{d^2U}{ds^2} = (v - gs)R'' + \frac{\partial^2 \pi^E}{\partial^2 R}(R')^2 + (1-g)2R.$$

Inserting equations (29), (30) and (31) into the right-hand side gives

$$\frac{d^2U}{ds^2} = \frac{R}{(1-\gamma)(\rho - s)}\left\{\frac{(2-\gamma)(v-gs)}{(1-\gamma)(\rho-s)} + \left[2(1-g) - \frac{\alpha^{1-\gamma}R^{\gamma-1}}{\rho - s}\right]\right\}.$$

Using equation (7) to substitute $\rho - s$ for $\alpha^{1-\gamma}R^{\gamma-1}$ this further simplifies to

$$\frac{d^2U}{ds^2} = \frac{R}{(1-\gamma)(\rho - s)}\left[\frac{(2-\gamma)(v-gs)}{(1-\gamma)(\rho-s)} + 1 - 2g\right].$$

Then, substituting s^{**} from equation (14) for s in the term in the square brackets shows that the term is negative when $1 + \gamma(g-1) > 0$. This holds under the parameter restrictions imposed by Assumption 1. This suffice to prove that $d^2U(R^{**}(s),s)/ds^2 \mid_{s=s^{**}} < 0$. Consequently, equation (14) characterizes the unique type-contingent maximum for the agency's unconstrained decision problem.

Because $U(R^{**}(s),s)$ is continuous, constraints of minimum and maximum subsidies are linear, and the optimal unconstrained subsidy $s^{**}(v)$ is increasing in v (see equation (14)), the optimal subsidy rate is given by $s^{*}(v) = 0$ for $v \leq \underline{v}$, $s^{*}(v) = s^{**}$ for $v \in (\underline{v}, \bar{v})$, and $s^{*}(v) = \bar{s}$ for $s \geq \bar{s}$. This is the optimal subsidy allocation rule given $d = 1$. If the agency does not receive an application ($d = 0$), $s^{*}(v) = 0$ for all v by assumption. Thus, the agency's optimal subsidy allocation rule in stage two is a function $s^{*} : \{0,1\} \times V \rightarrow \{0, s^{**}, \bar{s}\}$, i.e., conditional on v and d, the action of the agency in stage two is unique.

In stage one the firm decides whether to apply or not given $\phi(v)$, $s^{*}(v)$, and $\pi^{B*}(s^{*})$. Since in a PBE the firm's choice must maximize the profits and the firm's beliefs must be consistent with the agency's strategy, $d^{*} = 1$ only if condition (21) holds and $d^{*} = 0$ otherwise. Clearly, the agency's best response to $d^{*} = 1$ is $s^{*}(v) = \{0, s^{**}, \bar{s}\}$, and $d^{*} = 0$ implies $s^{*}(v) = 0$ for all v. Thus, we have found a PBE that satisfies the five equilibrium criteria defined in Section 3. Since the utility maximizing action in each stage of the game is unique for each $v \in V$, the equilibrium is also unique.

ii) When $F \in \left(\hat{F}, \bar{F}\right]$, the firm will be able to raise funding and invest in stage four only if it gets a subsidy rate which is at least $\hat{s}$. Conditional on $s^{*}(v) > \hat{s}$, the proof is identical to step i) above. We may focus on the range of parameter values where $s^{*}(v) \leq \hat{s}$. From equations (14) and (15) we see that $s^{*}(v) \leq \hat{s}$ for $v \leq \hat{v}$ where $\widehat{v} := \rho g - (\rho - \widehat{s})\left[1 + \gamma\left(g - 1\right)\right]$ equalizes $s^{**} = \hat{s}$.

For $v \leq \hat{v}$, $s^{**}(v) \leq \hat{s}$. The firm is not able to invest if $s = s^{**}$ since the cost of finance $\pi^{B*}\left(s^{**}\right)$ would be prohibitively high. Therefore, $s^{*}(v) = \hat{s}$, implying $R^{*}(\hat{s}) = R^{**}\left(\hat{s}\right) > 0$ and $\pi^{B*}\left(\hat{s}\right) = \rho\left(R^{**}(\hat{s}) + F\right) - \hat{s}R^{**}(\hat{s})$, might constitute a pair of optimal agency and financier decisions for $v \leq \hat{v}$. But this requires that the agency's participation constraint (13) holds, i.e.,

$$U(R^{**}(\hat{s}), \hat{s}) = vR^{**}(\hat{s}) + \pi^{E}(R^{**}(\hat{s}), \hat{s}) - g\hat{s}R^{**}(\hat{s}) \geq 0.$$

Because by definition $\pi^{E}(R^{**}(\hat{s}), \hat{s}) = 0$, we have $U(R^{**}(\hat{s}), \hat{s}) \geq 0$ if $v - g\hat{s} \geq 0$. Inserting $\hat{s}$ from equation (15) into the this condition yields

$$v - g\rho - g\left(\frac{1-\gamma}{\rho F\gamma\alpha + \alpha^{-\gamma}}\right)^{\frac{1-\gamma}{\gamma}} \geq 0. \tag{33}$$

Let $v^{0} := g\rho + g\left[\left(1-\gamma\right)/\left(\rho F\gamma\alpha + \alpha^{-\gamma}\right)\right]^{\frac{1-\gamma}{\gamma}}$. Then if $v^{0} \leq \hat{v}$, there exist $v \in [v^{0}, \hat{v}]$ such that $s = \hat{s}$ is both optimal and feasible for the agency. From the definitions of v^{0} and $\hat{v}$, $v^{0} \leq \hat{v}$ if

$$\rho g - (\rho - \widehat{s})\left[1 + \gamma\left(g - 1\right)\right] \geq g\rho + g\left(\frac{1-\gamma}{\rho F\gamma\alpha + \alpha^{-\gamma}}\right)^{\frac{1-\gamma}{\gamma}}.$$

After substitution of $\hat{s}$ from equation (15) for the above inequality, the inequality simplifies to $\gamma \leq 1$. This holds under Assumption 1. As a result, $s^{*}(v) = \hat{s}$ constitutes the optimal agency decision for $v \in [v^{0}, \hat{v}]$.

For $v < v^{0}$, any $s \in [0, \hat{s})$ would result in prohibitively high cost of finance $\left(\pi^{B*}(s) = \rho\left(R^{*}(s) + F\right) - sR^{*}(s)\right)$, and thus in $R^{*}(s) = 0$ and $U(R^{*}(s), s) =$

0. Our fifth criterion for PBE stipulates that in this case $s^*(v) = 0$.

As a result, we have shown that when $F \in \left(\hat{F}, \bar{F}\right]$ and $d = 1$, $s^*(v) = 0$ for $v < v^0$, $s^*(v) = \hat{s}$ for $v \in [v^0, \hat{v}]$, $s^*(v) = s^{**}$ for $v \in (\hat{v}, \bar{v})$, and $s^*(v) = \bar{s}$ for $v \geq \bar{v}$. If the agency does not receive an application $(d = 0)$, $s^*(v) = 0$ for all v. Therefore, the agency's optimal subsidy rate decision in stage two is a function $s^* : \{0, 1\} \times V \to \{0, \hat{s}, s^{**}, \bar{s}\}$.

In stage one the firm decides whether to apply or not given s^*, $\phi(v)$, and π^{B*}. Since in a PBE the firm's choice must maximize the profits and the firm's beliefs must be consistent with the agency's strategy, $d^* = 1$ only if condition (22) holds and $d^* = 0$ otherwise. Clearly, the agency's best response to $d^* = 1$ is $s^*(v) = \{0, \hat{s}, s^{**}, \bar{s}\}$ and to $d^* = 0$, $s^*(v) = 0$ for all v, so we have found a PBE. Since the utility maximizing action in each stage of the game is unique for each $v \in V$, the equilibrium is also unique.

iii) When $F > \bar{F}$, the agency will reject any application since it knows that the firm would not be able to raise funding and invest even if it received a maximum feasible subsidy rate $\bar{s}$. In theory, when $F > \bar{F}$, all feasible subsidy levels $s \in [0, \bar{s}]$ amount to a rejection of an application. However, our fifth criterion for PBE stipulates that in this case $s^*(v) = 0$ for all v. Since condition (17) is independent of v, the firm knows when $F > \bar{F}$. Hence the firm does not apply for a subsidy it will not receive for sure, i.e. $d^* = 0$. But $F > \bar{F}$ implies by construction that market funding without a subsidy becomes so expensive $(\pi^{B*}(0) = \rho(R^*(0) + F))$ that the firm cannot profitably raise funding and invest, i.e., $R^*(0) = 0$.■

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