Patrick M Crowley

Long cycles in growth: explorations using new frequency domain techniques with US data

SUOMEN PANKKI · FINLANDS BANK ·

EUROJÄRJESTELMÄ
EUROSYSTEMET

Bank of Finland Research
Discussion Papers
6 • 2010

Suomen Pankki
Bank of Finland
PO Box 160
FI-00101 HELSINKI
Finland
☎ +358 10 8311

http://www.bof.fi
E-mail: Research@bof.fi

Bank of Finland Research
Discussion Papers
6 • 2010

Patrick M Crowley*

Long cycles in growth: explorations using new frequency domain techniques with US data

The views expressed in this paper are those of the author and do not necessarily reflect the views of the Bank of Finland.

* College of Business, Texas A&M University, Corpus Christi, TX, USA. Email: patrick.crowley@tamucc.edu.

Douglas Maraun (University of East Anglia, UK) is to be thanked for providing Matlab code for the CWT plots. Karin Blank (NASA) is to be thanked for providing assistance in interpreting HHT/EMD results and providing marginal spectrum Matlab code. This research is part of continuing research projects using frequency domain methods that began during a visit to the Bank of Finland in 2004-2005. The Bank of Finland is to be thanked for their continuing support and their kind hospitality during this and subsequent visits.

http://www.bof.fi

ISBN 978-952-462-586-9
ISSN 0785-3572
(print)

ISBN 978-952-462-587-6
ISSN 1456-6184
(online)

Helsinki 2010

Long cycles in growth: explorations using new frequency domain techniques with US data

Bank of Finland Research Discussion Papers 6/2010

Patrick M Crowley
Monetary Policy and Research Department

Abstract

In his celebrated 1966 Econometrica article, Granger first hypothesized that there is a 'typical' spectral shape for an economic variable. This 'typical' shape implies decreasing levels of energy as frequency increases, which in turn implies an extremely long cycle in economic fluctuations and therefore in growth. Spectral analysis is however based on certain assumptions that render these basic frequency domain techniques inappropriate for analysing non-stationary economic data. In this paper three recent frequency domain methods for extracting cycles from non-stationary data are used with US real GNP data to analyse fluctuations in economic growth. The findings, among others, are that these more recent frequency domain techniques neither provide evidence to support the 'typical' spectral shape, nor an extremely long growth cycle following Granger.

Keywords: business cycles, growth cycles, frequency domain, spectral analysis, long cycles, Granger, wavelet analysis, Hilbert-Huang Transform (HHT), empirical mode decomposition (EMD), non-stationarity

JEL classification numbers: C13, C14, O47

Ovatko Yhdysvaltain talouden pitkät kasvujaksot vain tilastollisten menetelmien synnyttämiä mielikuvia?

Suomen Pankin keskustelualoitteita 6/2010

Patrick M. Crowley
Rahapolitiikka- ja tutkimusosasto

Tiivistelmä

Taloustieteen nobelisti Clive Granger esitti reilut 40 vuotta sitten, että taloudellisen aikasarjan vaihtelu syntyy taloudellisille muuttujille ominaisten jaksottaisten komponenttien vaikutuksesta. Tällä Grangerin ajatuksella taloudellisen sarjan spektrin tyypillisestä muodosta on ollut huomattava vaikutus aikasarjaekonometriassa. Spektrin tyypillinen muoto tarkoittaa, että trendi hallitsee aikasarjan vaihtelua ja että lyhyen aikavälin vaihteluiden merkitys on suhteellisesti pienempi. Spektraalianalyysin soveltaminen epästationaaristen taloudellisen aikasarjojen vaihtelun hajotelmaan ei kuitenkaan ole perusteltua. Tässä tutkimuksessa sovelletaan kolmea suhteellisen uutta taajuusalueen menetelmää Yhdysvaltain bruttokansantuotteen kasvujaksojen estimointiin. Näillä menetelmillä saadut tulokset eivät tue Grangerin väitettä spektrin tyypillisestä muodosta eivätkä näin ollen tästä seuraavia äärimmäisen pitkiä talouskasvun trendejä.

Avainsanat: suhdannevaihtelut, kasvujaksot, taajuusalue, spektraalianalyysi, Granger, väreanalyysi, Hilbertin-Huangin muunnos, empiirisen tyyppiarvon hajotelma, epästationaarisuus

JEL-luokittelu: C13, C14, O47

Contents

The existence of a typical spectral shape suggests the following law (stated in nonrigorous but familiar terms): *The long-term fluctuations in economic variables, if decomposed into frequency components, are such that the amplitudes of the components decrease smoothly with decreasing period.* (page 155)

Granger (1966)

1 Introduction

Of all macroeconomic variables, perhaps the most important in our discipline is that of economic growth, and as macroeconomists we still think of growth in cyclical terms, particularly in reference to the 'business cycle'. But there are many other fluctuations apparent in growth, perhaps emanating from specific sectors or industries (– an idea that originated in the real business cycle literature), with the strength of the linkages between industries or sectors assumed to determine when these cycles might spillover, as they have done in the current economic downturn, to affect the entire macroeconomy. These shorter cycles are not recognized as important in the orthodox macroeconomics literature, which instead focuses on (mostly productivity) 'shocks'. Conversely, there is a tacit assumption in economics that a longer cycle exists (– sometimes referred to as a 'medium term cycle': see Comin and Gertler, 2003), a cycle that is assumed to be driven by technological change which occurs over several decades. At the extreme, some economists still argue that a very long cycle in growth exists with periodicity of at least 50 years.

In the economics literature frequency domain techniques were first introduced in a celebrated paper by Granger (1966), and the techniques used in his paper are still practiced by most economists. Indeed the quote which begins this paper is taken from Granger's original 1966 contribution, and implies that as period decreases (– in other words frequency increases), the amplitude or strength of fluctuations should decrease. Granger went much further than just stating this as a general rule, and thought that this pattern could be thought of as a 'law', which gives rise to a 'typical' shape for the spectrum of any economic variable. The implication from Granger's work is that there is an extremely long cycle at work in macroeconomic fluctuations. Indeed even recent work in economics has continued to make this assumption, and yet if more recent frequency domain techniques are used, as shown in this paper, Granger's results appear to no longer hold true.

This paper seeks to explore some of the above issues using the simple metric of the growth in US real GNP. To preview the general conclusions reached in the paper (among others), they are three-fold: firstly the business cycle does not appear to reside in one particular frequency mode in the frequency domain; secondly, the assumptions about the existence of a long cycle in growth (suggested in the opening quote from Granger's paper) are not borne out by the results obtained here; and thirdly, the transformation of real GNP reveals very different results in terms of identifiable cycles in the frequency domain.

The paper is organized into eight sections. Section 2 gives a short overview and presents the data used in the paper. Section 3 introduces spectral analysis, while section 4 introduces some of the more recent frequency domain techniques now in wide usage. Section 5 outlines both the discrete and continuous wavelet analysis approaches, while section 6 introduces and applies the empirical mode decomposition technique. Section 7 discusses and compares the findings of the different frequency domain techniques and lastly section 8 concludes.

2 Overview and data

2.1 Overview

Identifying the different frequencies at work in economic variables was pioneered by economists working in the early part of the last century. Economists such as Kitchin (1923), Keynes (1936), Schumpeter (1939), Mitchell (1946), and Burns and Mitchell (1946) studied the cycles in real GDP to determine how these cycles might interact and to assess what was driving these cycles. Zarnowitz and Moore (1986) (p. 522) neatly summarizes Schumpeter (1939)'s lengthy discussion (in volume 1, chapter 4) of the different types of cycles that early business cycle theorists hypothesized, namely:

i) the Kitchin (about 2 to 4 years), which were supposedly related to inventory investment;

ii) the Juglar (about 7 to 10 years), which roughly correspond to our current business cycle;

iii) the Kuznets (about 15 to 25 years), which purportedly relates to changes in factor growth and infrastructure cycles; and

iv) the Kondratieff (about 48 to 60 years), which was originally related to large swings in prices and perhaps technology (see Kondratieff, 1984).

Schumpeter (1939) even went as far as to construct a cyclical scheme such that there would be 3 Kitchin's per Juglar and 6 Juglars per Kondratieff. Nevertheless, other economists (notably Burns and Mitchell, 1946) could find little evidence for such a scheme and settled on a specific form of the Juglar cycle which he labelled the 'business cycle' and this terminology of the phases of the business cycle (expansion and recession) persists to this day. Other cycles have also been proposed, most notably the Wardwell, Spiethoff and Hicks cycles (see Hicks, 1950).

The first systematic frequency domain analysis of cycles in growth data was by Granger and Hatanka (1964)[1] with commentary by Adelman (1965) and then the celebrated article by Granger (1966) appeared in *Econometrica.*

[1] Crum's 1923 spectral analysis of monthly New York commercial paper rates is mentioned in Schumpeter (1939), where it was found that a prominent 40 month cycle appeared to exist so frequency domain analysis has a long lineage in economics.

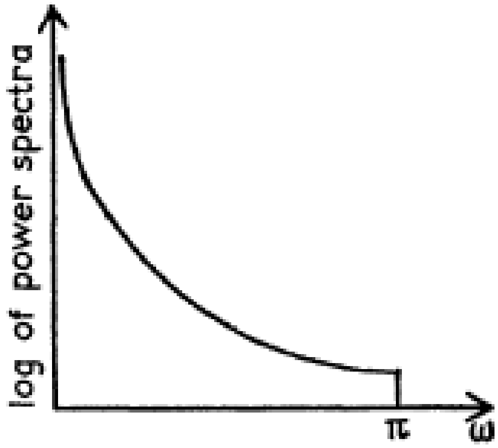

Figure 1: **'Typical' spectral shape of an economic variable (taken from Granger, 1966)**

This article claims that 'moreover, the same basic (spectral) shape is found regardless of the length of data available, the size of the truncation point used in the estimation procedure, or the trend removal method used'. The 'typical' spectral shape, in terms of the frequency of cycles detected in data in Granger's original article, is shown below in figure 1.

Granger's 'law', which serves as the quote at the beginning of this paper, follows directly. Granger's research was updated by Levy and Dezhbakhsh (2003b) and Levy and Dezhbakhsh (2003a), who state that 'Granger emphasizes that any peak of the spectrum of level series is at a very low frequency, and that the spectrum does not include peaks of decreasing size corresponding to cycles of different lengths'. This result seems at odds with economic growth data mainly because we would expect to be able to identify 'peaks' which correspond to the business cycle in the power spectrum for macroeconomic data.

Back when Granger's 1966 contribution was made, the notion of stationarity was not widespread – this is clearly obvious as Granger appears to be uncertain in his 1966 paper as to how best to detrend economic data and also notes in a subsequent plot of US bank clearing data that the 'peaks' in the spectrum correspond to seasonal variations and their harmonics, which is, given the frequencies corresponding to these peaks, unlikely to be the case. The notion of detrending by first differencing data was not widespread, so Granger's paper is largely based on the usage of level data. Interestingly the later papers updating the Granger results and generalizing them to international output growth apparently use level data to confirm what Granger found, which is beguiling to say the least.

By using simple data transformations (first differencing) of log output data, we can show that there are many other frequency cycles evident in growth data apart from the business cycle. Indeed once the empirical 'stylized facts' of US growth cycles were documented and measured by Zarnovitz (1985), Kontolemis (1997) and Zarnowitz and Ozyildirim (2002), theorists have set about trying to explain why there are periods of different rates of growth during the expansion phase – see for example Evans, Honkapohja and Romer (1998) and Boldrin (2005).

2.2 Data

The data used in this paper is obtained from two sources: the early segment up until 1946Q1 is sourced from Balke and Gordon (1986) and from this date onwards the data is sourced from the Bureau of Economic Analysis (BEA) of the US Department of Commerce. The data used is US real GNP on a quarterly basis from 1881 quarter 1 through 2009 quarter 1, which gives 513 observations.[2]

Three different versions of the data are used throughout most of the analysis, as follows:

i) level data (1881Q1–2009Q1);

ii) log quarterly differenced data (1881Q1–2009Q1); and

iii) log annually differenced data (1881Q1–2009Q1).

The data is shown in figures 1 to 3. Clearly the most volatile version of the series is the log annual version, and the least volatile is the level data. It is also notable how the current downturn in economic growth is evident in all 3 series, but is clearly not as prominent as downturns in the economy in the pre-World War II period.

3 Spectral analysis

3.1 The Fourier transform and basic spectral analysis

Spectral analysis had its origins in 1807 when Joseph Fourier published a paper[3] in which he used trigonometric functions to model the loss of heat in solid bodies. Although LaGrange, Bernoulli and Euler played significant roles in developing the Fourier transform, the principle workhorse of spectral analysis bears Fourier's name.

A primer on spectral analysis can be found in Koopmans (1995). Apart from the references already cited in the previous section, there are many studies in economics which refer to the use of spectral analysis but few which actually employ it – exceptions are Hughes Hallett and Richter (2008), Hughes Hallett and Richter (2006a), Hughes Hallett and Richter (2006b), Hughes Hallett and Richter (2004), Valle e Azevedo (2002), Berry (2001), Camba Mendez and

[2] In fact the Balke and Gordon (1986) data starts in 1876, but because the beginning part of the series starts in a particularly volatile period, the first 5 years of the series are ignored.

[3] 'Mémoire sur la propagation de la chaleur dans les corps solides'.

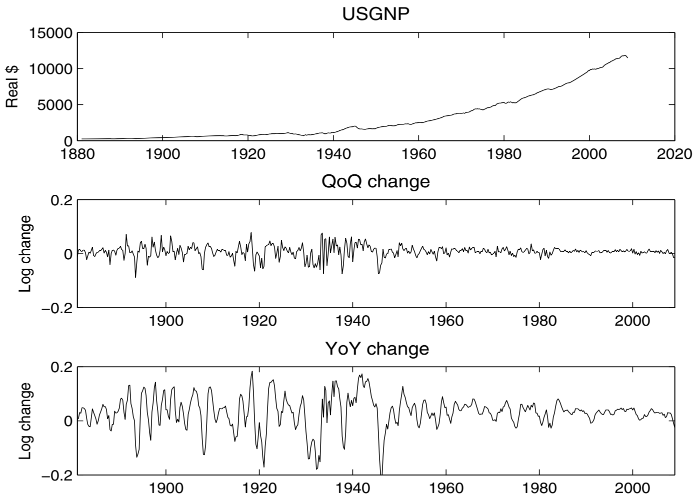

Figure 2: **US real GNP data: 1881–2009**

Kapetanios (2001), Conway and Frame (2000), Collard (1999), and Baxter and King (1999).

Simply put, spectral analysis uses trigonometric functions to fit to time series to determine the periodicity of cycles evident within the time series. As such it requires that all series be linearly generated and stationary (both globally and locally when using windowed analysis). In mathematical terms, spectral analysis is based on the Fourier series, which expresses a time series as the sum of an infinite series of harmonic functions

$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos(nx) + b_n \sin(nx)) \tag{3.1}$$

where

$$a_n = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x)\cos(nx)dx \tag{3.2}$$

and

$$b_n = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x)\sin(nx)dx \tag{3.3}$$

With finite time series the next step is to note that the autocovariance function of a covariance stationary process $x(t)$ is

$$\gamma(\tau) = E[(x_{t+\tau} - \mu)(x_t - \mu)] \tag{3.4}$$

where μ is the mean of the process. The spectrum of the series $x(t)$ is defined as the Fourier transform of its autocovariance function, and is given by

$$f_x(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \gamma(\tau)e^{-i\tau\omega}d\tau \tag{3.5}$$

with $-\pi \leq \omega \leq \pi$, where the frequency ω is measured in cycles per period (in radians). Here the autocovariance function is the inverse Fourier transform of the spectrum. That is

$$\gamma(\tau) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} f_x(\omega)e^{-i\tau\omega}d\omega \tag{3.6}$$

which, after setting $\tau = 0$ implies that $\gamma(0) = \sigma_x^2 = \int_{-\infty}^{+\infty} f_x(\omega)d\omega$. So the integral of the spectrum is the total unconditional variance of the series. Thus the spectrum plotted at each frequency, ω, represents the contribution of that frequency to total variance.

The simplest form of spectral analysis comprises the periodogram, which uses basic sine and cosine functions without further refinement. For US real GNP data the periodograms are shown as figures 3 to 5

The periodograms are quite different. In figure 3 the 'typical' spectral shape is apparent, which matches what Granger found, but in figures 4 and 5 this 'typical' spectral shape is clearly not apparent in the spectral features, with clear peaks at frequencies of around 10 years, 25 and 35 years, with a smaller peak at around a 55 year cycle. The 10 year cycle is clearly more prominent than the 25 and 35 year cycles in the quarterly change data, whereas this is reversed for the annual change data. There are other surprising features in figures 4 and 5. The first notable feature is that with the quarterly change data there is a prominent cycle at roughly 0.325 cycles per year, which corresponds to roughly a 3 year cycle, but this barely registers with the annual change data. The second notable feature is that there are no cycles apparent between 0.2 and 0.3 cycles per year (– which corresponds to between roughly 3 and 5 years) with the annual change data.

3.2 Modified spectral analysis

One of the problems of using simple periodograms is that there is leakage of frequencies into other frequency bands, and the shorter the time series

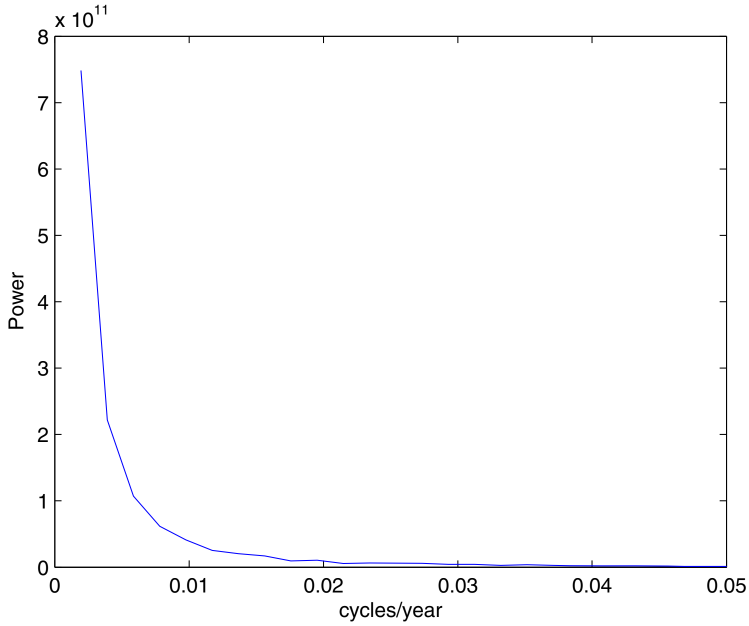

Figure 3: **Periodogram for US real GNP level data**

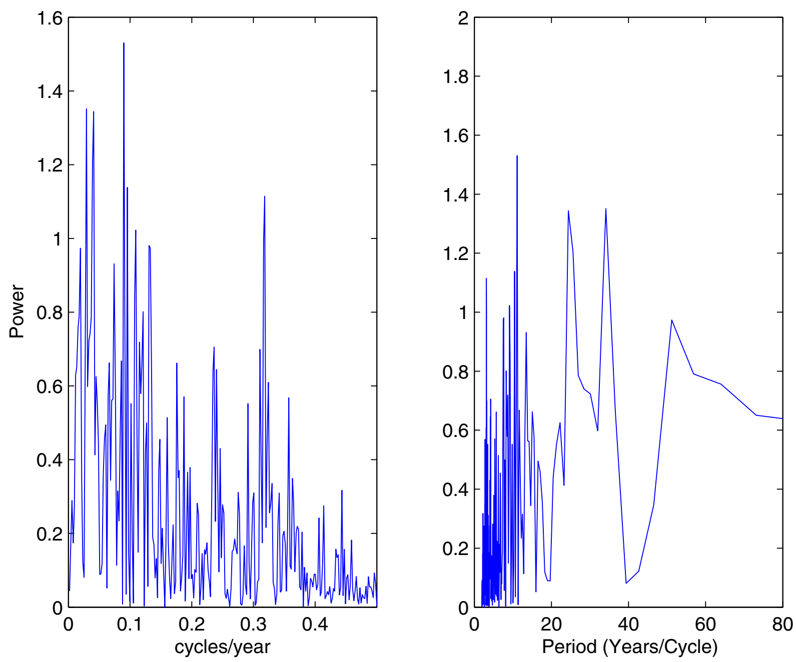

Figure 4: **Periodogram for log quarterly change in US real GNP**

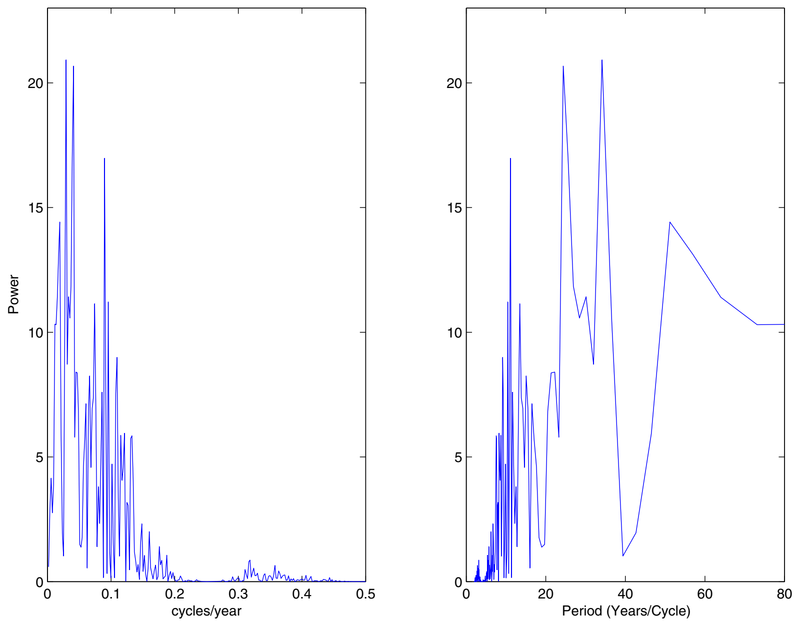

Figure 5: **Periodogram for log annual change in US real GNP**

under consideration the more notable this problem becomes. There are three solutions for dealing with the problem of leakage, and these are i) tapering, ii) padding the series and iii) spectrally smoothing the series by using a window. Most economists (for example Levy and Dezhbakhsh, 2003b) tend to opt for smoothing the series (Levy and Dezhbakhsh, 2003b, uses the Bartlett method with a Hanning window), so similar methods are adopted here. In this instance the Welch[4] method is used with three different types of windows and this leads to different results compared with the simple periodograms as shown in figures 6 to 8. It should be noted that the y axis now measures the average power under the spectral density curve (ie the integral) to give the so-called 'power spectral density' or PSD. The PSD should display peaks at the same frequencies as the basic spectrum.

The modified periodogram in figure 6 shows much the same 'typical' spectral shape as the basic periodogram, although the Hamming and Bartlett windows clearly give side artifacts compared to the Chebyshev window. Interestingly there is a peak in the spectrum, at around a 64 year cycle. This peak is also evident in the quarterly and annual log change data spectra which are plotted in figures 7 and 8. The next peak in the data for both the quarterly and annual data occurs at a frequency of 0.1406, which corresponds to cycles of 7.1 years in length, roughly the average of what one might expect for the business cycle. There are two further peaks in the spectra, which are particularly noticeable in the case of the quarterly data (also depending on the window used) at around a 3 year cycle and a 2 year cycle. Of course with

[4]A Welch window uses averaged periodograms of overlapped, windowed segments over the time series.

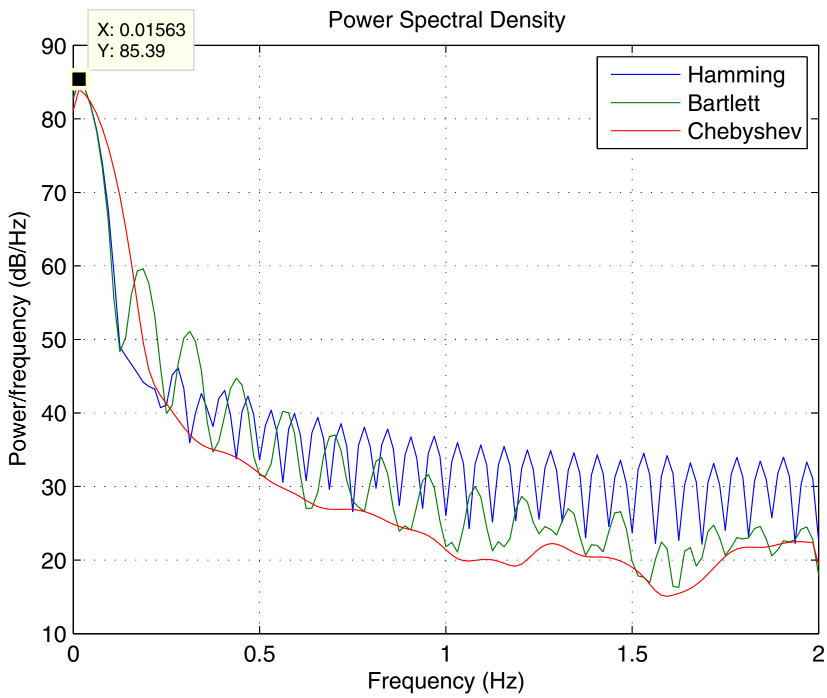

Figure 6: **Power spectral density for US real GNP level data**

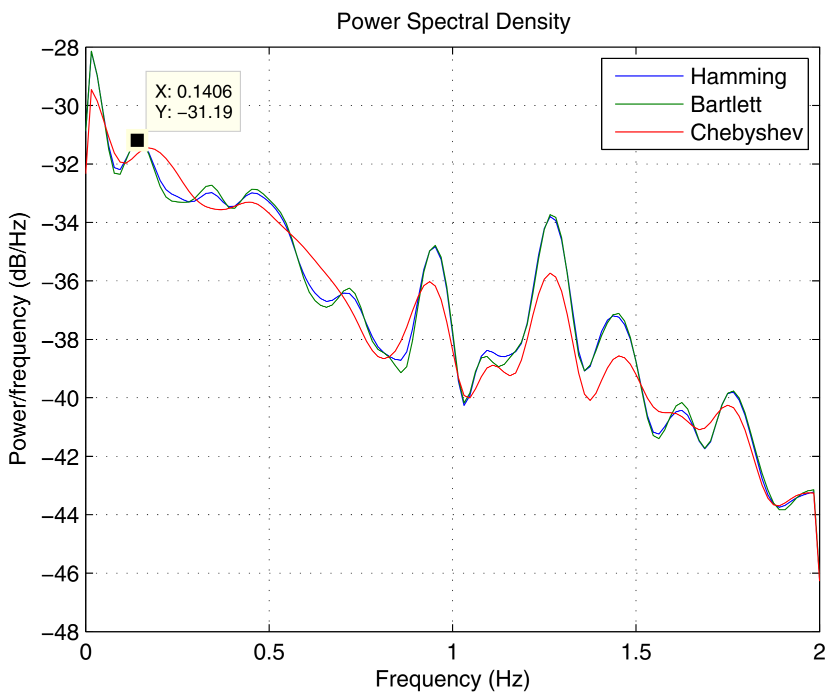

Figure 7: **Power spectral density for quarterly log change in US real GNP**

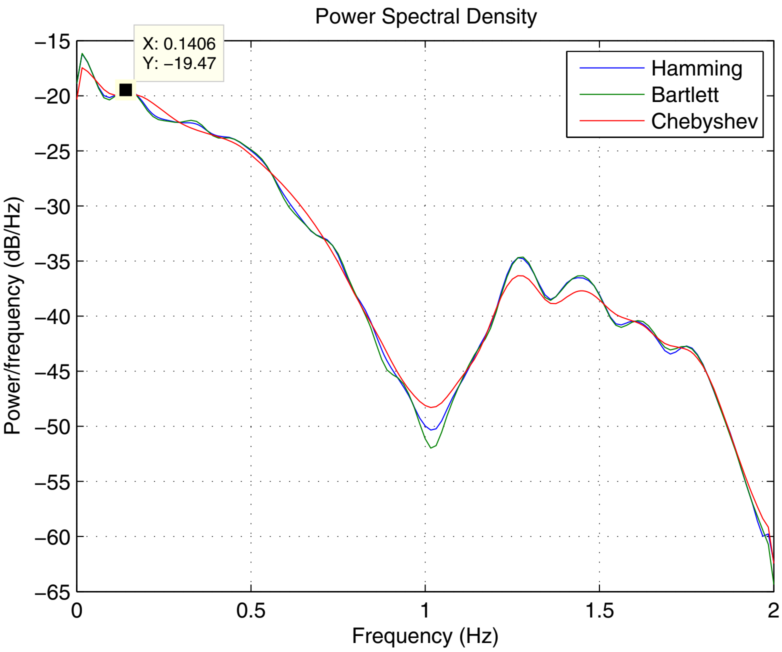

Figure 8: **Power spectral density for log annual change in US real GNP**

annual change data any annual cycles are likely filtered out, particularly as these data are seasonally adjusted, but the quarterly data seems to suggest that there are cycles at just over a year and one at around 3 quarters as well.

Figure 8 is useful in other ways as it shows the problems with the modified periodogram. The problems are manifest in the windowing needed to deal with the leakage of frequencies in the basic periodogram which gives rise to i) different results dependent on which windows are used; ii) the elimination of a large number of spectral peaks as the windows average over a range of frequencies – this is seen by a) the low density around the annual frequency in figure 8 and the disappearance of the 25 and 35 year cycles in the basic periodogram as these do not appear when averaged with neighboring frequencies and b) the disappearance of the gap between 0.2 and 0.3 in the basic periodogram (in figure 5) for the annual change data. Clearly even with modified spectral methods, there are substantial problems with their usage, and this has led to the development of new techniques in frequency domain analysis.

4 New frequency domain techniques

One of the basic problems with spectral analysis is the assumptions underlying its usage. Spectral analysis assumes that the series being analyzed is both locally and globally stationary – locally stationary in the sense of being intra-cyclical stationary – so that cycles should repeat in a deterministic way without any trend between the cycles, unless the intra-cyclical non-stationarity is due to further cyclical activity (for example a long cycle). This must be the case as the harmonic functions underlying the method are both locally and globally stationary given that we have at least one cycle to analyze. This implies that analyzing level real GNP data by using spectral methods as above is inappropriate as the trend in level GNP will be interpreted as the beginning of another very long cycle, leading to the strongest cycle being detected as at least 4 times the length of the series being analysed. As differenced series are usually stationary, it implies that analyzing growth using spectral analysis either by taking quarterly or annual differences will thus be the most appropriate way to proceed, but as outlined above, there are still problems with spectral leakage and also any local instationarities.that may arise.

One further innovation in spectral analysis is to pass a window through the series but instead of assuming (as conventional spectral analysis does) that cycles at certain frequencies persist throughout the series, do spectral analysis piecewise along the series and then observe the strengthening and weakening of certain frequencies through time. This is known as 'time-varying spectral analysis' and it falls under the general 'time-frequency' analysis category of methods. Of course if there are level shifts in the data, within a window, this can still be interpreted as a longer cycle in the data, which can lead to spurious results in the analysis.

As macroeconomic time series are non-deterministic (unlike harmonic functions) and largely stochastic in nature, it is instructive to use less well-known irregular functions for the analysis – functions that might not even be symmetric, for example, and to use functions that are finite in length rather than the infinite harmonic functions used by spectral analysis. This is the main innovation incorporated by wavelet analysis, which has several variants, but essentially involves the usage of wavelet functions – functions that are of finite length and have 'compact support' – which once selected are convolved with the data being analysed. By using functions of finite length this allows for global non-stationarity as if a trend can be extracted from the series as a whole, segment by segment, then each segment will automatically be stationary. The results of applying 2 types of wavelet analysis are presented in the next section of the paper.

A further development in 'time-frequency' analysis took place with the introduction of the Hilbert-Huang transform (HHT), which coupled with empirical mode decomposition (EMD) allows the researcher to extract the different frequencies (or 'modes') driving any variable in the time domain. This allows separation of the different frequency 'modes' driving the economic variable, which can then be analysed individually using time- or frequency-domain techniques.

Typical frequency domain techniques used in analysis of economic variables are summarized in table 1 in terms of their characteristics. For time series analysis, the basis is *a priori* in the sense that any mode of analysis assumes fitting some kind of equation. Time series analysis clearly requires stationarity, there is the assumption series are linearly generated, econometric theory underpins the analysis and asymmetric cycles can be modelled using time series techniques. Traditional spectral analysis analyses variables in the frequency domain, assumes stationarity, linear generation of time series, Fourier analysis underpins the analysis, but spectral methods do not account for cycle asymmetry. Both variants of wavelet analysis (discrete and continuous) use an *a priori* basis, analyse time series in both the time and frequency domain, do not require stationarity, but do assume that time series are linearly generated, have mathematical underpinnings from signal processing and lastly some of the compact functions allow for asymmetry in cycles (– this is more the case with dicrete rather than continuous wavelet analysis). Lastly the Hilbert-Huang transform combined with empirical mode decomposition (HHT/EMD) by employing a spline function with a sifting mechanism to identify cycles uses an *a posteriori adaptive* basis, which is implemented in the time domain, does not require stationarity or linear generation of the series, and makes no assumptions about the symmetry or otherwise of cycles. The big disadvantage to the HHT/EMD methodology is that unlike time-series, spectral, and wavelet analysis, there is no theoretical mathematical underpinning to the approach, so it can appear rather *ad-hoc.*

5 Wavelet analysis

Wavelet analysis first came into being due to a collaboration between a mathematician (Ingrid Daubechies) and a signal processor, (Stephane Mallat), although as Daubechies (1996) points out, the development of wavelets is an example where ideas from many different fields combined to merge into a whole that was more than the sum of its parts. The subject area of wavelets, developed since the mid-1980s, is connected to older ideas in many other fields, including pure and applied mathematics, physics, computer science, and engineering. The original theoretical background for wavelets can be found in Debauchies (1992), and the first applications using wavelet analysis can be located in Mallat (1989). Wavelet analysis (like Fourier analysis) has two versions, one which assumes continuous time functions (continuous wavelet analysis), and another which assumes sampling at discrete equally spaced points in time (discrete wavelet analysis). Both are presented below.

5.1 Discrete wavelet analysis

Consider a double sequence of functions

$$\psi(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-u}{s}\right) \tag{5.1}$$

Table 1: **Summary of frequency domain methods**

	Time series	**Spectral**	**Time-varying spectral**	**Wavelet**	**HHT/EMD**
Basis?	A priori	A priori	A priori	A priori	A posteriori adaptive
Domain?	Time	Frequency	Time-frequency	Time-frequency	Time
Stationary?	Yes	Yes	Yes within each window	No	No
Linearly generated?	Yes	Yes	Yes	Yes	No
Mathematical underpinning?	Yes	Yes	Yes	Yes	No, empirical
Asymmetric cycles?	Yes	No	No	Yes	Yes

where s is a sequence of scales, where scale here corresponds to a particular frequency range. The term $\frac{1}{\sqrt{s}}$ ensures that the norm of $\psi(.)$ is equal to one. The function $\psi(.)$ is then centered at u with scale s. In the language of wavelets, the energy of $\psi(.)$ is concentrated in a neighbourhood of u with size proportional to s, so that as s increases the length of support in terms of t increases For example, when $u = 0$, the support of $\psi(.)$ for $s = 1$ is $[d, -d]$. As s is increased, the support widens to $[sd, -sd]$. Dilation (ie changing the scale) is particularly useful in the time domain, as the choice of scale indicates the 'stretching' used to represent any given variable or signal. A broad support wavelet yields information on variable or signal variations on a large scale, whereas a small support wavelet yields information on signal variations on a small scale. As projections are orthogonal, wavelets at a given scale are not affected by features of a signal at scales that require narrower support. Lastly, if a wavelet is shifted on the time line, this is referred to as translation or shift of u. Any series $x(t)$ can be built up as a sequence of projections onto father and mother wavelets indexed by both j, the scale, and k, the number of translations of the wavelet, where k is often assumed to be dyadic. As shown in Bruce and Gao (1996), if the wavelet coefficients are approximately given by the integrals

$$s_{J,k} \approx \int x(t)\phi_{J,k}(t)dt \tag{5.2}$$

$$d_{j,k} \approx \int x(t)\psi_{j,k}(t)dt \tag{5.3}$$

$j = 1, 2, ...J$ such that J is the maximum scale sustainable with the data to hand, then a multiresolution representation of the signal $x(t)$ is can be given by

$$x(t) = \sum_k s_{J,k}\phi_{J,k}(t) + \sum_k d_{J,k}\psi_{J,k}(t) + \sum_k d_{J-1,k}\psi_{J-1,k}(t) + ... + \sum_k d_{1,k}\psi_{1,k}(t) \tag{5.4}$$

where the basis functions $\phi_{J,k}(t)$ and $\psi_{J,k}(t)$ are assumed to be orthogonal, that is

$$\begin{array}{c} \int \phi_{J,k}(t)\phi_{J,k'}(t) = \delta_{k,k'} \\ \int \psi_{J,k}(t)\phi_{J,k'}(t) = 0 \\ \int \psi_{J,k}(t)\psi_{J',k'}(t) = \delta_{k,k'}\delta_{j,j'} \end{array} \tag{5.5}$$

where $\delta_{i,j} = 1$ if $i = j$ and $\delta_{i,j} = 1$ if $i \neq j$. The multiresolution decomposition (MRD) of the variable or signal $x(t)$ is then given by the set of coefficients (– known as 'crystals')

$$\{s_J, d_J, d_{J-1}, ...d_1\} \tag{5.6}$$

The interpretation of the MRD using the DWT is of interest as it relates to the frequency at which activity in the time series occurs. For example with

Scale crystals	Quarterly frequency resolution
d1	2–4 quarters
d2	4–8=1–2yrs
d3	8–16=2–4yrs
d4	16–32=4–8ys
d5	32–64=8–16yrs
d6	64–128=16–32yrs
d7	etc

Table 2: Frequency interpretation of MRD scale levels

a quarterly time series table 2 shows the frequencies captured by each scale crystal

Note that as quarterly data is used in this study, to capture the conventional business cycle length scale crystals need to be obtained for 5 scales. This requires at least 64 observations, but to properly resolve at the longest frequency it would help to have 128 observations, and as we have 513 observations this is easily accomplished – in fact we can easily resolve to 6 scales which should then provide evidence of cycles of up to 32 years in length. If conventional business cycles are usually assumed to range from 12 quarters (3 years) to 32 quarters (8 years),[5] then crystals d3 and d4 will be assumed to contain the business cycle.

In the wavelet literature there are various methods that can be used for decomposing a series or signal. The first transform to be used extensively in applications was the Discrete Wavelet Transform (DWT). Although extremely popular due to its intuitive approach, the DWT suffers from two drawbacks: dyadic length requirements for the series to be transformed and the fact that the DWT is non-shift invariant. In order to address these two drawbacks, the maximal-overlap DWT (MODWT)[6] was introduced by Shensa (1992) and a phase-corrected version was added and found superior to other methods of frequency decomposition[7] by Walden and Cristan (1998). The MODWT gives up the orthogonality property of the DWT to gain other features, given in Percival and Mofjeld (1997) as:

- the ability to handle any sample size regardless of whether the series is dyadic or not;
- increased resolution at coarser scales as the MODWT oversamples the data;

[5] This is given our discussion above, and as assumed by Levy and Dezhbakhsh (2003a) when evaluating output growth rate spectra.

[6] As Percival and Walden (2000) note, the MODWT is also commonly referred to by various other names in the wavelet literature such as non-decimated DWT, time-invariant DWT, undecimated DWT, translation-invariant DWT and stationary DWT. The term 'maximal overlap' comes from its relationship with the literature on the Allan variance (the variation of time-keeping by atomic clocks) – see Greenhall (1991).

[7] The MODWT was found superior to both the cosine packet transform and the short-time Fourier transform.

- translation-invariance – in other words the MODWT crystal coefficients do not change if the time series is shifted in a 'circular' fashion; and
- a more asymptotically efficient wavelet variance estimator than the DWT.

Both Gençay, Selçuk, and Whicher (2001) and Percival and Walden (2000) give a thorough and accessible description of the MODWT using matrix algebra. Crowley (2007) also provides a good 'intuitive' introduction to wavelets, written specifically for economists, and references the (limited) contributions made by economists using discrete wavelet analysis.[8] The first real usage of wavelet analysis in economics was by James Ramsey (NYU) and can be found in Ramsey and Lampart (1997), and the first application of wavelets to economic growth (in the form of industrial production) was by Gallegati and Gallegati (2007) and in the form of GDP in a working paper by Crowley and Lee (2005) and then more recently in a published article by Yogo (2008).

In figures 9 to 12 the MODWT using a tap 8 Daubechies wavelet[9] is presented for US growth data,[10] together with plots of the energy breakdown by scale crystal. In the quarterly data (figures 9 and 10) the strongest cycle occurs in crystal d1 which corresponds to 2 to 4 quarter cycles, but if most of this cycle is attributed to noise, then the d2 crystal corresponding to 1 to 2 year cycles in growth is the strongest cycle detected in the data, with progressively lower variance attributed to each detail crystal increment. In the annual data (figures 11 and 12) the quarterly changes are filtered out, with the results showing the strongest cycle occurs in the d3 crystal, corresponding to 2–4 year cycles. Figures 9 and 11 also clearly show two moderations in volatility in cycles – one in higher order (lower frequency) crystals in 1950, and the 'great moderation' in lower order (higher frequency) cycles only in the early 1980s, with longer cycles remaining largely intact with similar variances.

These results might be a little surprising to most macroeconomists, not only because they give completely different results from traditional and modified spectral analysis, but also because they show different results, depending on whether quarter-on-quarter data or year-on-year data is used. As stated above, the reason for the dramatic difference with spectral methods is simply due to the inappropriate assumptions for macroeconomic data that spectral analysis requires. The reason for the results being dependent on whether the series is transformed by taking quarterly or annual changes is not entirely clear.

The most important results are that i) there is no long cycle apparent in the data; and ii) the energy distribution (which is equivalent to the spectrum) does not appear to be similar to the results obtained by Granger, that is, the 'typical' spectral shape.

---

[8] An up to date list of contributions in economics using frequency domain techniques can also be accessed online at http://faculty.tamucc.edu/pcrowley/Research/frequency_domain_economics.html

[9] Note that this is a wavelet function that covers 8 observations for the basic d1 crystal. It therefore does not emphasize short-term fluctuations (unlike a 4 tap wavelet basis function would).

[10] No results are presented for level data but the interested reader can find the results in an appendix.

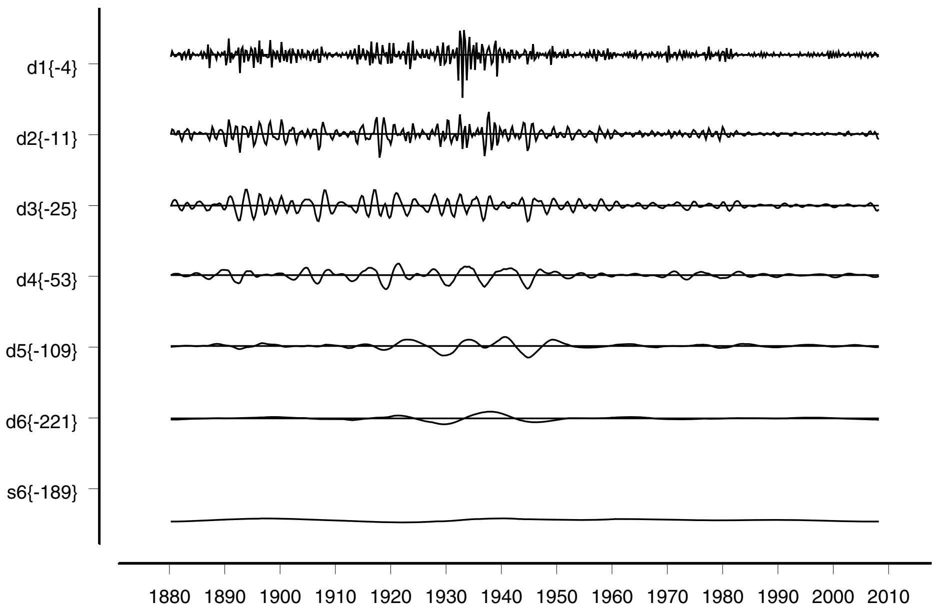

Figure 9: **MODWT decomposition for quarterly log change in US real GNP**

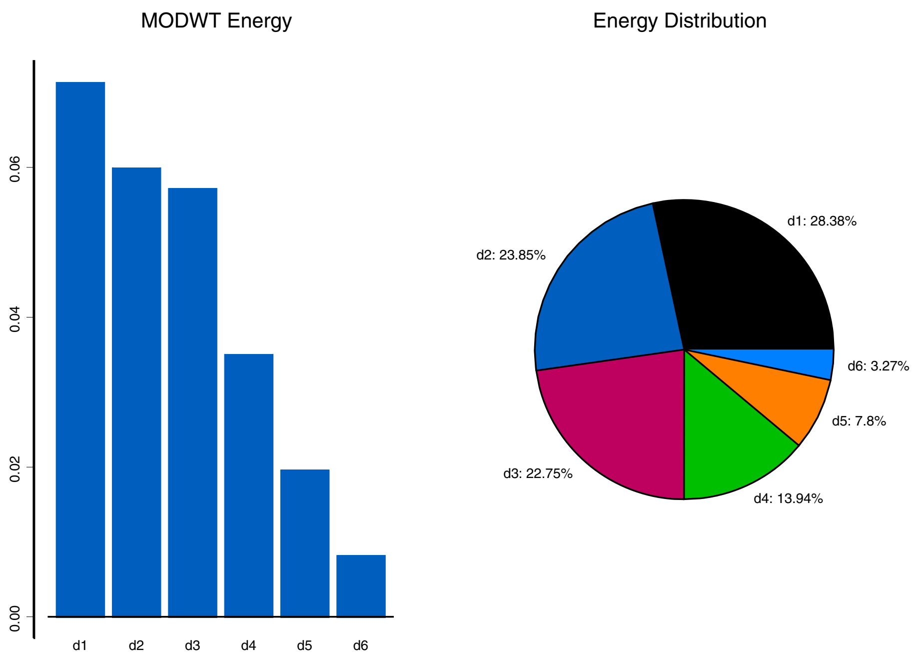

Figure 10: **Variance decomposition by scale for log quarterly change in US real GNP**

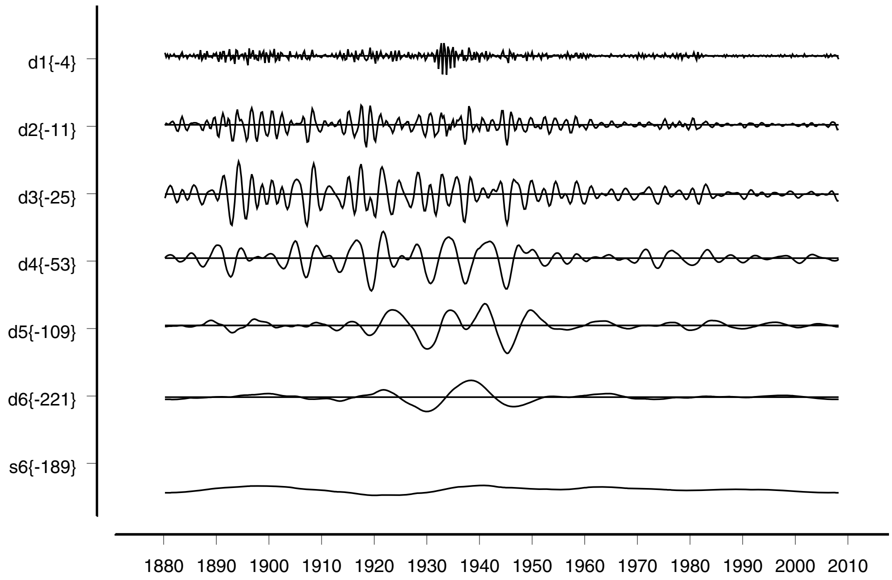

Figure 11: **MODWT decomposition for annual log change in US real GNP**

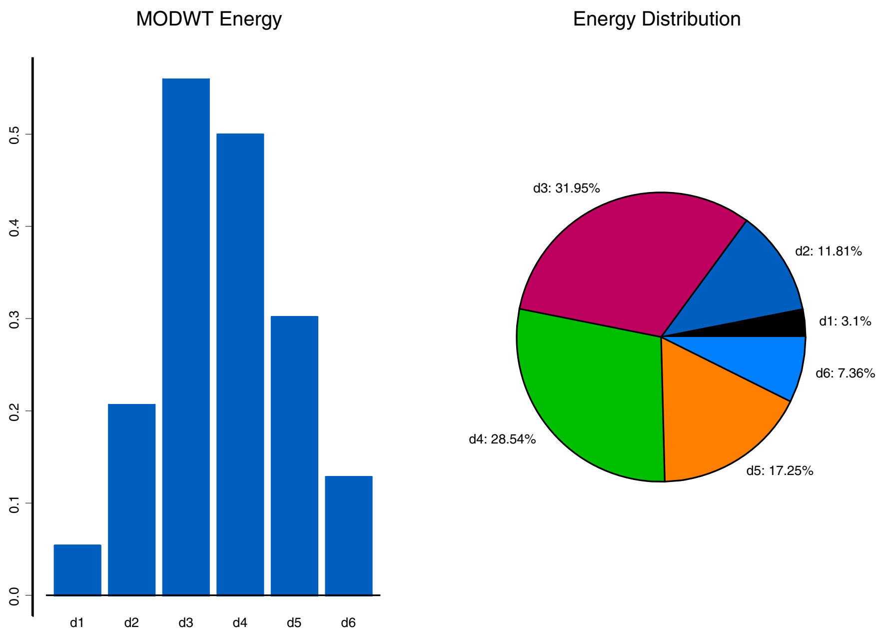

Figure 12: **Variance decomposition by scale for log annual change in US real GNP**

5.2 Continuous wavelet analysis

As wavelet analysis is a form of time-frequency analysis, it is quite natural that wavelet applications can produce measures associated with spectral analysis. Continuous wavelet transforms (CWTs), rather than looking at a range of frequencies (which are included in each detail crystal) to increase the time resolution, have the ability to look at greater frequency resolution (but with less certainty as to time resolution). This is equivalent to temporal narrow-band filtering. Perhaps the best introduction into the theoretical CWT literature can be found in Lau and Weng (1995), Holschneider (1995) and Chiann and Morettin (1998), while Torrence and Compo (1998) probably provides the most illuminating examples of empirical applications to time series from meteorology and the atmospheric sciences.

In brief, a representation of a covariance stationary process in terms of its frequency components can be made using Cramer's representation, as follows

$$x_t = \mu + \int_{-\pi}^{\pi} e^{i\omega t} z(\omega) d\omega \tag{5.7}$$

where $i = \sqrt{-1}$, μ is the mean of the process, ω is measured in radians and $z(\omega)d\omega$ represents a complex orthogonal increment processes with variance $f_x(\omega)$, where it can be shown that

$$f_x(\omega) = \frac{1}{2\pi}\left(\gamma(0) + 2\sum_{\tau=1}^{\infty} \gamma(\tau)\cos(\omega\tau)\right) \tag{5.8}$$

where $\gamma(\tau)$ is the autocorrelation function. $f_x(\omega)$ is also known as the spectrum of a series as it defines a series of orthogonal periodic functions which represent a decomposition of the variance into an infinite sum of waves of different frequencies. Given a large value of $f_x(\omega_i)$, say at a particular value of ω_i, $\widehat{\omega}_i$, this implies that frequency $\widehat{\omega}_i$ is a particularly important component of the series.

Given a time series $x(t)$ and an analysing wavelet function $\psi(\theta)$, then the continuous wavelet transformation (CWT) is given by

$$W(t,s) = \int_{-\infty}^{\infty} \frac{d\tau}{s^{\frac{1}{2}}} \psi^* \left(\frac{\tau - t}{s}\right) x(\tau) \tag{5.9}$$

For an easier computation making use of FFT algorithms this can be rewritten in Fourier space. For a discrete numerical evaluation we get

$$W_k(s) = \sum_{k=0}^{N} s^{\frac{1}{2}} \widehat{x}_t \widehat{\psi}^* (s\omega_k) e^{i\omega_k t \partial t} \tag{5.10}$$

where $\widehat{x}_k$ is the discrete Fourier transform of x_t

$$\widehat{x}_k = \frac{1}{(N+1)} \sum_{k=0}^{N} x_t \exp\left\{\frac{-2\pi i k t}{N+1}\right\} \tag{5.11}$$

Here we use a Morlet wavelet, which is defined as

$$\psi(\theta) = e^{i\omega\pi}e^{-\frac{\pi^2}{2}} \tag{5.12}$$

This is a symmetric wavelet, and is widely used for CWT analysis in the wavelet literature. Given our analysis above, it is also then possible to calculate conventional spectral measures, such as the spectral power

$$WPS(t,s) = E\{W(t,s)W(t,s)^*\} \tag{5.13}$$

Wavelet power spectra measures the strength of cycles at various frequencies – it is the analogue measure of energy for a DWT or variance in a time series context.

Figures 13 and 14 plot the power spectrum against an AR1 background, where thin black contours indicate significant departures at the 90% level from an AR1 process. The vertical axis (labelled 'Scale') shows the frequency in years. Maraun and Kurths (2007) have shown that some of supposedly small significant areas might simply be 'artifacts', and so they conduct an 'areawide' test which averages power over scale and time and then rejects 90 per cent of the 'artifact' areas of the plot. In other words, the black contours can be interpreted as indicating a highly significant 'zone' rather than a few significant points. The arch drawn in the plot shows the 'cone of influence',[11] so points outside the cone are to be interpreted as being less reliable than those placed within the cone (because of instabilities that can result from fitting wavelets to fluctuations at the end of the series).

Figures 13 and 14 give similar results to discrete wavelet analysis, but there are also some differences as well. The color plots display the log intensity (power) of cycles at each frequency, with black lines denoting statistically significant power, and the thicker black lines denoting area-wise significance. Here in figure 13 there is clearly a large amount of volatility in growth between 3 and 20 year cycles from 1920 to 1950, and then in 1950 shorter cycles up to roughly a 6 year periodicity wane, and then from around 1970 onwards longer cycles also wane. The 'great moderation' is apparent at cycles up to around 12 years but is clearly not as dramatic as the change in 1950. The plot for the annual change in real GNP shown in figure 14, and once again there are significantly powerful cycles from around an 18 month to 8 year frequency, but the 'great moderation' now shows up much more clearly in the data.

The power spectra are plotted relative to the strongest fluctuations in the data, and as these occurred in the first part of the data span, the wavelet power spectra are repeated using only the post-World War II data in figures 15 and 16. Several interesting features appear in the power spectra. First, the 'great moderation' is now clearly shown in the data (but only in 1985), but similarly to the results for the discrete wavelet analysis, the waning of cycles is only at higher frequencies.below around a 4 year periodicity. Secondly, there are three distinct cycles (which are particularly apparent in the annual data) which have

[11] This indicates the central area of the graph where the full length wavelets are applied to the data, so are free of any bias resulting from the use of boundary coefficients to enable wavelet application.

collectively been lengthening since the 'great moderation' – these are shown as red bars. Although these red bars move out of the 'cone of influence' and therefore we are not so certain about their resolution, the current frequencies of these cycles are at 6, 16, and 32 years.

Note here that the CWT approach does not provide a residual (as no 'father wavelet' equivalent) is used in the analysis, so no assessment of cycles longer than 32 years is possible. It is clear, nevertheless, that continuous wavelet analysis does not support the notion of a 'typical' spectral shape for growth in US real GNP.

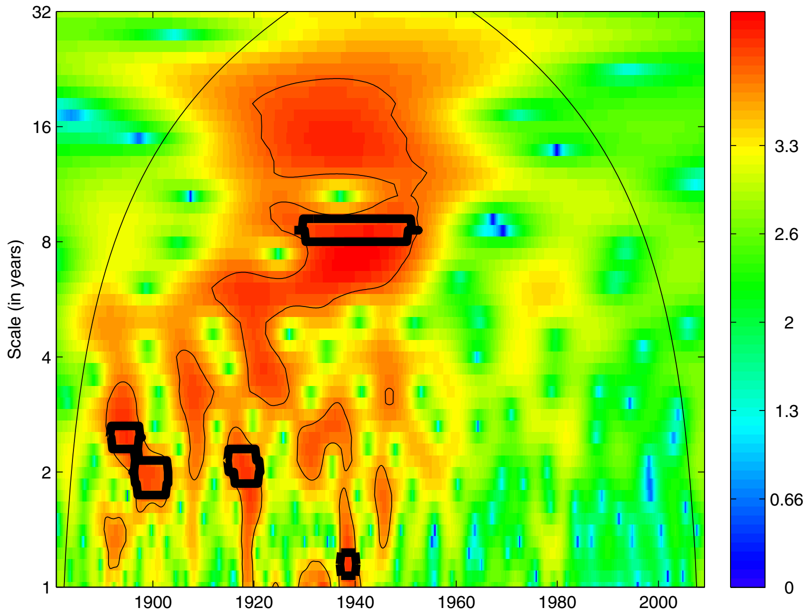

Figure 13: **Wavelet power spectrum for quarterly log change in US real GNP**

6 EMD/HHT

One of the problems with wavelet analysis and traditional spectral analysis is that it is not always clear how many frequencies are at work driving a particular variable. Although wavelet analysis is an improvement over spectral analysis as it can handle non-stationarity due to the localized nature of wavelet functions, it still requires a linear generating process, and also suffers from leakage between scales.

A completely different approach called the Empirical mode decomposition (EMD)/Hilbert-Huang transform (HHT) method appeared in the signal processing literature about a decade ago, introduced by Huang, Shen, Long, Wu, Shih, Zheng, Yen, Tung, and Liu (1998). The method has subsequently been applied to many areas in physics, mechanics, engineering, astronomy and

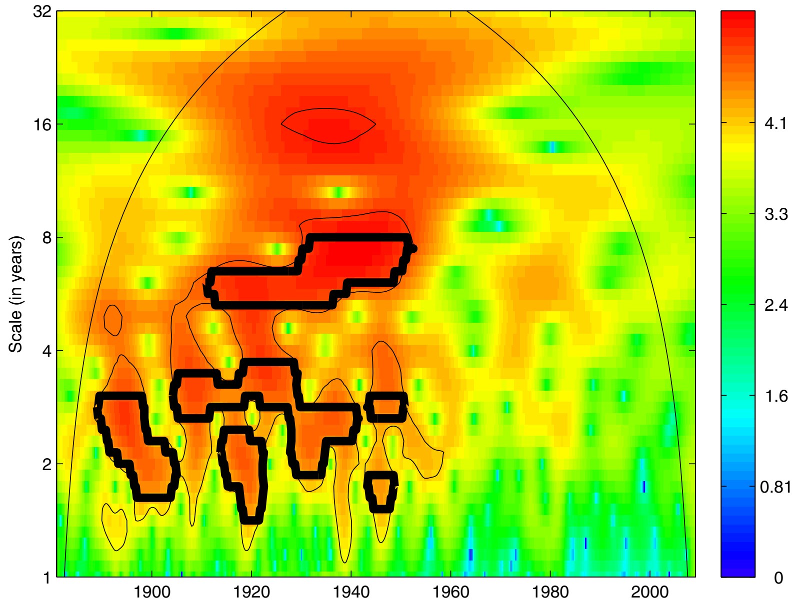

Figure 14: **Wavelet power spectrum for annual log change in US real GNP**

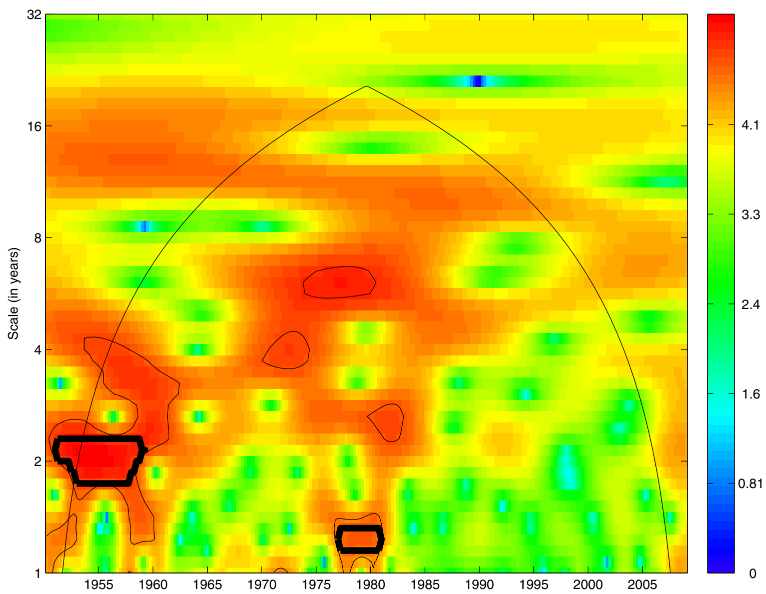

Figure 15: **Wavelet power spectra for post-WWII quarterly log change of US GNP**

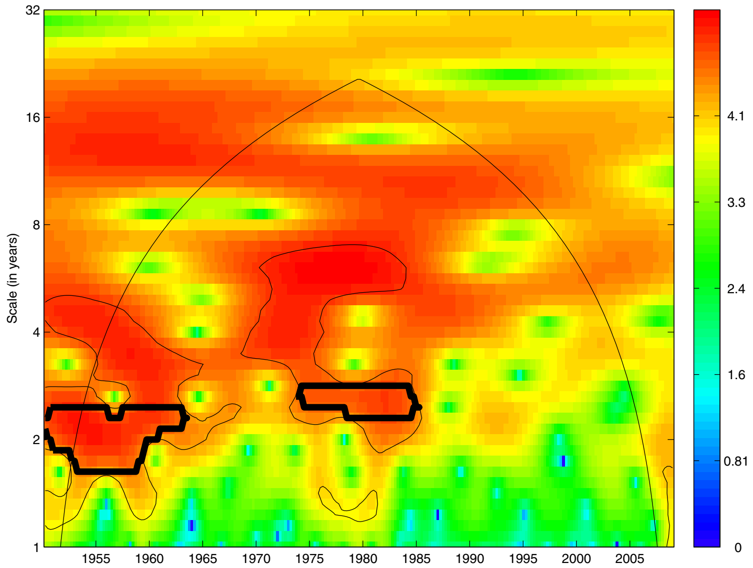

Figure 16: **Wavelet power spectra for post-WWII annual log change in US GNP**

the environmental sciences, and yet to date there are only two applications of the technique in economics and finance, one applied to US mortgage rates by the originators of the technique (see Huang, 2003) to highlight the advantages of using it over traditional spectral methods, and now a couple of other papers by the author (see Crowley, 2009 and Crowley and Schildt, 2009).

Unlike traditional spectral analysis and wavelets, the EMD/HHT method is entirely empirically based – it has no formal mathematical foundation, but rather attempts to break down the series according to how many frequencies are apparent in the data – in other words it let's the data speak for itself rather than imposing certain *a priori* beliefs about which frequencies are present at any time within a series. It relies on a sifting process based around extrema or the curvature of movements in the data. The sifting process is as follows:

i) identify maxima and minima of time series $x(t)$

ii) generate upper and lower envelopes with cubic spline interpolation $e_{\min}(t)$ and $e_{\max}(t)$

iii) calculate point by point mean of upper and lower envelopes

$$m(t) = (e_{\max}(t) + e_{\min}(t))/2 \tag{6.1}$$

iv) take mean away from time series

$$d(t) = x(t) - m(t) \tag{6.2}$$

v) if it is an 'intrinsic mode function' (IMF) (– that is, it satisfies certain stopping criteria), denote $d(t)$ as ith IMF and replace $x(t)$ with residual $r(t)$

$$r(t) = x(t) - d(t) \tag{6.3}$$

if it is not an IMF, replace $x(t)$ with $d(t)$
vi) repeat steps i) to v) until residual satisfies stopping criterion.

To summarize this sifting procedure, see figure 17, taken from Huang, Shen, Long, Wu, Shih, Zheng, Yen, Tung, and Liu (1998).

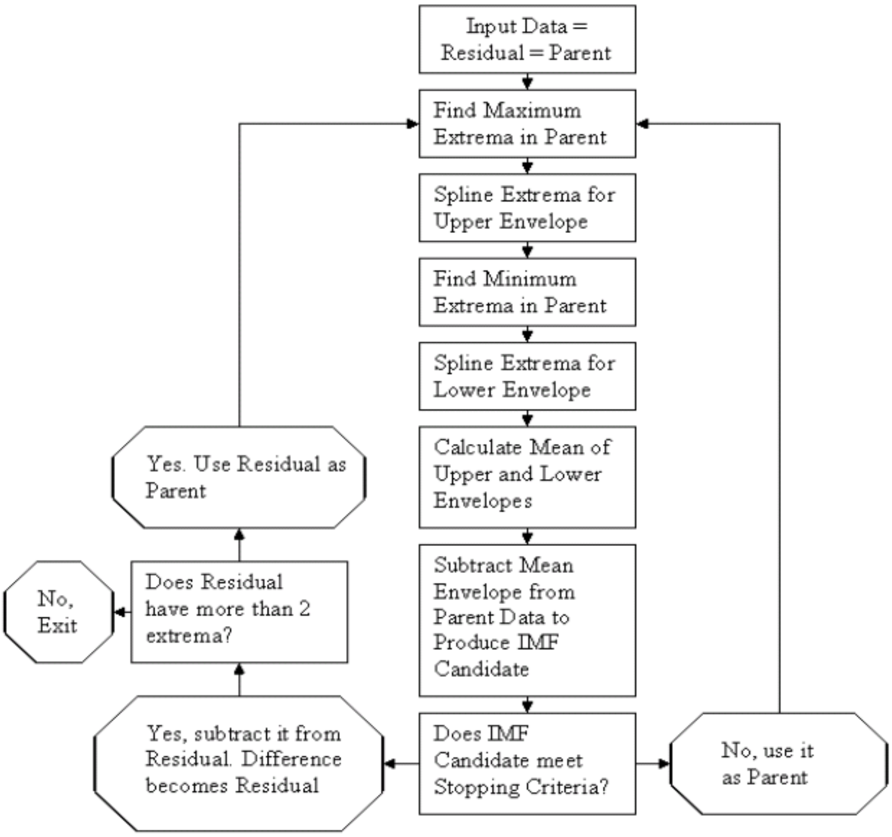

Figure 17: **Flow diagram of EMD sifting procedure to obtain IMFs**.

Once the IMFs have been extracted from the series, the Hilbert spectrum can be applied to the series – this spectrum enables an estimate of the instantaneous frequency of each IMF from the series and then these can be combined in a marginal spectrum, mimicking the spectral density from Fourier analysis.

In mathematical terms, for any function $x(t)$ of L^p class, its Hilbert transform $y(t)$ is

$$y(t) = \frac{1}{\pi} P \int_{-\infty}^{+\infty} \frac{x(\tau)}{t - \tau} d\tau \tag{6.4}$$

where P is the Cauchy principal value of the singular integral. The Hilbert transform $y(t)$ of any real-valued function $x(t)$ will yield the analytic function

$$z(t) = x(t) + iy(t) = a(t) \exp [i\phi(t)] \tag{6.5}$$

where $i = \sqrt{-1}$, $a(t)$ represents the amplitude and $\phi(t)$ the phase ($\phi(t) = \arg(x(t))$). $a(t)$ is then given by

$$a(t) = \left(x^2 + y^2\right)^{1/2} \tag{6.6}$$

and

$$\phi(t) = \tan^{-1}\left[\frac{y}{x}\right] \tag{6.7}$$

Instantaneous frequency, ω, then is given by

$$\omega = \frac{d\phi}{dt} \tag{6.8}$$

The instantaneous frequency introduced here is physical and depends on the differentiation of the phase function, which is fully capable of describing not only interwave frequency changes due to nonstationarity but also the intrawave frequency modulation due to nonlinearity. The Hilbert transform as applied to each IMF can now be expressed as

$$z(t) = \sum_{j=1}^{t} a_j(t) \exp\left[i \int \omega_j(t) dt\right] \tag{6.9}$$

so that the instantaneous amplitude $a_j(t)$ can be separately extracted from the instantaneous phase $\omega_j(t)$ for each IMF and combined into a Hilbert (amplitude) spectrum, $H(\omega, t)$. The power (or energy) spectrum is given by $[H(\omega, t)]^2$ so accordingly the marginal average power spectrum is

$$h(\omega) = \frac{1}{T} \int_0^T H^2(\omega, t) dt \tag{6.10}$$

With EMD/HHT we first start with level data. Figure **??** shows the original level GNP data with the residual superimposed (– these both rise through time), plus all the IMFs superimposed on one another (– these are all stationary). There is clearly one IMF (– the IMF that appears to be sine wave-like), IMF7, that is required when combined with the residual to mimic the exponential nature of the trend in real GNP, so both the residual and this IMF are excluded from what follows.

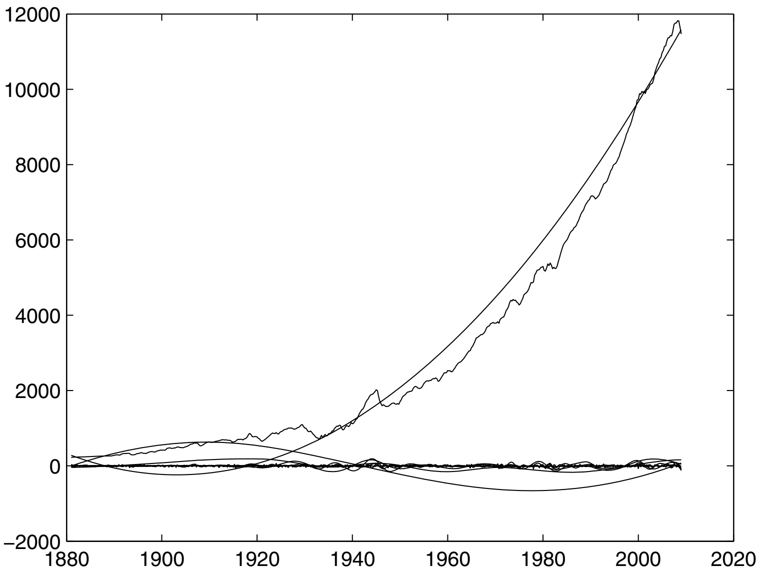

Figure 18: **IMFs for level real US GNP**

The other IMFs are now separated out in figure 19, with IMFs 1 through 6 clearly showing downturns which coincide with recessions. What is particularly interesting about the IMFs is that none of them appear to show the 'great moderation' and if anything IMFs 3, 4 and 5 show increased volatility, rather than less, as would be expected.

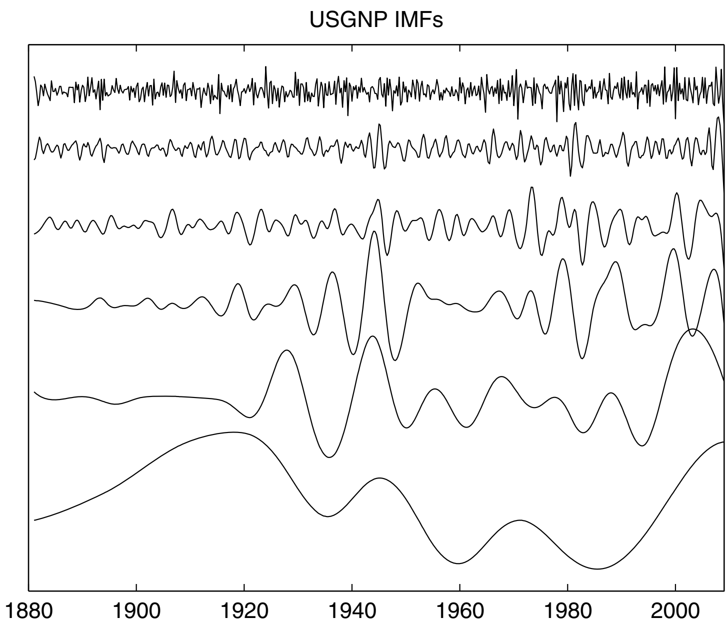

Figure 19: **Level US real GNP IMFs 1–6**

Having obtained the IMFs for level US real GNP we can evaluate the instantaneous frequency of each IMF using the Hilbert spectrum. This is shown in figure 20 where the degree of separation of instantaneous frequencies is clearly not good, indicating some difficulties in resolving the cycles extracted in each IMF.[12]

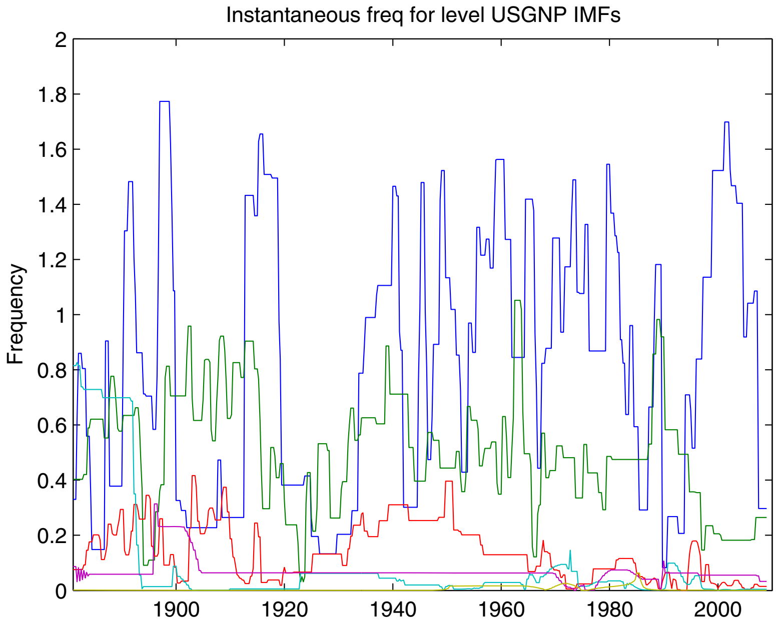

Figure 20: **Instantaneous frequency for level US real GNP IMFs**

Now assuming that IMF1 is white noise, we can test whether the other IMFs carry significantly more energy than what would be found with a white noise signal. Figure 21 shows that all the other IMFs are highly significant at the 95% level compared with white noise, with the highest amount of energy in the longest cycle.

[12]To do this decomposition, ensemble EMD (see Wu and Huang, 2008) was applied (– with 0.2 variance and n=200), and these results were better than those obtained with simple EMD or with other parameters.

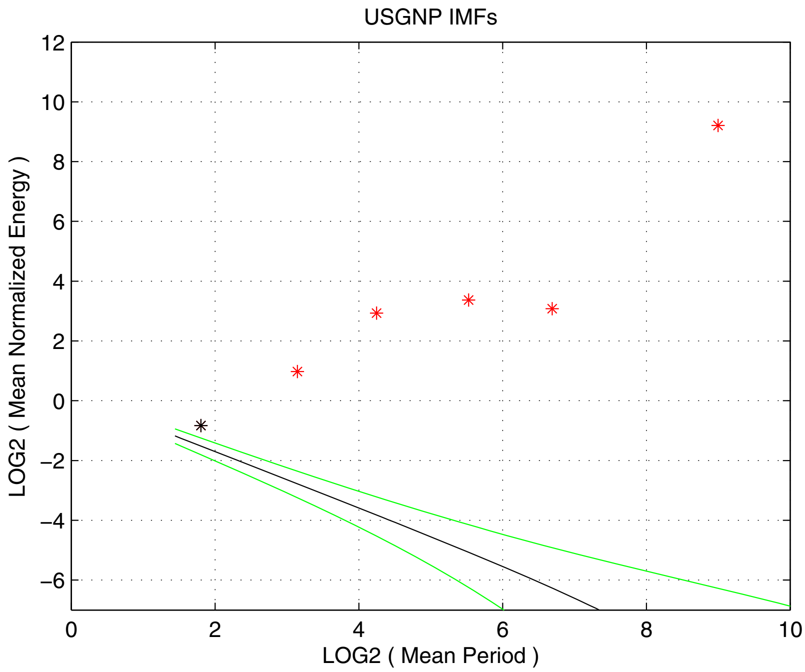

Figure 21: **Significance test of level US real GNP IMFs against white noise**

Lastly, we can combine the IMFs to calculate, as in equation 29 above, the marginal spectrum by combining all the IMFs together. The result of doing this is shown in figure 22. Here cycles are clearly concentrated at frequencies below 0.5, or 2 year cycles, and there are essentially 3[13] prominent peaks in the spectrum – the main one at 0.114, which implies cycles of 8.77 years, one at a frequency of 0.137, implying 7.3 year cycles, one at frequencies of 0.00965, implying 100 year cycles and one at 0.2 or 5 year cycles.

[13]Apparently there is a very long cycle in growth here as well, but appears to be due to the fact that certain of the IMFs appear to drop to a frequency of zero, so here it is ignored.

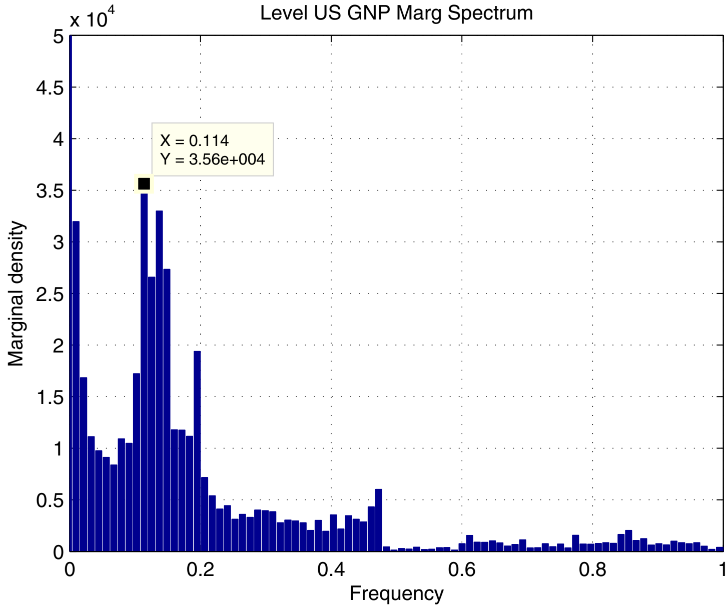

Figure 22: **Marginal spectrum for level US real GNP**

When EMD/HHT is applied to the log change US real GNP data, we obtain the following IMFs, displayed in figure 23. In the figures the original data is shown in the first row, then 6 IMFs are found for both the quarterly and annual log change data, and then the residual is plotted in red in the last row of the plot. The residual clearly doesn't contain any remaining cycles and is virtually constant for the quarterly data. The 'great moderation' is only apparent in IMFs 1 and 2, but is hardly apparent in the other IMFs. Particularly in the annual data, the longer cycles represented by IMFs 3 and 4 continue beyond the 1980s with similar levels of volatility.

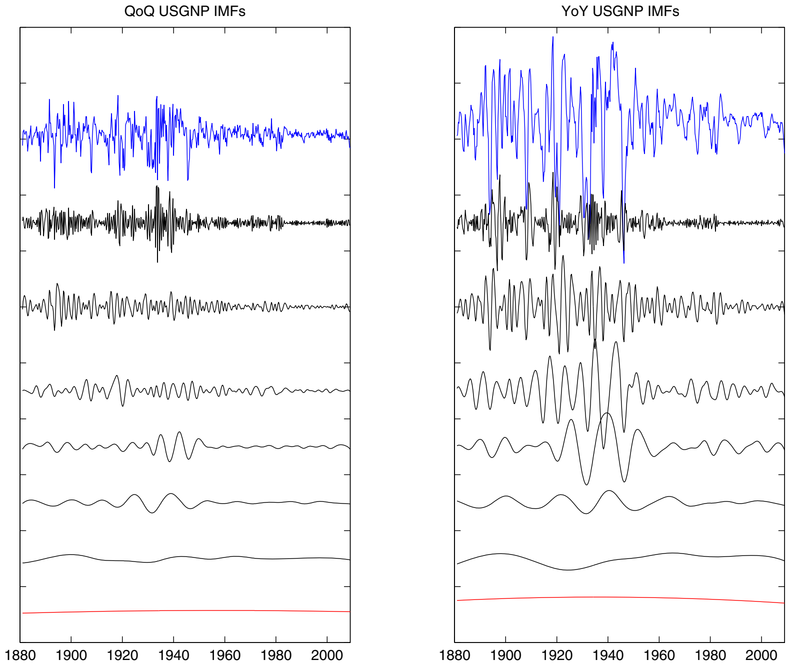

Figure 23: **IMFs for quarterly and annual log change in US real GNP**

Given the IMFs obtained above, the instantaneous frequencies of these IMFs can then be plotted in each case – these plots appear in figure 24. The frequency plots overlap occasionally, indicating that the frequency resolution of individual IMFs might not be optimal,[14] but what is most revealing is that the longest cycle detected after removing the trend in the level data has around a 10 year periodicity in the quarterly data, and a 40 year periodicity in the annual data (although this cycle is very weak according to figure 23). Although there is some 'mode mixing' after 2000 in the annual data, there is strong indication of a cycle at around 10 years and another at around 5 years.

[14]This problem has been addressed by a recent variant of the EMD, known as Ensemble EMD – details can be found in Wu and Huang (2008) and accompanying MATLAB code is located at http://rcada.ncu.edu.tw/research1_clip_program.htm. Ensemble EMD did not appear to resolve the problem in this instance though.

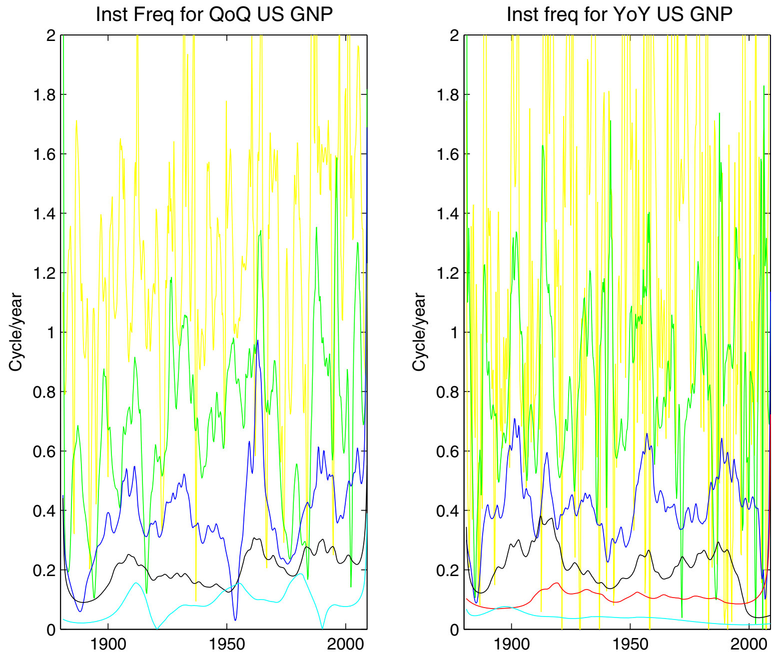

Figure 24: **Instantaneous frequency for IMFs of US quarterly and annual change in real GNP**

Next, on the assumption that IMF1 is 'white noise', it is possible to test the 'energy' or 'power' of the remaining IMFs against what would be found at the same frequencies if the whole series were indeed white noise and was then decomposed using the EMD/HHT method. This is done in figure 25 where the green lines denote the 75 and 95 per cent significance levels while the black line represents the 90 per cent level. The results are clearly all significant when cast against a background spectra of white noise, with perhaps the exception of IMF3 for the quarterly series, which has an average period of 16 years. The most significant cycles appear in the annual data, and have average periodicities of 9, 16 and 30 years.

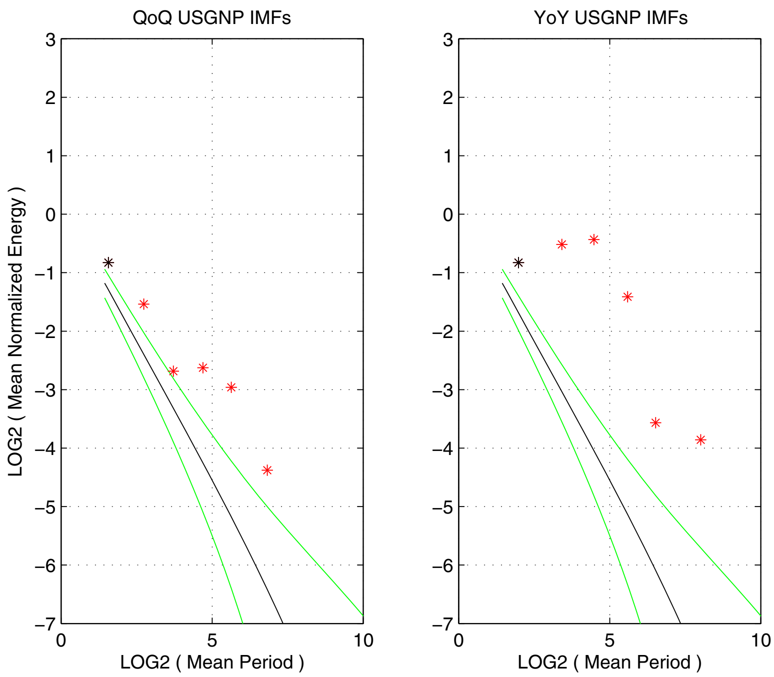

Figure 25: **Significance test of IMFs against white noise for quarterly and annual log change in US GNP**

As shown in equation (5.6) the amplitude spectra can also be extracted from each IMF, giving some idea of the energy contained in each IMF through time. These amplitude spectra are plotted for both quarterly and annual log change data in figure 26. In the upper part of the plot it is clear that the major reduction in volatility took place between 1945 and 1955 with IMFs 1 to 5 becoming much less volatile, but IMF6's amplitude is virtually unchanged. This is even more obvious when looking at the lower panel which shows the annual change: here IMFs 1, 3, 4 and 6 become noticeably less volatile between 1945 and 1955, but IMFs 2 and 5 although moving to slightly lower amplitudes during this time, do not move decisively until the 'great moderation' in the mid-1980s. The 'great moderation' itself can be seen most clearly in IMFs 1 to 3 in the upper panel around 1985 which uses the quarterly data, but IMFs 4 to 6 do not change (– which mirrors the result we obtained with discrete wavelet analysis).

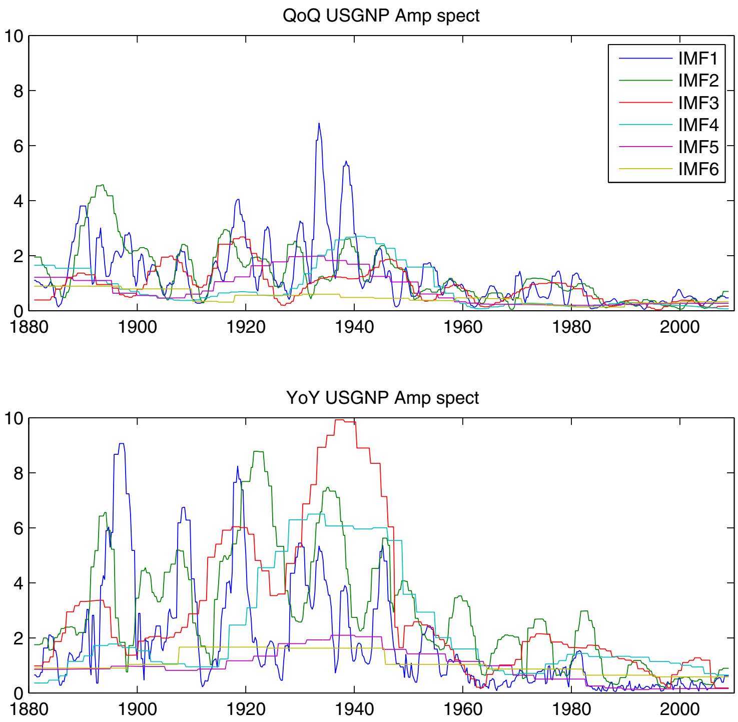

Figure 26: **Amplitude spectrum for quarterly and annual log change US real GNP data**

Another way of presenting this data is to calculate the Hilbert marginal spectrum for the entire series, which then ignores the time aspect of the data but allows a comparison with the spectra obtained from traditional Fourier analysis by combining the spectra obtained from all the different IMFs obtained from EMD/HHT into one figure.

Figure 27 provides the marginal spectra for the IMFs derived from both quarterly and annual log changes. The first thing that is striking about the spectra is that once again neither has what Granger referred to as a 'typical' spectral shape. The second thing that is striking about the two spectra is that they look quite different – the quarterly change spectrum increases in marginal density from very long cycles up to frequencies of roughly 0.45 (or a single cycle every 2.22 years), whereas the annual change spectrum does not display this pattern at all.

The two highest peaks in each marginal spectrum are marked and it is clear that the quarterly change spectrum has peaks at roughly 20 year and 2.22 year cycles, with other peaks occurring at 5, 4.5 and 2.7 year cycles. The annual change panel on the right hand side shows peaks at around 11, 3.33 and 1.85 year cycles.with a fair amount of variation around both the 3.33 and 1.85 year cycles.

At first sight these results seem puzzling as the quarterly and annual change results are quite different, but partly this will be due to the different IMFs that are derived from the different fluctuations in growth when measured on a quarterly compared with an annual difference basis. Despite the substantial differences, three results remain though, one being that there are many frequency cycles at work in the US real GNP growth data, and not just those reported at business cycle frequencies, and the others being that the ‘typical’ spectral shape does not appear evident using the EMD methodology, and that the longest strong cycle detected is only slightly longer than 20 years – beyond this there is little detected.

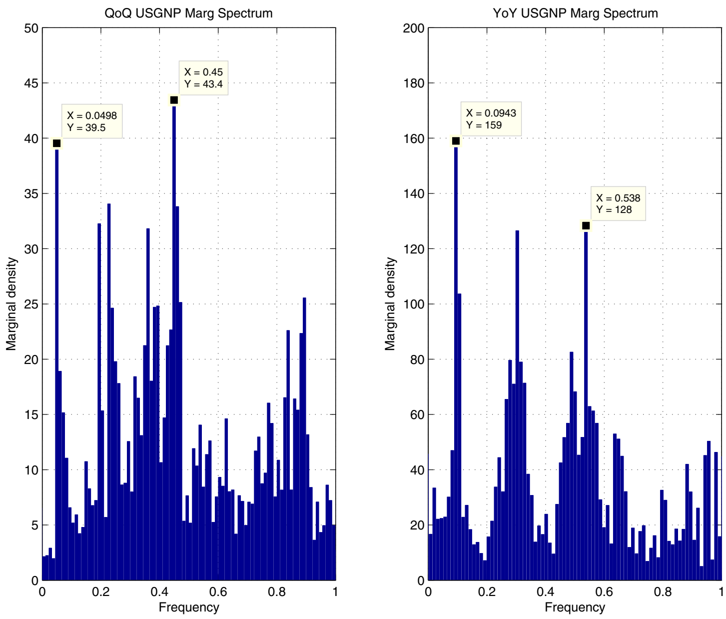

Figure 27: **Marginal spectra for log quarterly and annual change in US real GNP**

As with wavelet analysis, one of the main benefits of using HHT/EMD is that it highlights the non-static nature of the frequency of different cycles at work in the data. Unlike wavelet analysis, however, these IMFs are approximations to the actual frequencies at work, and unlike wavelet crystals, which because of ‘leakage’ might contain too much information, IMFs might miss frequency information because of the way in which cycles are identified (– ie not by an approximating function but by a spline function coupled with a sifting mechanism).

7 Discussion

As with time-series econometrics, each new method applied to the same set of data uncovers new aspects of the data and usually brings some commonalities with existing techniques and some differences in results, often with some conflicting elements that then prompt further research. It is no different here with frequency domain methods. Clearly the limitations of basic spectral analysis require usage of more modern frequency domain techniques that can deal with local and global non-stationarities, but also these new techniques suffer from other problems which then need to be understood and addressed.

Certainly wavelet analysis is widely recognized as a major technological advance over traditional spectral analysis, particularly when dealing with non-stationary and stochastic rather than stationary and deterministic time-series. Choices still have to be made with regard to the wavelet function and whether to use discrete or continuous wavelet analysis, plus statistical inference is not impossible but poses challenges. EMD/HHT is also an important innovation that will hopefully be further developed as a frequency domain tool, but there are still problems regarding frequency resolution due to mode mixing and the sifting procedure used in the decomposition of the series into IMFs. Nevertheless both wavelet analysis and EMD/HHT offer new insights into irregular cyclical activity in macroeconomic variables and their usage could shed new light on economic issues. Such is the advancement of knowledge, and strong resistance to these new techniques would not serve any great purpose.

From conventional time-domain macroeconomics, if nothing else we would expect to see clear indications of cycles in growth at business cycle frequencies of between roughly 3 and 8 years in periodicity. But, as shown below in table 3 cycles between 3 and 8 years in periodicity are not always the dominant cycles selected by these methods, and for many of the methods the strongest cycle does not lie within the expected business cycle frequency bandwidth. For reasons already explained, this is not surprising for the level data, but it is surprising for both the quarterly and annual data. It is instructive to note that with the exception of the level data, the quarterly and annual growth data all have cycles within the recognized business cycle frequency range of 3 to 8 years.

In the frequency domain, with the exception of spectral analysis, it clearly makes a substantial difference whether quarterly or annual changes are used to calculate growth rates. In one sense this is a suprising result, as one might expect more conformity between the quarterly and annual results, but it is obvious that quarterly differencing emphasises short-term movements and annual differencing emphasises longer term movements. This is particularly apparent with basic spectral analysis and both types of wavelet analysis, but it is interesting that this expected finding is reversed with EMD/HHT, where annual change data has shorter cycles than quarterly data.

	Level	Quarterly	Annual
Trad Spectral	Almost infinite	10,25,35,3yrs	35,25,10yrs
Mod Spectral	64yrs	64,7.1,3,2yrs	64,7.1yrs
DWT	NA	2–4Q, 1–2,2–4yrs	2–4,4–8,8–16yrs
CWT	NA	Currently 2–4, 6, 16, 32yrs	Currently 6, 16, 32yrs
EMD/HHT	8.8,7.3,100yrs	20,2.2,5yrs	11,3.3, 1.8yrs

Table 3: Summary of Results: 3 most dominant cycles

8 Conclusions

In this paper the assertions made in Granger (1966) of a ‘typical’ spectral shape regarding long-term fluctuations of macroeconomic data were re-evaluated using US real GNP with four generally available frequency domain methods. There are several economic conclusions from the research and these are summarized below:

i) the cycles that drive economic growth in the US are at a variety of different frequencies, and not just the business cycle frequency range;

ii) Granger’s original ‘law’ regarding the ‘typical’ spectral shape appears to be the result of an artifact of the methodology being used rather than a stylized fact of frequency domain analysis. Granger’s ‘typical’ spectral shape suggested very long cycles with large amounts of energy, with shorter cycles possessing less energy – none of the frequency domain methods used here except spectral analysis indicates that this is the case;

iii) the longest cycle evident in US real GNP data is of around 35 years, so there is no evidence of a Kondratieff cycle in the US data (this confirms results already obtained by Adelman, 1965, and Howrey, 1968);

iv) the ‘great moderation’ is only evident in shorter cycles, it does not seem to have significantly affected the amplitude of longer cycles;

v) unlike the time domain, quarterly and annual change data do not generally yield the same results in the frequency domain.

In terms of the methodologies used, there are also some important conclusions:

a) spectral analysis assumes stationary linearly generated processes, so it is not appropriate to use level data with this method;

b) even when transforming the data, if using spectral windowed analysis (ie for smoothing purposes or for time-varying spectral analysis), local non-stationarities can introduce long cycles into the data;

c) even when using methodologies that allow for non stationarities (either global or local), different results can be obtained (– for example with wavelet analysis and HHT/EMD); and

d) given non-stationary level data, HHT/EMD appears to be the most promising approach for extracting cycles, although because of mode-mixing and frequency resolution more research needs to be done to.refine the method so that it more effectively isolates the different inherent frequencies.

In terms of future research, obviously the frequency domain approach holds promise and could emerge as an alternative methodology for analyzing economic growth to the usual time-series approach taken in the standard economics literature. Nevertheless there needs to be much more research undertaken to analyze the interaction between the variables which cause growth, as well as the components of real GDP and how they interact in terms of cyclical fluctuations.

References

Adelman, I (1965) **Long cycles: Fact or artifact?** American Economic Review 55(3), 444–464.

Balke, N – Gordon, R (1986) **Appendix b: Historical data.** In Gordon, Robert J (ed): The American Business Cycle: Continuity and Change. NBER, University of Chicago Press, Chicago, IL, USA, 781–850.

Baxter, M – King, R (1999) **Measuring business cycles: Approximate band-pass filters for economic time series.** The Review of Economics and Statistics 81(4), 575–593.

Bruce, A – Gao, H-Y (1996) **Applied wavelet analysis with S-PLUS.** New York, NY, USA, Springer-Verlag.

Burns, A – Mitchell, W (1946) **Measuring business cycles.** New York, NY, USA, NBER.

Camba Mendez, G – Kapetanios, G (2001) **Spectral based methods to identify common trends and common sycles.** Working Paper 62, ECB, Frankfurt, Germany.

Chiann, C – Morettin, P (1998) **A wavelet analysis for time series.** Nonparametric Statistics 10, 1–46.

Collard, F (1999) **Spectral and persistence properties of cyclical growth.** Journal of Economic Dynamics and Control 23, 463–488.

Comin, D – Gertler, M (2003) **Medium term business cycles.** Working Paper 10003, NBER, Cambridge, MA, USA.

Conway, P – Frame, D (2000) **A spectral analysis of new zealand output gaps using fourier and wavelet techniques.** DP2000/06.

Crowley, P (2007) **A guide to wavelets for economists.** Journal of Economic Surveys 21(2), 207–267.

Crowley, P (2009) **Whistling down the wire: new results on cycles in growth using time-frequency analysis with US GNP data.** Research mimeo.

Crowley, P – Lee, J (2005) **Decomposing the co-movement of the business cycle: A time-frequency analysis of growth cycles in the euro area.** Bank of Finland Discussion Paper 12/2005, Helsinki, Finland.

Crowley, P – Schildt, T (2009) **An analysis of the embedded frequency content of macroeconomic indicators and their counterparts using the Hilbert-Huang transform.** Bank of Finland Discussion Paper 23/2009. Suomen Pankki – Bank of Finland, Helsinki, Finland.

Debauchies, I (1992) **Ten lectures on wavelets.** Montpelier, VT, USA: Capital City Press.

Daubechies, I (1996) **Where do wavelets come from? A personal point of view.** Proceedings of the IEEE 84(4), 510–513.

Evans, G W – Honkapohja, S – Romer, P (1998) **Growth cycles.** American Economic Review 88(3), 495–515.

Gallegati, M – Gallegati, M (2007) **Wavelet variance analysis of output in G-7 countries.** Studies in Nonlinear Dynamics Econometrics 11(3), 1435–1455.

Gençay, R – Selçuk, F – Whicher, B (2001) **An introduction to wavelets and other filtering methods in finance and economics**. San Diego, CA, USA: Academic Press.

Granger, C (1966) **The typical spectral shape of an economic variable.** Econometrica 34(1), 150–161.

Granger, C – Hatanka, M (1964) **Spectral analysis of economic time series.** NJ, USA: Princeton University Press.

Greenhall, C (1991) **Recipes for degrees of freedom of frequency stability estimators.** IEEE Transactions on Instrumentation and Measurement 40, 994–999.

Hicks, J (1950) **A contribution to the theory of the trade cycle.** Oxford, UK: Oxford University Press.

Holschneider, M (1995) **Wavelets: an analysis tool.** Oxford, UK: Oxford University Press.

Howrey, E (1968) **A spectrum analysis of the long-swing hypothesis.** International Economic Review 9, 228–260.

Huang, N – Shen, Z – Long, S – Wu, M – Shih, H – Zheng, Q – Yen, N-C – Tung, C – Liu, H (1998) **The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series methods.** Proceedings of the Royal Society of London A 454, 903–995.

Hughes Hallett, A – Richter, C (2004) **Spectral analysis as a tool for financial policy: An analysis of the short-end of the british term structure.** Computational Economics 23, 271–288.

Hughes Hallett, A – Richter, C (2006a) **Is the convergence of business cycles a global or regional issue? The UK, US and Euroland.** International Journal of Finance and Economics 11(3), 177–194.

Hughes Hallett, A – Richter, C (2006b) **Is the convergence of business cycles a global or regional issue? The UK, US and Euroland.** International Journal of Finance and Economics 11, 177–194.

Hughes Hallett, A – Richter, C (2008) **Have the eurozone economies converged on a common cycle?** International Economics and Economic Policy 5(1–2), 71–101.

Keynes, J (1936) **The general theory of employment, interest and money.** London, UK: Macmillan.

Kitchin, J (1923) **Cycles and trends in economic factors.** Review of Economics and Statistics 5, 10–17.

Kondratieff, N (1984) **The long wave cycle.** Richardson and Snyder, New York, NY, USA. Translated from the 1925 original in Russian by Guy Daniels.

Kontolemis, Z (1997) **Does growth vary over the business cycle? Some evidence from the G7 countries.** Economica 64(255), 441–460.

Koopmans, L (1995) **The spectral analysis of time series.** Probability and Mathematical Statistics, Vol. 22, Academic Press, New York, NY, USA.

Lau, K-M – Weng, H-Y (1995) **Climate singal detection using wavelet transform: How to make a time series sing.** Bulletin of the American Meteorological Society 76, 2391–2402.

Levy, D – Dezhbakhsh, H (2003a) **International evidence on output fluctuation and shock persistence.** Journal of Monetary Economics 50, 1499–1530.

Levy, D – Dezhbakhsh, H (2003b) **On the typical spectral shape of an economic variable.** Applied Economic Letters 10(7), 417–423.

Mallat, S (1989) **A theory for multiresolution signal decomposition: The wavelet representation.** IEEE Transactions on Pattern Analysis and Machine Intelligence 11(7), 674–693.

Maraun, D – Kurths, J (2007) **Nonstationary gaussian processes in wavelet domain: Synthesis, estimation, and significance testing.** Physical Review E 75(016707).

Mitchell, W (1946) **Business cycles: The problem and its setting.** New York, USA.

Percival, D – Mofjeld, H (1997) **Analysis of subtidal coastal sea level fluctuations using wavelets.** Journal of the American Statistical Association 92, 868–880.

Percival, D – Walden, A (2000) **Wavelet methods for time series analysis.** Cambridge, UK: Cambridge University Press.

Ramsey, J – Lampart, C (1997) **The decomposition of economic relationships by time scale.** 97-08.

Schumpeter, J (1939) **Business cycles.** McGraw Hill, New York, NY, USA.

Shensa, M (1992) **The discrete wavelet transform: Wedding the à trous and mallat algorithms.** IEEE Trans. Signal Processing 40, 2464–2482.

Torrence, C – Compo, G (1998) **A practical guide to wavelet analysis.** Bulletin of the American Meteorological Society 79(1), 61–78.

Valle e Azevedo, J (2002) **Business cycles: Cyclical comovement within the european union in the period 1960–1999. A frequency domain approach.** WP 5-02, Banco do Portugal, Lisbon, Portugal.

Walden, A – Cristan, C (1998) **The phase-corrected undecimated discrete wavelet packet transform and its application to interpreting the timing of events.** The Proceedings of the Royal Society of London: Mathematical, Physical, and Engineering Sciences 454(1976), 2243–2266.

Wu, Z – Huang, N (2008) **Ensemble empirical mode decomposition: A noise assited data analysis method.** Advances in Adaptive Data Analysis 1(1), 1–41.

Yogo, M (2008) **Measuring business cycles: A wavelet analysis of economic time series.** Economics Letters 100(2), 208–212.

Zarnovitz, V (1985) **Recent work on business cycles in historical perspective: A review of theories and evidence.** Journal of Economic Literature 23, 523–580.

Zarnowitz, V – Moore, G (1986) **Major changes in cyclical behavior.** iIn Gordon, E (ed) 'The American Business Cycle: Continuity and Change'. NBER Studies in Business cycles, Vol. 25, Cambridge, MA, USA.

Zarnowitz, V – Ozyildirim, A (2002) **Time series decomposition and measurement of business cycles, trends and growth cycles.** Working Paper 8736, NBER, Cambridge, MA, USA.

Appendix

A. MODWT of level real GNP

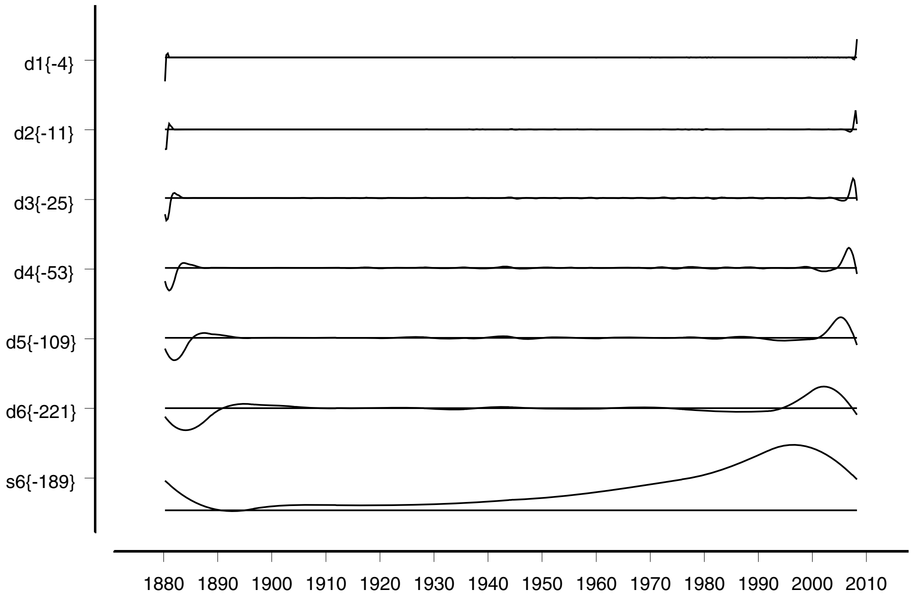

Figure 28: **MODWT for level US real GNP**

BANK OF FINLAND RESEARCH
DISCUSSION PAPERS

ISSN 0785-3572, print; ISSN 1456-6184, online

1/2010 Timo Korkeamäki – Elina Rainio – Tuomas Takalo **Law and stock markets: evidence from an emerging market.** 2010. 34 p. ISBN 978-952-462-568-5, print; ISBN 978-952-462-569-2, online.

2/2010 Iftekhar Hasan – Heiko Schmiedel – Liang Song **Growth strategies and value creation: what works best for stock exchanges?** 2010. 48 p. ISBN 978-952-462-570-8, print; ISBN 978-952-462-571-5, online.

3/2010 Iftekhar Hasan – Heiko Schmiedel – Liang Song **Return from retail banking and payments.** 2010. 53 p. ISBN 978-952-462-572-2, print; ISBN 978-952-462-573-9, online.

4/2010 Sungho Choi – Bill B Francis – Iftekhar Hasan **Cross-border bank M&As and risk: evidence from the bond market.** 2010. 47 p. ISBN 978-952-462-574-6, print; ISBN 978-952-462-575-3, online.

5/2010 Hanna Freystätter **Financial market disturbances as sources of business cycle fluctuations in Finland.** 2010. 46 p. ISBN 978-952-462-576-0, print; ISBN 978-952-462-577-7, online.

6/2010 Patrick M Crowley **Long cycles in growth: explorations using new frequency domain techniques with US data.** 2010. 49 p. ISBN 978-952-462-586-9, print; ISBN 978-952-462-587-6, online.

Suomen Pankki
Bank of Finland
P.O.Box 160
FI-00101 HELSINKI
Finland

. 2343