

---

# BANK OF FINLAND DISCUSSION PAPERS

---

3/97

**Antti Ripatti**

Research Department  
21.3.1997

Limited and Full Information Estimation  
of the Rational Expectations Demand for  
Money Model: Application to Finnish M1

**Suomen Pankki**  
**Bank of Finland**  
**P.O.Box 160, SF-00101 HELSINKI, Finland**  
**☎ + 358 0 1831**

**Antti Ripatti**

Research Department

21.3.1997

# Limited and Full Information Estimation of the Rational Expectations Demand for the Money Model: Application to Finnish M1

The views expressed are those of the author and not of the Bank of Finland. I thank Jouko Vilmunen, Pekka Ilmakunnas, Juha Tarkka and Matti Virén for useful comments and discussion. The usual disclaimer applies. Correspondence: Bank of Finland, Box 160, FIN-00101 Helsinki, Finland. e-mail: antti.ripatti@bof.fi

ISBN 951-686-543-7  
ISSN 0785-3572

Suomen Pankin monistuskeskus  
Helsinki 1997

# Limited and Full Information Estimation of the Rational Expectations Demand for Money Model: Application to Finnish M1

Bank of Finland Discussion Papers 3/97

Antti Ripatti  
Research Department

## Abstract

We compare parameter estimates of the intertemporal money-in-the-utility-function model estimated using the Generalized Method of Moments and the Full Information Maximum Likelihood method. The process driving the forcing variables is approximated with vector autoregression. The FIML estimates of the deep parameters are reasonable, although some of them differ from the corresponding GMM estimates. The simulation experiments suggest that the differences are not very big in practice and that they are connected with adjustment costs. The cross-equation restrictions are clearly rejected, as is typical for these kinds of models; exogeneity restrictions are rejected as well.

Keywords: money-in-the-utility-function model, demand for money, narrow money, Generalized Method of Moments, Full Information Maximum Likelihood

JEL classification: C22, C32, C52, E41

## Tiivistelmä

Tutkimuksessa vertaillaan rajoitetun informaation ja täyden informaation estimointimenetelmien tuottamia suppean rahan kysyntäfunktion parametriestimaatteja. Rahan kysynnän teoreettinen malli perustuu raha hyötyfunktiossa -lähestymistapaan, jossa taloudenpitäjä maksimoi odotettua hyötyä, jota hän voi saada kulutuksesta ja rahan hallussapidosta. Tarkasteluissa käy ilmi, että eri menetelmin tuotetut parameteriestimaatit ovat pääosin melko lähellä toisiaan. Suurimmat erot löytyvät parametreista, jotka liittyvät rahan määrän sopeuttamiseen ja sitä kautta rahan määrän lyhyen aikavälin kehityksen kuvaamiseen. Parametriestimaattien välisiä eroja kuvataan myös simulointikokein, jotka vahvistavat edelläesitetyt johtopäätökset. Suoritetuissa testeissä teoreettisen mallin tuottamat poikkeuslämpörajoitukset tulevat hylätyksi.

Asiasanat: raha hyötyfunktiossa, rahan kysyntä, suppea raha, yleistetty momenttimenetelmä, suurimman uskottavuuden menetelmä

JEL luokitus: C22, C32, C52, E41

# Contents

<b>1</b>	<b>Introduction</b>	<b>7</b>
<b>2</b>	<b>Money-in-the-Utility-Function Model</b>	<b>8</b>
<b>3</b>	<b>Limited and Full Information Estimators and Tests for Cross-Equation Restrictions</b>	<b>10</b>
3.1	GMM Estimation of the Parameters . . . . .	10
3.2	Campbell and Shiller Approach . . . . .	11
3.3	FIML Estimation . . . . .	13
<b>4</b>	<b>Empirical Results</b>	<b>15</b>
4.1	GMM Parameter Estimates and Campbell and Shiller Test . . .	15
4.2	FIML Parameter Estimates . . . . .	17
<b>5</b>	<b>Policy Simulations</b>	<b>23</b>
<b>6</b>	<b>Conclusions</b>	<b>25</b>
	<b>References</b>	<b>27</b>
	<b>Appendix The Data</b>	<b>28</b>

# 1 Introduction

In the companion study, Ripatti (1996), we presented an intertemporal money-in-the-utility-function model and estimated the log-linearized first order conditions in two steps by cointegration techniques and the Generalized Method of Moments (GMM) estimator. This is an example of the *limited information* approach to the estimation of 'deep' parameters, since we made no special assumptions on the process driving the forcing variables<sup>1</sup>. We used two money measures: narrow money (M1) and broad harmonized money (M3H). In contrast to the M3H model, estimation of the M1 model resulted in stable parameters. The estimates of the deep parameters are within a reasonable range.

In this paper we extend the analysis of M1 in two directions. First, we approximate the processes of the forcing variables by a finite order vector autoregression and estimate the same demand for money parameters as in the companion study, using the Full Information Maximum Likelihood (FIML) method. This gives us an exceptional opportunity to compare the GMM and FIML parameter estimates. Second, as a byproduct of the FIML approach, we can test the cross-equation restrictions implied by the theoretical model. Our application of FIML estimation utilizes the ideas of Campbell and Shiller (1987).

Although the GMM provides consistent estimates of the 'deep' parameters of preferences and technology, it is a limited information technique in the sense that all the assumptions of the theoretical model are otherwise utilized, but the process driving the forcing variables is not restricted<sup>2</sup>. Of course, this particular feature of the approach may prove advantageous, since it provides at least a partial hedge against the Lucas critique. Furthermore, tests of overidentification restrictions serve as a diagnostic tool to check whether the moment restrictions implied by the theoretical model are valid<sup>3</sup>.

However, even if we knew something about the process driving the forcing variables, we would not be able to utilize that information in the above GMM approach. The FIML estimation takes into account this kind of information, but a specific parametrization and distributional assumptions on the process of forcing variables are needed. If, in the estimation period, structural changes have occurred and we do not explicitly take them into account, the resulting parameter estimates are subject to the Lucas critique. Hence, there is a tradeoff between the two approaches. In this study, we approximate the process of the forcing variables with a vector autoregression.

By applying both approaches in this study, we are able to compare the parameter estimates produced by the limited and full information methods. This comparison could shed some light on the tradeoff between the GMM and FIML. However, the present study will not give a systematic account of this experiment (as does West 1986) eg by means of Monte Carlo simulations.

---

<sup>1</sup>We even relax the usual stationarity assumption.

<sup>2</sup>The GMM assumes stationarity of the variables. In this paper we relax that assumption and use cointegration techniques to estimate the parameters of the steadystate.

<sup>3</sup>There are several caveats to GMM estimation and the test for overidentification restrictions; see Newey (1985) and Hall (1993) and references therein.

Instead, it illustrates the differences in the parameter estimates by conducting two policy simulations and forecasting experiments, since this is the preferred context for application of the estimated model.

Once the process of the forcing variables is specified, one can test the cross-equation restrictions implied by the theoretical model. Using the approach proposed by Campbell and Shiller (1987), the test can be performed as a nonlinear Wald test. Within the FIML framework, one can use the likelihood ratio test and avoid the invariance problem of the nonlinear Wald test.

Section 2 introduces the intertemporal money-in-the-utility-function model and presents its main features. The GMM estimation is introduced in section 3.1. A reparametrization of the model and testable restrictions are derived in section 3.2. The FIML approach is reviewed in section 3.3. Section 4 presents the GMM and FIML parameter estimates<sup>4</sup>, compares them and section 5 illustrates their differences via two simulation experiments. The final section concludes.

## 2 Money-in-the-Utility-Function Model

We are keen on the dynamics of the relationship between money, consumption and interest rates. The strong persistence in nominal money balances — and even in the growth rate of nominal balances — suggests that changes in nominal balances involve adjustment costs. Consequently, we include adjustment costs in our model. The money-in-the-utility-function (MIUF) approach is analytically the simplest for our purposes and gives us room to illustrate the dynamics of the relationship<sup>5</sup>.

In the MIUF model, the household optimizes the discounted sum of expected utility from consumption and money (for details, see Ripatti 1996):

$$\max E_0 \sum_{t=0}^{\infty} \delta^t \left( u(C_t) + \zeta v \left( \frac{M_t}{P_t} \right) \right) \quad (1)$$

subject to the following budget constraint:

$$C_t + B_t + \frac{M_t}{P_t} + \frac{a(M_t, M_{t-1}, M_{t-2})}{P_t} \leq y + \frac{M_{t-1}}{P_t} + (1 + r_{t-1})B_{t-1}, \quad (2)$$

where  $y$  is exogenous income,  $C_t$  real value of consumption,  $B_t$  real value of bonds denominated in units of time- $t$  consumption,  $r_t$  real return on bonds,  $M_t$  money holdings,  $P_t$  price level and  $a(\cdot)$  adjustment costs. We specify the utility function in the constant-relative-risk-aversion form (CRRA) and adjustment

<sup>4</sup>The computations were done with PC-FIML 8.1 (see Doornik and Hendry 1994) and Gauss 3.2.11 with the MAXLIK library.

<sup>5</sup>See Ripatti (1996) for details of the following model.

costs as follows:

$$u(C_t) = \begin{cases} \frac{1}{1-\rho} C_t^{1-\rho} & \text{if } \rho \neq 1 \\ \log C_t & \text{if } \rho = 1 \end{cases}$$

$$v\left(\frac{M_t}{P_t}\right) = \begin{cases} \frac{1}{1-\omega} \left(\frac{M_t}{P_t}\right)^{1-\omega} & \text{if } \omega \neq 1 \\ \log\left(\frac{M_t}{P_t}\right) & \text{if } \omega = 1 \end{cases}$$

$$a(M_t, M_{t-1}, M_{t-2}) = \frac{\kappa}{2} [(M_t - M_{t-1}) - \nu(M_{t-1} - M_{t-2})]^2.$$

We assume that nominal bonds exists in our generic economy and that the following conditional covariance applies:

$$\text{cov}_t \left( \frac{P_t}{P_{t+1}} \frac{u'(C_{t+1})}{u'(C_t)}, [1 - a'_{M_t}(M_{t+1}, M_t, M_{t-1})] \right) = 0.$$

The first order condition for nominal bonds can then be written as

$$\begin{aligned} \Delta M_t = & \frac{1 + \nu}{(1 + \nu)\nu + I_t} E_t \Delta M_{t+1} + \frac{\nu I_t}{(1 + \nu)\nu + I_t} \Delta M_{t-1} \\ & - \frac{1}{\kappa[(1 + \nu)\nu + I_t]} (I_t - 1) + \frac{\zeta}{\kappa[(1 + \nu)\nu + I_t]} I_t C_t^\rho \left(\frac{M_t}{P_t}\right)^{-\omega}. \end{aligned} \quad (3)$$

where  $I_t \equiv 1 + i_t$ . The covariance condition holds if consumers are risk neutral and inflation is deterministic or if the 'net own-yield of money',  $1 - a'_{M_t}(M_{t+1}, M_t, M_{t-1})$ , is deterministic.

Due to the possibly non-stationary variables, the GMM is not suitable for estimation of the equation (3). We must use estimators that can be applied to models with nonstationary variables and (log-)linearize the equation (3). When we log-linearize the first order conditions around the steadystate, we obtain the following log-linear Euler equation:

$$\begin{aligned} \Delta m_t = & \frac{1}{I + \nu + \nu^2} \left[ i\omega \left( m - p - \frac{\rho}{\omega} c + \frac{1}{\omega i} i \right) + (1 + \nu) E_t \Delta m_{t+1} \right. \\ & \left. + I\nu \Delta m_{t-1} - \frac{i\omega}{\kappa M} \left( m_t - p_t - \frac{\rho}{\omega} c_t + \frac{1}{\omega i} i_t \right) \right], \end{aligned} \quad (4)$$

where the variable names without subscript are linearization points of the equation. Given that the variables of the first order condition (4) are integrated of order one (I(1)), the Euler equation (4) implies one cointegration vector.

### 3 Limited and Full Information Estimators and Tests for Cross-Equation Restrictions

#### 3.1 GMM Estimation of the Parameters

West (1988) and Sims, Stock and Watson (1990) show that for linear models<sup>6</sup> with nonstationary variables — like our's (4) — the parameters can be estimated with instrumental variables techniques and that the variance-covariance matrix can be estimated in the usual way, given that the nonstationary variables and instrument variables are mutually cointegrated and that the first differences of the nonstationary variables have nonzero drift terms. However, in practice this approach is misguided since the finite sample distribution is not invariant with respect to the values of the drift-term parameters. This approach also leads to tests whose power goes to zero as the sample size increases<sup>7</sup>.

We choose the following two step approach suggested by Dolado, Galbraith and Banerjee (1991): first we estimate the cointegration vector implied by the last term in parenthesis in equation (4) using the FIML approach of Johansen (1991). Given these cointegration vectors, we use the GMM to estimate the stationary part of equation (4). The details of the approach are described in Ripatti (1996).

In the GMM estimation we derive the orthogonality conditions from equation (4). Let  $\mathbf{x}_t$  be the  $s$  dimensional<sup>8</sup> vector of instruments. The orthogonality conditions are then

$$h(\theta, w_t) = \left[ (I + \nu + \nu^2)\Delta m_t - i\omega \left( m - p - \frac{\rho}{\omega}c + \frac{1}{\omega i}i \right) - (1 + \nu)\Delta m_{t+1} - I\nu\Delta m_{t-1} + \frac{i\omega}{\kappa M} \left( m_t - p_t - \frac{\rho}{\omega}c_t + \frac{1}{\omega i}i_t \right) \right] \mathbf{x}_t, \quad (5)$$

where  $\theta \equiv (\nu, I, \kappa M, \omega, \rho)$  is the parameter vector and  $w_t \equiv (\Delta m_t, \Delta m_{t+1}, \Delta m_{t-1}, m_t, p_t, c_t, i_t)'$  the vector of variables observed by the econometrician. The average value of these conditions is

$$g(\theta; w_1, \dots, w_T) \equiv \frac{1}{T} \sum_{t=1}^T h(\theta, w_t),$$

and the GMM objective function to be minimized is

$$Q(\theta) = g(\theta; w_1, \dots, w_T)' \hat{S}_T^{-1} g(\theta; w_1, \dots, w_T).$$

Since the agent uses all information available at time  $t$ , the orthogonality conditions should not be autocorrelated and the consistent estimator of  $S$  is  $\hat{S}_T = \frac{1}{T} \sum_{t=1}^T [h(\hat{\theta}, w_t)][h(\hat{\theta}, w_t)]'$ .

<sup>6</sup>They consider linear models in variables. Nagaraj and Fuller (1991) extends the analysis to linear models which are nonlinear in parameters.

<sup>7</sup>See references above and Campbell and Perron (1991).

<sup>8</sup> $s$  is equal or larger than the number of parameters to be estimated

In the GMM estimation, we encounter the problem of defining the instrument set,  $\mathbf{x}_t$ . The GMM estimator varies with the choice of instruments. According to the simulation experiments of Tauchen (1986) and Kocherlakota (1990), increasing the number of instruments decreases the estimators' variance but increases the bias in small samples.

### 3.2 Campbell and Shiller Approach

Campbell and Shiller (1987) merge rational expectations present value models and the cointegrated VAR model. Their idea relies on approximation of the processes of forcing variables using VAR and incorporating that information into the Euler equation. The approach is applicable only to linear (in variables) models.

We write our Euler-equation (4) in the error correction form with forward-looking dynamics:

$$\Delta m_t = \mu + \frac{\nu I}{(1+\nu)\lambda} \Delta m_{t-1} + \alpha E_t \sum_{i=0}^{\infty} \left(\frac{1}{\lambda}\right)^i \beta' \Delta X_{t+i} + \alpha(m_{t-1} + \beta' X_{t-1}), \quad (6)$$

where

$$A \equiv \frac{\omega i}{\kappa M(1+\nu)},$$

$$X_t \equiv [p_t \ c_t \ i_t]',$$

$$\alpha \equiv A + \frac{I}{1+\nu} + \nu - \lambda - \frac{\nu I}{(1+\nu)\lambda},$$

$$\beta \equiv \frac{A}{(\lambda-1)\alpha} \begin{bmatrix} 1 \\ \rho/\omega \\ -1/i\omega \end{bmatrix},$$

$$\mu = \frac{i\omega}{(1+\nu)(1-\lambda)} \left( m - p - \frac{\rho}{\omega} c + \frac{1}{i\omega} i \right).$$

The parameter  $\lambda$  represents the stable root of the characteristic equation

$$\lambda^3 - \lambda^2 \left( A + \frac{I}{1+\nu} + (1+\nu) \right) + \lambda(I+\nu) - \frac{\nu I}{1+\nu} = 0, \quad (7)$$

ie the root having the property  $|\frac{1}{\lambda}| < 1$ <sup>9</sup>.

One should note that all the components of (6) are stationary, given that the individual variables in  $X_t$  are I(1) and  $m_t$  and  $X_t$  are cointegrated. The cointegration vector  $\beta$  in (6) can be estimated separately. Because of super-consistency,  $\beta$  can be treated as asymptotically fixed in the subsequent analysis. Hence, by Wold's decomposition, a linear combination of  $m_t$  and  $X_t$  must have an infinite moving-average representation. The vector moving-average representation can be approximated in finite samples by a  $k^{\text{th}}$  order vector autoregression

$$\begin{bmatrix} \Delta m_t \\ e_t \end{bmatrix} = \sum_{i=1}^k \phi_i \begin{bmatrix} \Delta m_{t-i} \\ e_{t-i} \end{bmatrix} + \begin{bmatrix} \varepsilon_{m,t} \\ \varepsilon_{e,t} \end{bmatrix}, \quad (8)$$

<sup>9</sup>Given the parameter estimates by Ripatti (1996), the roots are  $-0.33, 0.93, 3.38$ .

where  $e_t = m_t + \beta X_t$ . This can be written in the companion form

$$Z_t = \Phi Z_{t-1} + \varepsilon_t,$$

where  $Z_t \equiv [\Delta m_t \ e_t \ \Delta m_{t-1} \ e_{t-1} \ \dots \ \Delta m_{t-k+1} \ e_{t-k+1}]'$ .

Let  $f$  and  $g$  be  $(2k \times 1)$  selection vectors with unity in the first and second elements and zeros elsewhere. Then we can write

$$\Delta m_t = f' Z_t \text{ and } e_t = g' Z_t.$$

Our information,  $H_t$ , which is less comprehensive than that of an economic agent, includes current and lagged values of  $\Delta m_t$  and  $e_t$  and can be written as

$$H_t = \{\Delta m_t, \Delta m_{t-1}, \dots, e_t, e_{t-1}, \dots\}.$$

Then

$$E(Z_{t+i}|H_t) = \Phi^i Z_t$$

and

$$E\left\{\sum_{i=0}^{\infty} \left(\frac{1}{\lambda}\right)^i \beta' \Delta X_{t+i} \middle| H_t\right\} = \left[g' \left(\mathbf{I} - \frac{\mathbf{I}}{\lambda}\right) - f'\right] \left(\mathbf{I} - \frac{\Phi}{\lambda}\right)^{-1} Z_t - g' Z_{t-1}.$$

We project both sides of equation (6) onto  $H_t$ :

$$\begin{aligned} E\{\Delta m_t | H_t\} &= f' Z_t \\ &= \frac{\nu I}{(1 + \nu)\lambda} f' Z_{t-1} + \alpha \left[g' \left(\mathbf{I} - \frac{\mathbf{I}}{\lambda}\right) - f'\right] \left(\mathbf{I} - \frac{\Phi}{\lambda}\right)^{-1} Z_t, \end{aligned} \quad (9)$$

which gives the 'theoretical' level of money demand growth,  $\Delta m_t^*$ . Finally, we project equation (9) onto  $H_{t-1}$ , which gives

$$f' \Phi = \frac{\nu I}{(1 + \nu)\lambda} f' + \alpha \left[g' \left(\mathbf{I} - \frac{\mathbf{I}}{\lambda}\right) - f'\right] \left(\mathbf{I} - \frac{\Phi}{\lambda}\right)^{-1} \Phi. \quad (10)$$

Equation (10) gives the (nonlinear) parameter restrictions. This parameter restriction could be tested with the nonlinear Wald test. However, the numerical value of the Wald test of nonlinear restrictions depends on the algebraic formulation of the nonlinear restrictions<sup>10</sup>.

A second drawback of our application of the Campbell-Shiller approach is that we must have estimates of parameters  $\nu$ ,  $I$ ,  $\lambda$  and  $\alpha$ . These can be obtained from the GMM estimation of the Euler equation. However, these parameters could be estimated together with the parameters in  $\Phi$ . The third problem follows from the fact that the forcing variables are estimated in error correction form, ie using  $e_t$ . The short-run dynamics are restricted by the parameters of the cointegration vectors. In such a case we do not allow cross-linkages between forcing variables.

<sup>10</sup>See eg Phillips and Park (1988) and Gregory and Veall (1985).

### 3.3 FIML Estimation

This section tries to solve the problems one faces in the Campbell-Shiller approach. Our aim is to estimate the parameters (other than cointegration parameters) of the model given the process of the forcing variables. Since the variables in the model are stationary, we can apply likelihood ratio test statistic to test the cross-equation restrictions implied by the rational expectations hypothesis. We can also specify the process of forcing variable in such a way that we can perform the policy experiments discussed in the introduction.

The first difference of forcing variables,  $\Delta X_t$ , in equation (6) is assumed to be stationary. Any stationary process has Wold decomposition, which can be approximated in small samples by a finite-dimension autoregressive process:

$$\Delta X_t = \sum_{i=1}^k \theta_i \Delta X_{t-i} + \epsilon_t. \quad (11)$$

For simplicity we drop the constant term from the vectors  $X_t$  and  $\beta$ . Since  $\Delta X_t$  is  $(3 \times 1)$  vector, each  $\theta_i$  is a  $(3 \times 3)$  matrix. Equation (11) can be written in the companion form, as above, as follows

$$V_t = \Theta V_{t-1} + \varsigma_t, \quad (11')$$

where  $V_t = [\Delta X_t' \cdots \Delta X_{t-k+1}']$  is  $(3k \times 1)$  vector,  $\Theta$   $(3k \times 3k)$  matrix and  $\varsigma_t = [\varsigma_{1,t} \varsigma_{2,t} \varsigma_{3,t} 0 \cdots 0]'$   $(3k \times 1)$  vector. As above, we use the  $(3k \times 3)$  selection matrix  $h = [\mathbf{I}_3 \mathbf{0}_3 \cdots \mathbf{0}_3]'$  to pick up the component  $\Delta X_t$  from  $V_t$ , ie

$$\Delta X_t = h' V_t.$$

We also define the information set of the econometrician  $H_t = \{\Delta X_t, \Delta X_{t-1}, \dots\}$  as above. The information set of the econometrician is strictly smaller than that of the economic agent,  $\Omega_t$  (here, the representative household), ie  $H_t \subset \Omega_t$ . Also  $E(V_{t+i}|H_t) = \Theta^i V_t$  applies.

Finally, when

$$\begin{aligned} E \left\{ \sum_{i=0}^{\infty} \left( \frac{1}{\lambda} \right)^i \beta^i \Delta X_{t+i} \middle| H_t \right\} &= E \left\{ \sum_{i=0}^{\infty} \left( \frac{1}{\lambda} \right)^i \beta^i h' V_{t+i} \middle| H_t \right\} \\ &= \sum_{i=0}^{\infty} \left( \frac{1}{\lambda} \right)^i \beta^i h' \Theta^i V_t = \sum_{i=0}^{\infty} \beta^i h' \left( \frac{\Theta}{\lambda} \right)^i V_t \\ &= \beta^i h' (\mathbf{I}_{3k} - \Theta/\lambda)^{-1} V_t, \end{aligned}$$

equation (6) can be written in the form

$$\begin{aligned} \Delta m_t &= \frac{\nu I}{(1 + \nu)\lambda} \Delta m_{t-1} + \alpha \beta^i h' (\mathbf{I}_{3k} - \Theta/\lambda)^{-1} V_t \\ &\quad + \alpha(m_{t-1} + \beta^i X_{t-1}) + \eta_t, \end{aligned} \quad (12)$$

where

$$\eta_t = \alpha \sum_{i=0}^{\infty} \left( \frac{1}{\lambda} \right)^i \beta^i \{ E[\Delta X_{t+i} | \Omega_t] - E[\Delta X_{t+i} | H_t] \}.$$

The error term,  $\eta_t$ , in equation (12) arises from the difference between the information sets of the econometrician and the household. Our equation to be estimated is in the same form as Binder and Pesaran (1995) or Blanchard (1983). We write these equations in a vector form in order to illustrate various restrictions implied by the model. Let us assume that  $k = 2$ . Then equations (11) and (12) can be stacked as follows

$$\underbrace{\begin{bmatrix} 1 & \varrho_{11} & \varrho_{12} & \varrho_{13} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{A_0} \underbrace{\begin{bmatrix} \Delta m_t \\ \Delta p_t \\ \Delta y_t \\ \Delta i_t \end{bmatrix}}_{\Delta v_t} = \underbrace{\begin{bmatrix} \frac{\nu I}{(1+\nu)\lambda} & \varrho_{21} & \varrho_{22} & \varrho_{23} \\ 0 & \theta_{111} & \theta_{112} & \theta_{113} \\ 0 & \theta_{121} & \theta_{122} & \theta_{123} \\ 0 & \theta_{131} & \theta_{132} & \theta_{133} \end{bmatrix}}_{A_1} \underbrace{\begin{bmatrix} \Delta m_{t-1} \\ \Delta p_{t-1} \\ \Delta y_{t-1} \\ \Delta i_{t-1} \end{bmatrix}}_{\Delta v_{t-1}} \\
 + \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \theta_{211} & \theta_{212} & \theta_{213} \\ 0 & \theta_{221} & \theta_{222} & \theta_{223} \\ 0 & \theta_{231} & \theta_{232} & \theta_{233} \end{bmatrix}}_{A_2} \underbrace{\begin{bmatrix} \Delta m_{t-2} \\ \Delta p_{t-2} \\ \Delta y_{t-2} \\ \Delta i_{t-2} \end{bmatrix}}_{\Delta v_{t-2}} \\
 + \underbrace{\begin{bmatrix} \alpha \\ 0 \\ 0 \\ 0 \end{bmatrix}}_a \underbrace{\begin{bmatrix} 1 \\ \beta \end{bmatrix}'}_{e_{t-1}} \underbrace{\begin{bmatrix} m_{t-1} \\ p_{t-1} \\ y_{t-1} \\ i_{t-1} \end{bmatrix}}_{e_{t-1}} + \underbrace{\begin{bmatrix} \eta_t \\ \epsilon_{p,t} \\ \epsilon_{y,t} \\ \epsilon_{i,t} \end{bmatrix}}_{\epsilon_t}. \quad (13)$$

We assume that  $\epsilon_t \sim \text{NID}(\mathbf{0}_{4 \times 1}, \Sigma_\epsilon)$ , where  $\Sigma_\epsilon$  need not be a diagonal matrix. The log-likelihood function of the system (13), that is to be maximized is the following:

$$\ell(\nu, I, \kappa M, \omega, \Sigma_\epsilon) = -\frac{T}{2} \log(|\Sigma_\epsilon|) - \frac{1}{2} \sum_{t=1}^T \epsilon_t' \Sigma_\epsilon \epsilon_t.$$

We have number of interesting hypotheses here. Since  $A_0$  is non-singular, we can write the model in the form of a Vector Error Correction Mechanism with two lags (VECM(2)):

$$\begin{aligned}
 A_0 \Delta v_t &= A_1 \Delta v_{t-1} + A_2 \Delta v_{t-2} + a e_{t-1} + \epsilon_t \iff \\
 \Delta v_t &= A_1^* \Delta v_{t-1} + A_2^* \Delta v_{t-2} + a^* e_{t-1} + \epsilon_t^*, \quad (14)
 \end{aligned}$$

where  $A_1^* \equiv A_0^{-1} A_1$ ,  $A_2^* \equiv A_0^{-1} A_2$  and  $a^* \equiv A_0^{-1} a$ . Our theoretical model restricts  $\varrho_{ij}$  to be a highly nonlinear function of  $\theta_{kij}$ . The number of restrictions is  $3k$ , ie here six. They are given by  $\alpha \beta' h' (\mathbf{I}_{3k} - \Theta/\lambda)^{-1}$ . Since all the variables in the system are stationary, the hypothesis can be tested with the likelihood ratio test. The test statistic is asymptotically  $\chi^2(3k)$  distributed. We can test our restricted model (14) against the unrestricted VECM(2) model with and without the cross-equation restrictions implied by the rational expectations assumption. Given the structure of  $A_0$ , the last three elements of the first column of  $A_1^*$  and  $A_2^*$  are zero and  $A_0^{-1} a = a$ . These features imply that in the system (14), the  $\Delta m_t$  should not Granger cause the forcing variables and that the forcing variables should be weakly exogenous with respect to the long-run parameters  $\alpha$  and  $\beta$ . Jointly this means that forcing variables should be *strong exogenous*, which is also a testable hypothesis.

## 4 Empirical Results

### 4.1 GMM Parameter Estimates and Campbell and Shiller Test

In the companion paper Ripatti (1996) we have estimated the cointegration part of the model using the FIML of Johansen (1988) and the dynamics part using the GMM of Hansen (1982). The Finnish data consist of monthly observations on narrow money (M1), the consumer price index, the GDP volume indicator and the one month money market rate. The estimation period is January 1980 – December 1995.

The estimated full sample cointegration vector is

$$\hat{e}_t = [1 \ \hat{\beta}']v_t = [(m - p)_t - y_t + 1.807i_t]. \quad (15)$$

The scale elasticity is restricted to unity ( $p$ -value 0.49). Given the above cointegration vector, the GMM estimates of the parameter are presented in table 1. The instrument set,  $\mathbf{x}_t$ , contains the constant,  $\Delta m_{t-3}$ ,  $\Delta p_{t-j}$ ,  $\Delta y_{t-j}$ ,  $\Delta i_{t-j}$  and  $\hat{e}_{t-j}$  ( $j = 2, 3$ ).

To implement the Campbell-Shiller test, we proceed in the following way. First, given the estimated cointegration vector (15), we estimate the parameters of (8). Second, we test for Granger non-causality. Third, given the parameter estimates of the first column of table 1, we compute the 'theoretical level' of money growth defined in (9). Fourth, using the nonlinear Wald test, we test for the parameter restriction implied by equation (10) using the GMM estimates presented in table 1.

Using the information criteria and the residual autocorrelation test, we end up with lag length three, ie  $k = 3$  in equation (8). The model is fairly stable. Instabilities might occur in the middle of 1980s (see figure 1).

For the test that  $\Delta m$  does not Granger cause  $e$ , the  $p$ -value is 0.49. Thus, as the theoretical model implies, there is no delayed feedback from money growth to the error correction term. The nonlinear Wald test for the hypothesis defined by (10) is asymptotically  $\chi^2(6)$  distributed. Given the GMM estimates of the parameters other than  $\Phi$ , the cross-equation restrictions are rejected; the value of the test statistic is 90.9 ( $p$ -value < 0.001). This is very usual in tests of such cross-equation restrictions.

There might be several reasons for rejection of the null hypothesis. First, one should remember, that the nonlinear Wald test is not invariant with respect to reparameterization of the restrictions. It appears that it is possible to obtain any (positive) numerical value for the Wald test of nonlinear restrictions by reparameterizing the restrictions. Second, we use GMM estimates for some of the parameters (parameters in the coefficient of the lagged money change). Hence, the computed standard errors are not the correct ones. Finally, the rejection of the model might be caused by economically unimportant factors like measurement errors and thus might not have economic significance (Campbell and Shiller 1987). The first two caveats can be handled by estimating the parameters under restrictions with FIML. Then the likelihood ratio test statistic can be computed. It is invariant with respect to reparameterization. In the next section we will concentrate on that issue.

Table 1: Parameter Estimates of the Euler Equations for M1

Parameter <sup>a</sup>	M1		
	1980:5 – 1995:12 <sup>b</sup>	1980:5 – 1986:12 <sup>c</sup>	1987:1 – 1995:12 <sup>d</sup>
$I = (1 + i)$	2.04 (0.014)	2.15 (0.10)	2.02 (0.005)
$\nu$	-0.34 (0.12)	-0.18 (0.12)	-0.38 (0.09)
$\kappa M$	3.71 (1.30)	15.46 (6.39)	1.77 (0.36)
$\omega$	0.53 (0.01)	0.48 (0.05)	0.54 (0.004)
$\rho^e$	0.53 (0.01)	0.48 (0.05)	0.54 (0.004)
Coefficient of the lead term	0.36 (0.02)	0.40 (0.19)	0.35 (0.01)
Coefficient of the lag term	-0.38 (0.04)	-0.50 (0.05)	-0.43 (0.03)
Coefficient of the error correction <sup>f</sup> term	-0.08 (0.09)	-0.02 (1.79)	-0.18 (0.02)
Significance level of the test for overidentification restrictions	0.87	0.70	0.13
Significance level of the parameter stability tests	AF <sup>g</sup> : 0.18; GH <sup>h</sup> : 0.53		

<sup>a</sup>Standard errors are in parentheses below the parameter value. The standard error of the 'derived' parameters, ie parameters that are computed from the original free parameters, are based on linear approximation with respect to the original parameters of the model. However, they do not account for the uncertainty of the cointegration parameters.

<sup>b</sup>Full sample

<sup>c</sup>Period of financial deregulation.

<sup>d</sup>Period of free capital markets.

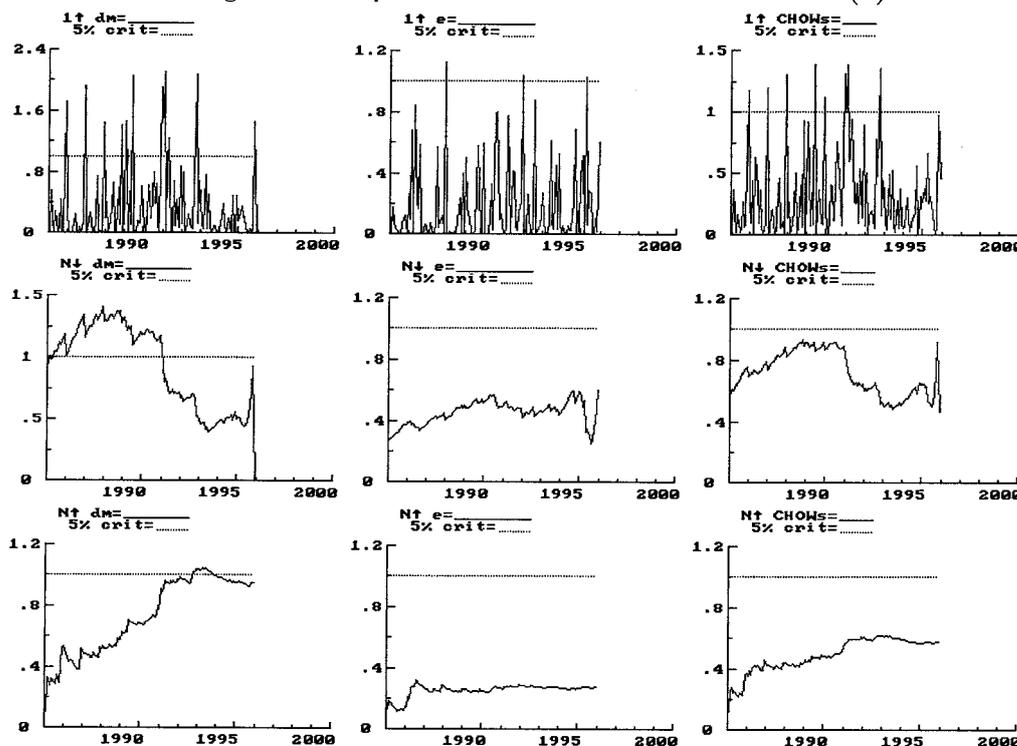
<sup>e</sup>In M1 system  $\rho = \omega$  due to the unit scale elasticity.

<sup>f</sup>This is the loading of the single cointegration vector, ie  $m_t - p_t - \frac{\rho}{\omega} y_t + \frac{1}{\omega i} i_t$ .

<sup>g</sup>Andrews and Fair (1988) test statistics.

<sup>h</sup>Ghysels and Hall (1990) test statistics, based on the weighting matrices of each sub-sample.

Figure 1: Sequence of Chow Tests of the Model (8)



The first row of graphs contains the sequence of Chow-tests, where the model estimated using the sample ending at  $t$  is compared to the model using the sample ending at  $t - 1$  ( $t = 1985M1, \dots, 1995M12$ ). The second row of graphs compares the full sample model with the model using the sample ending at  $t$  ( $t$  as above). In the third row of graphs, the model using the sample ending at 1984M12 is compared to the model using the sample ending at  $t$  ( $t$  as above). All the test statistics are scaled by one-off critical values from the F-distribution. The first column contains tests for the  $\Delta m_t$  equation, the second column for the  $e_t$  equation and the third column for the system. See Doornik and Hendry (1994) for details.

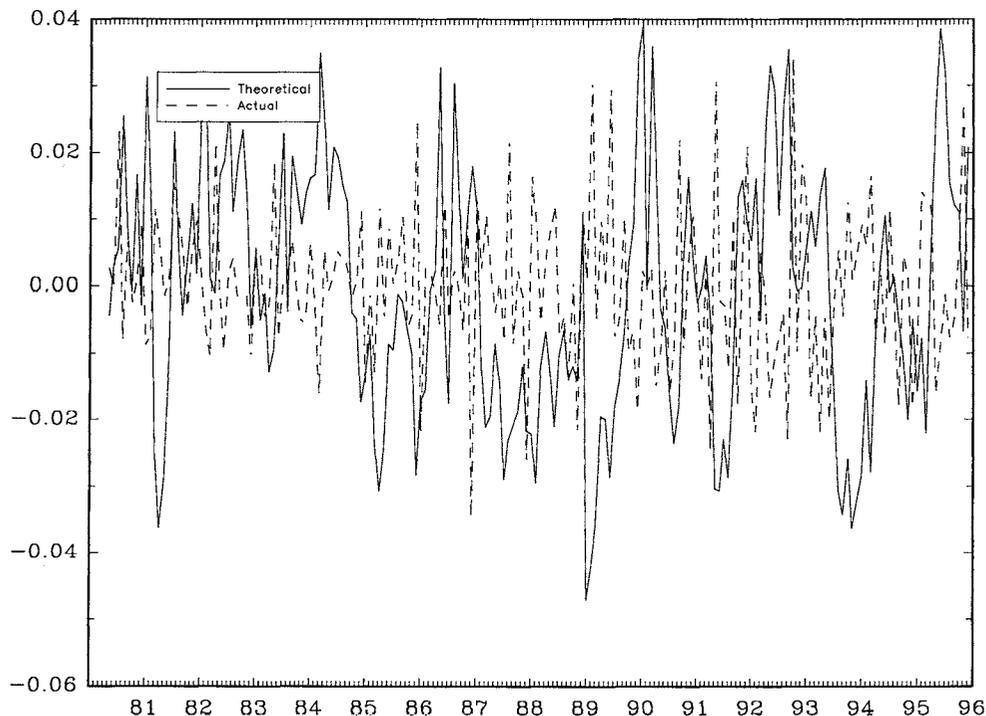
Since the test for the overidentification restrictions is not very informative, Campbell and Shiller (1987) recommend investigation of the graph of ‘theoretical’ (from equation (9)) and actual money growth (figure 2). This graph is not very convincing<sup>11</sup>. There are periods when these two lines diverge for several months, eg in 1984, 1989, 1993 and 1995.

## 4.2 FIML Parameter Estimates

In this section, we follow the Full Information Maximum Likelihood setup suggested in section 3.3. We estimate the model formed by equations (11) and (12); from these estimates we derive the deep parameters. We also compare the FIML and GMM estimates of the deep parameters. Finally, we test for the cross-equation and exogeneity restrictions implied by our theoretical model against the vector error correction model (VECM).

<sup>11</sup>We have produced the same type of graph and test statistic using lag length 12. The results remain roughly the same.

Figure 2: Actual and 'Theoretical' Money Growth

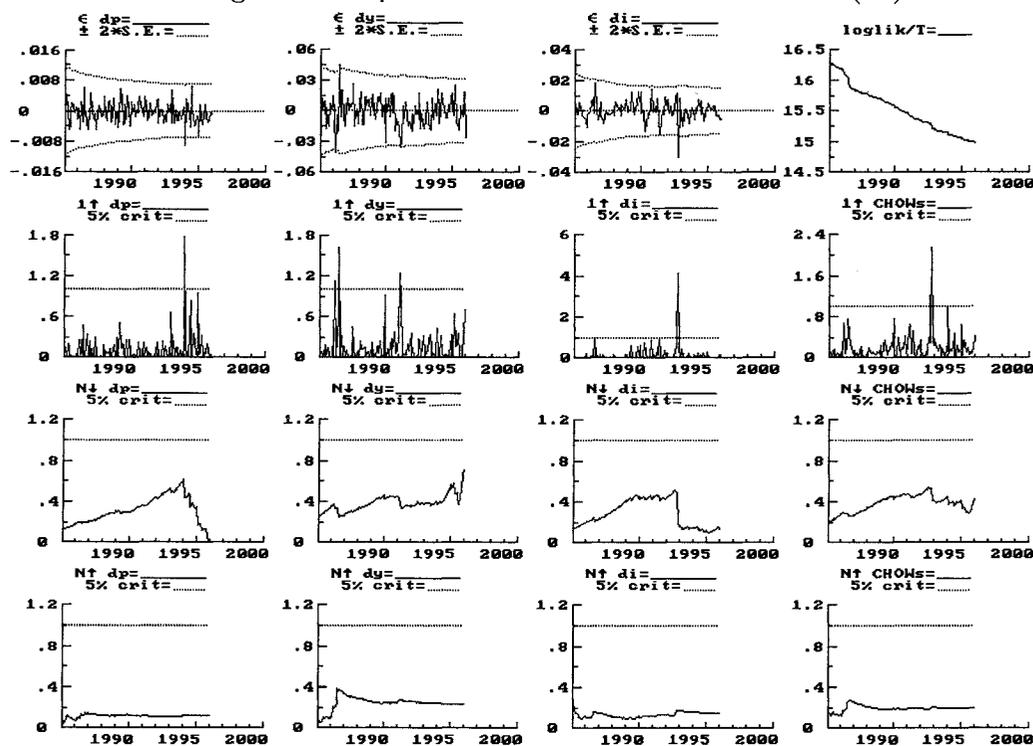


The error term in equation (12) arises from differences in the information sets of the economic agent and the econometrician. Because of this, the error terms in equations (11) and (12) are not independent. Hence, system estimation is needed. First, we need to determine the lag length  $k$  of the process in equation (11). Since the estimation of (12) is computationally burdensome, we cannot perform system-wide stability tests. Consequently, we concentrate on the stability of the process of the forcing variables.

We need three lags,  $k = 3$ , in (11) to obtain white noise residuals. The estimated system is fairly stable (see figure 3). However, there might be instabilities in the interest rate change equation at the start of the floating exchange rate regime. The introduction of the VAT in July 1994 is not modelled adequately, which is reflected in the recursive Chow tests.

The deep parameters are  $\nu$ ,  $\omega$ ,  $\rho$ ,  $I$  and  $\kappa M$ . The last two parameters, which refer to the linearization point, are not strictly deep parameters, ie they are not preference or technology parameters. We do not estimate the deep parameters directly. Instead, we estimate the coefficient of the term  $\Delta m_{t-1}$  and the parameters  $\alpha$  and  $\lambda$  in equation (12). The cointegration parameters  $\beta$  are as in the previous section (see equation (15)). The *deep* parameters are a highly nonlinear function of the estimated parameters. They are determined by equations (6) and (7). The characteristic equation (7) is a third order polynomial. The closed form solution of its roots is such a complicated function of the deep parameters that it is not analytically applicable to the estimation. Given the parameter estimates, we can compute the estimates of the deep parameters. We can use the implicit functions to compute the derivatives needed for the calculation of the standard errors of those parameters. The deterministic components, ie the constant and the seasonal and other dummies,

Figure 3: Sequence of Chow Tests of the Model (11)



The first row of graphs contains recursive residuals (first three columns) and recursive log-likelihood. The next row contains the sequence of Chow tests, where the model estimated using the sample ending at  $t$  is compared to model using the sample ending at  $t - 1$  ( $t = 1985M1, \dots, 1995M12$ ). The third row of graphs compare the full sample model with the model using the sample ending at  $t$  ( $t$  as above). In the fourth row of graphs, the model using the sample ending at 1984M12 is compared to the model using the sample ending at  $t$  ( $t$  as above). All the test statistics are scaled by one-off critical values from the F-distribution. The first column contains tests for the  $\Delta p_t$  equation, the second for the  $\Delta y_t$ , the third column for the  $\Delta i_t$  equation and the fourth for the system. See Doornik and Hendry (1994) for details.

are concentrated from the likelihood function.

The estimation turns out to be computationally burdensome. Parameter estimates and standard errors are presented in table 2<sup>12</sup>. The problem with the estimation is the tendency of  $\lambda$  to reach values below unity. We fixed  $\lambda$  at 3.38, which is the GMM estimate, and estimated rest of the parameters freely. The VAR approximation of  $\Delta X_t$  is quite modest. The interest rate changes and GDP growth are mainly explained by their own history. The residual correlation between the money change and GDP growth, as well as interest rate change, is surprisingly high. The third graph of grid plot 4 presents the likelihood surface of  $1/\lambda$  given the estimates of the other parameters.

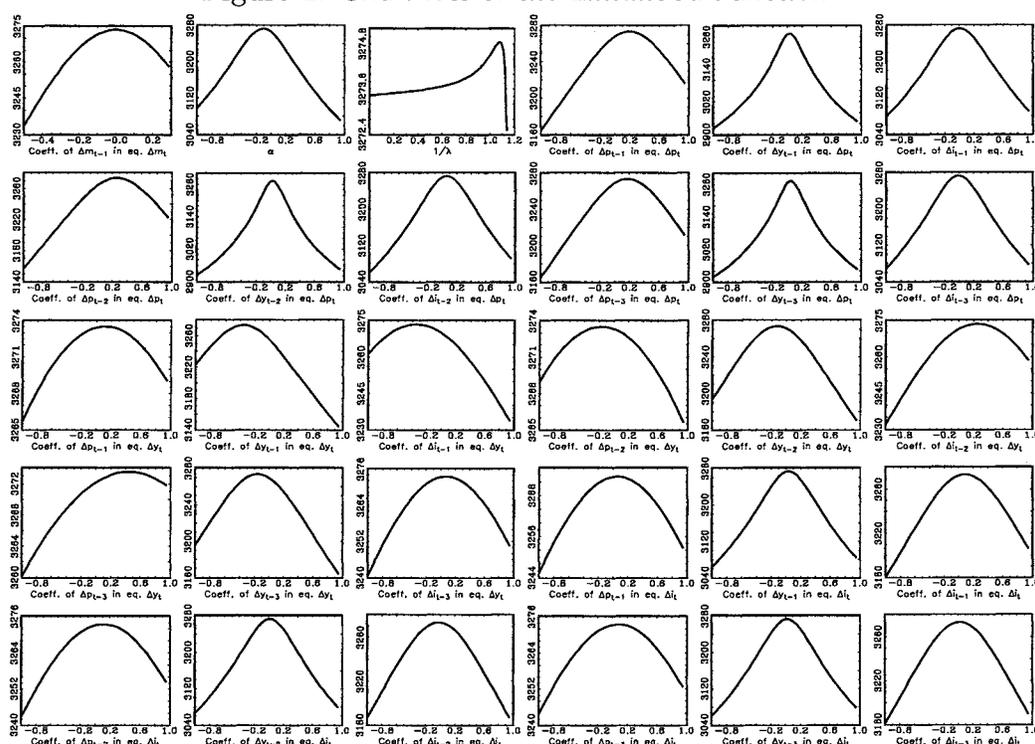
According to the residual diagnostics, the model is not quite satisfactory:

<sup>12</sup>We have estimated the coefficients of deterministic variables in the first step. They can be concentrated from the likelihood function since we have the same set of deterministic variables in every equation.

there is slight autocorrelation (in lags 1 and 12) in the money equation and (in lag 12) in the GDP equation. Normality is violated in the form of excess kurtosis in the price and interest rate equations. Due to these facts, the ML estimator should be considered as a Quasi ML estimator.

The estimate of the parameter  $\alpha$  is very close to the comparable parameter in the GMM estimation ( $-0.0931$ ). It is highly significant. The GMM estimate falls within the 95 per cent confidence interval. The coefficient of lagged money growth does not differ significantly from zero. The GMM estimate differs significantly from the FIML estimate. This is the major difference between the GMM and FIML estimates of the parameters. This parameter is important, since its existence is connected to our definition of the adjustment cost function. The parameters in the equations for  $\Delta p_t$ ,  $\Delta y_t$  and  $\Delta i_t$  determine the process of the forcing variables. The estimate of  $\Theta$  is reasonable. We have experimented with other lag lengths ( $k = 2, 4, 6, 9, 12$ ). These choices yield implausible estimates of the utility function parameters.

Figure 4: Grid Plots of the Likelihood Function



It is interesting to compare the GMM and FIML estimates of the deep parameters (table 3). First of all, the estimates are surprisingly close to each other, given that the GMM does not utilize information on the processes of the forcing variables. The standard errors of the FIML estimates are quite high compared to the standard errors for the GMM. The FIML estimate of the linearization point of interest rates,  $I$ , is significantly higher than the GMM estimate. It is also very high compared to what one would expect. The FIML estimates of the adjustment cost function parameter,  $\nu$ , does not differ significantly from zero. This simplifies our adjustment cost function in such a way that the lagged money change can be omitted. This yields the usual simple quadratic adjustment cost function in levels of money. The magnitude of the

Table 2: FIML Estimates of the Parameters

variable <sup>a</sup>	GMM Estimates	Equation			
		$\Delta m_t$	$\Delta p_t$	$\Delta y_t$	$\Delta i_t$
$\alpha$	-0.0931	-0.1074 (0.0306)			
$1/\lambda^b$	0.2959	0.2959 (3.3547)			
$\Delta p_t$		-0.0910 <sup>c</sup>			
$\Delta y_t$		-0.0070			
$\Delta i_t$		0.2528			
$\Delta m_{t-1}$	-0.3109	-0.00001 (0.0474)			
$\Delta p_{t-1}$		0.0057	0.2011 (0.0692)	0.1110 (0.3523)	0.0554 (0.2078)
$\Delta y_{t-1}$		0.0175	0.0348 (0.0183)	-0.3604 (0.0637)	0.0341 (0.0313)
$\Delta i_{t-1}$		0.0011	0.0084 (0.0317)	-0.3484 (0.1821)	0.0881 (0.0668)
$\Delta p_{t-2}$		-0.0017	0.2605 (0.0752)	-0.1669 (0.2731)	0.1026 (0.2141)
$\Delta y_{t-2}$		0.0033	0.0240 (0.0142)	-0.1274 (0.0766)	0.0124 (0.0291)
$\Delta i_{t-2}$		0.0144	0.0628 (0.0282)	0.2516 (0.1532)	-0.0260 (0.0863)
$\Delta p_{t-3}$			0.1759 (0.0592)	0.4432 (0.3333)	0.0736 (0.1578)
$\Delta y_{t-3}$			0.0485 (0.0117)	-0.1545 (0.0716)	0.0171 (0.0273)
$\Delta i_{t-3}$			-0.0085 (0.0342)	0.0706 (0.1471)	0.0157 (0.0898)
		Residual Diagnostics			
Normality		0.47	11.68	0.71	92.98
$\chi^2_{0.05}(2) = 5.99$					
Box-Pierce 12 lags		0.05	0.19	0.03	0.14
<i>p</i> -value					
		Residual Variance and Correlation			
$\Delta m_t$		0.02332			
$\Delta p_t$		0.09251	0.00181		
$\Delta y_t$		-0.17236	-0.05712	0.04012	
$\Delta i_t$		-0.13544	0.07726	-0.04564	0.00884

<sup>a</sup>The standard errors are in the parenthesis below the parameter estimates. They are computed from the inverse of cross-product of first derivatives.

<sup>b</sup>Parameter  $\lambda$  is estimated in the inverse form. In the FIML estimation it is fixed to the GMM estimate.

<sup>c</sup>The parameters below without standard error are computed from the parameters of  $\Theta$ .

parameter  $\kappa M$  is close to that of the GMM estimate. It is very precisely estimated and the GMM estimate does not fall within the 95 per cent confidence interval. The utility function parameters,  $\omega$  and  $\rho$ , are somewhat smaller than the GMM estimates. Their standard error is very high; the GMM estimates and even zero fall within the 95 per cent confidence interval.

Table 3: FIML and GMM Estimates of the ‘Deep’ Parameters

	GMM	FIML <sup>a</sup>
$I = (1 + i)$	2.04 (0.014)	3.02 (0.16)
$\nu$	-0.34 (0.12)	-0.00002 (0.54)
$\kappa M$	3.71 (1.30)	2.17 (0.03)
$\omega$	0.53 (0.01)	0.27 (1.10)
$\rho$	0.53 (0.01)	0.27 (1.10)

<sup>a</sup>The standard errors of the deep parameters have been computed using the implicit functions (6) and (7) and their derivatives at the point estimates of the parameters. The deep parameters are highly correlated.

Let us denote the various (nested) restrictions as follows:

- $\mathcal{H}_{\text{VECM}}$  unrestricted VECM( $k$ ),
- $\mathcal{H}_{\text{EXO}}$  VECM( $k$ ) with Granger non-causality (plus weak exogeneity of cointegration parameters) restrictions,
- $\mathcal{H}_{\text{RE}}$  cross-equation restrictions implied by the theoretical model.

The hypotheses are nested as  $\mathcal{H}_{\text{RE}} \subseteq \mathcal{H}_{\text{EXO}} \subseteq \mathcal{H}_{\text{VECM}}$ . They are illustrated by equations (13) and (14) (with the assumption that lag length is two). Our test setups and results are presented in table 4.

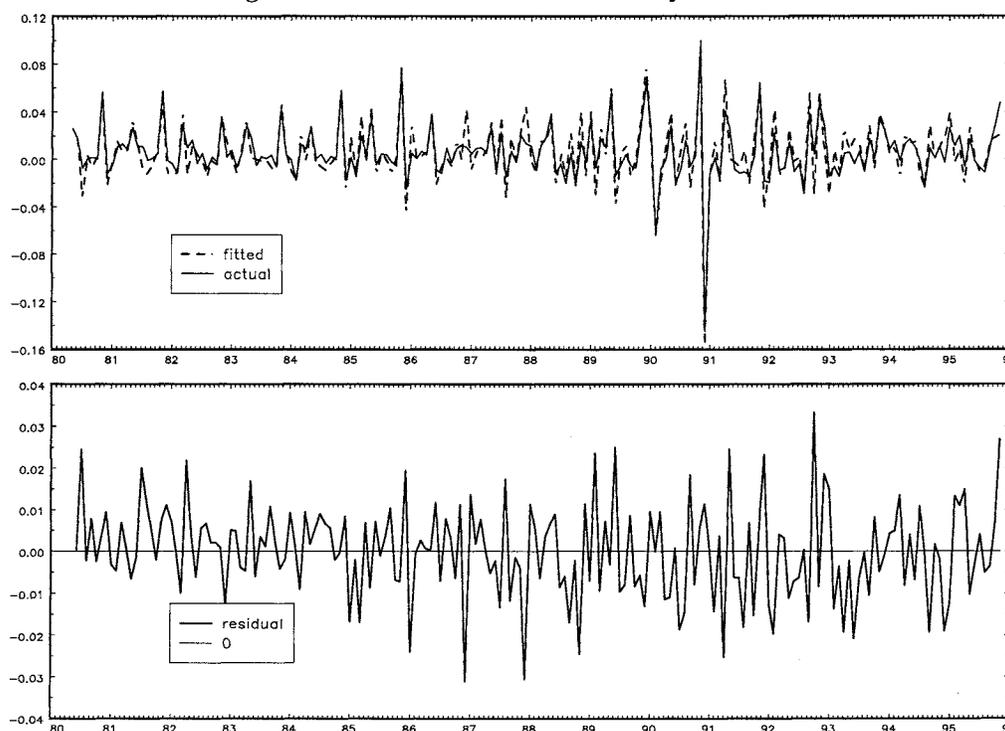
Table 4: Tests for Exogeneity and Cross-Equation Restrictions

Hypotheses		Test	Degrees of	$p$ -value
$H_0$	$H_1$	statistic	freedom	
$\mathcal{H}_{\text{RE}}$	$\mathcal{H}_{\text{EXO}}$	740	9	< 0.001
$\mathcal{H}_{\text{RE}}$	$\mathcal{H}_{\text{VECM}}$	768	23	< 0.001
$\mathcal{H}_{\text{EXO}}$	$\mathcal{H}_{\text{VECM}}$	28.5	14	0.012

The message of the likelihood ratio test statistics is typical: The cross-equation restrictions implied by the theoretical model are clearly rejected. The Cambell-Shiller discussion of the interpretation of the result is valid here also.

The rejection of the cross-equation restrictions might well be due to factors that are not economically important, like measurement errors etc. However, one should also note that the rejection of the exogeneity restriction is also on the borderline. This implies that our approach would benefit from the modelling of the behaviour of the other sectors, eg price formation and monetary policy. Even though the cross-equation restrictions are rejected, the fit of the model is fairly good. The view given by figure 5 is entirely different than the view given by figure 2 of the Campbell-Shiller approach.

Figure 5: Actual and Fitted Money Growth



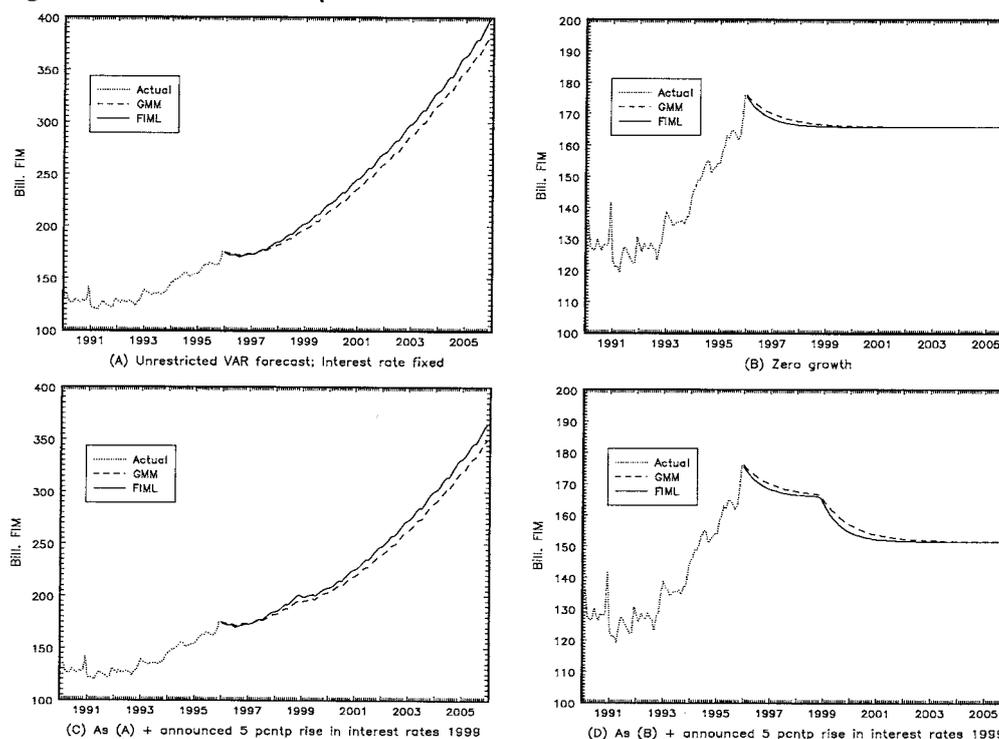
## 5 Policy Simulations

We can use our model version (12) for forecasting. The forecasting performance is not necessarily very good compared with the VECM since we rejected the cross-equation restrictions, ie the unrestricted VECM fitted the data better than the restricted model. Policy simulations serve as an alternative way of illustrating the differences between the GMM and FIML estimates of the deep parameters. We consider two forecasting and policy experiments: First, all the forcing variables are fixed at last-observation levels, ie we use zero growth scenario. Second, unrestricted<sup>13</sup> VAR is used to produce forecasts of  $\Delta p$ ,  $\Delta y$  and  $\Delta i$ . We use both GMM and FIML estimates of the deep parameters to produce the conditional forecasts of M1. These experiments are repeated with the preannounced 5 percentage point rise in interest rates in 1999. Parameter

<sup>13</sup>Instead of unrestricted VAR, we restrict the constant to zero in the interest rate change equation. The unrestricted estimate of the constant is negative, which implies negative interest rates over time.

estimates as reported in table 3 and equation (6) are used to compute the simulated paths, which are showed in figure 6.

Figure 6: Simulation Experiments with GMM and FIML Parameter Estimates



The unrestricted VAR forecasts yield expanding paths for M1. The forecast based on GMM estimates of the parameters is lower than the forecast based on FIML estimates. However, it takes almost two years for the divergence to become visible. The simulated M1 paths based on zero-growth of forcing variables converge to the same level since both techniques are based on the same steady-state estimate. The dynamics differ: the paths based on GMM estimates converge slower toward the steady-state. This is probably due to the significant estimate of the lagged money change. The adjustment costs, according to the GMM estimates, are then higher.

The preannounced 5 percentage point rise in interest rates in 1999 has an interesting impact. First, its discounted rise is visible only a few months earlier. That is due to the low discount factor, which is 0.29 based on the GMM estimates. Second, M1 converges to a level almost FIM 20 billion lower than with no change in interest rates. This clearly indicates that M1 is controllable via monetary policy. The one-month money market rate can be controlled by the Bank of Finland<sup>14</sup>.

The adjustment factors based on the FIML estimates are smaller than those based on the GMM estimates. These factors yield the different paths, which are visible in panels (B) and (D) in figure 6. This study does not show which of these estimates is closer to the true value. One would need to conduct simulation experiments to investigate the matter, which is beyond the scope of

<sup>14</sup>The main liquidity control instrument of the Bank of Finland is the tender. The maturity of the tenders is one month. Therefore, the Bank of Finland can control the one-month money market rate.

this study. The practical difference between the limited and full information estimates is not very large in our case. The computational and programming burden is however much larger for the full information method (FIML) than for the limited information method (GMM). For forecasting and policy simulation purposes the accuracy of the GMM estimates seem to be fairly close to the FIML estimates. Thus, the choice of method depends on the cost function of the applier.

## 6 Conclusions

In this paper, we compare the limited information (GMM) and full information (FIML) approaches to estimating the deep parameters of an intertemporal model of money demand. We illustrate the resulting differences in the parameter estimates with two simulation experiments. We also test for the cross-equation restrictions implied by the rational expectations hypothesis against the general VECM.

The theoretical underpinnings of the paper come from an extension of an intertemporal money-in-the-utility-function to incorporate dynamic adjustment costs from adjusting money balances. The estimated form is derived by log-linearizing the appropriate Euler equations. Hence, these adjustment costs allow for persistence in the growth rate of money. In this sense the model incorporates richer dynamics than, for example, Cuthbertson and Taylor (1987). The cost of this extension comes, quite naturally, from more the complicated algebra and increasing computational burden as well.

We estimate the steady-state parameters of the model using cointegration methods and the rest of the parameters — the dynamic part of the model — using GMM and FIML. In the full information estimation we approximate the process of the forcing variables with vector autoregression.

The GMM and FIML estimates of the parameters of the utility function are fairly close to each other. Larger differences occur in the estimated standard errors of these parameters. This shows up particularly well in the statistical significance of the estimated adjustment cost parameters; whereas the GMM estimate associated with the lagged money change in the adjustment cost function is significant, suggesting no overparameterization of the adjustment costs, the corresponding FIML estimate is clearly not significant. This in turn argues against the GMM conclusion and suggests a more parsimonious parameterization of the adjustment cost function. These differences in the estimated adjustment costs are also visible in the simulated paths of M1. The GMM estimates generate a simulated path that clearly converges at a slower rate to the steady state than does the corresponding one with the FIML estimates. Whether the differences that are visible in the two simulated paths are of any practical relevance is not discussed and cannot of course be determined on a statistical basis alone.

The stochastic specification of the forcing variables allows us to test the cross-equation restrictions implied by the model. Although the cross-equation restrictions are clearly rejected at the conventional significance levels, the fit of the model appears to be otherwise reasonable. This suggests that the cross-

equation restrictions are rejected due to economically uninteresting reasons. The exogeneity assumptions are rejected as well.

The stability of the parameters of the stochastic specification of the forcing variables is a crucial assumption in our application of the FIML estimation. Stability of these parameters is not tested in the present paper since such a test of the whole system is computationally very demanding in the present setup. In this sense, then, we are still ignorant of the empirical validity of the Lucas critique for our FIML estimates. On the other hand, if the FIML estimate of the structure of the adjustment cost function is the correct one, then that would cast doubts on the GMM estimator. Consequently, we can not justify superiority of either of these approaches.

## References

- Andrews, Donald W. K., and Ray C. Fair (1988) 'Inference in nonlinear econometric models with structural change.' *Review of Economic Studies* 55, 615–640
- Binder, Michael, and M. Hashem Pesaran (1995) 'Multivariate rational expectations models and macroeconomic modeling: A review and some new results.' In *Handbook of Applied Econometrics*, ed. M. Hashem Pesaran and M. R. Wickens, vol. 1 (Oxford: Blackwell) pp. 139–187
- Blanchard, Olivier J. (1983) 'The production and inventory behavior of the american automobile industry.' *Journal of Political Economy* 91, 365–400
- Campbell, John Y., and Pierre Perron (1991) 'Pitfalls and opportunities: What macroeconomist should know about unit roots.' *NBER Macroeconomics Annual*
- Campbell, J.Y., and R.J. Shiller (1987) 'Cointegration test of present value models.' *Journal of Political Economy* 95(5), 1052–1088
- Cuthbertson, Keith, and Mark P. Taylor (1987) 'The demand for money: A dynamic rational expectations model.' *Economic Journal* 97(Supplement), 65–76
- Dolado, Juan, John W. Galbraith, and Anindya Banerjee (1991) 'Estimating intertemporal quadratic adjustment cost models with integrated series.' *International Economic Review* 32(4), 919–936
- Doornik, Jürgen A., and David F. Hendry (1994) *PcFiml 8.0, Interactive Econometric Modelling of Dynamic Systems* (London: International Thompson Publishing)
- Ghysels, Eric, and Alastair Hall (1990) 'A test for structural stability of euler parameters estimated via the generalized method of moments estimator.' *International Economic Review* 31, 355–364
- Gregory, Allan W., and Michael R. Veall (1985) 'Formulating wald tests of nonlinear restrictions.' *Econometrica* 50, 1465–1468
- Hall, Alastair (1993) 'Some aspects of generalized method of moments estimation.' In *Handbook of Statistics, Vol. 11*, ed. G. S. Maddala, C. R. Rao, and H. D. Vinod, vol. 11 (London: Elsevier Science Publishers B.V.) pp. 393–417
- Hansen, Lars Peter (1982) 'Large sample properties of generalized method of moments estimator.' *Econometrica* 50(4), 1029–1054
- Johansen, Søren (1988) 'Statistical analysis of cointegrating vectors.' *Journal of Economic Dynamics and Control* 12, 231–254
- (1991) 'Estimation and hypothesis testing of cointegration vectors in Gaussian vector autoregressive models.' *Econometrica* 59, 1551–80

- Kocherlakota, Narayana R. (1990) 'On tests of representative consumer asset pricing models.' *Journal of Monetary Economics* 26, 385-304
- Nagaraj, Neerchal K., and Wayne A. Fuller (1991) 'Estimation of the parameters of linear time series models subject to nonlinear restrictions.' *The Annals of Statistics* 19(3), 1143-1154
- Newey, Whitney K. (1985) 'Generalized method of moments specification testing.' *Journal of Econometrics* 29, 229-256
- Phillips, Peter C. B., and Joon Y. Park (1988) 'On the formulation of wald tests of nonlinear restrictions.' *Econometrica* 56, 1065-1083
- Ripatti, Antti (1996) 'Stability of the demand for m1 and harmonized m3 in finland.' Discussion Papers 18/96, Bank of Finland
- Sims, Chrispher A., James H. Stock, and Mark W. Watson (1990) 'Inference in linear time series models with some unit roots.' *Econometrica* 58, 113-44
- Tauchen, George (1986) 'Statistical properties of generalized method of moments estimators of structural parameters obtained from financial market data.' *Journal of Business Econom. Statistics* 4, 397-425
- West, Kenneth (1988) 'Asymptotic normality when regressors have a unit root.' *Econometrica* 56(6), 1394-1417
- West, Kenneth D. (1986) 'Full versus limited information estimation of a rational expectations model: Some numerical comparisons.' *Journal of Econometrics* 33, 367-385

## A The Data

The empirical counterparts for the theoretical variables are as follows:

**Narrow Money:** Narrow monetary aggregate M1, mill. FIM, logarithm. Includes cash held by the public and transactions accounts at banks.

**Prices:** Consumer price index (1990=100), logarithm, published by Statistics Finland.

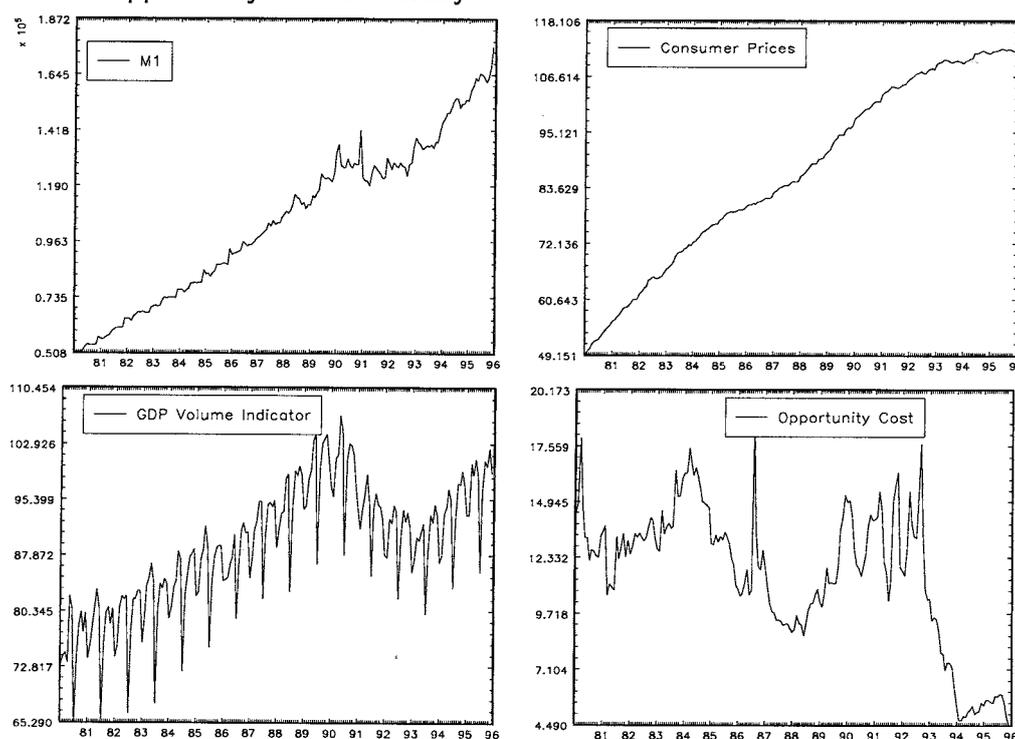
**Transactions:** Monthly GDP volume indicator (1990=100), logarithm, published by the Statistics Finland. A combined index of various indicators such as industrial production, retail sales, consumption of electricity, etc. Since the money measure includes consolidated money holdings of households and the corporate sector, one cannot use consumption as a scale variable. Instead, we use this GDP indicator and neglect the theoretical consequences.

**Opportunity cost of money:** Covered one-month Eurodollar rate for the markka for the pre-1987 period and one-month HELIBOR (money market rate) after that, divided by 100, published by the Bank of Finland. For after-tax version, see the explanation below.

Time period: January 1980 - December 1995. Graphs are presented in figure 7.

There are several exogenous shocks in this period also. They are modelled with the following dummy variables:

Figure 7: Narrow Monetary Aggregate, Consumer Price Index, GDP Volume Indicator and Opportunity Cost of Money



- JULY** The seasonal pattern of the GDP volume indicator has changed along with the construction cycle. An extra seasonal variable JULY has been added. It is the ratio of construction to total GDP, where monthly construction is measured by construction licences (Statistic Finland). The July value is multiplied by 1 and the August value by  $-1$ ; the values for the rest of the year are zero.
- REBATE** Tax rebates are normally paid in December. In the years 1991–1995, the pattern changed temporarily, and that is modelled by the dummy REBATE.
- DSPEC** Devaluation speculation raised interest rates in August 1986 and again in September – December 1991 and finally in April – November 1992, DSPEC. Devaluation speculation also measures the *currency substitution* effect.
- CGAINT** The increase in the capital gains tax in January 1989 is measured by the dummy CGAINT. It is 1 in December 1988, and  $-1$  at end — December 1990, since the special taxfree 24-month time deposit was introduced in December 1988.
- BSTRIKE1** The strike of bank office workers in February 1990 is measured by two dummies. BSTRIKE1 is 1 in January 1990 and  $-1$  in March 1990, while BSTRIKE2 is 1 in February 1990. The strike increased cash held by the public and interest rates were frozen. It was not anticipated before the very end of January.
- WTAX** Introduction of the withholding tax for bank accounts at the beginning of 1991 WTAX. A 15 per cent tax on bank accounts stimulated real competition between banks.

- TRAF The strike of harbour workers in June 1991 decreased industrial production during that month. The production gap was filled in the following month. That strike is modelled by the dummy TRAF.
- MFREST During the pre-1987 period, the Ministry of Finance regulated banks' CD issues. MFREST has a value of unity during that period and zero otherwise.

**BANK OF FINLAND DISCUSSION PAPERS**

ISSN 0785-3572

- 1/97 Peter Nyberg **Macroeconomic Aspects of Systemic Bank Restructuring**. 1997. 30 p. ISBN 951-686-541-0. (KASI)
- 2/97 Anne Brunila **Fiscal Policy and Private Consumption – Saving Decisions: Evidence from Nine EU Countries**. 1997. 36 p. ISBN 951-686-542-9. (TU)
- 3/97 Antti Ripatti **Limited and Full Information Estimation of the Rational Expectations Demand for Money Model: Application to Finnish M1**. 1997. 30 p. ISBN 951-686-543-7. (TU)