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Information acquisition during a Dutch auction

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INFORMATION ACQUISITION DURING A DUTCH AUCTION

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ABSTRACT. In this paper we consider equilibrium behavior in a Dutch (descending price) auction when the bidders that are uninformed of their valuations with probability q and can acquire information about their valuation with a positive cost *during the auction*. We assume that the information acquisition activity is covert. We characterize the equilibrium behavior in the setting where bidders are ex ante symmetric and have independent private values. We show that when the number of bidders is large the Dutch auction produces more revenue than the first price auction.

The theory of auctions usually assumes that the bidders know their valuations for the object to be auctioned. However, there are many instances where this may not be the case: When a venture capitalist is trying to sell a business that it owns it is not immediately clear how much the company is worth for a potential buyer. In addition, if the venture capitalist is unable to sell the company to some set of firms with a given price, he is pushed to lower the price that he asks (or refrain from selling). A lower price may attract the interest of some additional firms. Firms that were not initially interested in the company may want to assess how much the company is worth for them as the price is lowered. Similarly a company that contemplates entering into a takeover battle for one of its rivals must first evaluate how much the rival firm is worth.

Levin & Smith (1994) take up the question of endogenous entry in auctions. In their model the bidders have to incur a positive cost in order to participate into the auction. By paying the participation cost the bidders also learn their valuations for the object. After the bidders have decided about participation the number of participants is made common knowledge. The object is then auctioned to the participating bidders. In a symmetric equilibrium the bidders mix with respect to their decision to participate into the auction. Once the number of participants is known bidding follows the regular equilibrium behavior in the corresponding auction.

When a static auction is in question this approach is fine, as there is only one round of bidding. In a dynamic auction, such as a Dutch or an English auction,

the decision to participate can also be made *during* the auction. That is, if the bidders are allowed to participate into the auction after it has started. The example involving a venture capitalist above fits into this kind of a situation. Another example, that shares the descending nature of prices, is the After-Christmas sales. The sales typically start with a specific discount percentage. The discount percentage is then increased as the sales proceeds.[1] These examples suggest that participation decisions during the auction deserve attention for reasons that are not purely theoretical.

In this paper we study the bidding and information acquisition behavior in the following setting: There are N bidders that can be active in the auction. Each bidder knows his valuation with probability q. Each initially uninformed bidder may become informed by incurring a cost of $c > 0$. We assume that every bidder, informed or not, is allowed to participate into the auction. That is, a bidder may bid for the object even if he is not informed.[2] We study the Dutch auction where we assume that each uninformed bidder can decide the price at which he acquires information. If the object is not sold before the "information acquisition price" the bidder becomes informed about his valuation and incurs the cost c. The bidder may then end the auction immediately, or wait for the price to descend further.

We assume that the bidders' decision to acquire information is covert. Hence each bidder only knows the number of potential competitors. At any given time a bidder does not know how many other bidders have already acquired information or how many other bidders were initially informed.

We consider the case where the bidders have independent private values. We solve for the symmetric equilibrium in the Dutch auction. In equilibrium the uninformed bidders mix with respect to the price at which the information is acquired. The bidding is determined by a pure strategy (conditional on the acquired information). We then compare the revenues that the first price auction and the Dutch auction produce when the number of bidders grows large. We show that in this case the Dutch auction produces larger revenue than the first price auction.

Related literature.
Papers that are most related to the issue of information acquisition during the auction are by Bergemann & Välimäki (2005), Compte & Jehiel (2007), Rezende (2005). Compte & Jehiel (2007) consider information acquisition during an ascending price auction. They allow for information acquisition at any point during the auction in a setting where each bidder has a chance of being informed about his valuation. They show that the ascending price auction can generate higher revenue than the second price sealed bid auction. The setup of this paper coincides with the one analyzed by Compte & Jehiel (2007). Rezende (2005) also studies an

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[1]This year the sales after Christmas in some of Finland's department stores started with 40 percent discount. The discount increased up to 70 percent towards the end of January. The sales for clothing for example, typically involves many items and is not an exact match to the model presented here unless only one item remains in store. However, two features are in common: 1.) the price for the goods in the sale descends 2.) the buyers must incur the cost of inspecting if their size is left in the store.

[2]The analysis remains the same when c is interpreted as the cost of participating and becoming informed if it is assumed that the seller does not disclose any information about the number of participants and no uninformed bidding takes place in the equilibrium. If in equilibrium there are uninformed bidders who bid without information acquisition c cannot be interpreted as the participation cost.

ascending price auction, but in his model the bidders have initial estimates about their valuations and they may learn their exact valuation during the auction. He assumes that the bidders' initial estimates provide statistical information about their true valuation. Additionally, Rezende assumes that the cost of information acquisition is private information and that other bidders' drop out points are not observed before the auction ends, contrary to the paper by Compte and Jehiel. He characterizes the equilibrium information acquisition strategy and shows, like Compte and Jehiel, that the ascending auction is revenue superior (in some cases) to the second price auction.

Bergemann & Välimäki (2005) survey the literature on information and mechanism design and they emphasize the importance of further work on sequential information acquisition in dynamic auctions. Other related work on information acquisition in static auctions and mechanism design are by Milgrom (1981), Persico (2000) and Bergemann & Välimäki (2002). Both Milgrom (1981) and Persico (2000) study a situation where the decision to acquire information is made before any bidding takes place. Milgrom (1981) studies the incentives to acquire information in a second price auction while Persico (2000) studies the incentives that the first price and second price auctions provide for information acquisition. Bergemann & Välimäki (2002) study a general mechanism design problem and ask when it is the case that a mechanism provides ex-ante efficient information acquisition incentives and implements the efficient outcome ex-post.

This paper is organized as follows. In section 1 we first introduce the model. We then proceed constructively to determine the equilibrium bidding strategies and check that the incentive compatibility conditions are satisfied. Section 2 works out an approximation for the revenue difference between the Dutch auction and the first price sealed bid auction. We then show that the Dutch auction produces more revenue than the first price auction when n is large enough. Section 3 concludes.

1. THE MODEL

There are $n \geq 2$ bidders with i.i.d. valuations θ_i, generically denoted by θ. The valuations are distributed on $[0, \bar{\theta}]$ according to an absolutely continuous distribution function $F(\cdot)$ with a density $f(\cdot)$. Each bidder knows his valuation with probability $q > 0$. Hence the number of informed bidders is binomially distributed. The bidders who do not know their valuation may acquire information about their valuation by incurring a cost $c > 0$. The uninformed bidders know that their expected valuation is $v = \int x f(x) dx$. We assume that the bidders cannot distinguish between the uninformed and informed bidders. That is, we assume that the information acquisition is covert. We analyze the Dutch auction where the auctioneer begins with a high asking price which is lowered until some participant announces his willingness to buy the object at the current price. This participant wins the auction and pays the current price. We assume that the uninformed bidders can acquire information at any price during the auction.

A sketch of the Dutch auction equilibrium. To get a flavor of the equilibrium it is useful to first consider the uninformed bidders' information acquisition behavior which can be divided into three phases:

At phase 1 the price is high (above some $\bar{p}$) such that no information acquisition takes place.

At phase 2 the price reaches a point where information acquisition starts. Information acquisition takes place over an interval of prices $(\underline{p}, \bar{p})$ and all uninformed become informed at some price on the interval.

At phase 3 no further information acquisition takes place.

The bidding behavior in the three phases is the following:

At phase 1 only informed bidders with high valuations bid.

At phase 2 only the bidders that become informed and observe that their valuation is above a threshold valuation bid the price at which they acquired information. If the valuation is below the threshold the bidders wait.

At phase 3 all bidders compete with strategies that are similar to the first price auction.

We argue that the information acquisition must take place over an interval of prices. Consider what happens if the information acquisition were to take place at a specific price. If the price is "low" the informed bidders have an incentive to bid slightly before the price, since competition intensifies after the price is reached. If the price is "high", the uninformed bidders have an incentive to wait for others to acquire information first and acquire information if the price descends enough. This carries the information that the other bidders' valuations are not high and that the chances of winning the auction are good. On the other hand if the auction ends soon after the information acquisition price the bidder who decided to wait saves the information acquisition cost. Therefore, in equilibrium, the information acquisition price is decided by mixing over an interval of prices.

Since the information is acquired over an interval, it means that the problem that the informed agents face changes when the price arrives to the information acquisition range. This is because the amount of potential competitors increases. The equilibrium that we derive builds on the existence of a threshold valuation w_*. The bidder with this valuation is indifferent between bidding any price over the mixing interval. When the price is in the mixing interval, a bidder with a valuation higher than w_* wants to buy immediately and a bidder with a valuation lower than w_* wants to wait for the price to descend. Notice also that there is a gap in the informed bidders bidding that corresponds exactly to the mixing interval of the uninformed bidders. Our assumptions guarantee that all bidders are willing to acquire information in the auction. Therefore, in equilibrium, it is common knowledge that all bidders are informed once the price has reached the lower bound of the mixing interval.

To ease the notation in the paper we denote by $\tilde{G}(x) = (qF(x) + 1 - q)^{n-1}$ and by $G(x) = F(x)^{n-1}$. The corresponding density functions are denoted by $\tilde{g}(x)$ and $g(x)$. The interpretation of the $\tilde{G}(x)$ is the following: In phase 1 the competing bidders are either informed and have valuations below x - with probability $qF(x)$ - or they are uninformed - with probability $1 - q$. Similarly the interpretation of $G(x)$ is that in phase 3 all bidders - who in equilibrium are informed - have valuations below x. With these observations as our guide we now proceed to construct the equilibrium.

The Dutch auction equilibrium strategies. Let us start with defining a function $u^a(p)$ that gives the expected value of becoming informed (for free) and having the opportunity to buy the object at price p.

$$u^a(p) = \int_p^{\bar{\theta}} (\theta - p) f(\theta) d\theta.$$

We need to assume that the information acquisition cost c is not too large:

Assumption 1. *The information acquisition cost $c > 0$ satisfies*

$$c < u^a(v). \tag{1}$$

Let us define w_* as a threshold valuation that satisfies $c = u^a(w_*)$. Assumption 1 guarantees that w_* exist and that $v < w_*$.

The constructed equilibrium of this paper builds on the assumption that the cost of information acquisition is small enough to allow for all the uninformed bidders to acquire information in equilibrium.[3] It can be shown that this assumption is satisfied when n becomes large. There may be other reasons as to why remaining uninformed is not attractive such as an auction having an entry fee as in Levin & Smith (1994).

We next start to construct the equilibrium strategies. Let us define

$$u(\theta, \hat{\theta}) = \begin{cases} \tilde{G}(\hat{\theta})(\theta - \tilde{\beta}(\hat{\theta})), & \text{for } \hat{\theta} \geq w_* \\ G(\hat{\theta})(\theta - \beta(\hat{\theta})), & \text{for } \hat{\theta} < w_* \end{cases}$$

as the utility of an informed bidder with valuation θ that behaves like an informed with a valuation $\hat{\theta}$. Also define $U(\theta) = u(\theta, \theta)$. For $\theta < w_*$ the bidding strategy is fully determined by $U(0) = 0$ and for $\theta \geq w_*$ the bid function is determined up to a constant $\mathbf{K}$. It is straightforward to derive and check that incentive compatibility is satisfied[4] for the bid functions $\tilde{\beta}, \beta$ that are defined by

$$\tilde{\beta}(\theta) = \frac{\int_0^\theta x \tilde{g}(x) dx + \mathbf{K}}{\tilde{G}(\theta)}, \text{if } \theta \geq w_*$$

$$\beta(\theta) = \frac{\int_0^\theta x g(x) dx}{G(\theta)}, \text{if } \theta < w_*.$$

Let's then turn to the uninformed bidders' strategies and consider the value of acquiring information at the price $\beta(w)$. It is determined by

$$W(w) = \int_0^w U(\theta) f(\theta) d\theta + \int_w^{\bar{\theta}} u(\theta, w) f(\theta) d\theta - c\tilde{G}(w). \tag{2}$$

From the first order condition for equation (2) while observing that $\frac{\partial}{\partial w} u(\theta, w) = (\theta - w)\tilde{G}'(w)$ we get a condition for the threshold valuation:

$$\int_w^{\bar{\theta}} (\theta - w) f(\theta) d\theta = c, \tag{3}$$

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[3]Technically this condition amounts to assuming that $\int_v^{w_*} G(y) dy \geq \int_0^{w_*} G(y) F(y) dy$. One can check that when $w_* > v$ the inequality holds for n large enough. We elaborate on what happens when this assumption is relaxed in the discussion that follows the equilibrium characterization.

[4]See e.g. Krishna (2002).

which is can be satisfied by Assumption 1 for $w = w_*$. The uninformed that acquires information optimally at $\beta(w_*)$ gets

$$\mathbb{E}[\min\{U(\theta), U(w_*)\}] + \tilde{G}(w_*)\Big(\int_{w_*}^{\hat{\theta}}(\theta - w_*)f(\theta)d\theta - c\Big) = \mathbb{E}[\min\{U(\theta), U(w_*)\}].$$

Now the information acquisition starts at $\bar{p} = \tilde{\beta}(w_*)$ and it must also end at some $\underline{p}$.[5] The indifference condition for type w_* allows us to pin down the lower bound to $\underline{p} = \beta(w_*)$. We next need to find an information acquisition strategy that ensures that information acquisition at any price on $(\underline{p}, \bar{p})$ yields $\mathbb{E}[\min\{U(\theta), U(w_*)\}]$.

The uninformed that acquires information at a price $p \in (\underline{p}, \bar{p})$ obtains

$$\int_0^{w_*} U(\theta)f(\theta)d\theta + Q(p)^{n-1}\Big(\int_{w_*}^{\bar{\theta}}(\theta - p)f(\theta)d\theta - c\Big)$$

$$= \int_0^{w_*} U(\theta)f(\theta)d\theta + Q(p)^{n-1}(w_* - p)(1 - F(w_*)), \tag{4}$$

where the probability $Q(p) = \Big(qF(w_*) + (1-q)(F(w_*) + H(p)(1 - F(w_*)))\Big)$ is composed of three events. Either the uninformed is faced with an (initially) informed bidders with $\theta \leq w_*$ - with probability $qF(w_*)$ - or with uninformed bidders with valuations $\theta \leq w_*$ - with probability $(1-q)F(w_*)$. Finally the competing bidders may be uninformed and have valuations $\theta > w_*$ - with probability $(1-q)(1-F(w_*))H(p)$. Equating (10) with $\mathbb{E}[\min\{U(\theta), U(w_*)\}]$ allows us to solve for $H(\cdot)$ and obtain

$$H(p) = \frac{qF(w_*) + 1 - q}{(1-q)(1 - F(w_*))}\Big(\frac{w_* - \bar{p}}{w_- p}\Big)^{\frac{1}{n-1}} - \frac{F(w_*)}{(1-q)(1 - F(w_*))}. \tag{5}$$

Using the fact that $H(\underline{p}) = 0$ and $\underline{p} = \beta(w_*)$ we can solve for $\bar{p}$ and get that

$$\bar{p} = \frac{w_*\tilde{G}(w_*) - \int_0^{w_*} G(x)dx}{\tilde{G}(w_*)}.$$

Finally the indifference condition $\tilde{\beta}(w_*) = \bar{p}$ allows us to pin down the constant of integration $\mathbf{K}$ to be

$$\mathbf{K} = \int_0^{w_*} \tilde{G}(x) - G(x)dx.$$

We have just constructed the following equilibrium strategies.

Proposition 1. *The following strategies constitute a symmetric Bayesian Nash equilibrium of the Dutch auction.*

- *The informed bidders choose the amount they bid according to*

$$\tilde{\beta}(\theta) = \frac{\int_0^\theta x\tilde{g}(x)dx + \mathbf{K}}{\tilde{G}(\theta)}, \text{ if } \theta \geq w_*$$

$$\beta(\theta) = \frac{\int_0^\theta xg(x)dx}{G(\theta)}, \text{ if } \theta < w_*,$$

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[5]Information acquisition cannot continue to $p = 0$. This would necessitate either a mass point at zero or a guaranteed utility of zero to the uninformed bidders.

where $\tilde{G}(x) = (qF(x) + (1-q))^{n-1}$, $G(x) = F(x)^{n-1}$ and
$\mathbf{K} = \int_0^{w_*} \tilde{G}(x) - G(x)dx$.

- *The uninformed bidders choose the price p at which they acquire information from an interval $[\underline{p}, \bar{p}]$, according to the distribution function*

$$H(p) = \frac{\left(qF(w_*) + (1-q)\right)\left(\frac{w_* - \bar{p}}{w_* - p}\right)^{\frac{1}{n-1}} - F(w_*)}{(1-q)(1 - F(w_*))},$$

with the property that $H(\underline{p}) = 0$ and $H(\bar{p}) = 1$. After information is acquired the (un)informed bid according to the informed bidders' strategy. If the information acquisition price p is such that $\tilde{\beta}(\theta) \geq p$ then the uninformed bidder bids p.

The incentive compatibility for the informed bidders when competing against other informed bidders is a standard proof. The incentive compatibility for the information acquisition follows our assumption that information acquisition is always preferred to staying uninformed. It remains to show that the informed have no profitable deviations and that the same applies to the uninformed bidders' information acquisition strategy. We prove these with the following two lemmata.

Lemma 1. *The informed have no profitable deviations on $p \in (\underline{p}, \bar{p})$.*

Proof. Consider the expected utility of an informed bidder θ at price $p \in (\underline{p}, \bar{p})$. This is given by

$$Q(p)^{n-1}(\theta - p) = \tilde{G}(w_*)\left(\frac{w_* - \beta(w_*)}{w_* - p}\right)(\theta - p),$$

where we have substituted for $H(\cdot)$. Now it is immediate that the expected utility is constant if $\theta = w_*$. It is also immediate that the expected utility is increasing in p if $\theta > w_*$ and decreasing if $\theta < w_*$. The initially uninformed bidders with $\theta > w_*$ want to bid in immediately once they obtain information about their valuation, since their expected utility decreases when the price decreases. Conversely bidders with a valuation $\theta < w_*$ want to wait for the price to descend as their expected utility increases when the price decreases. It is also clear from this analysis that the critical type w_* is unique.[6] □

Lemma 2. *The uninformed have no profitable deviations from the information acquisition strategy.*

Proof. Suppose that the uninformed acquires at a price $\beta(\theta') < \beta(w_*)$ which implies that $\theta' < w_*$. Then using equation (2) and the definition of c allows us to compute

$$W(w_*) - W(\theta') = \int_{\theta'}^{w_*} U(\theta) - u(\theta, \theta')f(\theta)d\theta + \int_{w_*}^{\bar{\theta}} u(\theta, w_*) - u(\theta, \theta')f(\theta)d\theta$$

$$- \int_{w_*}^{\bar{\theta}} (\theta - w_*)f(\theta)d\theta(\tilde{G}(w_*) - G(\theta'))$$

$$= \int_{\theta'}^{w_*} U(\theta) - u(\theta, \theta')f(\theta)d\theta + \int_{w_*}^{\bar{\theta}} U(w) - u(w, \theta')f(\theta)d\theta \geq 0,$$

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[6]If $\theta' > w_*$ were the critical type, then the expected utility would decrease as the price increases conversely to the assumption of being a critical type.

since $U(\theta) - u(\theta, \theta') \geq 0$ by the incentive compatibility of the bidding functions. Then consider when the uninformed acquires at a price $\tilde{\beta}(\theta') > \tilde{\beta}(w_*)$ which implies that $\theta' > w_*$.

$$
\begin{aligned}
W(w_*) - W(\theta') &= - \int_{w_*}^{\theta'} U(\theta) f(\theta) d\theta + \int_{w_*}^{\theta'} u(\theta, w_*) f(\theta) d\theta \\
&\quad - \int_{\theta'}^{\bar{\theta}} u(\theta, \theta') f(\theta) d\theta + \int_{\theta'}^{\bar{\theta}} u(\theta, w_*) f(\theta) d\theta \\
&\quad - \int_{w_*}^{\bar{\theta}} (\theta - w_*) f(\theta) d\theta (\tilde{G}(w_*) - \tilde{G}(\theta')) \\
&= - \int_{w_*}^{\theta'} U(\theta) f(\theta) d\theta + \int_{w_*}^{\theta'} U(w_*) f(\theta) d\theta + \int_{w_*}^{\theta'} (\theta - w_*) f(\theta) d\theta \tilde{G}(\theta') \\
&\quad + \int_{\theta'}^{\bar{\theta}} (U(w_*) - u(w_*, \theta') f(\theta) d\theta.
\end{aligned}
$$

Now writing out the first line and observing that the constants of integration cancel we get that

$$
\begin{aligned}
&- \int_{w_*}^{\theta'} U(\theta) f(\theta) d\theta + \int_{w_*}^{\theta'} U(w_*) f(\theta) d\theta + \int_{w_*}^{\theta'} (\theta - w_*) f(\theta) d\theta \tilde{G}(\theta') \\
&= \int_{w_*}^{\theta'} \left(- \int_{w_*}^{\theta} \tilde{G}(x) dx + (\theta - w_*) \tilde{G}(\theta') \right) f(\theta) d\theta \\
&\geq \int_{w_*}^{\theta'} \left(- \int_{w_*}^{\theta} \tilde{G}(\theta') dx + (\theta - w_*) \tilde{G}(\theta') \right) f(\theta) d\theta = 0.
\end{aligned}
$$

The fact that bid functions are incentive compatibile gives us $U(w_*) - u(w_*, \theta') \geq 0$ and therefore

$$
W(w_*) - W(\theta') \geq 0.
$$

$\square$

Discussion. How significant is the assumption that all bidders have an incentive to acquire information in equilibrium? For some parameter values[7], an equilibrium can be constructed where there are essentially three kind of bidders: The (initially) informed bidders, the uninformed bidders that decide to acquire information and the uninformed bidders who decide to bid without acquiring information. In equilibrium the uninformed bidders first decide whether they want to acquire information or to bid "blind-folded". The first two phases are analogous to the ones presented in this paper. In the third phase the informed compete with each other until the price descends enough to incentivize the uninformed bidders to join the bidding. At the fourth phase the uninformed bidders that decided to bid blind-folded mix with respect to the price that they are willing to bid. Finally there is a fifth phase where all bidders are informed and compete in the same fashion as in the first price

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[7]It may also be the case that no information acquisition takes place in equilibrium for some parameter values. In this case the equilibrium takes the same form as in the first price auction. We elaborate this in the omitted proof section.

auction. This phase is attained only in the event that all bidders have become informed during the auction and observe that their valuations are low.[8]

We next spend a few lines to analyze the setup used in this paper. In the Dutch auction the uninformed bidders may postpone their information acquisition decision. This allows them to measure the level of competition prior to acquiring information. As the uninformed observe that the price descends the information acquisition becomes more attractive. A lower price implies that the competitors' valuations are drawn from an interval with a lower upper bound and hence the competitors' valuations are also smaller in expectation. Lower price also implies that the probability that the uninformed bidder has the highest valuation increases, which is good news for the uninformed. Note that this is in contrast with what is observed by Rezende (2005) in the context of an ascending auction. In his model the bidders do not observe the number of remaining bidders either. It is bad news for the uninformed to observe a price increase in the ascending auction, since it only conveys the information that it is less likely that his valuation is the largest among all bidders. In the ascending price auction, where the number of remaining bidders is not observed, no information about the intensity of competition is available to the (uninformed) bidders.

In the first price auction it is not possible to defer information acquisition. In fact, information acquisition quickly becomes unattractive, when the number of bidders increases. Miettinen (2010) analyzes the equilibrium bidding behavior in a first price auction with a large number of bidders. The main reason why the bidding functions differ in the two auctions is that the competition intensifies sooner in the Dutch auction than it does in the first price auction. In the Dutch auction the bidders know that the competition intensifies as the uninformed bidders start to acquire information.[9] In the first price sealed bid auction the uninformed bidders start bidding later than in the Dutch auction. Therefore, for a fixed number of informed bidders, the competition is less intensive in the first price auction.

We highlight this feature with the example in the figure below. Here the valuations are uniformly distributed and we graph the bidding functions for the informed bidders. The uninformed bidders' mixed strategies "fill the gaps". I.e. the mixing takes place on the interval where the informed bidders' bid function jumps. Therefore, the distribution of bids has no gaps in it.

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[8]It can be showed that if $q = 0$ then the equilibrium takes a form where all bidders initially use a mixed strategy to decide the price at which they acquire information. The structure of the equilibrium is similar, but there is no phase 1 where the informed bid prior to information acquisition.

[9]Note also that in a regular setting with independent private values, the bidders learn nothing about their opponents during the auction. Here, the bidders learn that all of their opponents are informed, if price descends below $\underline{p}$.

Bid functions for the informed in the FPA and Dutch Auction
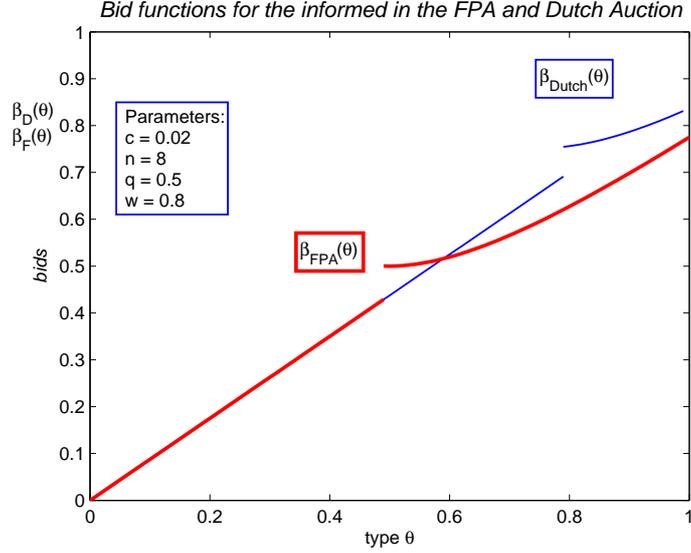
Notice that the first price auction bid function goes above the Dutch auction bid function for a small range of values. However, the Dutch auction bid function stays above the first price auction bid function once they have crossed.[10] On an intuitive level this implies that the revenue from the Dutch auction is larger than from the first price auction when the number of bidders is large enough. This is because with a large number of bidders the probability mass assigned to the events where the Dutch auction bids are above the first price auction bids converges to unity. At the same time the mass that is assigned to the events where the first price auction bids are above the Dutch auction bids becomes very small. We now address this issue formally.

2. The revenue

The Dutch auction revenue. To compare revenue from the Dutch auction with the revenue obtained from the first price auction we make the following observations.

In the first price auction (with a reserve price v) [11] the rent for the agent with type θ is given by

$$U^{FP}(\theta) = \int_v^\theta \tilde{G}(x)dx$$

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[10]If $n = 2$ in this example, the Dutch auction bid function never crosses the first price auction bid function. However, with $n = 3$ the crossing occurs.

[11]It is straightforward to show that the equilibrium bid function in the first price auction with a reserve price v is $\beta(\theta) = v\frac{\tilde{G}(v)}{\tilde{G}(\theta)} + \int_v^\theta \frac{\tilde{G}(x)dx}{\tilde{G}(\theta)}$. Here we consider a first price auction where the seller has a reserve valuation of v. We use this approach since we compare revenues from the two auction formats when the number of bidders is large. In this case the incentive to acquire information disappears in the first price auction and in equilibrium the uninformed bidders bid without information acquisition. However, this bidding take place at prices below v and comparison between the Dutch and the first price auction becomes cumbersome. To overcome this difficulty we approximate the revenue from the first price auction (from above) with the use of reserve price at v. A more detailed derivation of the equilibrium strategies can be found from the omitted proofs section.

and in the Dutch auction for $\theta > w_*$ by

$$U^D(\theta) = U(w_*) + \int_{w_*}^{\theta} \tilde{G}(x)dx.$$

Hence the more aggressive bidding in the Dutch auction pushes bids up by a constant

$$\Delta = \int_v^{w_*} \tilde{G}(x)dx - U(w_*) = \int_v^{w_*} \tilde{G}(x)dx - \int_0^{w_*} G(x)dx,$$

and results in a gap between the rent that the informed bidder with type $\theta > w_*$ gets in the first price and in the Dutch auction. The fact that the uninformed acquire information when the price gets low pushes rents down by a constant amount for all informed types above w_*. Therefore the seller is better of by Δ with probability $1 - \tilde{G}(w_*)$.

With probability $\tilde{G}(w_*) - G(v)$ all informed have $\theta < w_*$ but at least one uninformed learns that $\theta > v$. Then the seller cannot be worse off by more than $v - \beta(v)$.

Finally with probability $G(v)$ all types are below v and the seller loses at most v.

Hence the revenue difference between the Dutch and the FPA is approximated by

$$-vG(v) - (\tilde{G}(w_*) - G(v))(v - \beta(v)) + (1 - \tilde{G}(w_*))\Delta, \qquad (6)$$

where $v - \beta(v) = \frac{\int_0^v G(x)dx}{G(v)}$. We now show that this difference becomes positive for n large enough.

Proposition 2. *The revenue from the Dutch auction is larger than the FPA revenue for large but finite n when $q \in (0,1)$.*

Proof. It is clear that all the terms in (6) converge to zero when n increases without bounds. We need to be sure that the negative terms converge faster than the positive ones. Let's rewrite the expression for revenue difference as

$$\overbrace{-vG(v)}^{A(n-1)} - \tilde{G}(w_*)\overbrace{\frac{\int_0^v G(x)dx}{G(v)}}^{B(n-1)} + \tilde{G}(w_*)\int_0^v G(x)dx + (1 - \tilde{G}(w_*))\left(\overbrace{\int_v^{w_*} \tilde{G}(x) - G(x)dx}^{C(n-1)}\right)$$

The rates of convergence for these terms are

$$\frac{A(n) - A(n-1)}{A(n-1)} = F(v) - 1,$$

$$\frac{B(n) - B(n-1)}{B(n-1)} = \frac{\int_0^v G(x)(F(x)-1)dx}{G(v)F(v)} \frac{1}{B(n-1)}$$

$$\leq \frac{\int_0^v G(x)(F(v)-1)dx}{G(v)F(v)} \frac{1}{B(n-1)} = \left(1 - \frac{1}{F(v)}\right),$$

$$\frac{C(n) - C(n-1)}{C(n-1)} = \frac{\int_v^{w_*} (\tilde{G}(x)q - G(x))(F(x)-1)dx}{C(n-1)}$$

$$\underset{\text{for large n}}{\geq} \frac{\int_v^{w_*} (\tilde{G}(x)q - G(x))dx(F(v)-1)}{C(n-1)}$$

$$\geq \frac{\int_v^{w_*} (\tilde{G}(x) - G(x))dx q(F(v)-1)}{C(n-1)} = q(F(v)-1),$$

where the fist inequality for the term $C(n-1)$ holds for large n. Since the rate of decrease for $\tilde{G}(x)$ is $q(F(x)-1)$ and slower than the rate of decrease for $G(x)$ which is $(F(x)-1)$. So we have that $\tilde{G}(x)q > G(x)$ for n large. We disregard the analysis for the positive term $\tilde{G}(w_*)\int_0^v G(x)dx$.

Since $\tilde{G}(x) > G(x)$ for all x when $q \in (0,1)$ the term $C(n-1)$ is always positive. It is clear that $q(F(v)-1) > F(v)-1$, so the term $A(n-1)$ converges faster than $C(n-1)$. Finally comparing the convergence speed of $B(n-1)$ to $C(n-1)$ notice that as n increases the multiplier $\tilde{G}(w_*)$ of $B(n-1)$ decreases while the multiplier $(1-\tilde{G}(w_*))$ of $C(n-1)$ increases. By treating the multipliers as constants we approximate the respective multipliers of $B(n-1)$ upwards and of $C(n-1)$ downwards. Now we need that

$$q(F(v)-1) > 1 - \frac{1}{F(v)} = \frac{F(v)-1}{F(v)} \iff q < \frac{1}{F(v)},$$

which always holds. Therefore as n becomes large enough we have that

$$-vG(v) - (\tilde{G}(w_*) - G(v))(v - \beta(v)) + (1-\tilde{G}(w_*))\Delta > 0.$$

$\square$

Our analysis of the Dutch auction assumes that the information acquisition is instantaneous. This means that there is no delay between the information acquisition decision and the time when the information is received. In some cases it may be argued that information acquisition takes time. Compte & Jehiel (2007) show how to amend the ascending auction to account for the delay in the information acquisition. It is not immediately clear how, if at all, the Dutch auction could be modified to account for the delay in the information acquisition without altering the equilibrium strategies.

3. CONCLUSION

In this paper we've examined the bidding behavior in the Dutch auction with independent private values. Some bidders may be uninformed about their valuations and acquire information during the Dutch auction. We solve for equilibrium

strategies and show that the Dutch auction produces more revenue to the seller than the first price auction, when the number of bidders is large.

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The first price auction equilibrium strategies. We reproduce the analysis for the first price auction equilibrium strategies from Miettinen (2010). We focus on the case where the number of informed bidders is so large that information acquisition is an undesirable option for the uninformed bidders.[12] That is, the cost of information acquisition is larger than the expected payoff for an uninformed bidder who knows that there can be up to $n-1$ informed bidders in the auction. This means that the uninformed bidders choose their bids through a mixed strategy.[13] Note that the problem for the informed bidders is similar to what the informed bidders face in the Dutch auction. The only thing that is different is the constant of integration. The proof for this proposition can be obtained from the previous section by setting $\sigma = 0$.

Proposition 3. *The following strategies constitute a Bayesian Nash equilibrium of the first price auction, when the information acquisition for the uninformed is too costly.[14] The bidding function for the informed bidders is given by*

$$\hat{\beta}_{FPA}(\theta) = \frac{\int_0^\theta x\tilde{g}(x)dx + \hat{\mathbf{K}}}{\tilde{G}(\theta)}, \text{ if } \theta \geq v$$

$$\beta_{FPA}(\theta) = \frac{\int_0^\theta xg(x)dx}{G(\theta)}, \text{ if } \theta < v,$$

where $\tilde{G}(x) = (qF(v) + (1-q))^{n-1}$, $G(x) = F(x)^{n-1}$ and
$\hat{\mathbf{K}} = \int_0^v \tilde{G}(x) - q^{n-1}G(x)dx$. The uninformed bidders use a mixed strategy over the interval $[\tilde{p}, \hat{p}]$, where $\tilde{p} = \beta_{FPA}(v)$ and $\hat{p} = \hat{\beta}_{FPA}(v)$. The mixed strategy distribution function is

$$\underline{J}(\cdot) = \left[\left(\frac{v - \tilde{p}}{v - b} \right)^{\frac{1}{n-1}} - 1 \right] \frac{qF(v)}{1-q}.$$

Proof. Again we skip the proof that the bidding functions satisfy incentive compatibility. If there are uninformed bidders in the auction, then the winning price is always above $\tilde{p}$. This implies that, in equilibrium, the bidders that have valuations below v never win the auction unless all bidders are informed. Therefore, conditional on a bidder with valuation v having the highest bid, he knows that all other bidders must be informed and have valuations below v. Therefore, the bidders with valuations below v bid according to the regular FPA auction where all bidders valuations are in $[0, v]$.

We now show that the mixing takes place according to the proposed $\underline{J}(\cdot)$ and that there are no profitable deviations for the informed or uninformed on $(\tilde{p}, \hat{p})$. We

---

[12]We formalize this assumption below.

[13]Suppose that there is a price q at which all uninformed bidders bid. This price must be weakly below v to guarantee an expected payoff that is weakly above zero. If $q = v$, then in the case that there is only one uninformed bidder, this bidder can profitably deviate to some $q' < v$. If $q < v$ then any uninformed bidder can profitably deviate by bidding some $q' = q + \epsilon$ for an epsilon small enough. This guarantees virtually the same ex-post payoff as bidding q but with a higher probability, as the bidder avoids the ties that may occur by bidding q.

[14]It is enough that the expected payoff form the information acquisition is less than the expected payoff from the uninformed bidding.

start by assuming that the mixing by the uninformed takes place on the interval $[\tilde{p}, \hat{p}]$ such that $\tilde{p} = \beta(v)$ and $\hat{p} = \hat{\beta}(v)$ according to the distribution function $\underline{J}(\cdot)$. The uninformed bidders' expected payoff is equal to

$$\underline{V} = \overbrace{\left(qF(v) + (1-q)\underline{J}(p)\right)^{n-1}}^{\equiv \underline{G}(p)}(v - p),$$

for all $p \in [\tilde{p}, \hat{p}]$. The uninformed bidders' mixed strategy satisfies $\underline{J}(p) = 0$ for $p \leq \tilde{p}$ and $\underline{J}(\hat{p}) = 1$. This information allows us to determine $\underline{J}(\cdot)$ to be

$$\underline{J}(b) = \left[\left(\frac{v - \tilde{p}}{v - p}\right)^{\frac{1}{n-1}} - 1\right]\frac{qF(v)}{1-q}.$$

Since $\underline{J}(\hat{p}) = 1$ we get that

$$\left(\frac{qF(v) + 1 - q}{qF(v)}\right)^{n-1} = \left(\frac{v - \tilde{p}}{v - \hat{p}}\right).$$

Substituting $\tilde{p} = v - \dfrac{\int_0^v \left(qF(x)\right)^{n-1}dx}{\left(qF(v)\right)^{n-1}}$, $\hat{p} = v - \dfrac{\int_0^v \tilde{G}(x)dx - \hat{\mathbf{c}}}{\tilde{G}(v)}$ and solving for $\hat{\mathbf{K}}$ we find that

$$\hat{\mathbf{K}} = \int_0^v \left(qF(x) + 1 - q\right)^{n-1} - \left(qF(x)\right)^{n-1}dx \geq 0.$$

Substituting $\underline{J}(\cdot)$ into $\underline{G}(\cdot)$ we get that

$$\underline{G}(b) = \frac{v - \tilde{p}}{v - p}\left(qF(v)\right)^{n-1}.$$

Now let's check that none of the informed bidders want to bid $p \in [\tilde{p}, \bar{p}]$. The expected payoff for bidder with type θ from bidding $p \in [\tilde{p}, \bar{p}]$ is

$$\underline{G}(b)(\theta - p) = \frac{v - \tilde{p}}{v - p}\left(qF(v)\right)^{n-1}(\theta - p).$$

If the valuation $\theta = v$, the expected utility is constant for all $p \in [\tilde{p}, \hat{p}]$. If the valuation $\theta > v$ the expected utility increases with p implying that the bidder wants to bid more than $\hat{p} = \hat{\beta}(v)$. If the valuation $\theta < v$ the expected utility decreases with p implying that the bidder wants to bid less than $\tilde{p} = \beta(v)$.

Since the uninformed bidders do not acquire information their "valuation" is essentially v. Therefore, they do not want to bid above $\hat{p}$, since this is the optimal bid for the bidder of type v. Similarly they do not want to bid below $\tilde{p}$ since this is the optimal bid for the bidder of type v when all other bidders are informed and have valuations below v. □

We now formalize our assumption that the uninformed bidders do not acquire information in the FPA. The expected payoff for an uninformed bidder from information acquisition in the FPA is

$$\mathbf{R} = \int_0^v G(x)(x - \beta_{FPA}(x))f(x)dx + \int_v^{\bar{\theta}} \tilde{G}(x)(x - \hat{\beta}_{FPA}(x))f(x)dx - c$$

$$= \int_0^v \int_0^x \left(qF(y)\right)^{n-1}dyf(x)dx + \int_v^{\bar{\theta}} \int_0^x \left((qF(y) + 1 - q)^{n-1}dy - \hat{\mathbf{K}}\right)f(x)dx - c.$$

Now by the monotone convergence theorem we have that as $n \to \infty$ the expected revenue tends to $-\hat{\mathbf{K}}(1 - F(v)) - c \leq 0$.

Lemma 3. *For all $c > 0$ and $q > 0$ there exist $m \in \mathbb{N}$ such that if $n \geq m$ then* $\mathbf{R} < 0$.

The Dutch auction equilibrium with uninformed bidders. In this section we go through the equilibrium derivation that we refer to in the discussion following the equilibrium strategies. Here the assumption that information acquisition is always preferred to staying uninformed is relaxed.[15] In this case the bidders can be divided into three categories: The bidders that are initially informed, the uninformed bidders who decide to acquire information during the auction and the uninformed bidders who choose to bid without information acquisition. The sequencing of events in this case is the following:

- The nature chooses which bidders are informed with probability q
- The uninformed bidders randomize their decision to acquire information or to remain uninformed. We define by σ the probability that an initially uninformed bidder acquires information.
- The auction starts and the bidders abide by their strategies.

In this case the equilibrium behavior in the auction consists of five phases:

At phase 1 no information acquisition takes place and only initially informed bidders with high valuations bid

At phase 2 some uninformed acquire information and bid if they observe that their valuation is above a threshold w_*. Information acquisition takes place over an interval of prices $[\underline{p}, \bar{p}]$.

At phase 3 no information acquisition takes place and the informed bidders compete by bidding according a common bid function.

At phase 4 the still uninformed bidders bid by randomizing their bids over an interval of prices $[\underline{r}, \bar{r}]$.

At phase 5 all bidders are informed and have valuations below v. Bidding is conducted in a similar fashion as in the first price sealed bid auction.

To start let us recall assumption 1 which is still assumed to hold:

Assumption 1. *The information acquisition cost $c > 0$ satisfies*

$$c < u^a(v). \tag{7}$$

It is useful to simplify notation by (re)defining the following distribution functions:

$$\tilde{G}(\theta) = (qF(\theta) + 1 - q)^{n-1} \text{ for } \theta \geq w_*$$

$$\hat{G}(\theta) = \left(F(\theta)(q + (1-q)\sigma) + (1-q)(1-\sigma)\right)^{n-1} \text{ for } v \leq \theta < w_*$$

$$G_*(\theta) = (F(\theta)(q + (1-q)\sigma)^{n-1} \text{ for } \theta < v.$$

The interpretation of the probability $\tilde{G}(\cdot)$ is as before. At the first phase either the informed bidders have valuations below θ (with probability $qF(\theta)$) or the other bidders are uninformed (with probability $1 - q$).

---

[15]Let us recall that the condition for the information acquisition to be a desirable option in equilibrium requires that the inequality $\int_v^{w_*} G(y)dy \geq \int_0^{w_*} G(y)F(y)dy$ is satisfied

The events leading to probability $\hat{G}(\cdot)$ in the third phase are the following: Either the competing bidders are informed (initially or have acquired information) and have a valuations below θ (with probability $F(\theta)(q + (1-q)\sigma)$) or they decided to stay uninformed (with probability $(1-q)(1-\sigma)$).

Finally the only way to reach the fifth phase is that all bidders are informed (either initially or eventually acquired information) and have valuations below θ (with probability $F(\theta)(q + (1-q)\sigma)$.

Again let us define

$$
u(\theta, \hat{\theta}) = \begin{cases} \tilde{G}(\hat{\theta})(\theta - \tilde{\beta}(\hat{\theta})), & \text{for } \hat{\theta} \geq w_* \\ \hat{G}(\hat{\theta})(\theta - \hat{\beta}(\hat{\theta})), & \text{for } \hat{\theta} \in (v, w_*) \\ G_*(\hat{\theta})(\theta - \beta_*(\hat{\theta})), & \text{for } \hat{\theta} < v \end{cases}
$$

as the utility of an informed bidder with valuation θ that behaves like an informed with a valuation $\hat{\theta}$. Also define $U(\theta) = u(\theta, \theta)$. For $\theta < v$ the bidding strategy is fully determined by $U(0) = 0$ and for $\theta \geq v$ the bid functions are determined up to a constant $\mathbf{K_1}, \mathbf{K_2}$. It is straightforward to derive and check the incentive compatibility for the bidding functions.

$$
\tilde{\beta}(\theta) = \frac{\int_0^\theta x\tilde{g}(x)dx + \mathbf{K_2}}{\tilde{G}(\theta)}, \text{if } \theta \geq w_*
$$

$$
\hat{\beta}(\theta) = \frac{\int_0^\theta x\hat{g}(x)dx + \mathbf{K_1}}{\hat{G}(\theta)}, \text{if } \theta \in (v, w_*)
$$

$$
\beta_*(\theta) = \frac{\int_0^\theta xg_*(x)dx}{G_*(\theta)}, \text{if } \theta < v.
$$

Let's then turn to the uninformed bidders bidding at phase 4. The expected payoff of the uninformed bidder who decides to bid blind-folded at the price $p \in (\underline{r}, \bar{r})$ is

$$
V(p) = \left(F(v)(q + (1-q)\sigma) + (1-q)(1-\sigma)J(p)\right)^{n-1} (v - p),
$$

where $J(\cdot)$ denotes the mixed strategy distribution for the uninformed bidders. Here the probability that the price descends to p has the following interpretation: Either all informed bidders have valuations below v - with probability $F(v)(q+(1-q)\sigma)$ - or they are not informed and have decided to bid at a lower price - with probability $(1-q)(1-\sigma)J(p)$. Using the fact that $J(\underline{r}) = 0$ we get that

$$
V(\underline{r}) = (F(w)(q + (1-q)\sigma))^{n-1}(v - \underline{r})
$$

which allows us to solve $J(\cdot)$ and obtain

$$
J(p) = \left[\left(\frac{v - \underline{r}}{v - p}\right)^{\frac{1}{n-1}} - 1\right]\frac{F(v)(q + (1-q)\sigma)}{(1-q)(1-\sigma)}.
$$

Now we need that the indifference condition be satisfied for the bidder with a valuation v and hence we have that $\beta_*(v) = \underline{r}$. Since $J(\bar{r}) = 1$ we can solve for $\bar{r}$.

Since we also have that $\hat{\beta}(v) = \bar{r}$ we can solve for **K1** and obtain

$$\mathbf{K1} = \int_0^v \hat{G}(x) - G_*(x)dx.$$

Let's then consider the information acquisition decision by the uninformed. The value of acquiring information at price $\tilde{\beta}(w)$ is determined by

$$W(w) = \int_0^w U(\theta)f(\theta)d\theta + \int_w^{\bar{\theta}} u(\theta, w)f(\theta)d\theta - c\tilde{G}(w). \tag{8}$$

In an analogous fashion to the analysis in the main paper we get from the first order condition for equation (8) that[16]

$$\int_w^{\bar{\theta}} (\theta - w)f(\theta)d\theta = c. \tag{9}$$

The uninformed that acquires information optimally at $\beta(w_*)$ gets

$$\mathbb{E}[\min\{U(\theta), U(w_*)\}] + \tilde{G}(w_*)\Big(\int_{w_*}^{\hat{\theta}} (\theta - w_*)f(\theta)d\theta - c\Big) = \mathbb{E}[\min\{U(\theta), U(w_*)\}].$$

Now the information acquisition starts at $\bar{p} = \tilde{\beta}(w_*)$ and it must also end at some $\underline{p}$. Again by the indifference condition for type w_* we have that $\underline{p} = \hat{\beta}(w_*)$. We next need to find an information acquisition strategy that ensures that information acquisition at any price on $(\underline{p}, \bar{p})$ yields $\mathbb{E}[\min\{U(\theta), U(w_*)\}]$.

The uninformed that acquired information at a price $p \in (\underline{p}, \bar{p})$ obtains

$$\int_0^{w_*} U(\theta)f(\theta)d\theta + R(p)^{n-1}(w_* - p)(1 - F(w_*)), \tag{10}$$

where the probability

$$R(p) = \Big(F(w_*)(q + (1-q)\sigma) + (1-q)\sigma(1 - F(w_*))\hat{H}(p)) + (1-q)(1-\sigma))\Big)$$

is composed of three events. Either the uninformed is faced with an (initially) informed bidders with $\theta \leq w_*$ - with probability $F(w_*)(q + (1-q)\sigma$ - or with uninformed bidders that have valuations $\theta > w_*$ but who have not acquired information yet - with probability $(1-q)\sigma(1 - F(w_*))\hat{H}(p)$. Finally the bidders may be uninformed and decided not to acquire information - with probability $(1-q)(1-\sigma)$. Equating (10) with $\mathbb{E}[\min\{U(\theta), U(w_*)\}]$ we can solve for $\hat{H}(\cdot)$ and obtain

$$\hat{H}(p) = \frac{qF(w_*) + 1 - q}{(1-q)\sigma(1 - F(w_*))}\Big(\frac{w_* - \bar{p}}{w_* - p}\Big)^{\frac{1}{n-1}} - \frac{F(w_*)(q + (1-q)\sigma) - (1-q)(1-\sigma)}{(1-q)\sigma(1 - F(w_*))}. \tag{11}$$

Using the fact that $H(\underline{p}) = 0$ and $\underline{p} = \hat{\beta}(w_*)$ we can solve for $\bar{p}$ and get

$$\bar{p} = \frac{w_*\tilde{G}(w_*) - \int_0^v G_*(x)dx - \int_v^{w_*} \hat{G}(x)dx}{\tilde{G}(w_*)}.$$

---

[16]Remember that $\frac{\partial}{\partial w}u(\theta, w) = (\theta - w)\tilde{G}'(w)$.

Finally the indifference condition $\tilde{\beta}(w_*) = \bar{p}$ allows us to pin down the constant of integration $\mathbf{K_2}$ to be

$$\mathbf{K_2} = \int_0^{w_*} \tilde{G}(x)dx - \int_0^v G_*(x)dx - \int_v^{w_*} \hat{G}(x)dx.$$

We have just constructed the equilibrium strategies. It remains to check the incentives. The informed incentives need be checked in phases 2 and 4 where the uninformed use their mixed strategy. Also the uninformed bidders' incentives for information acquisition need to be checked. Finally we need to check that the uninformed get the same expected utility from information acquisition and bidding without acquiring any information.

Lemma 4. *The informed have no profitable deviations in phase 4, $p \in (\underline{r}, \bar{r})$.*

Proof. The expected payoff for an informed bidder in phase 4 is given by

$$\left(F(v)(q + (1-q)\sigma) + (1-q)(1-\sigma)J(p)\right)^{n-1}(\theta - p) = \left(F(v)(q+(1-q)\sigma)\right)^{n-1} \frac{v-r}{v-p}(\theta - p).$$

Here it is clear that this utility is constant for $\theta = v$. U is also increasing in p if $\theta > v$ and decreasing in p if $\theta < v$. $\qquad\square$

Lemma 5. *The informed have no profitable deviations in phase 2, $p \in (\underline{p}, \bar{p})$.*

Proof. The expected payoff for an informed bidder in phase 2 is given by

$$R(p)^{n-1}(\theta - p) = \left(qF(w_*) + 1 - q\right)^{n-1} \frac{w_* - \bar{p}}{w_* - p}(\theta - p).$$

Here it is clear that this utility is constant for $\theta = w_*$. U is also increasing in p if $\theta > w_*$ and decreasing in p if $\theta < w_*$. $\qquad\square$

Lemma 6. *The uninformed have no profitable deviations in phases 1 or 3.*

Proof. Almost analogously to the proof in the main text. Suppose that $\theta' < w_*$. Then using equation (2) and the definition of c allows us to compute

$$
\begin{aligned}
W(w_*) - W(\theta') &= \int_{\theta'}^{w_*} U(\theta) - u(\theta, \theta')f(\theta)d\theta + \int_{w_*}^{\bar{\theta}} u(\theta, w_*) - u(\theta, \theta')f(\theta)d\theta \\
&\quad - \int_{w_*}^{\bar{\theta}} (\theta - w_*)f(\theta)d\theta(\tilde{G}(w_*) - \hat{G}(\theta')) \\
&= \int_{\theta'}^{w_*} U(\theta) - u(\theta, \theta')f(\theta)d\theta + \int_{w_*}^{\bar{\theta}} U(w) - u(w, \theta')f(\theta)d\theta \geq 0.
\end{aligned}
$$

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Then assuming that $\theta' > w_*$ we get that

$$W(w_*) - W(\theta') = -\int_{w_*}^{\theta'} U(\theta)f(\theta)d\theta + \int_{w_*}^{\theta'} u(\theta, w_*)f(\theta)d\theta$$

$$- \int_{\theta'}^{\bar{\theta}} u(\theta, \theta')f(\theta)d\theta + \int_{\theta'}^{\bar{\theta}} u(\theta, w_*)f(\theta)d\theta$$

$$- \int_{w_*}^{\bar{\theta}} (\theta - w_*)f(\theta)d\theta(\tilde{G}(w_*) - \tilde{G}(\theta'))$$

$$= -\int_{w_*}^{\theta'} U(\theta)f(\theta)d\theta + \int_{w_*}^{\theta'} U(w_*)f(\theta)d\theta + \int_{w_*}^{\theta'}(\theta - w_*)\tilde{G}(\theta')f(\theta)d\theta$$

$$+ \int_{\theta'}^{\bar{\theta}} (U(w_*) - u(w_*, \theta'))f(\theta)d\theta \geq 0.$$

The inequality holds by an analogous argument as was presented in the main text.
$\square$

Consider then the difference between the expected utilities of the uninformed bidders and of the bidders who acquire information.

$$V - W = \int_0^v G_*(x)dx - \left(\int_0^{w_*} U(\theta)f(\theta)d\theta + \int_{w_*}^{\bar{\theta}} u(\theta, w_*)f(\theta)d\theta - c\tilde{G}(w_*)\right)$$

$$= \int_0^v G_*(x)dx(F(w_*) + (1 - F(w_*)))$$

$$- \left(\int_0^v \int_0^\theta G_*(x)dxf(\theta)d\theta + \int_v^{w_*}\int_0^\theta \hat{G}(x)dxf(\theta)d\theta + (1 - F(w_*))\tilde{G}(w_*)(w_* - \bar{p})\right)$$

$$= \int_0^v G_*(x)dxF(w_*)$$

$$- \left(\int_0^v \int_0^\theta G_*(x)dxf(\theta)d\theta + \int_v^{w_*}\int_0^\theta \hat{G}(x)dxf(\theta)d\theta + (1 - F(w_*))\int_v^{w_*} \hat{G}(x)dx\right)$$

$$= \int_0^v G_*(x)F(x)dx + \int_0^v G_*(x) - \hat{G}(x)dx(F(w_*) - F(v)) \qquad (12)$$

$$- \int_v^{w_*} \hat{G}(x)(1 - F(x))dx,$$

where we have integrated the terms $\int\int G(y)dyf(x)dx$ by parts and cancelled some terms. For the construction it is sufficient to show that we can find σ such that the difference is zero. This is not always the case. It is instructive to study how this difference behaves with respect to σ and q. The difference $V - W$ is increasing in σ

$$\frac{\partial V - W}{\partial \sigma} = \left[\int_0^v F(x)^2 Y(x)^{n-2}dx + \int_0^v F(x)Y(x)^{n-2} + (1 - F(x))\hat{Y}(x)^{n-2}dx(F(w) - F(v))\right.$$

$$\left. + \int_v^{w_*} (1 - F(x))^2 \hat{Y}(x)^{n-2}dx\right](n - 1)(1 - q) \geq 0,$$

where $Y(x) = F(x)(q + (1 - q)\sigma)$ and $\hat{Y}(x) = F(x)(q + (1 - q)\sigma) + (1 - q)(1 - \sigma)$. Then consider the difference in the limits as q goes either to 0 or 1. When $q = 0$

we find that

$$V - W_{\substack{q=0 \text{ and } \sigma=0}} = -v(F(w_*) - F(v)) - \int_v^{w_*} (1 - F(x))dx < 0.$$

Now consider the difference when either q or σ goes to 1.

$$V - W_{\substack{q=1 \text{ or } \sigma=1}} = \int_0^{w_*} G(x)F(x)dx - \int_v^{w_*} G(x)dx \geq 0,$$

where the inequality follows from the fact that assumption 2 is relaxed. However the difference cannot be pushed to change sign in all cases.[17] In this case the equilibrium behavior corresponds to the one derived in the case of the first price auction.

To see this suppose the parameters are such that for all $\sigma \in [0, 1]$ the equation (12) holds as $V - W \geq 0$. Then consider the case $\sigma = 0$. Notice that this limit case is equivalent to the setting for the FPA. One can easily check that the distribution functions $G_*(\cdot) = G(\cdot)$ and $\hat{G}(\cdot) = \tilde{G}(\cdot)$. Also the uninformed bidders get in expectation the same expected payoff as in the FPA case. Of course in this case the derivation for the information acquisition is unnecessary. The most relevant point is that the incentives to stay uninformed are satisfied since the equation (12) is positive when $\sigma = 0$.

Thus we have that when for all $\sigma \in [0, 1]$ equation (12) is positive the Dutch auction equilibrium is equivalent to the FPA equilibrium. If the equation (12) is negative for all $\sigma \in [0, 1]$ the Dutch auction equilibrium is the one derived in the main text of this paper, where all bidders acquire information during the information acquisition phase. Finally if it is the case that for some $\sigma \in (0, 1)$ the equation (12) equals zero, then the equilibrium derived here is obtained. I.e. some uninformed bidders may opt to stay uninformed and place bids blind-folded by randomizing over $[\underline{r}, \bar{r}]$.

As a final remark we show that for $\sigma = 1$ we have that $V - W \leq 0$ as n becomes large. To see this we write the inequality as

$$\int_v^{w_*} G(x)(1 - F(x))dx - \int_0^v G(x)F(x)dx \geq 0.$$

Now both these terms converge to zero when n approaches infinity. Consider the rate of convergence of the first term. This is equal to

$$\frac{\int_v^{w_*} G(x)(1 - F(x))(F(x) - 1)dx}{\int_v^{w_*} G(x)(1 - F(x))dx} \geq \frac{\int_v^{w_*} G(x)(1 - F(x))(F(v) - 1)dx}{\int_v^{w_*} G(x)(1 - F(x))dx} = (F(v) - 1).$$

The rate of convergence of the second term satisfies

$$\frac{\int_0^v G(x)F(x)(F(x) - 1)dx}{\int_0^v G(x)F(x)dx} \leq \frac{\int_0^v G(x)F(x)(F(v) - 1)dx}{\int_0^v G(x)F(x)dx} = (F(v) - 1),$$

which implies that for n large enough the inequality holds.

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[17]The difference $V - W$ remains positive for all $\sigma \in [0, 1]$ in the case of a uniform distribution for parameter values $c = 0.1$, $n = 3$, $q = 0.99$ and $v = 0.5$.

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