Patrick M. Crowley – Douglas Maraun – David Mayes

How hard is the euro area core? An evaluation of growth cycles using wavelet analysis

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The views expressed are those of the authors and do not necessarily reflect the views of the Bank of Finland.

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Abstract

Using recent advances in time-varying spectral methods, this research analyses the growth cycles of the core of the euro area in terms of frequency content and phasing of cycles. The methodology uses the continuous wavelet transform (CWT) and also Hilbert wavelet pairs in the setting of a non-decimated discrete wavelet transform in order to analyse bivariate time series in terms of conventional frequency domain measures from spectral analysis. The findings are that coherence and phasing between the three core members of the euro area (France, Germany and Italy) have increased since the launch of the euro.

Key words: time-varying spectral analysis, coherence, phase, business cycles, EMU, growth cycles, Hilbert transform, wavelet analysis

JEL classification numbers: C19, C63, C65, E32, E39, E58, F40

Kuinka yhtenäinen on euroalueen ydin? Suurten jäsenmaiden talouskasvun tarkastelu väreanalyysin keinoin

Suomen Pankin tutkimus
Keskustelualoitteita 18/2006

Patrick M Crowley – Douglas Maraun – David Mayes
Rahapolitiikka- ja tutkimusosasto

Tiivistelmä

Tässä työssä analysoidaan suurten euromaiden kasvujaksojen pituuksia ja vaiheistusta. Työn tilastollisissa tarkasteluissa käytetään moderneja moniulotteisten aikasarjojen vaihtelun ja keskinäisen riippuvuuden analysointiin tarkoitettuja spektraalianalyysin välineitä. Teknisesti kyse on harventamattomien tai ylipoimittujen epäjatkuvien väremuunnosten käytöstä koko havaintoaineiston tilastollisessa analyysissa. Tätä perusanalyysia laajennetaan ns. jatkuvaa väremuunnosta ja Hilbertin aallokepareja käyttäen maittaisten bruttokansantuotteiden keskinäisten riippuvuuksien tarkastelulla. Tutkimustulosten mukaan kolmen suuren euromaan – Italian, Ranskan ja Saksan – talouskasvun keskinäinen riippuvuus on voimistunut euroalueen perustamisen jälkeen. Näiden maiden talouskasvun vaihtelut ovat myös euron käyttöönoton jälkeen samanaikaistuneet.

Avainsanat: ajassa muuttuva spektraalianalyysi, koherenssi, suhdannevaihtelut, EMU, kasvujaksot, Hilbertin muunnos, aallokeanalyysi

JEL-luokittelu: C19, C63, C65, E32, E39, E58, F40

Contents

1 Introduction

The three largest member state members of the euro area are Germany, France and Italy – they therefore form the economic core of the euro area. Together their GDP represents over 65% of eurozone GDP, so inevitably common events or shocks in these three member states will impact European Central Bank (ECB) monetary policy. Given the fact that the Stability and Growth pact (SGP) currently allows these three member states limited freedom to manoevre on the fiscal policy front, there is undoubtedly a heavier burden placed on the stance of monetary policy within the euro area. But does the ECB need to be concerned about differences in growth dynamics at different frequencies between these three member states, as they are at the 'core' of EMU, and are these differences likely to be at longer or shorter cycles? This paper takes a first step in analysing the similarities and differences between the growth cycles of the three largest euro area members using time-frequency methods, and provides some tentative answers to this question. The results show, on the basis of two spectral methods, that the similarity and phasing of these growth cycles has increased since the launch of EMU.

Both discrete and continuous wavelet techniques are used in the paper, the former using a scale decomposition approach in the time domain (the discrete wavelet transform) and the latter using a spectral (ie frequency domain) analysis perspective (the continuous wavelet transform) to analyse the cycles and relationship between growth in France, Germany and Italy. Wavelet analysis is a useful tool for analysing time series, and probably represents the biggest breakthrough in frequency domain methods in several decades. These techniques are now used extensively in many disciplines which rely on time series for validation of theories or hypotheses,[1] and yet they are still largely ignored by economists.[2] This is indeed puzzling, as wavelet analysis has the ability to identify cycles in data at different frequencies through time – which is something that can be very revealing for empirical work in economics! Although to date, most of the applications with economic and financial data have been made by scholars in the finance area, there are some applications using wavelets in economics, and these are reviewed in Crowley (2005).

The paper is divided into three parts. In the next section, the notion of growth cycles is introduced, and is related to events in the European Union which had an explicit or implicit impact on these three member states. In the third section the discrete wavelet transform is introduced and used to analyse the dataset as a whole, and then in the following section the continuous wavelet transform is used to analyse the relationship between GDP in the three different countries over time. In the fourth section, a variation on the discrete wavelet transform using Hilbert wavelet pairs is used to again analyse the same data, but over time. A final section concludes by comparing and contrasting the results obtained by the different wavelet techniques.

[1] Disciplines such as signal processing, physics, meteorology, astronomy, medicine, engineering and biology.

[2] There are some exceptions to this: James Ramsay (New York University, USA) and Ramazan Gencay (Simon Fraser University, Canada) are notable in this regard.

2 Growth cycles, the EU event timeline and data

Granger (1966) first showed (and more recently confirmed by Levy and Dezhbakhsh (2003b)) that the spectrum for real GDP was downward sloping with longer frequencies being more prominent in the data. As Levy and Dezhbakhsh show with quarterly growth data, this general result carries through to many of the spectra for GDP data as well. In one sense this resolves the contradiction in the estimates of the duration of the business cycle, as the spectral methods point to strong medium term cycles influencing the conventional business cycle. But if the spectrogram is not smoothed, we also obtain the empirical observation that although as economists we measure the business cycle in terms of when recessions occur, the GDP series suggests that there are potentially many other cycles with different periodicities at work.[3] Of course the business cycle itself has various phases, as Kontolemis (1997) makes clear, but if they are of the four phase variety originally described by Hicks (1950), then they could occur at different frequencies to the business cycle itself, given that accelerating and decelerating growth cycles might not be in concordance with the conventional business cycle. Zarnovitz (1985) first suggested that these more frequent 'growth cycles' might have an important role to play in the business cycle itself, and set about studying them. In terms of dating these growth cycles, Zarnowitz and Ozyildirim (2002) conduct various time series decomposition approaches to identify the cycles, and construct a 'growth cycle' chronology for the US.

In attempting to characterise the EU event timeline and how it might impact the differences or similarities between growth cycles in France, Germany and Italy, we would expect least concordance in the 1970s and early 1980s, the period of the ill-fated European 'snake' exchange rate arrangements and clearly differentiated monetary policy. Only after the celebrated u-turn in French economic policy in 1983 did the ERM of the EMS provide the means to anchor monetary policy to the German Bundesbank's monetary policy (– the 'new' EMS). After the turbulence of the early 1990s with the near collapse of the ERM and German reunification, economic convergence was encouraged by the Maastricht criteria as an entry to EMU, with Italy being significantly affected by the ERM crisis and Germany encountering problems in terms of integrating ex-East Germany into the new unified Germany. Nevertheless the late 1990s saw all three member states preparing for EMU, and so there might be some similarities in growth during this period – indeed, Artis and Zhang (1999) and Artis and Zhang (1997) find evidence of the inception of a common European business cycle during this period. Once all three countries satisfied the Maastricht criteria and joined EMU, the period after 1999 saw stagnation in Germany (and mounting complaints about ECB monetary policy), but moderate growth in both France and Italy – thus we might expect similar growth cycles in both France and Italy over this period. In broad terms then, it makes most sense to treat France, Germany and Italy as three bivariate

[3] So as not to confuse the generally accepted notion of a business cycle with also the more general definition of a medium term cycle used by many economists, from this point onwards the term 'growth cycles' is used to describe cycles at different frequencies present in quarterly GDP growth data.

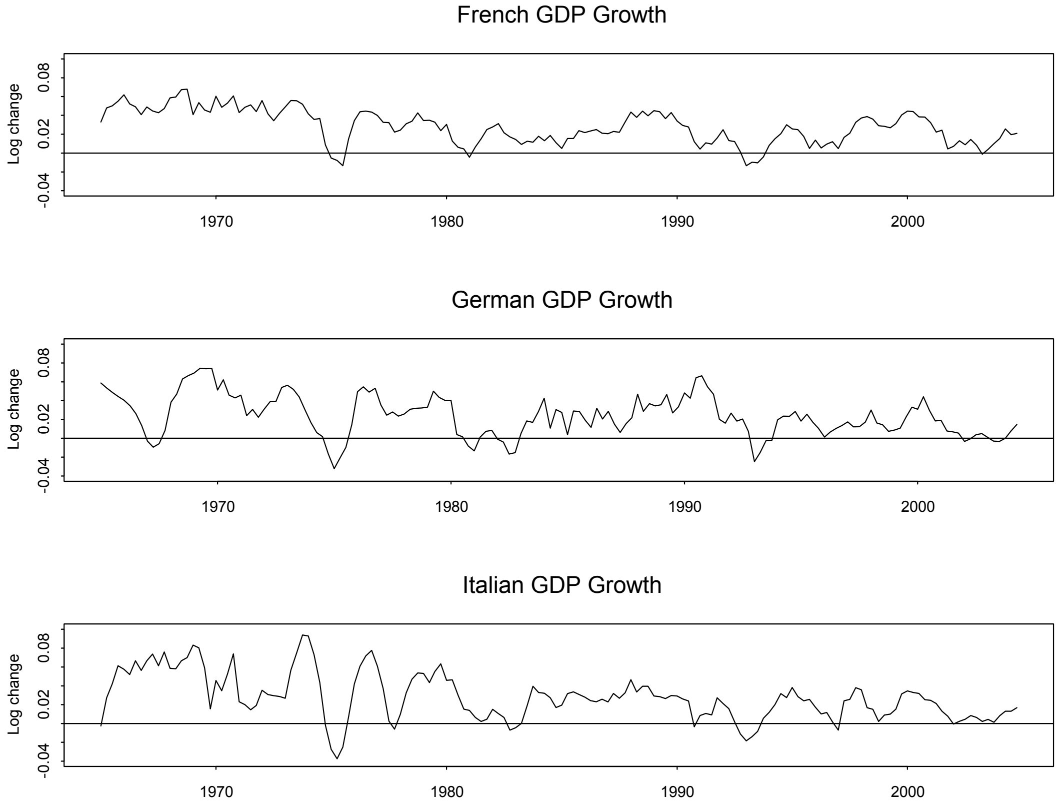

Figure 1: Log Change GDP growth for France, Germany and Italy

relationships, as although the 'axis of attraction' was Germany up until 1992, but from this point onwards, there is very little rationale for postulating a German 'dominance hypothesis'.

The data used in this study came from the ECB's Area Wide Model and have been updated manually. The data are quarterly from 1965 to 2005, and as such are constructed from annual data prior to 1970. Less emphasis should therefore be given to any high frequency cycles found in the data in the 1960s, as these will have arisen from the data contruction algorithm used by the ECB.[4] The data used in the analysis are plotted below in figure 1.

3 Scale decomposition by MODWT

3.1 Methodology

Wavelets are, by definition, small waves. That is, they begin at a finite point in time and die out at a later finite point in time. As such they must, whatever their shape, have a defined number of oscillations and last through a certain period of time or space. Clearly these small wavelike functions are ideally suited to locally approximating variables in time or space as they have the

[4] We have not been given access to this algorithm.

ability to be manipulated by being either 'stretched' or 'squeezed' so as to mimic the series under investigation. Thus the cycles within a data series can be extracted creating a new series which just incorporates the cycles in the original data at that particular frequency.

The main feature of wavelet analysis is that it enables the researcher to separate out a variable into its constituent multiresolution components. In order to retain tractability (– many wavelets have an extremely complicated functional form), assume we are dealing with symmlets,[5], then the father and mother pair can be given respectively by the pair of functions

$$\phi_{j,k} = 2^{-\frac{j}{2}}\phi(\frac{t-2^j k}{2^j}) \tag{3.1}$$

$$\psi_{j,k} = 2^{-\frac{j}{2}}\psi(\frac{t-2^j k}{2^j}) \tag{3.2}$$

where j indexes the scale, and k indexes the translation. It is not hard to show that any variable $x(t)$ can be built up as a sequence of projections onto father and mother wavelets indexed by both j, the scale, and k, the number of translations of the wavelet for any given scale, where if k is dyadic[6] we obtain the basic discrete wavelet transform (DWT). As shown in Bruce and Gao (1996), if the wavelet coefficients are approximately given by the integrals

$$s_{J,k} \approx \int x(t)\phi_{J,k}(t)dt \tag{3.3}$$

$$d_{j,k} \approx \int x(t)\psi_{j,k}(t)dt \tag{3.4}$$

$j = 1, 2, ...J$ such that J is the maximum scale sustainable with the data to hand then a multiresolution representation of $x(t)$ is given by

$$x(t) = \sum_k s_{J,k}\phi_{J,k}(t) + \sum_k d_{J,k}\psi_{J,k}(t) + \sum_k d_{J-1,k}\psi_{J-1,k}(t) + ... + \sum_k d_{1,k}\psi_{1,k}(t) \tag{3.5}$$

where the basis functions $\phi_{J,k}(t)$ and $\psi_{J,k}(t)$ are assumed to be orthogonal, that is

$$\begin{array}{c} \int \phi_{J,k}(t)\phi_{J,k'}(t) = \delta_{k,k'} \\ \int \psi_{J,k}(t)\phi_{J,k'}(t) = 0 \\ \int \psi_{J,k}(t)\psi_{J',k'}(t) = \delta_{k,k'}\delta_{j,j'} \end{array} \tag{3.6}$$

[5] Symmlets are symmetric wavelets. There are many varieties of wavelet forms, and these are reviewed in Crowley (2005).

[6] That is, k is a number which is an integer power of 2.

and $\delta_{i,j} = 1$ if $i = j$ and $\delta_{i,j} = 0$ if $i \neq j$. The multiresolution decomposition (MRD) of the variable $x(t)$ is then summarised as

$$\{S_J, D_J, D_{J-1}, ...D_1\} \tag{3.7}$$

where S_J is just the set of convolved father wavelet coefficients, and similarly for D_J with convolved mother wavelet coefficients.

The interpretation of the MRD using the DWT is of interest in terms of understanding the frequency at which activity in the time series occurs. For example with a quarterly time series (as we have here), table 1 shows the interpretation of the different scale crystals

Scale crystals	Quarterly frequency resolution
d1	1-2
d2	2-4
d3	4-8=1-2yrs
d4	8-16=2-4yrs
d5	16-32=4-8ys
d6	32-64=8-16yrs
d7	64-128=16-32yrs
d8	etc

Table 1: Frequency interpretation of MRD scale levels

Although extremely popular due to its intuitive approach, the classic DWT suffers from two drawbacks: dyadic length requirements and the fact that the DWT is non-shift invariant. In order to address these two drawbacks, the maximal-overlap DWT (MODWT)[7] is introduced. The MODWT gives up the orthogonality property of the DWT to gain other features (see Percival and Mofjeld (1997)), such as the ability to handle any sample size regardless of whether dyadic or not, increased resolution at coarser scales as the MODWT oversamples the data, translation-invariance,[8] and the MODWT produces a more asymptotically efficient wavelet variance estimator than the DWT.

Both Gencay, Selcuk, and Whitcher (2001) and Percival and Walden (2000) give a description of the matrix algebra involved in the MODWT, but the MODWT can be described in intuitive terms as simply moving a wavelet function along a series, data point by data point to obtain a detail crystal, rather than moving the wavelet function along to the next datapoints not already convolved with the data (– which would constitute

[7]As Percival and Walden (2000) note, the MODWT is also commonly referred to by various names in the wavelet literature. Equivalent labels for this transform are non-decimated DWT, time-invariant DWT, undecimated DWT, translation-invariant DWT and stationary DWT. The term 'maximal overlap' comes from its relationship with the literature on the Allan variance (the variation of time-keeping by atomic clocks) – see Greenhall (1991).

[8]In other words the MODWT crystal coefficients do not change if the time series is shifted in a 'circular' fashion.

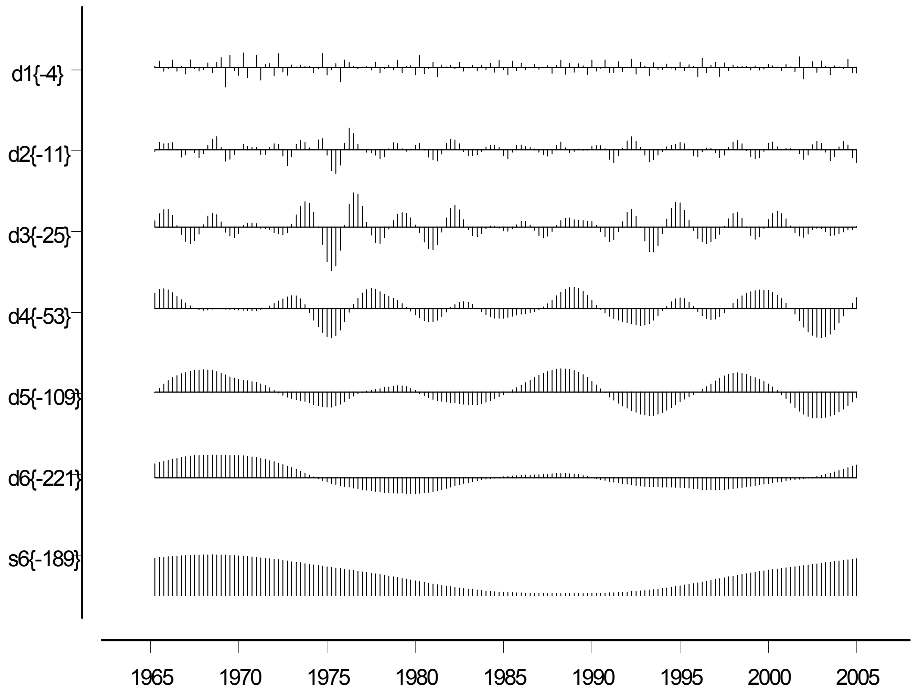

Figure 2: Phase-corrected MODWT for France

a DWT). Put another way, in terms of applying wavelet filters to the data, the MODWT simply skips downsampling after filtering the data, whereas the DWT downsamples. This means that the size of each crystal is the same as the length of the data under analysis with the MODWT, whereas the number of datapoints in each crystal halves for each crystal of a higher order with a DWT.

3.2 Empirical results

3.2.1 MODWT results

The MODWT using the nearly-symmetric Debauchies wavelet of length 8 was used to multi-scale decompose the productivity data using a total of 6 scales (– this therefore extracts fluctuations of up to 16 years in length). The results are phase corrected, and then shown graphically in stack plots as figures 2 to 4.

The MODWT stackplots are interesting in 4 respects:

i) Crystals d1 and d2 appear in all cases to exhibit some volatility, but since the early 1980s, the volatility at this frequency has tended to be low;

ii) France and Germany appear to have quite strong cycles at 4–16 year cycles (crystals d4–d6), but Italy does not.;

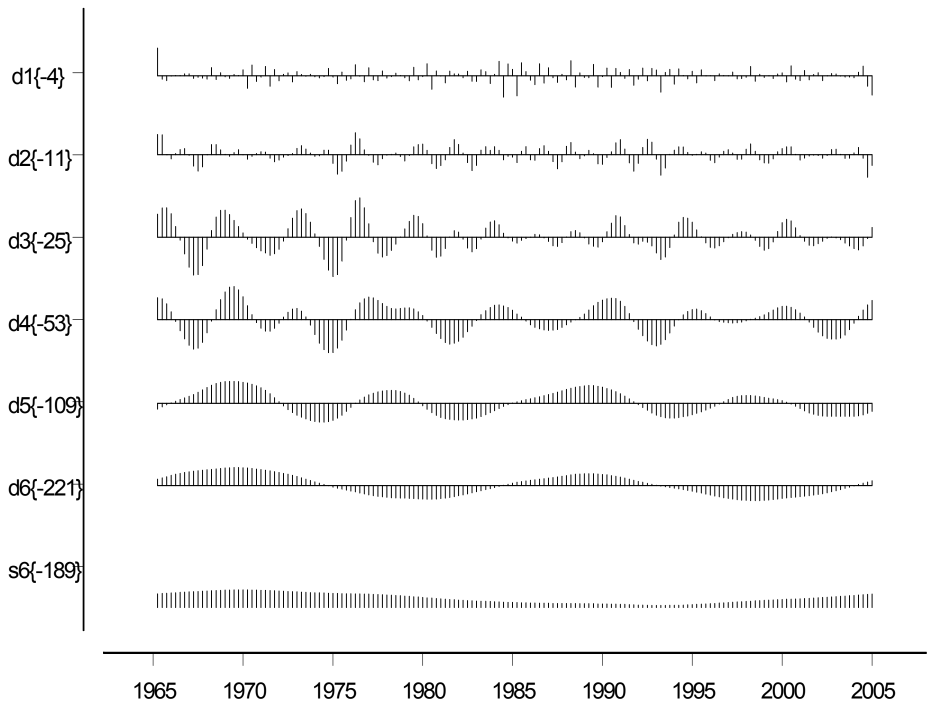

Figure 3: Phase-corrected MODWT for Germany

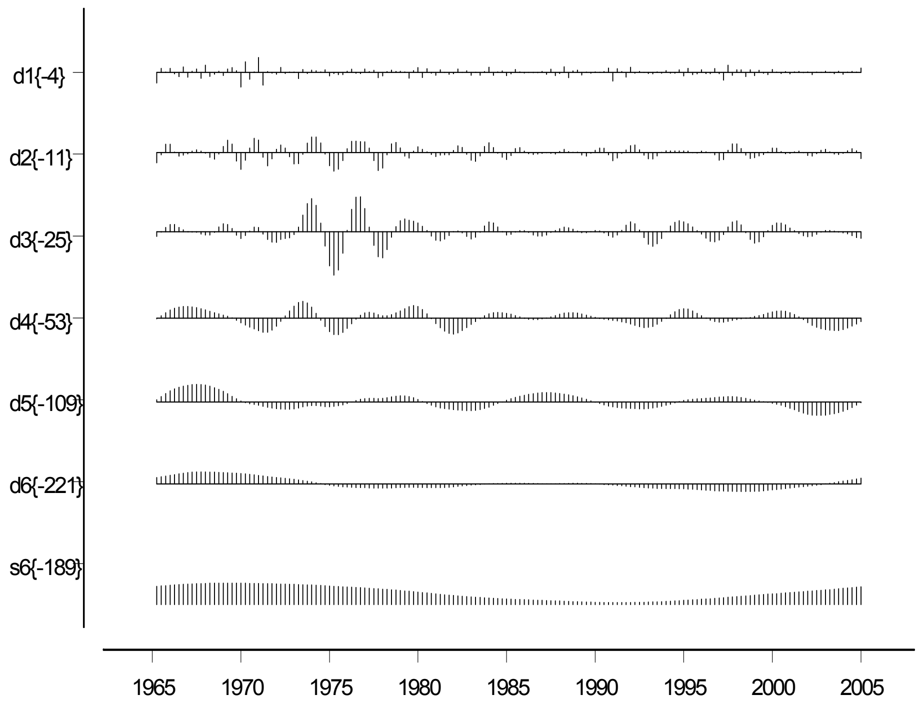

Figure 4: Phase-corrected MODWT for Italy

iii) German reunification shows up in the d2, d3 and d4 crystals for Germany, as increased volatility, but doesn't appear at the business cycle frequency of d5;

iv) In all cases their appears to be quite a bit of activity in the d6 and s6 crystals (8–16 years and above), which suggests even longer cycles might be at work here (– given what we observe here in the s6 crystal, the cycle appears to be roughly a 40 year cycle).

Another way of presenting this data is to present it in terms of the energy decomposition, where the energy of a crystal for scale j, E_j, is given by

$$E_j^d = \frac{1}{E} \sum_{k=1}^{\frac{n}{2^j}} d_{j,k}^2 \tag{3.8}$$

where d refers to the detail crystals and E is the total energy of the series. Orthogonal wavelets are energy (variance) preserving, so that

$$E = E_j^s + \sum_{i=1}^{j} E_i^d \tag{3.9}$$

where E_j^s is the energy of the smooth. Note that as the detail crystals have mean zero by construction, the energy distribution of the detail crystals amounts to a variance decomposition of the series by frequency band – but this is not so for the wavelet smooth, however, so this crystal is likely to contain a large amount of energy. The energy distribution of the crystals is shown by means of a histogram and a pie chart for each of the series in figures 5 to 7. The figures show in all cases that the wavelet smooth contains most energy (which is hardly surprising as this crystal does not have zero mean), but of the detail crystals d3 (1–2 year cycles) has most energy for Germany and Italy, followed by the d4 crystal (2–4 years), but for France the d5 crystal has most energy followed by the d4 crystal.

3.2.2 Correlation analysis

Given that wavelet analysis can decompose a series into sets of crystals at various scales, it is relatively straightforward to then take each scale crystal and use it as a basis for decomposing the variance of a given series into variances and covariances at different scales. Once covariance by scale has been obtained, the wavelet variances and covariances can be used together to obtain scale correlations between series. Confidence intervals can be derived for the correlation coefficients by scale[9] (these are also derived in Whitcher, Guttorp, and Percival (2000) with further details found in Constantine and

[9] The wavelet smooth correlations are point estimates as confidence intervals cannot be calculated for these cycles as they are theoretically infinite, given that the frequency cycles can range from 32 quarters to an infinite number.

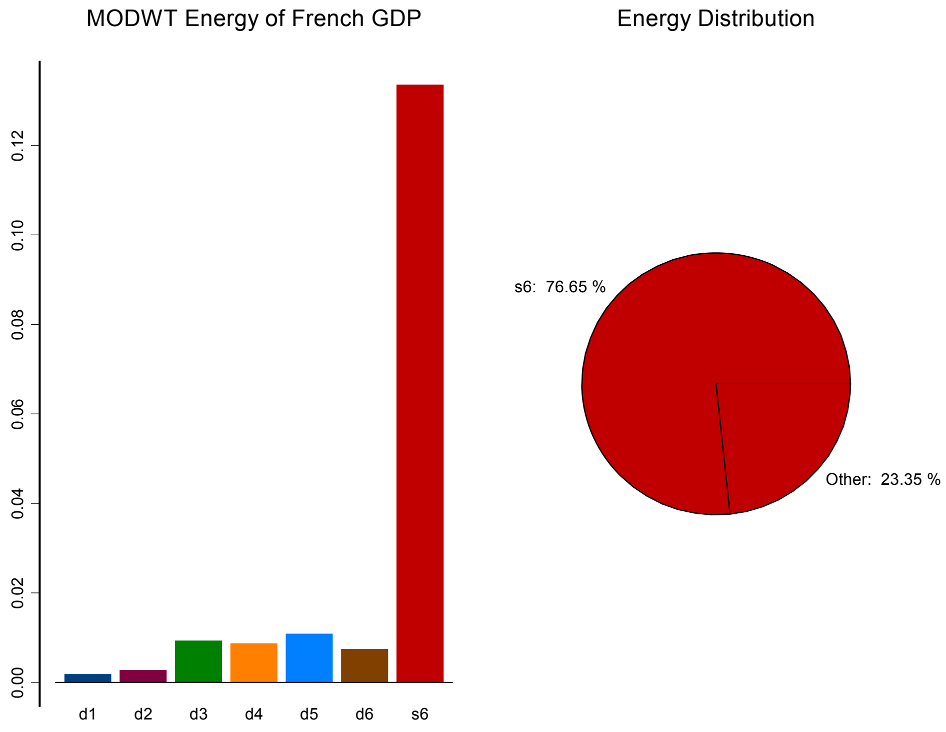

Figure 5: Energy distribution by Crystal for French GDP

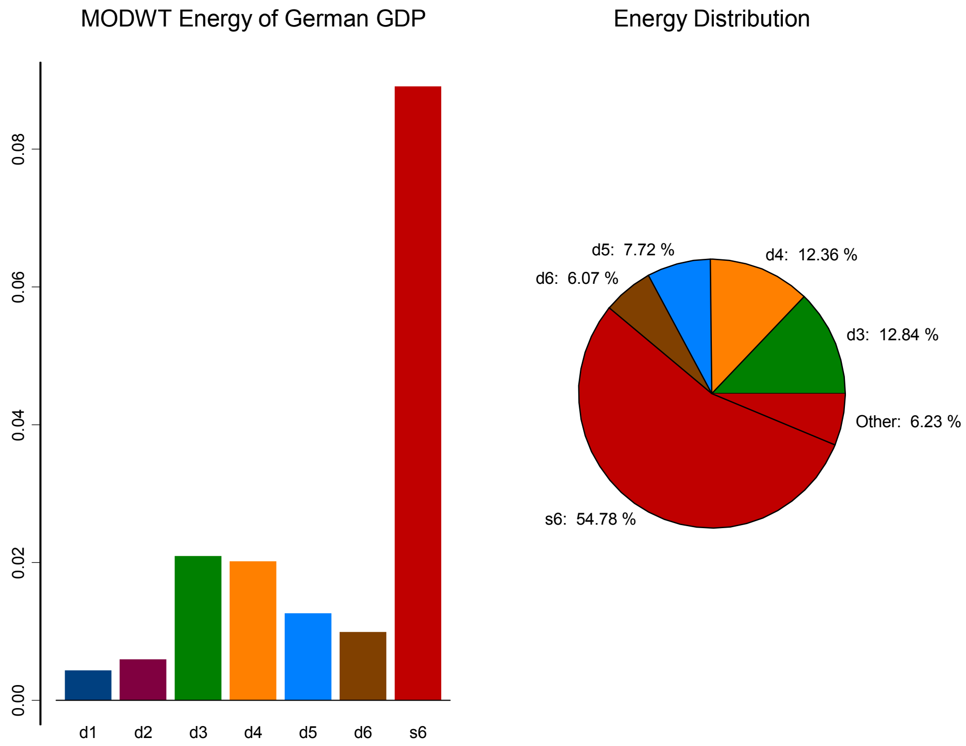

Figure 6: Energy distribution by Crystal for German GDP

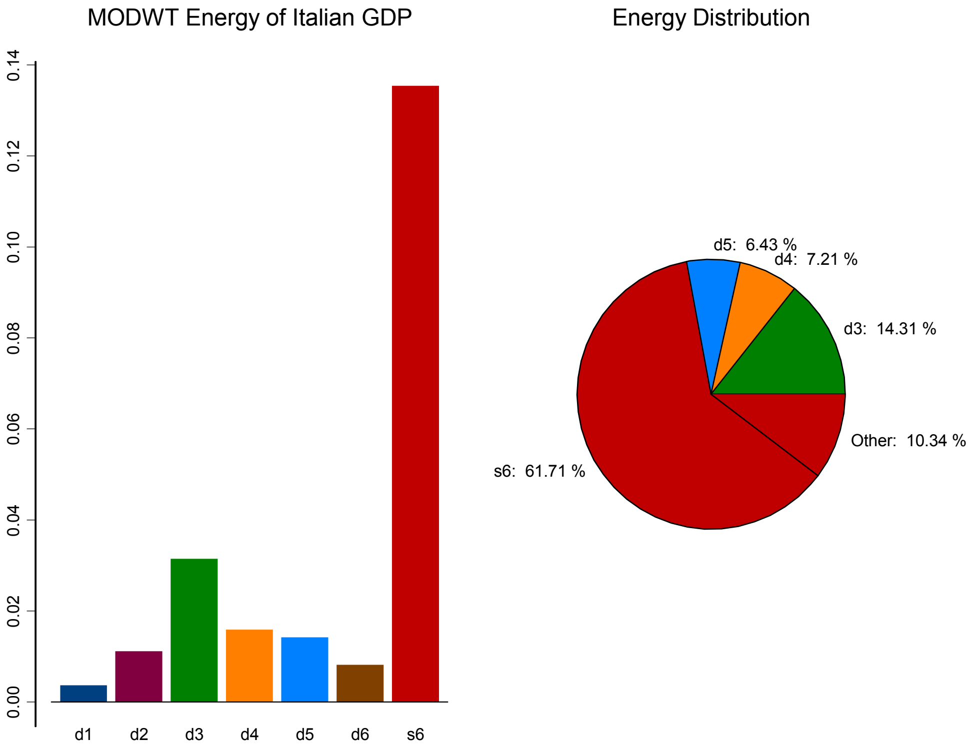

Figure 7: Energy distribution by Crystal for Italian GDP

Percival (2003)). Other more technical sources for this material are Percival and Walden (2000) and Gencay, Selcuk, and Whicher (2001). Figure 8 shows the wavelet correlations by scale for each of the series against one of the other series. Because of disposing with the boundary coefficients so as to ensure that the correlation estimates are unbiased, only 5 scales could now be resolved. The dark bars represent 95% confidence intervals. In nearly all cases the correlation coefficient between detail crystals was reported as positive – the only exception was the d1 crystal for Italy vs Germany which had a value of -0.05. In terms of positive correlations though, the d5 crystal correlation between France and Germany and Italy and Germany was not significantly positive, and the d4 and d5 crystals for Italy vs France were not significantly positive either. The highest correlation was recorded for the s5 wavelet smooth crystal (– cycles above 8 years), and this was 0.99 for Italy vs France. Among the detail crystals the highest correlation was recorded for Italy vs France at 0.88. Cross-correlations can also be plotted for each of these variables to see if any lags are apparent in the data. The cross-correlation plots are shown in figures 9 to 10.

The results highlight the leads and lags that occur in synchronization of GDP cycles for the 3 countries at different frequencies. In figure 9 France and Germany are fairly well synchronized, with contemporaneous correlations being the maximum values in all cases except for the d3 crystal where there is a one quarter lag in the d3 crystal over Germany's. In figure 10 for Italy vs Germany, the contemporaneous correlations for d1 is negative, and the highest correlation actually is a lead for Italy over Germany of one quarter (with a value

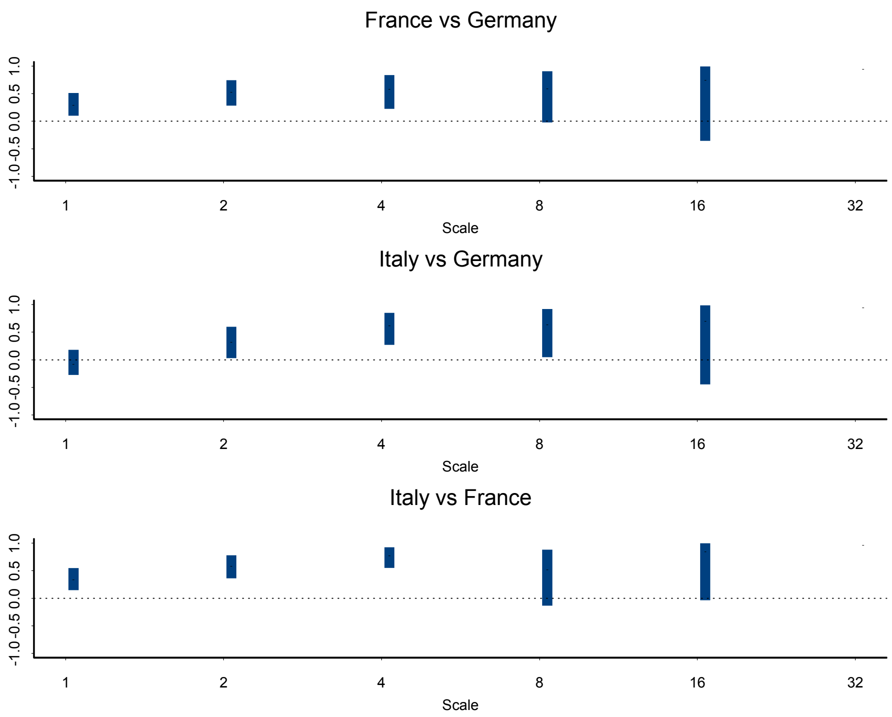

Figure 8: Wavelet correlations

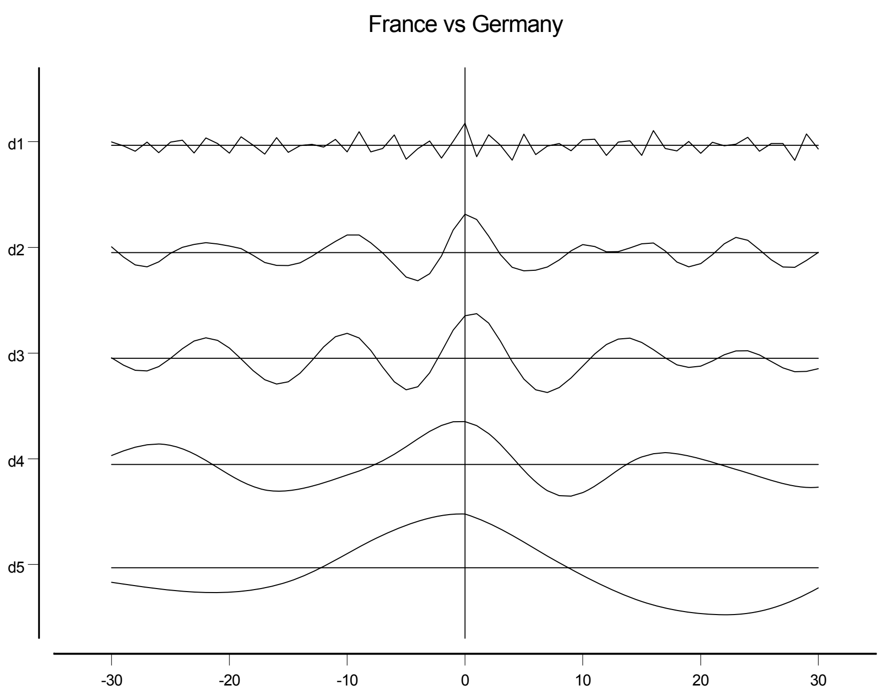

Figure 9: Wave cross-correlation for France vs Germany

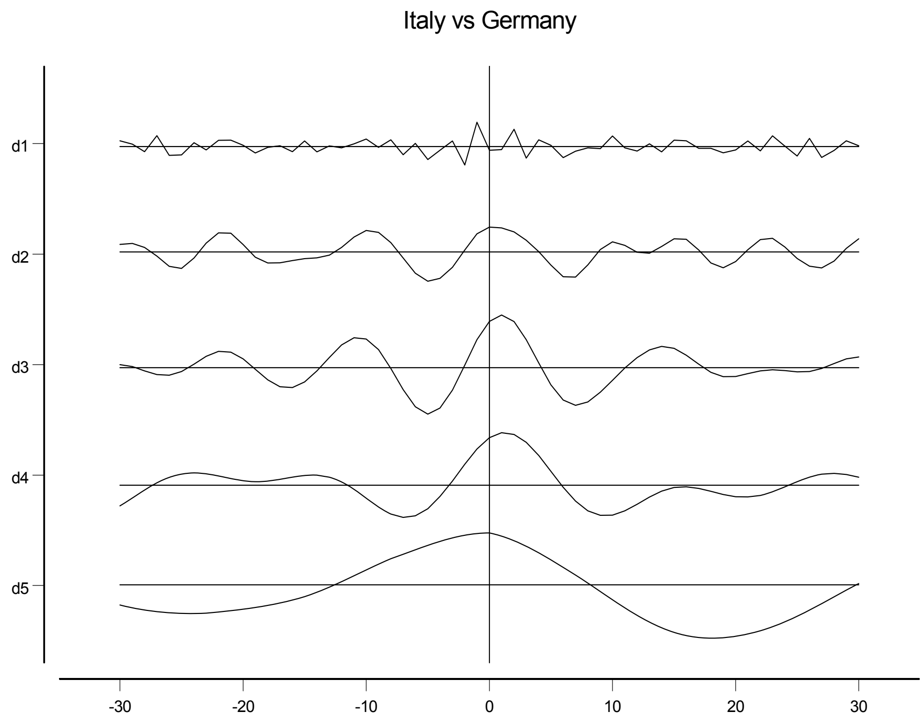

Figure 10: Wavelet cross correlations for Italy vs Germany

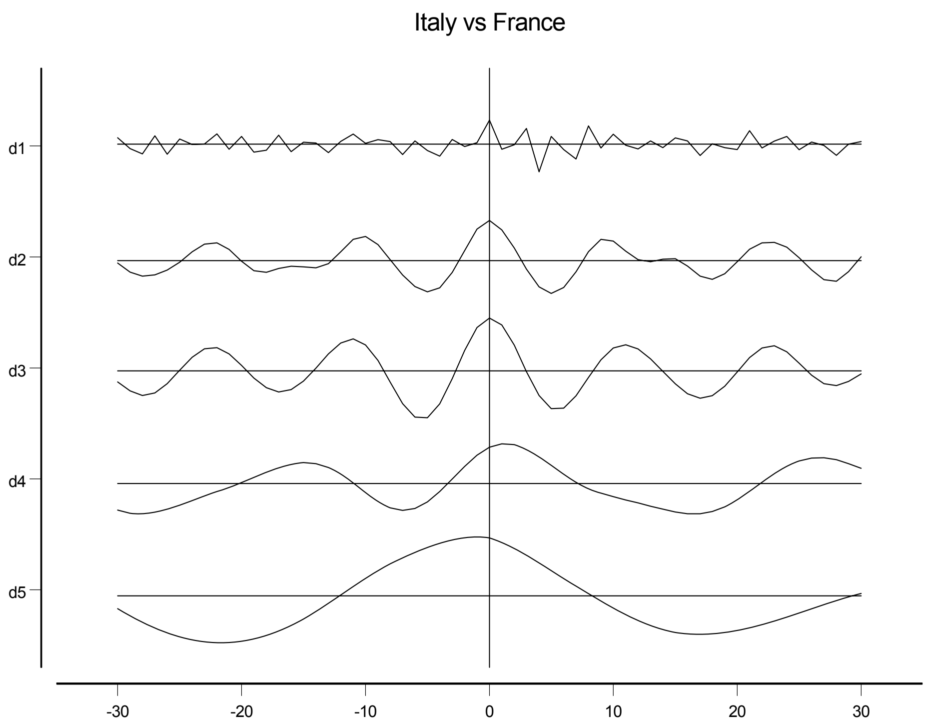

Figure 11: Wavelet cross correlations for Italy vs France

of 0.34) while d3 and d4 crystals are lagged by one quarter against Germany's crystals. The d5 crystal is at a maximum at its contemporaneous correlation. For Italy vs France, all crystals are at their maximum at contemporaneous correlations with the exception of the d4 crystal which leads by one quarter and the d5 crystal which lags by one quarter.

Perhaps the most noteworthy result though in this analysis is a general one: cycles between the 3 euro area core members have been remarkably well synchronized over the time period under consideration. Overall, despite the idiosyncratic shocks that have hit the three member states, there is clearly a commonality in growth patterns that is well synchronized between the three countries. The most obvious question to ask though is whether this synchronicity has changed over time, and in particular, whether the introduction of the euro and the concomitant union of monetary policies induced greater correlation and synchronicity between the core countries in the euro area. The following sections allow us to explore this question.

4 Continuous Wavelet Transforms (CWTs)

4.1 Methodology

Wavelet analysis is neither strictly in the time domain nor the frequency domain: it straddles both – the links between these domains is explored in some detail by Priestley (1996). It is therefore quite natural that wavelet applications have been closely related to the frequency domain and can produce measures associated with spectral analysis. Perhaps the best introduction into the theoretical side of this literature can be found in Lau and Weng (1995), Holschneider (1995) and Chiann and Morettin (1998), while Torrence and Compo (1998) probably provides the most illuminating examples of empirical applications to time series from meteorology and the atmospheric sciences.

Spectral analysis is perhaps the most commonly known frequency domain tool used by economists (see Collard (1999), Camba Mendez and Kapetanios (2001), Valle e Azevedo (2002), Kim and In (2003), Süssmuth (2002) and Hughes Hallet and Richter (2004) for some examples), and therefore needs no detailed introduction here. In brief though, a representation of a covariance stationary process in terms of its frequency components can be made using Cramer's representation, as follows

$$x_t = \mu + \int_{-\pi}^{\pi} e^{i\omega t} z(\omega) d\omega \tag{4.1}$$

where $i = \sqrt{-1}$, μ is the mean of the process, ω is measured in radians and $z(\omega)d\omega$ represents a complex orthogonal increment processes with variance $f_x(\omega)$, where it can be shown that

$$f_x(\omega) = \frac{1}{2\pi} \left(\gamma(0) + 2 \sum_{\tau=1}^{\infty} \gamma(\tau) \cos(\omega\tau) \right) \tag{4.2}$$

where $\gamma(\tau)$ is the autocorrelation function. $f_x(\omega)$ is also known as the spectrum of a series as it defines a series of orthogonal periodic functions which essentially represent a decomposition of the variance into an infinite sum of waves of different frequencies. Given a large value of $f_x(\omega_i)$, say at a particular value of ω_i, $\widehat{\omega}_i$, this implies that frequency $\widehat{\omega}_i$ is a particularly important component of the series.

To conduct wavelet analysis in discrete terms, we choose an orthogonal basis and then convolve the data with a wavelet filter to produce a set of coefficients (or crystals) which can be transformed back into the original series. In contrast to the DWT, we analyse a continuous set of scales (and thus choose a non-orthogonal basis accepting highly redundant results). So given a time series $x(t)$ and an analysing wavelet function $\psi(\theta)$, then the continuous wavelet transformation (CWT) is given by

$$W(s,t) = \int_{-\infty}^{\infty} \frac{d\tau}{s^{\frac{1}{2}}} \psi^* \left(\frac{\tau - t}{s} \right) x(\tau) \tag{4.3}$$

For an easier computation making use of FFT algorithms this can be rewritten in Fourier space. For a discrete numerical evaluation we get

$$W_k(s) = \sum_{k=0}^{N} s^{\frac{1}{2}} \widehat{x}_t \widehat{\psi}^* (s\omega_k) e^{i\omega_k t \partial t} \tag{4.4}$$

where $\widehat{x}_k$ is the discrete Fourier transform of x_t

$$\widehat{x}_k = \frac{1}{(N+1)} \sum_{k=0}^{N} x_t \exp \left\{ \frac{-2\pi i k t}{N+1} \right\} \tag{4.5}$$

where $\widehat{x}_k$,represents the Fourier coefficients. In this research we use a Morlet wavelet, which is defined as

$$\psi(\theta) = e^{i\omega\pi} e^{-\frac{\pi^2}{2}} \tag{4.6}$$

This is a symmetric wavelet, and is widely used in CWT analysis in the wavelet literature. Given our analysis above, it is also then possible to calculate conventional spectral measures, such as the spectral power

$$WPS(t,s) = E\{W(t,s)W(t,s)^*\} \tag{4.7}$$

With two variables, x and y, it is possible to also derive and empirically estimate the cross wavelet power spectrum

$$WCS^{xy}(t,s) = E\{W^x(t,s)W^y(t,s)^*\} \tag{4.8}$$

This gives rise to other multivariate spectral measures such as the coherence

$$WCO^{xy}(t,s) = \frac{|WCS^{xy}(t,s)|}{[WPS^x(t,s)WPS^y(t,s)^*]^{\frac{1}{2}}} \tag{4.9}$$

which can also be measured as the magnitude squared coherence,[10] being simply $[WCO^{xy}(t,s)]^2$. As wavelet analysis essentially identifies cycles in the data, if such cycles are detected then the phasing, $\Phi(s)$, between the cycles can also be calculated from

$$WCS^{xy}(t,s) = |WCS^{xy}(t,s)|\, e^{i\Phi(s)} \tag{4.10}$$

This measure is of particular interest to economists, as it is the analogue measure of co-correlation for the time domain, and therefore shows the degree of synchronization between cycles at different frequencies. By using these measures in conjunction with wavelet analysis, we can therefore evaluate how synchronization changes through time.

4.2 Empirical results

4.2.1 Power spectra

Wavelet power spectra measures the strength of cycles at various frequencies. Figure 12 shows the log power spectrum for log French GDP growth using a colour scale[11] and zero padding. The vertical axis measures the scale in years, and the conventionally measured business cycle frequencies (3–8 years) are delineated by dotted lines. Interestingly, although the business cycle likely contains some quite strong cycles, many of these cycles fall outside the 3–8 year range (which tends to confirm the results obtained by Crowley and Lee (2005) and Levy and Dezhbakhsh (2003a)) and indeed strong high frequency cycles (– one is clearly discernable at around the 2 year frequency) seem to exist, as well as longer periodicity cycles. A strong longer cycle can be detected in the late 60s and 1970s, but this has clearly waned in the 1980s and has remained weak in the 1990s. There are no specific missing cycle frequencies though, as there are no large areas of lower power in the plot, so splitting up the spectrum up in terms of frequency ranges (– which is what the discrete wavelet transform does), should not bias the results in any way.[12] The arch drawn in the plot shows the 'cone of influence',[13] so points outside the one are to be interpreted as being less reliable than those placed within the cone.

Figure 13 shows the German real GDP growth spectrum, and there are some clear short term cycles that strengthen from time to time, notably in the

[10] Andrew Hughes-Hallett recently referred to mean squared coherence as the R^2 of the frequency domain. We prefer to think of coherence as a measure of similarity of frequency content as the notion of an R^2 in the frequency domain can lead to some confusion in interpretation.

[11] Here we normalize the lowest value in the spectrum to one, and use a log scale for plotting contours.

[12] This is not certain though, as there is 'leakage' between different scales (that is, there is correlation between scales as shown in Maraun and Kurths (2004)). In addition, given limited data, to obtain resolution at lower frequencies, we have purposefully lowered the power of resolution at any given scale level.

[13] This indicates the central area of the graph where the full length wavelets are applied to the data, so are free of any bias resulting from the use of boundary coefficients to enable wavelet application.

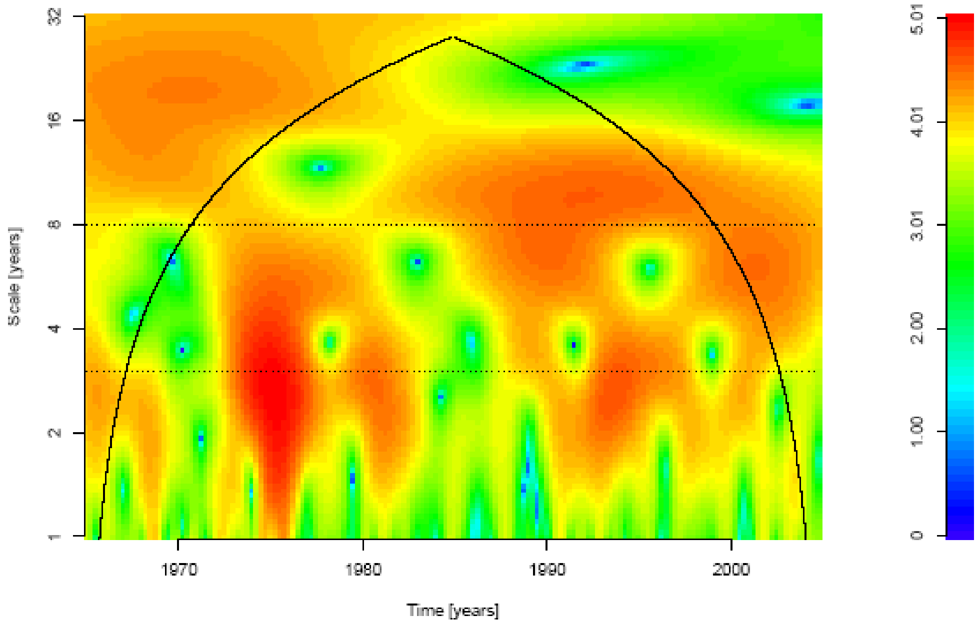

Figure 12: Wavelet power spectrum for French real GDP

late 1960s, mid-1970s and early 1990s. This might be expected and is likely related to the oil price shocks (these can be seen in the French plots as well), the turmoil caused in the EU by the French u-turn on policy in the early 1980s and German reunification in the early 1990s. A weaker long term cycle is also evident in the data for Germany compared to France, but the weakening of the cycle follows the same trend.

The Italian GDP growth spectrum in figure 14 is interesting in two respects: first, the absence of any waning in strength of a longer term cycle in the 1980s, and the particularly strong cycles at shorter frequencies appearing in the 1970s. The 1990s also appear to have been characterised by stronger cycles at shorter frequencies than business cycle frequencies and at the shorter end of the business cycle frequency range. Perhaps this was due to the growth effects of exchange rate changes following the departure of the lira from the ERM of the EMS in 1992.

4.2.2 Cross spectral analysis

Multivariate spectral analysis essentially combines the spectra to study the frequency content of pairs of series at particular frequencies, and also the phasing of any cycles located at those frequencies. Figure 15 shows a magnitude squared coherency and phase plot for French GDP growth against German GDP growth. 90% and 95% confidence intervals for the null hypothesis that coherency is zero are marked on the sidebar,[14] and these are plotted as contours in black in the figure. As might be expected, there

[14]These levels are 0.929 for the 90% significance level and 0.948 for the 95% significance level.

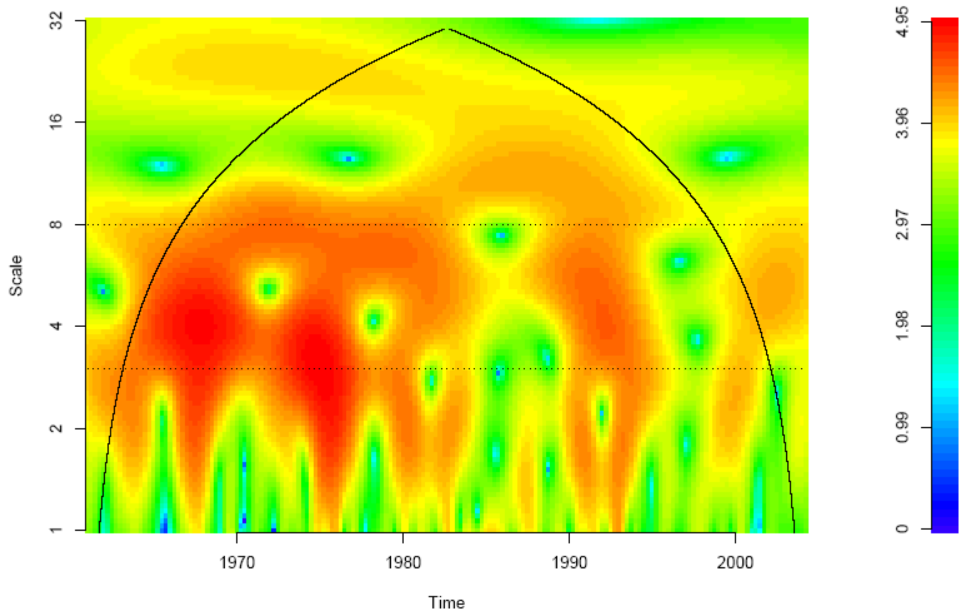

Figure 13: Wavelet power spectrum for German GDP

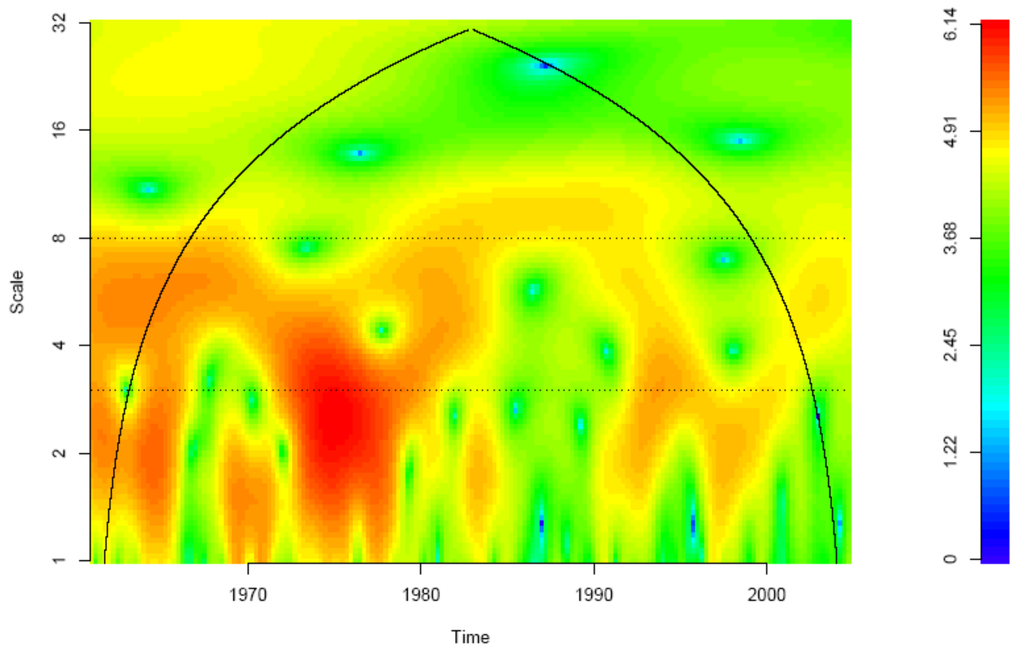

Figure 14: Wavelet power spectrum for Italian GDP

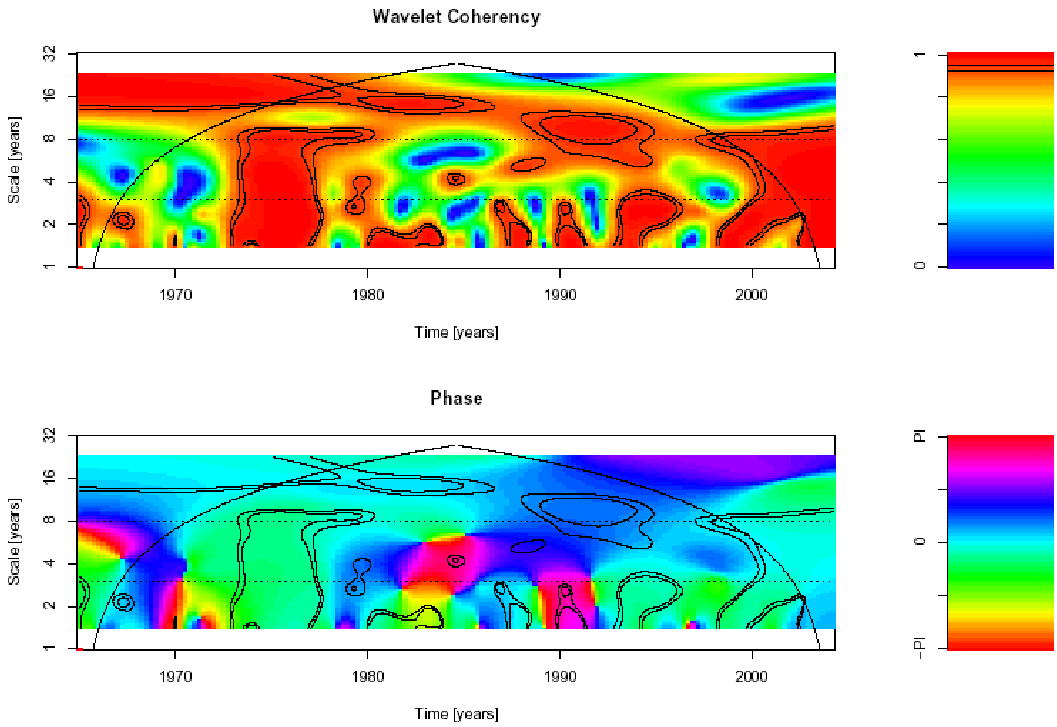

Figure 15: Cross spectral analysis: French vs German GDP

are large parts of the cross spectra that are highly coherent, but there are noticeably large patches of low coherence between the two series – since 1997 there appears to be a lack of coherence for cycles above 10 years[15] and also in the 1980s when there was a lack of coherence at the 6–8 year cycle frequency. There is also a lack of coherency at short term frequencies in the early 1970s (end of the Bretton Woods system) and in the early 1990s (German reunification). The phase plot shows that for most of the 1960s and 1970s, France's cycles lagged those of Germany, but then in the 1980s, after the u-turn in French economic policy (which is clearly apparent by the anticyclical phasing from around 1982-3 at the business cycle frequency, France started to lead Germarny, particularly at cycles longer than the business cycle frequency. Recently, French cycles appear to lead slightly at higher frequencies, and lag slightly at frequencies longer than the business cycle, although since 1999 business cycle frequencies appear to be synchronous.

Figure 16 shows less highly significant contours for coherence for Italian against German GDP growth cycles, but in recent years nearly all cycles appear to be in significantly high coherency areas. Phasing appears to have been much more of an issue for Italy though, with a period in the late 1960s when the Italian business cycle became anticyclical with Germany, and in the early 1990s, as German reunification caused a period of anticyclicality at fairly short frequencies. Throughout much of the late 80s and 90s, Italian cycles led those of Germany, but in recent years, cycles seem to be fairly synchronous at all frequencies.

[15] Although clearly we have to be concerned about this area lying outside the COI.

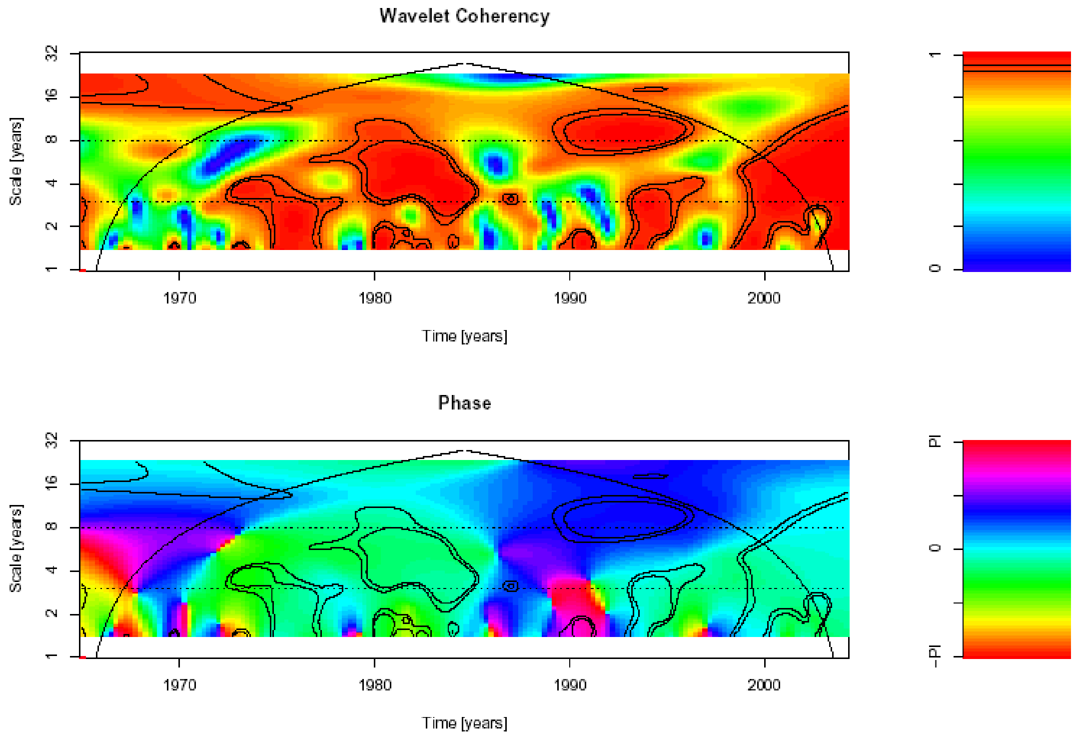

Figure 16: Cross spectral analysis: Italian vs German GDP

Lastly, when looking at figure 17 it is clear that wavelet coherency through the 1970s and 1990s was much higher for Italy with France than it was with Germany. In the first part of the 1980s though, Italy clearly had higher coherence with Germany at business cycle frequencies, and this is more likely due to the evolution of French economic policy at the time than to any specific events in Italy. From around 1987 onwards though, coherency is strong at all cycles until the mid-1990s, and this is likely explained by the commonality of experience that both countries had in the ERM of the EMS. In recent years coherency has once again been significant at business cycle frequencies, although cycles longer than 15 years appear to be less coherent. In terms of phasing, the late 1960s and early 1980s are clearly anticyclical at business cycle frequencies, so the late 1960s pattern of growth in Italy looks to be idiosyncratic. More recently, short frequency cycles appear to lag those of France and longer cycles tend to lead those of France, while cycles at business cycle frequencies once again appear to be in phase.

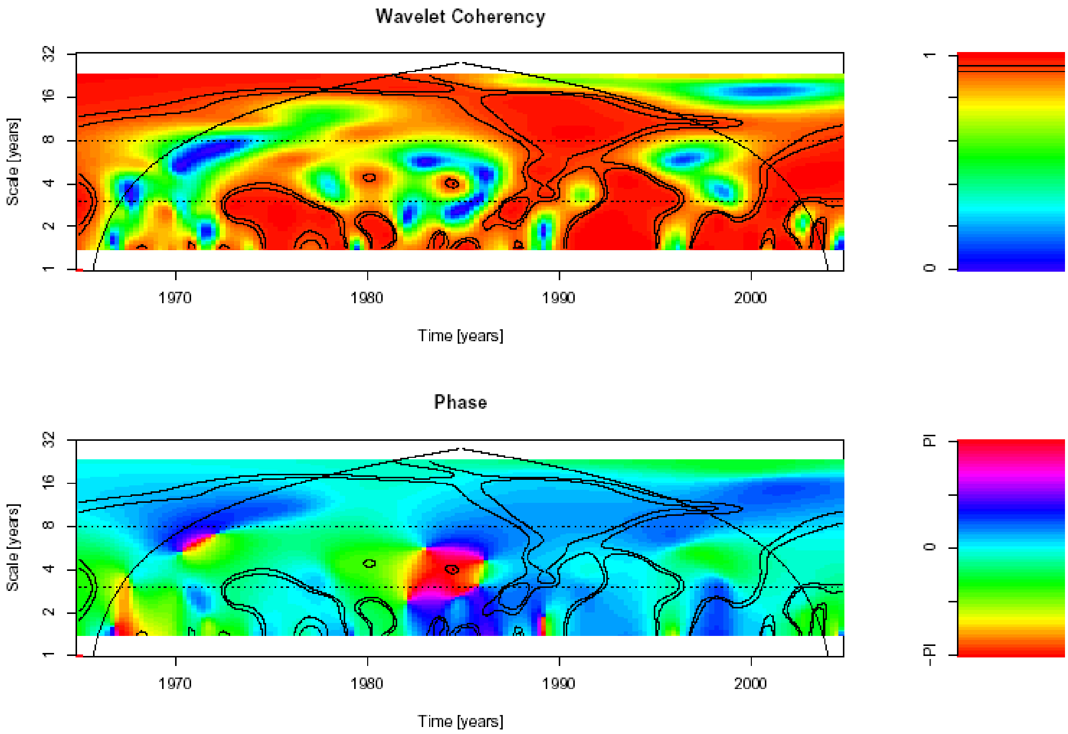

Figure 17: Cross spectral analysis: Italian vs German GDP

5 Multivariate spectral analysis using Hilbert wavelet pairs (HWP)

5.1 Methodology

The last technique employed in this paper uses a special version of the discrete wavelet transform by applying pairs of wavelets, not as continuous functions but as filters. The pairs of wavelets are identical in continuous time terms, except that one wavelet is displaced slightly compared to the other wavelet. The idea of applying 2 sets of wavelet filters to a series originated in work done by Kingsbury (2000) and is known as a dual-tree or complex wavelet transform. The wavelets here though have specific properties – that is that each filter contains a wavelet but the wavelet filter coefficients are different as one of the wavelet filters possesses coefficients that lead the other wavelet by a specified amount. The amount of this phasing difference accords to what is known as the Hilbert transform[16] – but in this implementation, there is an approximation made to the Hilbert transform. The advantage of this methodology is that it provides a discrete wavelet transform analogue for spectral analysis, as it produces the usual spectral measures that have been defined above. In a sense this is a combination of the DTWT and the CWT spectral methodology, but allows multivariate coherency and phase measures to be defined in the time-frequency domain. In Craigmile and Whitcher (2004), the basis for using

[16] A basic introduction to the Hilbert transform can be found in Bendat and Piersol (1986).

the Hilbert wavelet pairs (HWPs) is defined analytically and asymptotic theory used.

In technical terms, the maximal overlap discrete Hilbert wavelet transform (MODHWT) is implemented using a pair of mother and father wavelet filters such that the two sets only differ in their phase, and not in their gain functions. A good example of a Hilbert pair of functions would be the sine and cosine functions, that differ by only a quarter phase (or what is known as a half sample in the signal processing literature). As wavelets are not defined in trigonometric terms, Selesnick (2002) provides a way of obtaining near HWPs, which involves making the gain of two low pass (father) filters, denoted $A_0(f)$ and $B_0(f)$ relate in the following way

$$B_0(f) = A_0(f) \exp^{-i\theta(f)} \tag{5.1}$$

where $A_0(f)$ and $B_0(f)$ are filters which form a wavelet pair as long as $\theta(f) = \pi f$, so that they have a half sample delay between them - the same can clearly be done for the high pass or mother wavelets too. To characterise a HWP in practice, two parameters are required, denoted K and M, where K is the number of zero wavelet moments (which directly relates to the smoothness of the wavelet) and M represents the degree of approximation to the half sample delay (– as M increases this approximation improves). Care needs to be taken applying the HWP to high frequencies, as at high frequencies the relationship between the two filters is no longer characterised by the Hilbert transform.[17] To implement the MODHWT or its decimated equivalent the DHWT, define the high pass and low pass filters as

$$\widetilde{h}_l = \widetilde{a}_{1,l} + i\widetilde{b}_{1,l} \tag{5.2}$$

$$\widetilde{g}_l = \widetilde{a}_{0,l} + i\widetilde{b}_{0,l} \tag{5.3}$$

where equation 5.2 represents the mother wavelet filter and equation 5.3 the father wavelet filter, with both sometimes known as 'complex wavelets' because of the form of the equation representing the wavelet filters.

Given that we now have a Hilbert wavelet pair of two filters, if $x = (x_0, x_1, ..., x_{N-1})$, then convolution can occur with the data as follows

$$W_{1,t} = \widetilde{h} * x_t \tag{5.4}$$

using the wavelet filters defined above to give the detail crystal coefficients. As the wavelet filters are simultaneously moved along the series, phasing can also be studied by looking at the differences in crystal coefficients through time. There is also an analogous packet table available for the MODHWT as well.

In order to conduct time-varying spectral analysis, define $\{(W_t^X, W_t^Y)^T : t \in \mathbb{Z}\}$ as the MODHWT detail crystals from two series X_t and Y_t with a total of T observations in each series (– each crystal will also have T observations

[17] This is shown in the appendix in terms of a frequency function for the two filters.

in this MODHWT version of the analysis). The time-varying cross spectrum of X_t and Y_t can then be defined as

$$S_{XY}(\lambda_j, t) = E\left[W_{j,t}^X W_{j,t}^Y\right] \tag{5.5}$$

and then corresponding amplitude and phase spectra can be extracted as with conventional frequency domain analysis as per the previous section.[18]

5.2 Empirical results

In terms of implementation of the Hilbert wavelet pairs, the same data is used as for the CWT above, with an HWP(2,4) choice of wavelet pairs using a moving average window of 16 quarters. Periodic boundary conditions are applied, and 90% significance levels are shown as a horizontal line in the magnitude squared coherence plots.[19] Given the differences in the implementation of the wavelet analysis for the continuous and discrete wavelet methods, the results would not necessarily be expected to corroborate those of the previous section. Rather, given that this methodology uses a variation on the MODWT, the results would be more likely to closely mirror those of section 2.

Figure 18 shows the output from the MODHWT analysis for French vs German GDP growth in terms of mean squared coherence. In terms of the MODWT stack plots presented earlier, the results here seem to confirm the similarity of the cycles at lower frequencies, with high coherence in the 1970s at most frequencies, lower coherence in the 1980s but trending to higher levels in recent years. This pattern is particularly reflected in 1–2 year frequencies, but the jump in coherence is particularly noticeable in the mid-1990s. This trough throughout much of the 1980s is possibly due to the short run flexibility offered by the ERM of the EMS to diverge from German policies, but clearly beyond the mid-1990s, as has been well documented elsewhere, convergence occurred without the use of the exchange rate anchor, with French monetary policy becoming 'harder' than Germany's. With 2–4 year oscillations, coherency was also high in the 1970s, dipped to a low level at the beginning of the 1980s, and then has gradually increased to levels above 0.9 since 1999. Coherency at the main business cycle frequencies and above are remarkably high, showing that the business cycles of France and Germany now contain remarkably similar frequency content.

The phasing plots in figure 19 are interesting, and show some similarities and differences from figure 16, the CWT phasing plot. The acyclical nature of shorter cycles in the 1970s comes through clearly in both the 2–4 quarter, 1–2 year and 2–4 year phasing plots, but the 1983 economic policy u-turn only appears in the 1–2 year and 2–4 year cycles, and not in the 4–8 year cycles as

[18] More details of this approach and an illustration using atmospheric monsoon data can be found in Craigmile and Whitcher (2004).

[19] This was implemented using the waveslim package in R language. We acknowledge the assistance of Brandon Whitcher in this regard.

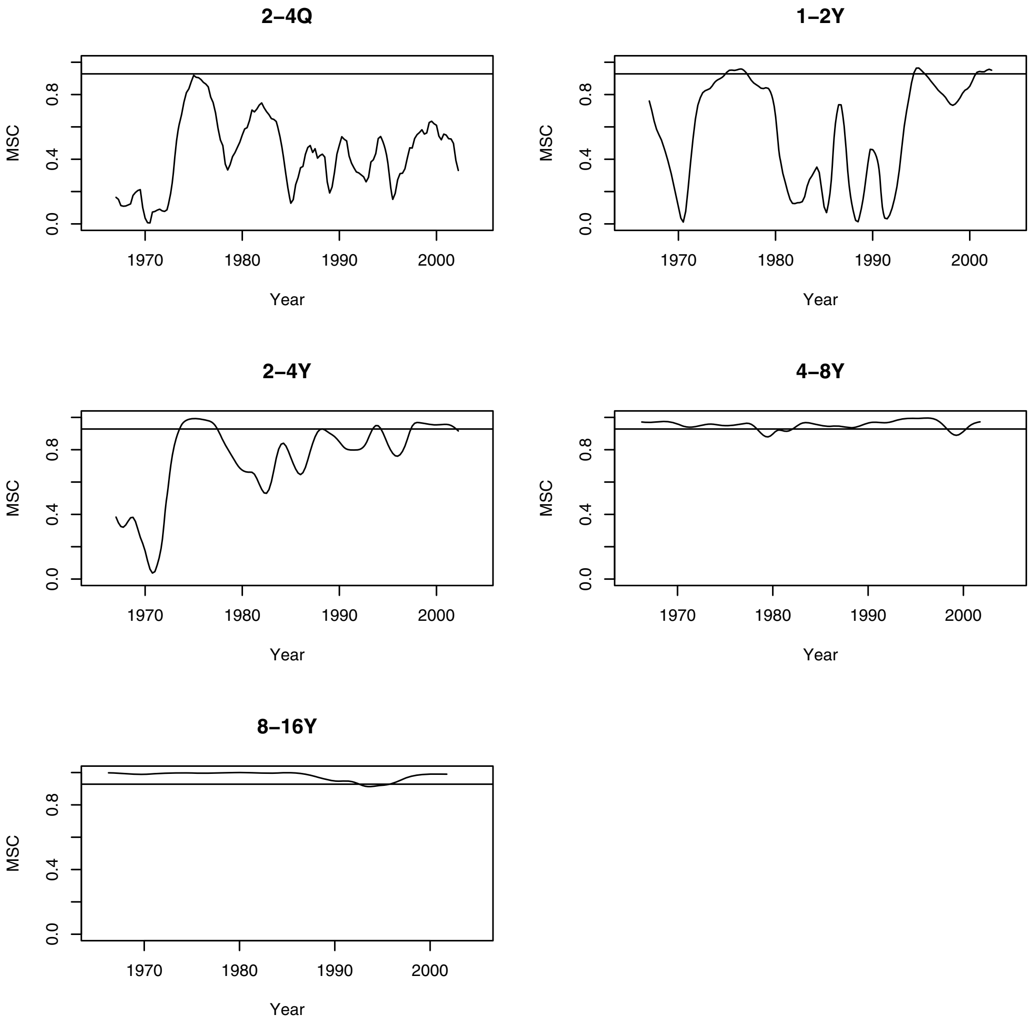

Figure 18: French vs German real GDP growth: coherence

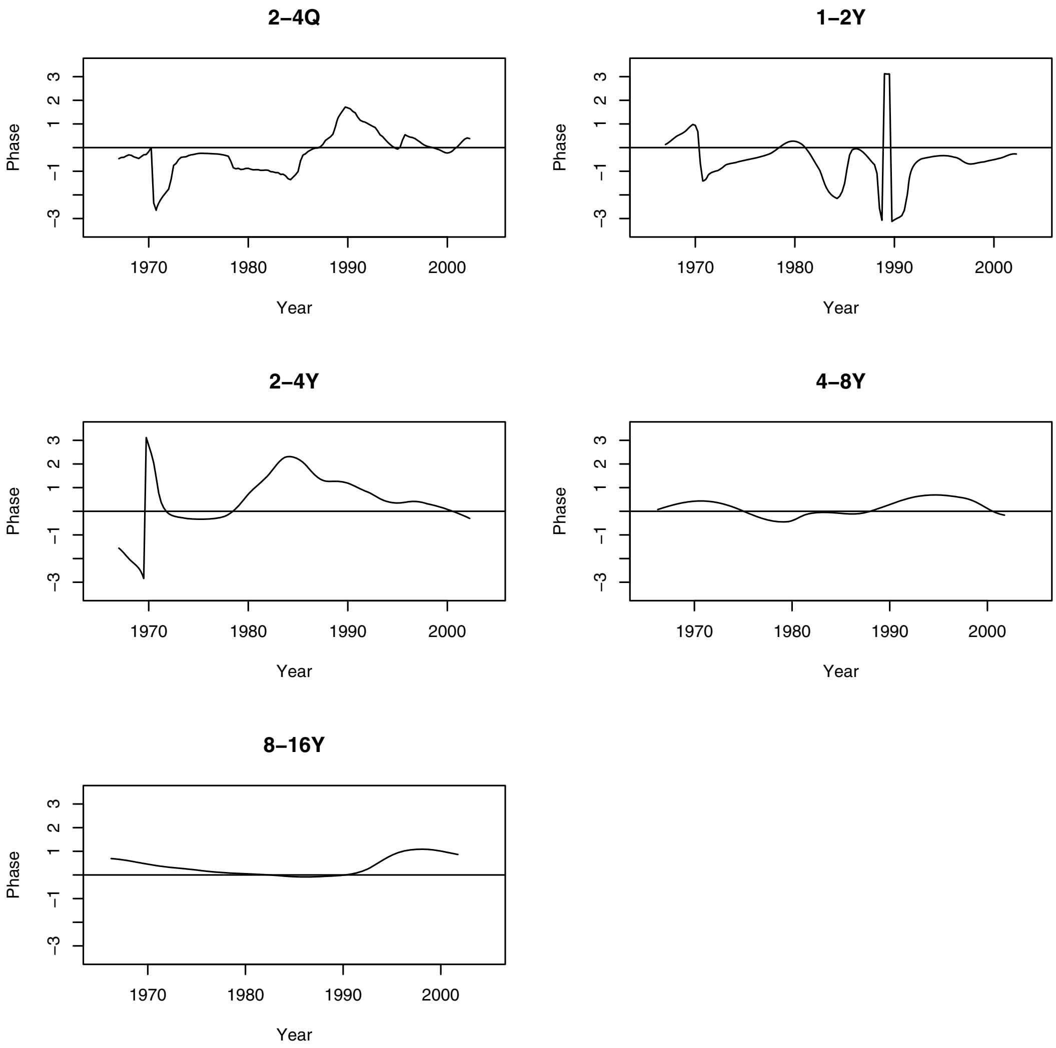

Figure 19: French vs German real GDP growth: phase

with the CWT plots. German reunification only appears as a phase problem in the shorter term cycles of 2–4 quarters and 1–2 years, and is not evident at business cycle frequencies. In recent years, the French 4–8 year cycle appears to have been leading the equivalent German cycle as has the 8–16 year cycle, although current estimates show that at all frequency cycles there is little phasing difference evident.

Coherence plots for Italy against Germany in figure 20 show low coherence at higher frequency cycles, although the magnitude against Germany is not that different to that of France. Coherency at the 1–2 year and 2–4 year frequencies clearly fell to lower levels at the end of the 1980s and into the early 1990s as might be expected, and interestingly coherence at the business cycle frequencies follows a similar pattern to that of France, although the variation in coherence is clearly larger than for that of France against Germany.

In terms of phasing of the Italian and German cycles (shown in figure 21), once again, the late 1980s and early 1990s clearly cause an anticyclical

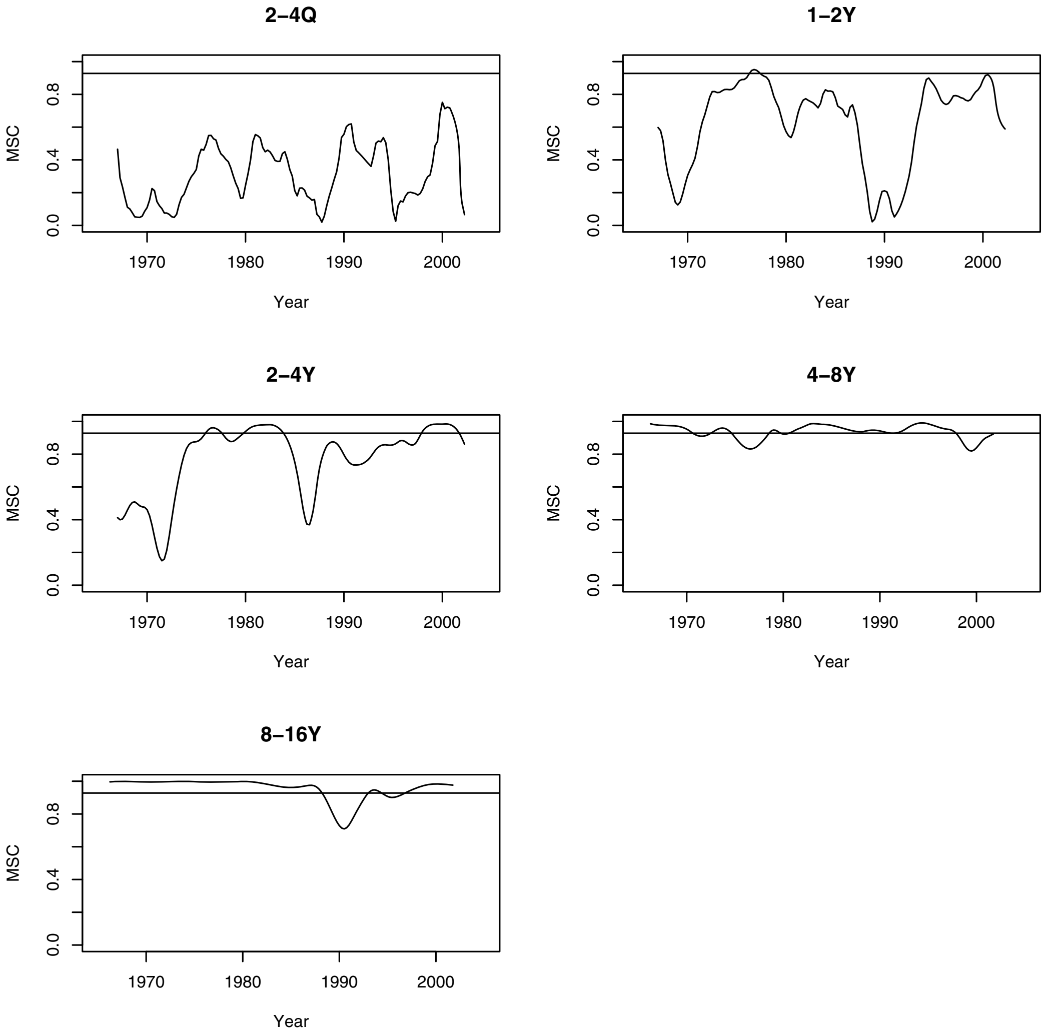

Figure 20: Italian vs German real GDP growth: coherence

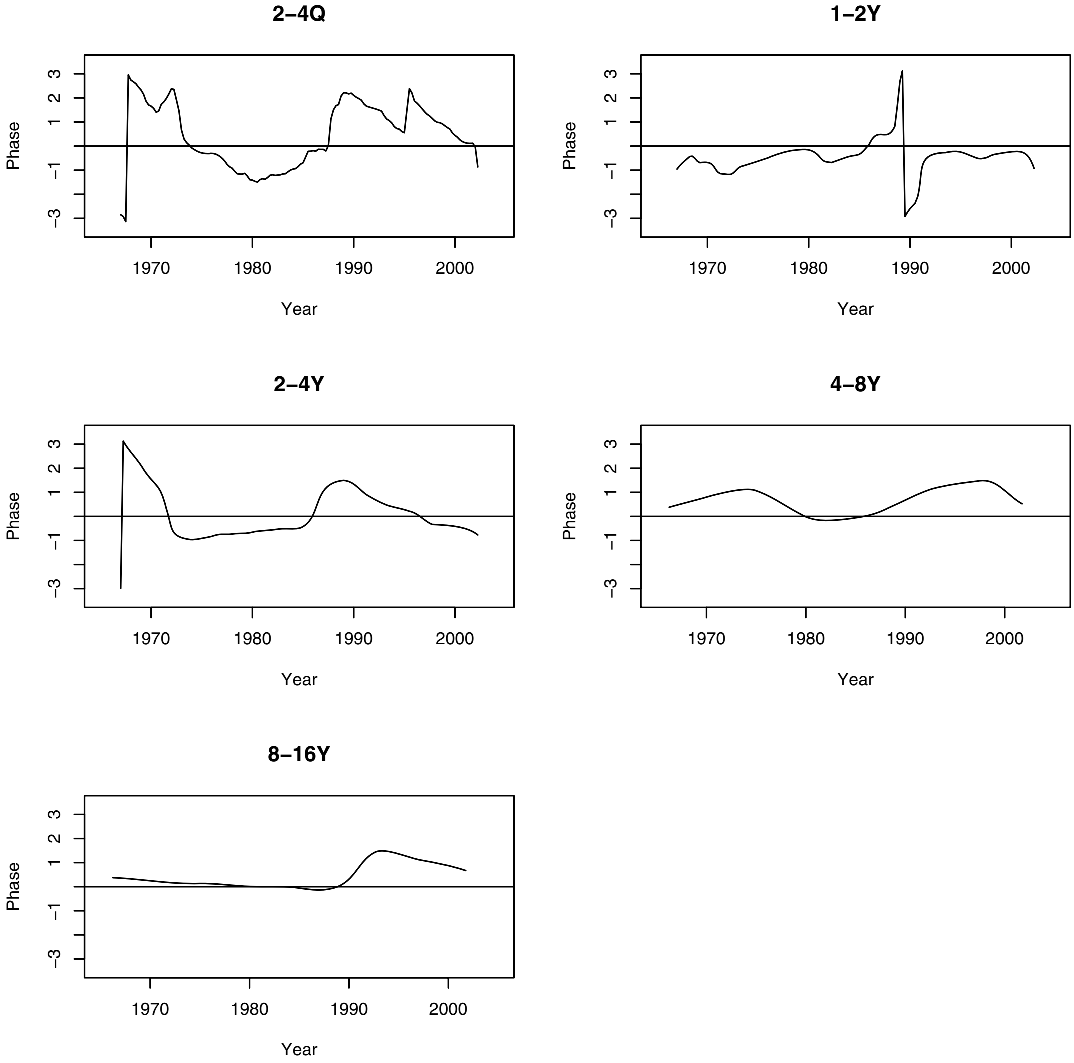

Figure 21: Italian vs German real GDP growth: phase

response, but this is apparent in all the shorter cycle frequencies and is seemingly reflected by the Italian business cycle accelerating ahead of the German one. One interesting difference between these plots and the CWT plots is that the Italian 4–8 year cycle was shown to lag the similar German cycle in the mid 1970s to mid 1980s. Here this is not the case, and this lag comes through mainly in the 2–4 year cycle instead.

In figure 22 the fall in coherence in the mid-1980s shown in the CWT plot is also evident in the 1–2 and 2–4 year cycles, and even comes through in the 4–8 year cycle. Coherency is high at most frequencies, but it is somewhat worrying to see the fall in coherency in 1–2 year cycles in the most recent data.

Lastly, in figure 23, the observation that the Italian and French growth cycles were much more synchronous in the 1970s and 1990s is apparent in the phase plots. In the 1980s both the 1–2 year and 2–4 year frequencies showed anticyclicality, but again this is more likely due to the change in French economic policies rather than any Italian policy or performance shifts.

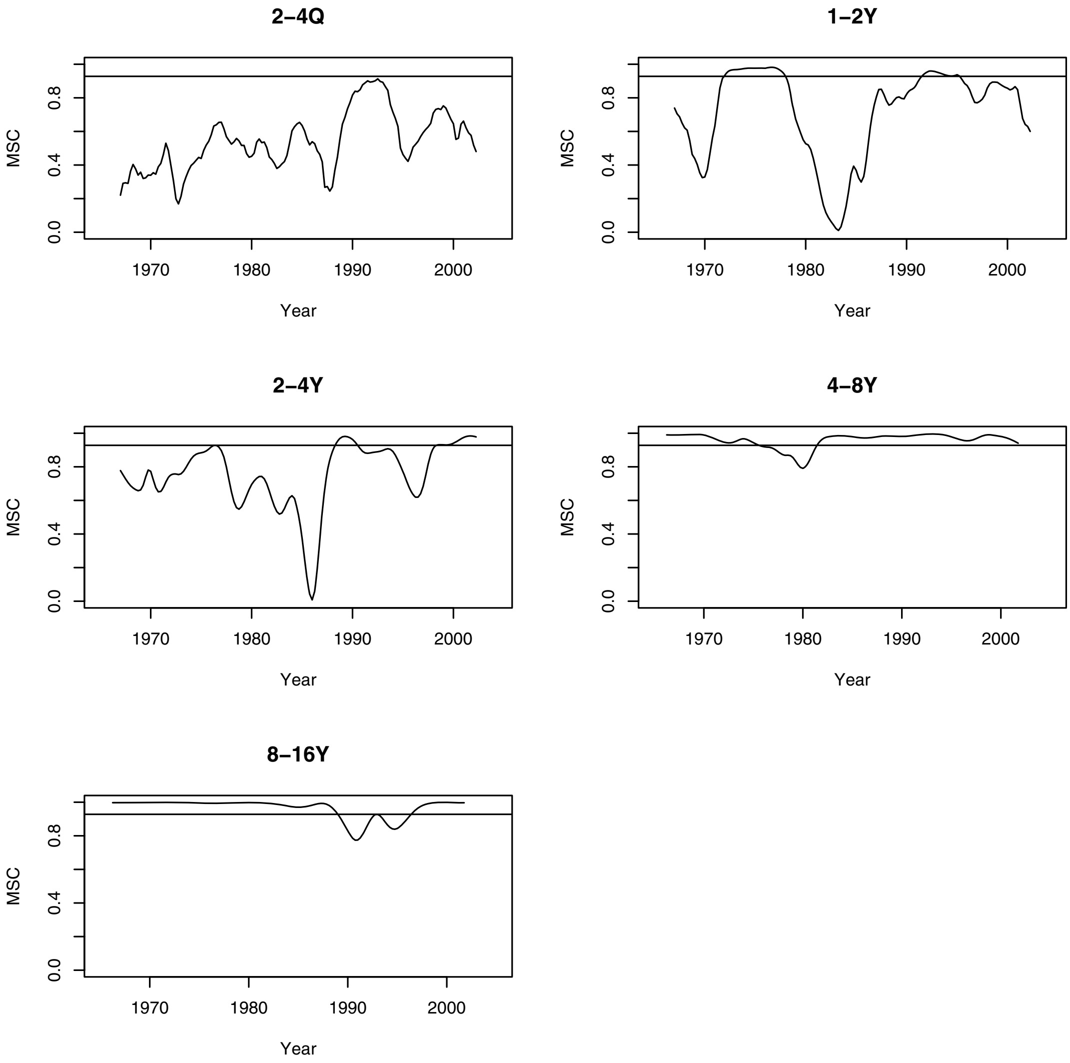

Figure 22: Italian vs French real GDP growth: coherence

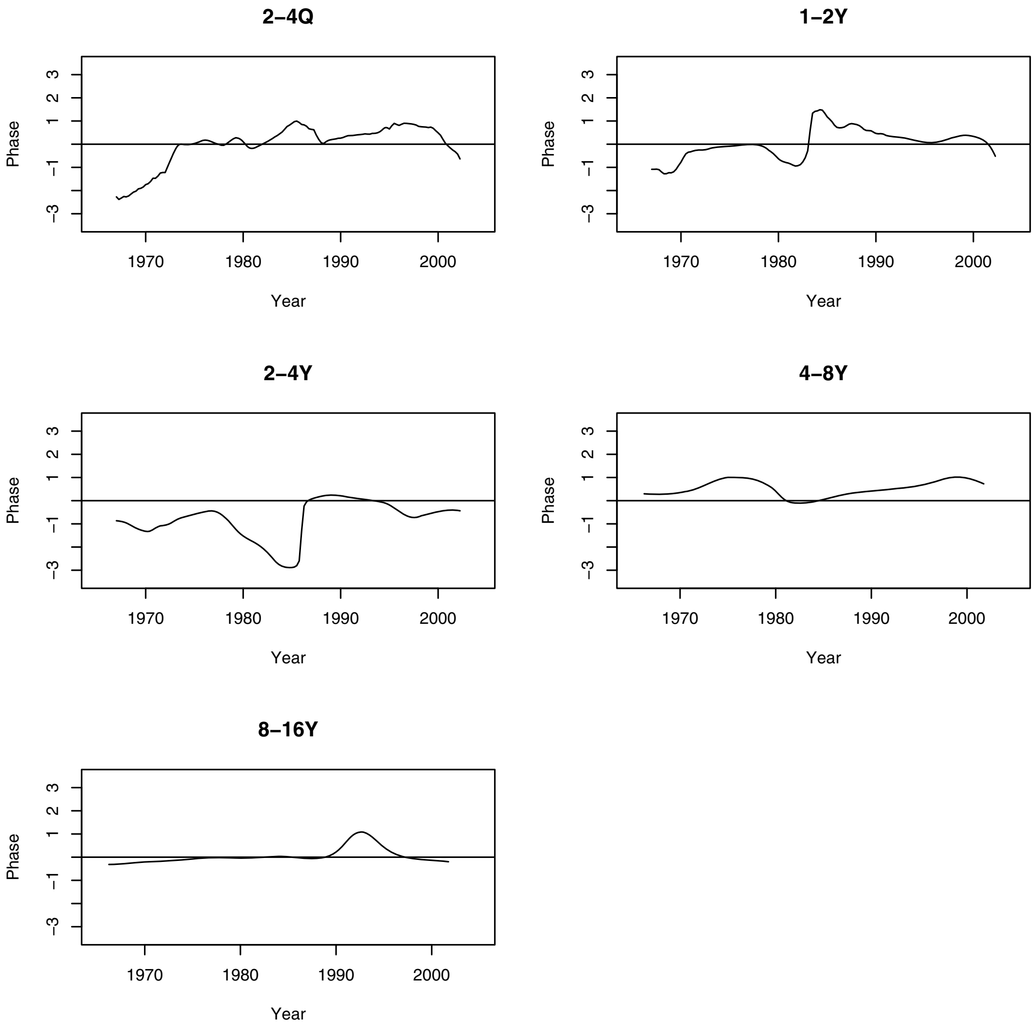

Figure 23: Italian vs French real GDP growth: phase

Once again, at 4–8 year frequencies, the Italian cycle is shown to lead the German cycle. It is interesting to note here that although there has been some variability at different phases, the number of cycles at each frequency has not changed (ie there has not been a complete phase shift at any point in time). Currently as well, the phasing is fairly well synchronised, with only the 4–8 year cycle leading the French cycle.

6 Discussion

There are two results that have implications for policy in terms of the properties of growth cycles between France, Germany and Italy:

i) coherence and phasing at business cycle frequencies is high between these three economies, but coherence is not always significant, although it appears to have increased under EMU; and

ii) coherence and phasing at other cycle frequencies is not so high, and synchronicity at higher frequencies is also less consistent.

iii) coherence varies according to idiosyncratic events in these three countries

In terms of the first finding, this is important, as clearly the largest economies of the euro area are relatively synchronised in their business cycles so aiding the implementation of monetary policy when it is perhaps most important – during downturns and recessions. Certainly since around 1995 there have been no extraordinary events that have caused synchronization to become anticyclical, and coherence has become significant at most cycle frequencies between these member states during recent years, which suggests that the ECB might be acting through monetary policy to 'couple' the synchronicity of cycles within the euro area.[20]

In terms of the second finding, this is not such good news for ECB monetary policy, as it implies that during the growth phase of the business cycle, there are shorter cycles at play which are not synchronised too well between the member states. What this implies for the conduct of monetary policy is clearly state-dependent, but it does suggest that economic growth might diverge between countries over these shorter cycles. What gives rise to these growth cycles is clearly a separate issue, but in the 1980s in particular, fiscal policies and exchange rate movements in the form of devaluations/revaluations in the ERM of the EMS were likely candidates. The analysis suggests that even during the 'halcyon' years of the ERM of the EMS,[21] there were different growth cycles at work at low and high frequencies in individual member states.

As for the third finding, this is not a strong effect at business cycle frequencies, and is secondary to our first finding. But it is possible to see the u-turn in economic policy in France and German reunification in this data, and to identify the growth frequencies which these events impact. Indeed one of the most interesting results is that the effects of German reunification appear to have been confined to the 1–4 year cycles of growth, and are no longer evident in the growth data.

In terms of the methodologies used here, they are used in a complementary fashion, but are not always completely consistent in the results they give. Nevertheless, there is clearly enough similarity in the spectral measures that are obtained from both methodologies to confirm most of the major features of growth cycles at different frequencies evident in the data. One of the advantages of using the CWT is the ability to obtain confidence intervals for our measure of coherence, which is currently not possible with the Hilbert wavelet pairs approach. On the other hand, boundary problems (– areas of the spectrum outside the cone of influence) are usually recognized to be much more of an issue with the CWT than with the discrete wavelet transform approach, so that if results obtained with the CWT approach are replicated with the Hilbert pairs method, then we should treat this as a confirmatory result.

[20] This 'coupling' phenomenon originates in the physical science literature – a good reference is Pikovsky, Rosenblum, and Kurths (2001).

[21] Roughly 1983 to 1990.

7 Conclusions

This paper attempted to compare growth cycles in the three major economies of the euro area, namely France, Germany and Italy. To do this, a maximal overlap discrete wavelet transform, a continuous wavelet transform, as well as a Hilbert wavelet pairs method using a discrete transform were used. The results were largely consistent between the three methods, which should be seen as confirmatory, and strengthens our results.

From the MODWT section of the paper, there are clearly cyclical patterns evident in the GDP data, and although the cycles extracted at business cycle frequencies were largely as expected, what was perhaps surprising here is that there appears to be a long cycle in the data with perhaps a 40 year cycle frequency.

The second and third methods used in the paper utilized spectral measures but here the results were largely consistent as well. There is a large degree of coherence at conventional business cycle frequencies between the three countries, although the coherence measure is not always significant. Further, these cycles are largely synchronous. Coherence at other frequencies is less consistent, with low coherence often found at higher frequency cycles. Phasing at all frequencies appears to be less of an issue between the Italian and French economies, but perhaps this is hardly surprising, given the fact that Germany was the anchor of the ERM of the EMS and also experienced the exceptional circumstances surrounding the reunification of the country in the 1990s. In terms of similarity of cycles and phasing, Germany and France appeared to have been more closely associated with each other than with Italy during the late 1960s and 1970s, but during the 1980s and 1990s, the French u-turn in economic policy and German reunification lead to closer association in cycles between the two countries not associated with these events.

In terms of more recent trends, although there is increased uncertainty associated with the results, it is likely that coherence is currently increasing between the three countries, albeit during a slowdown in all three economies when common turning points of the business cycle might be expected. Phasing at all frequencies seems to be roughly synchronous, suggesting that ECB policies are not going to differentially impact any single country.

In terms of non-business cycle growth frequencies, there are some concerns for policy. Clearly coherence at frequencies below a 4 year cycle are not consistently high, which does suggest different growth patterns between turning points. It is likely that monetary policy will be unable to respond to differences in cycles at these frequencies, although clearly these cycles are important in terms of growth dynamics, as was shown by the wavelet power spctra plots. Optimality of monetary policy in the frequency domain implies that monetary policy should be optimal for all frequency cycles for euro area member states, therefore implying high levels of coherence between cycles and zero phasing differences, which is clearly not currently the case. It is clear though that the situation has definitely improved since the inception of EMU, which in turn implies that the euro area core is likely more of an optimal currency area than it was before, suggesting a 'harder' core.

As for future developments, as synchronicity is an important issue in the timing of business cycles, ECB monetary policy could perform the function of a ‘coupler’, aligning synchronization of cycles between these countries. But ECB monetary policy will not cope with idiosyncratic developments, which could ‘decouple’ the synchronicity of business cycles – but in all cases where these could be identified in this study, the impact of these events was confined to cycles at frequencies shorter than the business cycle.

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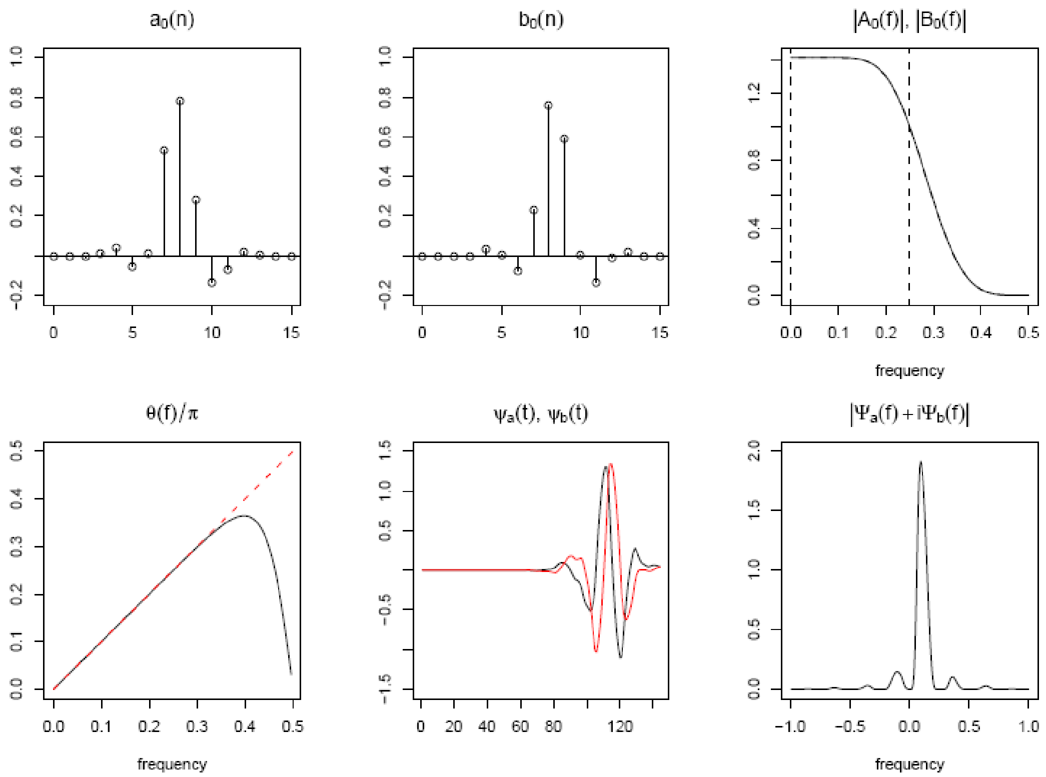
Appendix

A. Hilbert wavelet filter properties



Source: Craigmile and Whitcher (2004)

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