# BANK OF FINLAND DISCUSSION PAPERS 

$$
28 \cdot 2003
$$

Jukka Vauhkonen
Research Department
11.11.2003

## Are adverse selection models of debt robust to changes in market structure?

http://www.bof.fi

# BANK OF FINLAND DISCUSSION PAPERS 

$28 \cdot 2003$

Jukka Vauhkonen
Research Department
11.11.2003

# Are adverse selection models of debt robust to changes in market structure? 

The views expressed are those of the author and do not necessarily reflect the views of the Bank of Finland.

I thank Xavier Freixas, Vesa Kanniainen, Mikko Leppämäki, Juha-Pekka Niinimäki, Rune Stenbacka and Tuomas Takalo for very helpful comments.
http://www.bof.fi
ISBN 952-462-094-4
ISSN 0785-3572
(print)
ISBN 952-462-095-2
ISSN 1456-6184
(online)
Suomen Pankin monistuskeskus
Helsinki 2003

# Are adverse selection models of debt robust to changes in market structure? 

Bank of Finland Discussion Papers 28/2003

Jukka Vauhkonen

Research Department


#### Abstract

Many adverse selection models of standard one-period debt contracts are based on the following seemingly innocuous assumptions. First, entrepreneurs have private information about the quality of their return distributions. Second, return distributions are ordered by the monotone likelihood-ratio property. Third, financiers' payoff functions are restricted to be monotonically non-decreasing in firm profits. Fourth, financial markets are competitive. We argue that debt is not an optimal contract in these models if there is only one (monopoly) financier rather than an infinite number of competitive financiers.


Key words: security design, adverse selection, monotonic contracts, monotone likelihood ratio, first-order stochastic dominance

JEL classification numbers: D82, G35

# Ovatko haitalliseen valikoitumiseen perustuvat velkasopimusmallit robustisia markkinarakenteen muutoksiin nähden? 

Suomen Pankin keskustelualoitteita 28/2003

Jukka Vauhkonen
Tutkimusosasto

## Tiivistelmä

Useat velkasopimusten optimaalisuutta selittävät haitallisen valikoitumisen mallit perustuvat seuraaviin näennäisesti ei-kriittisiin oletuksiin. Ensiksikin vain yrittäjät tietävät investointiprojektiensa tuottojen todennäköisyysjakauman. Toiseksi jakaumat voidaan asettaa paremmuusjärjestykseen ensimmäisen asteen stokastisen dominanssin perusteella. Kolmanneksi rahoittajien tuottojen täytyy olla ei-väheneviä projektien tuottojen suhteen. Neljänneksi rahoitusmarkkinat ovat kilpailulliset. Tutkimuksessa esitetään, ettei velkasopimus ole optimaalinen näissä malleissa, jos oletus kilpailullisista rahoitusmarkkinoista korvataan oletuksella, että monopolirahoittaja on yritysten ainoa rahoituksen lähde.

Avainsanat: rahoitussopimusten teoria, käänteinen valikoituminen, monotoniset sopimukset, monotoninen uskottavuusosamäää, ensimmäisen asteen stokastinen dominanssi

JEL-luokittelu: D82, G35

## Contents

Abstract ..... 3
1 Introduction ..... 7
2 The model ..... 10
3 Profit-maximising separating contracts ..... 13
3.1 Financier's maximisation problem ..... 13
3.2 Properties of separating contracts ..... 14
3.3 Characterisation of profit-maximising separating contracts ..... 18
4 Conclusions ..... 22
Appendix 1. Proof of Lemma 6 ..... 23
Appendix 2 Proof of Proposition 3 ..... 25
References ..... 28

## 1 Introduction

One strand of the security design literature ${ }^{1}$ focuses on deriving the conditions under which the standard debt contracts are equilibrium financial contracts in adverse selection models. The main result of this literature, whose principal contributions are Innes (1993), Nachman and Noe (1994) and Wang and Williamson (1998), is that debt contracts are uniquely optimal financial contracts, when the following four key assumptions hold. First, entrepreneurs have private information about their return distributions. Second, higher quality borrowers have better return distributions in terms of the monotone likelihood ratio property. ${ }^{2}$ Third, financiers' admissible payoff functions are non-decreasing in firm return. Fourth, financial markets are competitive. In this paper, we show that debt contracts are not optimal in Wang and Williamson (1998), if the seemingly innocuous assumption of competitive financial markets is replaced by the assumption that the monopoly financier is the only source of finance. More importantly, we argue that other adverse selection models of debt, which satisfy the four key assumptions, are likely to fail in the same robustness test.

The idea that debt financing may alleviate the problem of asymmetric information between the firm and its potential financiers has its origins in the pioneering work of Myers and Majluf (1984). Myers and Majluf (1984) argue that debt financing minimises the underpricing losses from security issuance when there is asymmetric information on the value of the firm's current assets. However, as an explanation of the optimality of debt contracts their model has the shortcoming that the admissible securities consist only of debt and equity. To remedy this problem, the subsequent research has tried to generalise the results of Myers and Majluf (1984) by examining optimal security designs in asymmetric information models when contracts are endogenous.

When contracts are endogenous, the standard contract theory suggests that optimal contracts should be contingent on all relevant information. This generally implies that optimal contracts will be extremely complex (see Hart and Holmström 1987). However, most real world financial contracts are rather simple. The security design literature examines what kind of assumptions must be made, for example, on the nature of information and on the distributions of project returns to ensure that some simple contracts such as standard debt contracts are optimal. The central finding of Innes (1993), Nachman and Noe (1994) and Wang and Williamson (1998) is that only some seemingly mild and innocuous

[^0]assumptions need to be made to guarantee that standard debt contracts are optimal responses to the adverse selection problem.

Innes (1993) derives equilibrium financial contracts in competitive risk neutral capital markets, where high quality entrepreneurs have better profit distributions in the sense of the monotone likelihood property, and where financiers know the distribution of quality types in the population but not the quality of any particular borrower. The key result of Innes (1993) is that when the entrepreneur's investment size is fixed and when the financier's payoff is monotonically nondecreasing in firm profits, the equilibrium pooling contract is the standard debt contract. The signaling model of Nachman and Noe (1994) differs from the screening model of Innes (1993) in that informed borrowers try to signal their quality to financiers by their contract offers. Nachman and Noe (1994) show that the debt contract is the uniquely optimal pooling contract if and only if the cash flows of different types of borrowers are ordered by the strict conditional stochastic dominance (see ftn 2 below). Wang and Williamson (1998) introduce ex ante screening to the two quality type -version of the model of Innes (1993), and show that debt contracts are optimal separating contracts, when the monitoring technology allows financiers to commit to stochastic ex ante screening.

As our aim is to argue that the central results of the above models are not robust to a change in the market structure, we first provide a stylised illustration of the role of the key four assumptions in adverse selection models of debt.

A typical feature of adverse selection models with competition between financiers (or insurers as in Rotschild and Stiglitz 1976) is that lower quality borrowers have an incentive to mimic higher quality borrowers. In signaling models, low quality borrowers have an incentive to offer similar contracts as higher quality borrowers. In screening models, in turn, low quality borrowers have an incentive to choose the contracts directed at higher quality borrowers. Obviously, to reduce or eliminate the mimicking, financiers and higher quality borrowers try to design such contracts for high quality borrowers that are unattractive for lower quality borrowers. When project returns are ordered by the monotone likelihood ratio property, higher quality borrowers' returns are more concentrated on the upper end of the probability distribution of returns. In that case, low quality borrowers' incentives to mimic higher quality types are reduced when the contracts designed by or directed at higher quality types provide the borrower relatively low payoffs with low project returns and relatively high payoffs with high project returns. As shown by Innes (1994), Nachman and Noe (1994) and Wang and Williamson (1998), when project returns are ordered by the monotone likelihood ratio property and when the parties' payoff functions are
restricted to be monotonic ${ }^{3}$, the competitive contracts designed by or directed at higher quality types are the standard debt contracts.

It is rather easy to see that this explanation of the optimality of debt fails when the monopoly financier is the only source of finance. The objective of the monopoly financier is to extract all the expected project surpluses from borrowers. A consequence of assuming that the project distributions are ordered by the monotone likelihood ratio property is that lower quality borrowers' projects yield lower expected returns than those of higher quality borrowers. This implies that when the types of borrowers are unknown to the monopoly financier, it is high quality borrowers who have an incentive to mimic lower quality borrowers as the financier can extract lower expected surpluses from lower quality types. Obviously, to extract as much profits as possible, the monopoly financier designs the contract(s) in such a way that high quality types' benefits of mimicking lower quality types are as low as possible. As higher quality borrowers' returns are more concentrated on the upper end of the return distribution, the financier offers borrowers such monotonic contracts that provide them relatively high payoffs with low return realisations and relatively low payoffs with high return realisations. It is obvious that such contracts are very different from the standard debt contracts.

In this article, we formalise the above argument by extending the model of Wang and Williamson (1998). We show that their result of the optimality of standard debt contracts is not robust to a change in the market structure. More specifically, we show that the standard debt contracts are not optimal in their model, when the only source of finance is the monopoly financier. We utilise the model of Wang and Williamson (1998), because the intuition underlying our results is easy to see in their relatively simple model with only two types of borrowers. However, we emphasise that our argument is likely to hold also in other adverse selection -based models of debt, which are based on the above four key assumptions.

In Section 2 we present the model. In section 3 we examine the properties of separating equilibria, characterise the specific form of profit-maximising separating contracts, and compare our results to those of Wang and Williamson (1998). Section 4 concludes.

[^1]
## 2 The model

There is one major difference between our model and that of Wang and Williamson (1998). We assume that there is only one (monopoly) financier, whereas Wang and Williamson (1998) assume that there is a continuum of competive lenders. Otherwise, the models are identical except for some minor differences.

There are three types of risk-neutral agents: type g borrowers, type b borrowers, and the monopoly financier. There is a continuum [0,1] of borrowers with the share of type $g$ borrowers being $\alpha$ and the share of type $b$ borrowers being $1-\alpha$. Borrowers are endowed with an investment project which requires k units of funds to undertake, $0<\mathrm{k}<1$. Borrowers have no own funds and the only source of outside funds is the monopoly financier. If undertaken, projects yield random returns according to the distribution function $\mathrm{F}_{\mathrm{i}}(\mathrm{x})$ and the corresponding probability density function $\mathrm{f}_{\mathrm{i}}(\mathrm{x})$, where the subscript i denotes the type of a borrower, $\mathrm{i}=\mathrm{g}, \mathrm{b}$. We assume that $\mathrm{f}_{\mathrm{i}}(\mathrm{x})>0$ for $\mathrm{x} \in[0,1]$ and that $\mathrm{f}_{\mathrm{i}}(\mathrm{x})$ is continuous on $[0,1]$.

The following assumption is a typical way of ordering projects in adverse selection models.

Assumption 1. Type $g$ borrowers' projects are better than type borrowers' projects in the sense of the monotone likelihood ratio-property:

$$
\begin{equation*}
\frac{\mathrm{f}_{\mathrm{g}}(\mathrm{x})}{\mathrm{f}_{\mathrm{b}}(\mathrm{x})}<\frac{\mathrm{f}_{\mathrm{g}}(\mathrm{y})}{\mathrm{f}_{\mathrm{b}}(\mathrm{y})} ; \mathrm{x}, \mathrm{y} \in[0,1] ; \mathrm{x}<\mathrm{y} . \tag{2.1}
\end{equation*}
$$

As shown by Wang and Williamson (1998), the following property is a consequence of the above assumption.

Corollary 1. Type $g$ borrower's return distribution $F_{g}(x)$ first-order stochastically dominates type $b$ borrower's return distribution $F_{b}(x)$. Thus, for every nondecreasing function $u: \Re \rightarrow \Re$, we have (A1.1)
$\int u(x) \mathrm{dF}_{\mathrm{g}}(\mathrm{x}) \geq \int \mathrm{u}(\mathrm{x}) \mathrm{dF}_{\mathrm{b}}(\mathrm{x})$,
and (A1.2)
$\mathrm{F}_{\mathrm{g}}(\mathrm{x}) \leq \mathrm{F}_{\mathrm{b}}(\mathrm{x})$ for every x.

The first-order stochastic dominance implies, among other things, that any expected utility maximiser who prefers more over less prefers $\mathrm{F}_{\mathrm{g}}(\mathrm{x})$ to $\mathrm{F}_{\mathrm{b}}(\mathrm{x})$ and that the graph of $\mathrm{F}_{\mathrm{g}}(\mathrm{x})$ is uniformly below the graph of $\mathrm{F}_{\mathrm{b}}(\mathrm{x})$.

We assume that borrowers know their own types but the financier learns the type of a borrower only by paying a fixed cost of screening, c . The screening technology is such that it permits the financier to commit to screen only some fraction of the pool of loan applicants. A similar assumption is utilised also in Mookherjee and Png (1989) and Krasa and Villamil (1994) in the costly state verification context.

There are three types of potential (monopolistic) equilibrium configurations in our model. First, in the separating monopolistic equilibrium with a menu of contracts the financier offers a menu of two contracts, and both types of borrowers choose the contracts directed at them. In an alternative separating equilibrium, the financier offers only one contract, which is accepted only by type $g$ borrowers. The third potential equilibrium is the pooling equilibrium, where the financier offers only one contract and both types of borrowers accept that contract.

The time line of the model in the most interesting case, where the financier offers a menu of two contracts, and both types of borrowers choose the contracts directed at them is the following ${ }^{4}$


At date 0 , the financier announces the available contracts. The contracts are denoted by the pairs $\left(\mathrm{P}_{\mathrm{i}}(\mathrm{x}), \pi_{\mathrm{i}}\right), \mathrm{i}=\mathrm{g}, \mathrm{b}$, where $\mathrm{P}_{\mathrm{i}}(\mathrm{x})$ denotes type i borrower's payoff as a function of the project return x and where $\pi_{\mathrm{i}}, \mathrm{i}=\mathrm{g}, \mathrm{b}$, denotes the probability that the borrower who announces to be of type $i$ is screened.

We constrain the payoff functions to satisfy the following critical assumption which is standard in adverse selection models of debt.

Assumption 2. Borrowers' and the financier's payoff functions are constrained to be monotonically nondecreasing in the project return $x$ :

$$
x \leq y \Rightarrow P_{i}(x) \leq P_{i}(y) \text { and } x-P_{i}(x) \leq y-P_{i}(y) ; x, y \in[0,1], i=g, b
$$

[^2]One motivation for the use of the monotonicity constraint is provided by Innes (1993, p. 30). He assumes that the borrower may be able to sabotage the firm expost. That is, after observing a perfect signal of firm profits, the borrower may be in a position to sabotage the firm by burning as much of the profits as he chooses. Then, the borrower would choose to burn profits in any decreasing segment of their payoff function and a non-monotonic contract would never be chosen. ${ }^{5}$

At date 1, borrowers announce their types, that is, they inform the financier which of the two contracts they would prefer to choose.

At date 2, the financier screens the borrower who claims that he is of type i with probability $\pi_{\mathrm{i}}$. We assume that the financier can commit to punish a borrower who claims to be of type i but who turns out to be the other type by refusing to lend to him ${ }^{6}$. The financier obviously wants type i borrower to choose a type i contract. The role of stochastic screening is to induce borrowers to self-select and thus to choose the contracts targeted at them at the lowest possible screening costs.

At date 3 , the contracts are signed and the projects are started. At date 4 , the returns are realised and the payoffs divided between the parties.

The time line highlights two important features of our model. First, contracts are signed before the projects are screened, which is in contrast with some of the literature with ex ante screening (eg Broecker 1990). Second, screening takes place before the projects are executed, not after project execution as in the costly state verification models (Townsend 1979, Gale and Hellwig 1985). We argue that this timing structure, although not typical in the literature, is reasonable in an environment, where borrowers know their own types, where the projects of both types of borrowers are socially efficient and where the financier can commit to stochastic, perfect and costly ex-ante screening.

Finally, we make a simplifying restriction on the parameter values, which allows us to restrict our attention on the self-selective separating contracts. In the following assumption, parameters $\mu_{\mathrm{b}}$, c and $\overline{\mathrm{u}}$ denote, respectively, the mean investment returns for type $b$ borrowers, a fixed and strictly positive screening cost and a fixed and strictly positive reservation utility level of borrowers.

[^3]Assumption 3. $c<\min \left[\mu_{b}-\bar{u}-k, \alpha \bar{u} /(1-\alpha)\right]$,

The requirement that c is lower than the first term in the parentheses requires that the mean investment returns for type b borrowers satisfy $\mu_{\mathrm{b}} \geq \mathrm{k}+\mathrm{c}+\overline{\mathrm{u}}$. This condition implies that both types of projects are socially profitable. The requirement that c is lower than the second term in the parentheses is a technical assumption, which greatly simplifies our analysis. This assumption requires that the cost of screening must be sufficiently low, or, alternatively, that the share of type g borrowers, $\alpha$, is sufficiently high. Besides simplifying our analysis, the satisfaction of this assumption guarantees that the menu of separating contracts, which is our main interest, always yields the financier higher profits than the alternative contracts. ${ }^{7}$

## 3 Profit-maximising separating contracts

In this section, we derive the profit-maximising menu of separating contracts under the assumption that the monopoly financier is the only source of finance. We show that these profit-maximising contracts are very different from the standard debt contracts, which, as shown by Wang and Williamson (1998), are optimal in an otherwise similar model but with a large number of lenders. In Appendix, we characterise the properties of the profit-maximising separating contracts that attract only one type of borrowers and pooling contracts. In Proposition 3 below, we also show that the profit-maximising menu of separating contracts provides the financier bigger profits than the other alternatives provided that the screening cost is sufficiently low.

### 3.1 Financier's maximisation problem

By separating equilibrium with a menu of contracts we mean a pair of contracts $\left(\mathrm{P}_{\mathrm{i}}(\mathrm{x}), \pi_{\mathrm{i}}\right), \mathrm{i}=\mathrm{g}, \mathrm{b}$, which is a solution to the following constrained minimisation problem

$$
\begin{equation*}
\operatorname{Min}_{\left[\left(\mathrm{P}_{\mathrm{g}}(\mathrm{x}), \pi_{\mathrm{g}}\right),\left(\mathrm{P}_{\mathrm{b}}(\mathrm{x}), \pi_{\mathrm{b}}\right)\right]}\left\{\alpha\left[\int_{0}^{1} \mathrm{P}_{\mathrm{g}}(\mathrm{x}) \mathrm{dF}_{\mathrm{g}}(\mathrm{x})+\pi_{\mathrm{g}} \mathrm{c}\right]+(1-\alpha)\left[\int_{0}^{1} \mathrm{P}_{\mathrm{b}}(\mathrm{x}) \mathrm{dF}_{\mathrm{b}}(\mathrm{x})+\pi_{\mathrm{b}} \mathrm{c}\right]\right\} \tag{3.1}
\end{equation*}
$$

[^4]s.t.
\[

$$
\begin{align*}
& 0 \leq P_{i}(x) \leq x, x \in[0,1] ; \quad i=g, b .  \tag{3.2}\\
& x \leq y \Rightarrow P_{i}(x) \leq P_{i}(y) ; \quad x, y \in[0,1] ; i=g, b .  \tag{3.3}\\
& x \leq y \Rightarrow x-P_{i}(x) \leq y-P_{i}(y) ; \quad x, y \in[0,1],  \tag{3.4}\\
& \int_{0}^{1} P_{i}(x) d F_{i}(x) \geq\left(1-\pi_{j}\right) \int_{0}^{1} P_{j}(x) d F_{i}(x) ; \quad i, j=g, b ; j \neq i  \tag{3.5}\\
& \int_{0}^{1} P_{i}(x) d F_{i}(x) \geq \bar{u} ; \quad i=g, b . \tag{3.6}
\end{align*}
$$
\]

According to (3.1), the financier's profits are maximised when the sum of the borrowers' expected payoffs and the screening costs is minimised. Condition (3.2) is the limited liability constraint. Conditions (3.3) and (3.4) restate our Assumption 2 of the monotonicity requirements for borrowers' and the financier's payoff functions.

Conditions (3.5) and (3.6) are the standard incentive compatibility and the participation constraints, respectively. According to incentive compatibility constraints (3.5), type i borrower must receive higher expected payoffs when he truthfully reveals his type than the expected payoff he would receive if he falsified his type. The left hand side of (3.5) denotes borrower i's expected payoffs when he reports his true type. The right hand side denotes the expected payoffs when he cheats. With probability $1-\pi_{j}$, the cheating is not detected and he receives payments according to type j 's payoff function. With probability $\pi_{\mathrm{j}}$ cheating is detected and the project is not funded, in which case his payoff is zero. Participation constraints (3.6), in turn, state that the expected payoffs must be at least as large as the strictly positive reservation utility $\overline{\mathrm{u}}$.

### 3.2 Properties of separating contracts

In this section, we derive a number of lemmas that help us to solve the optimisation problem (3.1) subject to constraints (3.2)-(3.6). We first show that the separating contracts have the following properties. First, the participation constraints bind for both type $g$ and type b borrowers. Second, only type $g$ borrowers' incentive compatibility constraints bind. Third, only the borrowers who claim to be of type b are screened with a positive probability. These properties are established by the ensuing lemmas.

First, utilising the auxiliary Lemmas 1 and 2 we show in Lemma 3 that the type g borrower's incentive compatibility constraint (3.5) binds.

Lemma 1. In a separating equilibrium, if condition (3.5) is a strict inequality for $i$, then $\pi_{j}$ is zero.

Proof: Suppose not. In other words, suppose that (3.5) is a strict inequality for i and $\pi_{\mathrm{j}}>0$. Then, the financier can offer an alternative contract $\left(\mathrm{P}_{\mathrm{j}}^{*}(\mathrm{x}), \pi_{\mathrm{j}}^{*}\right)$ with $\mathrm{P}_{\mathrm{j}}^{*}(\mathrm{x})=\mathrm{P}_{\mathrm{j}}(\mathrm{x}), \pi_{\mathrm{j}}^{*}=\delta \pi_{\mathrm{j}}$, and $0<\delta<1$ for type i . This alternative separating contract reduces the costs of screening while the incentive compatibility and participation constraints remain unbinding. Therefore, in a separating equilibrium, if condition (3.5) is a strict inequality for $i$, then $\pi_{j}$ is zero. QED

The intuition underlying this lemma is clear. If condition (3.5) is a strict inequality and $\pi_{\mathrm{j}}$ is positive, then the financier can increase her profits by lowering the screening probability $\pi_{\mathrm{j}}$. If the decrease in $\pi_{\mathrm{j}}$ is small enough, the incentive compatibility condition for type i remains unbinding. The same argument holds for any $\pi_{\mathrm{j}}>0$. Thus, if (3.5) is a strict inequality for i , then $\pi_{\mathrm{j}}$ must be zero.

Lemma 2. In a separating equilibrium, if (3.5) is a strict inequality for $i$, then (3.6) must be an equality for $i$.

Proof: Suppose that (3.5) and (3.6) are both inequalities for i. In that case, by Lemma 1, (3.5) can be written as $\int_{0}^{1} \mathrm{P}_{\mathrm{i}}(\mathrm{x}) \mathrm{dF}_{\mathrm{i}}(\mathrm{x})>\int_{0}^{1} \mathrm{P}_{\mathrm{j}}(\mathrm{x}) \mathrm{dF}_{\mathrm{i}}(\mathrm{x})$. Now, the financier can offer type i an alternative separating contract $\left(\mathrm{P}_{\mathrm{i}}^{*}(\mathrm{x}), \pi_{\mathrm{i}}^{*}\right)$ with $P_{i}^{*}(x) \leq P_{i}(x), x \in[0,1]$ with strict inequality for some $x \in[0,1]$ and $\pi_{j}^{*}=\pi_{j}$. This alternative contract increases the financier's profits without violating the incentive compatibility and participation constraints. Thus, both (3.5) and (3.6) cannot be strict inequalities for i. QED

If both (3.5) and (3.6) are strict inequalities for i , then the financier can increase her profits by lowering payoffs to borrower i. The financier can lower the payoffs down to the point where either (3.5) or (3.6) becomes binding without violating the separating equilibrium conditions (3.2)-(3.6).

Lemma 3. In a separating equilibrium, the incentive compatibility constraint (3.5) is binding for $i=g$.

Proof: Suppose not, ie suppose that (3.5) is a strict inequality for $\mathrm{i}=\mathrm{g}$. Then, $\int_{0}^{1} \mathrm{P}_{\mathrm{g}}(\mathrm{x}) \mathrm{dF}_{\mathrm{g}}(\mathrm{x})>\int_{0}^{1} \mathrm{P}_{\mathrm{b}}(\mathrm{x}) \mathrm{dF}_{\mathrm{g}}(\mathrm{x})>\int_{0}^{1} \mathrm{P}_{\mathrm{b}}(\mathrm{x}) \mathrm{dF}_{\mathrm{b}}(\mathrm{x}) \geq \overline{\mathrm{u}}$. The first of these inequalities follows from Lemma 1. The second inequality is due to the monotonicity of $\mathrm{P}_{\mathrm{b}}(\mathrm{x})$ and the first-order stochastic dominance. The third inequality follows from the participation constraint of type $b$ borrowers. On the other hand, by Lemma 2, condition (3.6) can now be written as $\int_{0}^{1} \mathrm{P}_{\mathrm{g}}(\mathrm{x}) \mathrm{dF}_{\mathrm{g}}(\mathrm{x})=\overline{\mathrm{u}}$. This is a contradiction. QED

Thus, in a separating equilibrium type $g$ borrowers are indifferent between type $g$ and type b contracts. This is a standard result in adverse selection models: at least one type's incentive compatibility constraint is binding in the separating equilibrium.

Next, we show that in a separating equilibrium with a menu of contracts, borrowers claiming to be of type $b$ are screened with positive probability and that the participation constraints bind for both types of borrowers.

Lemma 4. In a separating equilibrium, $\pi_{b}>0$.

Proof: Suppose not. In other words, suppose that $\pi_{\mathrm{b}}=0$ and $\pi_{\mathrm{g}}>0$ (in a separating equilibrium with a menu of contracts at least one of the screening probabilities is positive). By Lemma 3, condition (3.5) for $\mathrm{i}=\mathrm{g}$ can now be written as $\int_{0}^{1} \mathrm{P}_{\mathrm{g}}(\mathrm{x}) \mathrm{dF}_{\mathrm{g}}(\mathrm{x})=\int_{0}^{1} \mathrm{P}_{\mathrm{b}}(\mathrm{x}) \mathrm{dF}_{\mathrm{g}}(\mathrm{x})$. Consider now an alternative pooling contract $\mathrm{P}^{*}(\mathrm{x})=\mathrm{P}_{\mathrm{b}}(\mathrm{x})$. The financier earns higher profits with this pooling contract, since borrowers' payoffs remain the same while the screening costs reduce to zero. Thus, $\pi_{\mathrm{b}}$ cannot be zero in equilibrium. QED

The above Lemma states that the borrowers claiming to be type $b$ are screened with positive probability in a separating equilibrium. In a separating equilibrium the financier uses its screening technology and the associated penalties (the financier declines to fund the project) to prevent type $g$ borrowers from pretending to be type $b$ borrowers. Without screening a positive proportion of type $b$ borrowers it is impossible to achieve self-selection.

Lemma 5. In a separating equilibrium, the participation constraint (3.6) is binding for $i=b$.

Proof: Consider first the case where information is symmetric. With symmetric information the financier offers contracts that provide borrowers only their reservation utility in expected value. That is, the payment schedules $\mathrm{P}_{\mathrm{g}}(\mathrm{x})$ and
$\mathrm{P}_{\mathrm{b}}(\mathrm{x})$ satisfy the participation constraints with equality: $\int_{0}^{1} \mathrm{P}_{\mathrm{g}}(\mathrm{x}) \mathrm{dF}_{\mathrm{g}}(\mathrm{x})=\int_{0}^{1} \mathrm{P}_{\mathrm{b}}(\mathrm{x}) \mathrm{dF}_{\mathrm{b}}(\mathrm{x})=\overline{\mathrm{u}}$. With asymmetric information but without screening these symmetric information contracts are not incentive compatible: type $g$ borrowers would choose type b contracts and earn expected payoffs in excess of their reservation utility. Thus, the financier has to use its screening technology to induce type $g$ borrowers to choose type $g$ contracts. The financier's objective is to induce self-selection with as small screening costs as possible. To minimise the screening costs, the financier has to make type b contracts as unattractive as possible from the point of view of type $g$ borrowers. It is obvious that the least attractive b type contracts must satisfy (3.6) as an equality. QED

The above proposition that the participation constraint binds for the lowest quality type is usual in adverse selection models. Next we establish a more interesting result. Given our parameter restriction (Assumption 3) the participation constraint binds also for $\mathrm{i}=\mathrm{g}$. Thus, provided that the monitoring cost is sufficiently low, the financier is able to extract all expected project surpluses in excess of reservation level from both types of borrowers.

Lemma 6. In a separating equilibrium, the participation constraint (3.6) is binding for $i=g$.

Proof: See Appendix.
Lemma 7. In a separating equilibrium, $\pi_{g}=0$.
Proof: By Lemma 6, in a separating equilibrium $\int_{0}^{1} \mathrm{P}_{\mathrm{g}}(\mathrm{x}) \mathrm{dF}_{\mathrm{g}}(\mathrm{x})=\overline{\mathrm{u}}$. On the other hand, by the first-order stochastic dominance we know that $\int_{0}^{1} \mathrm{P}_{\mathrm{g}}(\mathrm{x}) \mathrm{dF}_{\mathrm{g}}(\mathrm{x})>\int_{0}^{1} \mathrm{P}_{\mathrm{g}}(\mathrm{x}) \mathrm{dF}_{\mathrm{b}}(\mathrm{x})$. Therefore, by pretending to be type g borrowers type b borrowers would not reach their reservation utility in expected value. As a result, borrowers claiming to be of type $g$ need not be screened, since they indeed are type $g$ borrowers with certainty. QED

Lemma 8. In a separating equilibrium, the incentive compatibility constraint (3.5) is not binding for $i=b$.

Proof: Suppose that the incentive compatibility constraint (3.5) is binding for $\mathrm{i}=\mathrm{b}$. Then, $\int_{0}^{1} \mathrm{P}_{\mathrm{b}}(\mathrm{x}) \mathrm{dF}_{\mathrm{b}}(\mathrm{x})=\int_{0}^{1} \mathrm{P}_{\mathrm{g}}(\mathrm{x}) \mathrm{dF}_{\mathrm{b}}<\int_{0}^{1} \mathrm{P}_{\mathrm{g}}(\mathrm{x}) \mathrm{dF}_{\mathrm{g}}(\mathrm{x})=\overline{\mathrm{u}}$. The first equality follows from the incentive compatibility constraint (3.5) and Lemma 7. The inequality follows from the definition of the first-order stochastic dominance, and
the latter equality from Lemma 6 . The above chain of equalities and inequalities contradicts the participation constraint (3.6) for $i=b$. Consequently, (3.5) cannot be an equality for $\mathrm{i}=\mathrm{b}$ in a separating equilibrium. QED

Lemma 7 is in concert with the findings of De Meza and Webb (1988) and Wang and Williamson (1998). Also in their models only one type of borrowers is screened. The result of Lemma 8 that the incentive compatibility constraint does not bind for the lowest quality borrowers is a typical feature of adverse selection models.

### 3.3 Characterisation of profit-maximising separating contracts

In this section we study the structure of profit-maximising separating contracts. First we derive the profit-maximising contract forms for type b borrowers and then for type $g$ borrowers.

The financier's optimisation problem can be simplified by utilising the results of the previous section. First, since the participation constraints bind for both types of borrowers (Lemmas 5 and 6), the financier's expected profits from good and bad borrowers are essentially fixed at $\alpha\left(\mu_{\mathrm{g}}-\overline{\mathrm{u}}\right)$ and $(1-\alpha)\left(\mu_{\mathrm{b}}-\overline{\mathrm{u}}\right)$. Second, by Lemma $7, \pi_{\mathrm{g}}=0$. Using these auxiliary results, the financier's optimisation problem (3.1) reduces to a problem of minimising the screening costs $(1-\alpha) \pi_{\mathrm{b}} \mathrm{c}$ subject to binding participation constraints of both types of borrowers and to the binding incentive compatibility constraint of type g borrowers (Lemma 8). Thus, the profit-maximising separating contract for type b is the solution to the following problem:
$\operatorname{Max}_{P_{b}(x), \pi_{b}}\left(-\pi_{b}\right)$
subject to

$$
\begin{equation*}
\int_{0}^{1} \mathrm{P}_{\mathrm{b}}(\mathrm{x}) \mathrm{dF}_{\mathrm{b}}(\mathrm{x})=\overline{\mathrm{u}}, \tag{3.7}
\end{equation*}
$$

$$
\begin{equation*}
\overline{\mathrm{u}}=\left(1-\pi_{\mathrm{b}}\right) \int_{0}^{1} \mathrm{P}_{\mathrm{b}}(\mathrm{x}) \mathrm{dF}_{\mathrm{g}}(\mathrm{x}) . \tag{3.8}
\end{equation*}
$$

Proposition 1. A unique profit-maximising contract directed at type $b$ is an 'inverse debt' contract with $P_{b}(x)=x, x \in\left[0, \bar{P}_{b}\right] ; P_{b}(x)=\bar{P}_{b}, x \in\left[\bar{P}_{b}, 1\right]$ for some $\bar{P}_{b} \in(0,1)$.

Proof: Formulate the Lagrangean of the maximisation problem above:

$$
\begin{equation*}
\mathrm{L}=-\pi_{\mathrm{b}}+\lambda_{1}\left[\overline{\mathrm{u}}-\int_{0}^{1} \mathrm{P}_{\mathrm{b}}(\mathrm{x}) \mathrm{f}_{\mathrm{b}}(\mathrm{x}) \mathrm{dx}\right]+\lambda_{2}\left[\left(1-\pi_{\mathrm{b}}\right) \int_{0}^{1} \mathrm{P}_{\mathrm{b}}(\mathrm{x}) \mathrm{f}_{\mathrm{g}}(\mathrm{x}) \mathrm{dx}-\overline{\mathrm{u}}\right] . \tag{3.9}
\end{equation*}
$$

We maximise $L$ with respect to $\pi_{\mathrm{b}}, \mathrm{P}_{\mathrm{b}}(\mathrm{x})$ and $\lambda_{\mathrm{i}} \neq 0, \mathrm{i}=1,2$, subject to feasibility and monotonicity constraints (3.2), (3.3) and (3.4). We immediately notice that the first-order condition with respect to $\pi_{\mathrm{b}}$ implies that $\lambda_{2}<0$. In order to derive more results, we rewrite (3.9) as
$\mathrm{L}=-\pi_{\mathrm{b}}+\left(\lambda_{1}-\lambda_{2}\right) \overline{\mathrm{u}}+\int_{0}^{1}\left[\frac{\mathrm{f}_{\mathrm{g}}(\mathrm{x})}{\mathrm{f}_{\mathrm{b}}(\mathrm{x})}\left(1-\pi_{\mathrm{b}}\right)-\frac{\lambda_{1}}{\lambda_{2}}\right] \lambda_{2} \mathrm{f}_{\mathrm{b}}(\mathrm{x}) \mathrm{P}_{\mathrm{b}}(\mathrm{x}) \mathrm{dx}$

It can be shown that the other Lagrange constraint is also negative. Suppose conversely that $\lambda_{1}>0$. Then, the term inside the square brackets in (3.10) is positive. This, in turn, implies that $\mathrm{P}_{\mathrm{b}}(\mathrm{x})=0$ is optimal for all $\mathrm{x} \in[0,1]$. This cannot be true, since setting $\mathrm{P}_{\mathrm{b}}(\mathrm{x})=0$ would violate (3.7). Consequently, both Lagrange constraints must be negative. Now, examine the term in the square brackets. Given that $\lambda_{1}, \lambda_{2}<0$ there must exist some values of $x \in[0,1]$ for which this term is negative. Otherwise, $\mathrm{P}_{\mathrm{b}}(\mathrm{x})=0$ is optimal for all $\mathrm{x} \in[0,1]$. Likewise, there must exist some values of $\mathrm{x} \in[0,1]$ for which the term in square brackets is positive. Otherwise, $\mathrm{P}_{\mathrm{b}}(\mathrm{x})=\mathrm{x}$ would maximize L for all $\mathrm{x} \in[0,1]$. This is impossible, since it would violate condition (3.7). Because the term in the square brackets is continuous and increasing in $x$, there must exist a value $\bar{x} \in(0,1)$ for which this term is equal to zero. For $\mathrm{x} \in[0, \overline{\mathrm{x}})$, the term in square brackets is negative and for $\mathrm{x} \in(\overline{\mathrm{x}}, 1]$ it is positive. Now, given the feasibility and monotonicity constraints (3.2) and (3.3), we immediately see that the optimal payoff schedule $\mathrm{P}_{\mathrm{b}}(\mathrm{x})$ is $\mathrm{P}_{\mathrm{b}}(\mathrm{x})=\mathrm{x}, \mathrm{x} \in[0, \overline{\mathrm{x}}) ; \mathrm{P}_{\mathrm{b}}(\mathrm{x})=\mathrm{P}_{\mathrm{b}}(\overline{\mathrm{x}}), \mathrm{x} \in[\overline{\mathrm{x}}, 1]$. Denote $\mathrm{P}_{\mathrm{b}}(\overline{\mathrm{x}})$ by $\overline{\mathrm{P}}_{\mathrm{b}}$. This completes the proof. QED

By Proposition 1, the profit-maximising separating contract for type b can be characterised by the pair ( $\overline{\mathrm{P}}_{\mathrm{b}}, \pi_{\mathrm{b}}$ ), where $\overline{\mathrm{P}}_{\mathrm{b}}$ corresponds to the gross loan interest rate in standard loan contracts; only now this payment accrues to a borrower instead of a financier. Using (3.7) and (3.8) $\overline{\mathrm{P}}_{\mathrm{b}}$ and $\pi_{\mathrm{b}}$ are determined by the following two equations.

$$
\begin{equation*}
\int_{0}^{\overline{\mathrm{P}}_{\mathrm{b}}} \mathrm{xdF}_{\mathrm{b}}(\mathrm{x})+\overline{\mathrm{P}}_{\mathrm{b}}\left[1-\mathrm{F}_{\mathrm{b}}\left(\overline{\mathrm{P}}_{\mathrm{b}}\right)\right]=\overline{\mathrm{u}}, \tag{3.11}
\end{equation*}
$$

$\overline{\mathrm{u}}=\left(1-\pi_{\mathrm{b}}\right)\left\{\int_{0}^{\overline{\mathrm{P}}_{\mathrm{b}}} \mathrm{xdF}_{\mathrm{g}}(\mathrm{x})+\overline{\mathrm{P}}_{\mathrm{b}}\left[1-\mathrm{F}_{\mathrm{g}}\left(\overline{\mathrm{P}}_{\mathrm{b}}\right)\right]\right\}$.

Simplify the above equations by integrating by parts.
$\overline{\mathrm{P}}_{\mathrm{b}}-\int_{0}^{\overline{\bar{P}}_{\mathrm{b}}} \mathrm{F}_{\mathrm{b}}(\mathrm{x}) \mathrm{dx}=\overline{\mathrm{u}}$
$\overline{\mathrm{u}}=\left(1-\pi_{\mathrm{b}}\right)\left[\overline{\mathrm{P}}_{\mathrm{b}}-\int_{0}^{\overline{\mathrm{P}}_{\mathrm{b}}} \mathrm{F}_{\mathrm{g}}(\mathrm{x}) \mathrm{dx}\right]$
It is easy to show that there exists a unique pair $\left(\overline{\mathrm{P}}_{\mathrm{b}}, \pi_{\mathrm{b}}\right)$, which solves (3.13) and (3.14). Since (3.13) is continuous and increasing in $\bar{P}_{b}$, there exists a unique $\bar{P}_{b}$ solving (3.13). Inserting this unique $\overline{\mathrm{P}}_{\mathrm{b}}$ into (3.14) and solving for $\pi_{\mathrm{b}}$ yields a unique solution for $\pi_{\mathrm{b}}$.

Proposition 1 establishes a striking result that the profit-maximising contract for type $b$ is a mirror image of the standard debt contract. This result is close to the result of Boyd and Smith (1993). They examine a model with both adverse selection and costly state verification. They show that in the absence of costly state verification (only with an adverse selection problem), the equilibrium cannot have all contracts be debt contracts.

By Proposition 1, the result of Wang and Williamson (1998) of the optimality of debt is not robust to a change in the market structure. Furthermore, the models of Innes (1993) and Nachman and Noe (1994) are also likely to fail in the same robustness test, as their results are based on the same assumptions and mechanisms as Wang and Williamson (1998).

An intuition of the optimality of the inverse debt contract for type $b$ borrowers is the following. Inverse debt is the contract that provides borrowers the lowest payoffs in high-profit states among the contracts that satisfy the feasibility and the monotonicity constraints and the participation constraint of type $b$. As high quality borrowers have more probability weight in high-profit states than low-quality borrowers, the inverse debt contract is the least attractive feasible and monotonic type b contract for type g borrowers. Therefore, the inverse debt contract induces as much self-selection as possible by type $g$ borrowers and minimises the screening probability $\pi_{\mathrm{b}}$ that is needed to induce type g borrowers to truthfully reveal their types.

The next proposition characterises the properties of profit-maximising contracts for type $g$.

Proposition 2. Any feasible and monotonic contract that satisfies type g's participation constraint (3.6) with equality is the profit-maximising contract directed at type $g$.

Proof: By Lemma 6, the participation constraint (3.6) is binding for type $g$ borrowers in the separating equilibrium. In addition, any feasible and monotonic contract that satisfies (3.6) with equality is unattractive for type b borrowers (see the proof of Lemma 7). Thus, any feasible and monotonic contract that satisfies type g's participation constraint maximises the financier's expected returns from type $g$ borrowers and is unattractive for type b. QED

According to Proposition 2, both debt contracts and inverse debt contracts are among the continuum of optimal contracts for type $g$.

So far we have only shown that the contracts characterised in Propositions 1 and 2 are the profit-maximising separating contracts when both types of borrowers accept the contracts directed at them. However, there are potentially other separating contracts and pooling contracts that may yield the financier higher profits. First, there may exist a separating equilibrium, where the financier offers a single contract that attracts only one type of borrowers. Second, there may exist a pooling equilibrium, where the financier offers a single contract that attracts both types of borrowers. The next proposition shows that focusing on a menu of separating contracts implies no loss of generality as long as the monitoring cost is sufficiently low.

Proposition 3. When assumption 3 holds, the monopoly financier earns bigger profits by offering a тепи of separating (inverse debt) contracts rather than a pooling contract that attracts both types of borrowers or a separating contract that attracts only one type of borrowers.

Proof: See Appendix.

## 4 Conclusions

In this paper, we derive the profit-maximising financial contracts in an environment, where (1) there is one financier and many borrowers, (2) investment opportunities are heterogeneous, differing in their probability distribution of returns, (3) type $g$ borrowers' profit distribution is better than that of type $b$ borrowers in terms of the monotone likelihood ratio -property, (4) borrowers know their own types, (5) the net social value of all types of projects is positive, (6) the type of a firm can be learned only by paying a fixed ex-ante screening cost, (7) the screening technology permits commitment to stochastic ex-ante screening, (8) the financier can commit to punish borrowers found guilty of falsifying their types by denying a loan and (9) project returns are costlessly verifiable.

The main result of this inquiry is the following. When higher quality borrowers projects are better in the sense of the monotone likelihood property, the profit-maximising contracts are 'inverse debt' contracts. This finding is in contrast with the results of Wang and Williamson (1998), who establish that the standard debt is an optimal contract in an otherwise identical model but with a large number of small and independent financiers. Our results cast doubt on the robustness of the results of Wang and Williamson (1998), since in reality debt seems to be a prevalent contract form irrespective of the degree of competition between financiers.

Furthermore, we argue that also some other adverse selection -based models of debt may not robust to a change in the market structure (such as Innes 1993 and Nachman and Noe 1994). These models are based on the same key assumptions as Wang and Williamson (1998), and, thus, the role of debt is similar. Therefore, these models are likely to fail in the robustness test that is proposed in this article.

## Appendix 1

## Proof of Lemma 6

Lemma 6. In a separating equilibrium, the participation constraint (3.6) is binding for $i=g$.

Proof: With the help of Lemma 3 and Assumption 3, we show that the financier's profits are always strictly decreasing with respect to good borrowers' payoffs. Then, it is optimal for the financier to lower good borrowers' expected payoffs to a level where their participation constraint holds with an equality.

By Lemma 3, from the binding incentive compatibility constraint for type $g$ we get $\pi_{\mathrm{b}}=1-\left(\int_{0}^{1} \mathrm{P}_{\mathrm{g}}(\mathrm{x}) \mathrm{dF}_{\mathrm{g}}(\mathrm{x}) /\left(\int_{0}^{1} \mathrm{P}_{\mathrm{b}}(\mathrm{x}) \mathrm{dF}_{\mathrm{g}}(\mathrm{x})\right)\right)$. Insert this formula into the financier's profit function and denote $\int_{0}^{1} \mathrm{P}_{\mathrm{g}}(\mathrm{x}) \mathrm{dF}_{\mathrm{g}}(\mathrm{x})=\mathrm{EP}_{\mathrm{g}}$. Taking Lemma 5 also into consideration, the financier's profits can now be written as

$$
\begin{align*}
\Pi_{\mathrm{B}}= & \alpha\left(\mu_{\mathrm{g}}-E P_{\mathrm{g}}\right)+(1-\alpha)\left(\mu_{\mathrm{b}}-\overline{\mathrm{u}}\right) \\
& -\left(1-\frac{E P_{\mathrm{g}}}{\int_{0}^{1} \mathrm{P}_{\mathrm{b}}(\mathrm{x}) \mathrm{dF}_{\mathrm{g}}(\mathrm{x})}\right) \mathrm{c}(1-\alpha)-\alpha c \pi_{\mathrm{g}}\left(E P_{\mathrm{g}}\right)-1 . \tag{A1.1}
\end{align*}
$$

The first term in (A1.1) denotes the financier's income from type g borrowers' projects and the second term from type b borrowers' projects. The third term denotes the costs of screening type b borrowers. The fourth term refers to the costs of screening type $g$ borrowers. At this stage of analysis, we don't know whether $\pi_{\mathrm{g}}$ is zero or positive in a separating equilibrium. However, we do know that the derivative of $\pi_{\mathrm{g}}$ with respect to $\mathrm{EP}_{\mathrm{g}}$ is non-negative: as the expected payoffs to type g borrowers increase, the type g contracts become more attractive for type b borrowers. This makes the derivative non-negative: as $\mathrm{EP}_{\mathrm{g}}$ increases, the screening costs cannot decrease.

To analyse how the financier's profits behave when she lowers type g borrowers' payoffs, we take the derivative of (A1.1) with respect to $\mathrm{EP}_{\mathrm{g}}$.
$\frac{\partial \Pi_{B}}{\partial \mathrm{EP}_{\mathrm{g}}}=-\alpha+\frac{\mathrm{c}(1-\alpha)}{\int_{0}^{1} \mathrm{P}_{\mathrm{b}}(\mathrm{x}) \mathrm{dF}_{\mathrm{g}}(\mathrm{x})}-\alpha \mathrm{c} \pi_{\mathrm{g}}^{\prime}\left(\mathrm{EP}_{\mathrm{g}}\right)$.

The sign of this derivative is negative everywhere, if

$$
\begin{equation*}
\mathrm{c}<\frac{\alpha \int_{0}^{1} \mathrm{P}_{\mathrm{b}}(\mathrm{x}) \mathrm{dF}_{\mathrm{g}}(\mathrm{x})}{(1-\alpha)-\alpha \pi_{\mathrm{g}}^{\prime}\left(E P_{\mathrm{g}}\right) \int_{0}^{1} \mathrm{P}_{\mathrm{b}}(\mathrm{x}) \mathrm{dF}_{\mathrm{g}}(\mathrm{x})} . \tag{A1.3}
\end{equation*}
$$

By Assumption 3 and because $\pi_{g}^{\prime}\left(\mathrm{EP}_{\mathrm{g}}\right) \geq 0$, this condition is always satisfied. Thus, since the financier's profits are decreasing in good borrower's expected payoffs, she has an incentive to lower these expected payoffs to the point where good borrower's participation constraint is satisfied with an equality. QED

## Appendix 2

## Proof of Proposition 3

Proposition 3. When assumption 3 holds, the monopoly financier earns bigger profits by offering a menu of separating (inverse debt) contracts rather than a pooling contract or a separating contract that attracts only one type of borrowers.

Proof: We first derive the forms of profit-maximising pooling contracts and separating contracts that attract only one type of borrowers. Then, we show that the separating menu of contracts, characterised in Proposition 1, equation (3.12) and (3.13) and Proposition 2, provides the financier bigger profits than any separating contract that attracts only one type of borrowers or any pooling contract.

## Profit-maximising single separating contracts

Instead of the menu of two different separating contracts the financier can alternatively offer only a single contract that attracts only type $g$ borrowers. The optimal form of such contract follows from the following basic observations. First, any contract directed at type $g,\left(P_{g}(x), 0\right)$, where $P_{g}(x)$ is some monotonic payoff schedule such that $\int_{0}^{1} \mathrm{P}_{\mathrm{g}}(\mathrm{x}) \mathrm{dF}_{\mathrm{g}}(\mathrm{x})=\overline{\mathrm{u}}$, is unattractive for type b borrowers, since, by the definition of the first-order stochastic dominance, $\int_{0}^{1} \mathrm{P}_{\mathrm{g}}(\mathrm{x}) \mathrm{dF}_{\mathrm{b}}(\mathrm{x})<\int_{0}^{1} \mathrm{P}_{\mathrm{g}}(\mathrm{x}) \mathrm{dF}_{\mathrm{g}}(\mathrm{x})=\overline{\mathrm{u}}$. Second, any contract with $\int_{0}^{1} \mathrm{P}_{\mathrm{g}}(\mathrm{x}) \mathrm{dF}_{\mathrm{g}}(\mathrm{x})=\overline{\mathrm{u}}$ maximises the financier's expected profits among the single separating contracts, as the financier extracts all the surplus from type $g$ borrowers. The following result is a consequence of these observations.

Proposition A1. Any contract $\left(P_{g}(x), 0\right)$ with a monotonic payoff schedule $P_{g}(x)$ satisfying $\int_{0}^{1} \mathrm{P}_{\mathrm{g}}(\mathrm{x}) \mathrm{dF}_{\mathrm{g}}(\mathrm{x})=\overline{\mathrm{u}}$ is an optimal separating contract among the contracts that attract only one type of borrowers.

It is easy to see that the lender's expected payoff with any such contract is

$$
\begin{equation*}
\Pi_{\mathrm{S} 1}=\alpha\left(\mu_{\mathrm{g}}-\overline{\mathrm{u}}-\mathrm{k}\right) \tag{A2.1}
\end{equation*}
$$

## Profit-maximising pooling contracts

The analysis of pooling contracts is rather straightforward as there is no screening in the pooling equilibrium. The monopolistic pooling equilibrium contracts are characterised by a common payoff schedule $\mathrm{P}(\mathrm{x})$. The financier's problem is to choose an optimal pooling payment schedule $\mathrm{P}(\mathrm{x})$ to minimise (A2.2) subject to conditions (A2.3)-(A2.6).

$$
\begin{equation*}
\operatorname{Min}_{\mathrm{P}(\mathrm{x})}\left[\alpha \int_{0}^{1} \mathrm{P}(\mathrm{x}) \mathrm{dF}_{\mathrm{g}}(\mathrm{x})+(1-\alpha) \int_{0}^{1} \mathrm{P}(\mathrm{x}) \mathrm{dF}_{\mathrm{b}}(\mathrm{x})\right] \tag{A2.2}
\end{equation*}
$$

s.t

$$
\begin{align*}
& 0 \leq P(x) \leq x, \quad x \in[0,1]  \tag{A2.3}\\
& x \leq y \Rightarrow P(x) \leq P(y) ; \quad x, y \in[0,1]  \tag{A2.4}\\
& x \leq y \Rightarrow x-P(x) \leq y-P(x)  \tag{A2.5}\\
& \int_{0}^{1} P(x) d F_{i}(x) \geq \bar{u} ; \quad i=g, b \tag{A2.6}
\end{align*}
$$

The optimal pooling contract maximises the financier's profits, which is equivalent with the minimisation problem (), subject to feasibility, monotonicity and participation constraints (A2.3)-(A2.6).

Proposition A2. The unique optimal pooling contract $P(x)$ satisfies $P(x)=P_{b}(x)$, where $P_{b}(x)$ is characterised by Proposition 1 and equations (3.13) and (3.14).

Thus, the optimal pooling contract is the same inverse debt contract that the financier offers to type b borrowers in a separating equilibrium with a menu of contracts. Optimality follows from the fact that this contract provides the financier the highest possible payoffs from type $g$ borrowers among the contracts that satisfy type b borrower's participation constraint with equality. Thus, the optimal pooling contract is the least attractive contract from the good borrower's point of view among the contracts that give bad borrowers exactly their reservation utility.

The lender's expected profit from offering the profit-maximising pooling contract is

$$
\begin{equation*}
\Pi_{\mathrm{P}}=\alpha\left(\mu_{\mathrm{g}}-\int_{0}^{1} \mathrm{P}_{\mathrm{b}}(\mathrm{x}) \mathrm{dF}_{\mathrm{g}}(\mathrm{x})\right)+(1-\alpha)\left(\mu_{\mathrm{b}}-\overline{\mathrm{u}}\right)-\mathrm{k} \tag{A2.7}
\end{equation*}
$$

The first term denotes the financier's expected payoffs from type g borrowers. In pooling equilibrium, the lender cannot extract all expected project surpluses from type $g$ borrowers. The second term denotes the lender's expected payoffs from type b borrowers, from whom the lender is able to extract all expected project surpluses.

## Comparison of different contracts

In this section we compare the financier's expected profits under the three different cases. Let us start the comparison by defining the lender's profits from offering a menu of separating contracts.
$\Pi_{\mathrm{S} 2}=\alpha\left(\mu_{\mathrm{g}}-\overline{\mathrm{u}}\right)+(1-\alpha)\left(\mu_{\mathrm{b}}-\overline{\mathrm{u}}\right)-\mathrm{c}(1-\alpha)\left(1-\frac{\overline{\mathrm{u}}}{\int_{0}^{1} \mathrm{P}_{\mathrm{b}}(\mathrm{x}) \mathrm{dF}_{\mathrm{g}}(\mathrm{x})}\right)-\mathrm{k}$,
$\Pi_{\mathrm{s} 2}$ denotes the financier's expected profits when she offers an optimal menu of two separating contracts, and the borrowers choose the contracts directed at them. The first two terms denote the financier's expected returns from type $g$ and $b$ borrowers, respectively. The third term denotes the screening costs of screening a fraction of type b borrowers, where the probability of screening is obtained from the binding incentive compatibility constraint of type $g$ borrowers.

Now, it is easy to show that both differences $\Pi_{\mathrm{S} 2}-\Pi_{\mathrm{S} 1}$ and $\Pi_{\mathrm{S} 2}-\Pi_{\mathrm{P}}$ are positive when Assumption 3 holds. This completes the proof. QED

## References

Allen, F. - Winton, A. (1995) Corporate Financial Structure, Incentives and Optimal Contracting. In Handbook in Organizational Research and Management Science, Edited by R. Jarrow et. al, Elsevier Science B.V., Vol. 9.

Boyd, J. - Smith, B. (1993) The Equilibrium Allocation of Investment Capital in the Presence of Adverse Selection and Costly State Verification. Economic Theory, 3, 427-451.

Broecker, T. (1990) Credit-worthiness Tests and Interbank Competition. Econometrica, 58, 429-452.

Dowd, K. (1992) Optimal Financial Contracts. Oxford Economic Papers, 44, 672-693.

Gale, D. - Hellwig, M. (1985) Incentive Compatible Debt Contracts: The Oneperiod Problem. Review of Economic Studies, 52, 647-663.

Hart, O. - Holmström, B. (1987) The Theory of Contracts. In Advances in Economic Theory, Fifth World Congress, edited by T. Bewley. Cambridge University Press, New York.

Innes, R. (1993) Financial Contracting Under Risk Neutrality, Limited Liability and Ex ante Asymmetric Information. Economica, 60, 27-40.

Khalil, F. (1997) Auditing Without Commitment. RAND Journal of Economics, Vol. 28, No. 4, Winter 1997, 629-640.

Khalil, F. - Parigi, B. (1998) Loan Size as a Commitment Device. International Economic Review, Vol. 39, No. 1, February, 135-150.

Krasa, S. - Villamil, A. (1994) Optimal Contracts with Costly State Verification: The Multilateral Case. Economic Theory, 4, 167-187.

Mas-Colell, A. - Whinston, M. - Green, J. (1995) Microeconomic Theory. Oxford University Press, New York.

Mookherjee, D. - Png, I. (1989) Optimal Auditing, Insurance, and Redistribution. Quarterly Journal of Economics, 104, 399-414.

Myers, S. - Majluf, N. (1984) Corporate Financing and Investment Decisions when Firms Have Information Investors Do Not Have. Journal of Financial Economics, Vol. 13, 187-222.

Nachman, D. - Noe, T. (1994) Optimal Design of Securities under Asymmetric Information. Review of Financial Studies 7, 1-44.

Rotschild, M. - Stiglitz, J. (1976) Equilibrium in Competitive Financial Markets: An Essay in the Economics of Imperfect Information. Quarterly Journal of Economics, 80, 629-649.

Townsend, R. (1979) Optimal Contracts and Competitive Markets With Costly State Verification. Journal of Economic Theory, 21, 265-293.

Wang, C. - Williamson, S. (1998) Debt Contracts and Financial Intermediation with Costly Screening. Canadian Journal of Economics, 31, 573-595.

## BANK OF FINLAND DISCUSSION PAPERS

ISSN 0785-3572, print; ISSN 1456-6184, online

1/2003 Tanai Khiaonarong Payment systems efficiency, policy approaches, and the role of the central bank. 2003. 69 p. ISBN 952-462-025-1, print; ISBN 952-462-026-X, online. (TU)

2/2003 Iftekhar Hasan - Heiko Schmiedel Do networks in the stock exchange industry pay off? European evidence. 2003. 44 p. ISBN 952-462-027-8, print; ISBN 952-462-028-6, online. (TU)

3/2003 Johanna Lukkarila Comparison between Asian, Russian and Turkish financial crises. (In finnish). 2003. 57 p. ISBN 952-462-029-4, print; ISBN 952-462-030-8, online. (KT)

4/2003 Samu Peura - Esa Jokivuolle Simulation-based stress testing of banks' regulatory capital adequacy. 2003. 41 p. ISBN 952-462-035-9, print; ISBN 952-462-036-7, online. (RM)

5/2003 Peik Granlund Economic evaluation of bank exit regimes in US, EU and Japanese financial centres. 2003. 60 p. ISBN 952-462-037-5, print; ISBN 952-462-038-3, online. (TU)

6/2003 Tuomas Takalo - Otto Toivanen Equilibrium in financial markets with adverse selection. 2003. 45 p. ISBN 952-462-039-1, print; ISBN 952-462-040-5, online (TU)

7/2003 Harry Leinonen Restructuring securities systems processing - a blue print proposal for real-time/t+0 processing. 2003. 90 p. ISBN 952-462-041-3, print; ISBN 952-462-042-1, online (TU)

8/2003 Hanna Jyrkönen - Heli Paunonen Card, Internet and mobile payments in Finland. 2003. 45 p. ISBN 952-462-043-X, print; ISBN 952-462-044-8, online (RM)

9/2003 Lauri Kajanoja Money as an indicator variable for monetary policy when money demand is forward looking. 2003. 35 p. ISBN 952-462-047-2, print; ISBN 952-462-048-0, online (TU)

10/2003 George W. Evans - Seppo Honkapohja Friedman's money supply rule vs optimal interest rate policy. 2003. 22 p. ISBN 952-462-049-9, print; ISBN 952-462-050-2, online (TU)

11/2003 Anssi Rantala Labour market flexibility and policy coordination in a monetary union. 2003. 48 p. ISBN 952-462-055-3, print; ISBN 952-462-056-1, online. (TU)

12/2003 Alfred V. Guender Optimal discretionary monetary policy in the open economy: Choosing between CPI and domestic inflation as target variables. 2003. 54 p. ISBN 952-462-057-X, print; ISBN 952-462-058-8, online. (TU)

13/2003 Jukka Vauhkonen Banks' equity stakes in borrowing firms: A corporate finance approach. 2003.34 p. ISBN 952-462-059-6, print; ISBN 952-462-060-X, online. (TU)

14/2003 Jukka Vauhkonen Financial contracts and contingent control rights. 2003. 33 p. ISBN 952-462-061-8, print; ISBN 952-462-062-6, online. (TU)

15/2003 Hanna Putkuri Cross-country asymmetries in euro area monetary transmission: the role of national financial systems. 114 p . ISBN 952-462-063-4, print; ISBN 952-462-064-2, online. (RM)

16/2003 Kari Kemppainen Competition and regulation in European retail payment systems. 69 p. ISBN 952-462-065-0, print; ISBN 952-462-066-9, online. (TU)

17/2003 Ari Hyytinen - Tuomas Takalo Investor protection and business creation. 32 p. ISBN 952-462-069-3, print; ISBN 952-462-070-7, online. (TU)

18/2003 Juha Kilponen A positive theory of monetary policy and robust control. 26 p. ISBN 952-462-071-5, print; ISBN 952-462-072-3, online. (TU)

19/2003 Erkki Koskela - Rune Stenbacka Equilibrium unemployment under negotiated profit sharing. 28 p. ISBN 952-462-073-1, print; ISBN 952-462-074-X, online. (TU)

20/2003 Eric Schaling Learning, inflation expectations and optimal monetary policy. 49 p. ISBN 952-462-075-8, print; ISBN 952-462-076-6, online. (TU)

21/2003 David T. Llewellyn - David G. Mayes The role of market discipline in handling problem banks. 34 p. ISBN 952-462-077-4, print; ISBN 952-462-078-2, online. (TU)

22/2003 George W. Evans - Seppo Honkapohja Policy interaction, expectations and the liquidity trap. 32 p. ISBN 952-462-079-0, print; ISBN 952-462-080-4, online. (TU)

23/2003 Harry Leinonen - Kimmo Soramäki Simulating interbank payment and securities settlement mechanisms with the BoF-PSS2 simulator. 55 p . ISBN 952-462-082-0, print; ISBN 952-462-083-9, online. (TU)

24/2003 Marja-Liisa Halko Buffer funding of unemployment insurance in a dynamic labour union model. 28 p. ISBN 952-462-084-7, print; ISBN 952-462-085-5, online. (TU)

25/2003 Ari Hyytinen - Tuomas Takalo Preventing systemic crises through bank transparency. 25 p. ISBN 952-462-086-3, print; ISBN 952-462-087-1, online. (TU)

26/2003 Karlo Kauko Interlinking securities settlement systems: A strategic commitment? 38 p. ISBN 952-462-088-X, print; ISBN 952-462-089-8, online. (TU)

27/2003 Guido Ascari Staggered prices and trend inflation: some nuisances. 42 p . ISBN 952-462-092-8, print; ISBN 952-462-093-6, online. (TU)

28/2003 Jukka Vauhkonen Are adverse selection models of debt robust to changes in market structure? 29 p. ISBN 952-462-094-4, print; ISBN 952-462-095-2, online. (TU)


[^0]:    ${ }^{1}$ This literature is surveyd in Dowd (1992) and Allen and Winton (1995).
    ${ }^{2}$ For debt to be uniquely optimal (pooling) contract in Nachman and Noe (1994), the return distributions of different types must be ordered by the strict conditional stochastic dominance, which is a somewhat stronger condition than the monotone likelihood ration property or the firstorder stochastic dominance (see Nachman and Noe 1994, p. 18-21 for details).

[^1]:    ${ }^{3}$ In the absence of this restriction, the equilibrium contracts take the 'live-or-die' form (see Innes 1993) such that the financier receives all the returns if the return realisation is lower than some threshold level and nothing if the return is at least as high as the threshold level.

[^2]:    ${ }^{4}$ We show in Proposition 3 that the financier prefers a menu of two separating contracts to one separating contract or to pooling contracts provided that the cost of screening is sufficiently low.

[^3]:    ${ }^{5}$ See Innes (1993) for a discussion on the optimal financial contracts under adverse selection and without the monotonicity constraint.
    ${ }^{6}$ We assume that the financier can commit to use punishments which, in our static model, are not credible ex-post. We assume that the financier uses these punishments to create and maintain the reputation of being a tough financier. Of course, reputational issues cannot be satisfactorily analysed in a static model like this. See Khalil (1997) and Khalil and Parigi (1998) for analyses on optimal contracts when the principal cannot commit to an audit policy.

[^4]:    ${ }^{7}$ Note that assumptions that are close to assumption 1 are widely used in adverse selection models. In Rotschild and Stiglitz (1976), for example, the existence of the separating equilibrium requires the the proportion of low-quality types is high enough.

