BANK OF FINLAND DISCUSSION PAPERS

14 • 2003

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Research Department
12.5.2003

Financial contracts and contingent control rights

Suomen Pankin keskustelualoitteita
Finlands Banks diskussionsunderlag

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I am grateful to Leonardo Felli, Vesa Kanniainen, Mikko Leppämäki, Juha Tarkka, Otto Toivanen, and Jouko Vilmunen for helpful comments. Most of all, I would like to thank Tuomas Takalo, whose detailed comments considerably improved the manuscript.

Suomen Pankin keskustelualoitteita
Finlands Banks diskussionsunderlag

http://www.bof.fi

ISBN 952-462-061-8
ISSN 0785-3572
(print)

ISBN 952-462-062-6
ISSN 1456-6184
(online)

Suomen Pankin monistuskeskus
Helsinki 2003

Financial contracts and contingent control rights

Bank of Finland Discussion Papers 14/2003

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Abstract

According to empirical studies of venture capital finance, the division of control rights between entrepreneur and venture capitalists is often contingent on certain measures of firm performance. If the indicator of the company's performance (eg earnings before taxes and interest) is low, the venture capital firm obtains full control of the company. If company performance improves, the entrepreneur retains or obtains more control rights. If company performance is very good, the venture capitalist relinquishes most of his control rights. In this article, we extend the incomplete contracting model of Aghion and Bolton to construct a theoretical model that is consistent with these empirical findings.

Key words: incomplete contracts, financial contracting, contingent contracts, control rights, joint ownership

JEL classification numbers: G32

Rahoitussopimukset ja kontingentit päätösvaltaoikeudet

Suomen Pankin keskustelualoitteita 14/2003

Jukka Vauhkonen
Tutkimusosasto

Tiivistelmä

Pääomasijoitusrahoitusta käsittelevien empiiristen tutkimusten mukaan päätösvallan jako yrittäjän ja pääomasijoittajien välillä on usein ehdollinen yrityksen kannattavuutta kuvaaville mittareille. Jos yrityksen kannattavuutta kuvaava signaali (esimerkiksi yrityksen tuotot ennen veroja ja korkokuluja) on huono, pääomasijoittaja saa tyypillisesti kaiken päätösvallan yrityksessä. Jos yrityksen kannattavuus paranee, yrittäjä saa osan päätösvallasta. Jos yrityksen kannattavuus on erinomainen, pääomasijoittaja luopuu suurimmasta osasta päätösvaltaoikeuksiaan. Tässä työssä laajennetaan Aghionin ja Boltonin epätäydellisten sopimusten teoriaan perustuvaa mallia ja rakennetaan teoreettinen malli, joka selittää nämä empiiriset havainnot.

Avainsanat: epätäydelliset sopimukset, rahoitussopimukset, ehdolliset sopimukset, päätösvaltaoikeudet, yhteisomistus

JEL-luokittelu: G32

Contents

1 Introduction

In a stimulating paper, Kaplan and Strömberg (2000) examine in detail the characteristics of financial contracts between firms and venture capitalists. In this article, we present a model of the entrepreneur-investor relationship, which is consistent with their following two key findings about the allocation of control rights.

First, contrary to how control is typically specified in the theoretical literature, it is usually not an indivisible right that can be held at any given time by only one party. Rather, contracting parties typically agree on the division of many different control rights, such as voting rights, board rights and liquidation rights that can be adjusted through contingent provisions. Thus, in the real world, control is often closer to a set of *divisible* variables than a single binary variable.

Second, different control rights are frequently *contingent* on observable measures of the financial and non-financial performance of the firm. In particular, control rights are often allocated in the following way. If the signal of the firm performance (the firm's earnings before taxes and interest, for example) is low, the venture capital firm obtains full control of the firm. If the company performance improves, the entrepreneur retains or obtains more control rights. If the company performance is very good, the venture capitalist relinquishes most of his control rights.

In this paper, we extend the incomplete contracting model of Aghion and Bolton (1992) to build a model that, first, explains why control rights are often contingent and, second, in which control rights are not completely indivisible. There are some papers in which control rights are divisible and some in which they are contingent. Aghion and Bolton (1992) is the best-known example of a model of contingent but indivisible control rights. Kirilenko (2001) and Dessein (2002), in turn, develop models of continuous but not contingent control rights. However, except for this paper, none of these papers explains both of these observations.

Obviously, attempting to develop a model of divisible and contingent control rights is a demanding task. To simplify the problem, we approximate divisible and contingent control rights by the following *three-layered signal-contingent control allocation*, where control refers to the right to choose some interim action affecting the profitability of an investment project. If the signal of the firm performance is bad, the investor obtains full control of the firm. If the signal is intermediate, the parties share control (joint control). If the signal is good, the entrepreneur retains/obtains full control. In this article, we show that this three-layered signal-contingent control allocation may dominate other control allocations.

As our model is a quite straightforward modification of Aghion and Bolton (1992), we first briefly summarise their model. In their model, the entrepreneur with no initial wealth and a wealthy investor contract over the financing of an investment project, which yields two kinds of returns: monetary returns and non-monetary, non-verifiable and non-transferable private returns for the entrepreneur. The sizes of these returns depend on the realisation of the state of the world and the interim action, which is taken after the state of the world has been realised.[1] Because of private benefits, the parties may have conflicting interests over which action to take. This potential conflict cannot always be solved by ex-ante contracts since, by assumption, contracts cannot be contingent on the action or the state of the world. Then, it is critical which of the parties has the right to choose the action. By assumption, contracts can be contingent on a publicly verifiable signal of firm performance. The central result of Aghion and Bolton (1992) is that it may be optimal to make the control allocation dependent on the signal in the following way. If the realisation of the signal is bad, the investor obtains control and if the realisation of the signal is good, the entrepreneur retains control. In what follows, we refer to this allocation as a *two-layered signal-contingent control allocation*.

As summarised above, Aghion and Bolton (1992) is a model of all-or-nothing shifts of control. In their model, either the entrepreneur or the investor holds all control rights, and the party in control is changed if the realisation of the signal is higher than some threshold level. However, as Kaplan and Strömberg (2000) emphasize, changes in control right allocations are seldom so abrupt in reality. Rather, the entrepreneur's (investor's) share of various control rights is often continuously increasing (decreasing) in the performance of the firm. In this article, we take a step towards explaining continuous and contingent control rights by extending the two-layered signal-contingent control allocation model of Aghion and Bolton into a model of three-layered signal-contingent control right allocation and showing that this three-layered signal-contingent control allocation can dominate other control allocations.

The other main contribution of this paper is to show that sharing the control rights 50:50 can be a part of this optimal, three-layered signal-contingent control allocation. This result contrasts with most of the literature, where joint control is

[1] The state of the world can be interpreted, for example, as the original quality of the project or the ability of the entrepreneur. The action can be interpreted, for example, as a choice between defaulting or continuing, the choice of a new employee or the choice of how much to invest in perks.

typically never optimal[2] (see eg Hart 1995). The principal reason for the nonoptimality is that under the standard definition of joint control, each party has a right to veto the relationship into a standstill. In Aghion and Bolton (1992), for example, the entrepreneur can always force the firm into a standstill and extract the surplus in the contract renegotiation. Thus, the respective positions of the entrepreneur and the investor are extremely asymmetric in their model. Joint control, in effect, collapses into entrepreneur control with the entrepreneur holding all control and cash flow rights.

In this article, we define joint control not as a right to force the firm into standstill but as a right to force the firm into *stochastic control*. Under stochastic control, the party in control is determined stochastically and the assets are always in use. For simplicity, we assume that in case of disagreement each party obtains control with a probability ½. A consequence of this assumption is that the investor's disagreement payoff in renegotiations is always positive. A straightforward interpretation of the right to veto the firm into stochastic control is that in case of disagreement the party in control is chosen by tossing a coin. However, there are other arrangements that can implement the same outcome. Consider, for example, a board of directors where the decision is made by the majority rule, where the entrepreneur and the investor have an equal number of votes (but somewhat less than 50 per cent) and where the rest of the votes are held by a 'non-partisan' third party[3]. If the entrepreneur and the investor disagree over the decision, then the vote of the third party is decisive. Ex ante, the votes of the third party may be regarded as stochastic.

One may wonder why the entrepreneur, who has all the bargaining power ex ante and ex post in our model, would ever relinquish any control rights to the investor? Similarly as in Aghion and Bolton (1992), our explanation is the *financing constraint*. The *entrepreneur control* is not always feasible, as the entrepreneur's choice of action may not generate sufficient monetary returns (in expected value) to satisfy the investor's ex-ante participation constraint. In that case, the entrepreneur has to relinquish some or all of his control rights to the investor to induce her provide finance in the first place. Under *investor control*,

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[2] Joint control, though, is quite usual in practice. Among the best-known examples of joint control are joint ventures, where parties typically share control rights 50:50. Other control structures, which resemble joint control are partnerships and some venture capital financings. Partnerships differ from joint ventures in that decisions are typically made by a majority rule, which means that no fixed subset of the parties has a veto. Also some venture capital financings resemble joint control, in that neither the founders of the firm nor the venture capitalists have full voting control. In Kaplan's and Strömberg's (2000) data, the share of such financings was over 20 per cent.

[3] In venture capital financings, for example, there are typically various types of board members that are neither venture capitalists nor the insiders of the firm. The boards typically include, for instance, academics, executives from other firms, retired executives, lawyers, consultants, investment bankers, former managers of the firm, relatives etc. (Gompers and Lerner 1999, ch. 8).

the entrepreneur relinquishes all control rights to the investor. Joint control and signal-contingent control are more intermediate forms of control. In what follows we study the feasibility and optimality properties of all of these alternatives, and show that the optimal form of control crucially depends on the amount of needed finance (or, equivalently, on the degree of the conflict of interest between the parties).

Our main result is the following. If the cost of the project K is at an 'intermediate' level, the entrepreneur control contracts are not feasible, while under mild conditions the three-layered signal-contingent control dominates other forms of control. This result is consistent with the empirical findings of Kaplan and Strömberg (2000). In addition, we show that the investor control is optimal for a wider range of parameter values than as argued by Aghion and Bolton (1992) (see also Vauhkonen 2002).

The intuition of the optimality of the three-layered signal-contingent control allocation is the following. When the amount of the needed finance is sufficiently large, entrepreneur control is not feasible. Full investor control, in turn, is unattractive for the entrepreneur as the investor ignores the entrepreneur's private benefits when choosing the interim action. Under signal-contingent control allocation, the expected[4] share of control rights allocated to each party lies between these two extremes. This division of control rights provides both parties some protection from the expropriation by the other party. It protects the investor, as the entrepreneur cannot always choose his preferred action yielding high private benefits but low monetary returns. Simultaneously, it protects the entrepreneur from the investor always choosing an action that yields high monetary returns but only low private benefits.

Besides being consistent with contingent and divisible control rights, our model is consistent with the 'pecking order theory of control' (Aghion and Bolton 1992). If the size of the needed finance is small, the investor does not need much protection against entrepreneurial expropriation. In that case, the entrepreneur can retain all control rights. When the size of the needed finance is higher, the investor needs some control rights to guarantee her sufficient return to her investment. In that case, the three-layered signal-contingent control allocation is the optimal mode of control. When the size of the needed finance is very high, the entrepreneur must relinquish all control rights to the investor to induce her to finance the project.

Besides articles mentioned above, this article is related to Aghion, Dewatripont and Rey (1994) and Nöldeke and Schmidt (1995) who also consider the consequences of assuming that the renegotiation point is not exogenously set at (0, 0). Bolton and Scharfstein (1990), Diamond (1991), Hart and Moore (1994), and Hart and Moore (1998) are similar to this paper in that they focus on how the

[4] The actual division of control rights depends on the realisation of the signal.

allocation of control rights affects the trade-off between cash flows and private benefits. Another branch of related literature examines the optimal allocation of control and cash flow rights in venture capital finance. Probably closest to this paper are Chan, Siegel and Thakor (1990), Berglöf (1994), Hellmann (1998), who study how the convertible securities allocate control rights to the right persons (the entrepreneur and the venture capitalist) in different states of the world.

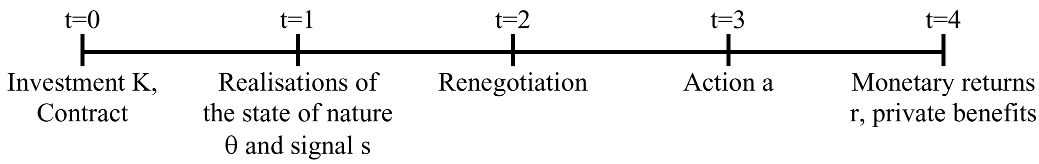
The outline of the paper is the following. In section 2, we present the model. In section 3 we highlight our major results in a simple numerical example. In sections 4–7 we examine the feasibility and optimality of different types of contracts. In sections 4 and 5, we examine entrepreneur control and investor control contracts, respectively. Joint control contracts are studied in section 6. In section 7, we examine the signal-contingent contracts. We conclude in section 8.

2 The model

We extend the model of Aghion and Bolton (1992) in two ways. First, to examine the optimality properties of the three-layered signal-contingent control allocation, we extend their model with two actions, two signals and two states of nature into a model with three actions, three signals and three states of nature. Second, we model joint control differently. Otherwise, the models are identical.

Consider a risk-neutral entrepreneur who has an opportunity to undertake an investment project but lacks funds to finance it. The funds for the investment, K, must come from a risk-neutral wealthy investor. There is a competitive market for finance. Thus, the entrepreneur reaps all the surplus of the project by making a take-it-or-leave-it offer to the investor. The investor, in turn, accepts the offered contract only if her expected monetary payoff is at least K.

The time structure of the model is the following.



The contract determines how the *cash flow rights* and the *control rights* of the project are divided between the entrepreneur and the investor. Cash flow rights determine how the monetary returns of the project, $r(a;\theta)$, are divided between a non-negative transfer to the entrepreneur, $r(s,r(a;\theta))$, and the residual allocated to the investor, $r(a;\theta)–t(s,r(a;\theta))$. Monetary returns depend on the realisation of the state nature θ and the interim action a. The transfer t(s,r(·) can be directly contingent of s and r only but not of a or θ, because contracts are incomplete.

Control rights refer to the right to choose the interim action a. Under unilateral forms of control (entrepreneur and investor control), the party in control has an exclusive right to choose the interim action. Under signal-contingent control, the party in control depends on the realisation of the signal. Under joint control, the action must be chosen unanimously.

This article provides a novel interpretation of joint control.

Assumption 1. *Under joint control, the entrepreneur first proposes some action. Then, the investor either accepts or rejects the proposed action. If the parties fail to reach an agreement, then the entrepreneur obtains control with a probability ½ and the investor with a probability ½.*

According to the standard definition of joint control, each party has a right to force the firm into a standstill in case of disagreement. As discussed in the Introduction, this assumption together with the assumption that the entrepreneur has all the bargaining power in renegotiations imply that joint control, in effect, collapses into entrepreneur control. We depart from the literature by assuming that each party has a right to force the firm into stochastic control. Under stochastic control each party obtains control with some given probability. For simplicity, we set this probability at ½.

Besides observable and verifiable monetary returns, the project yields some non-transferable, non-verifiable and non-monetary private benefits l(a;θ) for the entrepreneur. Although private benefits are non-monetary, we assume that they can be measured in monetary terms. Examples of private benefits are personal satisfaction of running the project, reputation, the entrepreneur's desire to keep the family business in operation although it may not be very profitable, and so on.

The fact that only the entrepreneur enjoys these private benefits creates a potential conflict of interests between the parties over the choice of action. The conflict of interests arises because the investor is only interested in cash flows, whereas the entrepreneur is interested in both cash flows and private benefits. The potential conflict of interests can be easily seen by comparing the parties' von Neumann – Morgenstern utility functions $U_E(a;s,\theta)$ and $U_I(a;s,\theta)$. It is assumed that the utility functions are linear and take the following form:

$$U_E(a;\, s,\theta) = E[t(s, r(a;\theta) + l(a;\theta))] \tag{2.1}$$

$$U_I(a;\, s,\theta) = E[r(a;\theta) - t(s, r(a;\theta))] \tag{2.2}$$

It is obvious that in some state of nature θ and for some arbitrary transfer schedule t(s,r(a;θ) the action that maximises the entrepreneur's utility $U_E(\cdot)$ may differ from the action that maximises the investor's utility $U_I(\cdot)$. Therefore, it matters who has the right to choose the action. In fact, the disagreement over the action choice can be so severe that the entrepreneur's preferred action schedule[5] may not compensate (in expected value) the investor her initial investment K even if she has all the cash flow rights (ie, if $t(\cdot) = 0$). That is, for some parameter values, $E(r(a_E(\cdot),\theta)) < K$. In that case, the feasibility of financing requires that the entrepreneur relinquishes at least some control rights to the investor to ensure that her participation constraint is satisfied.

Contracts are incomplete in two ways. First, the realisation of the state of nature is unverifiable for third parties. Therefore, contracts cannot be contingent on θ. The entrepreneur and the investor, however, observe the realisation of θ ex-post. This provides a rationale for the ex-post renegotiation. The variable θ can be interpreted, for example, as the quality of the project. As the project goes on, both parties are likely to learn this quality. Although observable ex post, the quality may not be easily measurable and describable. Therefore, it may be impossible to write contracts that are contingent on it. However, it may be possible to write a contract that is contingent on some publicly verifiable signal s, which correlates with θ. For example, firm's short-term profits are likely to correlate with the quality of the project. If the correlation between the signal and the state is sufficiently high, it may be useful to design contracts that are contingent on signals. Second, the action is too complex or too difficult to describe in the contract. As a consequence, contracts cannot be contingent on actions.

After the realisation of θ, the initial contract can be renegotiated. As typical in incomplete contracting environments, renegotiation may be socially useful and actually take place in equilibrium (see, for example, Salanie 1997). We assume that the entrepreneur has all the bargaining power in renegotiations.[6] Thus, the entrepreneur can make a take-it-or-leave-it offer to the investor after θ has been realized. In renegotiations, the entrepreneur proposes a new monetary transfer schedules for the investor. If the investor accepts the new contract, the old contract is torn up. Obviously, the investor accepts the new contract if and only if her payoff is at least as high as under the old contract.

In Aghion and Bolton (1992), there are only two states of nature, two actions in the action set and two possible outcomes of the signal. Their model is designed to examine the optimality of 'all-or-nothing' shifts of control. However, to study smoother shifts in control right allocations we need a larger number or a

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[5] The entrepreneur's preferred action in state θ for a given transfer schedule t(s,r(·)) is given by $a_E(s,\theta) = \arg\max_a \{E[t(s,r(a;\theta))+l(a;\theta)]\}$.

[6] Our main results remain valid even if this assumption is relaxed. See the arguments in Aghion and Bolton (1992, p. 479, ftn. 7).

continuum of possible states and signals. We examine the simplest extension with three actions, three states, and three signals. Thus, we assume that the sets of actions, states of nature, and signals are, respectively, $A = \{a_g, a_m, a_b\}$, $\Theta = \{\theta^g, \theta^m, \theta^b\}$, $S = \{s^g, s^m, s^b\}$.

The first-best action $a^*(\theta^i)$ in state θ^i, $i = g, m, b$, maximises the sum of monetary returns and private benefits. In other words $a^*(\theta^i) = \arg\max_{a_j \in A} \{E(r(a_j;\theta^i)) + l(a_j;\theta^i)\}$, $i, j = g, m, b$. This formulation shows that the first-best action may be different in different states of the world. We assume that the parameters of the model are such that $a_g = a^*(\theta^g)$ is the first-best action in state θ^g, $a_m = a^*(\theta^m)$ in state θ^m, and that $a_b = a^*(\theta^b)$ in state θ^b.

The signals are imperfectly correlated with the states of the world. Denote

$$\beta^{ki} \equiv \text{Prob}(s = s^k \mid \theta = \theta^i) \ (i, k = g, m, b). \tag{2.3}$$

We assume that the signals satisfy:

$$\beta^{gg}, \beta^{mm}, \beta^{bb} > 1/2 \tag{2.4}$$

We also assume that the project return $r \in \{0,1\}$. Given this set of return realisations, the expected monetary return of the project can be expressed as follows.

$$y_j^i = E(r \mid \theta = \theta^i, a = a_j) \equiv \text{Prob}(r = 1 \mid \theta = \theta^i, a = a_j) \tag{2.5}$$

Without loss of generality, we assume that each state of nature θ^i takes place with an equal probability 1/3. Thus, if the first-best action is chosen in each state of the world, the expected monetary return is $(1/3)(y_g^g + y_m^m + y_b^b)$. In what follows, we denote this return as the *first-best monetary payoff*.

Private benefits in different states and with different actions are denoted in the same way as the monetary returns. In other words, the level of private benefits in state θ^i when the action is a_j is denoted by l_j^i.

There is a conflict of interests between the investor and the entrepreneur only if the same action does not maximise both monetary returns and private benefits in a given state of nature. The next assumption guarantees that the potential conflict of interests is as stark as possible.

Assumption 2. *Private benefits* l_j^i *and expected monetary returns* y_j^i *satisfy (i)* $l_g^i > l_m^i > l_b^i$ *for* $i = g,m,b$, *and (ii)* $y_g^i < y_m^i < y_b^i$ *for* $i = g,m,b$.

According to (i), in each state of nature, action a_g yields more private benefits than action a_m, which, in turn, yields more private benefits than action a_b. According to (ii), in each states of nature, action a_b yields more monetary returns than action a_m, which, in turn, yields more monetary returns than action a_g.

In the next section, we highlight our main results in a simple numerical example. In subsequent sections, we generalise the insights of that example.

3 Numerical example

For a moment, assume that the initial contract allocates all the monetary returns to the investor and that the initial contract can be contingent on the state of the world. In later sections, these assumptions are relaxed.

Suppose that the monetary returns y^i_j and the private benefits l^i_j of an investment project depend on actions and states of the world in the following way.

		Action		
		a_g	a_m	a_b
State θ^g	return	$y^g_g = 100$	$y^g_m = 150$	$y^g_b = 200$
	private benefit	$l^g_g = 150$	$l^g_m = 80$	$l^g_b = 0$
	total surplus	250	230	200
State θ^m	return	$y^m_g = 0$	$y^m_m = 60$	$y^m_b = 90$
	private benefit	$l^m_g = 90$	$l^m_m = 50$	$l^m_b = 0$
	total surplus	90	110	90
State θ^b	return	$y^b_g = 0$	$y^b_m = 30$	$y^b_b = 60$
	private benefit	$l^b_g = 30$	$l^b_m = 20$	$l^b_b = 0$
	total surplus	30	50	60

Now, consider the entrepreneur's maximisation problem. The entrepreneur, who has all the bargaining power, allocates the control rights with the aim of maximising his expected payoffs while satisfying the investor's participation constraint. We show below that the way the entrepreneur allocates the control rights depends on the cost of the project, K (or, equivalently, on the severity of the conflict of interests between the parties).

For a moment, assume that the renegotiation is not possible. Suppose first that $K \leq (1/3)(y^g_g + y^m_g + y^b_g)$ or $K \leq 100/3$. In this case, the *entrepreneur control* is feasible and implements the first-best action schedule. To see this, consider the entrepreneur's problem of maximising his private benefits. The action that maximises the private benefit is a_g in all three states of nature. When the entrepreneur chooses a_g in all three states, the investor's expected monetary return is $(1/3)(y_g^g + y_g^m + y_g^b)$.[7] However, note that renegotiation will actually take place in states θ^m and θ^b. In state θ^m, the entrepreneur offers to choose the first-best action a_m in exchange for a payment $y_m^m - y_g^m = 60$ from the investor. Similarly, in state θ^b the entrepreneur offers to choose the first-best action a_b in exchange for a payment $y_b^b - y_g^b = 60$. Thus, whenever feasible, the entrepreneur control implements the first-best action schedule. The entrepreneur control is feasible whenever the investor's expected pre-renegotiation returns $(1/3)(y_g^g + y_g^m + y_g^b)$ (which are equal to her post-renegotiation returns as the entrepreneur has all the bargaining power) are at least as large as K.

The more interesting case is $(1/3)(y_g^g + y_g^m + y_g^b) < K$, when entrepreneur control is not feasible. In that case the entrepreneur must relinquish some or all of the control rights to the investor to guarantee that her participation constraint is satisfied. Suppose first that $(1/3)(y_g^g + y_g^m + y_g^b) < K \leq (1/3)(y_g^g + y_m^m + y_b^b)$ or $100/3 < K \leq 220/3$. For these values of K, the following *three-layered state-contingent control allocation* is feasible and implements the first-best action schedule. If the realisation of the state is θ^g, the entrepreneur retains control. If the realisation of the state is θ^m, the parties share control. If the realisation of the state is θ^b, the investor obtains control.[8]

To see this, consider which actions are implemented in each state. As shown above, entrepreneur control implements the first-best action a_g in state θ^g. It is also easy to see that investor control implements the first-best action a_b in state θ^b, since the investor always chooses the action that maximises her monetary payoffs. What is less obvious is that joint control implements the first-best action a_m in state θ^m. To see this, suppose that the entrepreneur proposes that the action a_m is taken. If the investor agrees, then the investor's and the entrepreneur's payoffs are y_m^m and l_m^m, respectively. If the investor disagrees, then, by Assumption 1, she forces the firm into stochastic control. Under stochastic control, the entrepreneur chooses his preferred action a_g with probability ½ and the investor her preferred action a_b with probability ½. In that case, the investor's and the entrepreneur's expected payoffs are $(1/2)(y_g^m + y_b^m)$ and $(1/2)(l_g^m + l_b^m)$, respectively.

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[7] Since the entrepreneur has all the bargaining power, the investor makes an initial lump-sum payment $(1/3)(y_g^g + y_g^m + y_g^b) - K$ to the entrepreneur if $(1/3)(y_g^g + y_g^m + y_g^b)$ is strictly bigger than K.

[8] Of course, it would be possible to set the numerical values of our example in such a way that the optimal state-contingent control allocation would be different. For example, it is easy to think of real world cases, where the investor control would be optimal in the good state of the world and the entrepreneur control in the bad state of the world.

Now, it is easy to see that joint control implements the first-best action a_m in state θ^m as the parameters in the example satisfy

$$y_m^m > (1/2)(y_g^m + y_b^m),$$

and

$$l_m^m > (1/2)(l_g^m + l_b^m).$$

To summarise, the state-contingent control allocation where the entrepreneur obtains control in the good state of the world, both (joint control) in the medium state of the world and the investor in the bad state of the world directly implements the first-best actions a_g, a_m and a_b in states θ^g, θ^m, and θ^b, respectively. Given these choices of actions, the investor's expected monetary payoff under the above state-contingent control allocation is $(1/3)(y_g^g + y_m^m + y_b^b)$. Thus, when $(1/3)(y_g^g + y_g^m + y_g^b) < K \leq (1/3)(y_g^g + y_m^m + y_b^b)$ or when $100/3 < K \leq 220/3$ the state-contingent control allocation is feasible and implements the first-best action schedule whereas entrepreneur control is not feasible. In what follows we also show that under mild conditions, the three-layered state-contingent control dominates other forms of control for intermediate values of K.

For very high values of K, only the investor control contracts that allocate control to the investor in all states of the world are feasible. When K is close to $(1/3)(y_b^g + y_b^m + y_b^b)$ or 350/3, the entrepreneur obtains finance only by relinquishing all the cash flow and control rights to the investor.

In what follows, we generalise the numerical example in two ways. First, we assume that state-contingent contracts are not possible because of contractual incompleteness. However, we show that signal-contingent contracts approximate state-contingent contracts if the signals are sufficiently well correlated with he states of the world. Second, we assume that the entrepreneur is interested not only in private benefits but also in monetary returns.

In the subsequent sections, we examine under which conditions entrepreneur control, investor control, and joint control are feasible. We then derive the conditions under which the three-layered signal-contingent control allocation dominates these other control allocations.

4 Entrepreneur control

This section is a straigtforward extension of the section III.A of Aghion and Bolton (1992) into a three-action, three-signal and three-state case.

Under entrepreneur control, the entrepreneur has the right to decide which action to choose at date 3. In this section we show that, whenever feasible, entrepreneur control implements the first-best plan of actions. Unfortunately, as shown below, some socially profitable projects cannot be implemented under entrepreneur control.

The entrepreneur's problem at date 0 is to design a transfer schedule t(s,r) such that the investor, anticipating the future actions and the outcomes of the possible future renegotiations, is willing to provide the funds at date 0. Without loss of generality we can concentrate on the affine transfer schedules of the form[9]

$t(s,r) = t^s r$, where $0 \le t^s \le 1$ for s = g, m, b.

Thus, the variable t^s denotes the entrepreneur's share of the final monetary returns, which depends on the realisation of the signal s.

Let us examine when the entrepreneur control is feasible and when it implements the first-best action plan. As shown in our example, the initial contract may implement the first-best actions either directly or indirectly through renegotiation. Recall that in our example where $t^s = 0$ for all s, the entrepreneur's preferred action was a_g in all three possible states of the world. In consequence, the first-best action a_g was directly implemented in state θ^g. In fact, the entrepreneur control directly implements the first-best action a_g in state θ^g for all admissible transfer schedules.

Proposition 1. *Under entrepreneur control, any transfer t^s, $0 \le t^s \le 1$, directly implements the first-best action a_g in state θ^g.*

Proof. For a given t^s, the entrepreneur strictly prefers action a_g to actions a_m and a_b, if $t^s y_g^g + l_g^g > \max\left[t^s y_m^g + l_m^g,\ t^s y_b^g + l_b^g\right]$. By the definition of the first-best action, $y_g^g + l_g^g > \max\left[y_m^g + l_m^g,\ y_b^g + l_b^g\right]$ and by assumption 2(i), the above inequality is satisfied for all t^s, $0 \le t^s \le 1$. QED

[9] As shown by Aghion and Bolton (1992 p. 480, p. 482), this simplification is possible since the final returns can take only two values (0 or 1) and since in some states of the world the action plan that maximises the total surplus (y + l) is not the same as the action plan that maximises the private benefits.

In contrast to state θ^g, all transfers t^s, $0 \le t^s \le 1$, do not directly implement the first-best actions in states θ^m and θ^b. To see this, consider the entrepreneur's problem of choosing the action in state θ^i with some arbitrary transfer schedule (t^g, t^m, t^b). When the realisation of the signal is s, the entrepreneur's preferred action in state θ^i is given by

$$a_E(s,\theta^i) = \arg\max_{a_j \in A} \left\{ t^s y_j^i + l_j^i \right\}; \; i, j = g, m, b. \qquad (4.1)$$

It is easy to see that when the realisation of t^s is close to zero, the entrepreneur prefers the inefficient action a_g, which provides him high private benefits, to the efficient action θ^m or θ^b. In that case, the first-best action is implemented only if the investor bribes the entrepreneur to choose the efficient action in contract renegotiations.

For illustration, suppose that in state θ^m (a similar analysis applies to state θ^b), the realisation of t^s is so low that the entrepreneur prefers the action a_g. Thus, the entrepreneur's and the investor's pre-renegotiation payoffs are $t^s y_g^m + l_g^m$ and $(1 - t^s) y_g^m$, respectively. In renegotiations the investor bribes the entrepreneur to choose the first-best action a_m. As the entrepreneur has all the bargaining power in renegotiations, the investor's and the entrepreneur's post-renegotiation payoffs are $y_m^m + l_m^m - (1 - t^s) y_g^m$ and $(1 - t^s) y_g^m$, respectively. Analogously, we can show that in state θ^b the first-best action a_b is implemented in renegotiations in a similar fashion. Therefore, whenever feasible, the entrepreneur control always implements the first-best action schedule.

Unfortunately, as illustrated in our example, some efficient projects are not feasible under entrepreneur control since the entrepreneur cannot always guarantee the investor sufficient payoffs to satisfy her ex-ante participation constraint. The general result is presented in Proposition 1 below. Before stating the proposition, we first introduce some definitions.

Contracts are *renegotiation-proof*, if the transfer schedule (t^g, t^m, t^b) induces the entrepreneur to choose the first-best action in all states of the world for all possible signal realisations, that is, if the transfer schedule is such that $a_E(s,\theta^i) = a^*(s,\theta^i)$ for all s and i.

Contracts are *full renegotiation* contracts, if the initial transfers (t^g, t^m, t^b) are so low that the initial contract is always renegotiated in states θ^m and θ^b for all signal realisations.

Contracts are *partial renegotiation* contracts, if the initial contract is renegotiated in at least one of the states θ^m and θ^b with at least one of the signal realisations but not in both of the states with all signal realisations. Thus, partial renegotiation contracts include all contracts that are neither renegotiation-proof contracts nor full renegotiation contracts.

We also introduce the following auxiliary definitions:

$$t_{rp}^{m} \equiv \left(l_g^m - l_m^m\right)/\left(y_m^m - y_g^m\right),\ t_{rp}^{b} \equiv \max\left[\frac{l_g^b - l_b^b}{y_b^b - y_g^b}, \frac{l_m^b - l_b^b}{y_b^b - y_m^b}\right], \tag{4.2}$$

and $t_{rp}^{E} \equiv \max\left[t_{rp}^{m}, t_{rp}^{b}\right]$

$$\pi_{rp} \equiv \tfrac{1}{3}(1 - t_{rp}^{E})(y_g^g + y_m^m + y_b^b), \tag{4.3}$$

$$\pi_{fr} \equiv \tfrac{1}{3}(y_g^g + y_g^m + y_g^b), \tag{4.4}$$

$$\begin{aligned}\pi_{pr} \equiv\ & (1/3)\left[(\beta^{gg} y_g^g) + (\beta^{mg}\cdot(1 - t_{rp}^{m}) y_g^g) + (\beta^{bg}\cdot(1 - t_{rp}^{b}) y_g^g)\right] \\ & + (1/3)\left[(\beta^{gm} y_g^m) + (\beta^{mm}\cdot(1 - t_{rp}^{m}) y_m^m) + (\beta^{bm}\cdot(1 - t_{rp}^{b}) y_g^m)\right] \\ & + (1/3)\left[(\beta^{gb} y_g^b) + (\beta^{mb}\cdot(1 - t_{rp}^{m}) y_b^b) + (\beta^{bb}\cdot(1 - t_{rp}^{b}) y_b^b)\right],\end{aligned} \tag{4.5}$$

where π_{rp}, π_{fr}, and π_{pr} denote the entrepreneur's highest feasible monetary payoffs with renegotiation-proof, full renegotiation, and partial renegotiation contracts, respectively.

Proposition 2. *Entrepreneur control is feasible and implements the first-best action plan if and only if* $\max(\pi_{rp}, \pi_{fr}, \pi_{pr}) \geq K$. *If K belongs to the non-empty interval* $(\max(\pi_{rp}, \pi_{fr}, \pi_{pr}), (1/3)(y_g^g + y_m^m + y_b^b))$ *entrepreneur control is not feasible.*

Proof. See Appendix.

For illustration of this result, let us insert the numerical values of our example into the formulae of π_{rp}, π_{fr} and π_{pr}. We get

$\pi_{rp} = 115/9$,

$\pi_{fr} = 100/3$

$$\pi_{pr} = \frac{1}{3}\left[100\left(\beta^{gg} + \frac{1}{5}\beta^{mg} + \frac{1}{3}\beta^{bg}\right) + 70\cdot\frac{1}{5}\beta^{mm} + 60\left(\frac{1}{5}\beta^{mb} + \frac{1}{3}\beta^{bb}\right)\right].$$

When β^{gg}, β^{mm}, and β^{bb} converge to one, π_{pr} converges to 202/3. Thus, when signals are very informative, partial renegotiation contracts are feasible for higher values of K than renegotiation-proof or full-renegotiation contracts. However, even the partial-renegotiation contracts fail to implement the first-best action schedule when K is sufficiently close to the first-best monetary return 220/3.

The result that the partial-renegotiation contracts dominate full-renegotiation contracts for some values of K is somewhat surprising as the full renegotiation contracts allocate all the cash flow rights to the investor. The reason for this surprising result is that as the entrepreneur has all the bargaining power in the renegotiation phase he may extract too high a surplus when there is maximum renegotiation (for details, see Aghion and Bolton 1992, p. 483).

To summarise this section, all efficient projects are not feasible under entrepreneur control, since the entrepreneur must get a sufficient share of the monetary returns of the project (either directly or indirectly through renegotiation) to choose the first-best action schedule. For high values of K, this requirement is in conflict with the investor's participation constraint.

5 Investor control

In this section we study the feasibility of investor control contracts. We show that investor control contracts dominate entrepreneur control contracts for intermediate values of K. More specifically, we show that the investor control contracts implement the first-best action schedule even when $K \in \left(\max(\pi_{rp}, \pi_{fr}, \pi_{pr}), (1/3)(y_g^g + y_m^m + y_b^b)\right]$, that is, in the range of K where entrepreneur contracts are not feasible.

Let us start our analysis of investor control contracts by examining which transfers induce the investor to choose the first-best action in each state of nature. Note first that for any transfer t^s, $0 \le t^s < 1$, the investor's preferred action is a_b in all states of the world, since, by assumption 2(ii), $a_I(s,\theta^i) = \arg\max_{a_j \in A}((1-t^s)y_j^i) = a_b$ for all i and t^s, $0 \le t^s < 1$. The next proposition directly follows from that observation.

Proposition 3. *Given any t^s, $0 \le t^s < 1$, investor control directly implements the first-best action a_b in state θ^b.*

In states θ^g and θ^m, however, the investor's preferred action a_b differs from the first-best actions a_g and a_m. Thus, in those states the first-best actions a_g and a_m are implemented only if the entrepreneur offers to renegotiate his share of the monetary returns to bribe the investor to choose the first-best action.

On the basis of the renegotiation process, the investor control contracts can be divided into two categories: full-renegotiation contracts and partial renegotiation contracts.

Full renegotiation contracts

Full renegotiation contracts are always renegotiated in states θ^g and θ^m and implement the first-best actions with all realisations of the signal. First, consider the entrepreneur's problem of inducing the investor to choose the first-best action a_g in state θ^g. The investor chooses a_g only if the entrepreneur offers to lower his initial share of the monetary returns from t^s to $\hat{t}^s$, such that

$$(1-\hat{t}^s)y_g^g \geq (1-t^s)y_b^g. \tag{5.1}$$

By setting $\hat{t}^s = 0$, we get the lowest initial transfer t^s that satisfies this inequality. Denote that transfer by t_{fr}^g.

$$t_{fr}^g \equiv 1 - y_g^g / y_b^g, \tag{5.2}$$

which, by assumption 2, is bigger than zero.

Analogously, the lowest initial transfer t_{fr}^m that induces the investor to choose the first-best action a_m in state θ^m is

$$t_{fr}^m \equiv 1 - y_m^m / y_b^m, \tag{5.3}$$

which, by assumption 2, is also bigger than zero.

Let us combine (5.2) and (5.3) to find the lowest transfer t_{fr}^I that induces the investor to choose the first-best action in *all* states of the world and for all signal realisations:

$$t_{fr}^I \equiv \max\left[t_{fr}^g,\, t_{fr}^m\right] \tag{5.4}$$

Now, the investor's highest expected payoff under full-renegotiation investor control contracts is

$$\pi_{fr}^I \equiv \tfrac{1}{3}(1-t_{fr}^I)(y_b^g + y_b^m + y_b^b). \tag{5.5}$$

Thus, the full-renegotiation investor control contracts are feasible if and only if $\pi_{fr}^I \geq K$. If K belongs to the non-empty interval $\left(\pi_{fr}^I, (1/3)(y_g^g + y_m^m + y_b^b)\right]$, full-renegotiation investor control contracts are not feasible. In the numerical example, $\pi_{fr}^I = 125/3$. If $K \in \left(125/3, 220/3\right]$, full renegotiation contracts are not feasible and do not implement the first-best action schedule.

Aghion and Bolton (1992) limit their analysis of the investor control contracts to full-renegotiation contracts. Therefore, they conclude that investor control is feasible only if $\pi^{I}_{fr} \geq K$. However, as shown by Vauhkonen (2002), their reasoning is not valid, as they overlook the *partial-renegotiation investor control contracts*.

Next, we show that partial renegotiation investor control contracts implement the first-best action schedule for all $K \in [0,(1/3)(y^{g}_{g} + y^{m}_{m} + y^{b}_{b})]$, and, thus, strictly dominate the entrepreneur control contracts for some intermediate values of K.

Partial renegotiation contracts

As mentioned above, partial renegotiation investor control contracts are such contracts that are renegotiated in at least one of the states θ^{g} and θ^{m} for at least one of the signal realisations but not in both of the states for all signal realisations.

In the next proposition we show that if the signals are sufficiently well correlated with the states of nature, then the investor's expected payoff can converge to the first-best monetary payoff under investor control contracts.

Proposition 4. *When β^{gg}, β^{mm}, and β^{bb} converge to 1 and when the transfer schedule is $(t^{g}_{fr}, t^{m}_{fr}, 0)$ the investor's expected monetary payoff π^{I}_{pr} converges to the first-best monetary payoff $(1/3)(y^{g}_{g} + y^{m}_{m} + y^{b}_{b})$.*

Proof. By design, t^{g}_{fr} is defined as the entrepreneur's initial share of the monetary returns that he offers to cut to zero to bribe the investor to choose the first-best action a_g in state θ^{g}. The transfer t^{m}_{fr} is designed analogously in state θ^{m}. In state θ^{g}, the investor chooses the first-best action a_b with any transfer $0 \leq t^{s} < 1$. This implies that when $(t^{g}, t^{m}, t^{b}) = (t^{g}_{fr}, t^{m}_{fr}, 0)$ and when β^{gg}, β^{mm} and β^{bb} converge to 1, the investor's expected post-renegotiation payoffs in states θ^{g}, θ^{m} and θ^{b} converge, respectively, to y^{g}_{g}, y^{m}_{m} and y^{b}_{b}. This, in turn, implies that the investor's expected total monetary payoff converges to the first-best monetary payoff $(1/3)(y^{g}_{g} + y^{m}_{m} + y^{b}_{b})$. QED

It may be worth elaborating this result. Given the transfer schedule $(t^{g}_{fr}, t^{m}_{fr}, 0)$ and assuming as above that $t^{I}_{fr} = t^{g}_{fr} \geq t^{m}_{fr}$, the expression for the investor's expected payoff π^{I}_{pr} can be written as

$$\pi_{pr}^{I} = (1/3)\left[(\beta^{gg} y_g^g) + (\beta^{mg} \cdot (1 - t_{fr}^m) y_b^g) + (\beta^{bg} y_b^g)\right]$$
$$+ (1/3)\left[(\beta^{gm} \cdot (1 - t_{fr}^g) y_b^m) + (\beta^{mm} y_m^m) + (\beta^{bm} y_b^m)\right] \quad (5.6)$$
$$+ (1/3)\left[(\beta^{gb} \cdot (1 - t_{fr}^g) y_b^b) + (\beta^{mb} \cdot (1 - t_{fr}^m) y_b^b) + (\beta^{bb} y_b^b)\right].$$

We immediately see that when β^{gg}, β^{mm} and β^{bb} converge to 1, π_{fr}^{I} converges to the first-best monetary payoff $(1/3)(y_g^g + y_m^m + y_b^b)$.

To summarise this section, we showed that investor control contracts dominate the entrepreneur control contracts when $K \in (\max(\pi_{rp}, \pi_{fr}, \pi_{pr}), (1/3)(y_g^g + y_m^m + y_b^b)]$. The reason is that the investor's preferred action plan yields higher expected monetary returns than the entrepreneur's preferred action plan. That allows the financing of some high-cost projects, which are not feasible under entrepreneur control.

6 Joint control

Under joint control, the parties must take a unanimous choice of action. In this section, we show, first, that a renegotiation-proof joint control contract that allocates all monetary returns to the investor implements the first-best action in state θ^m under mild conditions. This is an important auxiliary result, which is utilised in section 7. Second, we show that joint control contracts never strictly dominate all other control allocations.

We consider two kinds of joint control contracts. Under renegotiation-proof contracts the initial transfer schedule directly induces a unanimous choice of action. The second possibility is that parties reach agreement in the renegotiation. Before examining these contracts, we need to examine what happens under joint control if the parties disagree over the choice of action.

If the parties disagree over the choice of action and if the renegotiations fail, then, by Assumption 1, each party obtains control with a probability ½. The party in control then chooses his or her most preferred action. As shown in section 5, the investor's preferred action in any state of the world for any realisation of the transfer t^s, $0 \leq t^s < 1$ is a_b. The entrepreneur's preferred action in state θ^i for some t^s, in turn, is given by $\arg\max_{a_j \in A}(t^s y_j^i + l_j^i) \equiv a_E(s, \theta^i)$. These action choices in combination with the initial transfer schedule (t^g, t^m, t^b) determine the parties' disagreement payoffs, which are the starting points of the possible future renegotiations.

Consider first the parties' disagreement payoffs in state θ^m. The entrepreneur's disagreement action is a_m if $t^s \geq t_{rp}^m$ and a_g if $t^s < t_{rp}^m$, where t_{rp}^m is defined in (4.2). Thus, the investor's expected disagreement payoff

$\pi_{dis}(s,\theta^m) = \frac{1}{2}(1-t^s)(y_m^m + y_b^m)$, if $t^s \geq t_{rp}^m$ and $\pi_{dis}(s,\theta^m) = \frac{1}{2}(1-t^s)(y_g^m + y_b^m)$, if $t^s < t_{rp}^m$. Note, in particular, that when $t^s = 0$, $\pi_{dis}(s,\theta^m) = \frac{1}{2}(y_g^m + y_b^m)$.

This result is utilised in the next proposition, which shows that joint control contract with $t^s = 0$ for all s directly implements the first-best action a_m in state θ^m under mild conditions.

Proposition 5. *When the parameters of the model satisfy $y_m^m \geq (1/2)(y_g^m + y_b^m)$ and $l_m^m \geq (1/2)(l_g^m + l_b^m)$, the transfer schedule $(t^g, t^m, t^b) = (0,0,0)$ directly implements the first-best action a_m in state θ^m.*

Proof. Suppose first that the investor and the entrepreneur unanimously agree that action a_m is taken. In that case, given that $(t^g, t^m, t^b) = (0,0,0)$, their payoffs are y_m^m and l_m^m, respectively. Suppose, alternatively, that the parties disagree over the choice of action. Then, given that $(t^g, t^m, t^b) = (0,0,0)$ and by Assumption 1, their expected payoffs are $(1/2)(y_g^m + y_b^m)$ and $(1/2)(l_g^m + l_b^m)$, respectively. Thus, the first-best action a_m in state θ^m is chosen unanimously when the parameters of the model satisfy $y_m^m \geq (1/2)(y_g^m + y_b^m)$ and $l_m^m \geq (1/2)(l_g^m + l_b^m)$. QED

The parameter restriction in the Proposition 5 that $y_m^m \geq (1/2)(y_g^m + y_b^m)$ and $l_m^m \geq (1/2)(l_g^m + l_b^m)$ hold simultaneously is rather mild, since, by the definition of the first-best action, at least one of these inequalities always holds.

In the next proposition we show that joint control is not feasible when K is sufficiently high. The Proposition directly follows from the following Lemma.

Lemma 1. *The first-best action a_b is always implemented in state θ_b if and only if the $t^s \geq t_{rp}^b$, where $t_{rp}^b \equiv \left[\frac{l_g^b - l_b^b}{y_b^b - y_g^b}, \frac{l_m^b - l_b^b}{y_b^b - y_m^b}\right]$.*

Proof. The entrepreneur proposes that action a_b is taken in state θ^b only if t^s is such that

$$t^s y_b^b + l_b^b \geq \max\left\{\frac{1}{2}\left[t^s\left(y_g^b + y_b^b\right) + \left(l_g^b + l_b^b\right)\right], \frac{1}{2}\left[t^s\left(y_m^b + y_b^b\right) + \left(l_m^b + l_b^b\right)\right]\right\},$$

ie if $t^s \geq \max\left[\frac{l_g^b - l_b^b}{y_b^b - y_g^b}, \frac{l_m^b - l_b^b}{y_b^b - y_m^b}\right] \equiv t_{rp}^b$. QED

Thus, if $t^s \geq t^b_{rp}$, a_b is always implemented. If $t^s < t^b_{rp}$, the parties disagree over the choice of action. In that case, a_b is implemented only with probability ½.

Proposition 6. *There are values of K such that joint control contracts do not implement the first-best plan of actions and, thus, are dominated by investor control contracts.*

Proof. By Lemma 1, joint control implements the first-best action a_b in state θ^b only if the entrepreneur obtains a positive share of the monetary returns. Therefore, the investor's largest expected monetary payoffs under joint control are bounded away from the first-best expected monetary payoffs $(1/3)(y^g_g + y^m_m + y^b_b)$. Thus, there are values of K close to the first-best expected monetary payoff such that joint control contracts are not feasible. The investor control contracts, in turn, implement the first-best action schedule for all $K \in (0,(1/3)(y^g_g + y^m_m + y^b_b)]$. Therefore, investor control contracts strictly dominate joint control contracts for some values of K close to $(1/3)(y^g_g + y^m_m + y^b_b)$. QED

By Propositions 4 and 6, joint control never strictly dominates other forms of control in our model. This is a standard result in the literature (see Hart 1995). In the next section we show, however, that joint control can be a part of the optimal signal-contingent control allocation.

7 Signal-contingent control

The control allocations examined in previous sections are not contingent on realisations of s. In this section, we examine signal-contingent control allocations, where the party in control depends on the realisation of s.

By Propositions 1, 3 and 5, an obvious candidate for efficiency is a control allocation rule, which allocates control to the entrepreneur when s = g, to the investor when s = b and to both (joint control) when s = m. The next Proposition is a straightforward corollary of Propositions 1, 3, and 5.

Proposition 7. *When the parameters of the model satisfy $y^m_m \geq (1/2)(y^m_g + y^m_b)$ and $l^m_m \geq (1/2)(l^m_g + l^m_b)$, the investor's expected payoff converges to the first-best monetary payoff when the initial contract with a transfer schedule $(t^g, t^m, t^b) = (0,0,0)$ allocates control to the entrepreneur when s = g, to the investor when s = b and to both when s = m.*

Proof. By propositions 1, 3 and 5, the above control allocation directly implements the first-best actions in each state of the world when the signals correspond to the state. Therefore, when $(t^g, t^m, t^b) = (0,0,0)$ and when β^{gg}, β^{mm} and β^{bb} converge to 1, the investor's expected monetary payoff converges to the first-best monetary payoff. QED

For illustration, note that the investor's expected payoff π_S with the signal-contingent contract specified in Proposition 7 is

$$\begin{aligned}\pi_S = (1/3)&\left[\beta^{gg}y_g^g + \frac{1}{2}\beta^{mg}(y_g^g + y_b^g) + \beta^{bg}y_b^g\right] \\ &+ (1/3)\left[\beta^{gm}y_g^m + \beta^{mm}y_m^m + \beta^{bm}y_b^m\right] \\ &+ (1/3)\left[\beta^{gb}y_g^b + \frac{1}{2}\beta^{mb}(y_g^b + y_b^b) + \beta^{bb}y_b^b\right].\end{aligned} \tag{7.1}$$

As β^{gg}, β^{mm}, β^{bb} converge to 1, π_S converges to the first-best monetary payoffs $(1/3)(y_g^g + y_m^m + y_b^b)$.

By Propositions 2 and 6, neither entrepreneur nor joint control is feasible for when K is sufficiently close to the first-best monetary return. Therefore, signal-contingent control dominates entrepreneur control and joint control when K is sufficiently close to $(1/3)(y_g^g + y_m^m + y_b^b)$. This argument, however, is not adequate to establish that signal-contingent control dominates investor control as, by Proposition 4, π_{pr}^I also converges to the first-best monetary payoff. However, it can be shown that there exist parameter values such that the difference $\pi_S - \pi_{pr}^I$ is positive. When $\pi_S - \pi_{pr}^I > 0$, then for some parameter values signal-contingent control strictly dominates investor control. By combining these findings, we can establish the main result of our paper.

Proposition 8. *There are values of K such that (i) entrepreneur and joint control are not feasible and (ii) signal-contingent control dominates investor control.*

Proof. See above.

We interpret the signal-contingent control allocation examined above as a control allocation associated with many venture capital financings. The benefit of the signal-contingent control allocation is close to the benefit of debt in Aghion and Bolton (1992). When the size of the needed finance is sufficiently high, the entrepreneur must relinquish some control rights to the investor to obtain finance. The signal-contingent control allocation allows the entrepreneur to keep some control rights and to reap some private benefits while allowing adequate protection to the investor.

8 Conclusions

The principal objective of this paper was to try to explain the smooth shifts in control observed by Kaplan and Strömberg (2000). In an incomplete contracting environment, we used a signal-contingent contract as a proxy of the smooth control contracts and showed that such a signal-contingent contract may dominate other control allocations under mild parameter restrictions. On the basis of this result, we regard that our model is consistent with the key empirical findings of Kaplan and Strömberg (2000).

The reader may wonder how such signal-contingent control allocation can be implemented by using standard financial securities. As discussed in Kaplan and Strömberg (2000), in real world venture capital financings there are many combinations of preferred equity, convertible securities, and multiple classes of common stock that implement any desired control allocation. One simple possibility is that the entrepreneur finances the project by issuing multiple classes of common stock. Initially the investor has all the control rights. Then, if the signal of the firm performance is intermediate some of the investor's equity with superior control rights converts into common stock such that the parties share the voting rights. If the signal is good, most or all of the investor's equity with superior control rights converts into common stock such that the entrepreneur obtains voting control.

We regard our model as a first step towards explaining continuous, contingent and divisible control rights. In this area, much remains to be explored in future studies. Another direction worth pursuing might be to try to build venture capital-specific models that are more consistent with the empirical findings of Kaplan and Strömberg (2000) than the existing venture capital-specific models (see Kaplan and Strömberg 2000, p. 30–32). Our general model lacks many features that are prevalent in the theoretical literature of venture capital financings, such as both parties' effort choices. Ideally, one would want to incorporate such venture capital specific characteristics into the theoretical models, which attempt to explain empirical observations from venture capital contracts.

Appendix

Proof of Proposition 1.

Note that this proposition and its proof are very similar to the Proposition 2 of Aghion and Bolton (1992) and its proof. The minor differences between the two propositions stem from the fact that our model studies the case with three actions, three signals and three states whereas Aghion and Bolton (1992) study the case with two-action, two-signal, and two-state case.

We consider one-by-one the feasibility conditions of the three different types of entrepreneur control contracts: renegotiation-proof contracts, full renegotiation contracts and partial renegotiation contracts. Recall that contracts are feasible if the investor's expected monetary payoffs are at least as high as the cost of investment K. To study the feasibility, we derive below the investor's highest possible expected monetary payoffs under different types of contracts.

Renegotiation-proof contracts:

First, we must derive the conditions when the contracts are renegotiation-proof. Contracts are renegotiation-proof if the initial transfers (t^g, t^m, t^b) are sufficiently high to induce the entrepreneur to choose the first-best action in all states of the world for all possible signal realisations. Consider state-by-state which transfers induce the entrepreneur to choose the first-best actions.

First, consider the state θ^g. By proposition 1, all t^s, $0 \leq t^s \leq 1$, directly implement the first-best action a_g in state θ^g.

Next, consider the state θ^m. The entrepreneur control directly implements the first-best action a_m, if and only if

$$t^s y_m^m + l_m^m \geq \max\left[t^s y_g^m + l_g^m,\ t^s y_b^m + l_b^m\right], \qquad \text{(i)}$$

which, by the definition of the first-best and by assumption 2 reduces to

$$t^s \geq (l_g^m - l_m^m)/(y_m^m - y_g^m) \equiv t_{rp}^m, \qquad \text{(ii)}$$

where t_{rp}^m is defined as the lowest t that directly induces the entrepreneur to choose the first-best action a_m in state θ_m.

Finally, consider the state θ^b. The analysis is similar as in state θ^m. Thus, the entrepreneur directly chooses the first-best action a_b in state θ_b if and only if

$$t^s \geq \max\left[\frac{l_g^b - l_b^b}{y_b^b - y_g^b}, \frac{l_m^b - l_b^b}{y_b^b - y_m^b}\right] \equiv t_{rp}^b, \qquad \text{(iii)}$$

where t_{rp}^b is defined in a similar fashion as t_{rp}^m.

Combining the results, we see that contracts are renegotiation-proof, if and only if the following condition is satisfied for all s:

$$t^s \geq \max\left[t_{rp}^m, t_{rp}^b\right] \equiv t_{rp}^E. \qquad \text{(iv)}$$

Thus, t_{rp}^E is the lowest transfer that induces the entrepreneur to choose directly the first-best action in all states of the world. Therefore, the investor's highest possible expected payoff with the renegotiation-proof entrepreneur control contracts is

$$\pi_{rp} = \frac{1}{3}(1 - t_{rp}^E)(y_g^g + y_m^m + y_b^b), \qquad \text{(v)}$$

and, thus, the renegotiation-proof contracts are feasible if and only if $\pi_{rp} \geq K$.

Full renegotiation contracts

This is the case we studied in our numerical example. We define full renegotiation contracts as the contracts which are always renegotiated in states θ^m and θ^b. Among these contracts, the investor's expected payoffs are maximised when $t^s = 0$ for all s. Given this transfer schedule, the entrepreneur's preferred action is a_g in all states of the world. Thus, the investor's pre-renegotiation expected return (which is the same as the post-renegotiation expected return) is

$$\pi_{fr} = \frac{1}{3}(y_g^g + y_g^m + y_g^b).$$

Thus, the full-renegotiation contracts are feasible if and only if $\pi_{fr} \geq K$.

Partial-renegotiation contracts

With partial renegotiation contracts, transfers are defined in such a way that the initial contract is renegotiated in some states of the world for some signal realisations. Thus, the family of partial renegotiation contracts consists of all contracts that are neither renegotiation-proof or full renegotiation contracts.

The transfer schedule that maximises the investor's expected monetary payoffs within this type of contracts is $(t^g, t^m, t^b) = (0, t^m_{rp}, t^b_{rp})$. Suppose, without loss of generality and similarly as in our numerical example, that $t^m_{rp} > t^b_{rp}$. Then, the investor's expected monetary payoffs π_{pr} can be written

$$\begin{aligned} \pi_{pr} = &(1/3)\left[(\beta^{gg} y^g_g) + (\beta^{mg} \cdot (1 - t^m_{rp}) y^g_g) + (\beta^{bg} \cdot (1 - t^b_{rp}) y^g_g)\right] \\ &+ (1/3)\left[(\beta^{gm} y^m_g) + (\beta^{mm} \cdot (1 - t^m_{rp}) y^m_m) + (\beta^{bm} \cdot (1 - t^b_{rp}) y^m_g)\right] \\ &+ (1/3)\left[(\beta^{gb} y^b_g) + (\beta^{mb} \cdot (1 - t^m_{rp}) y^b_b) + (\beta^{bb} \cdot (1 - t^b_{rp}) y^b_b)\right]. \end{aligned} \tag{vi}$$

Thus, the partial renegotiation contracts are feasible if and only if $\pi_{pr} \geq K$.

Combining the above results, we see that entrepreneur control contracts are feasible if and only if $(\pi_{rp}, \pi_{fr}, \pi_{pr}) \geq K$. QED

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