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Abstract

Many indicators of business and growth cycles have been constructed by both private and public agencies and are now in use as monitoring devices of economic conditions and for forecasting purposes. As these indicators are largely composite constructs using other economic data, their frequency composition is likely different to that of the variables they are used as indicators for.

In this paper we use the Hilbert-Huang transform, which comprises the empirical mode decomposition (EMD) and the Hilbert spectrum, in order to analyse the frequency content of comparable OECD confidence indicators and national sentiment indicators for industrial production and consumption. We then compare these with the frequency content of both industrial production and real consumption growth data. The Hilbert-Huang methodology first uses a sifting process (EMD) to identify the embedded frequencies within a time series, and the changing nature of these embedded frequencies (IMFs) can then be analysed by estimating the instantaneous frequency (using the Hilbert spectrum). This methodology has several advantages over conventional spectral analysis: it handles non-stationary and non-linear processes, and it can cope with short data series.

The aim of this paper is to decompose both indicator and actual economic variables to evaluate i) whether the number of IMFs are equivalent in both indicators and actual variables and ii) to see which frequencies are accounted for in indicators and which frequencies are not.

Keywords: economic growth, Hilbert-Huang transform, empirical mode decomposition, frequency domain, economic indicators

JEL classification numbers: C63, E21, E32

Hilbertin-Huangin muunnoksen käyttö makrotaloudellisten indikaattorimuuttujien ja niiden vastinmuuttujien aikasarjavaihtelun tilastollisessa analyysissa

Suomen Pankin keskustelualoitteita 33/2009

Patrick M. Crowley – Tony Schildt
Rahapolitiikka- ja tutkimusosasto

Tiivistelmä

Yksityiset tutkimuslaitokset ja viranomaistahot ovat rakentaneet useita suhdannevaihteluihin ja talouskasvuun liittyviä indikaattoreita, joita käytetään talouden seurannassa sekä tulevan talouskehityksen ennustamisessa. Nämä indikaattorit on useimmiten rakennettu yhdistelemällä muuta taloudellista tilastotietoa, mistä syystä indikaattoreiden taajuusalueen ominaisuudet ovat mahdollisesti erilaiset kuin niiden kuvaamien taloudellisten muuttujien vastaavat ominaisuudet. Tässä työssä käytetään ns. Hilbertin-Huangin muunnosta, jonka avulla analysoidaan teollisuustuotannon ja kulutuksen kehitykseen liittyvien vertailukelpoisten OECD:n luottamusindikaattoreiden ja kansallisten ilmapiiri-indikaattoreiden aikasarjavaihtelua taajuusalueen menetelmin. Empiirisen tyyppiarvon dekomponointi ja Hilbertin spektri ovat näistä taajuusalueen työkaluista tärkeimmät tämän työn kannalta. Näin estimoituja indikaattorimuuttujien aikasarjavaihtelun taajuuskomponentteja verrataan sekä teollisuustuotannon kasvun että kulutuksen määrän kasvun vastaaviin komponentteihin. Hilbertin-Huangin menetelmällä on useita etuja suhteessa tavanomaiseen spektraalianalyysiin. Menetelmän käyttö soveltuu yhtä hyvin stationaaristen ja ei-stationaaristen aikasarjojen samoin kuin lyhyiden aikasarjojen analysointiin. Tämän tutkimuksen tavoitteena on Hilbertin-Huangin menetelmän avulla yhtäältä arvioida, vastaavatko itse taloudellisten muuttujien ja näiden kehitystä kuvaavien indikaattorimuuttujien aikasarjavaihtelun taajuuskomponentit täysin toisiaan, ja toisaalta identifioida taajuuskomponentit, joiden suhteen indikaattoreissa ja varsinaisissa muuttujissa on poikkeavuuksia.

Avainsanat: talouskasvu, Hilbertin-Huangin muunnos, empiirisen tyyppiarvon hajoittaminen, taajuusalue, taloudelliset indikaattorit

JEL-luokittelu: C63, E21, E32

Contents

1 Introduction

Cyclical indicators aim to condense the information about economic activity by capturing the cyclical patterns in economic variables into one-dimensional variables that are tractable and possibly available prior to release of economic variables. In their book Burns and Mitchell (1946) introduce a system to measure cyclical fluctuations in economic activity developed by the National Bureau of Economic Research (NBER). Since then, such regularly collected information is now an integral part of the measurement ex-post of business cycles or to anticipate ex-ante cyclical fluctuations in economic activity. Thus, the indicators of business cycles (see for example Kydland and Prescott, 1990) and growth cycles (see for example Zarnowitz and Ozyildirim, 2002) are routinely used as monitoring devices in the analysis of macroeconomic conditions and forecasting in many countries.

A number of public and private institutions have developed cyclical indicators. The Organisation for Economic Co-operation and Development (OECD) has developed a comprehensive system of internationally comparable cyclical indicators. The data surveyed with harmonised business tendency surveys is used with data published by national agencies in construction of the OECD composite leading indicators. Through a standardisation procedure, both the business and the consumer confidence indicator data are made comparable between countries. The European Central Bank (2001) has dissected the information content of the composite indicators of the euro area and how closely indicators follow the cyclical pattern of the euro area reference series. As composite indicators are summary indicators, we might not only be interested in the cyclical behaviour of the aggregate series, but we might also be interested in determining what frequencies are at work in the indicators as well as the relevant macroeconomic variables, so that potential lead/lag relations between frequencies of particular interest can be identified.

In this empirical paper, we dissect the information content of the OECD standardised confidence indicators along with some national sentiment indicators and economic variables for the euro area, Japan and the US. To do this we employ the Hilbert-Huang transform (HHT) of Huang et al (1998). The first step is to extract the embedded frequencies of indicators and economic variables using empirical mode decomposition (EMD). Secondly, we analyse the embedded frequencies by using a Hilbert spectrum. As emphasised by Wu and Huang (2004), we use a recently developed variation of EMD that corrects some recognised problems associated with EMD. Finally, we evaluate the frequencies in the economic variables that are or are not being captured by the indicators.

The rest of the paper is organised as follows. Section 2 introduces the cyclical indicators and describes the data, while section 3 outlines the empirical method to

be used. Section 4 presents the empirical results and section 5 offers some conclusions.

2 Data

2.1 Cyclical indicators

The timeliness and extensive geographical breakdown of economic data provides a vast amount of information that is made available to today's economic policymaker. As is often the case in statistical work, an extensive source of economic data is to be reduced and summarised by computing a smaller number of derived measures that try to incorporate what is relevant and informative or even accidental in the cyclical behaviour of economic activity. The indicators of cyclical behaviour in economic activity, that is, cyclical indicators, are classified by the Conference Board (2001) into three categories of leading, coincident, and lagging based on the timing of their movements. Coincident indicators are series that define the business cycle by measuring aggregate economic activity and leading indicators are series that tend to shift direction in advance of the business cycle, while the lagging indicators, in contrast to the leading indicators, tend to change direction after the coincident series.

Burns and Mitchell (1946) document the measures of cyclical behaviour developed in the 1930s by the National Bureau of Economic Research (NBER). The leading economic indicator approach has been applied to measure business cycles ever since. While the literature on business cycle theory attempts to explain recurrent fluctuations, the empirical leading indicator approach is criticised for lack of constituent economic theory. With regard to criteria of an economic nature, Koopmans (1947) notes that the constituent series of cyclical indicators are chosen mainly on empirical grounds, rather than on the basis of economic theory.

It is desirable to properly assess the current and in particular future economic activity for effective economic policy-making. For these purposes business opinion is surveyed with so-called business tendency surveys of company managers concerning the current situation of their business and about their plans and expectations for the near future. In collaboration with the European Commission, the Organisation for Economic Co-operation and Development (OECD) has developed a system of harmonised business tendency surveys. The economic data published by national agencies is used together with the OECD's harmonised business tendency survey data in order to construct the OECD composite leading indicators (CLIs) for both the OECD member and the OECD non-member countries. The system of OECD (2008) leading indicators is based

on the growth cycle approach, which measures deviations from the long-term trend, whereas the leading indicator approach is based on repetitive but non-periodic fluctuations in economic activity.

The OECD released new cyclical indicators in December 2006 (OECD, 2006). These new standardised indicators enhance the existing product range in the OECD CLIs by providing both comparability between countries and the likelihood of fewer revisions. OECD CLIs are constructed to predict cycles in a reference series chosen as a proxy measure for the aggregate economy. The CLIs are released by the OECD on a monthly basis, and given their timeliness, are appropriate to model short-term movements in economic variables. However, short-term movements should be interpreted with caution, because local volatility and revisions that are a natural part of the calculation procedure can obscure the main patterns in economic activity.

The new set of OECD standardised cyclical indicators includes both Business Confidence Indicators (BCIs) and Consumer Confidence Indicators (CCIs). These new cyclical indicators are in addition to the existing CLIs, also published by the OECD. The BCIs are presumed to anticipate reference series with shorter and more stable lead times than the CLIs, and to date they appear to be subject to less revision than the CLIs.

The business confidence indicators are supposed to capture the cyclical patterns in real-economic activity. Usually the volume of industrial production or real gross domestic product (GDP) would be used as reference series for these indicators. Mourougane and Roma (2002) find confidence indicators useful in forecasting real GDP growth rates in the short run in a number of European countries. Industrial production data is also often used as a proxy measure for real GDP. While real GDP is a more comprehensive variable for analysing economy-wide fluctuations, industrial production data has the advantage of being available on a monthly rather than a quarterly basis.

An article by the European Central Bank (2001) dissects the information content of composite indicators of the euro area business cycle and the analysis suggests that composite indicators may, with hindsight, have followed developments broadly similar to those of the business cycle as measured by both industrial production and real GDP growth. However, the relationship between composite indicators and the business cycle may be not very stable over time, which obviously hampers the interpretation of the latest readings of such indicators. For example, Japanese survey forecasts of industrial production were found to be biased and inconsistent with rational expectations in comparison with the US by Aggarwal and Mohanty (2000). Furthermore, Fukuda and Onodera (2001) point out that several research institutes in Japan made serious errors in forecasting business cycles and prolonged recessions in the 1990s.

The consumer confidence indicators are presumed to capture cyclical patterns in household consumption behaviour. Andersen and Nielsen (2003) note that the

consumer confidence indicator is often taken as an indicator of the development in consumer spending, but in general it is not so clear which economic variable, if any, consumer confidence indicators describe. There is little agreement over the value of consumer confidence indicators for forecasting consumer spending over what is already captured by economic fundamentals. Consumer confidence indicators may potentially be used to point to future behaviour, since households are asked also about their future expectations of the economic activity. It is, however, ambitious to think that the indicator can describe not only the current, but also future economic development.

Few papers have investigated the usefulness of consumer confidence in modelling consumer spending (see for example Ludvigson, 2004). Carroll et al (1994) and Bram and Ludvigson (1998) report the predictive information content of lagged values of consumer confidence for US consumer spending. For Japan, Utaka (2003) finds that consumer confidence has only a short-term effect on fluctuations in economic activity. Other studies found in Chopin and Darrat (2000), examine the value of consumer attitudes for forecasting economic performance, but fail to produce a consensus about the value of attitudes from consumer measures for forecasting consumer behaviour. For example, Garner (1991) finds the consumer confidence indicator published by the University of Michigan an unreliable predictor of consumer spending.

For evaluation and proper use of an indicator, it is crucial to know how leading indicators are constructed and moreover how composite leading indicators are constructed. Furthermore, in the context of economic policy making, composite indicators, by construction, hide the driving factors behind current and short-term changes in economic activity. Nevertheless, it is clear that many of the components of these indicators may work at different frequencies, so that compiling composites might lead to internal frequency mismatches.

2.2 Data description

For our data set, we use a range of economic indicators for the euro area, Japan and the US. Both the national sentiment indicators and the OECD standardised confidence indicators are used. The national indicators are also used by the OECD in its construction of leading indicators so there is obviously going to be some overlap here. In the frequency domain, we study the comparable OECD confidence indicators and the nationally published sentiment indicators alike. The series used for each country are given in Table 2.1. The data is seasonally adjusted and the economic variables used as reference series are measured in real terms for each country.

We follow the OECD (1998) and European Commission (2007) in defining industrial production for manufacturing (henceforth referred to as industrial production), which is defined as a reference series for OECD business composite indicator construction. Both the industrial production data and the business confidence data are published on a monthly basis (except for Japan where we use the monthly series interpolated by the OECD). More information on the data used by the OECD could be found in OECD (2006).

We define private final consumption expenditure in GDP including non-profit institutions serving households (henceforth referred to as consumption) as the reference for consumer confidence. Since the data for consumption is available on a quarterly basis, we aggregate the consumer confidence indicator series from a monthly into a quarterly series. The indicator value in a quarter is calculated as the average of the monthly observations in that quarter for the consumer confidence indicator.

The data set covers the period up to the end of 2007 with the start of the sample period varying from country to country depending on availability of the data. More specifically, the industrial production data for the US starts in 1950, for Japan in 1978 and for the euro area at 1990. The consumption data for the US and Japan and the euro area start in 1973. In the short-term, the growth of economic variables against confidence indicators is obtained by transforming the data, that is, by taking month-on-month growth rates for industrial production and quarter-on-quarter rates for consumption. This most accurately corresponds to the measurement of the indicators, as they are constructed to measure changes in economic activity or spending rather than the absolute levels. In Figure 2.1 plots both the monthly industrial production and the quarterly consumption series with one period growth as measured by the log-change for the euro area, Japan and the US. The OECD standardised confidence indicators are correspondingly plotted in Figure 2.2.

Table 2.1 **Description of indicators and economic variables**

	Euro area	**Japan**	**US**
Business tendency survey	Harmonised EU Business tendency survey Indicator a) Source: European Commission, OECD	The Bank of Japan's quarterly Short-term Economic Survey of Enterprises in Japan (TANKAN) c) Source: Bank of Japan, OECD	The monthly Manufacturing Report On Business Source: The Institute for Supply Management
Consumer opinion survey	Harmonised EU Consumer opinion survey Indicator a) Source: European Commission	The Consumer Behavior Survey Source: Economic and Social Research Institute, Cabinet Office	University of Michigan Consumer Sentiment Index Source: University of Michigan
Business confidence indicator	Standardised Business Confidence Indicator a) Source: OECD	Standardised Business Confidence Indicator Source: OECD	Standardised Business Confidence Indicator Source: OECD
Consumer confidence indicator	Standardised Consumer Confidence Indicator a) Source: OECD	Standardised Consumer Confidence Indicator Source: OECD	Standardised Consumer Confidence Indicator Source: OECD
Industrial production	Volume Index of Production b) Source: Eurostat	Indices of Industrial Production Ministry of Economy, Trade and Industry	INDPRO, Industrial Production Index Source: Board of Governors of the Federal Reserve System
Consumption	Private Final Consumption Expenditure b) Source: Eurostat	Final Consumption Expenditure Source: Economic and Social Research Institute, Cabinet Office, IMF, OECD	Final Consumption Expenditure Source: Bureau of Economic Analysis

a) Euro area, 12 EMU countries up to 2006. 13 EMU countries, including Slovenia since 2007.
b) Euro area, 12 EMU countries up to 1989, 13 EMU countries including Slovenia since 1990.
c) Interpolated to monthly series by the OECD.

Figure 2.1 **Industrial Production monthly and Consumption quarterly with growth rates as measured by log-change**

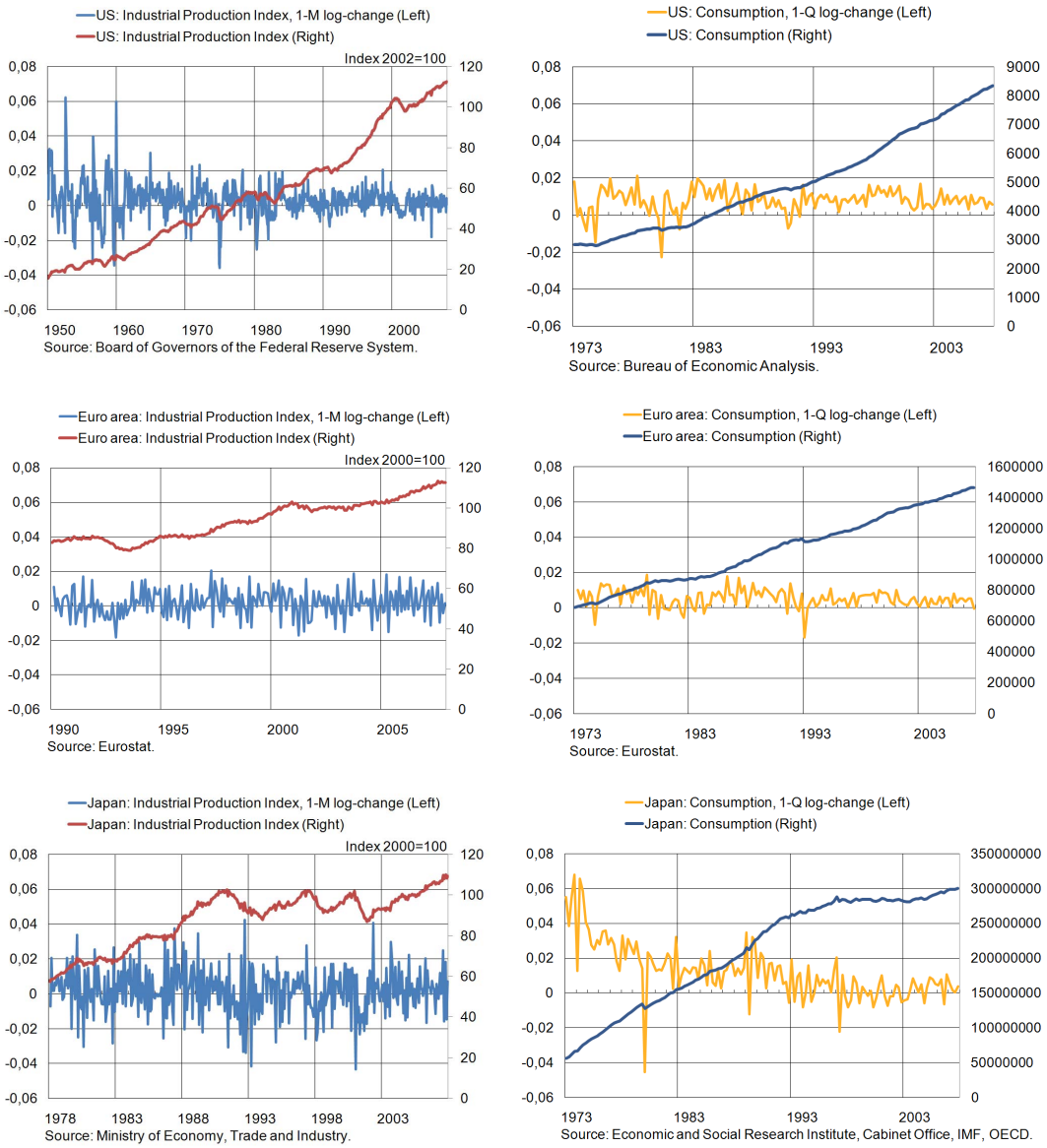
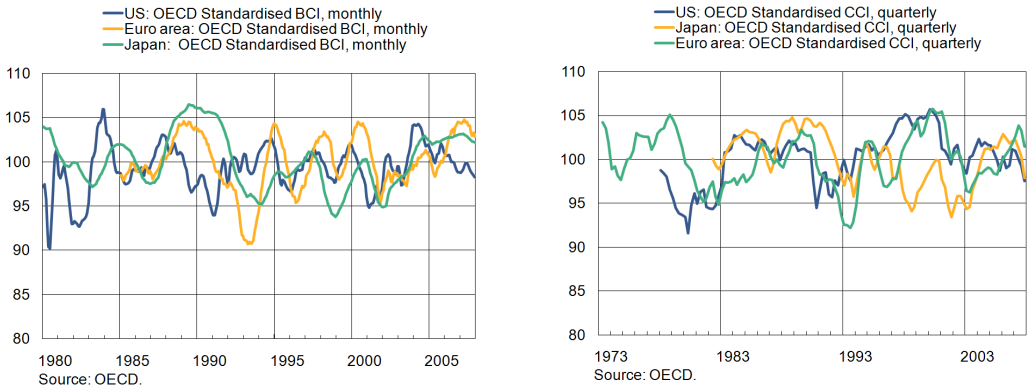


Figure 2.2 **The OECD Standardised Confidence Indicators**



3 The Hilbert-Huang transform

3.1 Frequency domain analysis: a brief synopsis

In terms of the investigation of time series, there are only two basic approaches – analysis in the time domain and analysis in the frequency domain. While most economists are familiar with time-domain techniques, most of the techniques employed in frequency domain analysis have been developed outside the domain of economics, but nonetheless are widely applied in the physical sciences and medical fields.

For most economists frequency domain analysis is synonymous with spectral analysis. But although spectral analysis has quite a long history in economics, beginning in fact with research in the 1920s alongside the original work done on the business cycle, most economists are only familiar with the oft-cited article by Granger (1966). Since then there has been relatively little research done using frequency domain techniques beyond this[1] up until relatively recently. Spectral analysis (which was originally invented by the mathematician Fourier in 1811 to solve the problem of how heat dissipates in a metal plate) essentially decomposes a periodic function into a sum of simple oscillating functions, namely sines and cosines. Applying a Fourier transform to a time series decomposes the time series by convolving sine and cosine functions to the series in order to represent the oscillations present in the series and subsequently estimate these frequencies. Any linear combination of sines and cosines that is used to represent the original time series is known as a Fourier series.

[1] Perhaps the two exceptions here would be James Ramsay (NYU) and John Pippenger (UC Santa Barbara).

In mathematical terms, spectral analysis can be defined quite simply as the sum of an infinite series of harmonic functions

$$f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos(nx) + b_n \sin(nx)) \tag{3.1}$$

where

$$a_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \cos(nx) dx \tag{3.2}$$

and

$$b_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \sin(nx) dx \tag{3.3}$$

These functions impose clear constraints on the series being analysed – the series should be stationary, and it is also clearly assumed to be generated by linear combinations of harmonic functions. Basic spectral analysis also assumes that different fluctuations in the series stay constant through time, which, other than for purely deterministic series, is clearly not the case. To mitigate this particular concern Windowed Fourier analysis was invented so as to essentially do spectral analysis on 'windows' which are passed down the time series. In more conventional statistical language this is called the short time Fourier transform (STFT) with a rectangular window, but of course if this is simply moved to different parts of the series we can get abrupt changes (leading to what is known as the Gibbs phenomenon) as well as incorrect estimates of frequencies, particularly if there are local non-stationarities in the series under study. Subsequent developments in spectral analysis focused on development of new windowing techniques and functions, some with overlapping windows, in which case weightings on observations can be specified, giving rise to more complicated windows, such as Hamming, Hann, Bartlett, Triangular, Gauss and Kaiser windows, for example.

If instead a window is passed down the length of the time series, the analysis is clearly not just confined to the frequency domain – indeed this type of frequency domain analysis is often called 'time-varying spectral' or 'time-frequency' analysis. Once this step had been taken in the 1970s, it was only a matter of time before more flexible techniques emerged, mostly from other disciplines (electrical engineering, acoustics and medical imaging) that rely more heavily on this type of analysis.

In the 1980s a chance meeting between a French mathematician (Daubechies) and signal processor (Mallat) gave birth to wavelet analysis, and in its discrete

form what Mallat called multiresolution decomposition. The idea of wavelet analysis is to apply different functions other than harmonic functions to time series, functions that are perhaps asymmetric (as with the Daubechies wavelet) or have certain features (such as a discrete step 'Haar' wavelet function). The mathematical background for wavelets can be found in Daubechies (1992). The basic idea behind wavelet analysis is to decompose a time series into components corresponding to the fluctuations embedded in the series over pre-defined ranges of frequencies. Much of the background for this approach and some applications in economics and finance can be found in Crowley (2007).

One perhaps might ask why wavelet analysis is seen as superior to time-varying spectral analysis using the STFT? The answer lies in the uncertainty principle, formulated by the quantum physicist, Heisenberg, which states that locating a particle in a small region of space makes the velocity of the particle uncertain; and conversely, that measuring the velocity of a particle precisely makes the position uncertain. Applying this to frequency domain analysis implies that the exact frequency and time information of a variable at some certain point in the time-frequency plane cannot be known, so that although we can resolve high frequencies very well in time, it is less easy to resolve their exact frequency, but conversely with low frequencies, they can be much better resolved in frequency, but much less well resolved in time. That means the best we can do is to look at the frequency components that exist over any given period of time, and use different 'basis' functions as a means of matching our series with some non-harmonic short function (or wavelet). Wavelets, because they use short functions (and in the discrete version of the multiresolution decomposition they utilize both a trend wavelet function and a 'fluctuation' wavelet – referred to in the literature as 'father' and 'mother' wavelets respectively), means they can also analyse non-stationary time-series, which is a further advantage over traditional spectral analysis.

Wavelet analysis, although only around now for 20 years, has had a huge impact on frequency domain analysis. But there are still some remaining issues with wavelet analysis. First, wavelet analysis relies on some arbitrary selection of wavelet 'basis' function, which inevitably can lead to some incomplete results if the matching of the series with the wavelet is suboptimal. Second, wavelet analysis produces a new series for each scale (or frequency range), where the range of each scale is pre-defined in dyadic terms. So if a particular series had embedded frequencies operating at both a 5 year and a 7 year cycle, then as the pre-defined scale ranges from 4 to 8 year cycle fluctuations, these two cycles would be combined and represented by a single series (or 'crystal') at one particular scale. Third, there is 'leakage' at the edges of each of these pre-defined scales – so if our pre-defined scale was as above, and we have a 4 year cycle, it might show up in two scale crystals, leading to the impression that there are two separate embedded frequency cycles when in fact there is only one.

Because of these problems, the Hilbert-Huang transform was introduced as an alternative to both spectral and wavelet analysis (as well as other frequency domain methods not discussed here such as waveform dictionaries and the Wigner distribution).

3.2 The basics of the Hilbert-Huang transform (HHT)

3.2.1 Introduction

The Hilbert-Huang transform (HHT) consists of two steps, of which only the first step is new – namely Empirical mode decomposition (EMD). EMD was pioneered by Huang (1998), a NASA oceanographer and research scientist, who developed an algorithm which is not mathematically based, as with spectral and wavelet analysis, but is instead an empirical method for extracting the embedded frequencies (or what signal processors call 'modes') in any given time series. The method does not rely on any 'basis' function as with spectral and wavelet analysis, and is a method that operates in the time domain rather than in the frequency domain. The second part of the HHT uses the Hilbert transform to obtain instantaneous frequency estimates using the 'mode' data around any given point in time. The resulting output is an improved decomposition and better resolution of a series into its constituent frequency components. Probably the best review of the HHT and further more recent developments in the methodology can be found in Huang and Wu (2008). Table 3.1 gives a basic comparison of empirical methods of analysis.

Very little has been done in economics using the HHT, but there are two notable exceptions, one in Huang and Shen (2005).where the HHT is applied to US mortgage rates and the other by Zhang, Lai and Wang (2007) which is an application to oil prices. The interested reader can also consult Crowley (2009) for more background on the method and some applications to economic and financial time series.

Table 3.1 **Comparison of empirical methods**

Feature	Time series	Spectral	Wavelet	EMD
Stationarity?	Yes	Yes	No	No
Linear?	Yes	Yes	No	No
Theory?	Yes	Yes	No	No
Adaptive?	Limited	No	Limited	Yes
Basis?	A priori	A priori	A priori	A posterior
Measurement?	Variance	Energy in frequency domain	Energy in time-frequency space	Energy in time-frequency space
Calculations?	Time domain	Frequency domain	Frequency domain	Time domain

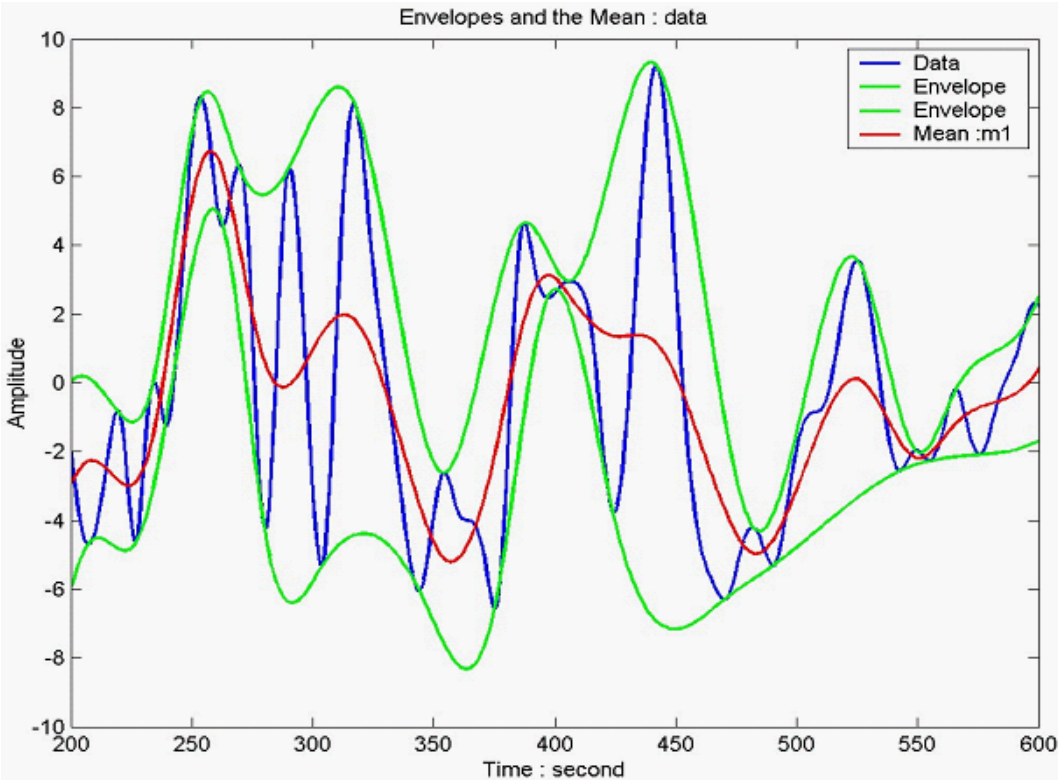
3.2.2 EMD Process

EMD employs a sifting process as follows:

1. Identify all local local extrema;
2. Connect all local maxima using a cubic spline to form an envelope, and do likewise for the minima;
3. Calculate the mean of the upper and lower envelopes and denote m_1.
4. Subtract the mean from the series itself, so in equation form, we are left with the so-called 'first protomode':
$$h_1 = x_t - m_1 \tag{3.4}$$
5. Repeat 2 to 4 above until a stopping criterion is met after k sifts, giving the first intrinsic mode function (IMF):
$$c_1 = h_{1k} \tag{3.5}$$
6. Subtract this first IMF from the original data to obtain r_1, the first 'residual':
$$x_t - c_1 = r_1 \tag{3.6}$$
7. Start the whole process from 1 again but using r_1 as new data.

Figure 3.1 illustrates steps 1 to 3 above for the EMD process.

Figure 3.1 **Illustration of EMD process**



The resulting process can be summarised as the decomposition of the original time series into a number of intrinsic mode functions (IMFs) and a residual that describes the trend. This can be mathematically expressed as follows

$$x_t = \sum_{j=1}^{n} c_j + r_n \tag{3.7}$$

The output from the whole process is therefore n IMFs and a residual which represents a (non-linear) trend, where the IMFs can be thought of as the equivalent of 'crystals' from wavelet analysis, except that they are not confined to lying in the limits imposed by using pre-defined scales.

3.2.3 The Hilbert transform and frequency analysis

The first part of the process described above is uniquely in the time domain, but then to obtain frequencies for each mode, the analysis of the output from this process must take place in the frequency domain. The Hilbert spectrum uses a

Hilbert transform to obtain an estimate of the instantaneous frequency of any time series.

For a time series x the Hilbert transform, y_t is given by

$$y_t = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{x_\tau}{t-\tau} d\tau \tag{3.8}$$

where P is the Cauchy principal value of the singular integral. Given the Hilbert transform of any arbitrary function $f(x_t)$ we obtain the analytical output z_t, where

$$z_t = a_t e^{i\theta_t} \tag{3.9}$$

Note that this output is of necessity complex. The term a_t then represents the amplitude at time t, and the phase of the frequency is given by θ_t, which can then be used to estimate the instantaneous frequency, ω, which is given as

$$\omega = \frac{d\theta}{dt} \tag{3.10}$$

Unfortunately there are two problems regarding the Hilbert transform, notably one of underestimating the true frequency (due to the so-called 'Nuttall theorem'), and also not completely extracting the frequency content of the data but leaving some information in the amplitude variable (due to the so-called 'Bedrosian theorem'). Because of this, there has been new research (see Huang, Wu, Long et al, 2008) which suggests that a direct quadrature method might be an improvement over the Hilbert spectrum for calculating instantaneous frequency. Here we use the direct quadrature method.

3.3 Ensemble EMD

In this paper we use an Ensemble EMD methodology to separate out the embedded frequencies in economic variables. Ensemble EMD (or EEMD) came about because of various problems with EMD associated with 'mode mixing'. 'Mode mixing' occurs when either:

- different frequencies are found to reside in the same IMF; or when
- similar frequencies are found to reside in different IMFs.

Wu and Huang (2004) note that various solutions to this problem (notably the 'intermittence test' in Huang, Shen and Long (1999)) were only partially

successful in mitigation, so a new approach was clearly needed. Research done by Flandrin, Rilling and Gonclaves (2004) and Flandrin and Gonclaves (2004a) with EMD showed that under certain circumstances the EMD acted like a filter bank (ie a constant band pass shape), equivalent to analysis by using wavelet analysis. Further research by Flandrin et al (2005) showed that when applied to white noise, the EMD was equivalent to the dyadic filter bank used by the discrete wavelet transform. Later Wu and Huang (2004) realised that by adding white noise to the data and running EMD many times this will properly reduce the problem of 'mode mixing', and yield greater resolution of the IMFs. This is mostly due to the fact that the cubic spline calculation requires well-defined extrema and by adding white noise to the data these extrema are more accurately located, giving significant improvements in the EMD results.

In theory, given N ensemble members, and if ε is the amplitude of the white noise added to the data, then it can be shown that the resultant standard deviation of the error (the difference between the variable and the sum of the corresponding IMFs) amounts to ε_n, where

$$\varepsilon_n = \frac{\varepsilon}{\sqrt{N}} \tag{3.11}$$

So when adding a certain amount of white noise to the data, the larger the ensemble (N), the lower the error, and so improved results are obtained. The steps of EEMD then are as follows:

a. add white noise to the series;
b. decompose the data with added white noise into IMFs using EMD;
c. repeat 1 and 2 N times, but with different white noise added each time; and
d. obtain the means of the 'ensemble' IMFs from each EMD decomposition.

Essentially the white noise added to each run will cancel out when averaged, and so the mean IMFs obtained will be less likely to suffer from 'mode mixing' and will more likely be better resolved, particularly at higher frequencies.

Wu and Huang (2008) spend considerable time on applications of EMD and show that if executed properly it can extract the embedded frequencies operating in most non-linear and non-stationary empirical data. It obviously has the potential to be of major benefit to any data analysis toolkit.

3.4 Estimation strategy with economic variables

Here, as we are mostly concerned with lower frequency IMFs, there should be less concern with using the EMD with economic variables. Nevertheless, the instantaneous frequencies obtained in the analysis occasionally displayed high levels of 'mode mixing' (– particularly in the case of some of the sentiment indicators), so most of the indicator variables were run first using basic EMD and then if 'mode mixing' occurs, these were re-run using EEMD with white noise with variance of 20% of the variance of the actual series added for the ensemble estimation and a value of N=400 for the ensemble.

4 Results

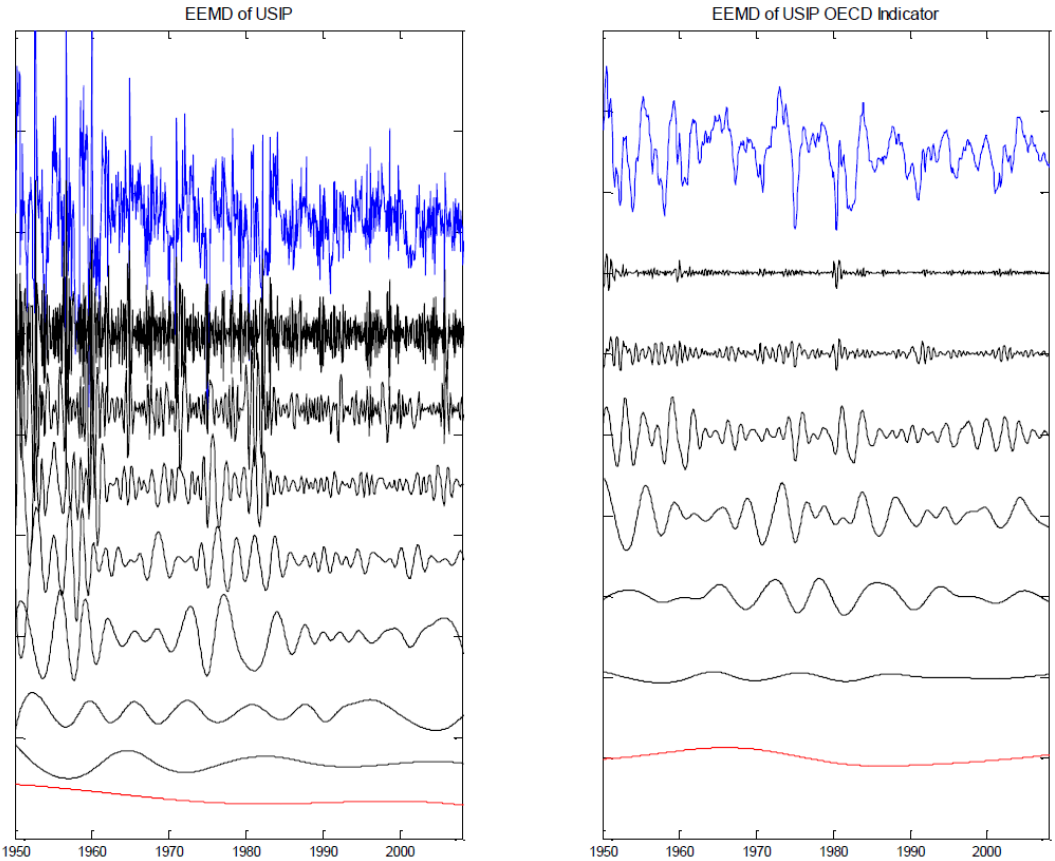
In all the results that follow, the data used for the economic (counterpart) variable to be analysed is the change in the logged value of the variable, and for the indicators, the actual level of the indicator is used for comparison. This is the most appropriate comparison, as most indicators are constructed so that they 'indicate' a rise or fall in the economic variable counterpart.

4.1 The US

4.1.1 Industrial production: US industrial production vs OECD business confidence indicator

The left hand panel of figure 4 shows the original series (in blue) for the economic variable ($\ln[USIP_t]-\ln[USIP_{t-1}]$), where $USIP_t$ is the US industrial production volume at month t, and the IMFs are shown beneath the variable in black (and on the same scale) and then the last series in the stackchart (in red) represents the residual (or adaptive trend). The same is done for the indicator variable (– in this case the OECD leading indicator for industrial production) in the right hand panel of figure 4.1.

Figure 4.1 **US industrial production vs OECD business confidence indicator: IMFs**



There are several things to note from the plot. First, 7 IMFs were extracted from USIP, where as only 6 IMFs were extracted from the OECD indicator. Obviously the more noise in the series to begin with, the more higher frequency IMFs will be extracted from the series. On inspection it appears that there is considerable correspondence in movements of IMF_i(USIP) with IMF_{i-1}(USIPIndicator), and indeed this shows up in the (maximal lag) correlation of the IMFs in table 3.1 where the strongest (significant[2]) correlation is between IMF7 for USIP and IMF6 (0.5609) for the indicator, and IMF5 for USIP against IMF4 for the indicator (0.6833). The second thing to note is that the business cycle is not separately identified within a particular frequency, and its location depends, as might be expected, on the shape of the downturn; so for the short sharp recession of the early 1980s this downturn is found in IMF5, but for the more shallower downturn in the early 2000s, this downturn is located in IMF6. Third it is also apparent that

[2] The p-value is computed by transforming the correlation to create a t statistic having n-2 degreees of freedom, where n is the number of rows being correlated and the variable is assumed to have a multivariate normal distribution. The confidence bounds are based on an asymptotic normal distribution of 0.5*log[(1+R)/(1-R)) with variance equal to 1/(n-3).

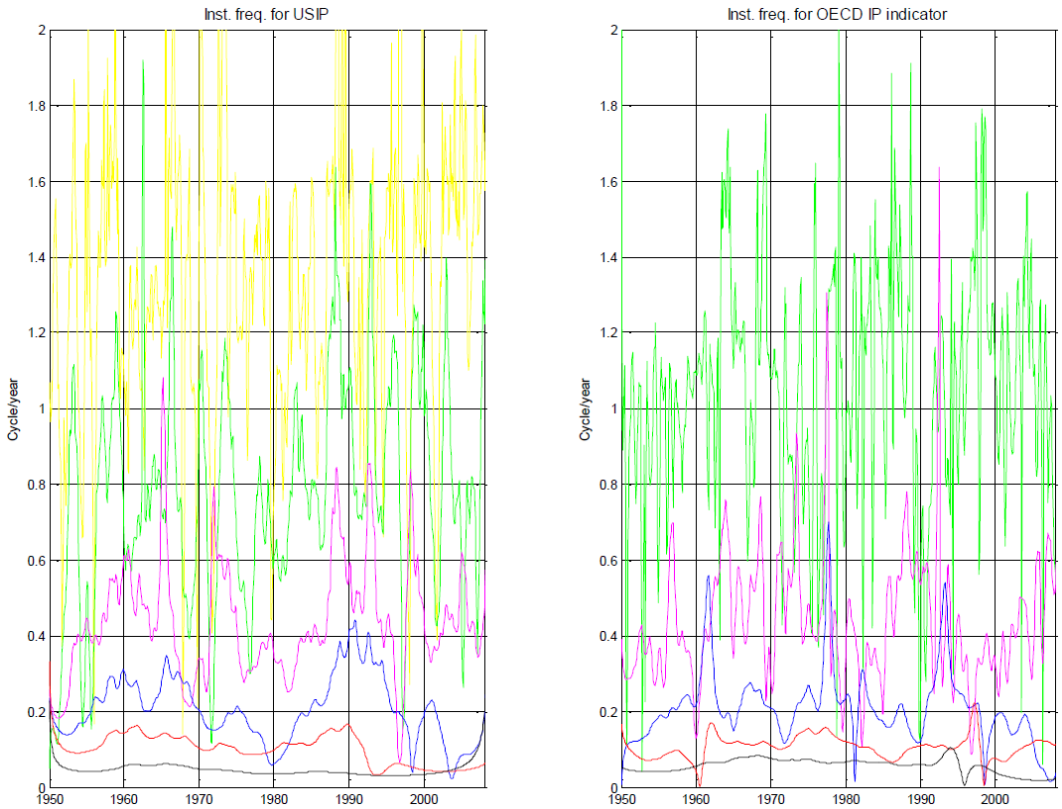
there are other cycles in the data besides the business cycle, and some of these fluctuations in the data are clearly of major significance in terms of amplitude of fluctuation. These cycles are what Zarnowitz and Ozyildirim (2002) have termed 'growth cycles', and we use the same terminology here for the remainder of the paper. Fourth, the residual is different for the indicator and for industrial production – the average level of industrial production increases appears to be falling over time, whereas the indicator perhaps appears to have a rather long cycle, although this is difficult to tell without more data.

Table 4.1 **Correlations of IMFs for US industrial production (USIP) and OECD business confidence indicator (overall = 0.5389) with frequencies**

Correlations of USIP	IMF2	IMF3	IMF4	IMF5	IMF6	Frequency
IMF2	0.4178*	-0.0206	-0.0207	0.0026	-0.0059	1.3477
IMF3	0.0998	0.366*	0.2186*	-0.0245	0.1038*	0.7802
IMF4	0.0466	0.3362*	-0.1329	-0.028	-0.1465	0.4379
IMF5	0.0206	0.2922*	0.6833*	0.2984*	0.1725*	0.2063
IMF6	0.0499	-0.0439	-0.1519	0.2989*	0.0296	0.1007
IMF7	-0.013	0.0213	0.0915*	0.0586	0.5609*	0.0486
Frequency	1.0464	0.4424	0.2074	0.1062	0.0564	

Correlations that are significant at the 5% level are denoted with an asterisk.

Figure 4.2 **US industrial production vs OECD business confidence indicator: instantaneous frequency**



The frequency content of the cycles found in the data and in the indicator are shown in figure 4.2. The separation of the IMFs in terms of frequency is clear for most of the series although in the 1980s there is 'mode mixing' and in the late 1990s it reoccurs with IMF5. The same patterns are evident in the OECD indicator, which implies that the indicator captures some of the changing frequency content associated with the different embedded cycles. Interestingly in terms of longer cycles there is a 10 year cycle evident in the data as well as a 20 year cycle. Nothing beyond this is detected.

4.1.2 Industrial production: US industrial production vs Business sentiment indicator

In figure 4.3 we show the IMFs obtained from the business sentiment index against the industrial production data. Note that this is a shortened time series so we obtain slightly different results for USIP. In figure 4.4 the instantaneous frequencies are also plotted.

Figure 4.3 **US industrial production vs Business sentiment indicator: IMFs**

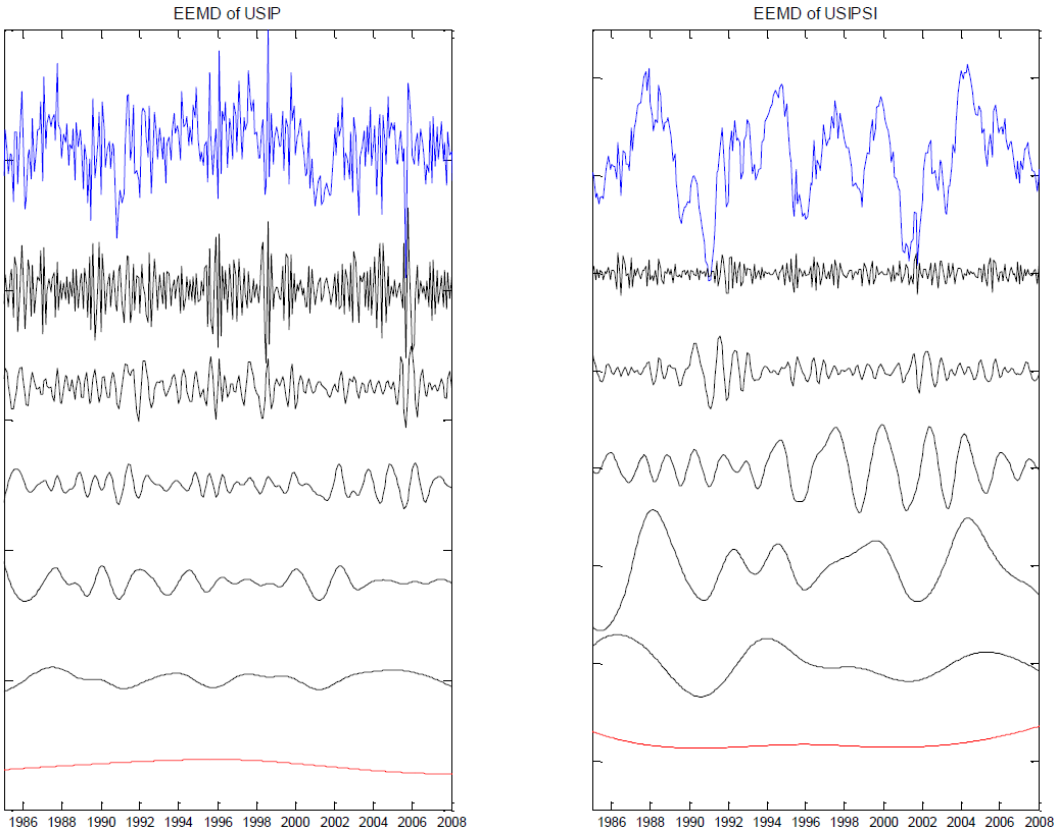
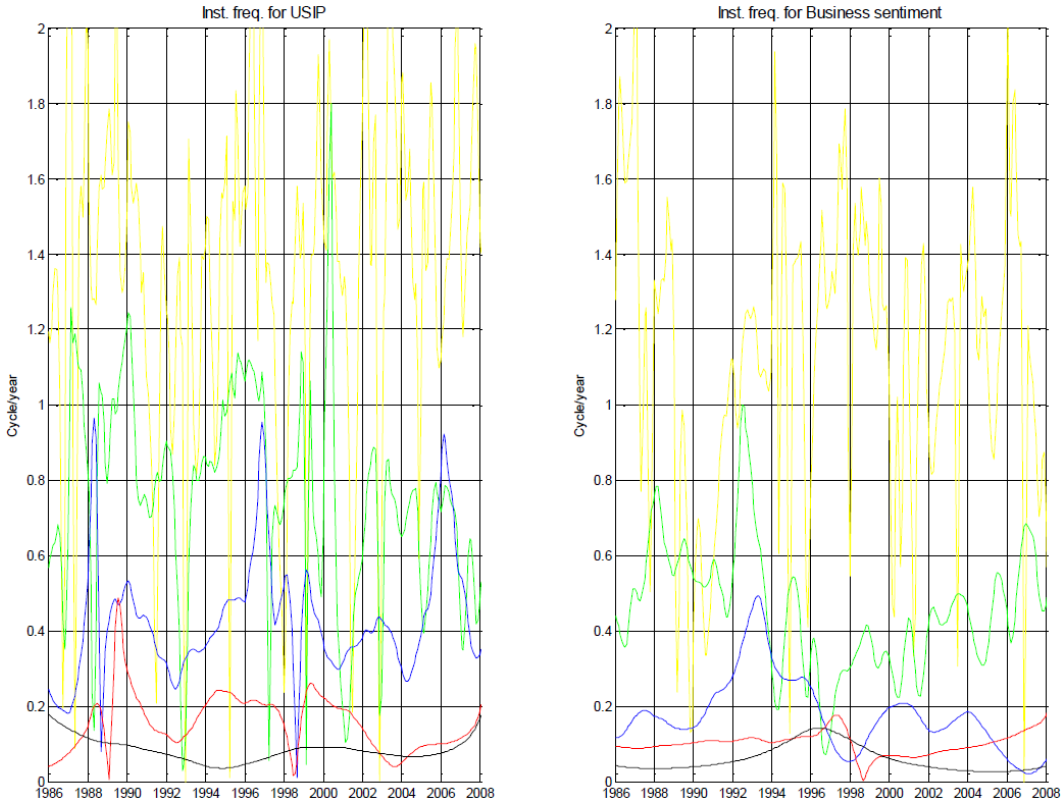


Table 4.2 **Correlations of IMFs for US industrial production (USIP) and business sentiment indicator (overall = 0.40) with frequencies**

Correlations of USIP	IMF2	IMF3	IMF4	IMF5	IMF6	Frequency
IMF2	0.2168*	0.049	-0.0067	-0.018	-0.0137	1.3563
IMF3	0.3169*	0.2445*	-0.0009	0.0632	0.0344	0.7433
IMF4	0.2332*	0.5131*	0.3141*	-0.0987	0.016	0.4260
IMF5	0.0324	0.1784*	0.6824*	0.4707*	-0.1473	0.1499
IMF6	-0.022	0.0647	0.1339*	0.5708*	0.5425*	0.0927
Frequency	1.1053	0.4624	0.1721	0.1013	0.0578	

Correlations that are significant at the 5% level are denoted with an asterisk.

Figure 4.4 **US industrial production vs business sentiment indicator: instantaneous frequency**

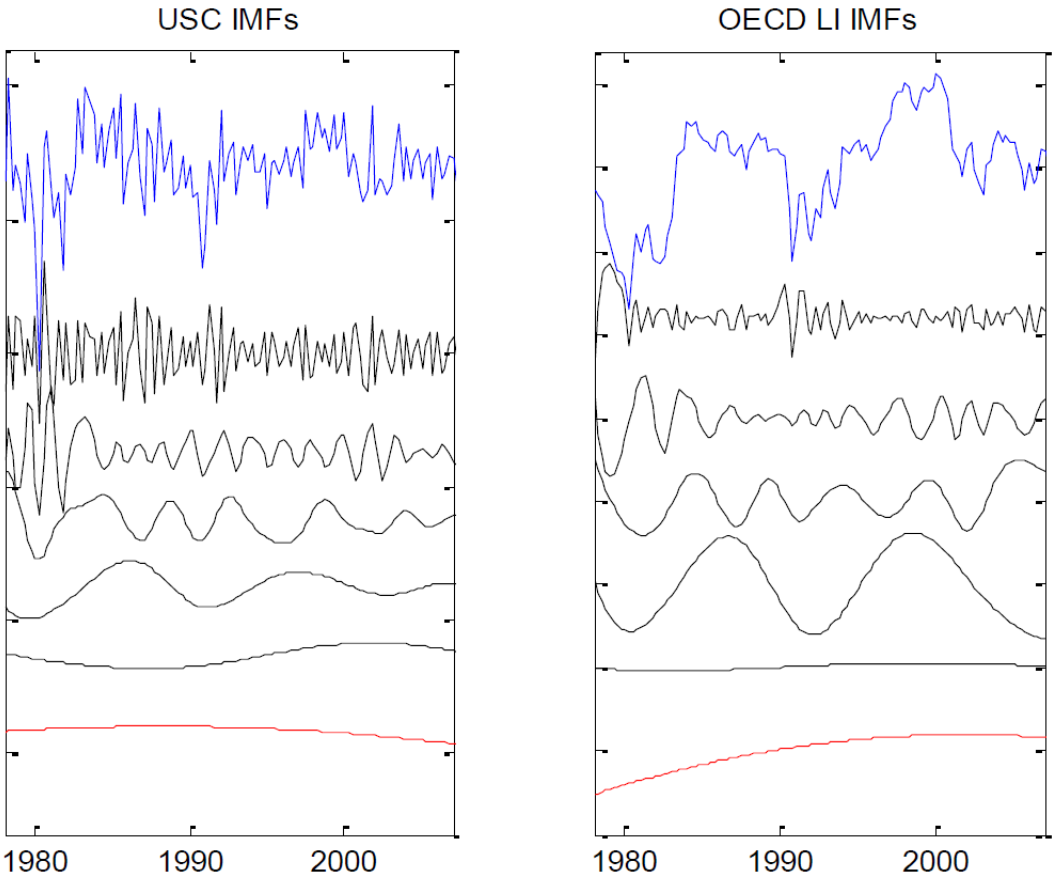


In this instance we extract the lower frequency for table 4.2, despite the fact there is very little evidence of a longer cycle in the short span of data that we have here. Nevertheless, we know from the full dataset that a longer (around 20 year) cycle is identified in the data. Here the business cycle is picked up in USIP in both IMF4 and IMF5, but in the business sentiment index the business cycle is located in IMF5, which appears to be a high-amplitude frequency component. It is noteworthy that IMF4 in USIP is also picked up quite well by IMF3 in the business sentiment index, but these growth cycles are less well identified than the business cycle as a whole. In terms of the frequency plots, very little mode-mixing occurs with USIP, but significant mode-mixing appears to occur in the mid-1990s for the sentiment index, but this appears to be a temporary phenomenon.

4.1.3 Consumption: US consumption vs OECD consumer confidence indicator

Figure 4.5 shows the IMFs from the EEMD of US consumption growth and the OECD leading indicator.

Figure 4.5 **US consumption vs OECD consumer confidence indicator IMFs**



Here the original series clearly shows the 'Great moderation' in terms of a narrowing of the amplitude of fluctuations in consumption growth, but this is not readily apparent when looking at the indicator. In terms of the IMFs, the business cycle is contained mostly in IMF4 for both the original series and the indicator, with the notable exception being the 1982 recession which shows up as a shorter frequency fluctuation in IMF2 in both cases. What is particularly interesting here is that the growth cycle in IMF2 and IMF3 are largely reflected in the OECD indicator growth cycles, and that when looking at the business cycle in IMF4, there is a clear dampening in the actual variable, but this is not apparent in the indicator variable. This suggests that although economic indicators predict a usual

size of amplitude in the business cycle, this does not tend to appear, perhaps due to offsetting measures, such as countercyclical fiscal and monetary policy.

When looking at the correlation between the cycles, it is clear that the correspondence between the IMFs for both consumption growth and the OECD consumption indicator is very good, except for the very high growth cycle frequencies.

Table 4.3 **Correlations of IMFs for US consumption (USC) and the OECD consumer confidence indicator (overall = 0.40) with frequencies**

Correlations of USC	IMF2	IMF3	IMF4	IMF5	Frequency
IMF2	0.0488	-0.0302	-0.0111	-0.0516	1.23
IMF3	-0.0618	0.5306*	0.0679	-0.1457	0.54
IMF4	0.1919	0.2255*	0.7229*	0.1321	0.25
IMF5	-0.0262	0.193	0.0259	0.7892*	0.08
Frequency	1.00	0.42	0.19	0.08	

Correlations that are significant at the 5% level are denoted with an asterisk

4.1.4 Consumption: USC vs consumer sentiment indicator

Here the Michigan index of consumer sentiment is used and the embedded frequency fluctuations compared with the growth of consumption expenditures. Figure 4.6 shows the IMFs obtained from EEMD. The IMF4s appear to correspond well to the business cycle, with the IMF3 picking up the 1974 recession for the sentiment index. One of the surprising aspects of the analysis is that other growth cycles (IMF2 and IMF3 for example) do not appear to be picked up as well as might be expected. This is confirmed in table 4.4 where consumption growth IMF3 is best picked up by 2 IMFs in the sentiment index (IMF2 and IMF3), but correlations are relatively low (but significant) in each case. This also comes out in the frequency plots in figure 4.7, where there appears to be frequency mis-matches between the IMFs, but of course this is because some of the features in consumption growth are captured by different IMFs in the consumer sentiment index.

What does this imply in economic terms? It implies that consumers know when shocks occur, but have less information about the medium to longer term impact on consumer spending of these shocks (whether permanent or transitory, for example), and this shows up in the results here. This is in direct contrast to the results obtained for industrial production, where producers appear to have a better idea about the medium to long term impact of shocks on output (perhaps through greater networking and greater monitoring of economic indicators than for consumers).

Figure 4.6 **USC vs consumer sentiment indicator: IMFs**

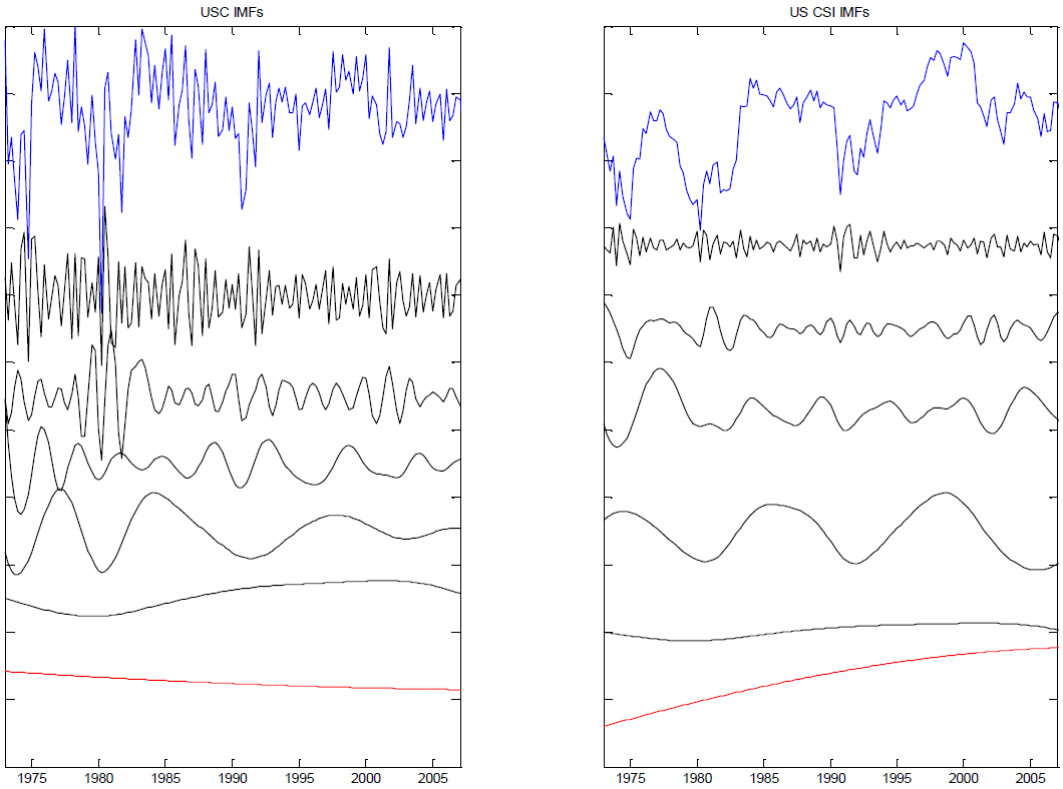
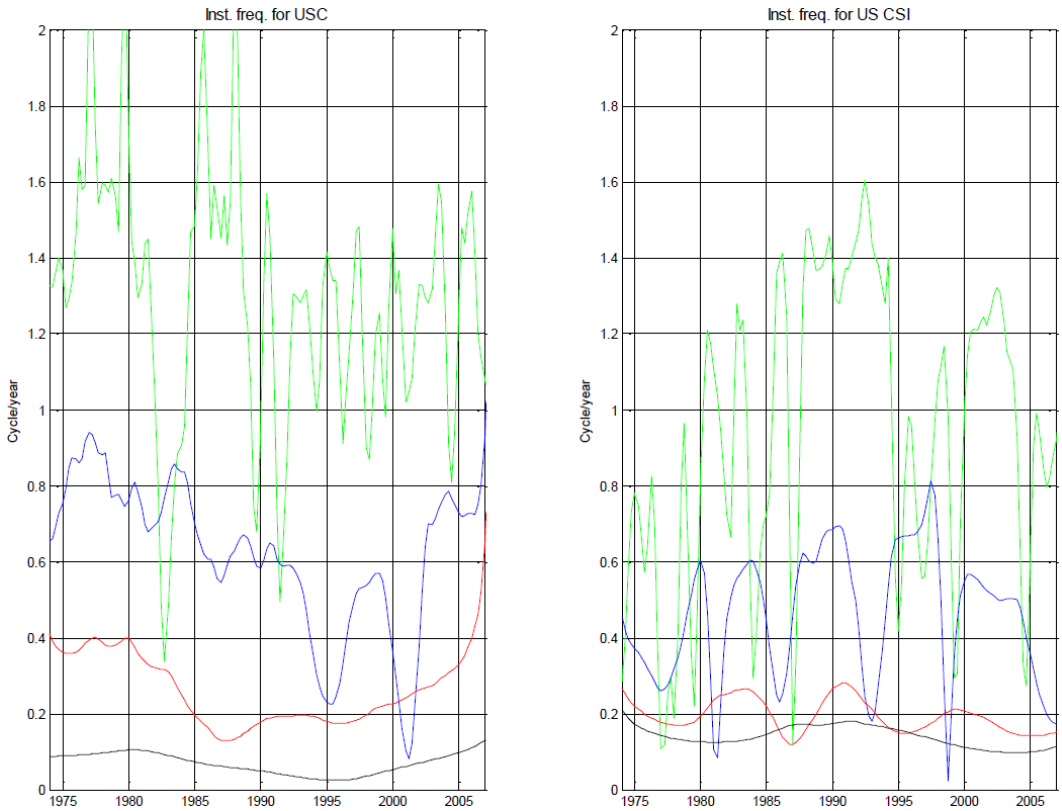


Table 4.4 **Correlations of IMFs for US consumption (USC) and consumer sentiment indicator (overall = 0.458) withfrequencies**

Correlations of USC	IMF2	IMF3	IMF4	IMF5	Frequency
IMF2	0.0934	0.0063	-0.0056	-0.01	1.293
IMF3	0.2127*	0.2913*	-0.0572	0.0141	0.662
IMF4	0.1465	0.5864*	0.4419*	0.1332	0.296
IMF5	0.0008	-0.0484	0.0799	0.4457*	0.0711
Frequency	0.916	0.458	0.197	0.143	

Correlations that are significant at the 5% level are denoted with an asterisk.

Figure 4.7 **US consumption vs consumer sentiment indicator: instantaneous frequencies**



4.2 Euro area

4.2.1 Industrial production: Euro area industrial production vs OECD business confidence indicator

Euro area industrial production and the OECD leading indicator are used here. Clearly these are synthetic data prior to 1999, and so care has to be taken with interpretation during the pre-EMU period. Figure 4.8 shows that in fact with the Euro area as with the US, there is a strong similarity between the IMFs of industrial production growth and the OECD leading indicator but at the (i-1)th IMF level, where i is the IMF number for industrial production growth. Another immediately apparent feature of euro area industrial production growth is that it has a strong high frequency cycle with length of around 2 years, and it also has a 10 year cycle, which presumably reflects the business cycle. Table 4.5 shows this clearly, with the strongest correlation being between IMF5 for industrial production and IMF4 for the OECD indicator. The business cycle is left in the residual for industrial production growth, but is clearly highly correlated (value 0.8487) with IMF5 in the indicator variable. It is a fairly weak cycle though

compared with the other growth cycles found in industrial production, and indeed is much more obviously represented in the composition of the leading indicator.

When looking at the frequencies of the IMFs in figure 4.9 it is clear that IMF2 for the OECD leading indicator has been constructed prior to 1999, as the frequency response is very different from other IMFs. Both a 5 and a 10-year cycle can be detected in the data, and this comes through in both industrial production growth and the OECD leading indicator.

Figure 4.8 **Euro area industrial production vs OECD business confidence indicator: IMFs**

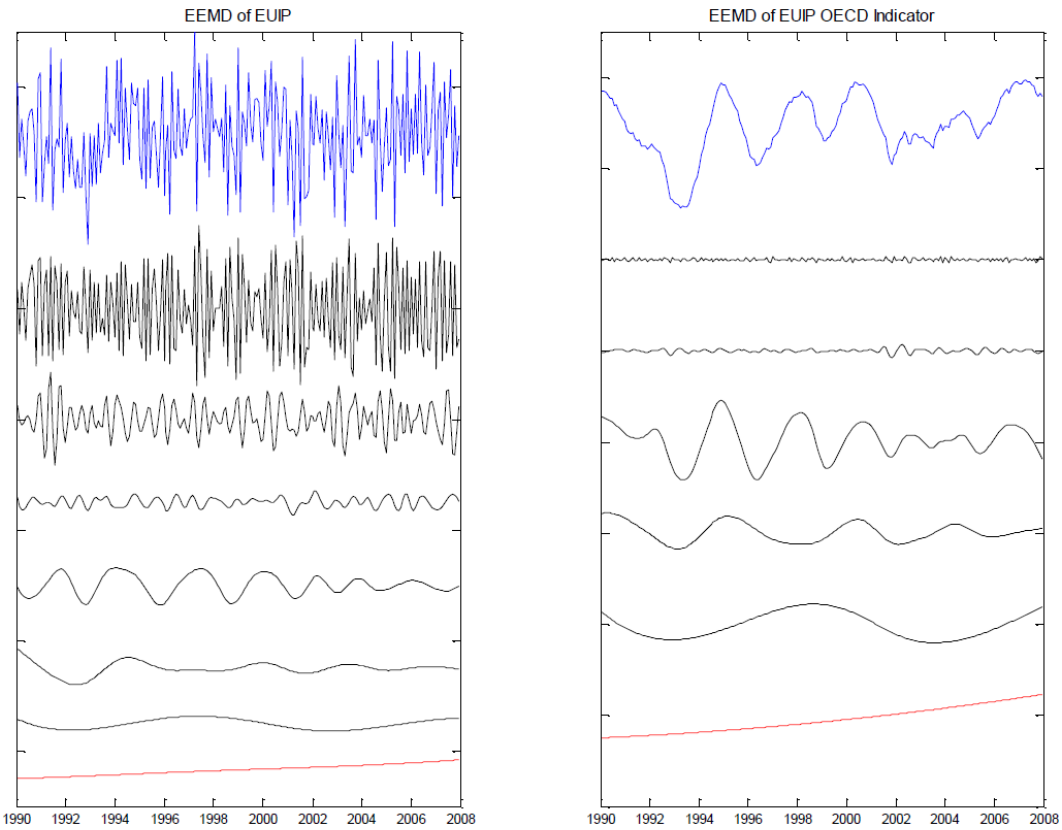
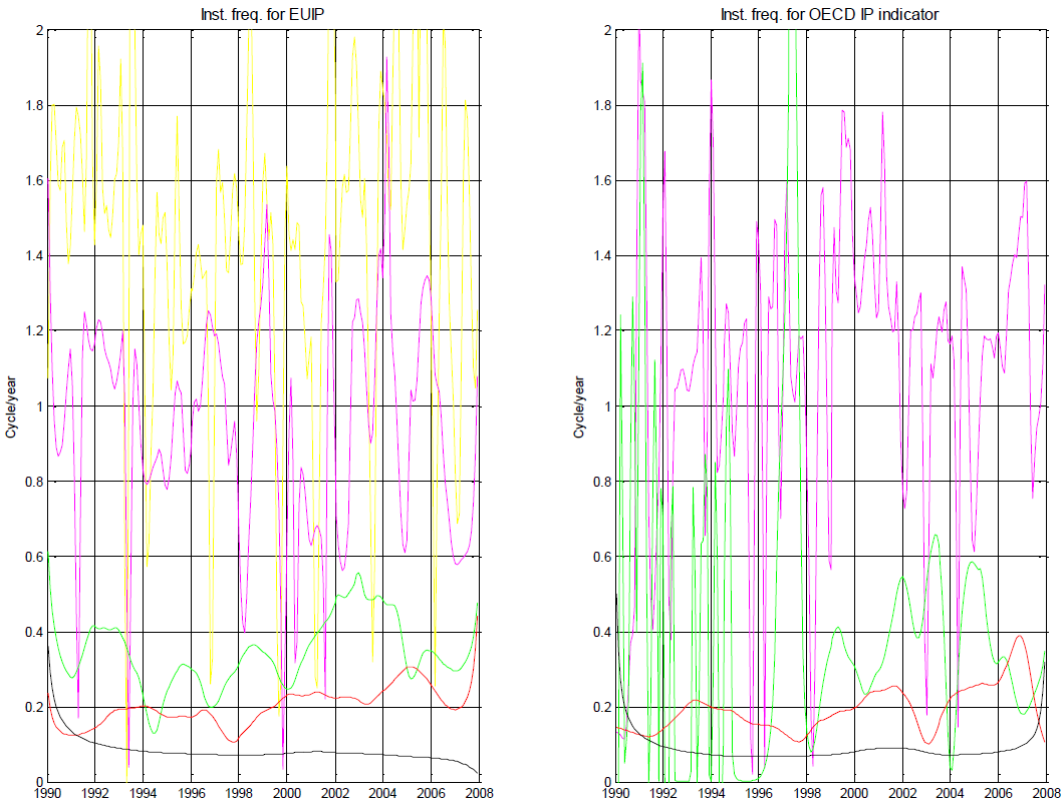


Table 4.5 **Correlations of IMFs for euro area industrial production (EUIP) and OECD Business confidence indicator (overall = 0.182) with frequencies**

Correlations of USC	IMF2	IMF3	IMF4	IMF5	Frequency
IMF2	0.1629*	-0.0045	-0.0061	-0.0175	1.44
IMF3	0.2013*	-0.0393	-0.0201	0.0253	0.94
IMF4	-0.0004	0.1613*	-0.0621	-0.047	0.345
IMF5	-0.002	0.3377*	0.5901*	0.2216*	0.201
IMF6	-0.027	0.08	0.095	0.8487*	0.083
Frequency	1.086	0.400	0.193	0.091	

Correlations that are significant at the 5% level are denoted with an asterisk.

Figure 4.9 **Euro area industrial production (EUIP) vs OECD business confidence indicator: instantaneous frequency**



4.2.2 Consumption: Euro area consumption vs OECD consumer confidence indicator

Here Euro area consumption growth is decomposed alongside the OECD leading indicator for the euro area. Figure 4.10 shows the IMFs and once again it is clear that the longer cycles present in consumption growth are evident in the leading indicator. Shorter cycles though are much less evident in the leading indicator, and this is also apparent from the size of the correlations reported in table 4.6, where shorter cycles have lower correlations than longer cycles. Here the instantaneous frequencies shown in figure 4.11 move together at lower frequencies, but less so at higher frequencies, although there is some mode mixing particularly when these movements in higher frequencies do not match.

Figure 4.10 **Euro area consumption vs OECD consumer confidence indicator: IMFs**

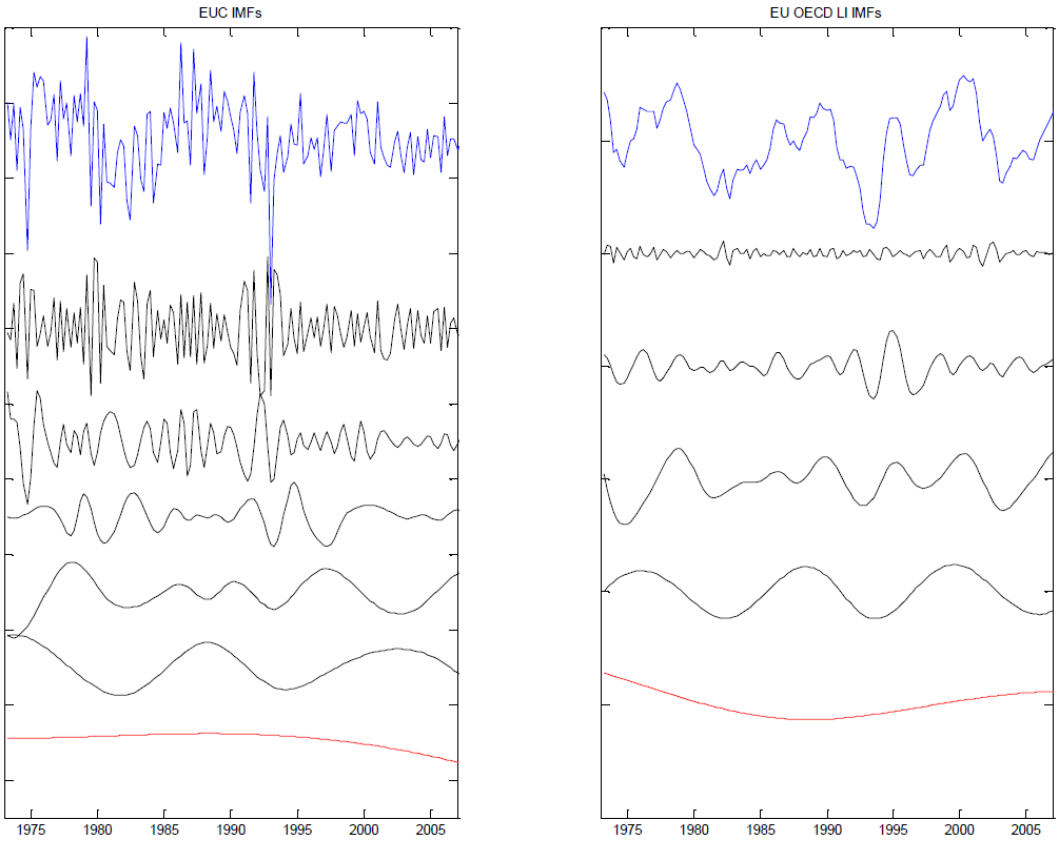
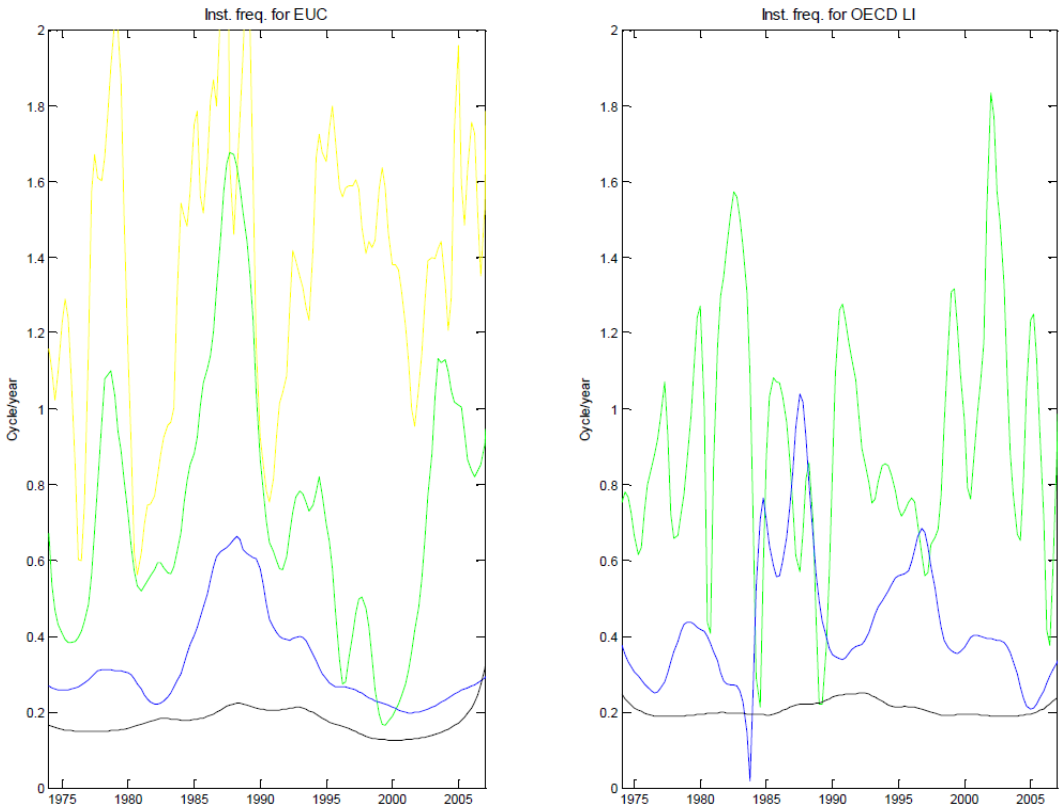


Table 4.6 **Correlations of IMFs for euro area consumption (EUC) and OECD consumer confidence indicator (overall = 0.182) with frequencies**

Correlations of EUC	IMF2	IMF3	IMF4	Frequency
IMF2	0.2198*	0.0012	-0.0156	0.9
IMF3	0.6155*	0.1612*	0.0321	0.5
IMF4	0.0577	0.6624*	0.1297	0.21
IMF5	-0.0061	-0.1884	0.7232*	0.11
Frequency	0.578	0.271	0.133	

Correlations that are significant at the 5% level are denoted with an asterisk.

Figure 4.11 **Euro area consumption vs OECD consumer confidence indicator: instantaneous frequency**



4.3 Japan

Japan's case is an interesting one, as it is a country that underwent a sustained period of stagnation and depression in the 1990s, so hopefully the EEMD approach will still be able to reveal any growth cycles that occurred during this prolonged contractionary period.

4.3.1 Industrial production: Japanese industrial production vs OECD business confidence indicator

Figure 4.12 shows the IMFs for Japanese industrial production and the OECD leading indicator. Unlike in the US or euro area case, here the OECD indicator is clearly non-stationary, but this poses little problem when applying the EEMD approach as the trend component is found in the residual. Here the actual long cycle containing the depression of the 1990s is contained in the JIP residual, but this has no direct corresponding IMF in the indicator, suggesting that the longevity of the depression was not anticipated by the indicator, and that policy

efforts to revive the economy, although showing up in the indicators, largely failed to appear in actual industrial production growth. The level of correlation between industrial production and the indicator is not high, and the highest significant correlation is reported to be between high frequency IMFs. Visual inspection of figure 4.12 suggests that the most prominent IMF from the leading indicator (IMF5) appears to lag that of the similar IMF (IMF4) in the industrial production data.

In terms of frequency, as table 4.7 shows, there is clearly a frequency mismatch that appears to have taken place across all cycles, and for longer frequencies the indicator suggests roughly a 20 year cycle in output growth, where as in fact the longest cycle found in the actual data is an 11 and a half year cycle. This is shown graphically in figure 4.13.

Figure 4.12 **Japanese industrial production vs OECD business confidence indicator: IMFs**

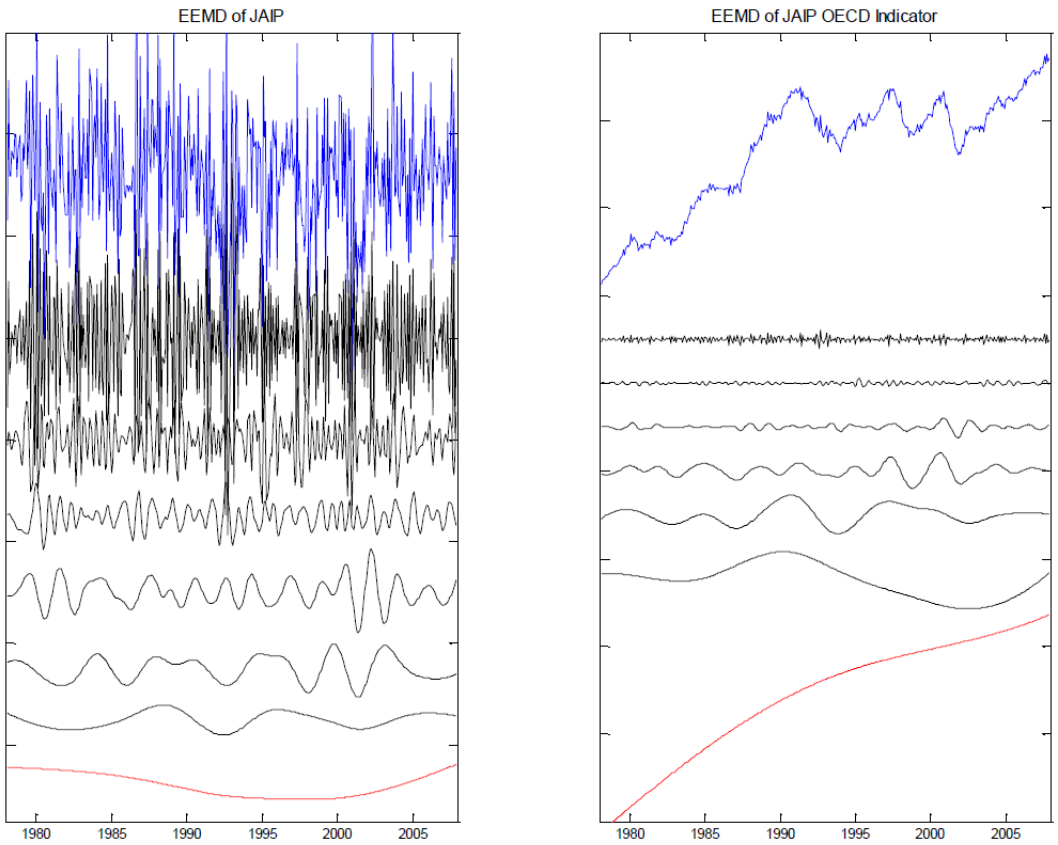
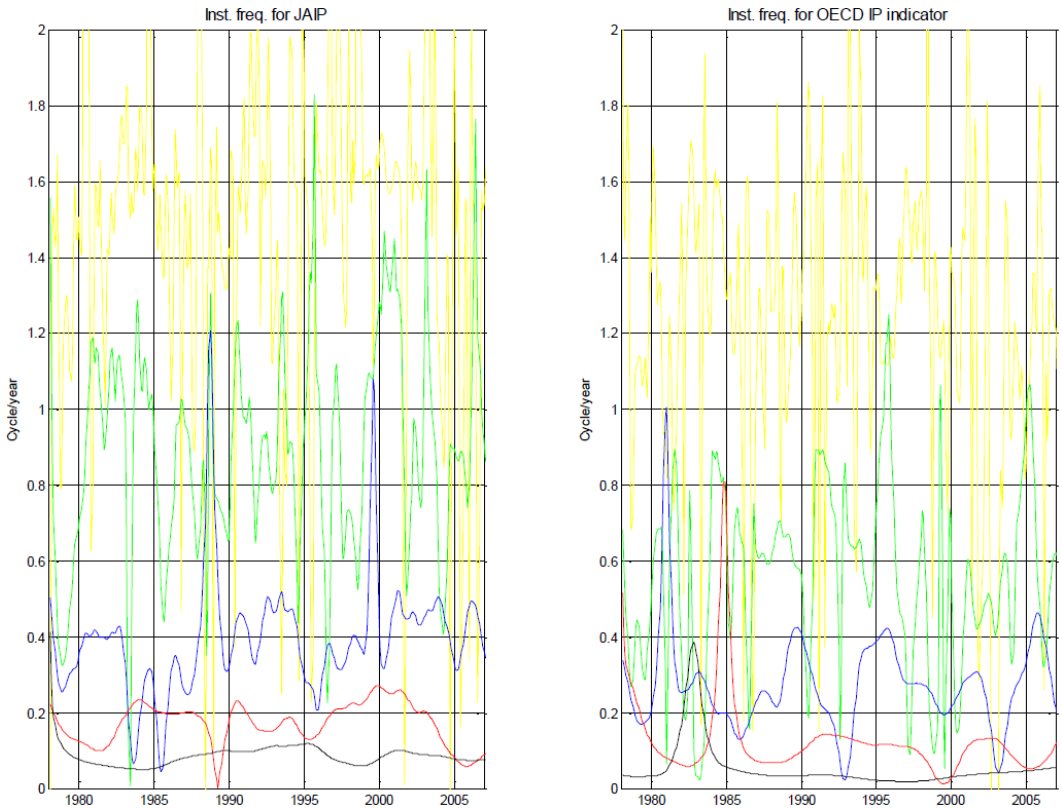


Table 4.7 **Correlations of IMFs for Japanese industrial production (JAIP) and OECD business confidence indicator (overall = -0.0293) with frequencies**

Correlations of JAIP	IMF2	IMF3	IMF4	IMF5	IMF6	Frequency
IMF2	0.212*	-0.0047	-0.0149	-0.0103	0.0088	1.475
IMF3	0.2356*	0.122*	0.0534	0.0202	0.0028	0.889
IMF4	0.0132	0.1168	0.057	-0.0063	-0.0173	0.393
IMF5	-0.0046	0.0439	-0.0176	0.0073	0.0037	0.170
IMF6	0.0045	-0.0049	-0.0167	0.1787*	0.1105	0.086
Frequency	1.198	0.548	0.270	0.126	0.053	

Correlations that are significant at the 5% level are denoted with an asterisk.

Figure 4.13 **Japanese industrial production vs OECD business confidence indicator: instantaneous frequency**



4.3.2 Industrial production: Japanese industrial production vs business sentiment indicator

Here we use the sentiment indicator for Japanese output and decompose it alongside industrial production growth. Figure 4.14 shows the IMFs from the decomposition and once again there appears to be little in common between the

IMFs although the there does appear to be a similar pattern in fluctuations between IMF6 for industrial production and IMF2 and/or IMF3 for the sentiment indicator. Table 4.8 shows the correlations and figure 4.15 plots the instantaneous frequencies. Surprisingly, there is only one significant correlation.

Once again there is a clear mismatch between industrial production growth and the business sentiment index in terms of frequencies of embedded cycles. Obviously why this is the case is an interesting question, as expectations of economic fluctuations are clearly not aligned with those in the actual data.

Figure 4.14 **Japanese industrial production vs business sentiment indicator: IMFs**

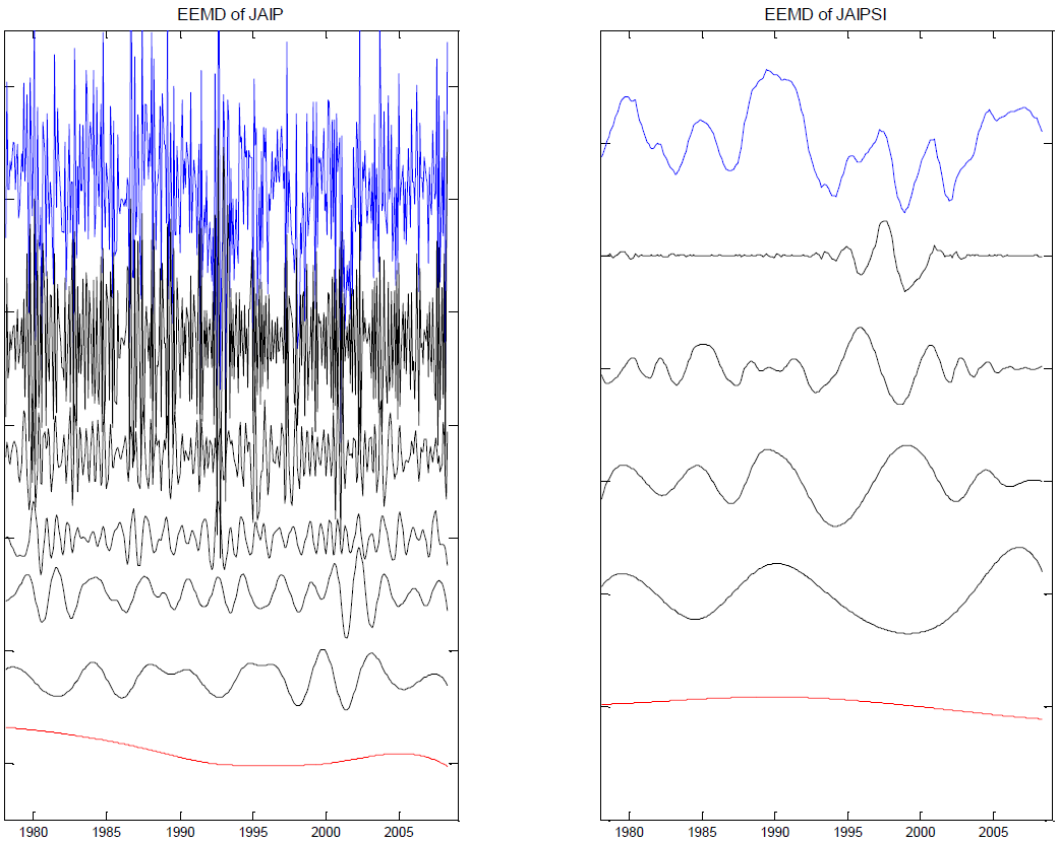
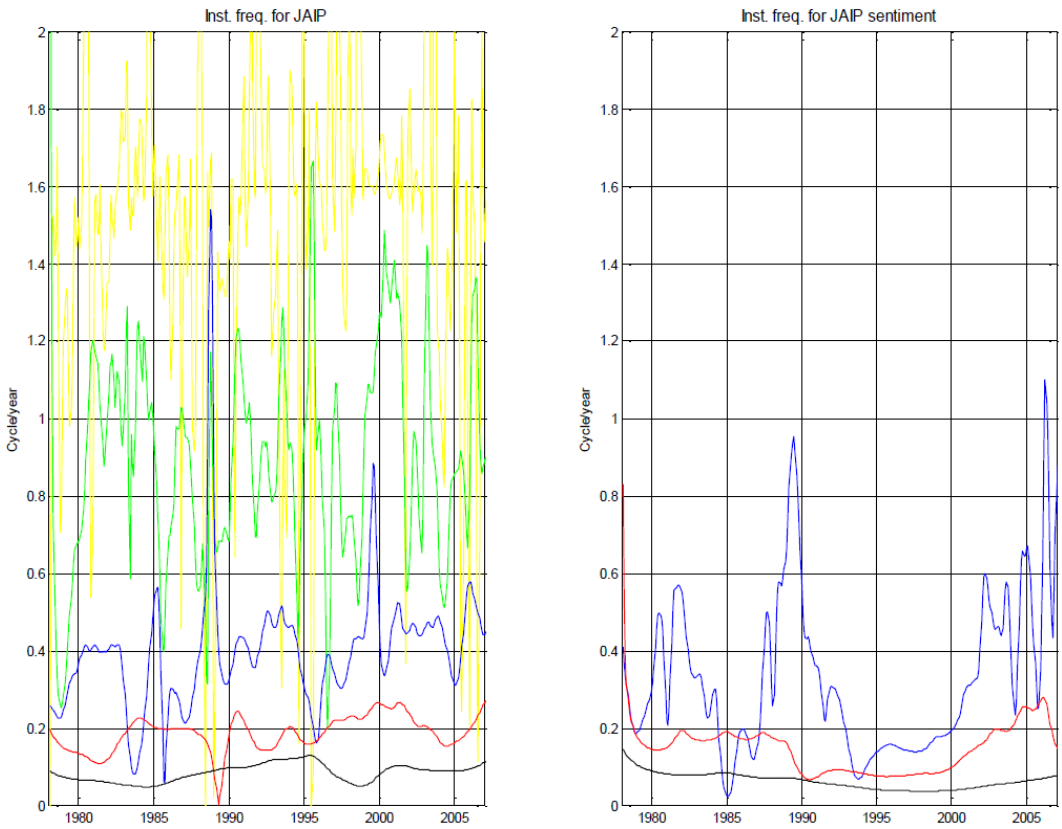


Table 4.8 **Correlations of IMFs for Japanese industrial production (JAIP) and business sentiment indicator (overall = 0.077) with frequencies**

Correlations of JAIP	IMF2	IMF3	IMF4	Frequency
IMF2	-0.0341	0.009	0.0135	1.475
IMF3	0.0198	0.0072	-0.0113	0.889
IMF4	-0.0524	0.0501	-0.0432	0.393
IMF5	0.2911*	0.0715	-0.0016	0.170
Frequency	0.3385	0.1501	0.0643	

Correlations that are significant at the 5% level are denoted with an asterisk

Figure 4.15 **Japanese industrial production vs business sentiment indicator: instantaneous frequency**



4.3.3 Consumption: Japanese consumption expenditure vs OECD consumer confidence indicator

Compared to industrial production, consumption growth yields much greater concordance between the indicator and the actual economic variable, as figure 4.16 shows. Here similar components for longer frequencies are extracted (with

also a very similar pattern found in the residual), but there appears to be much less similarities at higher frequencies. Table 4.9 shows correlations and frequencies, with very high correlations at lower frequencies, but as expected much lower correlations at lower frequency fluctuations. This also shows up with the instantaneous frequency plot shown in figure 4.17, where a 20 year cycle is clearly identified in both the indicator and in consumption growth, but higher frequency growth cycles appear to be mismatched between the IMFs.

Figure 4.16 **Japanese consumption vs OECD consumer confidence indicator: IMFs**

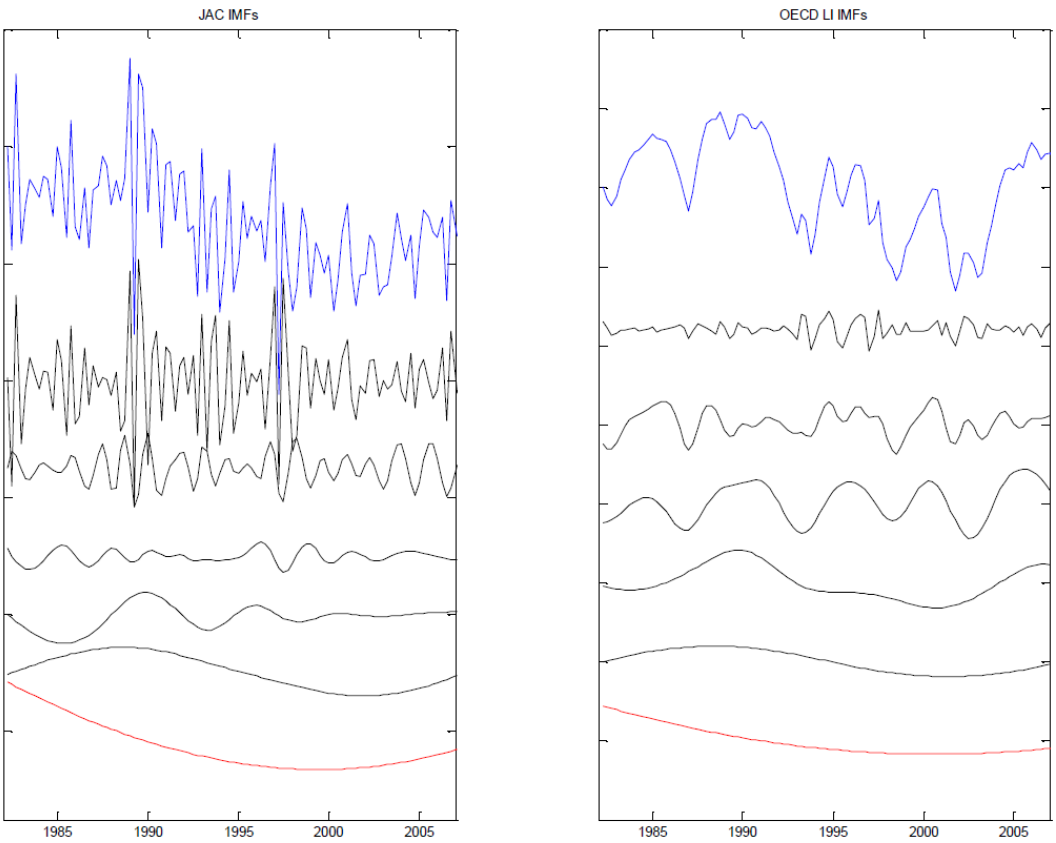
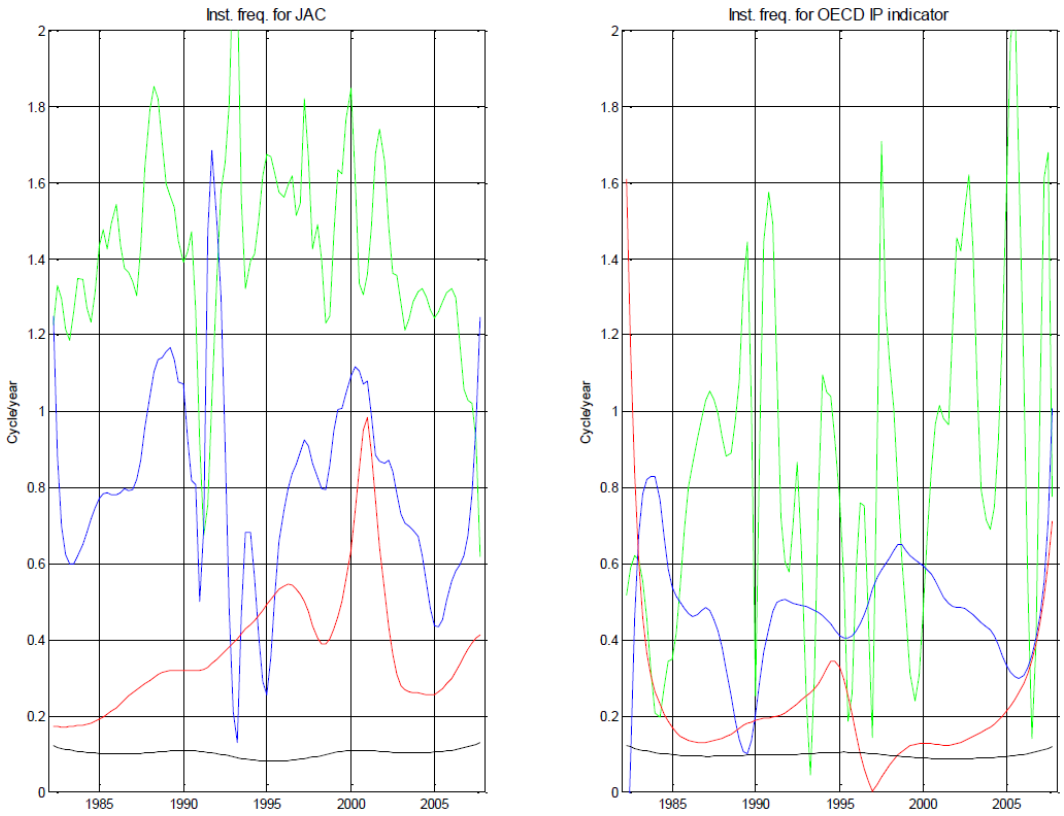


Table 4.9 **Correlations of IMFs for Japanese consumption (JAC) and OECD consumer confidence indicator (overall = 0.483) with frequencies**

Correlations of JAC	IMF2	IMF3	IMF4	IMF5	Frequency
IMF2	0.0008	-0.0461	-0.013	-0.0143	1.420
IMF3	0.1769	0.2457*	0.0302	-0.0617	0.803
IMF4	-0.0366	0.4997*	0.4299*	-0.1253	0.374
IMF5	0.066	-0.0128	0.7975*	0.9964*	0.101
Frequency	0.853	0.466	0.231	0.097	

Correlations that are significant at the 5% level are denoted with an asterisk.

Figure 4.17 **Japanese consumption vs OECD consumer confidence indicator: instantaneous frequencies**



4.3.4 Consumption: Japanese consumption vs consumer sentiment indicator

Here, in figure 4.18 we decompose over the same time period as for the OECD indicator. It appears to possess a similar correspondence between consumption growth and the consumer sentiment index in lower frequency cycles as the OECD indicator does, with less correspondence at the higher frequencies.[3] It is also notable that for Japanese data the amount of high frequency volatility appears to have been reduced in consumption growth, but that volatility in sentiment appears to have increased in both the IMF1 and IMF3 components. Table 4.10 shows that apart from the long cycle found in the data there is a significantly correlated shorter cycle of about 4 years in the data. Figure 4.19 shows that the frequency profile of this IMF (IMF4 in both cases) appears to be similar, although in terms of amplitude, figure 4.16 suggests that this cycle has been losing energy, despite the fact that the sentiment index IMF4 appears to still possess a fairly significant amount of energy.

[3] Once again if the lowest frequency IMF is extracted (here IMF5), the correlation between the two would be 0.99, with both possessing a frequency of roughly 0.05.

Figure 4.18 **Japanese consumption vs consumer sentiment indicator: IMFs**

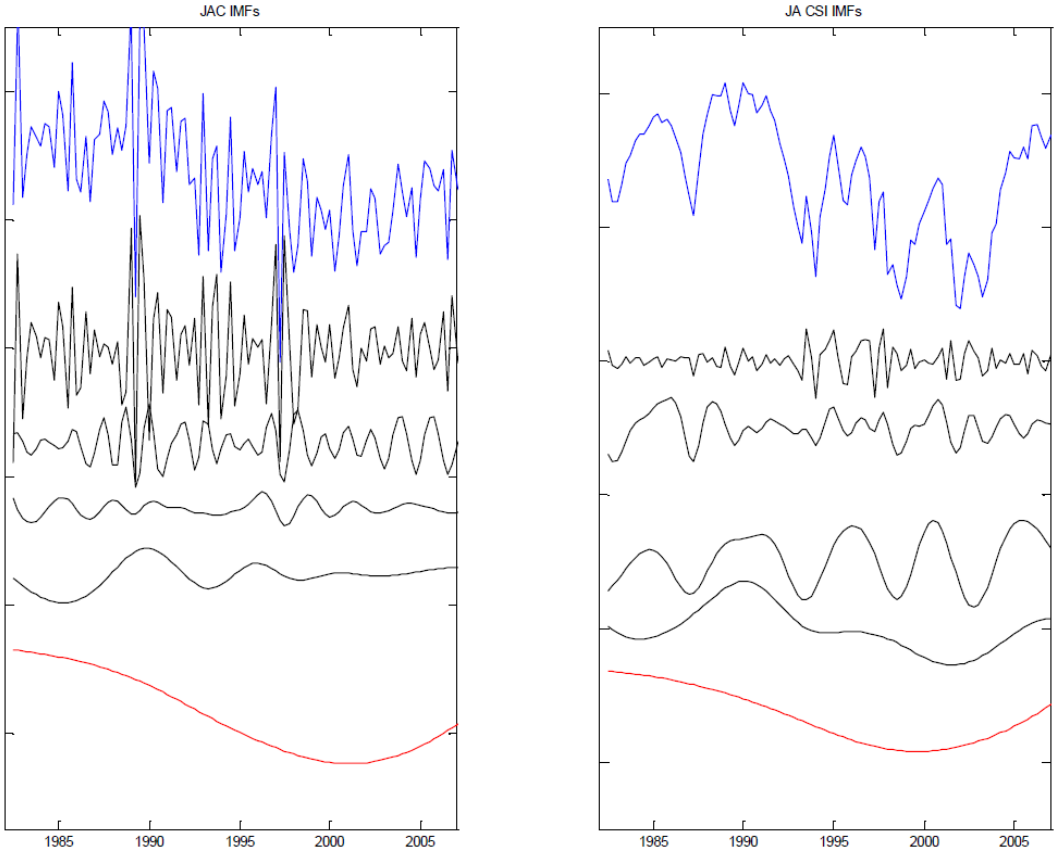
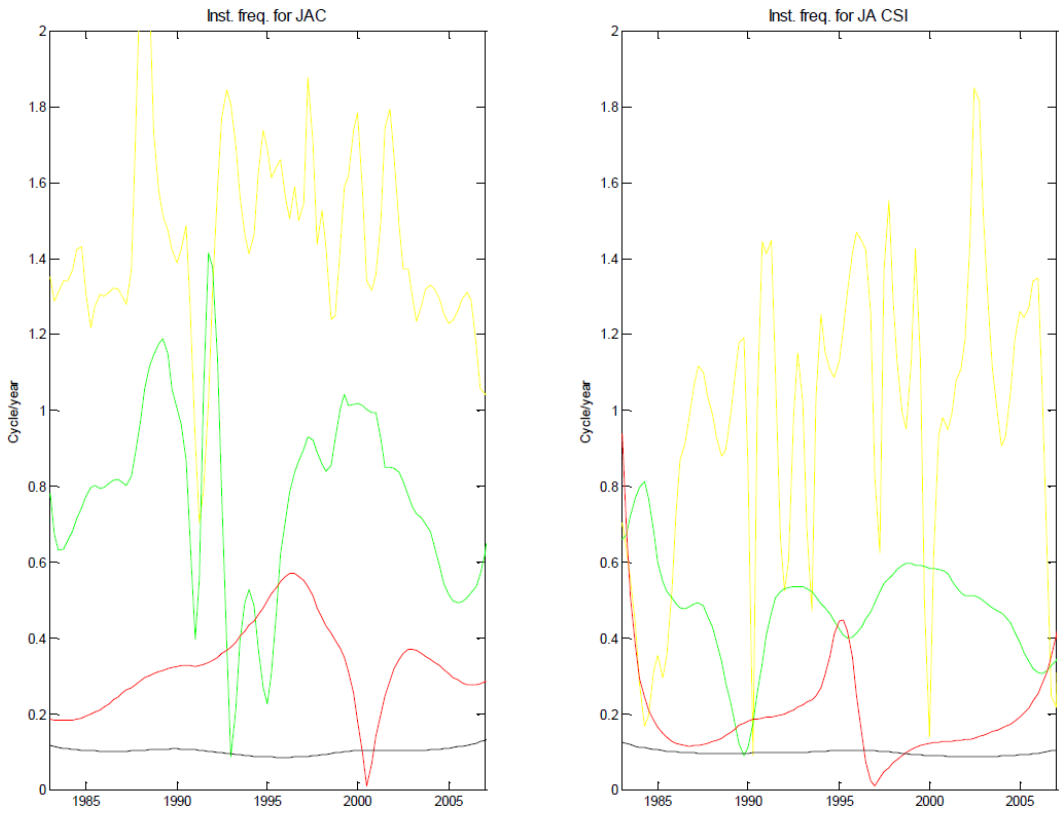


Table 4.10 **Correlations of IMFs for Japanese consumption and consumer sentiment indicator (overall = 0.5101) with frequencies**

Correlations of JAC	IMF2	IMF3	IMF4	Frequency
IMF2	-0.0243	-0.0238	-0.0074	1.443
IMF3	0.1264	0.2357	0.0202	0.764
IMF4	-0.1218	0.4444*	0.5218*	0.376
Frequency	0.959	0.465	0.247	

Correlations that are significant at the 5% level are denoted with an asterisk.

Figure 4.19 **Japanese consumption vs consumer sentiment indicator: IMFs**



5 Conclusions

In this paper industrial production and consumption growth, two major drivers of economic growth, were decomposed in the frequency domain using Ensemble Empirical Mode Decomposition (EEMD) alongside two different types of indicators for these variables, one a comparable confidence indicator obtained from the OECD and the other a nationally published sentiment indicator.

EEMD is an empirical methodology that permits the decomposition of economic variables into intrinsic mode functions (IMFs) that represent the cycles embedded in the variable in a meaningful way. In this study we show how the technique can be used to separate out the business cycle from other cycles that exist in macroeconomic variables, and estimate the frequency of the components.

A first result that pertains in particular to the US because of the long data series available, concerns the length of long cycles in the data. Although Granger (1966) found that long cycles had the greatest amplitude, this is not found in our results as pointed out in Crowley (2008). For US industrial production growth nothing beyond a 20 year cycle appears in the data, and for US consumption growth there is nothing beyond a 12.5 year cycle. This result is perhaps due to the

fact that EEMD does not require stationarity, where as spectral analysis does, so any attempt to frequency decompose economic variables using spectral analysis requires some kind of transformation to ensure stationarity – this was not done by Granger or most of the economists employing spectral analysis thereafter.

In relation to OECD confidence indicators and national sentiment indicators, the results we obtain show that there are generally many more cycles embedded in actual economic variables than in their corresponding indicators or sentiment measures. This is to be expected, as there is obviously a certain amount of noise operating in the measurement and reporting of actual economic variables, plus the reporting in sentiment indicators is much more likely to put more emphasis on medium and longer term trends in output growth. For the US and the euro area, OECD indicators have quite similar frequency content to industrial production and consumption growth, but for Japan there appears to be a frequency mis-match occurring, with little similarity between the frequency of cycles, particularly for industrial production.

One of the most interesting results that we obtained is that the sentiment measures of output from producers of goods and services appears to contain cycles that roughly correspond to the growth cycles found in actual economic variables, where as the sentiment measures of consumption expenditure by consumers appears not to contain as much information, but rather focuses much more on the business cycle.

A further insight that is gained by analysing data in the frequency domain is that it is clear that indicators capture longer term trends much better than shorter term trends, as the correlation between embedded cycles at shorter frequencies is generally lower than for those at lower frequencies.

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