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Equilibrium dynamics under lump-sum taxation in an exchange economy with skewed endowments

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Abstract

I explore the dynamics in overlapping generations models with pure exchange and lump-sum taxes, when the second period after tax endowment is negative, and contrast the characteristics of equilibria to those of models with positive after tax endowments. In particular, if the intertemporal elasticity of substitution is less than unity, there can be only a two cycle or stable (ie indeterminate) equilibria for certain parameter values. With this value for that elasticity chaos and a cycle of any order can occur in a model with regular endowments. In a sense the lump-sum taxation in this model operates as a stabilizing device. The precise stability condition holds with a small discount factor and in economies with relatively high taxes in the first period. If the intertemporal elasticity of substitution is greater than unity, the steady state equilibria are unstable, and thus determinate, as is the case with the regular model.

Key words: overlapping generations economy, saving, cycles, lump-sum taxation

JEL classification numbers: E21, E32

Vinoalkuvarantoisen vaihtotalouden dynamiikka könttäverojen tapauksessa

Suomen Pankin keskustelualoitteita 29/2004

Mikko Puhakka
Tutkimusosasto

Tiivistelmä

Tutkin tasapainodynamiikkaa limittäisten sukupolvien vaihtotalouden mallissa, jossa on könttäverot ja taloudenpitäjien toisen periodin verojen jälkeiset alkuvarannot ovat negatiiviset. Tällainen malli voi kuvata esimerkiksi taloutta, jossa on yksityinen eläkejärjestelmä. Vertaan tämän ”vinon” talouden suhdannevaihteluita vaihteluihin ”tavallisessa” taloudessa, jossa taloudenpitäjien molempien periodien alkuvarannot ovat positiiviset. Jos intertemporaalinen substituutiojousto on pienempi kuin ykkönen, voi vinossa taloudessa olla vain kaksi periodia kestäviä vaihteluita tai stabiileja tasapainoja. Näillä jouston arvoilla voi tavallisessa taloudessa olla kuinka monta periodia tahansa kestäviä vaihteluita ja kaaosta. Näin ollen könttäverojen voidaan katsoa stabiloivan taloutta. Vino talous on stabiili, jos taloudenpitäjän diskonttaustekijä on pieni. Samoin käy, jos talouden ensimmäisen periodin verot ovat suhteellisen korkeat. Jos intertemporaalinen substituutiojousto on suurempi kuin ykkönen, vinon talouden ja tavallisen talouden stationäärinen tasapaino on epästabiili.

Avainsanat: limittäisten sukupolvien talous, säästäminen, suhdannevaihtelut, könttäverot

JEL-luokittelu: E21, E32

Contents

1 Introduction

The possibility of cycles in overlapping generations models under pure exchange was observed by Gale (1973) and Cass, Okuno and Zilcha (1979). Grandmont (1985) analyzed the conditions for the existence of cycles. In particular, cycles of any order and chaos can emerge, if the elasticity of the marginal utility of the second period consumption (or the Arrow-Pratt measure of the relative risk aversion) is greater than one. The source for cycles in his model is thus the fact that the income effect dominates the substitution effect for some levels of interest rates in an old agent's utility function. These results depend crucially on the fact that there is a nonnegative second period after tax endowment.[1]

In this paper I explore the dynamics in overlapping generations models with pure exchange and lump-sum taxes, where the second period after tax endowment is negative, and contrast the characteristics of equilibria to those of models with regular endowments. This model can be thought of as describing eg certain features of economies with private retirement systems. I mostly work with the model, where the young and the old have the same periodic utility functions. In the model with regular endowments there can be cycles of any order and chaos, if the elasticity of intertemporal substitution is less than unity. Diamond (1965) and its derivatives are prominent examples of models with negative second period after tax endowment.[2]

There are examples in the literature, where exchange economy overlapping generations models with negative second period after tax endowment have been utilized for policy analyses. Aiyagari (1985) commented on the debate of how deficits affect inflation. In his model there are lump-sum taxes in both periods, but no endowment in the second period, and a positive primary deficit. He showed that one of the two steady states is locally stable. He did not, however, perform a complete dynamic analysis. Bhattacharya and Haslag (2001) present examples with no second period endowment, but positive taxes, to explore the effects of inflation tax on equilibrium. Neither did they perform a complete dynamic analysis of their model.

I show that cycles of higher order than two and chaos are not possible, if endowments are skewed. Since the after tax second period endowment is negative, there is a strong incentive to save. For relatively small interest factors the income effect will, however, dominate the saving behavior regardless of the value of the elasticity of the marginal utility. This means that at that range saving is a decreasing function of the interest rate.

[1] I call a model with nonnegative after tax endowments in both periods as regular.

[2] See also chapter 4 in de la Croix and Michel (2002), and chapter 20.1 in Azariadis (1993).

The dynamics in this model will significantly differ from that of the model with regular endowments. In the regular model chaos and a cycle of any order can occur, if the intertemporal elasticity of substitution is less than unity. With those values of the elasticity of substitution there can be only a two cycle or stable (ie indeterminate) equilibria in a model with skewed endowments. In a sense then the lump-sum taxation per se operates as a stabilizing device. The precise stability condition holds with a small discount factor and in economies with relatively high taxes in the first period. If the intertemporal elasticity of substitution is greater than unity, the steady state equilibria are unstable as is the case with the regular model. It is interesting to note that the results of this model rather closely resemble those of an overlapping generations model with Stone-Geary preferences (see Koskela and Puhakka, 2004).

The paper proceeds as follows. In section 2 I present an overlapping generations model and study the saving behavior. A particular attention is paid to the characteristics of the offer curves. Section 3 characterizes dynamical competitive equilibria with balanced budget. I explore equilibrium dynamics in detail and compare it to that of the model with regular endowments. There is also a brief concluding section.

2 The model and saving behavior

I consider an overlapping generations economy without population growth, perfect foresight, and government debt. I start with general preferences and move rather quickly to the specific examples. The lifetime utility function is

$$U(c_1^t, c_2^t) = u(c_1^t) + \beta u(c_2^t) \tag{2.1}$$

c_1^t (c_2^t) denotes consumption when young (old) and $\beta = (1+\rho)^{-1}$, where ρ is the rate of time preference. The periodic utility function is assumed to be increasing and strictly concave. Furthermore, it fulfills the following Inada conditions on the marginal utilities at low and high levels of consumption: $\lim_{c \to 0} u'(c) = \infty$ and $\lim_{c \to \infty} u'(c) = 0$.

A representative consumer's periodic budget constraints are

$$c_1^t + s_t = y - \tau_1 \tag{2.2i}$$

$$c_2^t = R_{t+1} s_t - \tau_2 \tag{2.2ii}$$

y is the first period endowment, τ_1 (τ_2) is the first (second) period lump-sum tax, s_t the saving, and R_{t+1} the interest factor from period t to period t+1. In particular, I have assumed that there is no endowment in the second period. Thus this model can be thought of as describing certain features of eg economies with private retirement systems.

The endowment point is described in Figure 1. If the interest factor is less than $\underline{R}$ (= $\tau_2/(y-\tau_1)$) the present value of lifetime income is negative. It is then clear that the interest factor must be at least $\underline{R}$ for the decision problem to have a reasonable solution. And furthermore, for interest factors slightly above $\underline{R}$, saving will be close to $y-\tau_1$. Using the terminology in Gale (1973) I consider a Samuelson case, which is needed for equilibria with positive levels of equilibrium debts.[3]

The first-order condition for optimal saving is (I drop the time subscripts for a moment)

$$-u'(y-\tau_1-s)+R\beta u'(Rs-\tau_2)=0 \tag{2.3}$$

The second order condition is

$$\Delta=u''(y-\tau_1-s)+R^2\beta u''(Rs-\tau_2)<0 \tag{2.4}$$

The dependence of saving on the interest factor is

$$\frac{\partial s}{\partial R}=-\frac{\beta u'(Rs-\tau_2)\left\{1+\dfrac{Rsu''(Rs-\tau_2)}{u'(Rs-\tau_2)}\right\}}{\Delta} \tag{2.5}$$

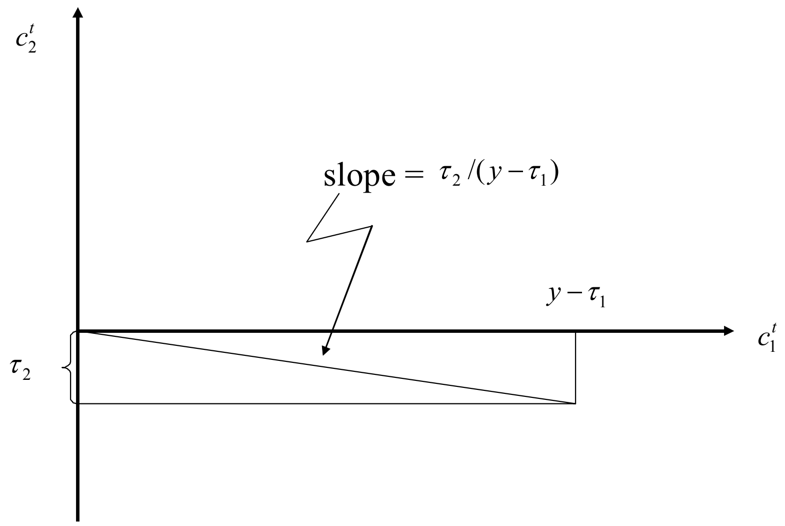
Since $\Delta<0$, the sign of this derivative depends on the sign of the numerator (Num), which is expressed as

$$\text{Num}=-\beta u'(Rs-\tau_2)\left\{1+\frac{(Rs-\tau_2)u''(Rs-\tau_2)}{u'(Rs-\tau_2)}\left(\frac{Rs}{Rs-\tau_2}\right)\right\} \tag{2.6}$$

and as

[3] Economy is Samuelsonian, if the marginal rate of substitution (MRS) at the endowment point is less than the gross rate of population growth (here unity). Since the endowment point is $(y-\tau_1,-\tau_2)$, there is no meaningful value for the MRS at the endowment point. As Figure 1 makes clear, consumer wants to have positive savings for any $R>\underline{R}$; in this sense the model is Samuelsonian.

Figure 1.



$$\text{Num} = -\beta u'(Rs - \tau_2)\left\{1 - \sigma(Rs - \tau_2)\left(\frac{Rs}{Rs - \tau_2}\right)\right\} \tag{2.7}$$

where $1/\sigma(c)$ is the intertemporal elasticity of substitution. It can now be seen that $\partial s/\partial R < 0$, if $\sigma(c) > 1$. But on the other hand since $Rs > Rs - \tau_2$, saving can be a decreasing function of the interest rate even though $\sigma(c) < 1$. This is an important characteristic of the saving behavior in this model with negative after tax endowments in the second period.

It is well known that the characteristics of the equilibrium dynamics depend on the properties of the saving function. In fact, if $\sigma(c) > 1$, many types of dynamics (including cycles of any order and chaos) can emerge in equilibrium in models with regular endowments. These are the issues I explore in the next section of the paper.

To develop further, and perhaps more precise results, I now assume the lifetime utility function to have the following form

$$U(c_1^t, c_2^t) = \frac{(c_1^t)^{1-\sigma}}{1-\sigma} + \beta \frac{(c_2^t)^{1-\sigma}}{1-\sigma} \tag{2.8}$$

$1/\sigma$ is the constant intertemporal elasticity of substitution. Given the budget constraints above the first-order condition for consumer's optimum, $(c_2^t)^\sigma / \beta (c_1^t)^\sigma = R_{t+1}$, leads to the following saving function

$$s(R;\bullet) = \frac{R^{\frac{1}{\sigma}}\beta^{\frac{1}{\sigma}}}{R + R^{\frac{1}{\sigma}}\beta^{\frac{1}{\sigma}}}(y - \tau_1) + \frac{\tau_2}{R + R^{\frac{1}{\sigma}}\beta^{\frac{1}{\sigma}}} \tag{2.9}$$

with the property

$$\frac{\partial s}{\partial R} = \frac{(\frac{1}{\sigma} - 1)R^{\frac{1}{\sigma}}\beta^{\frac{1}{\sigma}}}{\left[R + R^{\frac{1}{\sigma}}\beta^{\frac{1}{\sigma}}\right]^2}(y - \tau_1) - \frac{1 + \frac{1}{\sigma}R^{\frac{1}{\sigma}-1}\beta^{\frac{1}{\sigma}}}{\left[R + R^{\frac{1}{\sigma}}\beta^{\frac{1}{\sigma}}\right]^2}\tau_2 \tag{2.10}$$

I evaluate this at R = R̲ to get

$$\frac{\partial s}{\partial R}(\underline{R},\bullet) = \frac{\underline{R}^{\frac{1}{\sigma}}\beta^{\frac{1}{\sigma}}}{\left[\underline{R} + \underline{R}^{\frac{1}{\sigma}}\beta^{\frac{1}{\sigma}}\right]^2}\left\{(\frac{1}{\sigma} - 1)(y - \tau_1) - (\underline{R}^{-\frac{1}{\sigma}}\beta^{-\frac{1}{\sigma}} + \frac{1}{\sigma}\underline{R}^{-1})\tau_2\right\} \tag{2.11}$$

which reduces to

$$\frac{\partial s}{\partial R}(\underline{R},\bullet) = \frac{\underline{R}^{\frac{1}{\sigma}}\beta^{\frac{1}{\sigma}}}{\left[\underline{R} + \underline{R}^{\frac{1}{\sigma}}\beta^{\frac{1}{\sigma}}\right]^2}\left\{-(y - \tau_1) - \underline{R}^{-\frac{1}{\sigma}}\beta^{-\frac{1}{\sigma}}\tau_2\right\} < 0 \tag{2.12}$$

For any value of σ saving is a decreasing function of the interest factor for interest factors close to R̲. In general, saving is a decreasing function of the interest rate, if $\sigma > 1$. It can be seen from (2.10) that it is an increasing function, if $\sigma < 1$. This is in contrast to the case with regular endowments (ie when the after tax endowments are positive in both periods). In that case saving is an increasing function of the interest rate, when $\sigma < 1$, and it can be a decreasing function, if $\sigma > 1$. We also see that $s(\underline{R};\bullet) = y - \tau_1$.

To explore the limit of the saving function consider the function

$$g(R) = \frac{R^{\frac{1}{\sigma}}\beta^{\frac{1}{\sigma}}}{R + R^{\frac{1}{\sigma}}\beta^{\frac{1}{\sigma}}} = \frac{1}{1 + R^{\frac{\sigma-1}{\sigma}}\beta^{-\frac{1}{\sigma}}} \tag{2.13}$$

Figure 2. **The offer curve, when σ < 1**

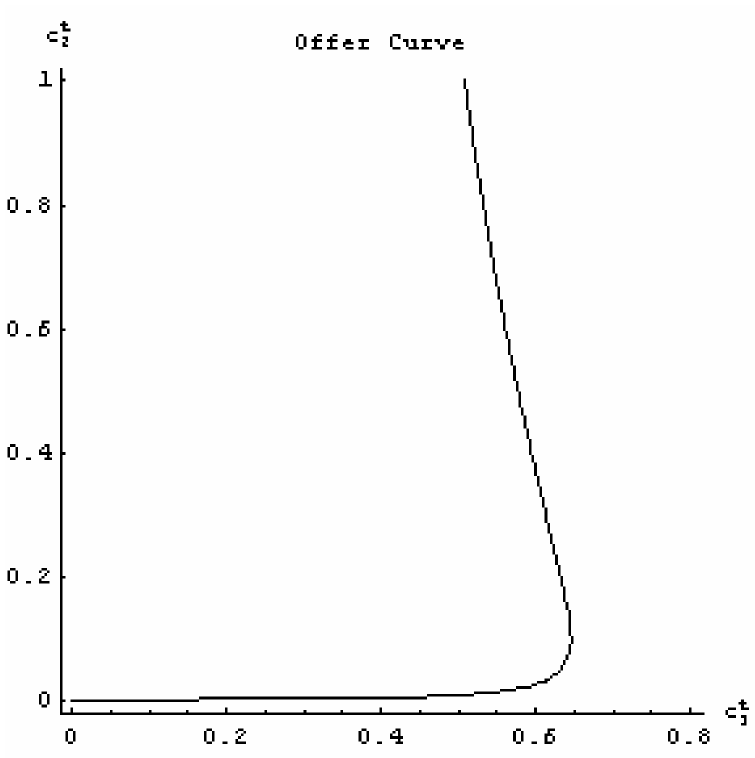
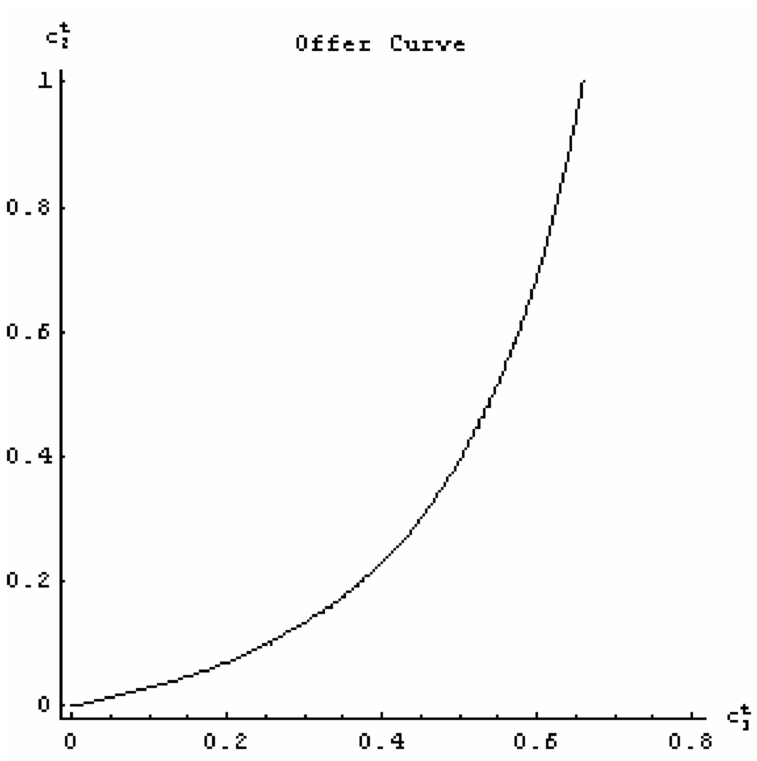


Figure 3. **The offer curve, when σ > 1**



Saving function can then be expressed as

$$s(R;\bullet) = g(R)\left[y - \tau_1 + \frac{\tau_2}{R^{\frac{1}{\sigma}}\beta^{\frac{1}{\sigma}}} \right] \tag{2.14}$$

From (2.13) we conclude that $\lim_{R\to\infty} g(R) = 0$, when $\sigma > 1$, and $\lim_{R\to\infty} g(R) = 1$, when $\sigma < 1$. This means that $\lim_{R\to\infty} s(R) = 0$, when $\sigma > 1$, and $\lim_{R\to\infty} s(R) = y - \tau_1$, when $\sigma < 1$. Also it then follows that the offer curve must turn backwards, when $\sigma < 1$.

The offer curve can be expressed as

$$\frac{(c_2^t)^\sigma}{\beta(c_2^t + \tau_2)} = \frac{(c_1^t)^\sigma}{y - \tau_1 - c_1^t} \tag{2.15}$$

I sketch the offer curves in Figures 2 and 3 for the parameter values $\sigma = 1/2$ and $\sigma = 2$ respectively. For the rest of the parameters I assume $y = 1$, $\tau_1 = \tau_2 = 1/10$. Note that now $\underline{R} = 1/9$.

3 Dynamical equilibria

To analyze the dynamics of the model, an outside asset is introduced into the economy by assuming that government borrows (lends) from (to) the public. Government debt (or assets, when negative) at the beginning of the period is denoted by b_t, and the constant primary deficit by d so that government's budget constraint is

$$b_{t+1} = d + R_t b_t, \quad \text{where } d = g - \tau_1 - \tau_2 \tag{3.1}$$

g denotes constant government expenditures per period. I study the case with nonnegative government debt, ie $b_t \geq 0$. Nonnegative debt allows the model to have a monetary interpretation, where the outstanding debt can be interpreted as the amount of outside money in the economy.

Since we want to concentrate on the fundamental dynamic implications of the model we assume public primary deficit, d, to be zero. If we had positive deficits, there would be two stationary equilibria in this economy at least with regular endowments.[4] Thus in the asset market equilibrium $s_t = b_{t+1}$, which leads to the

[4] The appearance of more than one steady state in models with positive deficits is basically known since Bailey (1956), see especially pp. 102–105 and his Figure 2. See also Aiyagari (1987).

following difference equation characterizing the equilibrium sequence of interest factors

$$s_{t+1} \equiv s(R_{t+2}) = R_{t+1}s(R_{t+1}) \equiv R_{t+1}s_t \tag{3.2}$$

I analyze equation (3.2) by the geometric techniques of the reflected generational offer curves developed by Cass, Okuno and Zilcha (1979). The offer curve, equation (2.15), can be rewritten as an equilibrium condition as

$$s_t^{\frac{1}{\sigma}}(s_{t+1} - \tau_2) = \beta^{\frac{1}{\sigma}} s_{t+1}^{\frac{1}{\sigma}}(y - \tau_1 - s_t) \tag{3.3}$$

which implicitly defines the reflected generational offer curve. When $\sigma = 1$, (3.3) describes equilibrium with logarithmic preferences.[5] If $R_{t+1} = (s_{t+1}/s_t) = \underline{R}$, and if $s_t = y - \tau_1$, then $s_{t+1} = \tau_2$. The steady state derived from (3.3) is

$$s^* = \frac{\tau_2 + \beta^{\frac{1}{\sigma}}(y - \tau_1)}{1 + \beta^{\frac{1}{\sigma}}} \tag{3.4}$$

which is obviously unique. I totally differentiate (3.3) to obtain

$$\frac{ds_{t+1}}{ds_t} = \left[\frac{-1 - \sigma \dfrac{s_t}{y - \tau_1 - s_t}}{\dfrac{\sigma s_{t+1}}{s_{t+1} - \tau_2} - 1} \right] \times \frac{s_{t+1}}{s_t} \tag{3.5}$$

If $\sigma \geq 1$, then this slope is always negative. If $\sigma < 1$, the slope can become positive. The slope at the steady state is

$$\left.\frac{ds_{t+1}}{ds_t}\right|_{s_{t+1} = s_t} = \left[\frac{1 + \sigma \dfrac{s^*}{y - \tau_1 - s^*}}{1 - \dfrac{\sigma s^*}{s^* - \tau_2}} \right] \tag{3.6}$$

Plugging in the steady state value for saving I get

5 The equilibrium of a general overlapping generations exchange economy with and without money, and with log-linear utility functions, is unique as shown by Balasko and Shell (1981). In the monetary economy the equilibrium is indexed by the price of money.

$$\left.\frac{ds_{t+1}}{ds_t}\right|_{s_{t+1}=s_t} = \frac{\beta^{\frac{1}{\sigma}}(y-\tau_1-\tau_2)+\beta^{\frac{1}{\sigma}}\sigma\left[\beta^{\frac{1}{\sigma}}(y-\tau_1)+\tau_2\right]}{\beta^{\frac{1}{\sigma}}(y-\tau_1-\tau_2)-\sigma\left[\beta^{\frac{1}{\sigma}}(y-\tau_1)+\tau_2\right]} \tag{3.7}$$

I first assume that $\sigma \geq 1$ (ie the slope of the offer curve is negative), and explore the stability of the steady state. For stability the slope should be bigger than minus unity. This is also the condition for indeterminacy.[6] By evaluating (3.7) at the steady state yields the following stability condition

$$2\beta^{\frac{1}{\sigma}}(y-\tau_1-\tau_2) < (1-\beta^{\frac{1}{\sigma}})\sigma\left[\beta^{\frac{1}{\sigma}}(y-\tau_1)+\tau_2\right] \tag{3.8}$$

which can be rewritten as

$$\left[\frac{1}{2}\beta^{\frac{2}{\sigma}}+(\frac{1}{\sigma}-\frac{1}{2})\beta^{\frac{1}{\sigma}}\right](y-\tau_1) < \left[\frac{1}{2}+(\frac{1}{\sigma}-\frac{1}{2})\beta^{\frac{1}{\sigma}}\right]\tau_2 .^{7} \tag{3.9}$$

By setting $\sigma = 1$ (ie with logarithmic preferences) this condition reduces to $\beta(y-\tau_1) < \tau_2$ or $\beta < \overline{R}$. This holds true for a small discount factor and in economies with relatively high taxes in the first period. As an example consider an economy, where $y = 1$ and $\tau_1 = \tau_2 = 1/3$. For stability it is then necessary that $\beta < 1/2$.[8] For smaller tax rates the discount rate must be smaller.

The reflected generational offer curve is described in Figure 4 for $\sigma = 2$ and with the same assumptions as I had above, when drawing figures for the individual's offer curves. With the given parameter values it turns out that the steady state in Figure 4 is unstable.

For the existence of a two cycle (or a periodic point with period two) it is necessary that the offer curve is downward sloping. That property, however, is not sufficient for periodic solutions of higher order. To have periodic points with period three and more, it is necessary that the offer curve must be hump-shaped. More precisely, if there are at least three cycles in the economy, there must be periodic solutions of any order higher than three according to Sarkovskii's

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[6] Guesnerie and Woodford (1992 especially chapter 5) discuss thoroughly the concept of indeterminacy in OG models. See also an illuminating survey by Woodford (1984).

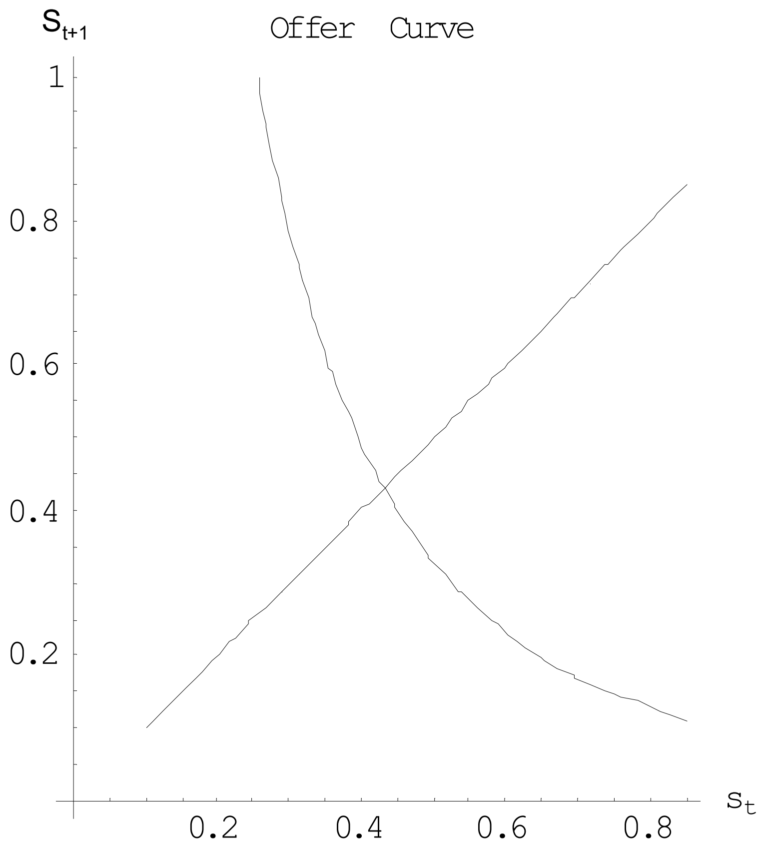
[7] This stability condition is similar to the condition obtained in a model with general Stone-Geary preferences; see Koskela and Puhakka (2004).

[8] A discount factor about one half in an overlapping generations model is reasonable. If one period is thirty years it corresponds to an annual discount factor about .98.

theorem.[9] Cycles in overlapping generations models are intimately connected with sunspot equilibria. Azariadis and Guesnerie (1986) showed that a two cycle is enough for the existence of a stationary sunspot equilibrium.

Since the offer curve in Figure 4 is not hump-shaped, only two cycles are possible. A model with lump-sum taxes and no second period endowment does not allow for more complicated dynamics than the two-cycle. This is in contrast to a model with regular endowments. In such a model with $\sigma > 1$ it is possible to have cycles of any period, and thus also chaotic behavior. In a sense lump-sum taxation has a stabilizing effect on the economy. Even though cycles of higher order than two and chaotic behavior are not possible, the stable equilibria, and thus indeterminacy, will still remain.

Figure 4. **Equilibrium offer curve, when $\sigma > 1$**



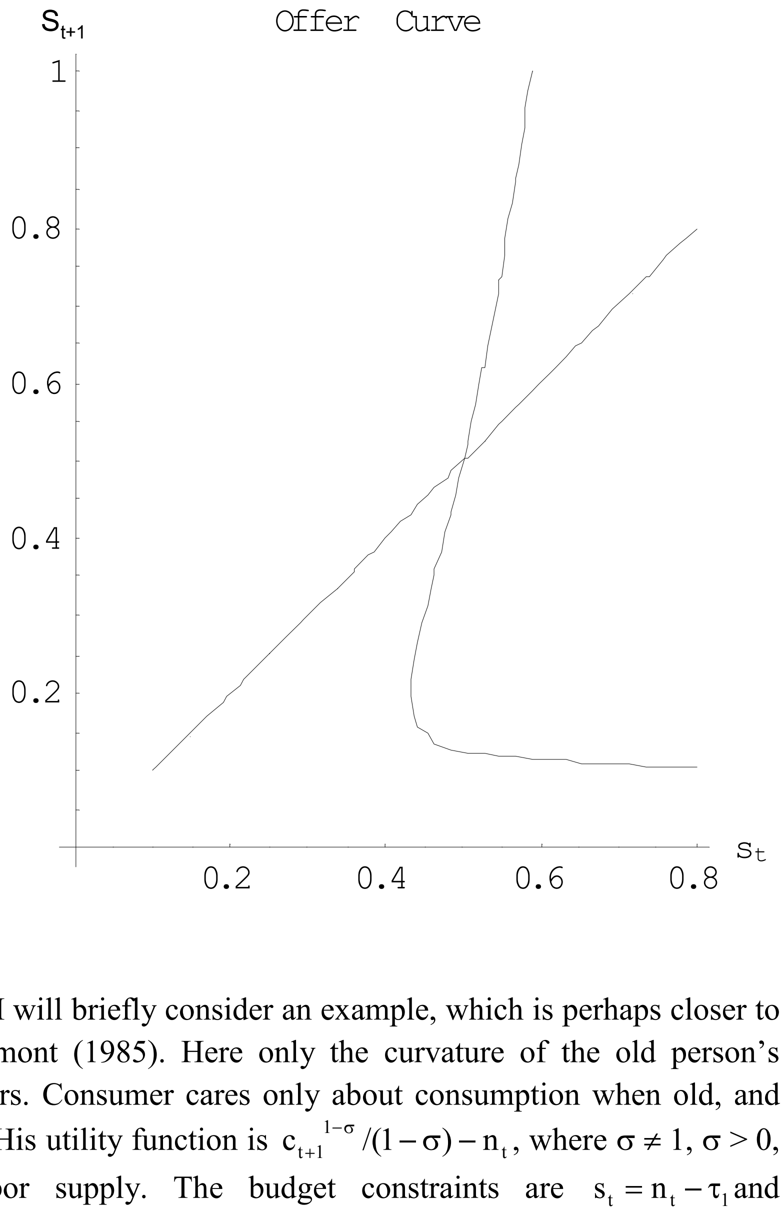
Suppose now that $\sigma < 1$, and let the slope be positive. For stability the slope should be less than unity, ie we get the condition

[9] An elementary discussion and elaboration of Sarkovskii's theorem can be found eg in Holmgren (1996), see in particular chapter 5. On the conditions for the existence of cycles of more than two periods in economic models, see Grandmont (1986).

$$(1+\beta^{\frac{1}{\sigma}})\sigma\left[\beta^{\frac{1}{\sigma}}(y-\tau_1)+\tau_2\right]<0 \qquad (3.10)$$

This obviously cannot hold, since all the terms on the left-hand side of (3.10) are positive. This means that the steady state is necessarily non-stable. This is reflected in Figure 5, which describes the equilibrium offer curve.

Figure 5. **Equilibrium offer curve, when σ < 1**



To close this section I will briefly consider an example, which is perhaps closer to the model of Grandmont (1985). Here only the curvature of the old person's utility function matters. Consumer cares only about consumption when old, and works, when young. His utility function is $c_{t+1}^{1-\sigma}/(1-\sigma)-n_t$, where $\sigma \neq 1$, $\sigma > 0$, and n_t denotes labor supply. The budget constraints are $s_t = n_t - \tau_1$ and $c_{t+1} = R_{t+1}s_t - \tau_2$. Equilibrium dynamics are now described by

$$\frac{s_{t+1}}{(s_{t+1} - \tau_2)^{\sigma}} = s_t \tag{3.11}$$

The unique steady state is s* = 1+τ_2. Differentiating (3.11) we get

$$\frac{ds_{t+1}}{ds_t} = \frac{(s_{t+1} - \tau_2)^{\sigma+1}}{(1-\sigma)s_{t+1} - \tau_2} \tag{3.12}$$

Note that the offer curve is downward sloping, when $\sigma > 1$. It can be upward sloping for $\sigma < 1$. The slope is $[1 - \sigma(1 + \tau_2)]^{-1}$ in the steady state. When $\sigma > 1$ the stability condition is $\sigma > 2/(1 + \tau_2)$. This gives a lower bound for the curvature parameter. When $\sigma < 1$ the stability condition is $\sigma(1 + \tau_2) < 0$, which obviously cannot hold.

4 Conclusions

This paper shows that the dynamic behavior of an overlapping generations model with pure exchange, no second period endowments, and lump-sum taxes is significantly different from that of the model with regular endowments. In particular, endogenous cycles of order higher than two are not possible in this model even though the elasticity of intertemporal substitution (or the elasticity of the marginal utility of the second period consumption) is less (higher) than one. Lump-sum taxes then in a sense are stabilizing devices in this type of an economy, since they rid the economy from cycles of order higher than three and chaos. There can be two cycles and stable equilibria. This means that a significant amount of indeterminacy is still present in the model.

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