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The effects of aging population on the sustainability of fiscal policy

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Abstract

We study the effects of aging population on the sustainability of fiscal policy in overlapping generations models with government debt and a pay-as-you-go pension system. The smaller the population growth rate, the lower the maximum sustainable level of deficits. When the utility function is of a specific form, an increase in the payroll tax rate and the replacement rate decreases the level of maximum sustainable deficits; except in the case when pension depends on the wage level prevailing during the working period. The ratio of the deficits in two economies with different population growth rates is characterized with numerical examples.

Key words: aging, pensions, overlapping generations, fiscal policy

JEL classification numbers: E21, E32

Väestön ikääntymisen vaikutuksia finanssipolitiikan kestävyyteen

Suomen Pankin tutkimus
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Mikko Puhakka
Rahapolitiikka- ja tutkimusosasto

Tiivistelmä

Tutkin tässä työssä väestön ikääntymisen vaikutuksia talouden tasapainon kanssa sopusoinnussa (kestävyys) olevaan finanssipolitiikkaan yksinkertaisissa limittäisten sukupolvien malleissa, joissa eläkkeet määräytyvät jakojärjestelmän mukaan. Primääristen budjettivajeiden suurin kestävä määrä on sitä pienempi, mitä vähäisempi on väestön kasvuvauhti. Kun taloudenpitäjien hyötyfunktio on erityistä muotoa ja kun eläkkeet määräytyvät eläkkeelläoloperiodin palkkojen mukaan, eläkkeet rahoittavan veroasteen ja korvausasteen nosto supistaa tasapainon kanssa sopusoinnussa olevien maksimivajeiden määrää. Luonnehdin numeroesimerkkien avulla väestön kasvuvauhdiltaan toisistaan eroavien talouksien vajeiden suhdetta. Kommentoin lyhyesti myös verojen vaikutusta työn tarjontaan ja verotulojen määräytymiseen. Tutkimustulokset kyseenalaistavat sen, voidaanko ikääntymisestä mahdollisesti seuraavat ongelmat ratkaista korottamalla veroja ja lisäämällä julkisia menoja.

Avainsanat: limittäisten sukupolvien talous, finanssipolitiikan kestävyys, jakojärjestelmä, korvausaste

JEL-luokittelu: E21, E32

Contents

1 Introduction

The demographic structures of many European and Western countries are changing substantially. IMF (May 2001) reports that the elderly dependency ratios in the Euro area are forecasted to double during the next fifty years. This is due to low fertility and longer expected lifetimes in these countries. The significant aging of population will put a lot of pressure especially on publicly funded pension systems and health care costs.

The aging of the population and the slowdown in the population growth rates will have a substantial effect on fiscal policy. The same IMF report (2001, p. 107) forecasts the long-term development of the primary budget deficits and the debt of the European Union, Japan and the United States. The levels of these deficits and debts are predicted to be the highest in about 2050. According to that forecast the same happens for the health care expenditures. There will be pressure for governments to run larger deficits because of these expenditures and the growing pay-as-you-go pensions.

Given these fundamental changes expected to occur in these countries and elsewhere it is important to understand their implications for fiscal policy. And in particular, it is an important question to ask, if these policies are sustainable, ie consistent with equilibrium. In addition these changes will have an impact on welfare and intertemporal allocation.

We use fairly simple two-period overlapping generations (OG) models with government debt and pay-as-you-go pension systems to study the effects of aging on the sustainability of fiscal policy. Pension systems we study include a straightforward pay-as-you-go system and the one, where pension depends on income (replacement). Replacement is important, since it is a part of many existing pension plans. It is interesting to note that replacement rates in many Western countries have risen since the 1930's as Tanzi and Schuknecht (2000, p. 40) report. We characterize those policies (primary deficits without pension costs and the level of debts) which are consistent with equilibrium. In particular, this characterization includes finding out what happens to the maximum level of sustainable deficits, when population ages. In most of the formal analyses the technique of the reflected generational offer curves developed by Cass, Okuno and Zilcha (1979) are utilized. Since the emphasis in this study is on the sustainability of deficits under the population aging we refrain from political economy considerations.[1]

In a rather general model a drop in the growth rate of population decreases the maximum level of sustainable deficits. With specific functional forms an increase in the payroll tax rate and the replacement rate decreases the level of maximum

[1] For an interesting political economy OG model with population growth, see Boldrin and Rustchini (2000). For a survey see Galasso and Profeta (2002).

sustainable deficits. An increase in the replacement rate does not, however, have any effect on equilibrium, if pension depends on the income prevailing in the working period. This type of replacement rule has been studied in OG models by Abel (2003) and in a slightly different manner by Diamond (1997). Their emphasis has not, however, been in the effects of the changes in population growth. If on the other hand pension depends on the wage prevailing in the retirement period an increase in the replacement rate lowers the maximum sustainable deficits. This type of replacement rule has been studied by Bohn (1999, 2002) and Cooley and Soares (1996).

We also provide more precise and quantitative information especially on the ratio of maximum sustainable deficits in two economies with different population growth rates. In the simplest specification of the model with pay-as-you-go system the ratio of the maximum sustainable deficits of these two economies is directly proportional to the ratio of the population growth rates. Including replacement makes the comparison a bit more complex, since the ratio depends also on the replacement rate. In the more general version of the model with linear utility in consumption and logarithmic utility for leisure the ratio without replacement is very small, and with replacement a little bit less than one.

Finally we comment on the possibility of increasing tax rates to generate more tax revenues. In the general specification of the model we show that if the tax rate exceeds one third the elasticity of labor supply with respect to the tax rate is greater than unity. This, in particular, means that raising tax rate leads to lower government revenues.

In a related literature Masson (1985), Nielsen (1992), Chalk (2000) and Rankin and Roffia (2003) study the sustainability of deficits and debt. They all use OG models with capital, but their emphasis is not on the effects of aging on the maximum levels of deficits and debt.

2 The model and individual behavior

I consider a perfect foresight overlapping generations model with population growth. Producer-consumers consume when young and old and produce only in youth. The person born at t has the following additively separable lifetime utility function

$$U(c_{t+1}, h_t) = u(c_1^t) + \beta u(c_2^t) - v(h_t) \tag{2.1}$$

where c_i^t denotes consumption of person born at the beginning of time period t in period i (= 1,2), and h_t labor supply in youth. $\beta = (1+\rho)^{-1}$, where ρ is the rate of time preference. Labor is transformed to output, y_t, in a linear fashion, ie $y_t = h_t$. u(c) is an increasing strictly concave function, and v(h) an increasing strictly convex function. $\overline{L}$ denotes the upper bound for the available time. Furthermore, I first make the following assumptions: $\lim_{c\to 0} u'(c) = +\infty$, $\lim_{c\to\infty} u'(c) = 0$, $\lim_{h\to 0} v'(h) = 0$ and $\lim_{h\to\overline{L}} v'(h) = +\infty$. Later on some of these assumptions are relaxed.

Consumer can save in the private asset with the gross return $\overline{R}$ or in government's debt instrument with the return R_{t+1}. The private asset is a storage technology (or investment), which yields a certain gross return, and which can also be interpreted as a private social security system. A representative consumer's periodic budget constraints are

$$c_1^t + s_t + k_t = (1 - \tau^y - \tau^p)h_t \tag{2.2i}$$

$$c_2^t = R_{t+1}s_t + \overline{R}k_t + \pi_{t+1} \qquad \text{(2.2ii)}^2$$

s_t is investment in government debt and k_t in storage. τ^p is the social security tax rate and τ^y the income tax rate during period t. We use the notation and make a natural assumption $\tau = \tau^y + \tau^p < 1$. π_{t+1} is the pension consumer receives in retirement. I assume that he takes this pension parametrically in his decision making. The proceeds of the social security tax are used for pension payments for the old. Fiscal policy is specified more precisely below.

Consumer does not invest anything in government bond if $R_{t+1} < \overline{R}$. When $R_{t+1} > \overline{R}$, he does not invest anything in the private asset. In this case the first-order conditions (dropping the time subscripts for a moment) will be

$$(1-\tau)u'[(1-\tau)h - s] - v'(h) = 0 \tag{2.3i}$$

$$-u'[(1-\tau)h - s] + R\beta u'(Rs + \pi) = 0 \tag{2.3ii}$$

We can solve the labor supply and saving from here, and then consumptions from the budget constraints. We denote the optimal saving function by $s(R;\tau,\pi)$. The second order condition requires the Hessian matrix,

$$H = \begin{bmatrix} (1-\tau)^2 u''(c_1) - v''(h) & -(1-\tau)u''(c_1) \\ -(1-\tau)u''(c_1) & u''(c_1) + R^2\beta u''(c_2) \end{bmatrix} \tag{2.4}$$

[2] Cass and Yaari (1966) considered a storage technology in an OG model as did also Wallace (1980).

to be negative semi-definite. In addition to the concavity assumptions this requires the following condition, which, because of concavity, also holds

$$|H| = \Delta \equiv -v''(h)[u''(c_1) + u''(c_2)] + R^2(1-\tau)^2\beta u''(c_1)u''(c_2) > 0 \qquad (2.5)$$

The dependence of labor supply and saving on the interest factor will crucially depend on the elasticity of the second period marginal utility with respect to consumption (or the Arrow-Pratt measure of relative risk aversion). They are both increasing functions of the interest factor, if that measure, $\sigma(c_2)$, where $c_2(= Rs + \pi)$ is less than unity. From now on I assume that $\sigma(c_2) < 1$. And this will indeed mean that $s_R(R;\tau,\pi) > 0$. This is an important assumption also for dynamic analysis, since without this assumption there can be cycles and chaotic behavior. Because my emphasis is on the sustainability of deficits I maintain this assumption. In addition, below we will not explicitly consider investment in private asset.

3 Public sector's budget constraint, fiscal policy and pensions

Government's fiscal policy, F, is defined as a sequence of expenditures and taxes $F = \{g_t, \tau^p, \tau^y\}$. g_t denotes expenditures (excluding expenditures on pensions) per young. Below we will also use the term deficit, when discussing the role of g_t. The aggregate budget constraint in real terms is

$$B_t = D_t + (1 + r_t)B_{t-1} \qquad (3.1)$$

where B_t denotes the aggregate public debt at the end of the period, D_t the deficit and r_t the real rate of interest.[3] The gross population growth rate is assumed to be $n > 0$. Thus there are n^t young and n^{t-1} old people around in period t. I define $b_t = B_t/n^t$ and $d_t = D_t/n^t$. Thus I can express the budget constraint in the per young form as

$$b_t = d_t + \frac{R_t}{n} b_{t-1} \qquad (3.2)$$

The primary deficit can then be defined as

[3] We could have specified a separate budget (trust fund) for pension costs, but that would not have affected the equilibrium; for such a separation, see Bohn (1999).

$$D_t \equiv n^t d_t = n^t g + n^{t-1}\pi_t - n^t \tau^p h_t - n^t \tau^y h_t \tag{3.3}$$

where we get

$$\frac{D_t}{n^t} \equiv d_t = g + \frac{\pi_t}{n} - \tau^p h_t - \tau^y h_t \tag{3.4}$$

Government's intertemporal budget constraint can now be written as

$$b_t = g + \frac{\pi_t}{n} - \tau^p h_t - \tau^y h_t + \frac{R_t}{n} b_{t-1} \tag{3.5}$$

For example a pure pay-as-you-go pension system is the one, where $\pi_t = n\tau^p h_t$. Under that system (3.5) reduces to

$$b_t = g - \tau^y h_t + \frac{R_t}{n} b_{t-1} \tag{3.6}$$

I consider other pension systems more precisely below. It is worth re-emphasizing that g includes other government expenditures except interest on public debt and expenditures due to public pension systems. Eg costs related to health care, which are included in g, are expected to rise significantly, when population ages. In the following analysis a lot of emphasis is put on understanding the limits of these expenditures in an aging economy. Expenditures, which are not consistent with equilibrium, are not sustainable.

4 Equilibrium in a general model with pure pay-as-you-go pensions

The asset market equilibrium condition is $b_t = s_t$. I characterize the equilibrium sequences of public debt by using consumer's first-order conditions.

Using consumer's budget constraints, the optimum choices, and the asset market equilibrium condition the first-order condition (2.3i) can be re-written as an equilibrium condition

$$(1-\tau)u'\left[(1-\tau)h_t - b_t\right] - v'(h_t) = 0 \tag{4.1}$$

which yields a contemporaneous relation between today's labor supply and the level of government debt. This relation is denoted by $h(b_t)$. Totally differentiating (4.1) we get

$$\frac{dh_t}{db_t} = \frac{(1-\tau)u''\left[(1-\tau)h_t - b_t\right]}{(1-\tau)^2 u''\left[(1-\tau)h_t - b_t\right] - v''(h_t)} > 0 \tag{4.2}$$

For future reference note that $(1-\tau)dh_t / db_t < 1$. If $b_t = 0$, there is a positive labor supply (denoted by h(0)), which fulfills (4.1), because of the Inada conditions.

I use this relation, the asset market equilibrium condition, $b_t = s_t$, and the interest factor from government's budget constraint (3.6) in the second first-order condition (2.3ii) above to derive the equilibrium evolution of the debt as follows

$$\begin{aligned} &b_t u'\left[(1-\tau)h(b_t) - b_t\right] = \\ &n\left[b_{t+1} - g + \tau^y h(b_{t+1})\right]\beta u'\left[n(b_{t+1} - g + \tau^y h(b_{t+1})) + n\tau^p h(b_{t+1})\right] \end{aligned} \tag{4.3}$$

I define the primary deficit as $d_t = g - \tau^y h(b_{t+1})$ and the following functions

$$A(b) \equiv bu'\left[(1-\tau)h(b) - b\right] \text{ and } B(b,d) \equiv n\left[b-d\right]\beta u'\left[n(b-d) + n\tau^p h(b)\right] \tag{4.4}$$

Now for any constant sequence of deficits equilibrium debt is a sequence, which fulfills the difference equation

$$A(b_t) = B(b_{t+1}, d) \qquad (4.5)^4$$

Many details of this model are rather well known so perhaps here it suffices to gather some basic facts, although below I'll do a more detailed analysis of dynamics, when population is aging.

I assume that we are dealing with the Samuelson case (for terminology, see Gale, 1973) here, which means that we need to assume that

$$u'\left[(1-\tau)h(0)\right] < n\beta u'\left[n\tau^p h(0)\right] \tag{4.6}$$

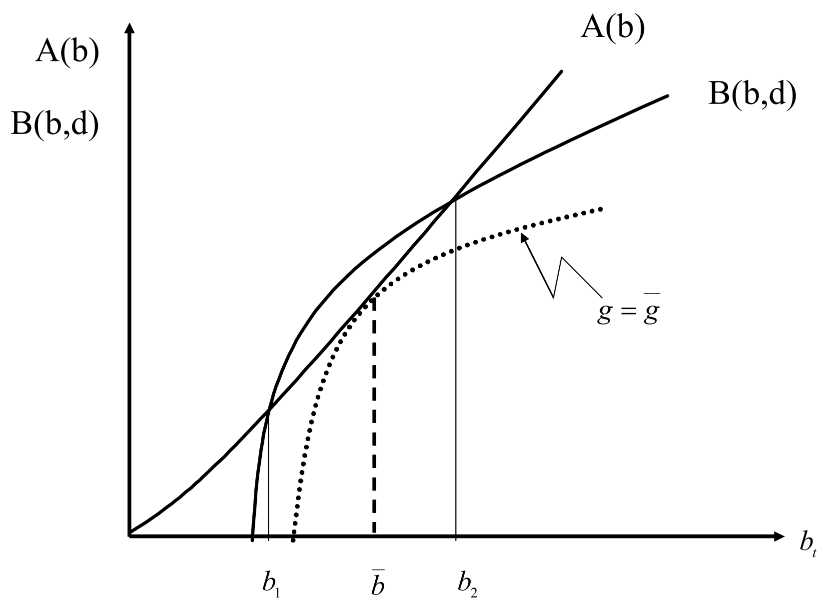
Due to our assumptions both functions are increasing in debt. When deficit is zero, the steady state equilibrium is determined from $u'\left[(1-\tau)h(b) - b\right] = n\beta u'\left[nb + n\tau^p h(b)\right]$. Given the Inada conditions and the Samuelson case it is

[4] This is exactly the way that Brock and Scheinkman (1980) do their analysis.

straightforward to see that the steady state is unique, since the left-hand side of the steady state equation is increasing and the right-hand side decreasing in b.[5]

Now I will analyze the dynamics in more detail. Given our assumptions particularly on the curvature of the second period utility function, one can see that the curve B(b, d) shifts down, when the level of deficits or government expenditures (g) is increased. This also means that there is a maximum level of expenditures (denoted by $\bar{g}$) consistent with equilibrium. Thus $\bar{g}$ is the maximum sustainable level of expenditures. The corresponding level of the steady state debt is denoted by $\bar{b}$. It is also well known that with positive, but not too large, primary deficits, there are usually two steady states at least in pure exchange economies with homogenous population.[6] The dynamics are described in Figure 1, where the steady state levels of debt for some level of deficits less than the maximum level are denoted by b_1 and b_2.

Figure 1.



What happens to the maximum level of expenditures, when the population growth slows down? First we note that a decrease in population growth rate, n, shifts the curve, B(b, d), downwards. Suppose that we are originally at the equilibrium, where the maximum level of government expenditures corresponding to n is $\bar{g}$. Since B(b, d) has shifted downwards because of a decrease in the population growth rate, we need to further decrease expenditures to restore equilibrium, and

[5] Azariadis (1993) characterizes equilibrium dynamics of an exchange economy with debt and deficits; see especially his chapter 19.

[6] This is basically known since Bailey (1956).

shift that curve upwards. Thus we have shown that other government expenditures, excluding pension costs, will have to decrease, when population growth rate drops.

To get more precise information about the effects of aging we will now turn to specific examples. They are still rather general, since in these examples there are also two steady state equilibria for many fiscal policies.

5 The sustainability of the fiscal policy

To get more intuition about the sustainability I will first consider a simple example with pure pay-as-you-go pension system. Consumer's preferences are

$$U(c_2^t, h_t) = \beta c_2^t - (1/2)h_t^2 \tag{5.1}$$[7]

The budget constraints are

$$s_t = (1 - \tau^y - \tau^p)h_t \tag{5.2i}$$

$$c_2^t = R_{t+1}s_t + \pi_{t+1} \tag{5.2ii}$$

The optimum labor supply is $h_t = R_{t+1}\beta(1-\tau)$ and saving $s_t = R_{t+1}\beta(1-\tau)^2$. The real after tax wage (here $R_{t+1}(1-\tau)$) elasticity is unity. It is interesting to note that the elasticity of labor supply with respect to the total tax rate is $\tau/(1-\tau)$, which is greater than unity for any tax rate exceeding one half.

The asset market equilibrium condition requires that $s_t = b_t$. We characterize the equilibrium sequences of public debt by using the reflected generational offer curve techniques of Cass, Okuno and Zilcha (1979). Using consumer's budget constraints, the optimum choices, and the asset market equilibrium condition we can rewrite government's budget constraint (3.6) above as

$$b_t = \left(\frac{1-\tau}{1-\tau^p}\right)g_t + \left(\frac{1-\tau}{1-\tau^p}\right)\frac{R_t}{n}b_{t-1} \tag{5.3}$$

Since from the optimum labor supply and the asset market equilibrium condition we can solve for the interest factor as

[7] Azariadis (1981) and Farmer and Woodford (1997) use this specification for preferences.

$$\frac{b_t}{\beta(1-\tau)^2} = R_{t+1} \qquad (5.4)$$

we end up with the following expression (indeed this is the reflected generational offer curve) for the evolution of the debt

$$b_t = \left(\frac{1-\tau}{1-\tau^p}\right)g + \frac{1}{(1-\tau^p)(1-\tau)}\frac{b_{t-1}{}^2}{\beta n} \qquad (5.5)$$

Figure 2 describes this curve. We compute the maximum sustainable expenditures from the following steady state expression for (5.5)

$$b^2 - (1-\tau^p)(1-\tau)\beta n b + (1-\tau)^2 \beta n g = 0 \qquad (5.6)$$

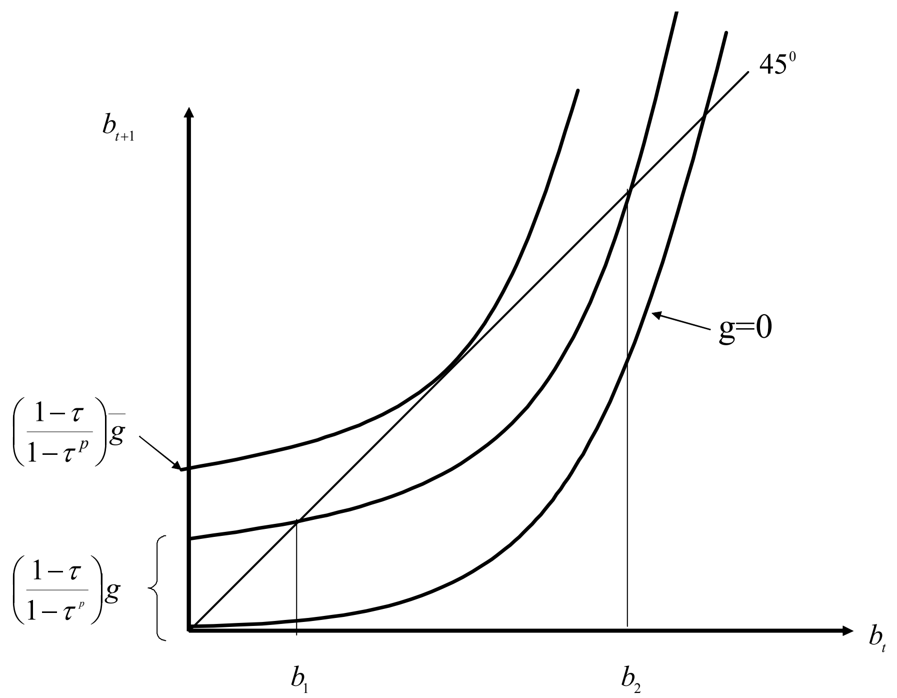
Solving the quadratic equation for the roots we get

$$b = \frac{1}{2}\left\{(1-\tau^p)(1-\tau)\beta n \pm \sqrt{(1-\tau^p)^2(1-\tau)^2\beta^2 n^2 - 4\beta n(1-\tau)^2 g}\right\} \qquad (5.7)$$

It can be seen that real solutions exist for all government expenditures such that

$$(1-\tau^p)^2(1-\tau)^2\beta^2 n^2 - 4\beta n(1-\tau)^2 g \geq 0 \qquad (5.8)$$

Figure 2.



Thus the maximum sustainable government expenditures (again denoted by $\bar{g}$) are $\bar{g} = (1-\tau^p)^2\beta n/4$.

We conclude that $\bar{g}$ is smaller the smaller is the population growth rate. And furthermore, the higher the social security tax the lower the level of the maximum sustainable government expenditure consistent with equilibrium. These are important results, since other government expenditures besides pension costs (eg health care costs) are expected to increase, when population ages. And there seems to be also pressure to increase different taxes. Of course, one should note that all economies are not necessarily operating at or even close to the maximum sustainable expenditures. But many contemporary aging economies will easily approach the limits, if their expenditures increase enough.

Furthermore, if two economies differ only with respect to their population growth rates, the ratio of their maximum sustainable expenditures ($\bar{g}_i/\bar{g}_j$) is directly proportional to their ratios of the gross population growth rates (n_i/n_j). Eg suppose that there is one percent difference in the annual gross growth rates such that $n_i^a = 1.01$ and $n_j^a = 1.02$ (superscript a refers to annual). Assuming that one period in the model is thirty years we get $\bar{g}_i/\bar{g}_j = 1.3478/1.8114 = .74$, which means that the maximum sustainable expenditures in the economy with slower population growth are about three fourths of that of the faster growing economy.

Now I consider an example, where pensions depend on income, ie an example with replacement. Tanzi and Schuknecht (2000, p. 40) report that the replacement rates have increased in many Western countries since the 1930's from below 20 per cent to even more than 60 per cent. The replacement rate is denoted by θ. It is the share of income a worker gets as a pension. In the first case pension in retirement will be a fraction θ of the working period's real income (wage). This case has been studied in OG models also by Abel (2003) and in a different manner by Diamond (1997).

The periodic budget constraints are now

$$s_t = (1-\tau^y-\tau^p)h_t \tag{5.9i}$$

$$c_2^t = R_{t+1}s_t + R_{t+1}\theta h_t \tag{5.9ii}$$

Note that before tax purchasing power of the wage (ie the real wage income) is $R_{t+1}h_t$ in retirement. From the budget constraints it is quite easy to see that the level of θ does not affect anything in the economy. This is due to the fact that this type of replacement rule essentially guarantees the same return as what the consumer gets by voluntarily investing in the government bond. The optimal behavior is $h_t = R_{t+1}\beta(1-\tau+\theta)$ and $s_t = R_{t+1}\beta(1-\tau+\theta)(1-\tau)$. Government's aggregate primary and per worker deficits are

$$D_t \equiv n^t d_t = n^t g_t + n^{t-1} R_t \theta h_{t-1} - n^t \tau h_t \tag{5.10i}$$

$$d_t = g + \frac{R_t \theta}{n} h_{t-1} - \tau h_t \tag{5.10ii}$$

The intertemporal budget constraint is now

$$b_t = g + \frac{R_t \theta}{n} h_{t-1} - \tau h_t + \frac{R_t}{n} b_{t-1} \tag{5.11}$$

After a little manipulation we end up with the following equilibrium relation for debt (reflected generational offer curve)

$$b_t = g(1-\tau) + \left(\frac{1}{\beta(1-\tau)}\right)\frac{b_{t-1}{}^2}{n} \tag{5.12}$$

from where we easily conclude that the replacement rate does not affect the behavior of debt over time.

The second case, studied here, is the one, where pension is computed from the wage prevailing in the retirement period. This case has been studied eg by Bohn (1999, 2002) and Cooley and Soares (1996). The budget constraints are thus

$$s_t = (1-\tau^y-\tau^p)h_t \tag{5.13i}$$

$$c_2^t = R_{t+1}s_t + \theta h_{t+1} \tag{5.13ii}$$

The optimal behavior is $h_t = R_{t+1}\beta(1-\tau)$ and $s_t = R_{t+1}\beta(1-\tau)^2$. The primary deficit per worker can now be expressed as

$$d_t = g + \frac{\theta}{n} h_t - \tau^p h_t - \tau^y h_t \tag{5.14}$$

Pensions are financed by particular payroll taxes, ie $\theta h_t = n\tau^p h_t$. We consider the replacement rate as a policy variable. Then the social security tax rate, τ^p, must adjust to reflect possible changes in θ. Government's intertemporal budget constraint can now be written as

$$b_t = g - \tau^y h_t + \frac{R_t}{n} b_{t-1} \tag{5.15}$$

Constructing again the reflected generational offer curve we get the following equilibrium representation for debt

$$b_t = \left(\frac{1-\tau}{1-\tau^p}\right) g + \frac{1}{\beta n(1-\tau)(1-\tau^p)} {b_{t-1}}^2 \tag{5.16}$$

Considering the fact that $\theta = n\tau^p$ we re-express the equilibrium relation (5.11) as

$$b_t = \left(\frac{1-\tau}{n-\theta}\right) ng + \frac{1}{\beta(1-\tau)(n-\theta)} {b_{t-1}}^2 \tag{5.17}$$

Steady state requires that $b^2 - \beta(1-\tau)(n-\theta)b + (1-\tau)^2 \beta ng = 0$. Solving the quadratic we get

$$b = \frac{1}{2}\left\{(1-\tau)(n-\theta)\beta \pm \sqrt{(n-\theta)^2(1-\tau)^2\beta^2 - 4\beta n(1-\tau)^2 g}\right\} \tag{5.18}$$

The real solutions exist for all government expenditures such that $(n-\theta)^2(1-\tau)^2\beta^2 - 4\beta n(1-\tau)^2 g \geq 0$. Thus the maximum sustainable government expenditures are $\bar{g} = (n-\theta)^2\beta / 4n$. We conclude that $\bar{g}$ is smaller the smaller is the population growth rate, and in particular, the higher the replacement rate the lower is the level of the maximum sustainable government expenditure consistent with equilibrium.

Again, if two economies differ only with respect to their population growth rates, the ratio of their maximum sustainable expenditures ($\bar{g}_i / \bar{g}_j$) is not directly proportional to the ratios of their gross population growth rates, but $\bar{g}_i / \bar{g}_j = (n_i - \theta)^2 n_j / (n_j - \theta)^2 n_i$. Suppose again that $n_i^a = 1.01$ and $n_j^a = 1.02$, and in addition assume $\theta = 2/3$. The last assumption seems to be quite realistic (see Tanzi and Schuknecht (2000, p. 40)). Assuming one period to be thirty years we get $\bar{g}_i / \bar{g}_j = (1.3478 - .6667)^2 1.8114 / (1.8114 - .6667)^2 1.3478 = .4758$. This means that the maximum sustainable expenditures in the economy with slower population growth are about half of that of the faster growing economy. It seems that inclusion of replacement will substantially decrease the ratio of sustainable deficits.

Next I will briefly study the robustness of these results. Although the above examples might seem to be even naively simple, it is actually the case that with rather general preferences the reflected generational offer curves have the convex shape I have drawn above in Figure 2.[8] And thus the results on the sustainability of the fiscal policies presented in these examples seem to be more general than what one would believe based on these examples alone.

6 The sustainability again: a more general case

The preferences are now assumed to be

$$U(c_1^t, c_2^t, h_t) = \beta c_2^t + \alpha \ln(1 - h_t) \tag{6.1}$$

The budget constraints are as above in (5.2i) and (5.2ii). Since labor supply in this specification has an upper bound of unity, it is worthwhile to note that the maximum amount of saving (and indeed government debt in equilibrium) is $1 - \tau$. Labor supply is now

$$h_t = 1 - \frac{\alpha}{R_{t+1}\beta(1-\tau)} \tag{6.2}$$

Solving the interest factor from (6.2) we get

$$R_t = \frac{\alpha}{\beta(1-\tau)(1-h_{t-1})} \tag{6.3}$$

The reflected generational offer curve can now be expressed as

$$b_t = \left(\frac{1-\tau}{1-\tau^p}\right) g + \frac{\left(\frac{1-\tau}{1-\tau^p}\right)\frac{\alpha}{\beta n} b_{t-1}}{(1-\tau-b_{t-1})} \tag{6.4}$$

By computing I get

$$\frac{\partial b_t}{\partial b_{t-1}} = \frac{\left(\frac{1-\tau}{1-\tau^p}\right)\frac{\alpha}{\beta n}}{(1-\tau-b_{t-1})} + \frac{\left(\frac{1-\tau}{1-\tau^p}\right)\frac{\alpha}{\beta n} b_{t-1}}{(1-\tau-b_{t-1})^2} = \frac{\left(\frac{(1-\tau)^2}{1-\tau^p}\right)\frac{\alpha}{\beta n}}{(1-\tau-b_{t-1})^2} > 0 \tag{6.5}$$

And the second derivative is

[8] For elaborations see Cass, Okuno and Zilcha (1979) and several chapters in Azariadis (1993). As Cass, Okuno and Zilcha point out, offer curves can have almost any shape, if there is a lot of heterogeneity among consumers.

$$\frac{\partial^2 b_t}{\partial b_{t-1}^{\;2}} = \frac{2\left(\frac{(1-\tau)^2}{1-\tau^p}\right)\frac{\alpha}{\beta n}}{(1-\tau-b_{t-1})^3} > 0 \tag{6.6}$$

Thus the reflected generational offer curve is a strictly convex function of debt. And furthermore, to have a possibility for positive levels of debt, we need to assume that the slope of the offer curve at the origin is less than unity, ie we need a condition $\alpha/(1-\tau^p)\beta n < 1$. Using again the terminology in Gale (1973) this is essentially the condition for the Samuelson case, which is needed for equilibria with positive levels of debt.

To find out the maximum sustainable level of expenditures I set the slope of the offer curve to be unity, since there must be only one level of debt, which is consistent with those expenditures, ie

$$\frac{\left(\frac{(1-\tau)^2}{1-\tau^p}\right)\frac{\alpha}{\beta n}}{(1-\tau-b)^2} = 1 \quad \Rightarrow (1-\tau)^2\alpha = \beta n(1-\tau^p)(1-\tau-b)^2 \tag{6.7}$$

Solving for debt we get

$$b = (1-\tau)\left(1 \pm \sqrt{\frac{\alpha}{\beta n(1-\tau^p)}}\right) \tag{6.8}$$

Since debt can be at most $1-\tau$, we choose the smaller value. We plug this value of debt into (6.4) to get the following equation

$$(1-\tau)\left(1-\sqrt{\frac{\alpha}{\beta n(1-\tau^p)}}\right) = \frac{1-\tau}{1-\tau^p}g + \frac{(1-\tau)\alpha\left(1-\sqrt{\frac{\alpha}{\beta n(1-\tau^p)}}\right)}{(1-\tau^p)\beta n\left(\sqrt{\frac{\alpha}{\beta n(1-\tau^p)}}\right)} \tag{6.9}$$

Solving the maximum level of expenditures from (6.9) we get

$$\bar{g} = (1-\tau^p)\left(1-\sqrt{\frac{\alpha}{\beta n(1-\tau^p)}}\right)^2 \tag{6.10}$$

Thus $\bar{g}$ is smaller the smaller is the population growth rate. And furthermore, the higher the social security tax rate the lower is the level of the maximum sustainable government expenditure consistent with equilibrium.

Now the ratio of the maximum sustainable expenditures ($\bar{g}_i / \bar{g}_j$) is again not directly proportional to the ratios of the gross population growth rates. To be able to make the comparison we need to pick a value for the preference parameter, α. We assume again that $n_i^a = 1.01$ and $n_j^a = 1.02$, and in addition that $\beta = 1/2$, and $\tau^p = 1/10$.[9] Since α is a preference parameter, we need to calibrate it. The labor supply in equilibrium is $h = 1 - \sqrt{\alpha} / \sqrt{\beta n(1-\tau^p)}$. This should be roughly one third, since that is the approximate time used for productive purposes. McGrattan and Rogerson (2004) report that the average weekly hours worked have stayed around 40 per worker since 1950. We see that hours depend on the rate of population growth. For $n_i^a = 1.01$ and $n_j^a = 1.02$ the respective preference parameters will be $\alpha_i = .5192$ and $\alpha_j = .60189$. I use the average of these, .56054, in my computations. The ratio of expenditures is now $\bar{g}_i / \bar{g}_j = (1 - 1.28402\sqrt{\alpha})^2 / (1 - 1.10762\sqrt{\alpha})^2$, which will yield $\bar{g}_i / \bar{g}_j = .05127$. This is a strikingly small number, which means that the maximum sustainable expenditures in the economy with slower population growth are about five percent of the faster growing economy.

Pensions with replacement are $\theta h_t = n\tau^p h_t$. The equilibrium reflected generational offer curve is now

$$b_t = \left(\frac{1-\tau}{n-\theta}\right) g + \frac{\left(\frac{1-\tau}{n-\theta}\right)\frac{\alpha}{\beta} b_{t-1}}{(1-\tau-b_{t-1})} \tag{6.11}$$

Proceeding as above we solve for the maximum sustainable level of debt

$$\bar{b} = (1-\tau)\left(1 \pm \sqrt{\frac{\alpha}{\beta(n-\theta)}}\right) \tag{6.12}$$

Choosing again the smaller value we end up with the following implicit equation for the maximum expenditures

[9] If one period is thirty years, the discount factor of one half corresponds to the annual discount factor of .97716.

$$(1-\tau)\left(1-\sqrt{\frac{\alpha}{\beta(n-\theta)}}\right)=\left(\frac{1-\tau}{n-\theta}\right)ng+\frac{(1-\tau)\alpha\left(1-\sqrt{\frac{\alpha}{\beta(n-\theta)}}\right)}{(n-\theta)\beta\left(\sqrt{\frac{\alpha}{\beta(n-\theta)}}\right)} \tag{6.13}$$

from where we get

$$(1-\tau)\bar{g}=\left(\frac{n-\theta}{n}\right)\left(1-\sqrt{\frac{\alpha}{\beta(n-\theta)}}\right)^2 \tag{6.14}$$

Thus again $\bar{g}$ is smaller the smaller is the population growth rate. And furthermore, the higher the replacement rate the lower is the level of the maximum sustainable government expenditure consistent with equilibrium.

Since the labor hours in equilibrium are $h=1-\sqrt{\alpha}/\sqrt{\beta(n-\theta)}$, we set them equal to one third and get the expenditure ratio as $\bar{g}_i/\bar{g}_j=(n_i-\theta)n_j/(n_j-\theta)n_i$. Assuming the replacement rate to be one half we get $\bar{g}_i/\bar{g}_j=(1.34785-.5)1.81136/(1.81136-.5)1.34785=1.53576/1.76751=.86888.$ This means that the maximum sustainable expenditures in the economy with slower population growth are about 15 per cent lower than in the faster growing economy.

7 Taxes and labor supply

It is often claimed that tax rates should be increased when population ages. The total taxes are $T=\tau h(\tau)$. When the tax rate changes there is a corresponding change in revenues

$$\frac{\partial T}{\partial \tau}=h(\tau)+\tau\frac{\partial h}{\partial \tau}=h(\tau)\left[1+\frac{\partial h}{\partial \tau}\frac{\tau}{h}\right] \tag{7.1}$$

T becomes smaller, if the elasticity of labor supply with respect to the tax rate $\left(-\frac{\partial h}{\partial \tau}\frac{\tau}{h}\right)$ is larger than unity.

We will next reconsider the above example from this point of view. The elasticity is

$$-\frac{\partial h}{\partial \tau}\frac{\tau}{h}=\frac{\alpha\tau}{(1-\tau)\left[R\beta(1-\tau)-\alpha\right]} \tag{7.2}$$

Since the hours of work are assumed to be one third we can easily compute that $R\beta(1-\tau)=(3/2)\alpha$. This means that the elasticity equals $2\tau/(1-\tau)$. This number is greater than unity for all tax rates which exceed one third. Practically in every Western market economy the tax rate (including all taxes) exceeds that number. From this perspective it is not clear that a plan to increase taxes can be a part of the solution to the aging problem.

8 Conclusions

Utilizing two-period overlapping generations models it has been shown that the effects of aging on the sustainability of fiscal policy are robust, and in many examples rather worrisome for beliefs about the power of expansive fiscal policy to solve the problems of aging. With pay-as-you-go pension system a drop in the growth rate of population decreases the maximum level of sustainable deficits. If pensions depend on income (replacement) prevailing in the working period, increases in the replacement rate do not have any effect on equilibrium. If on the other hand pension depends on the wage prevailing in the retirement period an increase in the replacement rate lowers the level of maximum sustainable deficit.

Numerical examples with two economies differing only with respect to their population growth rates show that with pay-as-you-go system the ratio of the maximum sustainable deficits of these two economies is directly proportional to the ratio of their population growth rates. Comparison with replacement gives the value of that ratio about half. In the more general version of the model the ratio without replacement is only about five percent, and with replacement almost ninety percent.

With the labor supply specification of the more general model the elasticity of supply with respect to the tax rate is greater than unity for all tax rates which exceed one third. So it is not clear at all that plans to increase taxes can be a part of the solution to the aging problem in many Western market economies.

Perhaps an important missing element in the model is the fact that consumer-workers cannot choose the exact time of their retirement. If they live longer than expected they might want to defer their retirement date. The possibility of that decision might alleviate the problems we have discussed above. But it bears emphasizing that many prevailing institutional restrictions (eg mandatory retirement age) do not really let workers to choose their retirement age in any case. A model along the lines of Sheshinski (1978) and Michel and Pestieau (2003) could be helpful in developing an OG model with voluntary retirement.

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