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28/98

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Research Department
14.12.1998

Developing the Interbank Payment System Efficiency of Public versus Private Investments

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Abstract

In this paper a game theoretic duopoly model is developed to analyse the development of an interbank payment system. There are two competing banks in the model, and payment services offered to the public are among their main products. The customer of the larger bank uses mainly intrabank payment services; these services are assumed to be of high quality. This creates a so-called network externality, meaning that many customers prefer to use the large bank for quality reasons. The development of interbank payment systems reduces the significance of this factor and hence benefits the small bank. A big bank has a sufficient incentive to develop the system only if a fee is charged for using payment systems. The role for public investment depends critically on the pricing of payment services. If banks offer payment services free of charge, their incentives to develop the system are strongly biased, and it would be efficient for the central bank to have an active role in developing the system. If instead payment services are directly priced, eventual distortions are much less serious, and the role of the central bank need not be as prominent.

JEL Classification Numbers: G18, G21, L13

Keywords: banks, payments systems, network externality, duopoly

Pankkien välisen maksuliikennejärjestelmän kehittäminen
Julkisten ja yksityisten investointien suhteellinen tehokkuus

Suomen Pankin keskustelualoitteita 28/98

Karlo Kauko
Tutkimusosasto

Tiivistelmä

Tutkimus käsittelee peliteoreettisen mallin avulla pankkien välisen maksujärjestelmän kehittämistä. Mallissa on kaksi kilpailevaa pankkia, joiden palveluvalikoimassa yleisölle tarjottavat maksupalvelut ovat keskeisellä sijalla. Suuremman pankin asiakas käyttää etupäässä pankin sisäisiä maksupalveluita, jotka ovat aina laadukkaita. Tämä aiheuttaa ns. verkostoeksternaliteetin, ja monet asiakkaat valitsevat mieluiten suuren pankin. Pankkien välisen maksujärjestelmän kehittäminen vähentää tämän tekijän merkitystä pienentämällä pankkien välisen ja pankin sisäisen maksujärjestelmän välistä laatueroa ja siis hyödyttää pientä pankkia. Suuren pankin kannalta on mielekästä kehittää pankkien välistä järjestelmää vain, jos maksupalveluista peritään maksu. Julkisten investointien asema riippuukin paljon maksupalvelujen hinnoittelukäytännöstä. Jos pankit tarjoavat ilmaisia maksupalveluita, niiden kannustimet kehittää järjestelmää ovat pahoin vääristyneet, ja keskuspankin olisi tehokasta osallistua järjestelmän kehittämiseen. Jos maksupalvelu on hinnoiteltu suoraan, mahdolliset vääristymät ovat paljon vähäisempiä, eikä keskuspankin tarvitse olla yhtä keskeisessä asemassa.

JEL-luokitusnumerot: G18, G21, L13

Asiasanat: pankit, maksujärjestelmät, verkostoeksternaliteetti, duopoli

Contents

1 Introduction

1.1 The purpose of the paper

In many countries, the central bank is responsible for both price stability and the smooth functioning of payment systems. For instance, the Bank of Finland Act explicitly states (paragraph three) that the central bank shall contribute to developing the payment system. According to the Maastricht treaty, the European Central Bank is obliged to contribute to the smooth functioning of payment systems.

But if the system is organised by the private sector with minimal central bank involvement, will there be a market failure? Because theorists have largely ignored this question, it is difficult to justify with solid economic arguments the existence of laws that oblige central banks to contribute to payment systems. This paper is a preliminary attempt to shed some light on this subject.

As we will show in this paper, the nature of optimal central bank involvement may depend on various factors. It turns out that optimal central bank policy may depend on whether payment services are a free service, and secondly, on the market structure of the banking industry.

According to the model to be presented in the following, the market outcome is seriously distorted if payment services are offered to the public free of charge as a marketing tool. If customers' needs for services were small, the private sector would invest heavily, and *vice versa*. The incentives of the private sector are totally distorted in two extreme cases, namely if either the market is highly concentrated or the market shares are of equal size. The central bank should have an important role both because private investment may be insufficient and because the central bank can affect the behaviour of the private sector.

Instead, if payment services are offered because of the fee revenue that can be earned, the situation is different. Banks can increase their income by improving the system when the number of interbank payments is large. Thus developing the system is profitable when there are a lot of interbank payments to be processed. This is a socially desirable incentive structure. Especially if banks' market shares are equal, there will be no serious market failure, and involvement of the central bank is not as essential as with free payment services. In fact, excessive investments by the central bank might even worsen the allocative distortions that could emerge with the use of private resources.

As with many formal models, the one presented in the following sections could be interpreted in various ways. There are certainly strong analogies with payment systems and the telecommunications industry, and it might be possible to interpret the model as a description of competition between, say, two mobile phone operators. In fact, there are hardly any details in the following model that are *absolutely* inconsistent with the realities of the telecommunications industry. Thus the model might have some implications for other industries as well.

1.2 Central banks and interbank payments in the real world

International comparisons reveal that there are clear differences between countries in the degree of central bank involvement in payment systems. In Germany and the U.S, for example, a significant part of the payment system is virtually run by the central bank. In some other countries, such as the U.K, the role of the central bank is limited to the final settlement between a few major payment system agents.

The efforts of the central bank are often essential to the smooth transacting of interbank payments. In practice, it would be hardly possible to create a reliable and efficient interbank payment system without any involvement by the central bank. At least the final (net) settlement between banks is effected with central bank money.

There are at least three kinds of investments the central bank can make.

1) The central bank can make purely technical investments, such as renovations of its own computer software and hardware. For instance, it could offer various types of alternative settlement systems for payments made with central bank money, or arrange automatic queuing facilities to facilitate clearing in case of illiquidity. These improvements might reduce the number of errors and speed up the process. The central bank could also make direct contributions to systems that are owned and operated collectively by the government and the private sector.

2) The central bank can adapt its rules and practices so that the interbank payment system functions better. For instance, the frequency of net clearings could be increased, which would enable banks to arrange faster payment services. Or, to take another example, if the customers of a bank make significantly more payments than they receive, the bank might not be able to pay the sum of these payments to other clearing parties unless the central bank provides it with sufficient liquidity.

3) The central bank can arrange combinations of regulations and subsidies that lead to improved payment services. For instance, the central bank might require that at least certain payments are processed with real time gross settlement (which might be more burdensome for banks) and at the same time subsidise participating banks by offering free services.

1.3 The literature

The number of papers analysing the monetary policy function of central banks is very large, and it is possible to identify different research traditions within the field, such as the discussion of central bank independence. By contrast, there are few contributions that provide us with theoretical insights concerning central banks' optimal payment systems policy. The functioning of different settlement systems is one of the few topics that has been analysed, but the focus has been on differences between various net and gross settlement systems instead of on market failures that require governmental intervention.

There are several theoretical contributions analysing the specifics of the banking industry, as either an oligopoly or a monopoly. In these contributions, the focus has been on banks' role as financial intermediaries between savers and investors. One of the central topics has been the strengths and weaknesses of intermediated finance compared to direct market-based financing. These models are often based on asymmetric information. Banks supposedly can monitor their debtors better than savers can. The role of branch networks and ATMs in differentiation and oligopolistic competition has been another central topic.

Moreover, there are several theoretical contributions that deal with payment systems. In most of these previous studies, the focus has been on the trade-off between risks and cost efficiency in clearing and settlement. If banks do not coordinate their actions, they normally maintain suboptimal balances with the settlement system, for instance on their central bank accounts. (See Angelini and Gianni 1994)

Here, the aim is to analyse the topic from another point of view, namely by analysing the situation as a struggle for market share. Risks and costs related to different interbank net and gross settlement systems are ignored. One of the few previous theoretical contributions concerning payment services as a competitive tool was that of McAndrews and Roberds (1997). They developed a duopoly model that describes cheque processing. A bank that can control the clearinghouse could either charge fees or delay cheque processing in order to affect adversely the ability of the competitor to offer services.

The model presented in the following is characterised by network externalities. Several articles dealing with these effects have been written. The concept was introduced to economic theory in the mid 1980s, one of the first contributions being made by Katz and Shapiro (1985). In the presence of network externalities, the utility provided by goods is greater if the number of other consumers using the same product is large. The telephone and e-mail are excellent examples of these kinds of effects; a telephone yields no consumer utility unless there are at least two telephones connected to the same network. The pricing policies of a monopoly company that owns the network have been analysed as one of the key issues.

Liebowitz and Margolis (1994) have written a review article covering this field. They emphasise the difference between network externalities and mere network effects, and argue that pure network externalities are not commonplace. Network effects, by contrast, are numerous. A network effect arises, for instance, because the availability of software improves if the number of consumers using a certain type of computer increases. If the same standard is preferred by the majority, the availability of services and compatible products is good, but this effect is not an externality in the strict sense of the word.

The impact of network externalities in competition between firms has been the topic of several recent theoretical contributions. Laffont, Rey and Tirole (1997) have presented a model describing the competition between two telephone operators. The model has several strong analogies with the model presented below. There are two telecommunications operators in their model. Each customer is interested in phoning both other customers of the same operator and customers of the other operator. In this model, nonlinear pricing and barriers to entry are analysed. Probably the most important analogy between this model and the model presented in this paper is the strategic motivation of the bigger competitor to hinder connections between service providers. By contrast, Laffont, Rey and Tirole do not analyse investments for developing the linkage between the networks. Neither is there a public body acting as the centre of the network in the Laffont-Rey-Tirole model.

2 Structure of the payment system in the model

The model describes a payment system consisting of the providers of payment services and the customers who use the system for payment transactions among themselves.

There are two profit maximising commercial banks that provide their customers with different banking services, including payment services. The payment services could be either giro, cheque or debit card payments, though the model probably best describes giro payments.

There are n customers, n being a very large number ($n>>0$). As to real life interpretations, it is likely that there are millions of customers. Because the number of customers is very large, no customer can affect the market shares with his own decisions. Every customer has a client relationship with either of the two banks. No customer uses two banks simultaneously. These customers have to make payments among themselves. The same customers are both payers and payees. Every customer has to make one payment to another customer in the same economy. The customers (consumers) maximise their personal welfare, and the quality of payment services is one of the factors that affects their utility.

The payment system consists of the following subsystems.

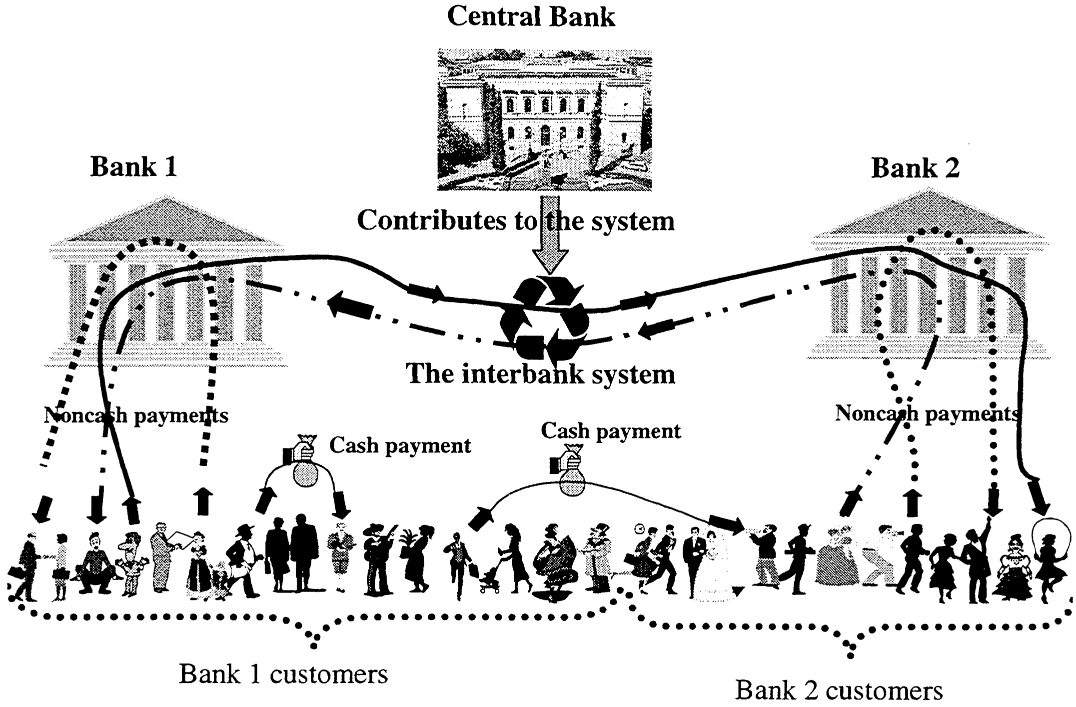
1) Intrabank systems used in giro, cheque and debit card payments between customers of the same bank. Both banks have an intrabank system. The quality of these systems is exogenous.

2) An interbank system used in giro, cheque and debit card payments between customers of different banks. The interbank system is both developed and operated cooperatively by the two commercial banks and the central bank. The quality of the interbank system is determined by the investments of the two private banks and the central bank.

3) Cash in circulation. Any consumer in the economy can make a cash payment to any other consumer in the economy. Customers do not need any banking services to make cash payments. Due to lost interest revenue, the inconvenience of queuing at the ATM and the risk of theft, the utility of using cash as a means of payment is negative. The utility of a cash payment equals $-\omega$. This cost is evenly divided between the payer and the payee.

Figure 1 illustrates the role of these different agents in the payment system.

Figure 1. **Structure of payment system**



There is no interbank payment centre owned and run by the banks collectively, and no collusion between banks. Exchange of payment information takes place either directly between the two banks or eventually through the central bank. The (net) clearing services are arranged by the central bank.

It is assumed that there are no capacity constraints on the system. Once it is established and developed, any number of transactions can be processed. In this sense, the model describes the present situation rather than the past. Computer systems are normally expensive to establish, but the marginal cost of running them is negligible.

3 Basic version – payments as a free service

In this version of the model, it is assumed that banks offer payment services free of charge. There could be a law that obliges them to offer free services, or eventually the banks themselves decide not to charge fees. It has also been proposed that banks might offer free or underpriced services because of tax incentives; customers prefer nontaxable benefits, such as access to free or subsidised services to taxable interest income (Tarkka 1995).

3.1 Assumptions

3.1.1 Players' moves

The model is a full information game, in which agents can always correctly calculate each other's decisions and moves. The moves are made in the following order.

1) The central bank decides its own investment in developing the interbank payment system. Commercial banks can immediately observe the level of investment.

2) Commercial banks decide simultaneously on their investments in development of the interbank payment system.

3) Customers observe the quality of the interbank system. Each customer chooses which bank he will use. No customer uses two banks. Customers choose their banks independently and simultaneously without cooperating. When choosing his bank, a customer does not know to whom he will make a payment. Every customer is equally likely to be the payee.

4) Each customer observes to whom he will make the payment. The payees of different customers are determined independently.

5) Customers make and receive their payments.

3.1.2 The functioning of the interbank payment system

Once the payment system has been developed, banks can use it at zero marginal cost. If there were a constant marginal cost for each transaction, the model might describe a situation where banks would be obliged to set the price at the marginal cost.

Developing the interbank payment system improves the quality of the service. Customers prefer a high quality system. The term 'quality' refers to the speed and reliability of the system. The quality of the interbank payment system is a function of the development efforts by the three agents, the payer's bank, the payee's bank and the central bank.

The quality of an interbank payment is independent of whether it goes from bank 1 to bank 2, or *vice versa*. In both cases, the payment must go through the same chain, which consists of three systems. Quality is beneficial to both the payer and the payee. Customers prefer receiving payments through a highly developed system.

Interbank payments are possible (though of low quality) even if nothing has been invested in developing the system.

The number of customers (n) is very large. The market share of bank 1 being s, the probability that a payment goes to a customer of bank 1 is s, and the probability that it goes to a customer of bank 2 is (1–s).

3.1.3 Customers' preferences in a Hotelling duopoly

The total utility of a customer is determined by the quality of payment services and by preferences as between the two banks.

Each consumer has to make a discrete choice between the two banks. Three factors are taken into account: First, the distance to the bank, secondly, a general preference parameter (G), and finally the number of other customers who are going to use the same bank. Customers can correctly calculate other's choices and the resulting market shares.

Customers are risk neutral. Risk neutrality matters because the consumer does not know in advance to whom he will make the payment, nor from whom he will receive a payment. The expected utility of consumer x is

$$W_x = \begin{cases} G + (2 - i_x) + (1 - s) \cdot a + s & \text{if the customer chooses bank 1} \\ -G + (i_x - 1) + s \cdot a + (1 - s) & \text{if the customer chooses bank 2} \end{cases}$$

where

- G is a preference parameter. The parameter is common to all customers. If $G > 0$, bank 1 is preferred by most customers. If $G < 0$, most customers prefer bank 2. If $G = 0$, customers are, on average, indifferent between the two banks.
- i_x is a customer-specific exogenous parameter, denoting the location of the customer.[1] The banks are located at the endpoints of the interval, bank 1 at point 1 and bank 2 at point 2. Getting service from a bank that is close to the customer provides the customer with higher utility. If $1 \leq i_x < 1½$, parameter i favours bank 1, if $1½ < i_x \leq 2$, parameter i favours bank 2, and if $i_x = 1½$, the parameter is neutral as between the two banks. (However, because the common parameter G may differ from zero, $i_x < 1½$ does not necessarily imply that consumer x would prefer bank 1)
- s= the endogenously determined market share of bank 1.
- a is the quality of the interbank payment system. The quality is always lower than the quality of intrabank payments The total utility provided by an interbank payment equals its quality a; this total utility is evenly divided between payer and payee. The expected value of the number of interbank payments to be paid by a customer of bank 1 is (1–s), and the expected value of the number of interbank payments to be *received* is also (1–s). Therefore, the expected utility of interbank payments is $2 \cdot (a/2) \cdot (1-s) = a(1-s)$. Analogically, the expected utility provided by interbank payments for a bank 2 customer equals $2 \cdot (a/2) \cdot s = s \cdot a$. a must be positive, but it cannot be greater than +1.
- If the customer has chosen bank 1, the probability that the payment would go to another customer of the same bank equals its market share (s). Analogically, the expected value of the number of intrabank payments to be *received* is s for a bank 1 customer. The utility of being involved in one intrabank transaction is assumed to equal ½. The customer benefits from intrabank payments as both the payer and the payee, implying that the

[1] The easiest interpretation of this parameter is that it describes the geographic distance, even though other interpretations are possible as well. For instance, bank 1 could use Swedish as the customer service language and bank 2 Finnish.

expected utility provided by intrabank transactions is $2 \cdot \frac{1}{2} \cdot s = s$ for a bank 1 customer. The expected utility provided by intrabank payments for a bank 2 customer is determined in an analogous way, being $2 \cdot \frac{1}{2} \cdot (1-s) = (1-s)$.

With the exception of the preference parameter i_x, all the parameters are common to all customers.

There is a pure network externality in the model. Using the same bank as the majority of customers provides the consumer with utility. This network effect is a direct externality, and it is not caused by the effects of other customers' choices on any prices.

The model describes a giro transfer system rather than a cheque-based system. In a cheque system, it is far from obvious why customers would benefit if the interbank system were improved. In fact, they might prefer a slow and unreliable system. At least the payer would gain marginal interest income if there were a lengthy delay between the moment of making the payment and the moment when the account is debited. If the system were so unreliable that a significant part of the cheques were lost in interbank clearing and the account of the payer were never debited, customers would be even better off. (However, if it were commonplace to debit the wrong account, customers would probably prefer a more reliable system.)

3.1.4 The quality of the interbank payment system

There are two kinds of noncash payments.

1) Intrabank payments between two customers of the same bank. The quality of an intrabank payment is exogenous, and equal to +1.

2) Interbank payments. The quality of an interbank payment is a.

The value of the parameter a is a function of the investments made by the two banks and the central bank. If the value of a is high, payments are processed fast and reliably. If a is close to zero, interbank payments are slow and unreliable.

Both private banks can affect the quality of the system by investing in it. In addition, investments by the central bank affect the quality. These investments have a declining marginal impact on the quality. With zero investment by any of the three agents, the marginal impact of investment on the quality is infinite. The investments are allowed to have different interaction effects on the quality. The assumptions for the a-function are presented in detail in the Appendix 1.

In the real world, banks may prefer slower interbank payments to faster ones. With slower processing of payments, banks can earn additional interest income on the float; if the account of the payer is debited several days before the account of the payee is credited, total interest payments to deposit customers are lower. Thus, system development is not necessarily a technological effort. One possible interpretation of the model is that the 'investment expenditure' consists partly of the interest loss caused by the decision to speed up the payment process with existing technological facilities.

3.1.5 Banks' revenues and profits

In addition to payment services, banks offer loan and deposit services as well, even though these are not explicitly modelled. In addition, each customer causes the bank some costs. Computer systems and the physical retail service network are more expensive to maintain if the bank has a lot of customers. The parameter α describes the exogenous net income per customer that a bank earns in collecting deposits, granting loans and maintaining the necessary infrastructure. The parameter is common to all customers and both banks. Therefore, the total net income of a bank equals α times the number of customers.

The profit of bank 1, when the cost of payment system development is not taken into account, is:

$$\Pi_1 = n \cdot s \cdot \alpha$$

and the profit of bank 2 is

$$\Pi_2 = n \cdot (1 - s) \cdot \alpha$$

The sum of banks' profits is always $n\alpha$. Thus, the struggle for market share is a zero sum game between duopolists. A bank can increase its profits only at the cost of its rival.

3.2 Solving the model

3.2.1 Banks' market shares

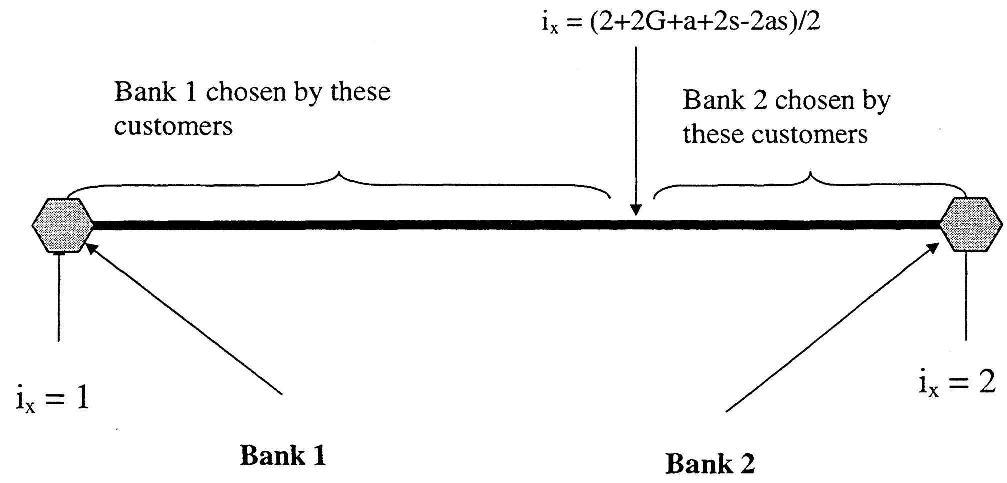
The utility of consumer x is determined according to the function described in the section 3.1.3.

The customer chooses bank 1 iff

$$\begin{aligned} & G + (2 - i_x) + (1 - s) \cdot a + s > -G + (i_x - 1) + s \cdot a + (1 - s) \\ & \Leftrightarrow i_x < (2 + 2 \cdot G + a + 2 \cdot s - 2 \cdot a \cdot s) / 2 \end{aligned} \tag{3.i}$$

The market share of bank 1 is determined according to the number of customers for whom the condition is valid.

Figure 2. **How customers choose a bank**



The density function for customers is extremely simple, ie the constant +n. Hence the market share is

$$s = \left\{\int_1^c n\, di\right\} / n$$

where c is the point where the condition for i_x (3.i) is no longer valid, ie $c = (2+2\cdot G+a+2\cdot s-2\cdot a\cdot s)/2$

The market share is given by

$$s = \frac{(2G + a)}{(2a)} = ½ + \frac{G}{a} \qquad (3.ii)$$

Thus, bank 2's market share is $1–s = 1–(2G+a)/(2a) = (–2G+a)/(2a)= ½–G/a$.

The market share of bank 1 is between 0 and 100 % iff $|2G| \leq a$. If $–a/2 < G < 0$, bank 1's market share is between 0 and 50 %. And if $0 < G < a/2$, the bank's market share is between 50 % and 100 %.

If $G < –a/2$ ($G > a/2$), the formula for s would predict a bank 1 market share less than 0 (greater than 1). In practice, in these cases, the market shares would be 100 % and 0 %, which would fall outside the duopoly case. The monopoly situation would be a completely meaningful case, but interbank payments would not take place in such a banking industry, and the following analysis would not apply. Hence, in the following, it is assumed that each bank has a positive market share, even though a market share may be close to zero.

As to real life interpretations, cases where G is close to either –a/2 or a/2, are not very interesting. Almost all countries have certain government regulations concerning minimum capitalisation of credit institutions; In the EU, for instance, a bank with equity capital of less than ECU 5 million is not allowed to enter the market. Thus, the smallest bank that can legally exist is a local institution with a moderate market share in its home area.

The situation is fundamentally symmetric, the only difference between the two banks being the eventually nonzero value of G. Therefore, all the following results are equally valid for both banks. To simplify the notation, the analysis is in most cases presented only for bank 1.

3.2.2 Banks' investments in the payment system

From the point of view of a bank, the quality of the payment system is relevant to profits because it affects market shares. The impact of payment system development on the market share of the bank can be calculated with the formula (3.ii)

$$\frac{ds}{da} = \left(\frac{-G}{a^2} \right)$$

If the two banks are equally popular (G = 0), then the quality of the payment system is of no relevance to market shares (ds/da = 0). Hence, the more popular the bank is, the less system development helps to increase market share.

$$\frac{d^2 s}{dadG} = \frac{-1}{a^2} < 0$$

If the market share is greater than 50 %, then developing the payment system causes a loss of customers to the rival. A well functioning interbank system would improve the payment services received by all the customers, but the effect would be even stronger for customers of the small bank because for them, interbank payments account for most of the payment traffic. Therefore, the improvement would strengthen the competitive position of the smaller bank. If the interbank payment system does not function smoothly, most customers find it useful to use the same bank as the majority. A customer of a small bank would be nearly isolated if the interbank payment system did not function properly. This would cause a substantial amount of disutility.

Result 3.a;
If market shares are equal, neither of the banks will invest in developing the system. If the market shares are not equal the smaller bank invests in developing the system but the larger does not.

Proof

$$\frac{d\Pi_1}{da} = n\alpha \frac{ds}{da} = -n\alpha \cdot \left(\frac{G}{a^2} \right)$$

If $G \geq 0$, the bank has no incentive to develop the system because

$$\frac{d\Pi_1}{da} = -n\alpha \cdot \left(\frac{G}{a^2} \right) \leq 0$$

If G < 0, then $d\Pi_1/da = -n\alpha\cdot(G/a^2) > 0 \Rightarrow$ bank 1 does have an incentive to invest in the system.

When the investment by the bank 1 (Λ_1) is zero, then, by assumption, $\partial a/\partial\Lambda_1 = \infty$ (See appendix 1)

If $d\Pi_1/da > 0$ and $\partial a/\partial\Lambda_1 = \infty$, then it is optimal for bank 1 to invest in developing the system.

QED

Unsurprisingly, the willingness to invest in developing the system is a decreasing function of the popularity parameter G.

Result 3.b:

If bank 1 is less popular than bank 2 (which will be the case when G < 0), then increasing the popularity of bank 1 (G) reduces its investment in developing the system.

Proof

The bank maximises its profits ($\Pi_1 - \Lambda_1$) according to the first order condition $\partial\Pi_1/\partial\Lambda_1 - 1 = 0$

Implicit differentiation gives $d\Lambda_1 / dG = -\{\partial^2\Pi_1 / \partial\Lambda_1\partial G\}/\{\partial^2\Pi_1 / \partial\Lambda_1^2\}$

The second order condition for profit maximisation implies $\partial^2\Pi_1 / \partial\Lambda_1^2 < 0$.

One can write $\partial\Pi_1/\partial\Lambda_1 = n\cdot\alpha\cdot(ds/da)\cdot(\partial a/\partial\Lambda_1)$

And $d^2\Pi_1/d\Lambda_1\, dG = n\cdot\alpha\cdot(d^2s/dadG)\cdot(\partial a/\partial\Lambda_1)$

Because

$d^2s/dadG = -1/a^2 < 0$, $n\cdot\alpha\cdot > 0$, $da/d\Lambda_1 > 0$ and $d^2\Pi_1/d\Lambda_1\, dG < 0$

it follows that $d\Lambda_1/dG<0$

QED

3.2.3 Banks' actual investments vs socially optimal investments

As concluded above, only the smaller bank invests in system development. And if neither of the banks is smaller than the competitor, there is no private investment at all. As a rule, this is not socially optimal. Because the marginal impact of investing in the system is extremely high when the investment is close to zero, both of the banks should invest equally heavily, if the system is to be developed at all. This would be the most cost-efficient way to reach any given level of a. Therefore, there is an obvious market failure. Moreover, in most cases, the investment by the smaller bank differs from the socially optimal level.

The struggle for the net interest income ($n\cdot\alpha$) is a zero-sum game between the two banks. Therefore, when one analyses the social welfare effects of payment system development, one can concentrate on the utility consumers get from using the system.

As to a bank 1 customer, his utility equals

$$W_x = G + (2 - i_x) + s + (1 - s) \cdot a$$

and the utility of a bank 2 customer equals

$$W_x = -G + (i_x - 1) + s \cdot a + (1 - s).$$

The total welfare in the economy equals

$$\psi = \sum_{i=1}^{z} W_i + \sum_{i=z+1}^{n} W_i + \Pi_1 + \Pi_2 - \Lambda_1 - \Lambda_2$$

where z = n·s = the number of the last customer who uses bank 1.

U_z (z = 1, 2) denotes the utility yielded by payment services in the case of a bank z customer. $U_1 = s + (1 - s)\cdot a$, and $U_2 = s\cdot a + (1 - s)$. Thus, if the customer uses the bank 1, then $W_x = G + (2 - i_x) + U_1$, and if the customer uses the bank 2, then $W_x = -G + (i_x - 1) + U_2$.

The subutility function U_z does not depend on the individual preference parameter i; thus it does not vary as between individuals. The derivative dU_z/da equals the whole impact of a on the welfare of a bank z customer. Because the values of the preference parameters (G and i) do not have any impact on the effect, all bank z customers have an equal value for dU_z/da. If customer i uses bank 1, then $dW_i/da = dU_1/da$.

The impact of payment system development on social welfare (ψ) is

$$\frac{d\psi}{da} = n \cdot \left[s \cdot \left\{ \frac{dU_1}{da} \right\} + (1 - s) \cdot \left\{ \frac{dU_2}{da} \right\} \right] \tag{3.iii}$$

Result 3.c:
The impact of the quality of the payment system on social welfare is maximal when G = 0

Proof

According to (3.iii)

$$\begin{aligned} \frac{d\psi}{da} &= n \cdot \left[s \cdot \left\{ \frac{dU_1}{da} \right\} + (1 - s) \cdot \left\{ \frac{dU_2}{da} \right\} \right] \\ &= n \cdot \left[s \cdot \left\{ \frac{½ - G}{a^2} \right\} + (1 - s) \cdot \left\{ \frac{½ + G}{a^2} \right\} \right] \\ &= n \left[\frac{½ - 2G^2}{(a^3)} \right] \end{aligned}$$

When this is differentiated with respect to G, one gets $-n4G/(a^3)$
This equals 0 when $G = 0$.
Because $d^3\psi/da\ dG^2 = -n4/(a^3) < 0$, this is the maximum of $d\psi/da$.

QED

This result is easy to understand intuitively. If customers are evenly distributed between the two banks, the number of interbank payments is maximal. Therefore, improving the system would be highly useful. However, no private investment is made because neither of the two banks can increase its profits by such investments. As concluded above, in this special case, system development has no impact on market shares.

Result 3.d:
Iff $|G| \geq a^{(3/2)}/2$, then the socially optimal investment in the system is zero.

Proof

$d\psi/da = n[½ - 2G^2/(a^3)]$; If $|G| > |a^{(3/2)}/2|$ then $d\psi/da < 0$, and therefore no investment would yield any social benefits.

QED

This result may sound rather counter-intuitive. If improved payment systems make interbank transactions fast and reliable, how could such a development be undesirable? The answer is related *to network externalities caused by consumer choice*. Suppose bank 1 has a very small market share. There is a customer who is indifferent between the small bank 1 and the large bank 2. The consumer might choose bank 2 at random. Then, a marginal improvement in interbank payments takes place; using bank 1 becomes slightly more attractive for the consumer because the problems created by the frequent need to make interbank transactions are marginally alleviated. The customer shifts to the small bank 1. This has a very small impact on personal utility, because the customer is nearly indifferent between the two banks. For other customers, this choice is more significant. This decision has a positive externality on all bank 1 customers and a negative externality on all bank 2 customers. The former group can exchange payments with this particular customer with less difficulties than before, but the latter group suffers from a comparable negative externality. If the market share of bank 1 is very small, the resulting negative externality is strong enough to more than offset the benefits of improved interbank payment services. In such a case, the overwhelming majority of customers would find it more difficult to exchange payments with the customer who decided to choose the smaller bank. In principle, the assumptions of this model imply that a banking monopoly would be ideal for the payment system, but improving the payment system strengthens the relative position of the smaller bank.

This effect is a good example of what Liebowitz and Margolis (1994) classify as direct network externality. The impact of consumer choice on other consumers is not channelled through the price mechanism. Instead, the choice itself has a direct impact on others' welfare.

As a rule, the investment by bank 1 is not socially optimal. Nevertheless, the investment by bank 1 is at its optimal level in two different cases. First, if bank 1's market share were large enough, increasing the market share of the minor bank 2 by improving the system would be socially undesirable because of its adverse impact on the payment system. Moreover, it would be unprofitable for bank 1 itself. Secondly, the investment by bank 1 is at its optimal level at a particular point where the bank has a market share between 0 and 50 %.

Result 3.e

There is no market failure in the investment by bank 1 in two cases, namely

1) **with one particular negative value of G, namely $G = (a/4)\{\alpha - \sqrt{[4a+\alpha^2]}\}$**
2) **when $G > a^{(3/2)}/2$**

Proof: There is no market failure if private and social benefits are equal, ie if

$$n\alpha \cdot \left(\frac{-G}{a^2}\right) = n\left[\frac{\frac{1}{2} - 2G^2}{(a^3)}\right].$$

This equation holds iff $G = (a/4)\{\alpha \pm \sqrt{[4a+\alpha^2]}\}$. If $G > 0$, profit maximising investment would be negative ⇒ The eventual candidate is $G = (a/4)\{\alpha - \sqrt{[4a+\alpha^2]}\}$.

The market share of bank 1 is between 0 and 50 % iff $(-a/2) < G < 0$

With the value of G given above this condition is

$(-a/2) < (a/4)\{\alpha - \sqrt{[4a+\alpha^2]}\}$

or $2+\alpha > \sqrt{[4a+\alpha^2]}$

which simplifies to $4 + 4\alpha + \alpha^2 > 4a + \alpha^2$; Because $a < 1$ and $\alpha > 0$, this holds with certainty.

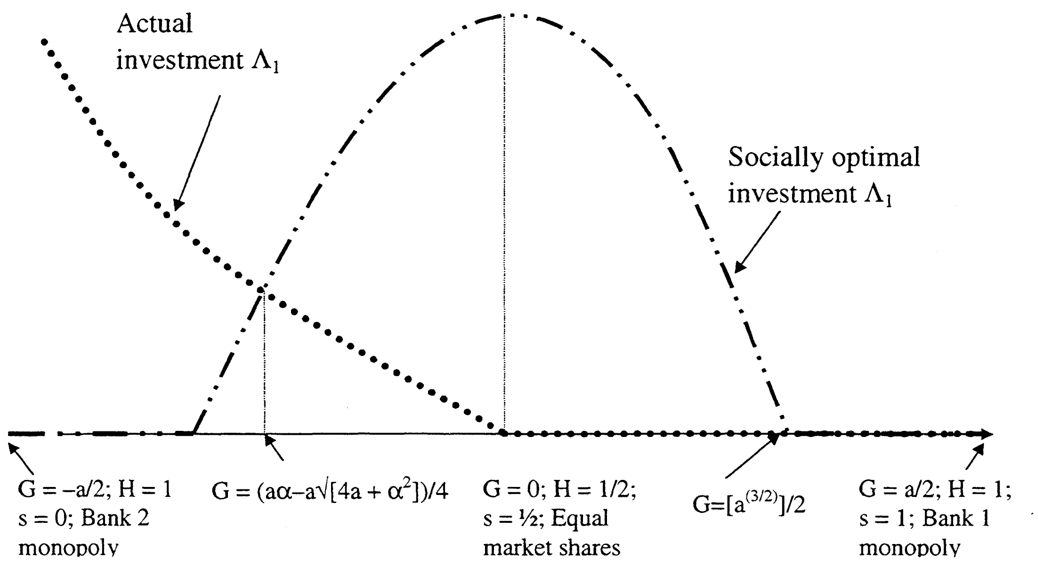
Because $a/4 > 0$ and $0 < \alpha < \sqrt{[4a+\alpha^2]}$, $(a/4)\{\alpha - \sqrt{[4a+\alpha^2]}\} < 0 \Rightarrow G < 0$. And therefore there is one value of G implying a market share s ($0 < s < \frac{1}{2}$) where bank 1 invests the socially optimal amount.

Moreover, there is another case where there is no market failure. The marginal impact of 'a' on bank 1 profits is negative if it has a greater-than-50 % market share. In this case, the bank does not invest in developing the system. This is socially optimal if $G > a^{(3/2)}/2 \Rightarrow d\psi/da < 0$ (Result 3.d). In this case, not to invest is optimal from the point of view of social welfare.

QED

Figure 3 may shed some light on market failures in bank 1 investments. The socially optimal investment reaches its maximum when market shares are equal (result 3c), and if $|G| \geq a^{(3/2)}/2$, the socially optimal investment is zero. If bank 1 has a small market share $[G < (a/4)\cdot(\alpha - \sqrt{[4a + \alpha^2]})]$, it invests more than the socially optimal amount (results 3a, 3d and 3e). And if its market share is greater than 50 %, it invests nothing (result 3a). The Herfindahl index (H) measures the degree of market concentration; in this case, it is defined as $s^2 + (1 - s)^2$.

Figure 3. **Actual investment by bank 1 and the socially optimal investment as a function of the market share (s), the parameter G and the Herfindahl-index (H)**



However, even though the investment by bank 1 may sometimes be at its socially optimal level, it is not possible to find examples where both banks would invest optimally, because it is never the case that they would both invest. Thus, there is no market structure (value of G) that would lead to an efficient allocation of resources.

3.2.4 Optimal central bank involvement

The role of investments by the central bank will be analysed in this section. The central bank investment can affect social welfare with its payment system investments in two different ways.

1: Directly, through the direct impact of investment on the quality of the system.
2: Through indirect effects. The central bank investment normally affects the investment made by the private sector.

In many cases, central bank investment reduces private investment. This is possible even when central bank investments strengthen the impact of private investments on the quality of payment systems (when $d^2a/d\Lambda_c d\Lambda_1 > 0$).

Result 3.f:
Let the market share of bank 1 be less than 50 %. A sufficient condition for increasing central bank investment to reduce investments by bank 1 is that either (or both) of the following two conditions holds:
1) $(\partial^2 a/\partial\Lambda_1\partial\Lambda_c\} \leq 0$.
2) $\Lambda_c = 0$

Proof: See appendix 2

It is fairly obvious that when central bank investment makes private investment inefficient ($\partial^2 a/\partial\Lambda_1\partial\Lambda_c \leq 0$), increasing public investment discourages private investment. It may be somewhat more difficult to understand why the result might always apply when central bank investment is close to zero ($\Lambda_c = 0$). The reason is simple: Central bank investments may discourage the private bank from investing in the system simply by affecting the quality of interbank payments (a). If the payment system already functions properly, the private bank 1 does not have to invest in the system itself.

The total impact of central bank payment system development on social welfare equals the difference between the utility provided by a marginal improvement in payments and the increase in the costs of developing the system. Mathematically, the social welfare impact of a marginal increase in central bank investment is

$$\frac{d\psi}{d\Lambda_c} = \left(\frac{\partial\psi}{\partial a}\right)\cdot\left[\left(\frac{\partial a}{\partial\Lambda_c}\right)+\left(\frac{\partial a}{\partial\Lambda_z}\right)\cdot\left(\frac{d\Lambda_z}{d\Lambda_c}\right)\right]-\left(\frac{d\Lambda_z}{d\Lambda_c}\right)-1 \qquad (3.iv)$$

where

- $(\partial\psi/\partial a)$ equals the improvement or deterioration in welfare due to the improvement in the payment system
- $(\partial a/\partial\Lambda_c)$ equals the direct impact of central bank investment on the quality of the payment system
- $(\partial a/\partial\Lambda_z)\cdot(d\Lambda_z/d\Lambda_c)]$ equals the indirect impact through the impact of central bank investment on private investment. z = 1 if bank 1 has a market share of less than 50 %; z = 2 if bank 2 has a market share of less than 50 %. (The larger bank does not invest.)
- $(d\Lambda_z/d\Lambda_c)$ equals the impact of central bank investment on private investment expenditure
- 1 = the marginal cost of investment by the central bank.

Because central bank investment affects the choices of the private sector, it is not surprising that in the maximisation of social welfare, the indirect effects of investments should be taken into account as well.

If it is certain that central bank investment reduces private investment, the optimal policy must clearly be the following.

Result 3.g:
If the smaller bank invests less (more) than the socially optimal amount in payment system development, the central bank should restrict (increase) its investment in the system in order to encourage (discourage) private investment, at least if $(d^2a/d\Lambda_1 d\Lambda_c\} \leq 0$.

Proof: See appendix 3;

On the other hand, it is much more difficult to draw robust conclusions concerning optimal central bank investment when $(\partial^2 a/\partial\Lambda_1\partial\Lambda_c) > 0$. If the level of central bank investment were very low, result 3.g would still be valid, because central bank investment would reduce private investment. However, with higher levels of central bank investment, the result would be the reverse.

If market shares are evenly distributed (G = 0), the central bank cannot do much to affect private sector investment. In this case, the banks would neither increase nor decrease their investment in response to central bank investment, because there would be no private investment anyway. There are no private reactions to be taken into account by the central bank.

It may be surprising that central bank investments in the system may be useful even when there is a very small private bank that has almost no customers but is about to get a handful of them. An improvement in the system causes a direct reduction in social utility because it affects market shares in a non-desirable way, but, on the other hand, central bank investment would have beneficial indirect effects. A private bank with a very small market share invests excessively in the payment system, and a feasible way to reduce private investment is to have public investment. At least a very small public investment could be justified in a highly concentrated market.

Result 3.h:
It is optimal for the central bank to invest at least something in the system with any value of G, at least if $\alpha = 1$ and $\partial^2 a/\partial\Lambda_1^2$ is close to zero.[2]

Proof See Appendix 4

The economic intuition behind this result goes as follows. When the fixed income per customer (α) has a suitable value, the smaller bank has moderate incentives to attract customers if it is possible with a reasonable cost. A marginally positive investment by the central bank can reduce the total investment rather efficiently, because the incentives of bank 1 to increase its market share are fairly weak. If, instead, private investment were close to zero because of a very low net income per customer, it would not be worthwhile to try to affect private investment. The impact of public investment on private investment is even stronger if the optimal amount of private investment is easily affected by changes in different parameter values. This is the case when $d^2a/d\Lambda_1^2$ is close to zero.

[2] Because $d\Pi_1^2/da^2 < 0$, $\partial^2 a/\partial\Lambda_1^2 = 0$ implies $\partial\Pi_1^2/\partial\Lambda_1^2 < 0$, which is a necessary condition for the existence of an optimal finite amount of investment.

Optimal central bank investment policies when it is always optimal to invest something can be clarified with figure 4. In figure 4, it is assumed that central bank investment in the system always reduces private investment and that the investment of the private bank responds strongly to changes in investment by the central bank. Now let us review how the central bank should alter its investment with different values of G.

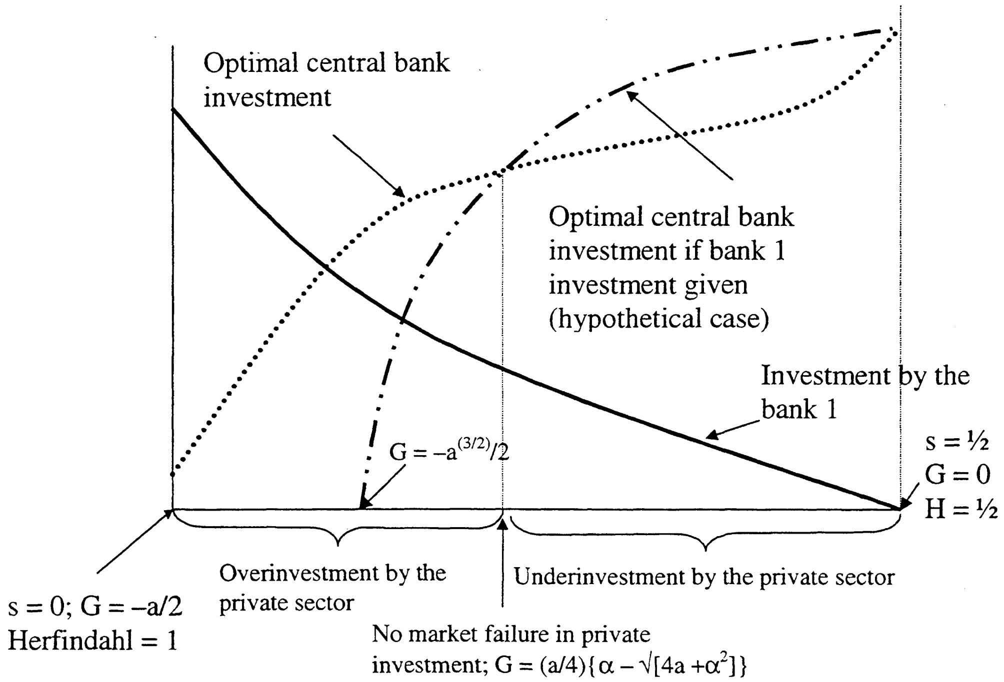
If bank 1 has a very small market share ($G \approx -a/2$), it invests excessively in payment system development, even though improvement of the system yields a negative social benefit and the investment causes costs. If the central bank cannot affect investment by bank 1, it should not invest in the system. However, a small investment may be justified because it would reduce private expenditure in the system.

If G is somewhat higher than $-a^{(3/2)}/2$, then an exogenous improvement in the payment system would improve social welfare. Therefore, at least a small investment by the central bank would be socially optimal even if private investment did not react to central bank investment. However, the social benefits of a better interbank payment system (higher a) are still lower than the marginal cost of improving the system with private investments. Therefore, the central bank should still try to restrict private investment by intensifying its own investment.

When bank 1's market share increases further and reaches the value ($G = (a/4)\cdot\{\alpha - \sqrt{[4a + \alpha^2]}\}$), there will be enough interbank payments to make the social benefits equal the private cost of investment, implying that the central bank has no incentive to try to affect private investment. Beyond this point, social benefits exceed private benefits, and bank 1 invests less than the socially optimal amount in developing the system. Therefore, the central bank should restrict its own investment in order to encourage the bank to increase its investment. As can be seen in figure 4, whenever the private bank 1 underinvests, optimal central bank investment exceeds the level that would be desirable if central bank investment did not affect private investment.

And finally, when $G = 0$, the private bank no longer invests, and the central bank cannot affect its behaviour. At this point, the number of interbank payments is maximal, and the only agent in the economy that invests in the payment system is the central bank. Therefore, it should invest substantially.

Figure 4. **Optimal central bank investment**
In this example optimal central bank investment always greater than zero



Needless to say, it is possible and even simpler to construct examples where at low values of G, optimal central bank investment is zero. In such cases, the optimal central bank investment curve would cross the horizontal axis at a point between $-a/2$ and $-a^{(3/2)}/2$.

4 Bertrand competition in payment services

4.1 Assumptions

This version of the model differs from the basic model in that it is no longer assumed *a priori* that payment services are offered free of charge. Banks as providers of payment services are now allowed to charge fees, if they prefer to do so.

Moreover, it is no longer assumed that the fixed net income per customer is always positive; α may be negative as well. The fixed cost caused by one customer may or may not be higher than the net interest revenue per customer.

When banks have invested in developing the system, they must decide on their pricing policies. Banks decide all prices simultaneously in a Bertrand-type competition. The price charged by bank z for one interbank payment is denoted p_z (z = 1 or 2). Intrabank payments can be priced as well, and the price of one intrabank payment is denoted b_z. Because of spatial differentiation, the two banks are not perfect substitutes, and they have a certain amount of market power. Bertrand competition does not lead to zero profits.

A bank may charge different prices for interbank and intrabank payments ($p_z \neq b_z$), if it prefers to do so. However, there is a marginal administrative cost (ε) that the bank must pay if it charges different prices for these two types of payments. This cost consists of the minor expenditures of adapting the information systems and informing employees concerning the two different prices. Compared to other expenses and revenues, this cost component is negligible. But if the bank is otherwise indifferent between a pricing policy characterised by $b_z = p_z$ and some other set of prices, then the bank prefers charging the same price for both interbank and intrabank payments. Instead, it is assumed that banks cannot charge any fees for receiving payments.

Banks cannot practice price discrimination, possibly because they cannot observe the exact location of different consumers on the interval [1,2]. Payment service fees charged by banks cannot be negative; it is not possible for banks to pay their customers for using the service. But it is possible not to charge any fees.

There is no demand for interbank payment services if their price exceeds the reservation price, which equals $(a + \omega)/2$, where ω is the sum of the disutility of using cash suffered by payer and payee. For intrabank payments, the reservation price is $(1 + \omega)/2$.

There is no price elasticity of demand for payment services, provided the price is lower than the reservation price. A bank's service fee is such an unimportant cost that it cannot affect the transactions in the real economy. However, the fee is relevant to the choice of payment medium. When customers have observed the fees, they decide which bank to use. At this stage, banks can no longer alter their prices. The central bank does not charge any fees.

If no customers use cash as a means of payment, the profit of bank 1 is

$$n \cdot s \cdot \alpha + n \cdot s(1-s) \cdot p_1 + n \cdot s^2 b_1$$

where n is the total number of customers in the economy, $n >> 0$, and s = the market share of bank 1; p_1 = the fee charged for one interbank payment by bank 1, and b_1 is the fee charged for an intrabank payment by bank 1.

In an analogous way, the profit of bank 2 is

$$n \cdot (1-s) \cdot \alpha + n \cdot (1-s) \cdot s \cdot p_2 + (1-s)^2 n \cdot b_2$$

The difference between the net interest income and fixed costs per customer (α) is still treated as exogenous, but as we shall see later (footnote 3), treating it as an endogenous variable would not affect the results significantly.

4.2 The Bertrand competition outcome

4.2.1 Banks' market shares

Now we will see how the market shares of banks are determined when the customers of both banks prefer interbank giro transfers to cash payments because prices are below their reservation levels. If both banks charge a price that is lower than the reservation price, customers prefer banks' payment system and the market shares are determined as follows.

Customer x chooses bank 1 iff

$$G+(2-i_x)+(1-s)\cdot(a-p_1)+s\cdot(1-b_1)>$$
$$-G+(i_x-1)+s\cdot(a-p_2)+(1-s)(1-b_2)$$

which implies

$$i_x<\frac{(2+a+b_2+2G-p_1+2s-2as-b_1s-b_2s+p_1s+p_2s)}{2}$$

The market share of bank 1 is determined according to the number of customers for whom the condition presented above is valid. The density function of customers is again the constant n. The market share is determined by

$$s=\frac{\left\{\int_1^c n\ di\right\}}{n}$$

where c is the point where the condition for i_x is no longer valid, i.e.

$$c=\frac{[2+a+b_2+2G-p_1+2s-2as-b_1s-b_2s+p_1s+p_2s]}{2}$$

The market share is then

$$s=\left\{\frac{(2\cdot G+a-p_1+b_2)}{(2a+b_1+b_2-p_1-p_2)}\right\} \tag{4.i}$$

Consequently, bank 2 market share is

$$\left\{\frac{(-2\cdot G+a-p_2+b_1)}{(2a+b_1+b_2-p_1-p_2)}\right\}$$

This formula implies effects that are easy to understand intuitively. If the bank charges high fees, its market share declines.

$ds/dp_1 = (2\cdot G - a + p_2 - b_1) / (2a + b_1 + b_2 - p_1 - p_2)^2$. Whenever the formula 4.i predicts positive values for bank 2 market share, $ds/dp_1 < 0$, which is reasonable. The impact of b_1 on the market share of bank 1 is always negative.

If bank 1 is unpopular (G<0), its market share remains small as well. On the other hand, high prices charged by the rival increase the market share.

4.2.2 The main case: Bertrand competition outcome with internal point solutions

As already mentioned, the highest possible price equals the reservation price, and the lowest possible price is zero. In this section (4.2.2), it is analysed how a bank sets its prices when neither of the two constraints on pricing is binding.

4.2.2.1 Why do banks charge the same price for both types of payments?

The optimal price for interbank payments is determined according to the first order condition

$$\frac{d\Pi_1}{dp_1} = 0.$$

This condition is satisfied by only one value of p_1, namely

$$p_1 = \frac{2a^2 - \alpha(b_1 + b_2 - p_2) - (b_2 + 2G)(b_1 - b_2 + p_2) - a(2\alpha + b_1 - 3b_2 - 4G + p_2)}{3a - \alpha + b_2 - 2G - 2p_2} \qquad (4.ii)$$

When p_1 is given this value,

$$\frac{d^2\Pi_1}{dp_1^2} = -\frac{n(-3a + \alpha - b_2 + 2G + 2p_2)^4}{8(2a + b_2 - p_2)^3(a + b_1 - 2G - p_2)^2}$$

Whenever $p_2 < 2a+b_2$ (which is implied by result 4.a and the reaction function 4.iii), and $d^2\Pi_1 / dp_1^2 < 0$, the extreme value is a maximum.

Result 4.a
If both prices (p_z, b_z) are between zero and the reservation level, the prices are equal.

Proof

When p_1 is optimised according to formula 4.ii, then

$$\Pi_1 = \frac{n(a + \alpha + b_2 + 2G)^2}{4(2a + b_2 - p_2)}$$

This implies $d\Pi_1/db_1 \equiv 0$

$\Rightarrow$ The value of b_1 is of no relevance to profits, provided the bank optimises its p_1 according to formula 4.ii.

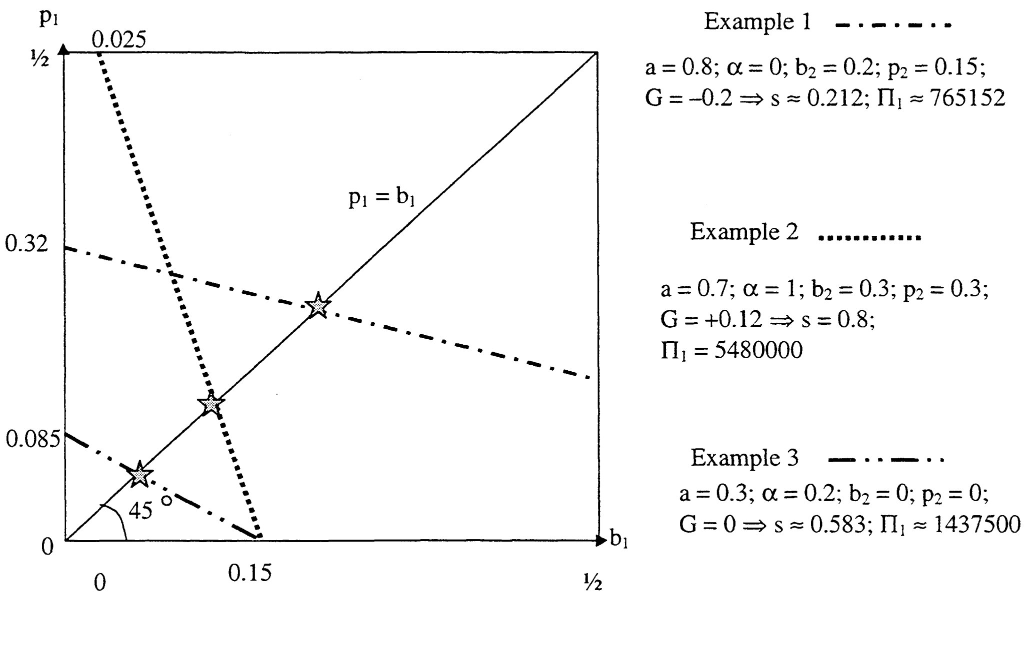
Therefore, in order to avoid the minor administrative cost ε, the bank optimises the fee for intrabank payments by choosing a combination of prices that satisfies $b_1 = p_1$, whenever such a combination is possible and neither the reservation price constraint nor the non-negativity constraint is binding.

QED

This result does *not* imply that when the price of an interbank payment (p_1) is optimised, any price for an intrabank payment (b_1) would be as good as any other. The price of an intrabank payment (b_1) is one of the factors that affect the optimal price of an interbank payment, and the two prices cannot be chosen independently. Instead, there are a huge number of *different combinations of b_1 and p_1* leading to the same maximum profit. This maximum profit can be reached with a variety of optimal combinations of p_1 and b_1, but it cannot be exceeded.

Each combination of parameters that is exogenous to the pricing of the bank (such as rival prices, α and a) gives a precisely defined set of combinations of the two prices (p_1 & b_1) leading to the same maximum profit. If one differentiates the expression 4.ii with respect to the price b_1, one observes that whenever both banks have a positive market share and they both charge positive prices, the optimal interbank payment fee is linearly dependent on the intrabank payment fee. The price p_1 is negatively related to the price of an intrabank payment. These combinations of the two prices can be presented graphically. A few examples are presented in figure 5. In the following, a set of profit maximising combinations of p_1 and b_1 will be called a 'pricing curve'.

Figure 5. **Examples of bank 1's pricing curves**



Unless the bank is bound by the reservation price constraint, there is one point on each pricing curve where the two prices are equal. The bank chooses this combination of prices in order to avoid the minor cost ε. In figure 5, these combinations are marked by stars.

The result that a large number of different combinations of the two prices lead to the same maximum profit may sound somewhat counter-intuitive. Nevertheless, the basic idea is farily simple. It is possible to interpret the pricing decision as a two-stage process.

1) First, the bank decides which average price level to offer, whether to be an expensive service provider or to sell services at low prices. This decision is extremely important, and the optimal choice is affected by many different factors, including the expected pricing policy of the rival.

2) Then the bank makes a less important decision in choosing which combination of the two prices to charge in order to implement the chosen average price level. When the bank increases the price of interbank payments and lowers the price of intrabank payments, the average price it offers to its customers may not change at all, at least not in expected value terms. When customers choose the bank, they still do not know with whom to exchange payments, and their decisions are affected by the expected value of the fee charged by the bank, not by a particular fee. If the market shares are exactly equal, the changes in the two prices must be equal, because an intrabank payment is as likely as an interbank payment. If, instead, the bank has a small market share, the change in the intrabank price must be much greater to offset a given change in the interbank payment fee, because the latter will be paid by most customers. In figure 5, one can see in the example 2 that a bank with a dominant market share has a nearly vertical pricing curve. The change in the interbank payment fee (p_1) has to be substantial to compensate for a much smaller change in the intrabank fee (b_1), because few customers will pay the interbank payment fee.

In a similar way, the total sum of payment service fees received by the bank does not change, and the profit of the bank remains unaffected. In light of this, it is not surprising that the market share of the bank is invariant to the point on a chosen pricing curve, provided the bank has chosen a combination of p_1 and b_1 that satisfies the profit maximisation condition. (Proof not shown here.)

Moreover, the profit of rival bank 2 is invariant to the point on the pricing curve chosen by bank 1. As long as bank 1 does not change the average price it charges, there is no change from the point of view of the market structure. (Proof not shown here.)

It is possible to demonstrate that a similar 'irrelevance effect' would obtain with the net interest income if it were endogenised, provided there is no price elasticity of demand with respect to financial intermediation.[3]

With formula 4.ii, we can calculate that, when the two prices are equal, the optimal pricing decision is

$$p_1 = b_1 = \frac{(a - \alpha + 2G + b_2)}{2}. \qquad (4.iii)$$

As one might expect, the bank charges high prices if the interbank system functions well, if the net interest revenue per customer is low, the bank is popular and its rival charges high prices. Interestingly, the optimal pricing policy depends on the intrabank payment fees charged by the rival but not on its fees for interbank payments.[4]

4.2.2.2 Prices, quantities and profits

When both banks have committed themselves to charging the same price (bank z's price denoted p_z) for both interbank and intrabank payments, the price will be determined according to the function 4.iii. Then, formula 4.ii holds as an identity.

When both banks optimise their service fees according to the expression 4.iii, prices are strategic complements, as they normally are in Bertrand competition. The outcome of the Bertrand competition is characterised by the following prices, market shares and profits.

Bank 1	Bank 2	
$p_1 = a - \alpha + 2G/3$	$p_2 = a - \alpha - 2G/3$	(4.iv)
$s = ½ + G/(3a)$	market share $(1-s) = ½ - G/(3a)$	
$\Pi_1 = n(3a + 2G)^2/(18a)$	$\Pi_2 = n(3a - 2G)^2/(18a)$	

With several different parameter values, the prices predicted by these formulas would lie between 0 and the reservation price level, $(a + \omega)/2$.

[3] Let δ_z be the net interest income, a transfer of wealth from the customer to bank z ($\alpha = \delta$ – the fixed cost per customer). The value of δ is freely chosen by bank z, but its value is limited by the competitive pressure. Let $p_z = b_z$.

Customer n chooses bank 1 iff

$$G + (2 - i_n) + (1 - s)\cdot(a - p_1) + s\cdot(1 - p_1) - \delta_1 > -G + (i_n - 1) + s\cdot(a - p_2) + (1 - s)(1 - p_2) - \delta_2$$

which gives $s = (a - \delta_1 + \delta_2 + 2G - p_1 + p_2)/(2a)$.

There is a very large number of different combinations of p_z and δ_z leading to the same market share. Let $T_1 = p_1 + \delta_1$. Then the total income of the bank equals $n\cdot(a + \delta_2 + 2G + p_2 - T_1)/(2a)\cdot T_1$. The profit does depend on the combination of p_1 and δ_1 used to implement any chosen value of T_1.

[4] The fact that the rival reacts to intrabank payments only has the following implication. If either of the two banks could pre-commit itself to a certain pricing policy, it would no longer be optimal for it to charge the same price for both types of payments. By committing itself to charging different prices for the two payments bank 1 could affect the pricing decisions of bank 2, which might be optimal for bank 1. The case of pre-commitment in pricing is not analysed in this paper.

An important difference between this result and the previous version of this model is the following. If there are no pricing decisions, which was the case in the third section, a small bank cannot try to enter the market by using the price weapon. Because the popular bank will charge relatively high prices whenever it is allowed to do so, a less popular rival can enter the market by undercutting prices. Now, there are three times as many eventual values of G that allow both banks to enter the market. The 'break even point' of a small bank is $G = \pm 3a/2$, whereas in the absence of price competition in the section 3, the break even point was $G = \pm a/2$.

Again, cases where the market share of either of the two banks is close to zero are not realistic. Due to minimum capitalisation requirements, such very small banks would not be allowed to enter the market.

4.3 The Bertrand competition outcome with binding constraints

4.3.1 The outcome with one binding non-negativity constraint

In the real world, payment services are often cross-subsidised, ie used to attract customers rather than as a significant source of revenue as such. Thus it is reasonable to study cases where no fees are charged, even though banks would be allowed to charge them. This section focuses on cases where either of the two banks does not charge prices although its rival does.

The formula 4.ii for the optimal interbank payment fee predicts negative values for p_1 with many different parameter values, especially when the net interest income per customer (α) is high. In such cases, it is of paramount importance for a bank to offer an attractive package of payment services in order to gain a maximal market share, because a large market share as such implies high revenues. In practice, payment services can be offered free of charge. Even relatively low values of α often lead to free payment services.

Whenever it is profitable not to charge a fee for an interbank payment, it would be reasonable to offer intrabank payments free of charge as well. There is no reason why a bank would implement a positive average price with a combination consisting of a positive price and a zero price.

The reaction functions of the banks are

$$p_2 = b_2 = \mathrm{Max}\left\{\frac{(a-\alpha-2\cdot G+p_1)}{2}, 0\right\}; \quad p_1 = b_1 = \mathrm{Max}\left\{\frac{(a-\alpha+2G+p_2)}{2}, 0\right\}$$

These pricing rules imply that cases where $p_1 = 0$ and $p_2 > 0$ cannot be observed unless bank 2 is more popular than bank 1 ($G < 0$), and the fixed net income per customer (α) is not excessively high. In such cases, the unpopular bank 1 would not charge fees, because charging them would imply a disastrous loss of market share. Bank 2 has insufficient incentive to offer payment services free of charge because it can charge reasonable prices and still maintain a sufficient market share.

To be more precise, the following four conditions must be satisfied to make a reasonable duopoly case where bank 1 charges no fees whereas bank 2 does charge positive prices.

1) $p_2 = (a - \alpha - 2G)/2 > 0$ iff $G < (a - \alpha)/2$;
2) $p_1 = 0$ iff $G \leq (a - \alpha + p_2)/2 \Leftrightarrow G < (-3a + 3\alpha)/2$.
3) These pricing policies imply that bank 1 has the market share $s = (3a - \alpha + 2G)/(4a)$. $\Rightarrow$ The situation is a duopoly if $(-3a + \alpha)/2 < G$. (If $G < (-3a + \alpha)/2$, bank 2 is a monopoly)
4) Unless $\alpha > 0$, bank 1, which does not charge any fees, cannot make a positive profit and would not enter.

These conditions cannot be satisfied simultaneously unless bank 1 is less popular than its competitor ($G < 0$).

If the parameter G is to satisfy these conditions, the lower bound to G is set by the third condition; unless G has a certain minimum value, there is a monopoly. If G is given higher and higher values, the upper bound of G is determined by either of the following two conditions:

- The first condition; $G < (a - \alpha)/2$. With a sufficiently high value of G, both banks offer free services.
- The second condition; $G < -3(a - \alpha)/2$. With a sufficiently high value of G both banks charge fees.

When all the four conditions are satisfied, bank 1 offers free services and bank 2 charges fees, the profit for bank 1 offering free services is

$$\Pi_1 = \frac{\alpha(3a - \alpha + 2G)n}{4a} \tag{4.v}$$

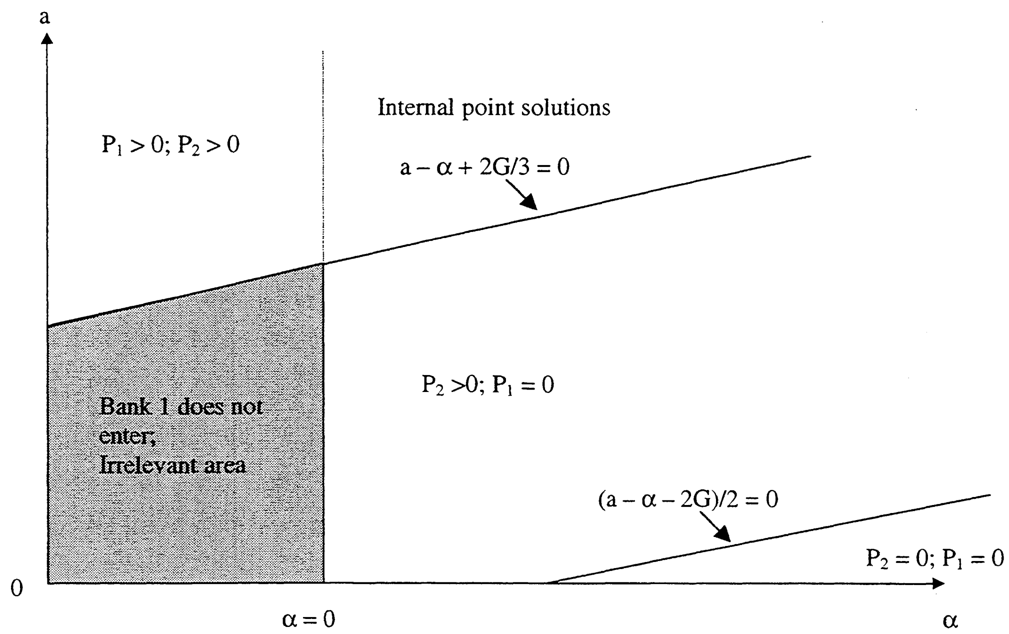
Unsurprisingly, if the fixed net income per customer (α) approaches zero, bank 1 cannot make a profit. The market share of bank 1 equals $s = (3a - \alpha + 2 \cdot G)/(4 \cdot a)$

In this case, bank 2 charges positive prices. When it has optimised its prices, its market share equals $1 - s = (a + \alpha - 2G)/(4a)$, and bank 2's profit is

$$\Pi_2 = \frac{n(a + \alpha - 2G)^2}{8a} \tag{4.vi}$$

Because the second bank charges fees for its payment services, its profit does not approach zero as the fixed net income per customer (α) approaches zero.

Figure 6. **Banks' pricing with different values of a and α with G < 0**



Even when reservation prices are assumed not to bind, there are several different combinations of different pricing policies. Figure 6 hopefully sheds some light on how banks' pricing decisions depend on two variables, a and α. As can be seen in the figure, a high net income per customer (α) makes banks unwilling to charge prices, whereas a well functioning interbank system has the opposite effect. If the exogenous net income per customer is very high, neither of the banks is willing to sacrifice any market share in order to earn fee revenue by pricing payment services above zero, thus, $p_1 = p_2 = 0$. If, instead, the quality of interbank payments is high, market shares do not react strongly to prices, and both banks prefer to charge prices. If both variables have moderate values, the popular bank can charge positive prices and still maintain its position in the market, whereas its rival must price at zero.

If the exogenous net income per customer (α) is negative, a bank must be able to maintain a positive market share even if it charges positive prices; otherwise it would not be able to cover the cost of having customer relationships, and it would not prefer to enter the market.

4.3.2 Pricing at the reservation level

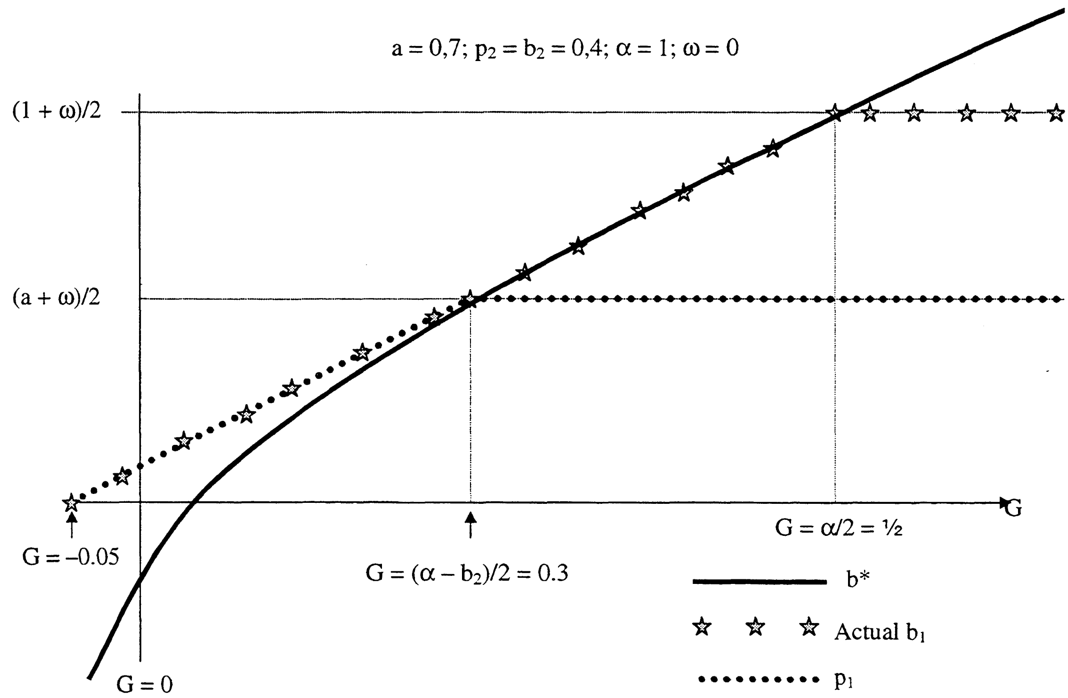
In principle, there is a very large number of different cases where the reservation price constraint is binding. A bank can be bound by either the reservation price for interbank payments or by both reservation price constraints.

Figure 7 describes how the pricing constraints affect the pricing decisions of the bank. If the bank is unpopular, it cannot afford charging any prices, because it would lose most of its customers. If it becomes more popular (G increases), it can charge higher and higher prices, as implied by the formula 4.iii.

At a certain point {when $G = (\alpha - b_2)/2$}, the reservation price constraint for interbank payments becomes binding. It becomes optimal to set the price of interbank payments (p_1) at the reservation level $(a+\omega)/2$, and the price of intrabank payments is determined according to the function b*, which is the optimal value of b_1 if p_1 is exogenously set to equal its reservation level.

With a certain value of G, even the value of b* exceeds the reservation level of b_1, and the bank must price both services at the reservation level.

Figure 7. **Different pricing rules with different values of G – an example**



It is also possible to construct examples where both banks' pricing decisions have at least one constraint.

In the following, the focus is on cases where both banks set both prices at the reservation level. The most important reason for this is that cases where only one, two or three of the four prices of the Bertrand game are constrained are, in practice, not solvable mathematically.

When all four prices are set at the maximum value [$p_1 = p_2 = (a + \omega)/2$ and $b_1 = b_2 = (1 + \omega)/2$], the market share of bank 1 is $s = (1 + a + 4G)/(2 + 2a)$. If $G = (-1 - a)/4$, then $s = 0$, if $G = 0$, then $s = ½$, and if $G = (1 + a)/4$, then $s = 1$.

The respective profits are

$$\Pi_1 = \frac{n(1+a+4G)\left[1+a^2+4\alpha+4G+2\omega+2a(1+2\alpha-2G+\omega)\right]}{8(1+a)^2} \qquad (4.vii)$$

and

$$\Pi_2 = \frac{n(1+a-4G)[1+a^2+4\alpha-4G+2\omega+2a(1+2\alpha+2G+\omega)]}{8(1+a)^2} \quad (4.viii)$$

4.3.3 Sabotage pricing

One potential outcome of the model is that a bank with a large and dominant market share would be able to make its competitor an unattractive choice for customers. If the dominant bank charges excessively high prices for interbank payments, its customers will use cash in payments to customers of the competing bank. This would affect the utility of both the payer and the payee equally strongly. In this hypothetical case, a typical customer of the smaller bank would receive payments above all from customers of the large bank, whereas a typical customer of the large bank would not be severely affected. Only a minor proportion of the latter's payments would go to customers of the small bank. Thus such a pricing policy would affect the average quality of payment services of a customer of the small bank much stronger than the quality of service perceived by a customer of the large bank.

Cases where sabotage pricing is profitable can be found if the disutility of cash payments (ω) is high and the quality of the interbank payment system (a) is good. Even though these cases are in principle possible, they will not be analysed in detail.

5 Investments when banks Bertrand-compete

5.1 The main case: neither of the two banks bound by constraints

5.1.1 Actual development efforts

When banks charge a fee for a payment service, the value of a affects banks' profits in two different ways.

1) As in section 3, it affects market shares. As to this effect, this version does not contain many new properties as compared to the version where prices were assumed to equal zero. The smaller bank invests in the system in order to increase its market share, whereas its big rival has no such incentive.

2) It affects the price banks can charge for a payment service. This effect could not have existed in the model of the section 3, but it is implied by the reaction function (4.iii).

Because the second effect could not exist in the absence of service fees, it is worth closer scrutiny.

Even though the demand for payment services is fixed and exogeneous, the quality of payment systems affects the equilibrium price (see 4.iii). This may sound counterintuitive, but the result has a reasonable explanation. If interbank payments are slow and unreliable, it is highly important for customers to use the same bank as the majority. And the larger the majority, the less attractive it is to belong to the minority. If a bank manages to attract customer A from the rival by offering payment services at a low price, customer B might follow, because he may have to exchange payments with A. Thus the market shares overreact to prices.

If instead interbank payments function well, the use of low prices as a competitive weapon has no cumulative effects, because it is no longer essential for customers to use the same bank as the majority. Gaining an additional customer has no cumulative effects. This result might be valid even more generally; *in the presence of direct network externalities, improving the compatibility of networks might relax price competition.*

This result has analogies with the model of Katz and Shapiro (1986). They present a discrete time model describing two competing firms selling at consecutive stages products characterised by direct network externalities. Each firm must make a one-off choice between supplying compatible or incompatible products. If products are incompatible, network externalities are limited to customers of the same company. All consumers have identical preferences, implying that the winning firm finally gets all the customers if technologies are incompatible. A central concept in the model is the so called 'installed base advantage', ie a product that was widely used in the past yields higher utility now. At the early stage of competition, firms supplying incompatible products compete fiercely with prices in order to provide customers with superior network externalities. In the beginning, incompatible products can even be sold at a loss with subsidised prices. Gaining the position as the dominant firms is essential to future (monopoly) profits. Product compatibility relaxes competition in the short term but intensifies it in the long term, because compatibility enables both firms to be permanently present in the market.

This finding may seem to be inconsistent with the results of Matutes and Padilla (1994). They demonstrated that in a spatial oligopoly, compatibility of ATM networks intensifies price competition between banks. If ATM networks are compatible, customers can make transactions through all ATMs, and the monopoly power provided by location and geographic distance partly disappears. The difference between the result of Matutes & Padilla and that presented above is due to the entirely different role of compatibility in the system. In the model of Matutes & Padilla, improving the compatibility erodes market power, because compatibility reduces the relative importance of geographic distance. In this model, compatibility has no impact on access to the services of the two banks. This makes it less important for each customer to try to use the same bank as the majority. Hence compatibility actually *increases* the relative significance of the location parameter (i).

The profit function of a bank and its dependence on the quality of interbank payments can be characterised by the following. If both banks charge positive prices that are not bound by the reservation price constraint, then, according to (4.iv), $\Pi_z = n(3a + 2G)^2/(18a)$, z = 1,2.

$$\Rightarrow \frac{d\Pi_1}{da} = \frac{n\left(\frac{9-4G^2}{a^2}\right)}{18}.$$

Result 5.a:

Iff a bank has a market share between 0 % and 100 % (which will be the case iff –3a/2 < G < +3a/2), it will invest in developing the system. If its market share is 0 % or 100 %, it will not invest.

Proof

If the market share of bank 1 is 100 %, then $s = ½ + G/(3a) = 1$, so that $G = 3a/2$. Therefore $d\Pi_1/da = n(9 - 4G^2/a^2)/18 = n(9 - 4\cdot(3a/2)^2/a^2)/18 = 0 \Rightarrow$ The bank has no incentive to invest in developing the system.

If the market share is 0 %, then $s = ½ - G/(3a) = 0$ so that $G = -3a/2$. Therefore $d\Pi_1/da = n[9 - 4\cdot(-3a/2)^2/a^2]/18 = 0 \Rightarrow$ The bank has no incentive to invest in the system.

If $0 < s < 1$, then $|G| < 3a/2$ which implies $d\Pi_1/da = (n/18)\cdot(9 - 4G^2/a^2) > 0 \Leftrightarrow 9a^2 > 4G^2$, which is true whenever $|G| < 3a/2$
Because the marginal impact of investment in payment system quality is infinity if nothing has been invested in the system, it cannot be optimal for the bank to invest nothing in the system when $-3a/2 < G < + 3a/2$.

QED

Unlike in the case of free payment services, a bank with a zero market share does not invest in developing the system. An unpopular bank that has difficulties in finding any customers uses low prices to attract customers rather than socially undesirable excessive investments in the payment system.

Investments by a small bank would both help the bank to gain a positive market share and to charge higher prices, but, paradoxically, the bank would not be interested in entering the market. The pricing formula (4.iii) does not predict that a bank with an almost zero market share would charge positive prices unless the fixed net income per customer (α) is negative and G is close to $\pm 3a/2$. Hence the difference between this result (5.a) and result 3.b is not simply due to the fact that payment services are now assumed to be costly. Instead, this analysis applies to cases that cannot be meaningfully analysed if one assumes that no fees are charged. As concluded above, there is now a much wider range of different values of G that lead to a duopoly situation where both banks have a nonzero market share.

When a bank has a market share of about 50 %, improvements in the payment system do not affect its customer base negatively. Instead, it can collect fees for a substantial amount of payment services. Thus it is not surprising that a medium-sized bank has the strongest incentive of all to invest in the system.

Result 5.b:
Private expenditure on payment system development as a function of G reaches its maximum when G = 0.

Proof

The optimisation condition for bank 1 is $(d\Pi_1/da)\cdot(da/d\Lambda_1) - 1 = 0$

Implicit differentiation gives

$$\frac{d\Lambda_1}{dG} = -\frac{\left(\frac{d^2\Pi_1}{dGda}\right)\cdot\left(\frac{da}{d\Lambda_1}\right)}{\frac{d^2\Pi_1}{d\Lambda_1^2}}$$

Optimisation implies that $d^2\Pi_1 / d\Lambda_1^2 < 0$ and by assumption $da/d\Lambda_1 > 0$
Hence, $d\Lambda_1/dG = 0$ if $d^2\Pi_1/dGda = 0$
$d^2\Pi_1/dGda = -n(4/9)\cdot G/a^2$, which cannot be 0 unless $G = 0 \Rightarrow$ There is only one extreme value in investment as a function of G.

The second order condition is:

$$\frac{d^2\Lambda_1}{dG^2} = -\frac{\left(\frac{d^3\Pi_1}{dG^2da}\right)\left(\frac{da}{d\Lambda_1}\right)\left(\frac{d^2\Pi_1}{d\Lambda_1^2}\right) - \left(\frac{d^3\Pi_1}{dGd\Lambda_1^2}\right)\left(\frac{d^2\Pi_1}{dGda}\right)\left(\frac{da}{d\Lambda_1}\right)}{\left[\frac{d^2\Pi_1}{d\Lambda_1^2}\right]^2}$$

Where $d^3\Pi_1/dG^2da = -(4/9)/a^2 < 0$. $da/d\Lambda_1 > 0$ and $d^2\Pi_1 / d\Lambda_1^2 < 0$.When $G = 0$ then $d^2\Pi_1/dGda = 0$.

It follows that $d^2\Lambda_1/dG^2 < 0$ and the extreme value is a maximum.

QED

The most surprising result may be the following, which is called the *symmetric incentives property.*

Result 5.c:
If both banks charge a price that lies between the reservation price and zero, they will always spend an equal amount in developing the system ($\Lambda_2 = \Lambda_1$).

Proof

The profit of bank 1 is $\Pi_1 = n\cdot(3a + 2\cdot G)^2/(18a)$
Analogically, the profit of bank 2 is $\Pi_2 = n\cdot(3a - 2\cdot G)^2/(18a)$

Differentiation with respect to a gives
$d\Pi_1/da = (n/18)\cdot(9 - 4G^2)/(a^2)$ and $d\Pi_2/da = (n/18)\cdot(9 - 4G^2)/(a^2)$, which are equal; $d\Pi_1/da = d\Pi_2/da$

The incentive for investing in the system is therefore always the same for both banks, and both banks invest the same amount in developing the system.

QED

This symmetric incentives property can be understood intuitively as follows. As in the model without payment service fees, the smaller bank can gain a larger market share by investing in the system. The big bank, by contrast, benefits in absolute terms much more than the small one from the impact of payment system development on the price of the service. The impact of development efforts on market shares is unfavourable, but the price effect more than offsets this negative effect. Overall, the big bank benefits as much as the small bank.

The symmetric incentives property has several interesting implications. For instance, it implies that both banks always react similarly to different exogenous factors that might affect the optimal amount of investments.

Due to the property of symmetric incentives, the resulting quality of payment system (a) is reached in a cost efficient way. The assumptions concerning the a-function imply that any given value of a is achieved in the most cost-efficient way if both private banks invest the same amount in developing the system. And when both banks have equally strong incentives to invest, their investments are equal.

In many duopoly models, it is interesting to know whether decision variables are strategic substitutes or complements. In this model, there is no generally valid answer to the question. In many cases, investments would be strategic substitutes, but this result is not universally valid. If investments by the two private agents have a negative interaction effect on the quality of the system ($d^2a/d\Lambda_2 d\Lambda_1 < 0$), investments would be strategic substitutes with many different parameter values. If they have a positive interaction effect, they would always be strategic complements. However, whether investments are stretegic substitutes or complements is of minor importance for optimal central bank investment.

5.1.2 Actual vs socially optimal investments

First, we shall review how payment system development affects the utility of customers. The welfare of a bank 1 customer equals

$$W_x = [G + (2 - i_x) + (1 - s) \cdot (a - p_1) + s \cdot (1 - p_1)]$$

and the impact of payment system development on the welfare of bank 1 customers equals $= 1 - s - dp_1/da + ds/da \cdot (1 - a)$.

The welfare of a bank 2 customer equals $W_x = -G + (i_x - 1) + s \cdot (a - p_2) + (1 - s) \cdot (1 - p_2)$ and the impact of a on the welfare of a bank 2 customer is $s - dp_2/da - (ds/da) \cdot (1 - a)$.

The impact of a change in *a* on consumer welfare in the whole economy can be calculated as in 3.iii. The total impact equals $n \cdot s \cdot \{1 - s - dp_1/da + ds/da \cdot (1 - a)\} + n \cdot (1 - s) \cdot \{s - dp_2/da - (ds/da) \cdot (1 - a)\} = -n[½ + 2G^2/(9 \cdot a^3)]$.

Interestingly, the impact of payment system development on consumer net welfare is negative. Although this may seem counter-intuitive, there is a natural explanation for it. The effect is basically due to the fact that price competition between banks is relaxed by improvements in the payment system, which is certainly undesirable from a consumer's point of view. As a whole, the price of a payment service increases proportionately with the quality of interbank payments. When the quality of interbank payments improves, intrabank payments do not improve, but the customer has to pay more even for them. Therefore, payment system development implies a transfer of wealth from customers to banks.

When the market is highly concentrated, an improvement in the quality of the payment system is of little use to customers, because few payments are processed through the interbank system. A well functioning system actually discourages the smaller bank from engaging in aggressive price competition, even though competition would benefit consumers. Thus, in a concentrated market, payment system improvement is especially undesirable for customers.

Needless to say, the impact of payment system development on *gross* consumer utility prior to payment of service fees, is positive for many different parameter values.

Result 5.d:
The total impact of payment system improvement on social welfare is positive ($d\psi/da > 0$) iff $|G| < 3a/(2\cdot\sqrt{[2 + 1/a]})$

Proof

It was demonstrated in the result 5,c that the impact of the quality of interbank payments (a) on profits equals $\{n/18\cdot(9 - 4G^2/a^2)\}$.

Moreover, it was demonstrated in the beginning of this section that the impact of the quality of interbank payments on consumer utility equals $-n[½ + 2G^2/(9\cdot a^3)]$.

Thus $d\psi/da = -n[½ + 2G^2/(9\cdot a^3)] + 2\cdot\{n/18\cdot(9 - 4G^2/a^2)\} =$
$n(9a^3 - 4G^2 - 8aG^2)/(18a^3)$ which is positive iff $|G| < 3a/(2\cdot\sqrt{[2 + 1/a]})$

QED

When a bank invests in developing the system, the improvement has several consequences. First, the investment affects the equilibrium price of payment services, thereby causing transfers of wealth from customers to the banking industry. These transfers of wealth do not cause any allocative distortions, and they are harmless from the point of view of social welfare. Secondly, customers receive improved interbank payment services, which is a positive effect. And finally, the investment affects market shares, which in most cases is an undesirable effect, because it hampers the functioning of the payment system by increasing the market share of the smaller bank.

Result 5.e:
Unless each bank has a 50 % market share ($G = 0$) or either of the banks has a 0 % market share ($|G| = \pm 3a/2$), the banks invest more than the socially optimal sum in payment system development.

Proof

The total externality of payment system development by bank 1 equals the sum of the impact on the rival's profits: $(n[½ - 2G^2/9a^2])$ and the impact on consumer welfare $(- n\cdot[½ + 2G^2/(9\cdot a^3)])$

The total externality equals $= -2n(1 + a)G^2/(9 \cdot a^3)$
Unless $G = 0$, this is negative, and private benefits exceed public benefits.

In addition, if $G = \pm 3a/2$, the bank invests nothing in the payment system, and the externalities of a hypothetical investment would be negative because $|G| > 3a/2 \cdot \sqrt{[2 + 1/a]}$). There would be a disparity between the private and social benefits of eventual investments. Nevertheless, there would be no disparity between the actual and socially optimal level of investment. When $G = \pm 3a/2$, the market shares are (100 %, 0 %), and, according to result 5.a, there is no private investment. As has been seen (result 5.d), no investment in the payment system would be desirable. In this case, there is no private investment, which is a socially optimal outcome.

QED

Hence, as a rule, banks overinvest in developing the system. As demonstrated in the previous result, a bank causes transfers of wealth from customers to both itself and its rival by investing in the system. Moreover, by investing in the system, a small bank can also increase its market share at the cost of the rival, which is also socially useless and even harmful.

When the two banks are equally popular, investing in the system does not affect market shares. However, the investment still has three effects on other sectors of the economy.

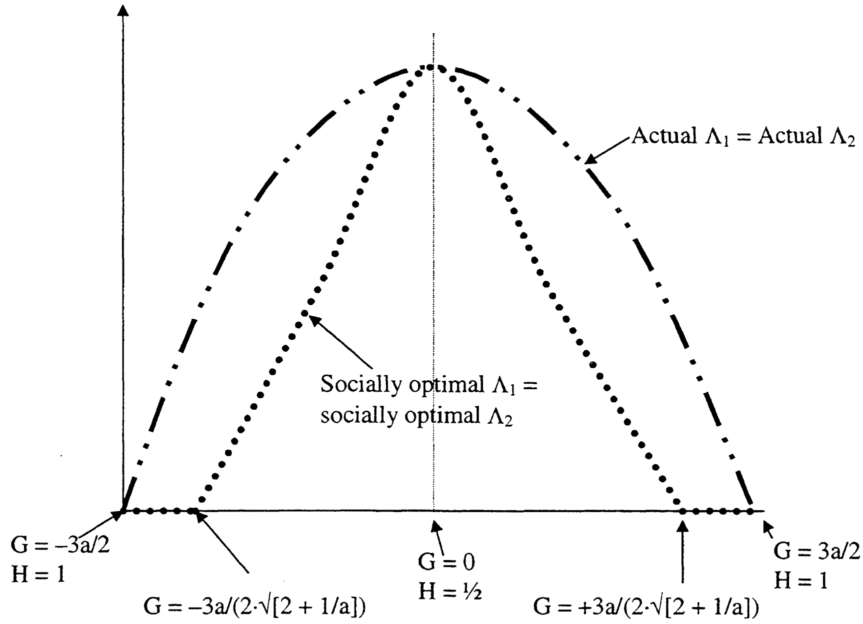
1) A transfer of wealth from customers to the investing bank

2) A gross increase in consumer utility yielded by improved payment services

3) A transfer of wealth from customers to the rival bank, caused by the increase in service fees. This is a mere transfer of wealth, which as such does not affect social utility.

When market shares are equal, effects 1 and 2 offset each other, and, in the aggregate, there is no net externality for the rest of the economy. Private and public benefits are equal, and private investment is at the socially optimal level. Interestingly, it is not possible to construct examples where the private sector would invest less than the socially optimal amount.

Figure 8. **Optimal and actual private investments**



If the two banks were to cooperate in payment services, the outcome might be worse. This is an implication of the symmetric incentives principle. Both banks would always prefer a higher level of rival investment. An additional investment by rival 2 would increase the profit of bank 1, but the costs would be borne by rival 2. If the two banks cooperated, they would agree on increasing their investment expenditure further. Hence there would be more over-investment.

5.1.3 Optimal central bank policies

Again, central bank investment in the payment system has two types of effects. First, it directly affects the quality of payment services. Secondly, it has an indirect effect through the reactions of the private sector. The symmetric incentives property significantly simplifies the analysis of the indirect effects.

Result 5.f:
Central bank investment affects both banks' investment in a similar way: $\partial\Lambda_1/\partial\Lambda_c = \partial\Lambda_2/\partial\Lambda_c$
This result is a direct corollary of the symmetric incentives property.

Because now both banks invest and because they react to each other's investments, the total impact of central bank investment on private investment is more complicated than in section 3. If investments by the two private banks are strategic complements, the indirect effects *strengthen* the impact of central bank investment on private investment. To take an example, an increase in central bank investment might encourage bank 1 to increase its investment. Due to symmetric incentives, bank 2 would also increase its own investment. And because of strategic complementarity, bank 1 would react to its rival's increased investment by increasing its own investment further. If instead investments are strategic substitutes, the situation is different, and the indirect effect through the reaction of bank 2 would weaken the total impact of central bank investment on bank 1's behaviour.

Nevertheless, qualitative conclusions concerning the impact of central bank investment on private investment are not reversed by indirect effects. If the increase in central bank investment is a (dis)incentive for a bank to invest in developing the system, rival reactions are not strong enough to reverse this effect.

Result 5.g:
Let $\partial\Lambda_z/\partial\Lambda_c$ denote the direct impact of central bank investment on private investment, if the investment by the rival were exogenous. Let $d\Lambda_z/d\Lambda_c$ denote the actual total impact, when all direct and indirect effects on investments by the two banks are taken into account. If the direct impact of central bank investment on private investment Λ_z is negative, then in no stable subgame perfect equilibrium indirect effects through rival reactions can reverse the effect. ($\partial\Lambda_z/\partial\Lambda_c < 0 \Rightarrow d\Lambda_z/d\Lambda_c < 0$). And iff $\partial\Lambda_z/\partial\Lambda_c > 0$, then $d\Lambda_z/d\Lambda_c > 0$.

Proof

See Appendix 5

This result simplifies the analysis concerning optimal central bank investment. Because the private sector typically invests more than the socially optimal amount (result 5e), it is reasonable for the central bank to try to reduce private investments by adapting its own investment behaviour. Therefore, it is of special interest to know whether central bank investment can reduce private investment and, if it can, under which circumstances. It turns out that it is not possible to reduce private investment by increasing public investment unless $d^2a/d\Lambda_z d\Lambda_c < 0$ and $\Lambda_c > 0$. Even in such cases, it is not certain that central bank investment would actually reduce private investment.

The easiest way to handle this problem mathematically is to analyse the possibilities of the central bank to do the opposite, ie to increase private investment, which would actually never be optimal. The reason for this is simple; it is possible to find sufficient conditions for $d\Lambda_z/d\Lambda_c > 0$.

Result 5.h:
The total impact of central bank investment on private investment is positive ($d\Lambda_z/d\Lambda_c > 0$) if either
$\partial^2a/\partial\Lambda_z\partial\Lambda_c \geq 0$ (Case 1) or
$\Lambda_c \approx 0$ and $G \neq 0$ (Case 2) or
both.

Proof

The f.o.c of bank 1 is $\partial\Pi_1/\partial\Lambda_1 - 1 = 0$.
Implicit differentiation gives

$$\frac{\partial\Lambda_1}{\partial\Lambda_c} = -\frac{\left[\frac{\partial^2\Pi_1}{\partial\Lambda_1\partial\Lambda_c}\right]}{\left[\frac{\partial^2\Pi_1}{\partial\Lambda_1^2}\right]}$$

By calculating the expression for $[\partial^2\Pi_1/\partial\Lambda_1\partial\Lambda_c]$ one obtains

$$= -\frac{8G^2\cdot\left(\frac{\partial a}{\partial\Lambda_c}\right)\left(\frac{\partial a}{\partial\Lambda_1}\right) + a(+9a^2-4G^2)\cdot\left(\frac{\partial^2 a}{\partial\Lambda_1\partial\Lambda_c}\right)}{\left[\frac{\partial^2\Pi_1}{\partial\Lambda_1^2}\right]\cdot 18a^3}$$

The second order optimisation condition for the bank implies that the denominator is always negative, which implies that the whole term is negative if the numerator is.

Beacause with any parameter values $8G^2\cdot(\partial a/\partial\Lambda_c)(\partial a/\partial\Lambda_1) > 0$, and because in any duopoly case $+9a^2 - 4G^2 > 0$, the numerator cannot be negative unless $(\partial^2 a/\partial\Lambda_1\partial\Lambda_2 < 0) \Rightarrow$ If $\partial^2 a/\partial\Lambda_z\partial\Lambda_c \geq 0$, public investment always increases private investment. (Case 1)

If Λ_c approaches zero, then $(\partial a/\partial\Lambda_c)$ approaches $+\infty$, and unless $G = 0$, $8G^2\cdot(\partial a/\partial\Lambda_c)(\partial a/\partial\Lambda_1) = +\infty$. $\Rightarrow$ If both $G \neq 0$ and Λ_c is close to zero, then the numerator must be positive. (Case 2)

According to the result 5.g $\partial\Lambda_1/\partial\Lambda_c > 0 \Rightarrow d\Lambda_1/d\Lambda_c > 0$.

QED

Expressing this result verbally in a more intuitive way, central bank investment can reduce private investment if public investment makes private investment inefficient $(\partial^2 a/\partial\Lambda_z\partial\Lambda_c < 0)$. By making the private investment inefficient the central bank can make the investment unattractive. If, instead, public investment strengthens the effects of private investment on the quality of payment systems $(\partial^2 a/\partial\Lambda_c\partial\Lambda_z > 0)$, it is not surprising that central bank investment is always a stimulus for private investment.

With very low levels of central bank investment, the impact of public investment on private investment is always positive. A marginal increase in central bank investment always encourages the private sector to increase its own investment. The private bank may be discouraged from investing if the quality of the existing system is so poor that no major improvements can be achieved at a reasonable cost. Thus, even when central bank investment makes private investment technically inefficient, it is not obvious that private investment would become unprofitable for the bank if the central bank increases its investment.

Nevertheless, a certain amount of public investment is justified if the market shares are roughly equal. The investment may have undesirable effects on the behaviour of the private sector, but when there are a lot of interbank payments, the direct benefits of public investment more than offset the non-desirable indirect effects.

Result 5.i;
It is not optimal for the central bank to invest in developing the system if $|G| \geq 3a/(2\cdot\sqrt{[2 + 1/a]})$; if G = 0, it is optimal to invest.

Proof

See Appendix 6

As a rule, banks overinvest in the system. Therefore, the central bank should try to reduce private investment. Whenever the marginal impact of central bank investment on private investment is negative (positive), it is reasonable for the central bank to increase (decrease) its investment in order to reduce private investment, except when there is no market failure because the market shares are equal (G = 0).

Result 5.j;
If $G \neq 0$ but it is optimal for the central bank to invest, then the following condition holds. If either $\partial^2 a/\partial\Lambda_1\partial\Lambda_c \geq 0$ or $\Lambda_c \approx 0$ or both, then the central bank should restrict its investment in order to reduce private investment.

Proof

The optimisation condition of the central bank is

$$\frac{d\psi}{d\Lambda_c} = \left(\frac{d\psi}{da}\right)\left(\frac{\partial a}{\partial\Lambda_c}\right) - 1 + \left[\left(\frac{d\psi}{da}\right)\cdot\left(\frac{\partial a}{\partial\Lambda_1}\right) - 1\right]\cdot\left(\frac{\partial\Lambda_1}{\partial\Lambda_c}\right) + \left[\left(\frac{d\psi}{da}\right)\cdot\left(\frac{\partial a}{\partial\Lambda_2}\right) - 1\right]\left(\frac{\partial\Lambda_2}{\partial\Lambda_c}\right) = 0$$

If $|G| < 3a/(2\cdot\sqrt{[2 + 1/a]})$ but $|G| > 0$, then $(d\Psi/da)\cdot(da/d\Lambda_z) - 1 < 0$.
(Result 5.e) If either $\partial^2 a/\partial\Lambda_z\partial\Lambda_c \geq 0$ or $\Lambda_c \approx 0$, then $d\Lambda_z/d\Lambda_c > 0$.
(Result 5.h) $\Rightarrow$ $[(d\psi/da)\cdot(\partial a/\partial\Lambda_1) - 1]\cdot(\partial\Lambda_1/\partial\Lambda_c) < 0$ and
$[(d\psi/da)\cdot(\partial a/\partial\Lambda_2) - 1]\,(\partial\Lambda_2/\partial\Lambda_c) < 0$

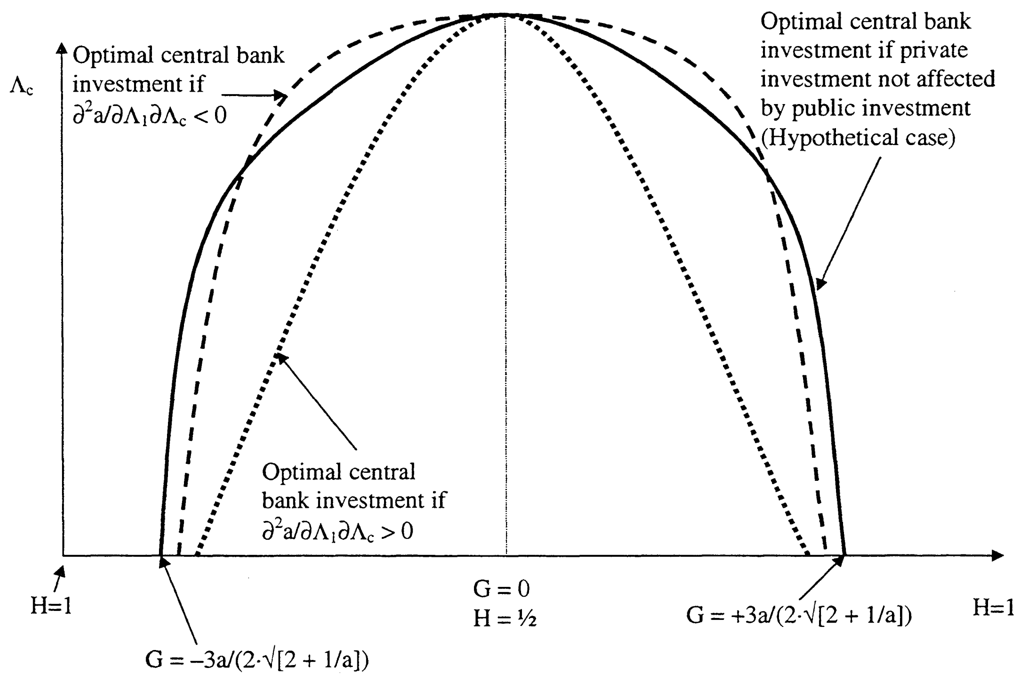
The optimisation condition $d\psi/d\Lambda_c = 0$ cannot hold unless $(d\psi/da)(\partial a/\partial\Lambda_c) - 1 > 0$, i.e. unless the direct impact of central bank investment on welfare is positive, and the value of Λ_c below the level that would be optimal in absence of privat sector reactions.
$\Rightarrow$ The central bank should restrict its investment.

QED

However, in some cases the reverse result might hold. If central bank investment is well above zero and public investment makes private sector investment inefficient, the central bank might have good reasons to *increase* its investment to discourage private investment. If the market shares are equal (G = 0), the central bank should not try to affect private investment because private investment is already at its desired level.

Optimal central bank policies are illustrated in figure 9.

Figure 9. **Optimal central bank investment when both banks charge prices**



5.2 Investments with binding constraints

5.2.1 Investments with a binding non-negativity constraint

The case where both banks price at zero has already been analysed in section 3. It is essentially irrelevant to banks' investment decisions whether prices are zero, because there is an exogenous constraint that forces banks not to charge any fees, or whether the banks deliberately choose to set their prices at zero.

Therefore, in this section, the focus is on banks' incentives to invest in the system when either of them charges fees for payment services, when its rival does not charge. Market outcomes for such cases have been analysed in section 4.3.1.

Now, it is assumed that

- Bank 1 has a small market share and it offers payment services free of charge in order to increase its clientèle.
- Bank 2 charges positive prices.

Both banks have a positive market share, or at least the less popular bank 1 would gain a nonzero market share if the functioning of the interbank payment system (a) were marginally improved.

The impact of payment system development on bank 1's profits equals $d\Pi_1/da = n\alpha(\alpha - 2G)/(4a^2)$, and the impact of payment system development on bank 2's profits equals $d\Pi_2/da = n\{a^2 - (\alpha - 2G)^2\}/\{8a^2\}$.

It is easy to demonstrate that if bank 2 dominates the whole market, it has no incentive to develop the payment system. This is understandable because developing the system would mainly erode its dominant market position.

Result 5.k:
If bank 2 has nearly a 100 % market share [which will be the case iff $G \approx (-3a + \alpha)/2$], then it cannot be optimal for it to invest in developing the payment system.

Proof

$d\Pi_2/da = n\{a^2 - (\alpha - 2G)^2\}/\{8a^2\}$; If $G = -(3a - \alpha)/2$, then $d\Pi_2/da = -n < 0$.
The profit decreases if the payment system begins to function better.

QED

As to the case where market shares that are nearly equal (higher values of G), it is difficult to draw any robust conclusions concerning the behaviour of bank 2. Bank 2 may have an incentive to invest in the system with a sufficiently high value of G, but this is not certain. Improvements in the payment system become less and less harmful to bank 2 as its market size decreases ($d^2\Pi_2/dadG = (\alpha - 2G)/2a^2$; $G < 0 \Rightarrow d^2\Pi_2/dadG > 0$), implying that at a certain point improving the interbank system *may* become profitable for bank 2. The improvement would lower its market share, but, it would also improve its possibilities to charge high prices. The effect of such pricing possibilities might eventually more than offset the adverse impact on market share.

Because the marginal impact of a very small investment (Λ_2) on the quality of the payment system (a) is disproportionately strong, bank 2 would invest at least something in the system whenever $d\Pi_2/da > 0$. However, it is possible to construct examples where $d\Pi_2/da < 0$ for any value of G.

Bank 1, instead, has incentives somewhat similar to those in the case where neither of the two banks charges fees. The bank can increase its market share by making it less burdensome for its own customers to make and receive payments. Moreover, it has an additional incentive. Improving the system encourages bank 2 to charge a higher price (p_2), which also helps bank 1 to increase its market share. Hence it is not surprising that bank 1 always invests in the system.

Result 5.l:
Bank 1, which does not charge fees, invests in the payment system.

Proof

$d\Pi_1/da = n\alpha(\alpha - 2G)/(4a^2)$
If $G < 0$, then $d\Pi_1/da > 0$.

When $\Lambda_1 = 0$, then $\partial a/\partial\Lambda_1 = \infty$
It cannot be optimal for bank 1 not to invest in the system.

QED

Result 5.m:
The incentives for bank 1 to invest in payment system development decrease as G increases.

Proof

$d^2\Pi_1/dadG = -2/(4a^2) < 0$. Thus, the higher the value of G, the less the incentives to invest in the system.

QED

In most cases, bank 1, which charges no fees, invests more in the system than bank 2. Surprisingly, it is also possible to find contrary examples.[5]

Result 5.n:
If the investment by the bank z (z = 1,2) is greater than zero, the impact of central bank investment on private investment, Λ_z, is negative at least if either $\Lambda_c \approx 0$ or $\partial^2 a/\partial\Lambda_z\partial\Lambda_c \leq 0$.

Proof

See Appendix 7

The payment services-related consumer surplus of a bank 1 customer equals $U_1 = s + (1-s)\cdot a$. The impact of a payment system improvement on the consumer surplus of a bank 1 customer equals $dU_1/da = (a^2 + \alpha - 2G)/(4a^2)$. The effect is positive, which is not surprising, because the customer benefits from the increased market share of bank 1, but does not have to pay anything for the improvement. In fact, consumer surplus equal consumer utility.

The payment services-related consumer surplus of a bank 2 customer equals $U_2 = s\cdot(a - p_2) + (1-s)\cdot(1 - p_2)$. And the impact of a payment system improvement on the consumer surplus of a bank 2 customer equals $dU_2/da = (a^2 - \alpha + 2G)/(4a^2)$, which can be either negative or positive.

The total impact of payment system development on consumer surplus in the whole economy equals

$$\frac{dU}{da} = n\cdot s\cdot\frac{dU_1}{da} + n\cdot(1-s)\cdot\frac{dU_2}{da} = \frac{\left[2a^3 + a(\alpha - 2G) - (\alpha - 2G)^2\right]n}{8a^3}$$

[5] The issue was tested with five thousand simplistic numerical simulations. When a,α and G were given evenly distributed random values that satisfied the four conditions, bank 2 had higher incentives to invest in the system than bank 1 in slightly more than 10 % of cases.

Result 5.o:
The investment by bank 1 exceeds its socially optimal level at least if bank 1 market share is close to 0, which would be implied by G = (–3a + α)/2.

Proof

The total externality caused by the investment is

$$\left(\frac{dU}{da}+\frac{d\Pi_2}{da}\right)\frac{\partial a}{\partial \Lambda_1}=$$
$$\left[\frac{\left[2a^3+a(\alpha-2G)-(\alpha-2G)^2\right]n}{8a^3}+n\frac{a^2-(\alpha-2G)^2}{8a^2}\right]\cdot\left(\frac{\partial a}{\partial \Lambda_1}\right)$$

When $G = (-3a + \alpha)/2$, $dU/da + d\Pi_2/da = -7/8 - 3/(4a) + a^2$

Whenever $0 \leq a \leq 1$, this is negative.
⇒ bank 1 investment causes a negative externality

QED

By investing heavily the small bank can take market share from its rival. This policy both affects customers adversely by increasing the number of interbank payments and reduces the rival's profits of the rival. Thus all the effects on other agents in the economy are negative. The central bank can discourage (encourage) bank 1 investment by increasing (decreasing) its own investment at least if $\partial^2 a/\partial\Lambda_1\partial\Lambda_c < 0$. {Result 5.n}

As already mentioned, the profit maximising investment by bank 2 is often zero, which is not always socially optimal. In these cases, it is difficult for the central bank to encourage it to invest.

5.2.2 Investment when banks price at the reservation level

In this section, it will be analysed how banks invest in the system if they both set their prices at the respective reservation levels. Prices, profits and market shares in such cases have been analysed in section 4.3.2.

This case has certain analogies with both the case analysed in section 3 and the case of internal point solutions. Both banks have an incentive to invest because a better system enables them to charge higher prices. On the other hand, an improved system would help the smaller bank to gain more market share, which would be beneficial for the small bank but not for the large one.

Mathematically, the situation can be analysed as follows. The impact of improved payment services on bank 1's profits equals

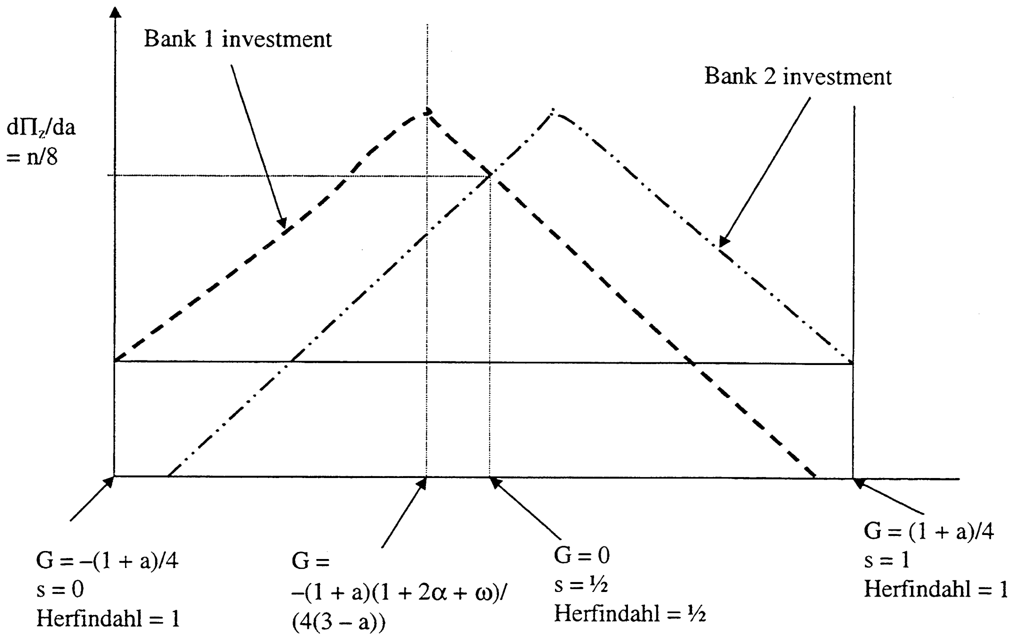
$$\frac{d\Pi_1}{da}=\frac{n\{1+3a^2+a^3-48G^2-8G[1+2\alpha+\omega]+a[3+16G^2-8G(1+2\alpha+\omega)]\}}{8(1+a)^3}$$

If G = 0, then $d\Pi_1/da = n/8 > 0$; If s = 0, then $G = -(1 + a)/4 \Rightarrow d\Pi_1/da > 0$; If s = 1, then $G = (1 + a)/4$, and $d\Pi_1/da < 0$.

The incentives of the bank to invest in the system are maximal when $d^2\Pi_1/dadG = 0$, which gives $G = -(1 + a)(1 + 2\alpha + \omega)/[4(3 - a)] < 0$, $[d^2\Pi_1/dadG^2 = -(3 - a)\cdot 4n/(1 + a^3) < 0$, which implies that this is a maximum].

Investment as a function of G is illustrated in figure 10.

Figure 10. **Investment when both banks price at the reservation price level**



As to the externalities of private investment, the impact of system improvement on consumer surplus is essential.

The consumer surplus of a bank 1 customer is

$$W_x = G + (2 - i_x) + (1 - s)\cdot\left[a - \frac{(\omega + a)}{2}\right] + s\cdot\left[1 - \frac{(1+\omega)}{2}\right]$$

and the consumer surplus of a bank 2 customer is

$$W_y = G + (i_x - 1) + s\cdot\left[a - \frac{(\omega + a)}{2}\right] + (1 - s)\cdot\left[1 - \frac{(1+\omega)}{2}\right]$$

The impact of payment system development on the total consumer surplus in the economy equals

$$n\cdot s\cdot\left(\frac{dU_1}{da}\right) + n\cdot(1-s)\cdot\left(\frac{dU_2}{da}\right) = \frac{n(1 + 3a + 3a^2 + a^3 - 32G^2)}{\{4\cdot(1+a)^3\}}.$$

Result 5.p;
Firms invest less than the socially optimal amount in payment system development at least when G = 0.

Proof

When $G = 0$, the impact of the quality of interbank payments on consumer surplus equals n/4. The impact of payment system quality on profits is $d\Pi_1/da = d\Pi_2/da = n/8$. The total effect on social welfare equals $n/4 + 2 \cdot n/8 = n/2$, which is much greater than the impact on private profits. Therefore, a private bank has insufficient incentive to invest in the system.

QED

Result 5.q:
If $G = 0$ and $\partial^2 a/\partial\Lambda_c\partial\Lambda_z > 0$ ($\partial^2 a/\partial\Lambda_c\partial\Lambda_z < 0$), then the central bank should increase (restrict) its investment in order to encourage the private sector to increase its own investment.

Proof

When $G = 0$, private investment is below its socially optimal level (result 5.p). $d^2\Pi_z/da^2 = 2Gn[1 + 2\alpha + 10G + \omega + a(1 + 2\alpha - 2G + \omega)]/(1 + a)^4$, which equals zero when $G = 0$.

The impact of central bank investment on bank z investment is

$$\frac{d\Lambda_z}{d\Lambda_c} = \frac{\left(\frac{d^2\Pi_z}{da^2}\right)\cdot\left(\frac{\partial a}{\partial\Lambda_z}\right)\left(\frac{\partial a}{\partial\Lambda_c}\right)+\left(\frac{d\Pi_1}{da}\right)\cdot\left(\frac{\partial^2 a}{\partial\Lambda_c\partial\Lambda_z}\right)}{\frac{\partial^2\Pi_z}{\partial\Lambda_z^2}} = \frac{-\left(\frac{d\Pi_1}{da}\right)\cdot\left(\frac{\partial^2 a}{\partial\Lambda_c\partial\Lambda_z}\right)}{\left(\frac{\partial^2\Pi_z}{\partial\Lambda_z^2}\right)}$$

Therefore, $d\Lambda_z/d\Lambda_c$ has the same sign as $d^2a/d\Lambda_c d\Lambda_z$

QED

A few words can also be said of market failures in asymmetric situations. Not surprisingly, a bank with a very small market share invests more than the socially optimal sum in developing the system. (When $G = -(1 + a)/4$, then $s = 0$; Then $dU/da + d\Pi_2/da = -n(3 - 2a + 2\alpha + \omega)/[4(1 + a)] < 0$, and investment by bank 1 is socially undesirable.) Both consumers and the rivalling bank suffer from the investment. The improvement increases the market share of the smaller bank, which, when all the effects are taken into account, reduces the average quality of payment services by increasing the number of interbank payments. Moreover, the rival bank 2 suffers a loss of market share, and the increase in profits of bank 1 is mainly due to the transfer of wealth from the larger to the smaller bank.

6 Conclusions and discussion

6.1 The main results

This paper has presented a simple model of duopolistic bank competition. The model describes a Hotelling-type vertically differentiated market where most customers prefer either of the two banks because of geographic proximity. The emphasis is on payment services rather than lending and borrowing, and especially on banks' incentives to invest in developing the payment system.

Basically, there seem to be two different factors that affect banks' incentives to invest in the payment system:

1) market shares and the degree of concentration in the market
2) interest rate margins and the intensity of competition in financial intermediation and other services.

These factors have rather complicated interaction effects. They largely determine banks' possibilities and incentives to charge fees for using the payment system, which is a key question.

Customers prefer intrabank payments to interbank payments, because intrabank payments are of superior quality. With intrabank payments, there are no delays due to complicated clearing arrangements between the two banks and no exchange of retail payment data between computer systems, which might not be fully compatible. This effect implies that there are economies of scale in the industry. Obviously, the number of intrabank payments is an increasing function of the number of customers who use the same bank; if a bank has only one customer, all the payments are interbank payments, and if it has a 100 % market share, there are no interbank payments.

The quality of intrabank payment services is assumed to be exogenous, whereas the quality of interbank payments is determined endogenously. The main issue in this paper concerns the incentives of banks to create an efficient and reliable system for interbank payments. As a rule, banks have incentives to invest in the system in socially non-optimal amounts, especially if they must offer payment services free of charge. In such a case, a small bank may invest excessively in the payment system; otherwise the bank would be unattractive to customers who have to make noncash payments, because there are few agents with whom it would be possible to make intrabank payments. A large bank, by contrast, would have no incentive to improve the interbank system. A well functioning interbank system would weaken its competitive advantage, which is based on its ability to offer a high relative share of intrabank payments.

If the two banks are of the same size, neither of them can increase its market share by improving the system. Thus, banks do not invest in the system, even though customers have pronounced needs for interbank payment services. The central bank should try to improve the situation with its own investments.

If banks can charge fees, their incentives are not as severely distorted. Both banks have incentives to develop the system, because it allows them to charge higher service fees. Especially if the two banks are roughly of equal size, there is no serious market failure. The number of interbank payments is high and banks invest substantial amounts in developing the payment system. The central bank should not try to affect the decisions of the private banks. Paradoxically, the degree of price competition depends inversely on the quality of the interbank system: if the system functions poorly, price competition is fierce.

One of the main conclusions of this model is that banks often have distorted incentives to invest in developing the payment system if they either cannot price the payment service or they voluntarily prefer not to charge fees for payment services. If financial market regulation or insufficient interest rate competition maintains abnormally wide interest rate margins, attracting more customers readily becomes the main objective of banks, because it is the most efficient way to increase profits. Thus antitrust policies against collusion regarding loan and deposit services can have beneficial, indirect effects on the allocation of resources in the payment system.

A banking monopoly might be optimal for the payment system, but a high degree of concentration would probably not be optimal in terms of allocative efficiency in financial intermediation. The monopoly bank would be able to maintain excessive interest rate margins, which would have serious distorting effects. Thus, there is a welfare tradeoff between the efficiency and reliability of the payment system vs the socially efficient allocation of financing.

There are three main restrictions incorporated in the model presented in this paper. Without these restrictions, many predictions of the model would probably be no longer valid.

1) In the real world, there are normally more than two banks. Typically, no bank has a dominant market position. If there were three banks, two of them could form a coalition in order to acquire greater market share at the cost of the third bank. (Common computer and accounting systems established by some local banks could be mentioned as examples.) Moreover, with multiple banks it would be possible to create examples where all the banks would have just a minor share of the market.

2) In the real world, the role of payment services as a way to attract customers and deposits is not as simplistic as in the model. For instance, many households and companies have accounts with numerous banks. Customers might use one bank for payment transactions, and another one for depositing large savings. Thus, it is not obvious that large savings and loan portfolios can be acquired by offering good payment services at low prices.

3 In this model, banks cannot develop their intrabank payment services. Probably large and small banks alike would have incentives to do so, but the large bank might be more interested in developing its internal systems; intrabank payments are the main type of payment for its customers.

Extending the analysis to take these considerations into account might be fruitful.

It is not likely that treating the net interest income as an endogenous variable would reverse the results. At least the intuition behind the results is not dependent on the assumption that the net interest income is exogenous. The same probably applies, at least to some extent, to the assumed zero price elasticity of demand for payment services.

6.2 History of the Finnish payment system

The historical development of the Finnish payment system can be compared with the predictions of the model.

In the late 19th and early 20th century, the most commonplace interbank payment media offered by banks were cheques and so-called postable cashier's drafts[6] (Korpisaari 1930, pp. 380–385). In the early stages of the postable cashier's draft system, some large banks discriminated against small banks by not accepting instruments issued by the latter. This practice continued until the Bankers' Association was established (Korpisaari 1920, p. 270). This observation is consistent with one of the main results presented in the section 3 of this paper: a large bank may try to make it more difficult for the customers of a small bank to make and receive interbank payments.

The savings bank group consisting of dozens of minor establishments made several attempts to improve payment services between the different member banks. The group launched a kind of a giro system already in the 1910s, although this experiment did not last for more than a few years (Urbans 1963, p. 392). Group members cooperated in the clearing and issuance of postable cashier's drafts. In 1940 the cooperation was developed further, and savings banks offered each other's customers all the basic deposit-related services. For instance, it became possible for a customer of savings bank A to make cash withdrawals in the office of savings bank B (Kalliala 1958, p. 60–61). There was no comparable arrangement between commercial banks. On average, commercial banks were much larger than typical savings banks. As implied by the model, the smallest players in the market were the first to cooperate in payment services.

The current giro system was established during the World War II.

For many years, the government had been planning and preparing to establish a postal giro system. The system was finally launched in December 1939, because the Winter War increased the number of retail payments made and received by the government. The government accounted for a disproportionate share of all retail payments. Thus the administrative decision to centralise government payments in the postal savings bank immediately created the 'critical mass' to make the system attractive even for private customers. Consequently, the market share of the Postal Savings Bank began to increase in deposit-taking as well. The market share of the Postal Savings Bank was insignificant in the 1930s, but the bank had more than a million deposit customers in 1946, which represented a substantial market share; the population of Finland was about 3,8 million. (Auer 1964, p. 295) This may be one of the best real world cases corroborating the view that payment services matter in attracting deposits.

The strengthening role of the Postal Savings Bank caused commercial banks substantial losses of market share. Being forced to react, the commercial banks established their own giro system in 1942. Interestingly, the system was established only between commercial banks; the savings banks did not participate. Because each of the commercial banks operated as an independent agent, and none of them had a dominant market position, one could conclude that the

[6] Postable cashier's drafts were used above all in Finland and Sweden. They were debt instruments issued by banks, redeemable upon request, and made payable to a particular payee. The payment was made in the following way. The bank of debtor A sold to its customer a draft payable to creditor B. Debtor A mailed the instrument to payee B. The bank of payee B cashed the instrument, and finally the issuing bank redeemed it with central bank money.

smallest players in the market were the most interested in launching their own interbank giro system.

Due to their close cooperation in payment systems, the savings banks were almost like a single institution. With their nearly 40 % market share, the savings banks did not decide to join the giro system until 1943. (Kuusterä 1995, p. 416) Thus the largest banking group was the last to join the interbank giro system. This observation is consistent with the model presented above. The two giro systems were linked up in 1948.

The model predicts that when there are no fees for using the payment system, a small bank has a strong incentive to develop and maintain a well functioning interbank retail payment system. When the Finnish financial market was tightly regulated in the 1950s, 1960s and 1970s, there was almost no interest rate competition, and payment services were offered free of charge as a marketing tool. None of the Finnish banks had a truly dominant position in the market. Interestingly, no bank made any attempt to abandon the banks' mutual giro system during these decades. Moreover, none of the banks was reluctant to adopt innovations in the mutual exchange of retail payment information, such as physical delivery of magnetic tapes between banks' computer centres in the 1960s and exchange of data via the telephone network in the 1970s. All the banks preferred to participate and to regularly update the technical infrastructure used in interbank payments.

When financial markets were tightly regulated, banks were above all interested in their market shares, possibly because the size of the customer base was essential to profits. Thus it is likely that participating in the giro system was essential to maintaining and increasing the customer base. This interpretation would be consistent with the model; unless a bank has a higher than 50 % market share, its market share would decline if it did not participate in the interbank payment system. If there had been a dominant bank in Finland in the past, it might have been reluctant to adapt any innovations that facilitated interbank payments. Eventually, it might even have dropped out of the giro system.

6.3 International comparison of the role of the central bank

One of the key issues of this paper has been the importance of market concentration for the optimal degree of central bank involvement in retail payment systems. Thus it might be interesting to take a closer look at the situation in different countries. Because the model describes a giro system better than a cheque-based system, the focus in international comparisons should be on countries where cheques play no major role in the payment system.

The following table describes the situation in nine different EU countries. Five member countries (France, Italy, UK, Greece and Ireland) are excluded because their payment systems are cheque-based, and one country (Luxembourg) because of the exceptional nature of its financial industry. The degree of central bank involvement in retail payments is based on a subjective classification, the main source of information being EMI (1996). The degree of market concentration is measured with the three-firm concentration ratio (Source of data: EMI).

	The role of the central bank limited	The role of the central bank important
Concentrated banking industry	Finland, Sweden, Denmark, Netherlands	
Intermediate degree of concentration	Belgium	Austria
A fragmented banking industry	Portugal	Spain, Germany

As we see, there seems to be a moderate, negative correlation between the degree of market concentration and the role of the central bank. This would, at least on the surface, be consistent with the conclusions drawn in section 3; if the market is concentrated, the central bank should not make heavy investments in the system.

Of course, due to the small size of the sample, this evidence has to interpreted with caution. Moreover, it may due to effects that have little to do with those analysed in this paper.

As to countries where cheques are the predominant payment medium, it is possible to find completely different cases. In some of them, the central bank plays a key role in the payment system, but the market itself may be highly fragmented (United States, Italy) or rather concentrated (France). In the UK, a cheque system coexists with an extremely limited role for the central bank.

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Appendix 1

The following notation is used.

Λ_1 = investment by bank 1,
Λ_2 = investment by bank 2 and
Λ_c = investment by the central bank.

The following assumptions characterise the 'a' function.

1) It is possible to make payments between the two banks even if nothing has been invested in the system. With any level of investment, $a > 0$.
2) Investing in the system always improves its quality, although this improvement is subject to diminishing returns; $\partial a/\partial\Lambda_1 > 0$; $\partial a/\partial\Lambda_2 > 0$; $\partial a/\partial\Lambda_c > 0$; $\partial^2 a/\partial\Lambda_1^2 < 0$; $\partial^2 a/\partial\Lambda_2^2 < 0$; $\partial^2 a/\partial\Lambda_c^2 < 0$;
3) If an agent invests nothing, its investment would have an excessively strong impact on the quality of the system. Iff $\Lambda_1 = 0$, then $\partial a/\partial\Lambda_1 = \infty$; Iff $\Lambda_2 = 0$, then $\partial a/\partial\Lambda_2 = \infty$; Iff $\Lambda_c = 0$, then $\partial a/\partial\Lambda_c = \infty$;
4) The cross derivatives may be positive, negative or zero, but they are always finite. $|\partial^2 a/\partial\Lambda_1\partial\Lambda_c| \neq \infty$; $|\partial^2 a/\partial\Lambda_2\partial\Lambda_c| \neq \infty$; $|\partial^2 a/\partial\Lambda_1\partial\Lambda_2| \neq \infty$.
5) The lowest possible value for any investment variable Λ is 0. No agent can make a negative investment.
6) The 'a' function depends in a similar way on Λ_1 and Λ_2. If $\Lambda_1 = A$ and $\Lambda_2 = B$, the value of 'a' is equal to the value of 'a' obtained when $\Lambda_2 = A$ and $\Lambda_1 = B$. As to derivatives, the value of $\partial a/\partial\Lambda_z$ (or $\partial^2 a/\partial\Lambda_c\partial\Lambda_z$ or $\partial^2 a/\partial\Lambda_z^2$) depends on the value of Λ_z, not on whether z = 1 or 2.[7]
7) Transferring a payment from one bank to another cannot improve the quality of the payment, as compared to an intrabank payment, which is not transferred between banks. Even in the best case, the quality of interbank payments is at least marginally lower than the quality of intrabank payments, ie $a < 1$.

[7] This does not necessarily imply that $a = a\{(\Lambda_1 + \Lambda_2), \Lambda_c)\}$

Appendix 2

Result 3.f:
Let the market share of bank 1 be less than 50 %. Increasing central bank investment decreases investments by bank 1 at least if either or both of the following two conditions holds.

1 $(\partial^2 a/\partial\Lambda_1\partial\Lambda_c\} \leq 0.$
2 $\Lambda_c = 0$

Proof

If the market share is < 50 %, then the investment Λ_1 is positive.

The bank's investment is determined according to the following optimisation condition.

$$\frac{d\Pi_1}{d\Lambda_1} = n\cdot\alpha\cdot\left(\frac{ds}{da}\right)\cdot\left(\frac{\partial a}{\partial\Lambda_1}\right) - 1 = 0$$

Implicit differentiation gives

$$\frac{d\Lambda_1}{d\Lambda_c} = -\frac{\left(\frac{d^2s}{da^2}\right)\cdot\left(\frac{\partial a}{\partial\Lambda_c}\right)\cdot\left(\frac{\partial a}{\partial\Lambda_1}\right)+\left(\frac{ds}{da}\right)\cdot\left(\frac{\partial^2 a}{\partial\Lambda_1\partial\Lambda_c}\right)}{\frac{d^2\Pi_1}{d\Lambda_1^2}}n\cdot\alpha$$

Where
$ds/da = -G/a^2 > 0$
$d^2s/da^2 = 2G/a^3 < 0$, and
by assumption $da/d\Lambda_c > 0$ and $da/d\Lambda_1 > 0$

Due to profit maximisation $\{d^2\Pi_1 / d\Lambda_1^2\} < 0$
If $(\partial^2 a/\partial\Lambda_1 d\Lambda_c\} \leq 0$, then $d\Lambda_1/d\Lambda_c < 0$
and if $\Lambda_c = 0$, then $\partial a/\partial\Lambda_c = \infty$, which implies

$$\frac{d\Lambda_1}{d\Lambda_c} = -\frac{-\infty}{\frac{d^2\Pi_1}{d\Lambda_1^2}} < 0$$

QED

Appendix 3

Result 3.g:
If the smaller bank invests less (more) than the socially optimal amount in payment system development, the central bank should restrict (increase) its investment in the system in order to encourage (discourage) private investment at least if $(d^2a/d\Lambda_1 d\Lambda_c\} \leq 0$.

Proof

Let z be the bank with the smaller market share.
The optimisation condition of the central bank is

$$\frac{d\psi}{d\Lambda_c} = \left(\frac{\partial\psi}{\partial a}\right)\cdot\left[\left(\frac{\partial a}{\partial\Lambda_c}\right)+\left(\frac{\partial a}{\partial\Lambda_z}\right)\cdot\left(\frac{\partial\Lambda_z}{\partial\Lambda_c}\right)\right]-\left(\frac{\partial\Lambda_z}{\partial\Lambda_c}\right)-1=0 \tag{3g.*}$$

If the central bank does not take into account indirect effects, then its optimisation condition would reduce to

$$\left(\frac{\partial\psi}{\partial a}\right)\cdot\left(\frac{\partial a}{\partial\Lambda_c}\right)-1=0$$

Because of indirect effects, this optimisation policy would make $d\Psi/d\Lambda_c$ equal 0 only iff

$$\left(\frac{\partial\psi}{\partial a}\right)\cdot\left[\left(\frac{\partial a}{\partial\Lambda_z}\right)\cdot\left(\frac{d\Lambda_z}{d\Lambda_c}\right)\right]-\left(\frac{\partial\Lambda_z}{\partial\Lambda_c}\right)=0$$

Because $(\partial^2 a/\partial\Lambda_1\partial\Lambda_c\} \leq 0$, it is known that $(d\Lambda_z/d\Lambda_c) < 0$ (Result 3.f)

If Λ_z is below its socially optimal level, then

$$\left(\frac{\partial\psi}{\partial a}\right)\cdot\left(\frac{\partial a}{\partial\Lambda_z}\right)-1>0;$$

If

$$\left(\frac{\partial\psi}{\partial a}\right)\cdot\left(\frac{\partial a}{\partial\Lambda_c}\right)-1=0,$$

then

$$\frac{d\psi}{d\Lambda_c} = \left(\frac{\partial\psi}{\partial a}\right)\cdot\left[\left(\frac{\partial a}{\partial\Lambda_c}\right)+\left(\frac{\partial a}{\partial\Lambda_z}\right)\cdot\left(\frac{\partial\Lambda_z}{\partial\Lambda_c}\right)\right]-\left(\frac{\partial\Lambda_z}{\partial\Lambda_c}\right)-1<0$$

$\Rightarrow (\partial\Psi/\partial a)\cdot(\partial a/\partial\Lambda_c) - 1 = 0$ implies that Λ_c would be higher than in the social optimum.

Thus, when the central bank takes into account indirect effects, its optimisation condition 3g.* implies $(\partial\Psi/\partial a)\cdot(\partial a/\partial\Lambda_c) - 1 > 0$.

Therefore, it would be optimal to restrict central bank investment. Analogically, if the smaller bank invests more than the socially optimal amount, the central bank should increase its investment.

QED

Appendix 4

Result 3.h:
It is optimal for the central bank to invest at least something in the system with any value of G at least if $\alpha = 1$ and $\partial^2 a / \partial\Lambda_1^2$ is close to zero.

Proof

The impact of central bank investment on social welfare is positive when:

$$\frac{d\psi}{d\Lambda_c} = \left(\frac{\partial\psi}{\partial a}\right)\cdot\left[\left(\frac{\partial a}{\partial\Lambda_c}\right)+\left(\frac{\partial a}{\partial\Lambda_1}\right)\cdot\left(\frac{d\Lambda_1}{d\Lambda_c}\right)\right]-\left(\frac{d\Lambda_1}{d\Lambda_c}\right)-1>0$$

which implies

$$\left(\frac{\partial\psi}{\partial a}\right)\cdot\left[1+\frac{\left(\frac{\partial a}{\partial\Lambda_1}\right)\cdot\left(\frac{d\Lambda_1}{d\Lambda_c}\right)}{\frac{\partial a}{\partial\Lambda_c}}\right]-\left\{\frac{\left(\frac{d\Lambda_1}{d\Lambda_c}\right)}{\left(\frac{\partial a}{\partial\Lambda_c}\right)}\right\}-\frac{1}{\left(\frac{\partial a}{\partial\Lambda_c}\right)}>0 \qquad \text{(3h.*)}$$

The bank 1 optimises $\partial\Pi_1/\partial\Lambda_1 - 1 = (ds/da)(\partial a/\partial\Lambda_1) - 1 = 0$. Implicit differentiation gives

$$\frac{d\Lambda_1}{d\Lambda_c} = -n\frac{\left(\frac{d^2 s}{da^2}\right)\cdot\left(\frac{\partial a}{\partial\Lambda_c}\right)\cdot\left(\frac{\partial a}{\partial\Lambda_1}\right)+\left(\frac{ds}{da}\right)\cdot\left(\frac{\partial^2 a}{\partial\Lambda_1\partial\Lambda_c}\right)}{\frac{\partial^2\Pi_1}{\partial\Lambda_1^2}}$$

If central bank investment is close to zero, then

$$\lim_{\Lambda_c\to 0}\left(\frac{\partial a}{\partial\Lambda_c}\right)=\infty$$

Division by $(\partial a/\partial\Lambda_c)$ gives

$$\frac{\left(\frac{d\Lambda_1}{d\Lambda_c}\right)}{\left(\frac{\partial a}{\partial\Lambda_c}\right)} = -n\frac{\left\{\left(\frac{d^2 s}{da^2}\right)\cdot\left(\frac{\partial a}{\partial\Lambda_1}\right)-\frac{\left[\left(\frac{ds}{da}\right)\cdot\left(\frac{\partial^2 a}{\partial\Lambda_1\partial\Lambda_c}\right)\right]}{\left(\frac{\partial a}{\partial\Lambda_c}\right)}\right\}}{\frac{\partial^2\Pi_1}{\partial\Lambda_1^2}} \qquad \text{(3h.**)}$$

which reduces to

$$\frac{\left(\frac{\partial \Lambda_1}{\partial \Lambda_c}\right)}{\left(\frac{\partial a}{\partial \Lambda_c}\right)} = -n\frac{\left\{\left(\frac{d^2 s}{da^2}\right)\cdot\left(\frac{\partial a}{\partial \Lambda_1}\right)\right\}}{\frac{\partial^2 \Pi_1}{\partial \Lambda_1^2}}$$

⇒ If central bank investment is close to zero, then (3h.*) can be rewritten as

$$\left(\frac{\partial \psi}{\partial a}\right)\cdot\left[1-\left(\frac{\partial a}{\partial \Lambda_1}\right)\cdot n\left\{\frac{\left(\frac{d^2 s}{da^2}\right)\cdot\left(\frac{\partial a}{\partial \Lambda_1}\right)}{\left(\frac{\partial^2 \Pi_1}{\partial \Lambda_1^2}\right)}\right\}\right]+n\left\{\frac{\left(\frac{d^2 s}{da^2}\right)\cdot\left(\frac{\partial a}{\partial \Lambda_1}\right)}{\left(\frac{\partial^2 \Pi_1}{\partial \Lambda_1^2}\right)}\right\} \qquad \text{(3h.***)}$$

Bank 1 optimisation implies $da/d\Lambda_1 = 1/(d\Pi_1/da) = -a^2/(G\cdot n\cdot\alpha)$.
On the other hand, $(\partial\Psi/\partial a) = n(½ - 2G^2/a^3)$, and
$\partial^2\Pi_1/\partial\Lambda_1^2 = [\alpha Gn\cdot(2\cdot(\partial a/\partial\Lambda_1)^2 - a\cdot(\partial^2 a/\partial\Lambda_1^2))]/(a^3)$

⇒ (3h.***) can then be rewritten as

$$n\cdot\frac{-2a^6(\alpha-1)+4a^4\alpha G-4\alpha^3\left(\frac{\partial^2 a}{\partial\Lambda_1^2}\right)G^4 n^2+a^3G^2\left[8\alpha-8+\alpha^3\left(\frac{\partial^2 a}{\partial\Lambda_1^2}\right)n^2\right]}{-4a^6\alpha+2a^3\alpha^3\left(\frac{\partial^2 a}{\partial\Lambda_1^2}\right)G^2n^2}$$

When $\partial^2 a/\partial\Lambda_1^2 = 0$ and $\alpha = 1$, this reduces to $n(-G/a^2)$; because $G < 0$, this is positive.

It follows that $d\psi/d\Lambda_c > 0$, and therefore the central bank should invest if $G < 0$.

QED

Appendix 5

Result 5.g:
Iff $\partial\Lambda_z/\partial\Lambda_c < 0$, then $d\Lambda_z/d\Lambda_c < 0$ in any stable subgame perfect equilibrium; and iff $\partial\Lambda_z/\partial\Lambda_c > 0$, then $d\Lambda_z/d\Lambda_c > 0$

Proof

Both banks optimise their investments according to the reaction function Λ.
The optimal investment depends on three factors
1) central bank investment
2) rival investment
3) the general popularity of the bank, G for the bank 1 and –G for the bank 2

The equilibrium of the investment stage is characterised by the following two equations.

$\Lambda_1 - \Lambda_1(\Lambda_c, \Lambda_2, G) = 0$; $\Lambda_2 - \Lambda_2(\Lambda_c, \Lambda_1, -G) = 0$;

$$|J| = \begin{vmatrix} 1 & -\dfrac{\partial\Lambda_1}{\partial\Lambda_2} \\ -\dfrac{\partial\Lambda_2}{\partial\Lambda_c} & 1 \end{vmatrix} = 1 - \left(\frac{\partial\Lambda_1}{\partial\Lambda_2}\right)\left(\frac{\partial\Lambda_2}{\partial\Lambda_1}\right)$$

Using Cramer's rule, one obtains

$$\frac{d\Lambda_1}{d\Lambda_c} = -\frac{\begin{vmatrix} -\dfrac{\partial\Lambda_1}{\partial\Lambda_c} & -\dfrac{\partial\Lambda_1}{\partial\Lambda_2} \\ -\dfrac{\partial\Lambda_2}{\partial\Lambda_c} & 1 \end{vmatrix}}{|J|} = \frac{\left\{\dfrac{\partial\Lambda_1}{\partial\Lambda_c} + \left(\dfrac{\partial\Lambda_1}{\partial\Lambda_2}\right)\left(\dfrac{\partial\Lambda_2}{\partial\Lambda_c}\right)\right\}}{|J|}$$

Result 5.c and the assumptions concerning the a-function imply $\partial\Lambda_1/\partial\Lambda_2 = \partial\Lambda_2/\partial\Lambda_1$. Unless $|\partial\Lambda_1/\partial\Lambda_2| = |\partial\Lambda_2/\partial\Lambda_1| < 1$, the equilibrium is not stable, since when $|\partial\Lambda_2/\partial\Lambda_1| \geq 1$, it follows that $1 - (\partial\Lambda_1/\partial\Lambda_2)(\partial\Lambda_2/\partial\Lambda_1) \leq 0$.

$\{\partial\Lambda_1/\partial\Lambda_c + (\partial\Lambda_1/\partial\Lambda_2)(\partial\Lambda_2/\partial\Lambda_c)\} = (\partial\Lambda_2/\partial\Lambda_c)[1 + (\partial\Lambda_1/\partial\Lambda_2)]$ is negative (positive) iff $(\partial\Lambda_2/\partial\Lambda_c)$ is negative (positive). Consequently, $d\Lambda_1/d\Lambda_c$ is negative (positive) iff $\partial\Lambda_1/\partial\Lambda_c$ is negative (positive).

QED

Appendix 6

Result 5.i:
It is optimal for the central bank to invest nothing in developing the system, iff |G| ≥ 3a/(2·√[2 + 1/a]). If G = 0, it is optimal to invest.

Proof

$$\frac{da}{d\Lambda_c} = \frac{\partial a}{\partial \Lambda_c} + \left(\frac{\partial a}{\partial \Lambda_1}\right)\cdot\left(\frac{d\Lambda_1}{d\Lambda_c}\right) + \left(\frac{\partial a}{\partial \Lambda_2}\right)\cdot\left(\frac{d\Lambda_2}{d\Lambda_c}\right)$$
$$= \frac{\partial a}{\partial \Lambda_c} + 2\cdot\left(\frac{\partial a}{\partial \Lambda_1}\right)\cdot\left(\frac{d\Lambda_1}{d\Lambda_c}\right)$$

Where

$$\frac{d\Lambda_1}{d\Lambda_c} = \frac{\left\{\frac{\partial \Lambda_1}{\partial \Lambda_c}\left[1+\left(\frac{\partial \Lambda_1}{\partial \Lambda_2}\right)\right]\right\}}{\left\{1-\left(\frac{\partial \Lambda_1}{\partial \Lambda_2}\right)^2\right\}} = \frac{\left(\frac{\partial \Lambda_1}{\partial \Lambda_c}\right)}{\left\{1-\left(\frac{\partial \Lambda_1}{\partial \Lambda_2}\right)\right\}}$$

(see appendix 5)

The effect on welfare is

$$\frac{d\psi}{d\Lambda_c} = \frac{\partial\psi}{\partial a}\left[\frac{da}{d\Lambda_c}\right] - 1 - 2\cdot\frac{d\Lambda_1}{d\Lambda_c}$$
$$= \frac{\partial\psi}{\partial a}\left[\frac{\partial a}{\partial \Lambda_c} + \frac{2\cdot\left(\frac{\partial a}{\partial \Lambda_1}\right)\cdot\left(\frac{\partial \Lambda_1}{\partial \Lambda_c}\right)}{\left\{1-\left(\frac{\partial \Lambda_1}{\partial \Lambda_2}\right)\right\}}\right] - 1 - \frac{2\left(\frac{\partial \Lambda_1}{\partial \Lambda_c}\right)}{\left\{1-\left(\frac{\partial \Lambda_1}{\partial \Lambda_2}\right)\right\}} \qquad (5i.*)$$

When $\Lambda_c \approx 0$, $\partial\Lambda_1/\partial\Lambda_c > 0$ (Result 5.h)

When $|G| = 3a/(2\cdot\sqrt{[2 + 1/a]})$, $d\psi/d\Lambda_c = -1 - 2(\partial\Lambda_1/\partial\Lambda_c)/\{1 - (\partial\Lambda_1/\partial\Lambda_2)\} < 0$

If $|G| > 3a/(2\cdot\sqrt{[2 + 1/a]})$, then the impact of central bank investment on welfare becomes even more negative, because
$d\psi/da[\partial a/\partial\Lambda_c + 2\cdot(\partial a/\partial\Lambda_1\}\cdot(\partial\Lambda_1/\partial\Lambda_c)/\{1 - (\partial\Lambda_1/\partial\Lambda_2)\}] < 0$

⇒ Increasing central bank investment above zero is not optimal if $|G| \geq 3a/(2\cdot\sqrt{[2 + 1/a]}$.

If G = 0, then according to result 5.e, $(d\psi/da)\cdot(\partial a/\partial \Lambda_1) - 1 = 0$, and 5i.* reduces to $\partial\psi/\partial a[\partial a/\partial \Lambda_c] - 1$.

When $\Lambda_c = 0$, $\partial a/\partial \Lambda_c = \infty$. Because $\partial\psi/\partial a > 0$, $\partial\psi/\partial a[\partial a/\partial \Lambda_c] - 1 > 0$.
Thus, it cannot be optimal not to invest when G = 0.

QED

Appendix 7

Result 5.n:
If investment by bank z (z = 1, 2) is greater than zero, the impact of central bank investment on private investment, Λ_z, is negative at least if either $\Lambda_c \approx 0$ or $\partial^2 a/\partial\Lambda_z\partial\Lambda_c \leq 0$.

Proof

The f.o.c. of bank 1 is $d\Pi_1/d\Lambda_c = 1$

Implicit differentiation gives

$$\frac{d\Lambda_1}{d\Lambda_c} = -\frac{\left[\dfrac{\partial^2\Pi_1}{\partial\Lambda_1\partial\Lambda_c}\right]}{\left[\dfrac{\partial^2\Pi_1}{\partial\Lambda_1^2}\right]} = \frac{2\left(\dfrac{\partial a}{\partial\Lambda_c}\right)\cdot\left(\dfrac{\partial a}{\partial\Lambda_1}\right) - a\left(\dfrac{\partial^2 a}{\partial\Lambda_1\partial\Lambda_c}\right)}{-2\left(\dfrac{\partial a}{\partial\Lambda_1}\right)^2 + a\left(\dfrac{\partial^2 a}{\partial\Lambda_1^2}\right)}$$

When either $\Lambda_c = 0$ (and consequently $\partial a/\partial\Lambda_c = \infty$) or $\partial^2 a/\partial\Lambda_1\partial\Lambda_2 \leq 0$, $d\Lambda_1/d\Lambda_c < 0$

Bank 2 f.o.c is $d\Pi_2/d\Lambda_2 - 1 = 0$

$$\frac{d\Lambda_2}{d\Lambda_c} = -\frac{\left[\dfrac{\partial^2\Pi_2}{\partial\Lambda_2 d\Lambda_c}\right]}{\left[\dfrac{\partial^2\Pi_2}{\partial\Lambda_2^2}\right]}$$

$$= \frac{2(\alpha - 2G)^2\cdot\left(\dfrac{\partial a}{\partial\Lambda_2}\right)\left(\dfrac{\partial a}{\partial\Lambda_c}\right) + a\left[+a^2 - (\alpha - 2G)^2\right]\left(\dfrac{\partial^2 a}{\partial\Lambda_c\partial\Lambda_2}\right)}{-2(\alpha - 2G)^2\cdot\left(\dfrac{\partial a}{\partial\Lambda_2}\right)^2 - a\left\{a^2 - (\alpha - 2G)^2\left(\dfrac{\partial^2 a}{\partial\Lambda_2^2}\right)\right\}} \quad (5n.*)$$

Because $G < 0$, $s = (3a - \alpha + 2G)/(4a) < 1/2$, which implies $3a - \alpha + 2G < 2a$, which in turn implies $2G - \alpha < -a$. Because $-a < 0$, $2G - \alpha < 0$.

It follows that $|2G - \alpha| > |a| \Rightarrow + a^2 - (\alpha - 2G)^2 < 0$

This, in turn, implies
$-2(\alpha - 2G)^2\cdot(\partial a/\partial\Lambda_2)^2 - a\{a^2 - (\alpha - 2G)^2\}(\partial^2 a/\partial\Lambda_2^2)$

$d\Lambda_2/d\Lambda_c$ is negative when the numerator of 5n.* is positive. This is true at least when either $\Lambda_c = 0$ or $\partial^2 a/\partial\Lambda_2\partial\Lambda_c \leq 0$ or both.

QED

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