

Stock Market Bubbles, Inflation and Investment Risk

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This paper proposes an autoregressive regime-switching model of stock price dynamics in which the process creates pricing bubbles in one regime while error-correction prevails in the other. In the bubble regime the stock price depends negatively on inflation. In the error-correction regime it depends on the price-dividend ratio. We find that the probability of regime-switch depends on exogenous inflation and lagged price. The model is consistent with Shleifer and Vishny's theoretical noise trader and arbitrageur model and Modigliani's inflation illusion phenomenon. The results emphasize the importance of inflation and the price-dividend ratio when assessing investment risk.

KEYWORDS. Dividend-Price Ratio, LMARX Model, Stock Return Distribution, Time Varying Risk, Time-Varying Expected Returns. JEL: C51,C52, G11,G12

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1. Introduction

A bubble is a well-known empirical phenomenon in stock markets, but there is no consensus about the mechanisms behind it. When a pricing bubble appears, prices rise rapidly, making the listed stocks substantially overvalued. For instance the Japanese stock market development in the late 1980s and the US market in 1929 and 1999 - 2000 are usually described as bubbles. The bull and bear markets of the twentieth century have suggested that large stock market swings reflect irrational behavior (see e.g. Barsky and De Long, 1990). Generally a bubble is followed by a crash. A characterization of large crash on the stock market is that its financial impact is considerable, hence, by definition, bubbles and crashes are of profound importance to risk management of investment portfolios.

The aim of this paper is to study stock market pricing bubbles econometrically, and their relation to inflation. The idea to analyze the effect of inflation comes from several studies suggesting that inflation correlates negatively with the price-earnings ratio (e.g., Ritter and Warr, 2002), although the real dividend growth rate does not depend significantly on inflation. In addition, Shiller (1997) has studied public attitudes toward inflation using a survey method. He concluded that people associate high inflation rate with economic disarray and lower purchasing power.

Here we empirically investigate bubbles and crashes using an autoregressive regime-switching technique. We suggest a model where in one regime the price depends on a fundamental factor, the price-dividend ratio, and in the other (bubble) regime, the price is independent of fundamentals. The probability of a crash depends through a probit function on the change in stock price and on (exogenous) inflation.

Previously, van Norden & Shaller (1999) have studied bubbles using a regime switching technique according to which the probability of a crash depends positively on the price-dividend ratio. Also Maheu and McCurdy (2000) have characterized stock market dynamics by a model that

incorporates duration dependence to capture bull and bear markets. In this model, the regime-switching mechanism is Markovian and the probability that the bubble survives depends on the duration of the bubble period. Rahbek and Shephard (2002) have employed a similar type of model for the exchange rate so that the process follows a random walk in one regime while it is a mean-reverting autoregressive process in the other regime. Hess (2003) has recently evaluated competing Markov regime-switching model setups for the Swiss stock market. He finds that the stochastic movement is optimally tracked by time varying first and second moments including memory effects.

We utilize another regime-switching model - a logistic mixture autoregressive model proposed by Wong and Li (2001), where the probability of a regime is a logistic function of some observable variables. The model of Wong and Li has desirable properties for risk analysis: it is relatively simple and it generates a wider range of shape-changing predictive distributions than the other methods.

The proposed econometric model can be interpreted in the context of the behavioral model of Shleifer and Vishny (1997), which implies that the stock market is not fully efficient and prices are not completely determined by rational expectations of forthcoming cash flows. In Shleifer and Vishny's model there are two kinds of investors: noise traders and arbitrageurs. Noise traders are individuals who have erroneous beliefs about the future returns of risky assets. They react to noisy information, which is irrelevant to future cash flows. Arbitrageurs are investors who have rational expectations about the future cash flows of risky assets and try to exploit the noise traders' incorrect beliefs.

Black (1986) posits that noise trading will be cumulative, leading asset prices to deviate even more from their fundamental values, which after some point in time will induce arbitrageurs to become more active in the market. In the model of Shleifer and Vishny (1997) rational arbitrageurs cannot fully eliminate the influence of irrational investors on the market prices because resources of

arbitrageurs are limited due to risk aversion, short horizon and an agency problem. However, it is not necessarily true that bubbles exist when noise traders are present in the market. That is, the existence of noise traders simply implies that prices are more volatile and is not sufficient enough to prove that bubbles exist. In fact, bubbles are a subset of market noise where noise traders dominate.

An assumption of van Norden & Shaller (1999) that the probability of bubble collapse increases with the size of the bubble contradicts the model by Shleifer and Vishny (1997), which claims that resources of arbitrageurs are most limited when mispricing is largest. Abreu and Brunnermeier (2003) argue that timing the bubble is a difficult task for arbitrageurs. Since a single trader alone cannot burst the bubble, he faces the following trade-off: if he attacks the bubble too early, he forgoes profits from the subsequent run-up caused by momentum traders; if he attacks too late, he will suffer from the subsequent crash. Brunnermeier and Nagel (2004) question the efficient markets notion that rational speculators always stabilize prices. Their empirical results are consistent with models in which rational investors may prefer to ride bubbles because of predictable investor sentiment and limits to arbitrage.

In our model the bubble regime can be interpreted as one where the stock market is dominated by noise traders. We assume that noise traders are investors who have a nonrational attitude toward inflation and do not rely on the relationship between dividend and stock price. The error-correction regime can be interpreted as being dominated by arbitrageurs. The assumption of arbitrageur dominance contradicts a common explanation of bubbles as being caused by lack common knowledge of rationality and ensuing speculation (Smith, Suchanek and Williams (1988)). Our assumption is consistent with an experimental study of Lei, Noussair and Plott (2001), in which it was shown that bubbles and crashes also appear in nonspeculative market, where some participants behave irrationally.

The paper is organized as follows. Section 2 discusses the irrationality of bubbles. Section 3 introduces the bubble model, Section 4 presents and section 5 discusses the empirical results. The final section concludes.

2. Bubbles – rational or not?

Economists use different definitions of a bubble. The common element is that asset prices increase at a rate that is greater than can be explained by market fundamentals. According to the standard valuation model, the asset price is the present value of the stream of dividends that the owner expects to receive. It is then implicitly assumed that dividends and prices are cointegrated (e.g., Campbell and Shiller, 1988). There is compelling empirical evidence against the rational valuation model. Empirical studies by Shiller (1981) and West (1988) imply that with respect to the standard Efficient Market Model stock price volatility is too large in comparison to the volatility of dividends. Experimental studies of bubbles are made in e.g., Smith, Suchanek and Williams (1988), and Lei, Noussair and Plott (2001).

There are also rational models for stock market bubbles (see e.g. Blanchard and Watson (1982)). A characteristic property of the rational bubbles is that the price/dividend –ratio has a unit root. There is some empirical evidence supporting this hypothesis. First, standard unit root tests usually can not reject the null hypothesis that the log price/dividend –ratio has a unit root. Second, Maheu and McCurdy (2000) have found evidence that partially support the rational bubble hypothesis and that the probability that bubble collapses is a decreasing function of the duration of bubble, which is a typical property for many rational bubble models.

However, there are lots of theoretical and empirical arguments against rational bubbles (Campbell Lo and MacKinley (1997) pp. 258 – 260; Santos and Woodford (1997)). Santos and Woodford

have presented theoretical results that rule out the possibility of rational bubbles under very general conditions implying that the price-dividend ratio can not have a unit root.

We believe that a more plausible explanation for the fact that the stock prices deviating from fundamentals is caused by irrational noise traders (Shleifer and Vishny (1998)). Under this explanation the price/dividend –ratio is a slowly mean reverting process. Maheu and McCurdy (2000) have pointed out that the negative duration dependence, which is characteristic for rational bubbles, is consistent with this model.

3. The bubble model

We propose to model the bubble phenomenon using an autoregressive two-regime model, where in one regime the change in the log stock price does not depend on fundamentals and in the other it depends on the log price-dividend ratio. The latter regime assumes, like standard stock valuation models, that log returns are determined by a linear co-integration relationship between dividends and prices.

As in Shleifer and Vishny (1997) the latter regime is dominated by rational arbitrageurs. In practice they are professional portfolio managers who face short horizons because their performance is evaluated at regular and short intervals. An investment strategy exploiting mispricing, due to noise traders, can be very risky, because noise traders' beliefs can change to become even more extreme in the short run and an arbitrageur has to liquidate his position before the stock price changes towards its equilibrium. According to the Shleifer and Vishny model the resources of arbitrageurs are most limited when mispricing is largest.

McMillan (2004) has investigated nonlinear relationships in UK stock market returns. He has found evidence for the Shleifer and Vishny model (1997), suggesting that the mean reversion of stock return is weakest when the deviation from equilibrium of the price-dividend ratio is large.

Due to irrationality of noise traders, the stock price is almost unpredictable in the short run, consequently there is no easy and quick way to make large profits in the stock market at low risk. This assumption imposes some restrictions on the model. A regime-switch can cause a level shift in the stock price, and hence the regime indicator variable, s_t , has to be unobservable. Otherwise this switch would provide an easy profit opportunity to an informed investor. This restriction excludes all threshold models (e.g., Tong, 1990) where the regime is determined by some observable variables.

Several empirical papers (see e.g. Lo and MacKinlay (1999) and the references therein) have found evidence of weak positive autocorrelation of short run stock index return. In order to control the effect of this autocorrelation to estimation results we have assumed in our model that the log price change depends also its lagged value.

To summarize, we propose a bubble model where in one regime ($s_t = 0$) the change in the log stock price, Δp_t , is independent of fundamentals but depends on inflation, $\Delta\pi_t$, ($\epsilon_t \sim \text{NIID}(0,1)$)

$$\Delta p_t = \alpha_1 + \beta \Delta p_{t-1} + c_1 \Delta \pi_t + \sigma_1 \epsilon_t \quad (\text{Bubble regime}) \quad (1)$$

and in the other regime ($s_t = 1$), the change in the log stock price depends on the log price-dividend ratio, $p_{t-1} - d_{t-1}$,

$$\Delta p_t = \alpha_2 + \beta \Delta p_{t-1} + \theta(p_{t-1} - d_{t-1}) + c_2 \Delta \pi_t + \sigma_2 \epsilon_t \quad (\text{Error-correction regime}) \quad (2)$$

The latter regime assumes, like standard stock valuation models, that log returns are determined by a linear co-integration relationship between dividends and prices.

When seeking for the regime-switching mechanism we build on the following observations. Modigliani and Cohn (1979) have advanced a hypothesis that people suffer from the so-called "inflation illusion". According to this hypothesis, market participants discount real dividends by a nominal interest rate, which has a strong dependence on inflation. Recently, Ritter and Warr (2002) have found evidence supporting that hypothesis. Secondly, Sharpe (2002) suggests that market expectations of real earnings growth are negatively related to expected inflation. These studies imply that inflation is negatively related to the log dividend-price ratio, and that a low inflation rate has a tendency to create optimistic expectations, or outright bubble behavior.

These studies are compatible with Shiller's (1997) survey-based study on the public attitude toward inflation. People dislike high inflation much more than it is rationally anticipated. There is significant difference between answers of economists and non-economists: non-economists' attitude toward inflation is much more negative. Non-economists associate a high inflation rate with economic disarray and lower purchasing power. A low inflation rate is associated with economic prosperity and social justice.

Here we assume that differences between economists and non-economists also characterize differences between noise traders and arbitrageurs. Under this assumption, Shiller's (1997) findings imply that noise traders' participation in the stock market is higher during periods of low inflation. Consequently, inflation would be a suitable explanatory regime switch variable. We also assume that noise traders' participation in the stock market is more influential during periods of fast rising stock prices. This is modeled by letting the switching probability also depend on the lagged stock price change.

Realistically, regime switching should be characterized by uncertainty, best modeled as a stochastic process. A final requirement is that there exists a working estimation method for the model. Here we assume that the probability of the error-correction regime ($s_t = 1$) is given by a function of past and present inflation observations and lagged change of stock price

$$Prob(s_t = 1) = \Phi(\varphi_1 + \varphi_2(\pi_t - \pi_{t-12})^2 + \varphi_3 \Delta p_{t-1}) \quad (3)$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function. In this form the model is a logistic mixture autoregressive model with an exogenous variable (LMARX model of Wong and Li, 2001), where the probability of a regime is a direct function of observable variables. A generalization of this model would be a Markov Regime Switching model with time-varying transition probabilities (e.g., Diebold, Lee and Weinbach, 1994; Filardo, 1994), where the regime indicator variable would follow a Markov process. The proposed bubble model is a special case of this more general model when the rows of the time-varying transition matrix are the same.

Our model is close to Psaradakis, Sola and Spagnolo (2004) Markov error-correction model where stock prices and dividends are cointegrated in one regime and move independently in the other regime. That model is based on a standard present value model of the stock price which implies that if dividends follow geometric random walk then the fundamental value of stock is proportional to current dividends. Psaradakis, Sola and Spagnolo (2004) have used untransformed variables in their statistical analysis and we have used logarithmic transformation of stock price and dividends. Our analysis is based on a well-known log-linear approximation of a standard present value model which is advanced by Campbell and Shiller (1988). In this approach log price dividend ratio can be represented as a function of expected future growth rate of dividends and stock returns under very general conditions.

Like Psaradakis, Sola and Spagnolo (2004) we assume that the fundamental value of the stock price is proportional to current dividend. This is consistent with an assumption that the fundamental value of log price-dividend ratio is constant. Psaradakis et al. assume a linear cointegration relationship between stock price and dividends in the error-correcting regime. We assume log-linear cointegration relationship in the error-correcting regime.

4. Empirical Results

The data which we use is available on the home page of Shiller (<http://www.econ.yale.edu/~shiller/>). The stock price and dividend indices are value weighted indices of the 500 largest companies of the USA. The inflation rate is calculated from the Consumer Price Index of the USA. The data set consists of monthly stock price, dividends and the consumer price index. Monthly dividend data are computed from the S&P four-quarter tools for the quarter with linear interpolation to monthly figures. Stock price data are monthly averages of daily closing prices.

4.1. Estimation and Testing

Standard numerical maximum likelihood estimation techniques are used to produce the results reported in Table 1. Note that inflation has statistically significant influence on the change of stock prices only in the bubble regime. An influence of inflation is strongly significant and the regression coefficient of inflation does not deviate significantly from number -1 . The estimated model can be written as ($\epsilon_t \sim \text{NIID}(0,1)$)

$$\Delta p_t = 0.011 + 0.146\Delta p_{t-1} - \Delta \pi_t + 0.026\epsilon_t \quad (\text{Bubble}) \quad (4)$$

$$\Delta p_t = 0.201 + 0.146\Delta p_{t-1} - 0.066(p_{t-1} - d_{t-1}) + 0.047 \epsilon_t \quad (\text{Error-correction}) \quad (5)$$

The probability of being in the error-correction regime is determined

$$Prob(s_t = 1) = \Phi(-1.64 + 243.3(\pi_t - \pi_{t-12})^2 - 12.4\Delta p_t) \quad (6)$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function. The influence of price/dividend ratio on the regime-switching probability is not statistically significant. This is consistent with an assumption (Shleifer and Vishny, 1997) that the resources of arbitrageurs are most limited in a situation where mispricing is largest.

According to our estimation results the log price change is weakly positively autocorrelated in both regimes, while this effect is statistically strongly significant we do not think that it provides for an informed investor large exploitable profit opportunities. In our model log price dividend ratio covariates positively with past stock price change which implies that rapid increase of the stock price is connected with stock market crashes. Thus, we suggest that exploitable profit opportunities which are caused by weak positive autocorrelation can appear only over short time period and these profit opportunities are quite marginal after controlling transaction costs.

The model has a natural interpretation. In the noise trader (bubble) regime, the change in stock price does not depend on fundamentals and is negatively related to inflation. Thus, noisy traders do not understand the relationship between stock price and present value of expected dividends. They have negative attitude toward inflation. In the error correction regime where the stock market is dominated by rational arbitrageurs the change in stock price does not depend on inflation but on fundamentals. Thus, arbitrageurs are not subject on the inflation illusion.

In order to test the sufficiency of the model we used a likelihood-ratio test against an alternative that the change in the log stock price depends on the log price-dividend ratio in both regimes. The null hypothesis of the simpler model (4-5) could not be rejected at the 5% level.

One problem, which affects the statistical inference of the log price-dividend ratio, is share repurchases. Liang and Sharpe (1999) have studied the effect of share repurchases and emission. Their sample includes the 144 largest firms of the S & P 500 index. They concluded that share

repurchases have a significant influence on the dividend price ratio at the end of 1990. Interestingly, parameter estimates (see Table 2) do not change substantially when the last ten years of the estimation period are dropped albeit there was also IT-bubble in that period.

4.2. Comparison with Linear Models

For the sake of comparison we model the log stock price by a linear error-correction model including the inflation rate as an explanatory variable (see Table 3):

$$\Delta p_t = \alpha + \beta \Delta p_{t-1} + \theta(p_{t-1} - d_{t-1}) - \Delta \pi_t + \sigma \varepsilon_t, \quad (7)$$

where $\varepsilon_t \sim \text{IID}(0,1)$ is a) symmetric (normal distribution), b) asymmetric (a mixture of two normal distributions). The augmented Dickey-Fuller unit root test with a constant and inflation as regressors can not reject the null of unit root in the log price-dividend ratio at the 10% level. On the other hand, inflation again is a highly significant factor with a regression coefficient of inflation not deviating significantly from -1 .

The result of unit root test contradicts the theoretical arguments presented in Santos and Woodford (1997) ruling out a unit root in the price-dividend –ratio, under very general conditions. There is also evidence (see e.g. Caner and Hansen (2001)) that Standard unit-root tests have a low power against a nonlinear stationary process. Hence, a linear error-correction model is implausible.

Unfortunately, there does not exist any straightforward way to test the null hypothesis of a linear model against two regime alternatives (e.g., Andrews and Ploberger, 1994). Instead we compared the linear models (7) and the bubble Model (4-6) using the model selection criteria AIC and BIC. The first linear alternative has a symmetric Gaussian innovation distribution. In the second case the innovation distribution is assumed to be asymmetric: a mixture of two normal distributions with

constant mixing probability. Table 4 shows that the bubble model is clearly better than the linear alternatives according to both criteria.

4.3. Model Diagnostics

For diagnostics check we studied the quantiles, v_t , of the conditional distribution. If the Bubble Model were true, v_t would be independent and approximately standard uniform. In the case of the symmetric linear model (7) the most divergent observation is from September 1946, whose studentized value is - 4.43. According to the Jarque-Bera test and the normal probability plot (see Figure 1) the empirical residuals of the symmetric linear model differ significantly from normality. The value of Jarque-Bera statistics is 137.59 with p-value lower than 10^{-12} . For the bubble model (4-6) the value of Jarque-Bera statistics is 1.73, not significant at the 5% level. These calculations show that the bubble model is much better in modeling the short-term risk of stock investment than the linear model or alternatively the estimation period includes extremely unusual observations by accident.

Quantile residuals (e.g., Dunn and Smyth, 1996), u_t , are based on the fact that the inverse normal distribution transformations of standard uniform variables, $u_t = \Phi^{-1}(v_t)$, are standard normal variables themselves. Because many regression diagnostic tests are based on the assumption of normality, it is reasonable to use quantile residuals in a diagnostic check. We tested serial autocorrelation and conditional heteroskedasticity of quantile residuals in the same way as for the linear model in Tables 5 and 6. The conclusion of these tests: there is no clear evidence of serial correlation or conditional heteroskedasticity.

5. Discussion of the Results

In the bubble model, given the prior probability, $z_t = Prob(s_t = I)$, of being in the error-correction regime, the Bayesian ex-post probability of staying in the regime is

$$z_t \varphi(\Delta p_t, \mu_{2t}, \sigma_2^2) / [(1-z_t) \varphi(\Delta p_t, \mu_{1t}, \sigma_1^2) + z_t \varphi(\Delta p_t, \mu_{2t}, \sigma_2^2)], \quad (8)$$

where $\mu_{1t} = 0.011 + 0.146\Delta p_t - \Delta \pi_t$, $\mu_{2t} = 0.201 + 0.146\Delta p_t - 0.066(p_{t-1} - d_{t-1})$ and $\varphi(x, \mu, \sigma^2)$ is the density of a $N(\mu, \sigma^2)$ -variable. These probabilities are plotted in Figure 2. It can be seen that the process is much more often in the bubble regime (82 per cent of time) than in error-correction regime. This means that most of the time the stock price does not react to the information on dividends. This is consistent with the short-run unforecastability of the stock price.

A regime switch from the bubble regime to the error-correction regime can cause a jump in the stock price. The sign and magnitude of the jump is mainly determined by the log price-dividend ratio of the previous month $p_{t-1} - d_{t-1}$. On average, the stock price grows faster than the dividend in the bubble regime. It means that the jump of the process after a switch from the bubble regime to the error-correction regime is more likely to be negative (causing a market crash) than positive. A bubble usually starts slowly, and gradually builds up to the peak over a period of several years. After it has peaked prices almost always fall abruptly.

The relationship between inflation and stock price in the bubble model is totally different from that in the linear error-correction model. In both models inflation is strongly significant. Inflation decreases probability of bubbles and the stock price is very sensitive to inflation in the bubble regime. In other words, bubbles seem to be coupled with low-inflation periods.

Another clear difference between the regime-switching model and the linear model is the way that the stock price behaves under a high-deflation (negative inflation) period. The data do not

include any high-deflation period. According to the linear model, the price-dividend ratio is high during a deflation period. Under the regime-switching model this relationship is negative, because the probability of the bubble depends on the square of inflation. It is well known that deflation is often related to general economic depression. It is unreasonable to assume that the stock price is overvalued in that situation.

A characteristic property of the bubble model is its time-varying conditional moments (see e.g. Wong and Li, 2001 for the properties of LMAR models). An important issue is what implications this could have for risk management. Time-varying higher moments indicate that a statistical analysis, which focuses only on the two lowest conditional moments, may underestimate the importance of the log price-dividend ratio. The model suggests that high values of the log price-dividend ratio precede a crash in the stock market. Fat tails and asymmetry in return distribution over the long horizon can be an indication of nonlinear bubble phenomena.

6. Conclusions

An econometric model for stock market bubbles has been proposed. It is consistent with the assumption that noise traders have a significant impact on stock prices. The proposed model has two regimes: a bubble regime and an error-correction regime. The probability to stay in the bubble regime, where noise traders dominate, depends negatively on inflation. Therefore low inflation is related to high valuation ratios and weak mean reversion on the stock market. This indicates that noise traders' participation in the stock market is higher during periods of low inflation.

The link between bubbles and low inflation is very strong: the probability of a bubble regime is negatively related to inflation and the stock price change is very sensitive to inflation in the bubble regime. This is consistent with Shiller's (1997) survey-based study of public attitudes toward inflation, which claims that low inflation is associated with positive economic expectations. We

suggest that a negative relationship between valuation ratios and inflation is mainly caused by nonrational noise traders.

The proposed model has several implications for risk management. First, it emphasizes the importance of inflation. Second, previous studies argue that the conditional expectation of stock return depends weakly on the price-dividend ratio.

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APPENDIX**Table 1.** Parameter estimates of the bubble model. Data is from 1/1946 to 10/2003.

Parameter	Estimate	STD
α_1	0.0111	0.0015
β	0.146	0.037
σ_1	0.026	0.0015
α_2	0.201	0.092
θ	-0.066	0.030
σ_2	0.047	0.0040
φ_1	-1.64	0.428
φ_2	243.33	89.83
φ_3	-12.39	4.33

Table 2. Parameter estimates of the bubble model. Data is from 1/1946 to 10/1993.

Parameter	Estimate	STD
α_1	0.0102	0.0015
β	0.170	0.043
σ_1	0.025	0.0015
α_2	0.253	0.069
θ	-0.083	0.022
σ_2	0.047	0.0033
φ_1	-1.61	0.276
φ_2	253.93	84.69
φ_3	-16.72	7.03

Table 3. Parameter estimates of the symmetric linear model.
Data is from 1/1946 to 10/2003.

Parameter	Estimate	STD
α	0.033	0.011
β	0.214	0.037
θ	-0.008	0.003
σ	0.0335	0.001

Table 4. Likelihood ratio and information criteria for bubble model and for linear models.

Model	Bubble	Symmetric linear	Asymmetric linear
Likelihood ratio	1414.9	1372.1	1400.0
AIC	-2811.8	-2736.2	-2786.0
BIC	-2770.9	-2718.0	-2754.2

Table 5. The ljung-Box Q-test for the quantile residuals of the bubble model.

Lags	1	2	3	4
Test Statistics	0.00	2.42	2.86	8.62
Critical values	3.84	5.99	9.49	18.31
p - values	0.99	0.30	0.58	0.57

Table 6. The Engel's ARCH test for the quantile residuals of the bubble model.

Lags	1	2	4	10
Test Statistics	0.20	0.21	2.88	10.76
Critical values	3.84	5.99	9.49	18.31
p - values	0.65	0.90	0.58	0.38

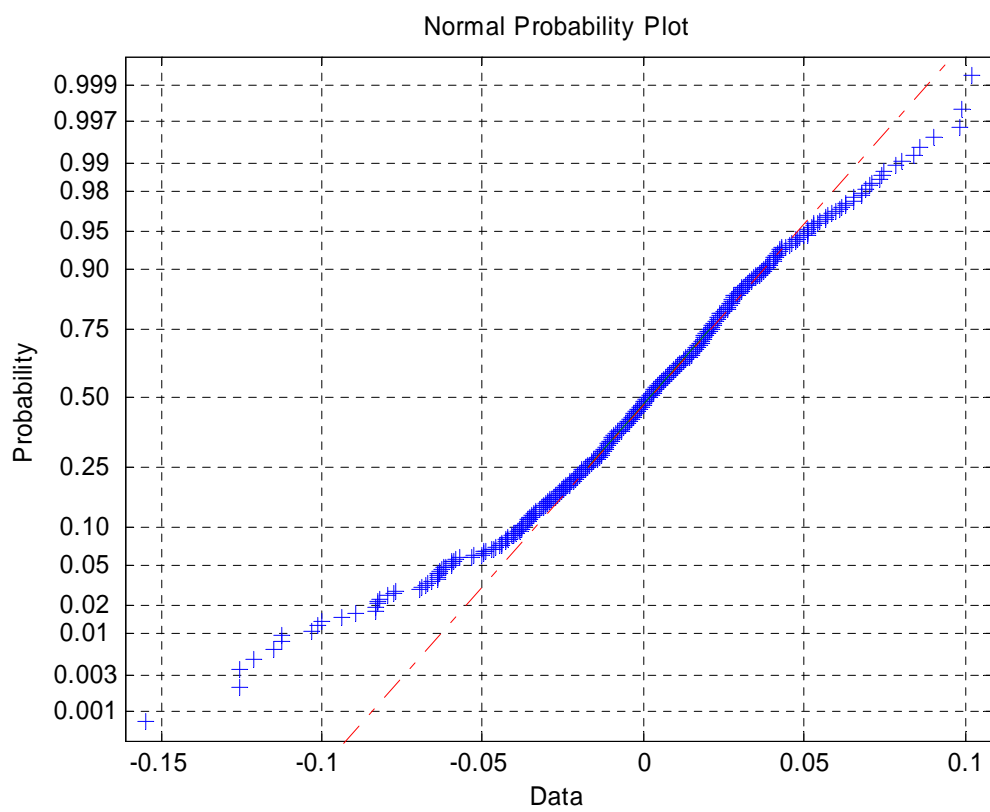


Figure 1. The normal probability plot of the symmetric linear model.

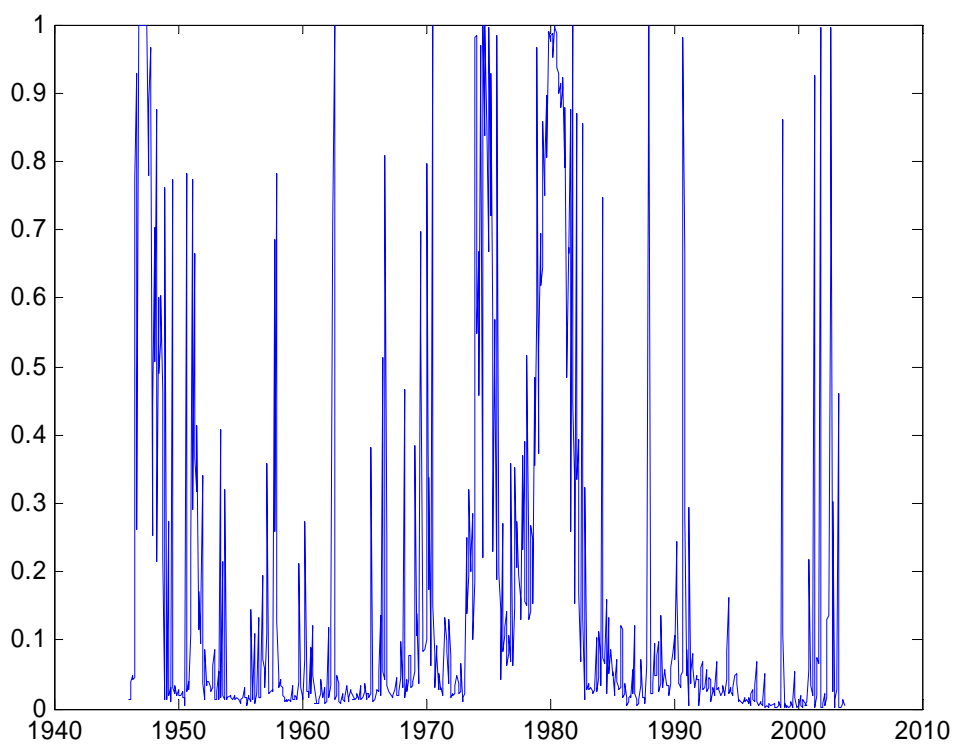


Figure 2. The ex-post probabilities to stay in the error-correction regime.