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INFLATION EXPECTATIONS UNDER LEVEL SHIFTS IN THE INFLATION PROCESS

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Inflation expectations under level shifts in the inflation process

Abstract

The aim of this paper is to study the dynamics of inflation and survey inflation expectations under sudden random level shifts in the inflation process. We suggest that the recently introduced mixture autoregressive MAR model is suitable for modelling this kind of behaviour. We arrived at a model where the inflation expectations are not fully rational in a sense that survey participants of the Livingston Survey adjust their expectations too conservatively in response to new evidence.

Keywords: Survey forecasters; conservative expectations; level shifts; MAR model

1 Introduction

The time variation of the short real interest rate plays a central role in long-term investment management and many asset pricing models. According to Campbell & Viceira (2001), time variation in the short-term real interest rate creates a particularly important source of investment opportunities for a long-term investor. They argue that short-term investments become risky, because they have to be rolled over in the future at an uncertain real interest rate. The time-variation of the ex-poste short-term interest rate is depend on two components: the ex-ante real interest rate and the forecast error of inflation.

In order to understand the time-variation of the ex-poste interest rate, is important to study dynamics of these components. In this paper we focus on the latter component, using the Livingston survey on inflation forecasts. An important question is how expectations deviates from model-consistent rational expectations.

A characteristic property of fluctuation of the ex-poste short real interest rate are infrequent structural shifts in both mean and variance which cannot incorporated into a linear model with constant parameters. Garcia & Perron (1996) have found evidence that the U.S ex-poste real interest rate has undergone three level shifts under the post-war era. The timing of these level shifts coincided with level shifts of the inflation rate.

Furthermore, Rapach & Wohar (2005), using Bai & Perron (2003) methodology, have found evidence of structural breaks in the mean real interest rate for 13 industrialized countries. According to their results the timing of the break in the real interest rate often coincides with breaks in the inflation rate, thus an upward (downward) level shift in the inflation process coincides with a downward (upward) level shift in the ex-poste real interest rate.

An explanation for these observations is that economic agents under(over)predict inflation under time periods with upward(downward) level shift in the inflation process. There is survey-based empirical evidence which is supportive for this interpretation. According to theses studies, survey inflation expectations are not perfectly rational (e.g. DeBondt & Bange (1992), Roberts (1998) or Thomas (1999)). These studies claim that survey participants adjust their forecasts too slowly in response to new information and inflation forecast errors are significantly positively autocorrelated. These findings are consistent with experimental studies that people adjust their existing beliefs too slowly with respect to new information (Phillips & Edwards (1966) or Beach et al. (1994)).

However, autocorrelated forecast errors can appear over the small

sample in spite of rational expectations. Evans & Lewis (1995) have suggested that serially correlated forecast errors may result from the so-called "peso problem". According to the peso problem explanation forecast errors may be serially correlated over periods when rationally anticipated infrequent shift in the inflation process does not materialize.

The main objective of the paper is to investigate how forecasting errors are related to a sudden level shift in the mean of the inflation rate under slowly adjusted expectations, which are not perfectly rational, using the Livingston survey on inflation forecasts. There are empirical results which suggest that the U.S inflation process has undergone structural level shifts during the post-war period (Evans & Lewis (1995) or Garcia & Perron (1996)), which cannot be incorporated into a linear model with constant parameters. Rapach & Wohar (2005) have also found similar evidence of structural shifts in the inflation rate from other industrialized countries.

In response to these findings we consider a non-linear model for the inflation rate where level shifts occur randomly. A plausible model for this purpose is a recently introduced mixture autoregressive (MAR) model (Wong & Li (2000), (2001) or Le et al. (1996)) where probabilities of shifts are constant or a direct function of observable variables. The MAR model is a mixture of K Gaussian autoregressive models. In the first version of the MAR model Wong and Li (2000) assumed that the mixing probabilities are constant over time. Because this assumption is too restrictive in many applications, Wong & Li (2001) generalized the MAR model to a case where the mixing probabilities change over time as a logistic function of lagged values of process or some exogenous variables. In this paper the MAR model always means this general model, which Wong & Li (2001) called the LMARX model.

In our model the probability of level shift is a direct function of past inflation. In the case of the inflation rate the same kind of model is previously employed by Lanne (2005). He has used a two-variate MAR model with time-varying volatility in order to investigate a long-term relationship between U.S inflation and interest rate. According his results, a MAR model seems to fit the data much better than a linear VAR model.

In the case of inflation expectation formation we have employed a so-called "stubborn" expectations model (Roberts (1998)), where inflation expectations are a weighted average of what they were in the last period and the rational expectation. The previous studies (Roberts (1998), Mankiw (2001), Mankiw & Reis (2001) and Carroll (2003)) imply that this kind of model characterizes quite well how survey participants formulate their inflation expectations. This model is consistent with an

assumption that economic agents under(over)predict inflation over time periods with upward(downward) level shift in the conditional mean in the inflation rate.

The paper is structured as follows. The data are summarized in section 2. In section 3 we discuss the inflation expectations. We study in this section how survey data are useful to measure inflation expectations and inflation uncertainty. In section 4 we present our model. Estimation results are presented in section 5. Finally section 6 concludes.

2 Data

We use the data from the Livingston Survey as the proxy for inflation expectations. The annualized semi-annual forecasts are asked from survey participants every June and December. The survey participants are professional economists. The first observation is from June 1946 and the last observation is from December 2003. This survey is available on the Philadelphia Fed's web page (<http://www.phil.frb.org>). The data contain mean and standard deviation across one-step forecasts at each time point. We denote these variables by the symbols π_t and s_t . Thus the variable π_t is a forecast for the inflation rate at time point $t + 1$. An extensive description of this survey is found in Croushore (1997).

The U.S inflation data is computed from the CPI-U (Consumer Price Index-All Urban Consumers) published by the U.S Bureau of Labor Statistics. We use the semi-annual log difference of this index multiplied by two as the annualized quarterly inflation rate. We denote this variable by the symbol i_t .

3 Model for the survey expectations

In this study we used expectations based on the Livingston survey as a measure for the inflation expectations. There is some discussion how well survey expectations characterize actual inflation expectations. Roberts's (1995,1997) results indicate that survey expectations are a good proxy for actual inflation expectations. He found that some macroeconomic models perform better when survey expectations are used in place of rational expectations.

Survey data is not only a useful measure for the expected inflation. There is also evidence that survey data provide a good proxy for inflation uncertainty (Bomberger (1996,1999) and Giordani & Söderlind (2003)). Bomberger (1996,1999) has studied the relationship between inflation uncertainty and disagreement among forecasters. He has found evidence that the variance across survey forecasters in the Livingston survey tracks inflation uncertainty better than an ARCH model. Fur-

thermore, he does not find evidence of remaining ARCH effects once the survey variance is included. Giordani & Söderlind (2003) have presented further evidence that disagreement among forecasters is a good measure for inflation uncertainty.

Furthermore, Bomberger (1996) has found that the bias and serially correlation of survey participants' forecast errors are significantly reduced when the disagreement is used to correct for heteroskedasticity. In order to avoid biased estimates for the degree of rationality of survey participants we suggest a model where the conditional standard deviation of inflation is proportional to the standard deviation of survey forecasts.

There is empirical evidence that survey expectations are not perfectly rational (e.g. Roberts (1998) or Thomas (1999)). Roberts (1998) concluded that the surveys reflect an intermediate degree of rationality. He proposed a so-called "stubborn" expectations model, where inflation expectations π_t are a weighted average of what they were in the last period π_{t-1} and the rational expectation $E_t(i_{t+1})$. This model can be written as

$$\pi_t = \alpha E_t(i_{t+1}) + (1 - \alpha)\pi_{t-1}, \quad (1)$$

where $0 \leq \alpha \leq 1$. $E_t(i_{t+1})$ is the expectation with respect to the information set Ω_t , which contains all available information at time t . The coefficient α determines the degree of rationality of expectations. If $\alpha = 1$ expectations are perfectly rational. In the other cases expectations are less than perfectly rational. Roberts (1998) found that this kind of model characterizes well both consumers' and economists' inflation expectations. Model (1) can also be expressed in the form

$$\pi_t - E_t(i_{t+1}) = -(1 - \alpha)(E_t(i_{t+1}) - E_{t-1}(i_t)) + (1 - \alpha)(\pi_{t-1} - E_{t-1}(i_t)).$$

Thus the bias of the inflation expectations $\pi_t - E_t(i_{t+1})$ is positively autocorrelated and depends negatively on the change of conditional expectations ($E_t(i_{t+1}) - E_{t-1}(i_t)$), when $\alpha < 1$. This form implies that the bias of inflation expectations is largest in times where a structural shift occurs in the conditional mean of the inflation rate. The coefficient α determines how quickly expectations adjust to the shift in the inflation process. In the special case, where the change of the conditional expectations ($E_t(i_{t+1}) - E_{t-1}(i_t)$) is zero, the inflation expectations π_t slowly converge to their rational value $E_t(i_{t+1})$ when $\alpha > 0$. This calculus implies that the inflation expectations π_t perform quite well under a stable inflation environment with slowly changing conditional expectations $E_t(i_{t+1})$ and can fail in times when a level shift occurs in the

inflation process.

There are different interpretations for this kind of expectation formation. Roberts (1998) has proposed an explanation that professional forecasters avoid adjusting their forecasts too rapidly because they might not want to differ too much from the consensus of other forecasters. Pons-Novell (2003) has found evidence for this kind of herding behaviour in the case of some groups of forecasters.

Mankiw & Reis (2001) explain this kind of behaviour assuming that individuals form their expectations rationally, but they do not do this very often due to costs of acquiring and processing information. Carroll (2003) has used a similar model in order to describe how information spreads through the population gradually via news of media.

The model is also consistent with experimental psychology which has shown that people adjust their existing beliefs in the right direction but too slowly with respect to new information (Phillips & Edwards (1966) or Beach et al. (1994)). One reason for this kind of conservatism is that forecasters are overconfident, so they have a tendency to overestimate the precision of their prior knowledge (e.g. Harvey & Harries (2004)).

There is some evidence to support this assumption. Psychological research suggests that most individuals are overconfident. There is also evidence of overconfidence from many professional fields (e.g. Harvey & Harries (2004)). An indication of overconfidence is the findings of Giordani & Söderlind (2003), which suggest that professional forecasters systematically underestimate inflation uncertainty.

4 A non-linear model for inflation

In the previous section we presented a model for aggregated inflation expectations which imply that inflation expectations are most biased in time periods with a sudden level shift in the inflation process. In this section we present a model which is consistent with level shifts which are characteristic of actual inflation. The proposed model is a non-linear MAR model for inflation with randomly occurring level shifts.

In this model the probability of an upward level shift is a direct function of past inflation. In the case of U.S inflation this kind of model is previously employed by Lanne (2005). The main difference between our and Lanne's model is the use of survey data. We suggest that survey data contains information about the probability distribution of future inflation, which cannot include past inflation.

It is reasonable to assume that survey participants also have useful information about monetary policy or other economic variables which is uncorrelated with current and past inflation. Thus the innovation term v_t of aggregated inflation expectations can be correlated with fu-

ture inflation. In the case of fully rational expectations the regression coefficient of this innovation term is one. Furthermore, the survey also contains information about the uncertainty of future inflation. Based on the results of Bomberger (1996,1999) we assume that the conditional variance of inflation is proportional to the variance of forecasters s_t in the Livingston Survey.

The proposed model for inflation i_t can be express in the form

$$\begin{aligned} i_t &= \mu_t + z_t \lambda_t + \theta s_{t-1} \varepsilon_t + \phi v_{t-1} \\ \mu_t &= \beta_0 + \sum_{j=1}^p \beta_j i_{t-j} \\ \lambda_t &= \varphi_0 + \sum_{j=1}^p \varphi_j i_{t-j} \end{aligned}$$

$$\varepsilon_t \sim NID(0, 1), \quad (2)$$

where the variable z_t is a nonobservable switching variable, whose value is one when the level shift occurs and zero otherwise. The variable is the conditional mean μ_t of the inflation rate when shifts do not occur. The variable λ_t is a magnitude of the level shift when it occurs. The magnitude of shift depends on the constant term and past inflation. The probability of shift is determined by the probit function of past inflation, thus

$$z_t \sim \text{bernoulli}(\Phi(\alpha_1 + \alpha_2 i_{t-1})),$$

where $\Phi(x)$ is the standard normal cumulative distribution function.

It is easy to see that the proposed MAR model is a generalization of the linear $AR(p)$ model. This model can be interpreted as a generalization of the self-exciting threshold autoregressive (SETAR) model (e.g. Tong (1990)), where the switching between regimes occurs whenever some observable variable crosses a certain treshold value. The MAR model can be interpreted as a SETAR model, where the treshold value is a non-observable random variable.

The proposed MAR model also has similarities with the usually employed Markov switching (MS) model (Hamilton (1989)), where the unobservable switching variable follows a discrete-state Markov chain. In the MS model Markovian dependence for the switching variable is assumed. In our model the switching probability is time-variant via the inflation rate, which is a highly autocorrelated variable. Several empirical studies use a MS model to describe the dynamics of the inflation process (e.g. Evans & Lewis (1995) or Garcia & Perron (1996)).

For our purpose an advantage of the MAR model compared to the MS model is that the conditional expectation can be written as a simple functional form of past observations. This is a very important property

because our model for expectation formation is based on the conditional expectation of the underlying process. An extension of our model is the MS model with time-varying transition probabilities (see e.g. Diebold et al. (1994) or Filardo (1994)), but we think that it is too complicated for our purpose.

5 Statistical analysis

We estimated our model over a period which includes the inflation expectations and the standard deviations of the inflation expectations from 1959:1 to 2003:1 and the inflation observations from 1959:2 to 2003:2. The number of observations is thus 89. These three time series are presented graphically in figures 1, 2 and 3. We estimated our models by the conditional maximum likelihood method, where the values before the estimation period work as fixed initial values. [Figures 1, 2 and 3]

A common problem of many non-linear models is that standard asymptotic properties of common test statistics, such as the likelihood ratio test statistics, do not hold, when a linear model is tested against the non-linear model (Andrews & Ploberger (1994)). Namely, a non-linear model usually includes nuisance parameters which are not identified under the null hypothesis of linearity. In the case of our model testing linearity is the same as testing restrictions $\varphi_j = 0$ for all j jointly. Under these restrictions parameters α_1 and α_2 are unidentified and the standard asymptotic test theory is not valid.

However, previous studies of the U.S inflation rate have exhibited time series properties which cannot be incorporated into a linear model with constant parameters (e.g. Garcia & Perron (1996)). Furthermore, by using information criteria we will show that our non-linear model fits the data much better than an alternative linear model. Diagnostic checks also indicate that our model performs quite well.

Based on the earlier sections we give a non-linear model for inflation i_t (2) and aggregated inflation expectations π_t (1)

$$\begin{aligned} i_t &= \mu_t + z_t \lambda_t + \theta s_{t-1} \varepsilon_t + \phi v_{t-1} \\ \pi_t &= \alpha E_t(i_{t+1}) + (1 - \alpha) \pi_{t-1} + v_t \\ \varepsilon_t &\sim NID(0, 1) \\ v_t &\sim N(0, \sigma_{\pi t}^2) \end{aligned}$$

and

$$z_t \sim \text{bernoulli}(\Phi(\alpha_1 + \alpha_2 i_{t-1})),$$

where $E_{t-1}(i_t)$ is the model-consistent rational expectation for inflation (rational expectation without information about the innovation term v_t) and the variable z_t is a non-observable switching variable. Due to non-linear dynamics of the MAR model the conditional expectation $E_{t-1}(i_t)$ is a non-linear function of past inflation

$$E_{t-1}(i_t) = \mu_t + \Phi(\alpha_1 + \alpha_2 i_{t-1})\lambda_t.$$

Hence, we now have a two-variate model whose both components are nonlinear. Actually, the aggregated expectations π_t follow in our model the smooth transition autoregressive (STAR) model (e.g. Granger & Teräsvirta (1994)). The proposed non-linear structure is unusual but it has relevant interpretation when the switching variable is assumed to be nonobservable and aggregated expectations are less-than-perfectly rational.

Using a standard numerical estimation technique (further details in Appendix A) we arrived at the following model with standard estimation errors in brackets

$$\begin{aligned} i_t &= \mu_t + z_t \lambda_t + 1.091 s_{t-1} \varepsilon_t + v_{t-1} \\ &\quad (0.106) \\ \mu_t &= 1.214 + 0.464 i_{t-2} \\ &\quad (0.222) \quad (0.062) \\ \lambda_t &= 3.458 + 0.660(i_{t-1} - i_{t-3}) \\ &\quad (0.477) \quad (0.158) \\ \pi_t &= 0.260 E_t(i_{t+1}) + (1 - 0.260)\pi_{t-1} + v_t \\ &\quad (0.039) \quad (0.039) \\ \sigma_{\pi_t}^2 &= 0.331^2 + 0.258^2 s_t^2. \\ &\quad (0.070) \quad (0.072) \end{aligned}$$

and

$$z_t \sim \text{bernoulli}(\Phi(-2.500 + 0.329 i_{t-1})).$$

(0.402) (0.067)

According to our estimates, the probability of level shift is strongly dependent on past inflation with t-value 4.910. The magnitude of a level shift depends on the constant and the growth rate of inflation. These results imply that the model can generate time periods with accelerated growth rate in the inflation rate, which are characteristic for the U.S inflation rate. The estimated model is also consistent with long time periods of low and stable inflation, which are also characterical for the U.S inflation rate. The annualized equilibrium level in the low-inflation

regime is 2.265 and the probability that a level shift occurs at this value is only 4%.

Using the Bayesian formula the ex-post probability that the inflation process has undergone a level shift at a specific time point can be written as

$$p_t \phi(i_t, \mu_t + \lambda_t, (\theta_{s_{t-1}})^2) / \{(1-p_t) \phi(i_t, \mu_t, (\theta_{s_{t-1}})^2) + p_t \phi(i_t, \mu_t + \lambda_t, (\theta_{s_{t-1}})^2)\},$$

where $p_t = \Phi(\alpha_1 + \alpha_2 i_{t-1})$ and $\phi(x, \mu, \sigma^2)$ is the density of the $N(\mu, \sigma^2)$ -variable. These probabilities are plotted in figure 5. It can be seen that the process occurs most of the time in the low regime especially before 1970 and after 1990. [Figure 5]

The degree of rationality of the aggregated survey inflation expectations deviates from full rationality very significantly: the deviation from one of the estimated parameter values 0.260 is 18.974 times its standard estimation error. This result gives more precision to previous findings that survey expectations are not perfectly rational. However, we have also found that the regression coefficient of the innovation term v_{t-1} does not significantly differ from one. These estimation results imply that the expectations π_t react too conservatively to inflation news and on average effectively to incremental information.

The studentized values of all parameters in our model are highly significant. However, due to the above identification problems these studentized values cannot provide a formal justification for the proposed non-linear structure. Due to these identification problems we compare a linear model and the MAR model using the information criteria AIC and BIC. A caveat of these comparisons is that small sample properties of such criteria are not available in the case of such statistical problem.

Using standard statistical tests we arrived at a linear AR(3) model for inflation where the AR(1) and AR(2) coefficients are the same. The estimates of this model are given in Appendix A. Table 1 shows that the MAR model performs much better than the linear model according to both information criteria.

Model	Linear	MAR
Log-likelihood	-228.55	-215.325
AIC	471.10	450.65
BIC	488.52	475.54

Table 1: The values of information criteria

There are some similarities between the linear and the non-linear model. The estimated degree of rationality 0.249 in the case of a linear model is rather close to the estimated degree of rationality in the case of a non-

linear model. This comparison indicates that the degree of rationality of survey forecasters is not so sensitive to model specification. Furthermore, the regression coefficient of the innovation term v_{t-1} is the same in the case of the linear model and the non-linear model.

We also study goodness-of-fit of our MAR model by standard diagnostic tests. A problem of diagnostic studies is that standard diagnostic tests usually assume that the conditional distribution of the process is Gaussian. Unfortunately this assumption does not hold in the case of the MAR model, where the conditional distribution is a mixture of two Gaussian distributions. Due to the non-normality of the conditional distribution we investigate the diagnostic properties of our MAR model by the quantile residuals (Dunn & Smyth (1996))

$$q_t = \Phi^{-1}(F_t(y_t)),$$

where F_t is the conditional cumulative distribution of observation y_t and Φ is the standard normal cumulative distribution function. When the data-generating process is true and the conditional distribution is continuous, the distribution of the quantile residuals q_t is $N(0, 1)$.

The quantile residuals and the standardized residuals of the inflation expectations e_t are graphically presented in figures 6 and 7. The diagnostic study of residuals does not find evidence for serial correlation, the non-normality and conditional heteroskedasticity in the case of both residuals. The cross-correlation function ρ_{eq} (Box et al. (1994) pp. 408 – 411) with 95% confidence limits in figure 7 does not show clear cross-correlations between these residuals. Further details of the test results are given in Appendix A. [Figures 6, 7 and 8]

A characteristic property of our MAR model is that the conditional variance of inflation is positively related to the level of inflation, because one source of uncertainty concerns the inflation regime. This result is consistent with previous studies, which have found that high inflation raise inflation uncertainty (e.g. Ball (1992)). How strong an effect the level of inflation has on the inflation uncertainty depends also on the relationship between the inflation rate i_t and the variable s_t . According to the findings of Mankiw et al. (2003), disagreement of forecasters is positively related to the change of the inflation rate. The conditional variance of the inflation rate is plotted in figure 9. The conditional variance $Var_t(i_{t+1})$ is easy to calculate based on the variance decomposition

$$Var_{t-1}(i_t) = E_{t-1}(Var_{t-1}(i_t|z_t)) + Var_{t-1}(E_{t-1}(i_t|z_t)).$$

We get

$$\begin{aligned} Var_{t-1}(i_t) &= p_t\theta^2 s_{t-1}^2 + (1 - p_t)\theta^2 s_{t-1}^2 + 2p_t(1 - p_t)\lambda_t^2 \\ &= \theta^2 s_{t-1}^2 + 2p_t(1 - p_t)\lambda_t^2. \end{aligned}$$

By the above formula we can see that the conditional value of inflation depends on the standard deviation of survey forecasters s_{t-1} , the probability p_t and the magnitude of level shift λ_t , which depends on the growth of the inflation rate. The first term in this formula characterizes uncertainty within inflation regime and the second term characterizes the uncertainty concerning the inflation regime. In the case of a linear model only the first term has an impact on inflation uncertainty.

6 Conclusions

We have built a non-linear two-variate model for inflation and inflation expectations. In this model we use survey-based measures for the inflation expectations and the uncertainty of inflation. The model based on an assumption that economic agents adjust their expectations slower than is rationally anticipated.

The model is consistent with earlier empirical findings that an upward (downward) level shift in the inflation process coincides with a downward (upward) level shift in the ex-poste real interest rate. A basic property of the model is the randomly occurring level shifts in the inflation process and slowly adjusted inflation expectations. According to diagnostic checks and information criteria our model fits the data quite well. The model also has plausible interpretations.

An interesting further question is what implication our model has for strategic asset allocation using a model for the survey-based ex-ante real interest rate. Cambell & Viceira (2002) have pointed out that time variation of real interest rate and inflation are an important source of long-term investment risk. We suggest that the possibility of level shifts in the inflation process under slowly adjusted expectations have substantial influence on the long-term inflation and real interest rate risk, which play crucial role for a long-run investor.

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Appendix A: Statistical analysis

All statistical analysis is done by MATLAB 6.1. We maximize the log-likelihood function of the non-linear model by standard numerical methods. The conditional log-likelihood function of the non-linear model can be written as

$$l(\theta) = -\frac{n}{2} \ln(\pi) - \sum_{t=1}^n \ln(\sigma_{\pi(t-1)}) - \sum_{t=1}^n v_{t-1}^2 / \sigma_{\pi(t-1)}^2 + \sum_{t=1}^n \ln((1-p_t)\phi(i_t, \mu_t + v_{t-1}, (\theta s_{t-1})^2) + p_t\phi(i_t, \mu_t + \lambda_t + v_{t-1}, (\theta s_{t-1})^2)),$$

where $p_t = \Phi(\alpha_1 + \alpha_2 i_{t-1})$ and $\phi(x, \mu, \sigma^2)$ is the density of the $N(\mu, \sigma^2)$ variable. For comparison we also estimate a two-variate linear model for inflation and aggregated inflation expectations

$$\begin{aligned} \pi_t &= 0.249E_t(i_{t+1}) + (1 - 0.249)\pi_{t-1} + v_t \\ &\quad (0.043) \qquad\qquad\qquad (0.043) \\ i_{t+1} &= 0.574 + 0.521(i_t + i_{t-1}) - 0.244i_{t-2} + 1.547s_t\varepsilon_{t+1} + v_t \\ &\quad (0.270) (0.053) \qquad\qquad\qquad (0.094) \qquad\qquad (0.117) \\ \sigma_{\pi_t}^2 &= 0.316 + 0.290s_t^2, \\ &\quad (0.066) (0.060) \end{aligned}$$

where $E_t(i_{t+1}) = 0.574 + 0.521(i_t + i_{t-1}) - 0.244i_{t-2}$ is the model-consistent rational expectation for inflation i_{t+1} .

We tested the serial correlation of the residuals by the Ljung-Box Q -test (Box et al. (1994) pp. 314 – 317). The conditional heteroskedasticity we test by Engle's ARCH test (Engle (1982)) and the normality by the Jarque-Bera test. The value of the Jarque-Bera test statistics is 0.18 in the case of the standardized residuals of the inflation expectations and 3.55 in the case of the quantile residuals of the inflation. Because the critical value at the 5% level of the test statistics is 5.99, the null hypothesis of normality cannot be rejected in either case. The results of the other tests are presented in the tables below. These tables do not show clear evidence of misspecification of our model.

Lags	1	2	4	10
Test statistics	0.28	0.28	0.39	7.44
Critical values	3.84	5.99	9.49	18.31
p -values	0.60	0.87	0.98	0.68

Table 4: The Ljung-Box Q -test for the standardized residuals of the inflation expectations

Lags	1	2	4	10
Test statistics	1.41	3.95	4.05	6.86
Critical values	3.84	5.99	9.49	18.31
p -values	0.24	0.14	0.40	0.74

Table 5: The ARCH test for the standardized residuals of the inflation expectations

Lags	1	2	4	10
Test statistics	0.53	0.53	0.82	5.40
Critical values	3.84	5.99	9.49	18.31
p -values	0.47	0.77	0.94	0.86

Table 6: The Ljung-Box Q -test for the quantile residuals

Lags	1	2	4	10
Test statistics	3.27	3.75	4.26	8.88
Critical values	3.84	5.99	9.49	18.31
p -values	0.07	0.15	0.37	0.54

Table 7: The ARCH test for the quantile residuals

Appendix B: Figures

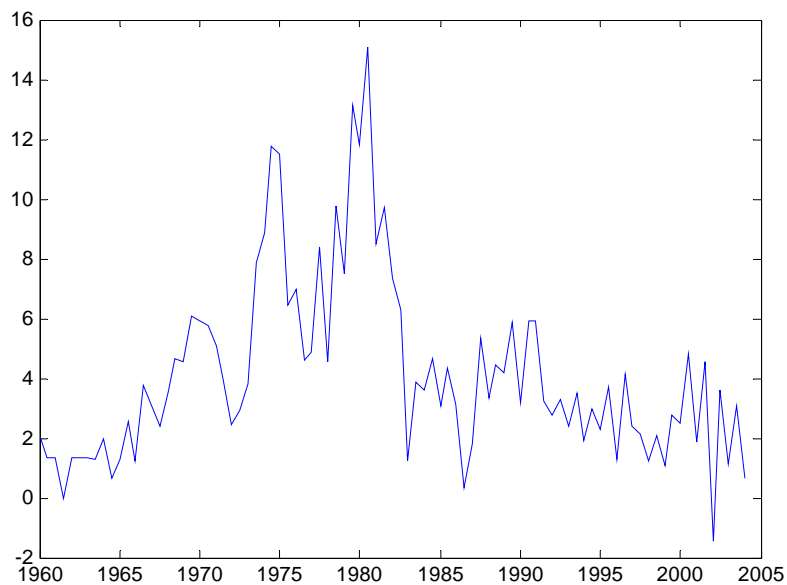


Figure 1: U.S inflation rate from 1959:2 to 2003:2

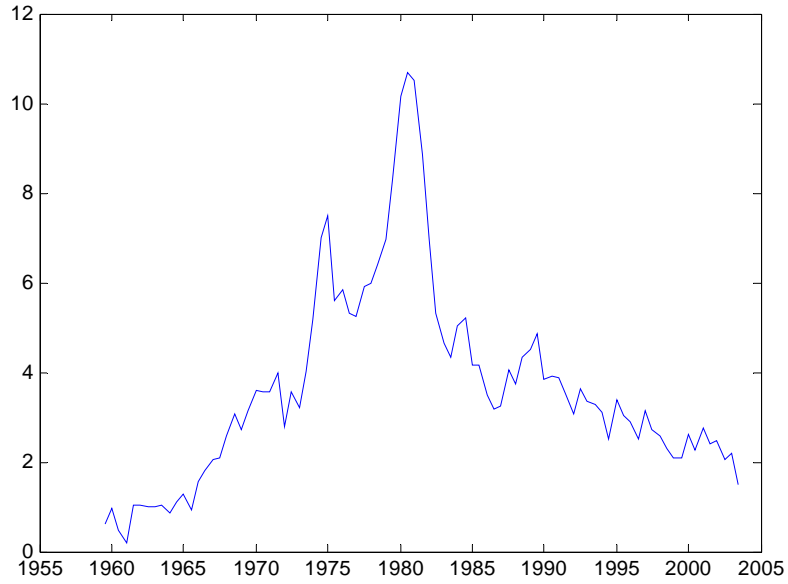


Figure 2: Survey inflation expectations from 1959:1 to 2003:1

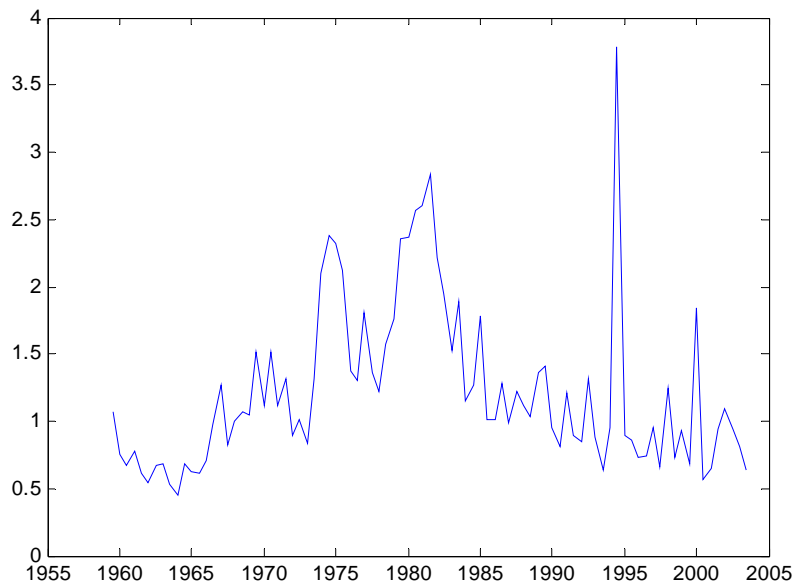


Figure 3: The standard deviation of survey forecasters from 1959:1 to 2003:1

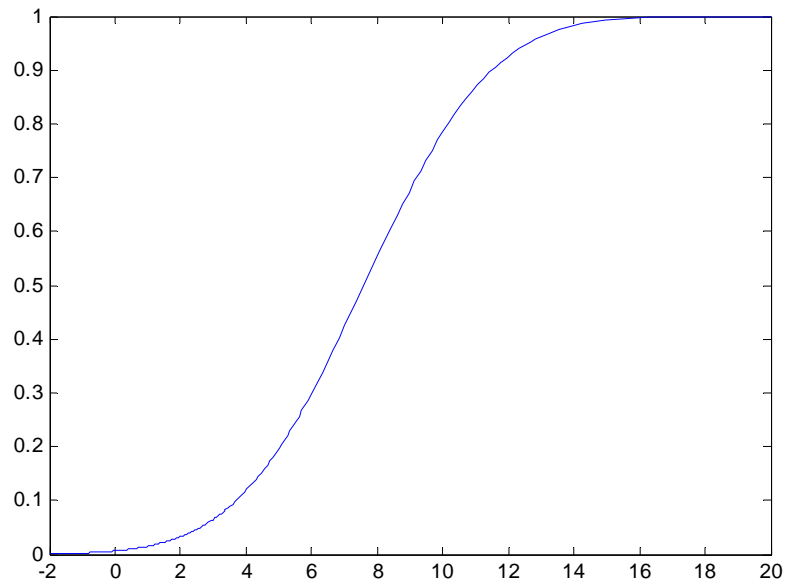


Figure 4: The probability of the high-inflation regime as a function of inflation rate

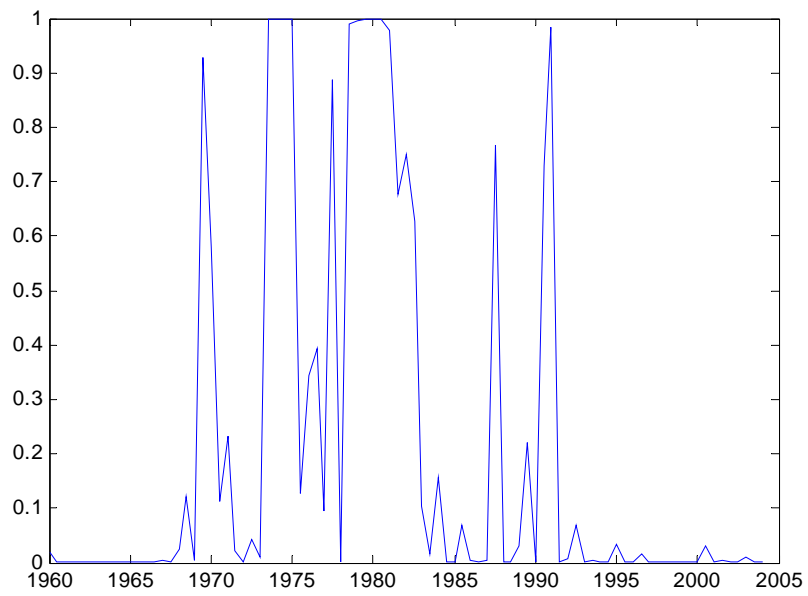


Figure 5: The ex-poste probability of the high inflation regime

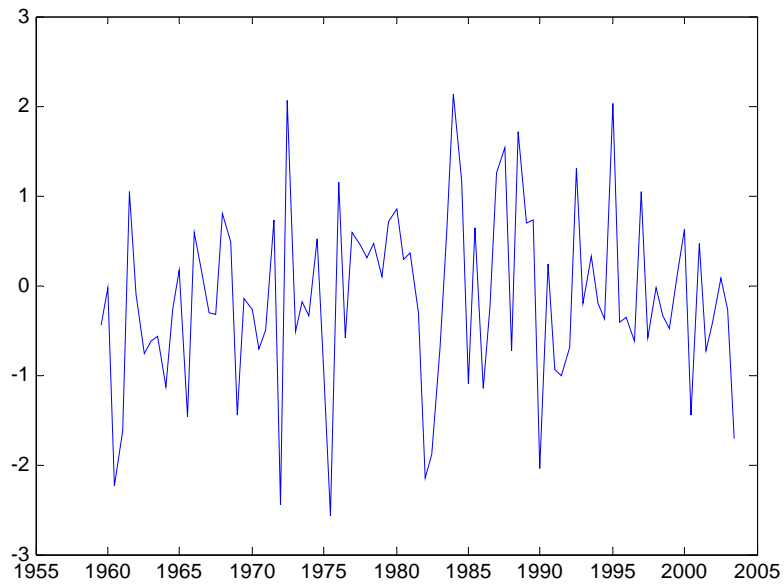


Figure 6: The standardized residuals of inflation expectations

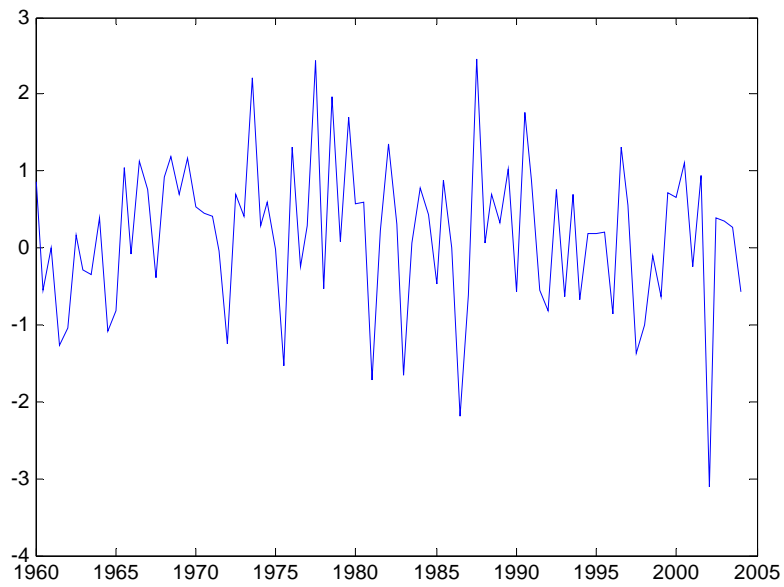


Figure 7: The quantile residuals

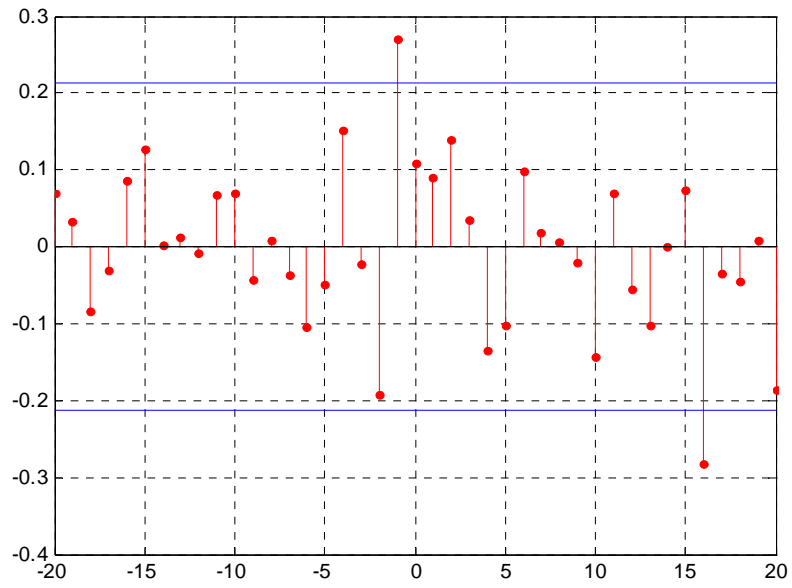


Figure 8: The cross-correlations between residuals

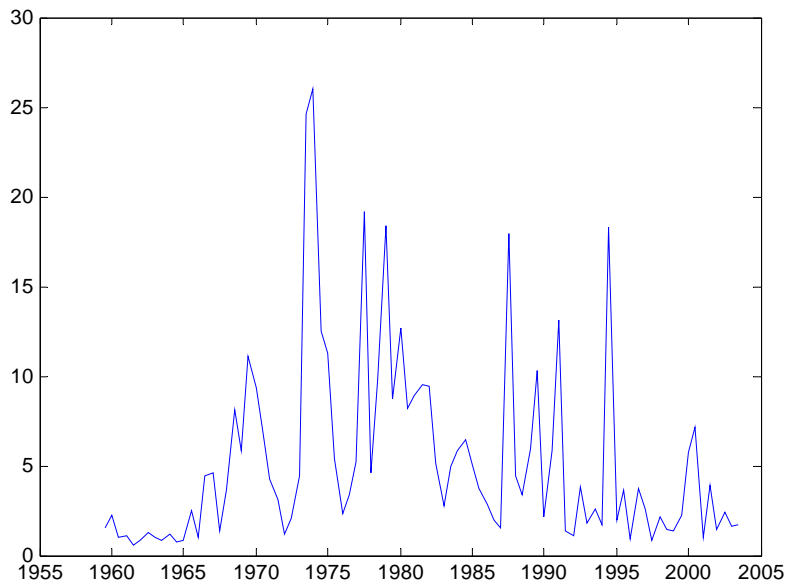


Figure 9: The conditional variance of inflation